Title
TIDAL GRAVITATIONAL FORCES: THE INFALL OF 'NEW' COMETS AND COMET SHOWERS

Permalink
https://escholarship.org/uc/item/0349j1xz

Authors
Morris, D.E.
Muller, R.A.

Publication Date
1985
TIDAL GRAVITATIONAL FORCES:
THE INFALL OF "NEW" COMETS AND COMET SHOWERS

D.E. Morris and R.A. Muller

January 1985
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
TIDAL GRAVITATIONAL FORCES:
THE INFALL OF "NEW" COMETS AND COMET SHOWERS

D. E. Morris and R. A. Muller

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

"This work was prepared for the US Department of Energy
under Contract Number DE-AC 03-76SF00098"
Abstract

The tidal gravitational field of the Galaxy directed into the galactic plane changes the angular momentum of comets in the Oort cloud. For comet orbits with semimajor axis greater than $2 \times 10^4$ AU, the change of angular momentum in one orbit is sufficient to bring comets from the Oort cloud into the visible region, causing the infall of "new" comets. The limiting size orbit is weakly dependent on the angle between the major axis of the comet orbit and the galactic plane. The flux of comets into the inner solar system caused by the galactic tidal field will be continuous and nearly isotropic. This effect appears to exclude any determination of the trajectories of passing stars by analysis of the angular distribution of new comets. The production of intense comet showers by the tidal field of a solar companion or of an interstellar cloud is considered. We show that the direction of a solar companion cannot be found from the present distribution of observable comets. The frequency of comet showers induced by encounters with interstellar clouds is found to be much lower than that from passing stars, and the tidal fields of interstellar clouds are not strong enough to cause comet showers of sufficient intensity to result in earth impacts.
1. Introduction

Comets with periods longer than recorded history are called "new" comets. They are members of the solar system, as shown by the absence of strongly hyperbolic "original" orbits (Oort, 1950; Marsden and Sekanina, 1973), calculated by subtracting the perturbations by the planets as the comet approached the observable region. (A typical comet is visible within ~3 AU of the sun according to Everhart 1967a, 1967b). New comets may be passing for the first time through the inner solar system when they are observed (Marsden, Sekanina and Everhart, 1978). The origin of new comets was explained by Oort (1950). As the source he postulated the existence of a cloud of comets in orbits around the sun with semi major axis between $2.5 \times 10^4$ and $1 \times 10^5$ AU, and with all possible eccentricities. He showed that interactions with passing stars will cause small random changes of velocity, angular momentum, and energy, although most comets will remain bound to the sun. Some of the comets in highly elliptical orbits will lose nearly all of their small angular momentum due to the interactions with passing stars and will enter the visible region.

In this paper we point out that new comets will be brought into the visible region even without considering perturbations by passing stars. When the field of the galaxy is included, the comets move in a non-central gravitational field, and so their angular momentum about the sun is not constant. The quasi-static gravitational field of any distant slowly moving mass will have a similar effect. The case of a solar companion star is considered, as well as the consequences of an encounter with a dense molecular cloud. We first summarize Oort's analysis of the effects of passing stars, followed by our analysis of the effects of tidal forces. We will use the convenient units of solar mass $M_{\odot}$, distance in AU, and time in years. In these units the gravitational constant $G = (2\pi)^2$. 
2. The infall of "new" comets caused by passing stars

Oort estimated the effects of passing stars in the impulse approximation. This is valid because a star with velocity $\approx 30$ km/sec would pass through the comet cloud in about 30,000 yr, whereas the orbital periods of the affected comets are typically several million years. Oort showed that passing stars will randomize the eccentricity of orbits of comets with semi-major axes $a > 2.3 \times 10^4$ AU over the life of the solar system. Some of the comets in the randomized cloud will have highly eccentric orbits, but if they have perihelion distance $q > 3$ AU they will not become visible, that is they will not pass close enough to the sun to be seen.

Comet orbits with perihelion distance $q$ smaller than about 15 AU do not have stable semimajor axes since comets which approach or cross the orbits of Jupiter or Saturn gain or lose energy by interacting with the major planets. The interaction takes place near the perihelion of the very large comet orbit ($a >> q$). Because the mass of the planet is much smaller than that of the sun, the fractional change in velocity of the comet caused by the planet is small; therefore the fractional changes in angular momentum $\Delta L/L$ and in perihelion distance $\Delta q/q$ are also small. The mean change of velocity for comets crossing Jupiter's orbit is $\approx 10^{-3}$, with a corresponding energy change per unit mass of about $5 \times 10^{-4}$ G AU$^{-1}$ (Everhart, 1968; Everhart and Raghavan, 1970; Marsden and Sekanina, 1973). Since the kinetic and potential energy of the weakly bound comets in the Oort cloud are nearly equal but of opposite sign, the mean change of energy caused by Jupiter is larger than the very small binding energy of the comets ($1-5 \times 10^{-5}$ G AU$^{-1}$). If energy is gained, the comet may escape from the solar system entirely, while if energy is lost in the interaction with Jupiter, the new semimajor axis of the comet's orbit is reduced to a fraction of the previous value but the perihelion distance $q$ is nearly unchanged. In the new smaller orbit, the comet returns to the inner solar system, where interaction with the major planets again changes the energy but not the angular momentum, to first order. The comet returns to nearly the same $q$ after each orbit, and interacts with the major planets again and again, losing or gaining energy randomly during each
perihelion passage until it is destroyed by planetary collision, volatized by solar heating, or ejected into hyperbolic orbit (Weissman, 1980). One consequence of this process will be a relatively large flux of comets with $3 \text{ AU} < q < 15 \text{ AU}$, resulting in a continuing bombardment of the outer planets and their moons. Thus one must show extreme care when relating the number of comets in the inner solar system to the number of craters seen on the moons of the outer planets.

