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Publication Date
2014

Peer reviewed

Journal of Macroeconomics 42 (2014) 174–183

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Progressive taxation and macroeconomic (in)stability with utility-generating government spending

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A R T I C L E I N F O

Article history:
Received 27 January 2014
Accepted 11 August 2014
Available online 3 September 2014

JEL classification:
E32
E62

Keywords:
Progressive income taxation
Equilibrium (in)determinacy
Utility-generating government spending
Business cycles

A B S T R A C T

We examine the theoretical interrelations between progressive income taxation and macroeconomic (in)stability in an otherwise standard one-sector real business cycle model with utility-generating government purchases of goods and services. When private and public consumption expenditures are complements in the household utility and the tax schedule is progressive, we analytically show that the economy exhibits indeterminacy and sunspots if and only if the degree of government-spending preference externality is higher than a critical threshold. Unlike traditional Keynesian-type stabilization policies, raising the tax progressivity may destabilize this version of our model by generating endogenous cyclical fluctuations. Moreover, the economy always displays saddle-path stability and equilibrium uniqueness under utility substitutability between private and public consumptions and progressive taxation.

1. Introduction

The relationship between government purchases of goods and services versus agents' private consumption is an important aspect in understanding the demand-side effects of a fiscal policy rule within dynamic general equilibrium macroeconomic models.[2] In particular, whether private and public consumption expenditures enter the household's utility function as Edgeworth complements or substitutes may affect the model's local dynamics. Recent work in this area includes Cazzavillan (1996), Zhang (2000), Raurich (2003), Fernández et al. (2004), Chen (2006), Guo and Harrison (2008), Lloyd-Braga et al. (2008), Hori and Maebayashi (2013), among others. Building upon these existing studies, we consider a prototypical one-sector real business cycle (RBC) model with two prevalent features observed in developed economies: progressive income taxation[3] together with utility-generating public spending, and analytically explore the interrelations between tax progressivity and

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[1] Tel.: +886 2 8674 1111x67165; fax: +886 2 2673 9880.
[2] Previous research that studies the supply-side effects of public expenditures in representative-agent models incorporate productive flow of government spending (*e.g.* Barro (1990), Barro and Sala-i-Martin (1992), Glomm and Ravikumar (1997), Turnovsky (1999), Palivos et al. (2003), Slobodyan (2006), Hu et al. (2008), Kamiguchi and Tamai (2011), and Chen and Guo (2013a,b)), or stock of public capital (*e.g.* Futagami et al. (1993), Glomm and Ravikumar (1994), Turnovsky (1997), Baier and Glomm (2001), Greiner (2006, 2007), and Agénor (2011)).
[3] By contrast, Cazzavillan (1996), Zhang (2000), Raurich (2003), Fernández et al. (2004), Chen (2006), Guo and Harrison (2008) all postulate a constant tax rate of income; and Hori and Maebayashi (2013) consider a flat consumption tax. On the other hand, Lloyd-Braga et al. (2008) incorporate progressive consumption taxation into a finance-constrained macroeconomy with heterogeneous agents.

http://dx.doi.org/10.1016/j.jmacro.2014.08.002
0164-0704/

equilibrium (in)determinacy.[4] Our analysis is valuable not only for its theoretical relevance, but also for its broad implications for the design, evaluation and implementation of tax policies.

In this paper, we systematically study the (de)stabilization effects of Guo and Lansing's (1998) progressive tax formulation in an otherwise standard one-sector RBC model with balanced budget and utility-generating public expenditures. Per the empirical findings of Ni (1995), our model examines a constant-relative-risk-aversion (CRRA) Cobb-Douglas utility specification that postulates government spending as a positive preference externality. As it turns out, the (local) stability properties of the model's unique interior steady state depend crucially on (i) the utility complementarity or substitutability between private and public consumptions, (ii) the slope parameter of the tax schedule that governs its progressivity attribute, and (iii) the degree of government-purchases preference externality.

When government spending is complementary to private consumption in the household utility and the tax policy is progressive, we derive the *necessary and sufficient* condition under which our model exhibits an indeterminate steady state and endogenous cyclical fluctuations driven by animal spirits or sunspots. In particular, the degree of preference externality from public expenditures needs to be higher than a critical value that can be analytically expressed as a function of other structural parameters. Start the economy from its steady state, and consider a slight deviation caused by agents' optimistic anticipation about an expansion in future economic activities. Acting upon this belief, the representative household will reduce consumption and raise investment today. This in turn leads to another dynamic trajectory with higher future output, private consumption, and income tax rate because of progressive taxation. Through the government's balanced-budget constraint, the level of public spending also rises, which will then produce a further increase in future private consumption since private and public consumption expenditures are Edgeworth complements. We show that the after-tax return on investment is monotonically increasing along this alternative transitional path if and only if the government-spending preference externality exceeds the requisite threshold. As a result, agents' initial rosy expectations on the economy's future are validated as a self-fulfilling equilibrium. Moreover, in sharp contrast to previous studies with useless or wasteful government purchases of goods and services,[5] raising the tax progressivity *ceteris paribus* may transform our model's steady state from a saddle point into a sink provided the degree of public-consumption preference externality is sufficiently strong. It follows that unlike traditional Keynesian-type stabilization policies, a more progressive tax schedule may destabilize the economy by generating belief-driven business cycle fluctuations.

