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Berkeley, California

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ABSTRACT

The charged mode of decay of 189 \( \Lambda \) and 77 \( K^0 \) particles produced by 1.12-Bev/c \( \pi^- \) mesons in the copper wall of the Berkeley 10-inch liquid hydrogen bubble chamber have been analyzed and their asymmetries calculated. The asymmetries have been examined for several interesting subgroups of the sample and for several directions of quantization. A fore-aft asymmetry in \( \Lambda \) decay has previously been reported in cosmic-ray interactions with complex nuclei and in some machine events of a similar nature. This asymmetry has recently been reviewed and summarized by Salmeron and Zichichi. They find a 3-standard-deviation effect, \( \bar{P}_{\text{fore-aft}} = -0.56 \pm 0.15 \). In contrast, the asymmetries found here for both \( \Lambda \) and \( K^0 \) are consistent with the assumption of the conservation of parity in the production process \( \pi^- + p(\text{in nucleus}) \rightarrow \Lambda + K^0 \).

The cross section for the associated production of \( \Lambda K^0 \) and \( \Sigma^0 K^0 \) has been determined as 10.5 ± 0.9 mb per copper nucleus. This value is somewhat larger than that expected on the assumption that the nucleons interact in first approximation as free nucleons possessing Fermi momentum according to an \( \Lambda^{2/3} \) scaling of the hydrogen cross sections. This scaling predicts a value for this cross section of 7.4 ± 0.8 mb/nucleus.

Subsequent interactions of the hyperons with another nucleon in the same nucleus in which they were produced are frequent. A lower limit for the probability of scattering of the \( \Lambda \) can be set at 0.23 ± 0.07. This is in accordance with the observed \( \Lambda + p \) cross sections.
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Introduction

If parity is not conserved in strong interactions, then the final-state particles may have a component of polarization in the plane of production. An asymmetry in this plane may result from the parity-nonconserving decay of unstable particles. In particular, with respect to the line of flight of the particle, a fore-aft asymmetry is observed. Such an asymmetry has been observed in the decay of \( \Lambda \)'s produced in complex nuclei by cosmic rays, and in corresponding machine experiments. The situation has been reviewed by Salmeron and Zichichi. They have applied criteria which supposedly eliminate all biases and give an over-all result for \( e_P_{\text{fore-aft}} \) of \(-0.56 \pm 0.15\), a 3-standard-deviation effect. No such asymmetries in the plane of production have been found at Berkeley in \( \pi^- + p \rightarrow \Lambda + K^0 \) hydrogen events. Many \( V \)-particles which were produced in the copper wall of the bubble chamber are also observed. An unbiased sample of these has been analyzed in a search for possible asymmetries.

*Work done under the auspices of the U. S. Atomic Energy Commission.
Sample, Analysis, and Asymmetries of the Wall Vees

Three sets of film of fifty rolls each were selected from the 1.12-Bev/c \( \pi^- \) experiment in the Berkeley 10-inch liquid hydrogen bubble chamber. This sample yielded 189 \( \Lambda \rightarrow p + \pi^- \) and 77 \( K^0 \rightarrow \pi^+ + \pi^- \) decays from 276 events of the type \( \pi^- + Cu \rightarrow \text{vee} \). These 276 events are divided as follows: 23 double \( \Lambda K^0 \), 220 single \( \Lambda \) or \( K^0 \), 5 \( e^\pm \) pairs, 4 three-body \( K^0 \) decays, 4 \( K^- \) interactions (arising from the 1% \( K^- \) in the beam), one \( \Lambda + p \rightarrow \Lambda + p \) elastic scattering, and 19 "junk" (unmeasurable events and coincidences, \( \pi^-p \) scatterings, and \( \pi^- \rightarrow \mu \) decays). This sample represents 40% of the 1.12-Bev/c \( \pi^- \) film and about 20% of all the associated production film of the 10-inch chamber at Berkeley.