Oort (1950), and Weissman (1980) estimated that orbits with $q$ less than about $15 \text{ AU}$ are perturbed sufficiently by Saturn and Jupiter so that comets will rapidly be removed from orbits in the Oort cloud ($a > 2.5 \times 10^4 \text{ AU}$) and transferred into orbits with smaller $a$, or ejected from the solar system. The region in velocity space which corresponds to orbits with $q < 15 \text{ AU}$ is called the "loss cone." Since orbits with $q < 15 \text{ AU}$ are unstable, the criterion for "new" comets to enter the visible region ($q < 3 \text{ AU}$) is that the change in angular momentum in a single orbit must be sufficient to reduce $q$ from $15 \text{ AU}$ to $3 \text{ AU}$. In our units the relation between $q$ and $L$ for nearly parabolic orbits is

$$L = 2\pi m (2q)^{1/2},$$

where $m$ is the mass of the comet, so for a change of $q$ from $15$ to $3$ we find the minimum required angular momentum change in one orbit is given by

$$\Delta L \geq 19 m \text{ AU}^2/\text{yr}.$$  

The necessary change in $L$ can be provided by any perturbation acting on the comet while it is far from the sun. Because of the large lever arm, only a very small change of velocity is required to change the angular momentum, which is already small for the highly eccentric orbits of interest. The effect of a single passing star on comet orbits is usually small because of the short interaction time, but the passage of many stars during a single orbital period $P \geq 4 \times 10^6 \text{ yrs}$ of a comet with $a \geq 2.5 \times 10^4 \text{ AU}$ can cause a random walk in angular momentum. The comet's angular momentum can fall below $15 m \text{ AU}^2/\text{yr}$ giving $q < 3 \text{ AU}$ and bringing the comet into the visible region.

A comet in a smaller orbit ($a < 2 \times 10^4 \text{AU}$) can only be brought into the visible region by rare close stellar encounters which can absorb nearly all of the comet's angular momentum in a single interaction. Oort showed that a star passing within $4.5 \times 10^4 \text{ AU}$ (as expected, on average, every $10^7 \text{ years}$) could change the angular momentum of comets near its path so that a comet with $a$
Then new comets would appear 1 to 2 million years after the passage of the star, when comets which were near its path reach perihelion. They would appear to come from points near a great circle marking the path of the star. Oort plotted the directions of the perihelia of long period comets, but did not find such a concentration. Several other such searches have been made (Tyror,1957; Hurnik,1959; Oja,1975; Hasegawa,1976; Yabushita,1979; Tomanov,1979; Bogart and Noerdlinger,1982).

Oort neglected the effect of the gravitational field of the galaxy, which, although small, acts continuously to change the angular momentum of comets in large eccentric orbits. Other authors have considered the effect of tidal force on the stability of comet orbits (Chebotarev 1964,1965,1966; Smoluchowski and Torbett 1984; Hills 1984), or considered the effects of the gravitational tidal force toward the center of the galaxy on angular momentum of comets but did not consider the dominant component of the field directed into the galactic plane (Byl,1983). After submitting this paper for publication, we learned that Harrington (1985) had considered the effect of the dominant component of the galactic field in increasing the perihelion of observable comets at previous and subsequent passages, although he did not identify this as an important mechanism for delivering new comets from the Oort cloud into the planetary system.

3. Effect of the tidal field of the galaxy.

We find that the galactic tidal field alone is sufficient to ensure that the rate at which comets with semimajor axes \( a > 2.5 \times 10^4 \) AU enter the planetary system is at the maximum allowed by statistical equilibrium since the change of angular momentum \( L \) in one orbit is sufficient to bring a comet which had perihelion distance \( q > 15 \) AU into the center of the solar system at its next perihelion passage. For these comets the loss cone is kept full by the tidal field alone. Passing stars scatter as many comets out of the loss cone as they scatter into it. However, passing stars do govern the rate at which comets with semimajor axes \( a < 2.5 \times 10^4 \) AU enter the planetary system.
The principal component of the tidal acceleration $dA/dz$ is directed along the $z$ axis normal to the plane of the galaxy. Let us consider a comet in a highly eccentric orbit ($q \ll a$) with aphelion direction at angle $\phi$ from the $z$ axis (see Figure 1). The change of angular momentum $L$ of a comet during time $dt$ will be:

$$dL = m \Delta v \times R = m \ z \ \frac{dA}{dz} \ d \sin \phi \ dt$$

$$= m \ dA/dz \ d^2 \ \sin \phi \ \cos \phi \ \ dt$$  \hspace{1cm} (1)$$

where $m$ is the comet mass and $d$ is its distance from the sun. The time averaged value of $d^2$, $<d^2> = 2.5 \ a^2$, in the limit of high eccentricity. (This can be easily shown by integration of the motion of a free falling object from rest in a $1/r^2$ field.) The orbital period in years is $P = a^{3/2}$. Therefore the change of angular momentum in one orbit is

$$\Delta L = 1.25 \ m \ a^{7/2} \ \frac{dA}{dz} \ \sin 2\phi$$  \hspace{1cm} (2)$$

