Title
A TRANSPORT EQUATION THEORY OF ELECTRON BACKSCATTERING AND X-RAY PRODUCTION

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ELECTRON BACKSCATTERING AND X-RAY PRODUCTION
A TRANSPORT EQUATION THEORY OF
observed currents 

to rewrite the equation in terms of fluxes which can easily be related to
as convoluted with respect to this variable. It is also convenient
by Fourier transformation with respect to this variable. Dependence on it can easily be calculated.

Integrating over the azimuthal angle removes it from the equation as the
beam has infinite extent in the x,y plane (parallel to the specimen surface).
Assuming solutions are stationary with respect to time and that the electron
example Shimmizu et al. [1]. The transport equation can be simplified by
the behavior of many electron trajectories by Monte-Carlo methods (see for
the integro-differential equation directly the usual approach has been to simulate
rather than solve this
velocity v being scattered to a final velocity v'. Rather than solve this

\[ \int \int \int \int d \mathbf{r} \left[ \nabla \cdot ( \bar{v} \rho (t, \mathbf{r}, \bar{v} \rho ) ) + \frac{10}{\epsilon} \right] = \frac{\partial}{\partial t} \int \int \int \int d \mathbf{r} \bar{v} \rho (t, \mathbf{r}, \bar{v} \rho ) \]

To calculate the rate of X-ray production or backscattering a knowledge

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BACKSCATTERING AND X-RAY PRODUCTION
A TRANSPORT EQUATION THEORY OF ELECTRON
energy and it is also a good approximation to assume that they are only scattered.

matrix relating forward and backward traveling fluxes. As electrons only lose

bashen [8] uses a similar approach but derives a nonlinear equation for the

\[ C_{ij}(\theta, \phi) = \sum_{\theta', \phi'} W_{ij}(\theta, \phi) \delta(\theta, \phi) \]

\[ A_{ij}(\theta, \phi) = \sum_{\theta', \phi'} W_{ij}(\theta, \phi) \delta(\theta, \phi) \]

\[ \frac{\partial}{\partial \theta} \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{W_{ij}(\theta, \phi)}{A_{ij}(\theta, \phi)} \]

where

\[ \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{Z P}{I P} \]

Variables}

Equation 3 is put into a matrix form by segmenting the angle and energy calculations [7].

This has also been a problem with earlier Monte Carlo calculations. It has also been a problem with earlier Monte Carlo

electron state and this means that energy distributions were calculated assuming

Integrating path length was nearly always used instead of energy to describe the

in in infinite foil with a source at the center where calculated [4],[5].

applying the boundary conditions is not easy and sometimes only spatial

length. It is also usual to expand in Legendre polynomials [4],[5],[6], but

By Brown et al. [3], [4], [5], [6], [7],[8,[9], [10], [11], [12]; but

et al. [2] derive the corresponding Fokker-Planck equation and this has been solved

In many treatments further approximations are made at this stage. Before

\[ \sum_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) \]

\[ \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) \]

\[ \int_{\phi} \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{Z P}{I P} \cos \theta \]

\[ \int_{\phi} \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{Z P}{I P} \cos \theta \]

\[ I(\theta, \phi) = \int_{\phi} \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{Z P}{I P} \cos \theta \]

\[ I(\theta, \phi) = \int_{\phi} \int_{\phi} W_{ij}(\theta, \phi) \delta(\theta, \phi) = \frac{Z P}{I P} \cos \theta \]
10. \( I^g(0) = V_{1} I^g(0) \)

As can be seen if it tends to infinity the backscattered flux becomes

\[ (9) \]
\[ I^g = I^g(0) \]

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This can be solved for \( I^g(0) \), subject to the boundary conditions:

\[ (8) \]
\[ I^g(0) \]

\[ (8) \]
\[ I^g(0) \]

\[ (8) \]
\[ I^g(0) \]

\[ (8) \]
\[ I^g(0) \]

The eigenvalues and eigenvectors can be partitioned further forward and backward scattering before applying the boundary conditions, which can be evaluated by diagonalizing the matrix. As there is a symmetry between forward and backward scattering, the resulting matrix differential equation has a simple solution.

The resulting matrix differential equation describes the energy transfer between the energy states due to the inelastic scattering and the "absorption" due to inelastic scattering and those on the surface. The diagonal matrixes represent the elastic scattering and the off diagonal represent the coupling between the energy states due to the inelastic scattering. The next equation is assumed to be written as a coupled system of equations from one energy level to the next equation and can be rewritten as a coupled system.
energy distribution of backscattered electrons. For the elastic scattering
90° were necessary for convergence at 10–15 energy intervals were needed for
in calculations using this theory it was found that 10 angle elements in
x-ray yield as opposed to backscattering.