The events were measured on the Franckenstein measuring projector. The analysis was done by using IBM 650 programs. The IBM 650 computer first reconstructed the geometry, calculating the direction cosines of the beginning and end of each track. The momentum of each track was computed from the curvature in the 10-kilogauss magnetic field. A second program corrected the momentum for \( dE/dx \) losses or computed the momentum from range for stopping tracks. Two calculations were made for each track, assuming a \( \pi^- \) in the first case and a \( p \) in the second case. A third program fitted the decay both to a \( K^0 \) and to a \( \Lambda \) interpretation by least-squares fits. For each interpretation a \( \chi^2 \) function was calculated. By choosing the smallest \( \chi^2 \), we identified the vee as a \( K^0 \), as a \( \Lambda \), as ambiguous (fits either \( K^0 \) or \( \Lambda \)), or as a "beast" (fits neither \( K^0 \) nor \( \Lambda \)). The ambiguous events were resolved by a measurement of the ionization produced by the positive tracks, as determined by the ratio of gaps per unit length of the positive track relative to the negative track. A minimum ionization indicates a \( \pi^+ \) track from \( K^0 \) decay in general, whereas a nonminimum ionization occurs from the proton from \( \Lambda \) decay. The "beasts"
were remeasured and those failing to fit were examined and placed in the appropriate category (e^± pairs, three-body decays, etc.).

In addition, the double vee events were examined for consistency by means of the following checks.

The point of production (in the wall) of the \( \Lambda \) or \( K_{1}^{0} \) was calculated for all vees. True double-vee events had to have the same point of production and to fit either a \( \Lambda K^{0} \) or a \( K_{1}^{0} \Lambda \) interpretation for the two vees. Also, rough agreement of the correlation of momentum and angle of production for the known beam momentum and some reasonable Fermi momentum was found to hold in many cases, although many \( \Lambda \)'s were observed at large angles (> 30 deg). These are attributed to the nuclear scattering of the \( \Lambda \)'s upon coming out of the nucleus (see the discussion below).

After the event was fitted, the direction cosines of the negative decay pion were computed in the \( K \) or \( \Lambda \) rest frame, with a right-handed coordinate system. These direction cosines are \( \varphi \), \( \rho \), and \( \xi \), where \( \varphi \) measures the right-left, \( \rho \) the fore-aft, and \( \xi \) the up-down asymmetries with respect to a "defined plane," which is the plane of Tracks 1 and 2 as shown in Fig. 2. This "defined plane" is only approximately the production plane, since the struck nucleon in the copper nucleus possesses Fermi momentum. In some cases the \( \Lambda \) undergoes a subsequent scattering in the same nucleus, thus further altering the production plane. These direction cosines are defined by the equations

\[
\varphi = \frac{\vec{P}_3 \cdot [\hat{z} \times (\hat{1} \times \hat{z})]}{P^*},
\]

\[
\rho = \frac{\gamma(\vec{P}_3 \cdot \hat{z}) - \eta E_3}{P^*},
\]

\[
\xi = \frac{\vec{P}_3 \cdot (\hat{1} \times \hat{z})}{P^*}.
\]
where

\[ P_1 = \text{momentum}, \]
\[ E_1 = (P_1^2 + M_1^2)^{1/2} = \text{energy}, \]
\[ \gamma = \frac{E_2}{M_2}, \quad \eta = \frac{P_2}{M_2}, \]

and where, for example,

\[ \hat{i} = \text{unit vector in direction 1}, \]
\[ \hat{Z} = \text{vector in direction 2}, \]
\[ \vec{P}_3 = \text{momentum vector in direction 3}, \]
\[ P^* = \text{rest-frame momentum of decay particles}. \]

An asymmetry coefficient \( (a_1) \) for the decay distribution

\[ f(\cos \theta) = \frac{1 + a_1 \cos \theta}{2}, \]

where \( a_1 = (aP)_1 \), \( a = \text{parity mixing parameter} \), and \( P = \text{average polarization of the decaying particles} \), can be found from the relations\(^8\)

\[ a_1 = \frac{2(N_+ - N_-)}{N_+ + N_-}, \]

\[ \delta a_1 = \pm 4 \frac{Pq}{N_+ + N_-} = \pm \frac{2}{N^{1/2}} \]

\[ |a_1| < \frac{1}{2} \]

where \( p = \frac{N_+}{N}, \quad q = 1 - p, \quad N = N_+ + N_- \), \( N_+ = \text{number of events with } \cos \theta_i > 0 \), and \( N_- = \text{number of events with } \cos \theta_i < 0 \) for a given \( \cos \theta_i = \rho, \sigma, \) or \( \xi \).