As we discussed earlier, a comet can be brought into the inner solar system ($q < 3$) from $q > 15$ if $\Delta L > 19 \ m$, and this will be the case if $a$ exceeds a minimum value

$$a_m = 2.18 \ (\sin 2\phi \ \frac{dA}{dz})^{-2/7}$$  \hspace{1cm} (3)$$

The magnitude of $dA/dz$ can be estimated from the period of oscillation normal to the galactic plane $P_g = 7 \times 10^7$ years, giving $dA/dx = (2\pi/P_g)^2 = 8 \times 10^{-15}/\text{yr}^2$. The component of the tidal acceleration towards the center of the galaxy considered as a point mass is smaller by about an order of magnitude, and may be neglected. We note that only those comets which have a $z$ component of angular momentum $L_z < 11m$ can enter the observable region if we neglect
perturbations by passing stars, since the galactic tidal field which acts along the z direction cannot change \( L_z \). Equation 2 indicates that \( \Delta L \) increases rapidly with \( a \), so the "inner edge" of the (observable) Oort cloud will be quite sharp, with the loss cone completely filled for comets with \( a > a_m \), and very few comets with \( a < a_m \) entering the visible region due to the Galactic tidal field.  

The angular dependence of the lower limit of semimajor axis for observable comets from the Oort cloud is plotted in Figure 2. At an aphelion direction of 45° from the galactic pole, \( a_m = 2.3 \times 10^4 \) AU, and increases to \( 2.8 \times 10^4 \) AU at 15° and at 75°. We see that \( a_m \) exceeds \( 3.6 \times 10^4 \) AU for about 10% of the area of the celestial sphere. It should be noted that the effects of passing stars have been ignored. Passing stars alone, in the absence of a tidal field would also cause an apparent sharp inner boundary in the comet cloud (Hills 1981). This boundary is usually at \( a = 2 \times 10^4 \) AU unless a star has made an unusually close passage to the sun in recent past (Oort 1950, Hills 1981). Fortuitously, the inner boundaries predicted on the basis of passing stars and the tidal field are nearly at the same semimajor axis. Because of perturbations by passing stars, the observed value of \( a_m \) will not increase towards the Galactic plane and Galactic pole as predicted for the tidal field acting alone.

The value of \( a_m \) calculated by Oort from stellar perturbations depends on the number, size distribution and velocities of stars in the stellar neighborhood, and also depends on solar system parameters: the size of the orbits of Jupiter and Saturn and their masses and the mass of the sun. The value of \( a_m \) found from the galactic tidal field depends on the local value of the density of the galaxy, and has the same dependence on solar system parameters as the case of stellar perturbation considered by Oort. The galactic tidal field is primarily due to the gravitational attraction from the matter located in the region between the comet and the sun. One might think that this matter consists of stars which pass occasionally through this region, leading to a fluctuating tidal field. Oort assumed a density of 0.02 stars/pc³ with average mass \( = 1.4 \, \text{M}_\odot \) and velocity \( = 31 \, \text{km/s} \). Then one would expect 0.5 stars to pass within the length of the comet orbit \( 2a = 5 \times 10^4 \) AU during \( 4 \times 10^6 \) yrs, the period of a comet with \( a = 2.5 \times 10^4 \) AU. In fact, at least half of the local
density is dark matter that is likely to consist of small objects with nearly uniform density on the scale of the sun-comet system (Bahcall, 1984; Bahcall, Hut and Tremaine, 1985; for a review see Blumenthal et al., 1984), thus the approximation of a constant tidal field is reasonable.

4. Production of comet showers by slowly moving perturbers.

We may apply a similar analysis to the effects on comet orbits of the tidal gravitational field of a slow moving body such as a companion star, or of a massive object at large distances such as a giant molecular cloud. This gives physical insight into the process by which a slow moving perturber produces a comet shower. The situation and analysis is complementary to the impulse approximation, appropriate only for brief interactions, which has been used by authors since Oort (1950) to estimate the effects of passing stars.