As the depth integration is analytic there is no extra effort in calculating

\[
\left( \frac{d\theta}{n} + f_\gamma \right)
\]

\[
(0) \sum_{I}^{\infty} \frac{\theta - \lambda^{2}}{I_{1}^{2} + I_{1}^{2}}
\]

would be

an angle dθ with the specimen normal the observed rate of x-ray production
produced at the rate f1 by electrons at energy level 1 and the detector makes
an x-ray yield can be calculated. If x-rays with absorption coefficient n are
from the energy distribution of electrons at different depths various other
quantities such as the rate of x-ray production, the secondary electron and
energy is given by

The matrixes now have indices corresponding to energy. The backscattering at
the surface is given because the solution is similar to that given previously except that
this method is proportional to the number of energy levels. The partitioning
of the previous energy. The time taken to diagonalize the supermatrix
of these matrices was divided which solution for each energy is found using the eigenvector
matrixes. This is done by expressing the diagonal

The solution for the supermatrix is diagonal. This can be done
in two stages, the first step is to bring it to a block diagonal form analogous
to diagonalization. The second process is diagonalized

\[
(0) \sum_{I}^{\infty} \frac{\theta - \lambda^{2}}{I_{1}^{2} + I_{1}^{2}}
\]

and this is the solution matrix of Dabney’s nonlinear equation. The forward
backward traveling fluxes at depth z are:

\[
(0)^{d} \left[ \frac{\lambda_{z}}{\exp \left( -\frac{z}{\lambda} \right)} \right] = (z)^{d}
\]
enough absorption of low energy electrons at greater depths in the sample, the distribution of electrons near the incident energy and at large depths and is too low at small depths. This is to be expected as the discrepancy is that the single energy step calculation decreases too slowly with the results of Spencer's [5] calculations scaled to 30 eV. The main difference can also be calculated using this approach and this is shown in Fig. 4. The energy distribution of 30 eV electrons for copper as a function of depth follows a cosine law.

The distribution is dominated by single scattering at small angles. The 30 key electrons at normal incidence are presented. For this specimen, the Monte Carlo calculations. In Fig. 3, the relative backscattering for independent of atomic number. This agrees with the experimental results of Kane [7].

For 50 eV electrons, at grazing incidence the backscattering is more or less constant. At grazing incidence the backscattering is always stronger and so there is less 2 7 agreement between calculated points and experimental points. The agreement between calculated points and experimental points is good, the discrepancy being mainly due to the simple expressions assumed for density in the calculation.

It was found that the results were not very sensitive to small changes in the screening parameter or invasion electron potential. However, the theory is quite general and a screened Rutherford cross section was used for the inelastic scattering. A cross section was determined from the Bethe loss law using the formula:

$$
\left( \frac{2}{m^*} \right) \left( \frac{d\sigma}{dE} \right) = \left( \frac{m^*}{2} \right) \frac{E}{E - \hbar^2 \frac{\alpha}{m^*}}
$$

where

$$
\alpha = \frac{1}{\exp \left( \frac{E}{1.6 \times 10^6} \right) - 1}
$$

and

$$
\frac{d\sigma}{dE} = \frac{m^*}{2} \frac{E}{E - \hbar^2 \frac{\alpha}{m^*}}
$$

was used.
REFERENCES

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On an overlayer of backscattering observations, the great scatter of elastic scattering for light elements giving rise to more energy dispersion.

In Figs. 5, 6, and 7, the energy distribution of backscattered electrons of fluxes at various depths in a semi-infinite specimen.

Similar effects due to the treatment of absorption can be seen in calculations probably being because Spencer did not use the correct boundary conditions.

About 5 energy levels are needed for convergence the remaining discrepancy, if more energy levels are considered this can be corrected and
Fig. 1. Backscattered fraction \( n_b \) as a function of atomic number \( Z \). The open circles are calculated points, the crosses are experimental points and the dots are the calculated points for \( Z \) less than 20. The open circles with the number 2. The open circles with the number 3. The open circles with the number 4.

Fig. 2. Backscattered fraction \( n_b \) as a function of angle of incidence 0°. The open circles are experimental points and the dots are calculated points for \( Z \) less than 20.

Fig. 3. Angular distributions of backscattered electrons.
Fig. 7. Energy distribution of backscattered electrons for copper. 30°X normal incidence.

Fig. 6. Energy distribution of backscattered electrons for aluminum. 30°X normal incidence.

Fig. 5. Energy distribution of backscattered electrons for copper. 30°X normal incidence.

Fig. 4. Energy distribution of backscattered electrons for zirconium. As a function of depth, dE/dz.
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