The asymmetries in the production plane were also looked at in a rotated frame \( \rho', \sigma' \), in which the rotation is back through the production angle \( \cos^{-1}(\hat{i} \cdot \hat{Z}) \), so that \( \rho' \) is parallel to incident beam direction. The asymmetries were calculated for the \( K' \)s and \( \Lambda' \)s for the following subdivisions of the sample:
(a) all events,
(b) all events within the geometrical cutoff used for the cross-section calculation (see below),
(c) all $\Lambda$ with $p_\Lambda < 700$ Mev/c (as was done in some prior works$^{2,3}$) and
(d) an "enriched" sample of events in which the $\Lambda$ lies in a relatively small band about the $\pi^- + p$ (hydrogen) kinematical region (which is expected to yield a sample containing a larger fraction of nonscattered $\Lambda$'s).

Table I summarizes the experimental results.

In all cases, the assignment of $a_\parallel = 0$ is satisfactory within statistics—that is, consistent with no asymmetries.$^9$ There is no evidence here for the nonconservation of parity in the production process.

The absence of a large up-down ($a_\parallel$) asymmetry requires examination. Two effects necessarily reduce the up-down asymmetry. First, the $\Sigma^0 K^0$ production events decay into $\Lambda$'s having only one-third of the original polarization of the $\Sigma^0$. Second, the defined plane used for calculating the asymmetry is not, in general, the plane of production, because the struck nucleon possesses Fermi momentum. One might reasonably expect an up-down asymmetry of the order of two-thirds of the value observed in hydrogen for $\Lambda$'s, which value is $(a_\parallel)^\Lambda = 0.57 \pm 0.066$. The asymmetry can also be reduced if the $\Lambda$'s scatter upon another nucleon on emerging from the nucleus in which they are produced. This reduction occurs even without spin-dependent scattering forces, and is caused by the random orientation of the $\Lambda + N$ scattering plane relative to the production plane, so that the defined $\xi$ axis bears little relation to the original polarization direction of the $\Lambda$. We might reasonably expect, therefore, an up-down asymmetry for the $\Lambda$'s of approximately 0.3. The data are in accord with this value.
Table I. Tabulation of Asymmetry Coefficients
(for negative decay pion)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>All events</th>
<th>Events within geometry</th>
<th>&quot;Enriched&quot;</th>
<th>$P_{A&lt;700}$ MeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\rho$</td>
<td>$77 , K^0$</td>
<td>$189 , \Lambda$</td>
<td>$45 , K^0$</td>
<td>$104 , \Lambda$</td>
</tr>
<tr>
<td></td>
<td>(error $\pm .23$)</td>
<td>(error $\pm .13$)</td>
<td>(error $\pm .30$)</td>
<td>(error $\pm .20$)</td>
</tr>
<tr>
<td>$a_\sigma$</td>
<td>-.08</td>
<td>+.14</td>
<td>-.04</td>
<td>+.06</td>
</tr>
<tr>
<td>$a_\xi$</td>
<td>+.03</td>
<td>+.14</td>
<td>+.04</td>
<td>-.06</td>
</tr>
<tr>
<td>$a_\rho'$</td>
<td>-.23</td>
<td>+.12</td>
<td>-.49</td>
<td>-.04</td>
</tr>
<tr>
<td>$a_\sigma'$</td>
<td>+.34</td>
<td>+.20</td>
<td>+.40</td>
<td>+.25</td>
</tr>
</tbody>
</table>