Hills (1981) suggested that the comets in the Oort cloud with \( a \geq 2 \times 10^4 \) represent only a "halo" containing \( \approx 1\% \) of the comets in a more massive "inner Oort cloud" consisting of comets with \( 3 \times 10^3 \text{ AU} \leq a \leq 2 \times 10^4 \text{ AU} \), and that the outer cloud comets have been supplied by scattering of inner cloud comets into the loss cone by relatively close stellar passages, followed by scattering by the major planets into higher energy orbits. He estimated the density of the inner cloud by several methods. From the number of stars that have passed within a given distance of the sun during the life of the solar system he estimated that \( \approx 3\% \) of the inner cloud comets have been scattered into the loss cone, and of these he estimated \( \approx 16\% \) ended up in the present outer Oort cloud. Unless a larger fraction of the outer cloud comets have been lost than estimated by Hills (\( \approx 35\% \)), this appears to place an upper limit on the density of the inner cloud. Hills also pointed out that if the comets diffused out to their present orbits, the number of comets per unit of binding energy should decrease as the binding energy decreases. A lower limit for the number of comets in the inner cloud can be derived by setting the number of comets per unit energy to be uniform.
Comets in the inner Oort cloud cannot ordinarily be observed since they are scattered by the major planets before they can lose sufficient angular momentum to reach the visible region except following a close stellar encounter. Evidence of their presence might be obtained by infrared, radar, or other observation of the number of comets with perihelion distance $15 \text{ AU} < q < 30 \text{ AU}$ compared with the number with smaller $q$, since numerical simulations (Weismann 1984) should be able to predict the radial distribution of perihelia of comets from the outer Oort cloud alone.

Hills (1981) found that the close passage of a star at a distance $=3 \times 10^3 \text{ AU}$ would produce an intense comet shower from the inner cloud, which would result in several earth impacts. Hills (1984) estimated that it would be necessary to fill the loss cone of comets with semimajor axis $= 4000 \text{ AU}$ in order to produce a sufficiently intense shower to result in earth impacts, and we will adopt this value.

4.1 The tidal field of a solar companion.

We first looked into the effect of the gravitational field of a nearly stationary nearby star on comet orbits with the intention of finding the approximate area of the sky where an unseen solar companion (Davis, Hut and Muller, 1984; Whitmire and Jackson, 1984) might be located, by the differing effects on the orbits of comets with different aphelion directions.

The tidal gravitational field of the companion may be approximated by the value of its tidal field at the position of the sun, with the change of angular momentum in one orbit of a comet given by equation 2. It is instructive to compare the gravitational effects of such a solar companion with that of the galaxy. For a companion with mass $M = 0.08 \text{ } M_\odot$, with orbital period of 28 million years and eccentricity $e \approx 0.7$ and presently near aphelion, the present distance $D = a (1 + e)$ would be about $1.6 \times 10^5 \text{ AU}$ and the acceleration produced near the sun $A = (2\pi)^2 \frac{M}{r^2}$, since the
gravitational constant $G = (2\pi)^2$ in our units ($M_\odot$, AU and year). Then $dA/dr = 1.5 \times 10^{-15} \text{yr}^{-2}$ or $< 1/5$ of that due to the galaxy. The angular velocity of the companion near aphelion is very small, so the direction of the gravitational tidal force of the companion, unlike that of a passing star, would remain almost constant during the orbit of a comet with $a = 2.5 \times 10^4$ AU ($P = 4 \times 10^6$ years).

Adding the tidal acceleration from such a companion near aphelion to that of the galaxy gives a resultant acceleration directed within $11^\circ$ of that due to the galaxy alone, regardless of the direction of the companion, so this method cannot locate the the position of such a solar companion when it is near aphelion. In fact, even if a companion were closer and provided a field sufficiently strong to give a resultant at a large angle to the galactic field, this could not be found from the angular distribution of comets. Data is available for only 108 new comets with accurately determined initial orbits, 98 of these from Marsden, Sekanina and Everhart (1978) and 10 from Everhart and Marsden (1983). Out of these, only 17 have $a$ in the required range. Since these few comets are spread over the celestial sphere it would not be possible to identify a narrow belt along a great circle in which such comets were absent. Furthermore, this discussion neglects the randomizing effects of passing stars, which fill the loss cone for comets with $a > 2 \times 10^4$ and bring in comets with all aphelion directions.

The overall distribution of angular momenta of the comets in the Oort cloud has been randomized by passing stars over the life of the solar system. Some comets will be located close to the companion star and will be strongly perturbed. We note however, that these comets are in orbits with $a >> a_m$, for which the loss cone is kept full by the tidal field of the Galaxy. The perturbing field of the companion will scatter as many of these comets out of the loss cone as it scatters into the loss cone, so the number of comets entering the visible region will not change. We conclude that the distribution of angular momentum of comets entering the visible region cannot serve to locate a solar companion as attempted by Delsemme and Soonthornthum (1985). The observed nonuniform distribution of comet aphelion directions and angular momenta may be
ascribed to observational selection (Everhart, 1967a).

We see that the tidal field of the galaxy and the weaker tidal field of a companion near aphelion bring in the maximal number of comets with \( a > 2.3 \times 10^4 \) AU, while comets that have \( a < 2 \times 10^4 \) AU cannot be brought in within Jupiter's orbit. However when the companion approaches perihelion its tidal field near the sun would increase as \( 1/r^3 \) and become many times greater than that of the galaxy, and \( a_m \) will be reduced to a fraction of its usual value. This will permit comets in orbits with smaller values of \( a \) to enter the inner solar system, although they are otherwise unable to do so because they undergo too small a change of angular momentum per orbit.

The tidal acceleration from the companion star in the vicinity of the sun \( \frac{dA}{dz} = -2 \frac{GM}{Q^3} = -8\pi^2 \frac{M}{Q^3} \). So, from equation (3), with \( \sin 2\phi = 1 \) and taking \( \Delta L \geq 26\) m AU\(^2/\)yr to bring comets into earth crossing orbits \( (q \leq 1) \),

\[
a_r = 2.38 \left( \frac{dA}{dz} \right)^{2/7} = 0.68 \frac{Q}{6/7} \frac{M}{L^{-2/7}},
\]

For orbits with \( a > a_r \) (the reduced value of \( a_m \)) the loss cone will be filled and the maximal number of comets will be brought in by the tidal field of the companion causing a "comet shower." We can use equation (4) to find the region of the Oort cloud which can supply the shower. If \( a_r \) is reduced to a sufficiently small value, the result can be a very large increase in flux of comets in earth crossing orbits, with the consequence that several comets will hit the earth as first discussed by Hills (1981).

The angular velocity of a companion will be small enough at perihelion that it may be treated as stationary during the orbit of a comet which has a semimajor axis \( a < 0.3 \) Q, where Q is the perihelion distance of the companion star. We show this as follows. The companion star in a nearly parabolic orbit has angular velocity at perihelion \( \Omega = 2^{3/2} \pi/Q^{3/2} \) (This is \( \sqrt{2} \) larger than
that of a body in a circular orbit with \( r = Q \). For a comet with \( a = 0.3 \, Q \), the period of the comet is \( P = a^{3/2} = (0.3 \, Q)^{3/2} \), so \( \Omega \, P < 1.5 \), and the companion star moves through less than 90° during one orbit of the comet. As noted earlier, only one orbit of the comet need be considered.

The arrival directions of comets averaged over the duration of the shower will be nearly isotropic since the loss cone will be filled down to \( a_r \) given by equation (4) and the angle \( \phi \) will change enough as the companion passes perihelion to smooth out the directional dependance of \( \sin 2\phi \).

Let us calculate \( a_r \) for some companion masses and orbital parameters which have been considered by previous authors. Davis, Hut and Muller (1984) considered a companion with \( M = 0.08 \, M_\odot \) and \( Q = 3 \times 10^4 \, \text{AU} \), in this case eq.(4) gives \( a_r = 9.6 \times 10^3 \, \text{AU} \). Since the major axis \( 2a \) of the limiting comet orbit is comparable to the distance of the companion, for a more accurate result we must calculate the torque directly from the gravitational field of the companion instead of using the approximation of a uniform tidal field. The minimum value of \( a_r = 7 \times 10^3 \, \text{AU} \) is found at \( \phi = 26 \), while in the hemisphere opposite to the companion star \( a_r \) will be somewhat larger than the value given by equation (4). If we substitute into eq. (4) the value \( a_r = 4 \times 10^3 \, \text{AU} \) estimated by Hills (1984) to be necessary to produce a sufficiently intense comet shower to result in earth impacts, we find \( Q \leq 2.5 \times 10^4 \, M_\odot^{1/3} \) and \( e \geq 1 - 0.28 \, M_\odot^{1/3} \). Then, for \( M = 0.08 \, M_\odot \) we find \( Q \leq 1.08 \times 10^4 \, \text{AU} \) and eccentricity \( e \geq 0.88 \). This result is different than the estimate \( Q \leq 3 \times 10^4 \, \text{AU} \) and \( e \geq 0.67 \) to give \( a_r = 3 \times 10^3 \, \text{AU} \) by Davis, Hut and Muller (1984) based on the impact approximation, but is similar to the estimate \( Q < 9 \times 10^3 \, \text{AU} \) and \( e > 0.9 \) for a companion with \( M = 0.07 \, M_\odot \) based on the two body scattering cross section calculated from the sphere of action of the companion (Whitmire and Jackson, 1984).

Hills (1984) used 3 body calculations to find the dependence of \( \Delta q \), the change \( q \) of a comet in one orbit, on the perihelion distance of the companion. The calculation assumed \( M = 0.05 \, M_\odot \) or \( M = 0.005 \, M_\odot \), and comets with \( a = 4,000 \, \text{AU} \) and \( e = 0.999 \). The initial value of \( q \) was taken to be \( q_0 = (1-e) \, a = 4 \, \text{AU} \), and \( <\Delta q> \), the required mean change in \( q \) was taken to be 9.3
Hills concluded that for $M = 0.05 \, M_\odot$, the required value of $Q \leq 1.04 \times 10^4$ AU and $e \geq 0.88$ for this case, very close to the estimate $Q \leq 0.92 \times 10^4$ and $e \geq 0.9$ from eq.(4). The main advantage of our analysis is that it gives physical insight into the process of induction of comet showers by a companion star.

To find the required orbital eccentricity for a companion of larger mass, Hills applied a scaling law valid in the impulse approximation, which is not appropriate in this situation, with the result $Q \leq 3 \times 10^4$ AU and $e \geq 0.66$ for $M = 0.2 \, M_\odot$. In contrast, we find $Q \leq 1.5 \times 10^4$ AU and $e \geq 0.84$ since $Q$ is proportional to $M^{1/3}$ from eq (4). When the mass is larger, $a_r < 0.3 \, Q$ and the approximations of a stationary companion and a uniform tidal field near the sun, used in eq. (4), will be more accurate. In the case of a small companion with $M < 0.05 \, M_\odot$ this is not so, but this range of $M$ is excluded as an explanation of periodic comet showers because of the fluctuations of the perihelion distance of a companion in a highly eccentric orbit which would be caused by passing stars (Hut 1984).