*Note: All values are given with their respective errors.*
Qualitatively, more $\Lambda$'s were observed at angles greater than 30 deg to the beam direction than is expected on the basis of independent-particle interactions identical with elementary $\pi^- + N$ interactions and the effects of the Fermi momentum. In order to check this quantitatively, the distribution of angle, $N(\theta_{\Lambda})$, of the $\Lambda$'s was calculated on the basis of individual $\pi^- + N$ interactions in the nucleus with the nucleon possessing Fermi momentum. The $\Lambda$ laboratory-system angles ($\theta_{\Lambda}$) versus the center-of-mass angle of the $\Lambda(\theta_1)$ were calculated from the kinematics for Fermi momentum $P_F = 0, 100, 200$, and $300$ Mev/c for angles of the Fermi momentum vector with respect to the beam direction ($\theta_F$). By use of suitable angular intervals in $\theta_F$ and $\theta_1$, the distribution $N(\theta_{\Lambda})$ versus $\theta_{\Lambda}$ was calculated for each $P_F$. This was done by weighting the kinematics with the known energy dependence for hydrogen total cross sections in $\pi^- + p \rightarrow \Lambda (\Sigma^0) + K^0$ (as a function of the center-of-mass momentum of the outgoing $\Lambda$ or $\Sigma^0$) and with known angular distributions (from the Berkeley data). The $\Lambda$'s arising from $\Sigma^0$ decays were treated as isotropically populating a band of angles in the laboratory system about the $\Sigma^0$ production angle (lab). Finally these distributions were folded with a Fermi momentum distribution. Two distributions were used:

(a) A Gaussian $f(T)dT = \exp \left(-p^2/p_0^2\right) dT$, where $T = \text{kinetic energy}$, with $p_0^2$ corresponding to kinetic energy of 20 Mev, a value observed in lighter nuclei, i.e. $Z < 6$. 13, 14

(b) A completely degenerate Fermi gas

$$f(T) dT = \begin{cases} 1 & \text{for } T < T_0, \\ 0 & \text{for } T > T_0, \end{cases}$$

with $T_0 = 33.2$ Mev, corresponding to a maximum of 250 Mev/c, which approximates that given by energies in infinite nuclear matter. 15 The distributions $N(\theta_{\Lambda})$ are relatively insensitive to the momentum distributions
and are plotted in Fig. 3 along with the observed distribution shown in histogram form, for 104 $\Lambda$ (same sample as used in cross sections; see below). We see that the observed and predicted distributions disagree, and we attribute the discrepancy to the scattering of the $\Lambda$'s on emerging from the nucleus. Such scattering of the $\Lambda$'s is consistent with the known $\Lambda N$ cross sections. We find that the fraction of scattered $\Lambda$'s is greater than the fraction 0.23±0.07 of the observed values lying outside the predicted distribution. We conclude that a large fraction of the $\Lambda$'s must scatter, and thus expect the value of $a_5$ will be small. The result is consistent with either a small or zero value. It is to be noted that Blumenfeld et al. found no up-down asymmetry in the same sample that gave the fore-aft asymmetry. However, their up-down distribution was not isotropic, but was peaked at $\cos \theta = \pm1$. This is not possible for the known value of one-half for the spin of the $\Lambda$, which gives a linear distribution in $\cos \theta$. Also, Bowen et al. and Boldt et al. have not observed up-down asymmetries, although in their experiments the polarisation may be smaller because of the higher energies involved.

**Cross Section**

In order to determine a cross section one must evaluate many effects. These are scanning efficiency, detection efficiency, neutral-decay corrections, and path length of the beam in the material. In order to have a sample of events in which the scanning and detection efficiencies can be evaluated and in which the total path length of the beam is known, definite geometrical cutoffs need to be applied. A determination of the "lifetime" of the particles indicated an almost complete lack of decays near the entrance to the chamber (i.e., within 1 cm of the wall), which necessitated using events that decayed beyond 1 cm. The number of beam tracks was determined for a central region of the chamber corresponding to the bubble chamber's thin-window area, and for high-quality
film (less than 30 tracks per picture, no large bubbles, good illumination, etc.).