A similar calculation may be carried out for the case of a more massive companion. For example, if the companion were a neutron star with mass $M = 1.4 \, M_\odot$, the total mass would be $2.4 \, M_\odot$ and the semimajor axis of the orbit $a = 1.23 \times 10^5$ AU for a period of $2.8 \times 10^7$ yrs. In order that $\Delta L$ of comets with $a = 4 \times 10^3$ AU be sufficient to bring them into earth crossing orbits, the required minimum separation between the sun and companion star from equation (4) would be $Q = 2.8 \times 10^4$ AU, so the eccentricity $e \geq 0.77$. The present (aphelion) distance would be $2.23 \times 10^5$ AU and the present tidal field of the companion would be about the same as that of the galaxy. From eq. (3), this would reduce $a_r$ less than 18%, an undetectable amount.

4.2 The tidal fields of interstellar clouds and giant molecular clouds.

Encounters with interstellar clouds (IC) have been proposed by Rampino and Stothers
(1984) as a cause of quasi-periodic intense comet showers leading to earth impacts. They cite Talbot and Newman (1977) (abbreviated as TN 1977 in the following) for data on ICs. Briefly, the abundant "standard" clouds are several parsecs in size, have typical density $N = 10^{-30}$ atoms cm$^{-3}$ and mean time between encounters $\tau = 1.3 \times 10^7$ yrs. The most dense clouds (Lynds dark nebulae) have a similar size range, and densities $10^2 \leq N \leq 5 \times 10^3$, but they are rare, and the mean time between encounters with such clouds is large. It is easy to show that encounters with "standard" clouds of low density cannot produce comet showers, that cloud density $N > 10^3$ atoms cm$^{-3}$ is needed to produce an intense comet shower leading to earth impacts, and that the tidal field of a cloud during a distant encounter is too weak to produce such showers. In consequence, showers produced by encounters with interstellar clouds are much less frequent than showers produced by passing stars.

The tidal field inside a diffuse object such as a molecular cloud is $dA/dr = (32 \pi^3/3) \rho$ in the approximation of a spherically symmetric cloud of uniform density $\rho$, since $A = G M r^2$ and $M = 4\pi/3 r^3 \rho$, where $r$ is the distance from the center of the cloud. We recall that $G = (2\pi)^2$ in our units (M$_\odot$, AU and years). Outside the cloud $dA/dr = (32\pi^3/3) \rho (R/r)^3$ where $R$ is the cloud radius and. In more convenient units, $\rho = 2.8 \times 10^{-18} \text{N M}_\odot \text{AU}^{-3}$ where $N$ is in hydrogen atoms/cm$^3$, so inside the cloud $(dA/dr) = 9.3 \times 10^{-16} \text{N}$. The static approximation used in Eq.(3) is only applicable when the duration of the interaction is longer than the period of a comet with semi-major axis $a = a_m$ in the tidal field of the cloud, i.e. when $R/v > 10^{-6} a^{3/2} = 9 N^{-3/7}$ for a penetrating encounter. When this condition is not satisfied, the effective duration of the interaction during an encounter is limited to $T = R/v$, which replaces the comet's period $P = a^{3/2}$ in the derivation of eq.(2). (For a non-penetrating encounter $R/v$ is replaced by $S/v$ where $S$ is the impact parameter in parsecs.) When $R/v < 9 N^{-3/7}$ we can use the impulse approximation in which we integrate $dL$ over the time $R/v$. A large fraction of comets in highly eccentric orbits with a given $a$ will be found near aphelion at any given time, since a comet in a highly eccentric orbit is at a distance $d \geq a$ during $=0.82$ of its period, and the time averaged value of $d$ in this part of the orbit
is $1.78\, a$ (Rickman 1976). We use the approximations $d = 1.78\, a$, and $\sin \phi \cos \phi = 1/2$. A comet can be brought in from $q \geq 15$ into an earth crossing orbit ($q \leq 1$) if $\Delta L/m > 26$. Substituting in (3) we find

$$a_s = 1.35 \times 10^5 \left(\frac{v}{R\, N}\right)^{1/2} \text{AU} = 6 \times 10^5 \, R^{-1/2} \, N^{-1/2} \, \text{AU}$$

(5)

where $R$ is in parsecs and $v$ in km/s. We take $v = 20$ km/s, the velocity of the sun with respect to the local standard of rest (TN 1977). Then the condition $T = R/v < a_s^{3/2}$ is satisfied if $N < 1.8 \times 10^5 \, R^{-7/3}$ in these units. It may be noted that the value of $a_s$ during a close or penetrating encounter with an interstellar cloud depends directly on the cloud density rather than the cloud mass.

Values of $a_s$ were calculated for each class of IC given in Table I of TN (1977). Fig. 3 gives the cumulative rate of encounters with the classes of IC which would reduce $a_s$ below a given value, along with $\tau$ the mean interval between encounters. For comparison we give the rate of encounters as well as $\tau$ as a function of $a_s$ for comet showers induced by passing stars, after Hills (1981). Although the abundant "standard" clouds are encountered frequently, because of their low density the calculated value of $a_s \approx 7 \times 10^4 \, \text{AU}$. Since the loss cone for comets in the Oort cloud with $a > 2 \times 10^4 \, \text{AU}$ is kept filled by the Galactic field, encounters with "standard" clouds do not produce any increase in comet flux. In fact even an encounter with a cloud with $N = 10^2$ (Lynds dark nebula class $j=1$) would give $a_s = 2.4 \times 10^4 \, \text{AU}$ and would not produce a comet shower. Encounters with denser clouds (Lynds classes $j=2$ to $j=6$) with $2 \times 10^2 \leq N \leq 5 \times 10^3$ would reduce $a_s$ sufficiently to produce comet showers, but at very long intervals (see Fig 3). Values of $a_s$ and $\tau$ were calculated for GMCs based on mean density $N = 290 \,(R/10 \, \text{parsec})^{-0.75}$ atoms $\text{cm}^{-3}$ and size $10 \leq R \leq 40$ parsecs, from Sanders, Scoville and Soloman (1985). The results are unchanged if the GMCs are taken to be "clumpy" with higher density subclouds ($R \approx 2 \, \text{pc}$ and $N \approx 10^3$) as suggested by Blitz and Shu (1980).
Hills (1984) estimated that it is necessary to bring in comets with \( a = 4000 \) AU, in order to produce an intense shower causing earth impacts, which he calls a "death shower". This requires a cloud density \( N \geq 10^4 \) atoms cm\(^{-3}\) according to eq 5, if \( r = 2 \) pc and \( v = 20 \) km/s. Clouds of such high density are very rare; not even a single encounter would be expected over the life of the solar system according to Fig. 1 of TN (1977).

It has been suggested that the tidal field of a GMC, during a close passage, may be strong enough to strip off weakly bound comets in the outer Oort cloud (Biermann 1978, van den Bergh 1982, Bailey 1983). However, the density and tidal field of a typical GMC is too small to perturb the more tightly bound comets in the postulated inner Oort cloud into earth crossing orbits to produce a death shower.

In an encounter with a large GMC (\( R > 20 \) pc), \( R/v > a_s^{3/2} \), the effective duration of interaction is limited to one comet period and the static tidal approximation may be used. Then from Eq. 3 the smallest comet orbits with filled loss cone have \( a = a_m = 2.18 \) (dA/dx\(^{-2/7}\) = 4.3\( \times 10^4 \) N\(^{-2/7}\) = \( 10^4 \) AU for a large GMC of typical density (\( R = 30 \) pc and \( N = 125 \) cm\(^{-3}\)). This result is nearly the same as for smaller clouds, where the impulse approximation is applicable.

The volume fraction of GMCs \( f = 0.08 \) and mean free path \( \approx 3.5 \times 10^3 \) parsec in the inner galaxy, with only half as many GMCs at the sun's galactic radius. This gives an encounter rate less than one per \( 4 \times 10^8 \) yrs, so encounters with GMCs are not a significant cause of comet showers. The encounter rate vs \( a_s \) of molecular clouds based on CO measurements given by TN (1977) also falls below the dashed line in Fig 3, but is not plotted since the estimates of cloud size and density are out of date.

The passage of a dense IC or GMC at a distance several times its radius would produce a
much weaker tidal field than during a penetrating encounter since the tidal field falls as \(1/r^3\). Therefore the relatively frequent distant passages of GMC’s discussed by Thaddeus and Chanan (1985) cannot produce comet showers.

If, contrary to the estimates of Hills (1981) the density of comets in the inner Oort cloud increases very rapidly with binding energy, then an intense shower with earth impacts could possibly be produced when \(a_r = 10^4\) AU. Even then, a cloud with \(N > 10^3\) (Lynds class \(j=4\)) would be required, and the frequency of encounters would be about one in \(4.5 \times 10^8\) yrs, still only 1/10 of the rate of showers with the same \(a_s\) which are induced by passing stars (see Fig 3). Then the rate of star induced death showers would be comparable to the rate of periodic mass extinction events, practically eliminating the need to invoke perturbations by interstellar clouds, or by a companion star, to produce several death showers per \(10^8\) years, although the star induced showers would occur at random intervals. We conclude that encounters with dense interstellar clouds or with GMCs cannot be the reason for mass extinctions at \(\approx 30\) Myr intervals (Raup and Sepkoski 1984) if caused by earth impacts from intense comet showers.

**Summary and Conclusions**

The tidal gravitational field of the Galaxy perpendicular to the galactic plane changes the angular momentum of comets in the Oort cloud. Some of these comets lose angular momentum and their perihelion distances fall below 15 AU. For comet orbits with semi-major axis \(a < 2 \times 10^4\) AU the change in \(L\) and \(q\) in a single orbit is insufficient to bring the comets into the visible region (\(q < 3\) AU). These comets are perturbed near perihelion by Jupiter and Saturn so that they are rapidly removed from their large orbits which have very small binding energies, and are either ejected into hyperbolic orbits or brought into smaller short period orbits. For larger orbits with \(a > 2 \times 10^4\) AU, scattering by Jupiter and Saturn is not effective in screening the inner solar system from the flux of comets since the change in angular momentum in a single orbit is sufficient to bring comets from the Oort cloud with \(q > 15\) AU into the visible region where \(q < 3\) AU, filling the loss
cone and resulting in the infall of "new" comets. This effect of the tidal gravitational field directed into the plane of the galaxy seems to have been overlooked in the past. The limiting value of $a$ is on the same order as that determined by the perturbations by passing stars. The lower limiting value of semimajor axis depends only weakly on the angle between the major axis of the comet orbit and the galactic plane. The flux of comets into the inner solar system caused by the galactic tidal field will be continuous and nearly isotropic.