From the sample of 189 \( \Lambda \) and 77 \( K^0 \) decays, the application of the above criteria left 104 \( \Lambda \) and 45 \( K^0 \) events. These decays showed an exponential distribution in proper time. The maximum-likelihood solutions for the lifetimes are in good agreement with the accepted values\(^2\),\(^2\)\(^9\) (104 wall \( \Lambda \) gave \( \tau_\Lambda = 3.1 \pm 0.3 \times 10^{-10} \) sec; 45 wall \( K^0 \) gave \( \tau_K = 1.0 \pm 0.2 \times 10^{-10} \) sec).

The scanning efficiencies were determined by rescanning some of the film. In the sample used, one-fifth had been rescanned in a normal manner similar to the first scan. An additional one-fifth was rescanned, with a careful search for \( \Lambda \)-events near the entrance wall. This is a poorly illuminated region of the chamber (see Fig. 1). The number of events found in this region (decay length less than 1 cm from wall) agrees favorably with the number required to decay in there by calculation from the number of observed decays beyond 1 cm. An equivalent of 80±20 events was found versus the 85±10 required.

The scanning efficiency for the wall \( \Lambda \)-events was determined to be 0.93±0.04 (versus 0.98±0.02 for the events in hydrogen\(^1\)).

The geometrical escape correction or detection efficiency, which is the probability that the production and decay of the \( \Lambda \) occurs within the acceptable region of the chamber, was calculated. The calculation was made by using the midpoint thickness of the wall as the average point of production (so that the 1 cm from the wall for the counting of decay time becomes 1.35 cm from the production point). The calculation averages over the known true beam distribution, over the random orientations of the production and decay planes and over a set of laboratory-system angles and momenta. These angles and momenta were picked from a suitably divided momentum-versus-angle histogram. The true counts in each histogram interval were calculated and an average \( \langle \Delta E \rangle \) was determined:
\[
(\Delta E)_i = \frac{\sum N_i}{\sum N_i} \frac{1}{(DE)_i},
\]
where \(N_i\) = number of events observed in interval \(i\), 

\((DE)_i\) = calculated detection efficiency for interval \(i\).

The results are \(DE_\Lambda = 0.684\) and \(DE_{K^0} = 0.602\). It turns out that these D. E.'s are essentially the probability that the decay occur beyond 1.35 cm from the production point. Under these conditions, the detection efficiency for observing each double is just the product of the probabilities for each associated \(\Lambda\) and \(K^0_1\) to decay beyond this point, so that we have

\[
(\Delta E)_{\Lambda K} = \frac{N_{\Lambda K}}{\sum_{\text{events}} (DE)_{\Lambda} (DE)_{K}},
\]

with the result \(DE_{\Lambda K} = 0.410\). Table II gives the results of the corrections for scanning and detection efficiencies. The corrections due to the branching ratios (BR) into the neutral or charged decay modes now can be made. The observed ratios are:

\[
\text{BR} \left( \frac{\Lambda \rightarrow p + \pi^-}{\text{all } \Lambda} \right) = \frac{N_{\Lambda K}/(SE) \times (DE)_{\Lambda K}}{(N_{\Lambda K} + N_K)/(SE) \times (DE)_K} = \frac{47 \pm 11}{(18 + 26) / 0.602} = 0.60 \pm 0.16,
\]

\[
\text{BR} \left( \frac{K^0_1 \rightarrow \pi^+ + \pi^-}{\text{all } K^0} \right) = \frac{N_{\Lambda K}/(SE) \times (DE)_{\Lambda K}}{(N_{\Lambda K} + N_{\Lambda})/(SE) \times (DE)_\Lambda} = \frac{47 \pm 11}{(18 + 83) / 0.684} = 0.30 \pm 0.08.
\]
Table II. Corrections to data

<table>
<thead>
<tr>
<th>Observed Number, N</th>
<th>Efficiencies</th>
<th>Actual Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detection</td>
<td>Scanning</td>
</tr>
<tr>
<td>$18 \Delta K^0$ Doubles</td>
<td>0.410</td>
<td>0.93</td>
</tr>
<tr>
<td>$(86-3\Delta K^+) = 83$ single $\Delta$</td>
<td>0.684</td>
<td>0.93</td>
</tr>
<tr>
<td>$(27-1\Sigma K^0) = 26$ single $K^0_1$</td>
<td>0.602</td>
<td>0.93</td>
</tr>
</tbody>
</table>
These branching ratios are in agreement with the hydrogen results and the predictions according to the $|\Delta| = 1/2$ rule$^{18,19}$ (i.e., $BR_\Lambda = 0.67$, $BR_{K^0} = 0.33$). Using the 0.67 and 0.33 values we can calculate $n_{true}$ associated "$\Lambda K$" observables from

$$\pi^- + p \rightarrow (\Lambda \Sigma^0) + K^0,$$  

for $\Sigma^0 \rightarrow \Lambda + \gamma$.

We obtain

$$n_{true}^{\Lambda K} = \frac{N_{\Lambda K}}{BR_\Lambda BR_{K}} = \frac{9}{2} (47 \pm 11) = 212 \pm 50$$

$$= \frac{N_K + N_{\Lambda K}}{BR_{K}} = 3(94 \pm 14) = 282 \pm 42$$

$$= \frac{N_\Lambda + N_{\Lambda K}}{BR_{\Lambda}} = \frac{3}{2} (178 \pm 18) = 267 \pm 27.$$

Upon averaging these we obtain $n_{true}^{\Lambda K} = 254 \pm 20$, which is taken as the "best" value. This number, coupled with the track count and path length, gives the cross section. The good-quality film in the sample used has 38,153 good frames with an average of 12.7$\pm$0.2 tracks per frame through the defined thin-window area. The path length ($L$) per track is very nearly the thickness of the wall, $0.63 \pm 0.01$ cm (1/4 inch). Thus for the density $\rho$ of copper of $8.6 \pm 0.1$ g/cc, $A = 64$ (atomic weight), and the $K^-, \mu^-, e^-$ contamination of the beam of approximately 6%$^{20}$ (i.e., 94% $\pi^-$s) we obtain (taking into account that the average $\pi^-$ flux in the wall is 2% higher than in the chamber)
\[
\sigma_{\Delta K^0 + \Sigma K^0} = (pL)^{-1} \frac{A}{N_{\text{Avogadro}}} \frac{n_{\text{events}}}{n_{\pi-\text{tracks}}} \\
= (8.6 \times 0.63)^{-1} \frac{64}{6.03 \times 10^{23}} \times \frac{(254.4^8)}{(3.82 \times 10^4)(12.7)} \frac{0.98}{0.94} = 10.5 \pm 0.9 \text{ mb/Cu nucleus}
\]

This result can be compared with an \( A^{2/3} \) law utilizing the hydrogen data for \( p^- + p \rightarrow \Delta_1 \Sigma^0 + K^0 \). Averaging the energy dependence of the known cross sections for \( \theta' \) and \( \theta_F \) over the Fermi momentum distribution (see discussion of \( N(\theta_\Lambda) \) above), we obtain, for an incident \( p^- \) momentum of 1.12 Bev/c on copper,

\[
p^- + p \rightarrow \Delta + K^0: 0.64 \pm 0.08 \text{ mb}, \\
p^- + p \rightarrow \Sigma^0 + K^0: 0.37 \pm 0.06 \text{ mb}.
\]

Thus, on the basis of an \( A^{2/3} \) scaling of these cross sections, we obtain

\[
\sigma_{\Delta K^0 + \Sigma K^0} = 7.3(0.64 + 0.37) = 7.3 \pm 0.7 \text{ mb/nucleus}
\]

and a ratio of 1.44 \( \pm \) 0.20 for the observed to predicted cross sections.

Several factors can contribute to the increase of the observed over the predicted cross section. The first factor is a volume-effect deviation from an \( A^{2/3} \) law. A strict \( A^{2/3} \) law would apply only if the pions interact only once in traversing the nucleus. Some increase in associated production can then follow from pions that undergo a small-angle elastic scattering and then make

\footnote{Four events are estimated to be due to interactions in hydrogen less than 2 mm from the edge of the wall.}
an associated production. The second factor is a contribution from 
\( \pi^- p \rightarrow \Lambda K^0 \) events, which are energetically possible in approximately half 
the cases because of the Fermi momentum. The cross section for such a 
process is not known, but is expected to be small. The third factor is a 
contribution from \( \pi^- + n \) interactions. The \( \pi^- + n \) interactions can yield only 
\( \Sigma^- K^0 \) events. However, a \( \Sigma^- \) can be absorbed to give a \( \Lambda \) by the reaction 
\( \Sigma^- + p \rightarrow \Lambda + n \) and thus appear as a \( \Lambda + K^0 \).