Comet showers may be produced by the tidal field of a solar companion as it approaches perihelion if there is a substantial density of comets in orbits with $a < 2 \times 10^4$ AU. Comets in such orbits are not brought into the visible region at present because the change in angular momentum per orbit caused by the Galactic tidal field is small and the comets are screened off by scattering by the major planets. These comets can reach earth crossing orbits only if they lose sufficient angular momentum in a single orbit. For a companion star with $M = 0.08 M_\odot$ this condition is satisfied for comets with $a = 4000$ AU if the the perihelion distance of the companion $Q \leq 1.08 \times 10^4$ AU and orbital eccentricity $e \geq 0.88$. For comets with $a = 10,000$ AU, the companion must have $e \geq 0.7$.

To determine whether either of these conditions result in a sufficiently intense comet shower to produce an earth impact, we must know the radial distribution of comets. At present this is estimated by model dependent calculations of the formation of the Oort cloud (Hills 1981, Bailey 1983), but it can be probed directly by measurements of the density of comet perihelia over the range 3 to 30 AU. If the density of comets in the inner Oort cloud increases rapidly enough with binding energy to provide an intense shower with earth impacts when $a_r = 10^4$ AU, then passing stars would produce such showers at an average rate of one per $4.4 \times 10^6$ years. Such a high rate of stellar induced intense showers would practically obviate the need to invoke perturbations by a solar companion or by molecular clouds to produce several intense comet showers per $10^8$ yrs, although the showers produced by stars would not be periodic.
The direction of a solar companion cannot be found from the present distribution of observable comets. The tidal field of a companion far from perihelion would be smaller than the field of the galaxy. Even when the companion is near perihelion the induced comet shower would be nearly isotropic, and the shower comets would then be scattered away by the major planets within a few million years.

The frequency of comet showers induced by interstellar clouds is much lower than that from passing stars. The tidal field of an interstellar cloud (including a GMC) with density $N \lesssim 10^3$ atoms cm$^{-3}$ appears to be insufficient to produce an intense comet shower leading to earth impacts, even in a close encounter, and the frequency of encounters with dense clouds is very low. The gravitational interactions during more distant passages are too weak to produce any increase in the usual flux of comets.

Acknowledgements

It is a pleasure to acknowledge stimulating discussions with P. Hut, F. Crawford, J. Kare, P. Tans, C. Pennypacker, R. Kahn, J. Graham and other colleagues. We wish to express our appreciation to J. Machol who carried out computer analysis of comet orbits and prepared the figures, and also to B. Boyd for assistance. We thank S. Tremaine for pointing out the equivalence of the galactic tidal field to the attraction of the matter density located between the sun and comet, and to J. G. Hills and an anonymous referee for useful comments.
References:


Figure 1. A comet at distance $d$ in a highly eccentric orbit with semimajor axis $a$ inclined at angle $\phi$ to the galactic pole. The perihelion distance $q$ is shown larger than scale for clarity. The differential acceleration of the comet with respect to the sun caused by the galactic field is $A_g = z \cdot dA/dz$. The component of this acceleration which provides the change of angular momentum of the comet is $z \cdot dA/dz \sin \phi$, as shown.
Figure 2. Comets in orbits with semimajor axis outside of the shaded region can lose sufficient angular momentum in a single orbit from the galactic tidal field to enter the visible region within $\sim 3$ AU of the sun. For orbits with smaller semimajor axis (inside the shaded area of the figure), angular momentum is lost more slowly. Such comets are scattered by the major planets into orbits with small semimajor axis in which they do not continue to lose angular momentum, or into hyperbolic orbits. Consequently they do not reach the inner solar system, except as a result of scattering by passing stars which have been ignored in this analysis.
Figure 3. A Comet shower will be produced when comets in the inner Oort cloud are brought into earth crossing orbits by a perturbing star or interstellar cloud (IC). The boundary between the outer Oort cloud and the inner cloud near semi-major axes \( a = 2.3 \times 10^4 \) AU is marked by the dashed vertical line. The semi-major axis \( a_s \) of the smallest comet orbit which can be perturbed into an earth crossing orbit by a given class of IC is indicated, along with the cumulative rate of encounters with all classes of ICs which give smaller \( a_s \). The corresponding mean time between encounters \( \tau \) is also given. It is much longer than 30 Myrs, the interval between mass extinctions. The solid line gives the rate of stellar encounters which will bring in comets from a given \( a_s \), after Hills (1981). This rate is several times greater than that due to ICs, so passing stars induce many more comet showers than do ICs. Reduction of \( a_s \) below \( 4 \times 10^3 \) AU is required to produce an intense "death shower" causing earth impacts (Hills 1984). ICs are not dense enough to accomplish this.
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.