Thus, the observed cross section of 10.5±0.9 mb/nucleus is 
reasonably well explained on the basis of elementary \( \pi-N \) interactions according 
to an \( A^{2/3} \) law with some modifications which depend on multiple-step processes, 
such as described above.
ACKNOWLEDGMENTS

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I am much indebted to Frank S. Crawford, Jr., Myron L. Good, Frank T. Solmitz, and M. Lynn Stevenson for their encouragement and many suggestions and support given in completing this experiment. Thanks are due to J. Donald Cow and the bubble chamber crew, Hugh Bradner and the scanners, and Edward J. Lofgren and the Bevatron crew for the technical assistance that made this experiment possible.
References and Footnotes


6. These programs are described in internal memoranda (unpublished) by the Alvarez Group, Lawrence Radiation Laboratory.

7. The least-squares fit is a one-constraint fit. A wall vee has two tracks (one positive and one negative) for which both the momentum and direction are known. Nothing is measurable on the neutral Λ or Κ. Of the four constraints of momentum-energy conservation, three are used to eliminate the two angles (for direction) and the momentum specifying the neutral particle, leaving one constraint for applying a least-squares analysis. The results of the analysis gave a $\chi^2$ distribution for the events in good
agreement with the theoretical distribution for one constraint ($\chi^2 = 1$). The $K_1^0$ mass used was 498.0 MeV/c$^2$, in accord with the latest data on the $K^0$-$K^+$ mass difference (F. S. Crawford, M. Cresti, M. L. Good, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters 2, 112 (1959)).

8. The $a_i$ and $\delta a_i$ can be found from a more detailed calculation,\(^1\)

$$a_i = \frac{3}{N} \sum_a (\cos \theta_i)_a$$

and

$$\delta a_i = \pm \sqrt{\frac{3-a_i^2}{N}}.$$ 

However, as the $a_i$ here are consistent with zero, and the histograms in six intervals in $\cos \theta_1$ confirm isotropy,\(^9\) only the cruder calculation given in the text was made.

9. The breakdown of the $a_i$ into six intervals confirms that the distributions $f(\cos \theta_1)$ are essentially isotropic except for one "hole" in the $K_1^0$ fore-aft ($\rho$) direction cosine. This "hole" occurs between $+0.67$ and $+1$, and is probably a bad statistical fluctuation. It is possible that some events may have been missed here, as these correspond to $K^0 \rightarrow \pi^- + \pi^+$ with high $\pi^-$ and low $\pi^+$ momentum such that the $\pi^+$ can decay into a $\mu^+$ which may or may not decay within the chamber. If it does not, then the "vee" looks roughly like a $\pi^+ - \mu^+ - e^+$ decay in flight and may not have been recorded. All listed $\pi^+ - \mu^+ - e^+$ decays were reexamined to see if any could be $K^0$ decays. The results were negative and we must attribute the "hole" to statistics, as there is no other apparent bias.


Fig. 1. One of the two stereo views of a double-vee event, number 209909.

The left-hand vee is a $\Lambda$ of momentum $549 \pm 39$ Mev/c and the right-hand vee is a $K^0$ of momentum $614 \pm 15$ Mev/c. The through tracks are beam $\pi^-$ mesons of 1.12 Bev/c incident from the bottom of the picture.

Fig. 2. Track numbering.

Fig. 3. Distribution of angle, $N(\theta_{\Lambda})$, of $\Lambda$. 
Negative \((\pi^-)\)

Positive \((\pi^+ \text{ or } p)\)

Neutral \((K^0 \text{ or } \Lambda)\)

Incident \(\pi^-\) direction

Fig. 2
Fig. 3