When government spending is substitutable with private consumption in the household utility and the tax policy is progressive, we find that the mechanism described in the proceeding formulation that makes for multiple equilibria, *i.e.* an increase of the equilibrium after-tax marginal product of capital in response to higher expenditures of today's investment, will not be realized in that higher public expenditures now lower the marginal utility of private consumption. It follows that our model economy always exhibits saddle-path stability and equilibrium uniqueness in this setting. Finally, the same stability/uniqueness result continues to hold when there is no government-purchases preference externality, regardless of the level of tax progressivity under consideration; or when the income tax rate is a fixed constant, no matter whether private and public consumptions are Edgeworth complements or substitutes.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions. Section 3 examines the theoretical interrelations between tax progressivity, government-spending preference externality and our model's local stability properties. Section 4 concludes.

2. The economy

We incorporate utility-generating government purchases of goods and services into an otherwise standard one-sector real business cycle (RBC) model under the progressive income tax policy á *la* Guo and Lansing (1998). Households live forever, and derive utilities from private consumption, public expenditures and leisure. Based on the empirical findings of Ni (1995), our analysis considers a constant-relative-risk-aversion (CRRA) Cobb-Douglas utility specification that postulates government spending as a positive preference externality. On the production side, each competitive firm produces output using a constant returns-to-scale technology with capital and labor as inputs. We further assume that there are no fundamental uncertainties present in the economy.

2.1. Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm produces output Y_t, using physical capital K_t and labor hours H_t as inputs, with a constant returns-to-scale Cobb-Douglas production function

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

[4] See Benhabib and Farmer (1999) for an excellent survey of the RBC-based indeterminacy literature.

[5] See, for example, Schmitt-Grohé and Uribe (1997), Guo and Lansing (1998), and Christiano and Harrison (1999), among others.

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{2}$$

$$w_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{3}$$

where r_t is the capital rental rate and w_t is the real wage. In addition, α and $1-\alpha$ represent the capital and labor share of national income, respectively.

2.2. Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes a discounted stream of utilities over its lifetime

$$\int_0^\infty \left[\frac{\left(C_t^{\theta_1} G_t^{\theta_2}\right)^{1-\sigma}}{1-\sigma} - A\frac{H_t^{1+\gamma}}{1+\gamma}\right] e^{-\rho t}\, dt, \; A, \sigma > 0, \; \sigma \neq 1, \quad \text{and} \quad \gamma \geqslant 0, \tag{4}$$

where $\rho > 0$ is the subjective discount rate, C_t is private consumption, H_t is hours worked, and γ governs the inverse of the labor supply elasticity.[6] Moreover, G_t denotes the flow of government spending on goods and services that are determined outside the individual household's control. Per the empirical results of Ni (1995), the instantaneous utility function in our model (i) is increasing and strictly concave with respect to private consumption, thus $\theta_1 > 0$ and $\theta_1(1-\sigma) < 1$; (ii) is increasing in public consumption, thus $\theta_2 > 0$ indicating the presence of a positive preference externality; and (iii) exhibits linear homogeneity in "effective consumption" $C_t^{\theta_1} G_t^{\theta_2}$, thus $\theta_1 + \theta_2 = 1$ (see also Bean (1986) and Campbell and Mankiw (1990)).[7]

We also note that when $\sigma = 1$, the household's preference in (4) exhibits additive separability between private and public consumption expenditures, hence the marginal utility of C_t is independent of G_t. It follows that the inclusion of utility-generating government spending will not have any impact on the model's equilibrium conditions and local dynamics. Therefore, our subsequent analysis only considers the cases with $\sigma \neq 1$. Specifically when $\sigma < (>) 1$, the marginal utility of private consumption increases (decreases) with respect to government spending, hence C_t and G_t are Edgeworth complements (substitutes).

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (1-\tau_t)(r_t K_t + w_t H_t) - \delta K_t - C_t, \quad K_0 > 0 \text{ given}, \tag{5}$$

where K_t is the household's capital stock and $\delta \in (0,1)$ is the capital depreciation rate. Households derive income by providing capital and labor services to firms, taking factor prices r_t and w_t as given. As in Guo and Lansing (1998), we postulate that the income tax rate τ_t takes the form

$$\tau_t = 1 - \eta\left(\frac{Y^*}{Y_t}\right)^{\phi}, \quad \eta \in (0,1) \quad \text{and} \quad \phi \in [0,1), \tag{6}$$

where Y_t represents the household's taxable income $(= r_t K_t + w_t L_t)$, and Y^* denotes the steady-state level of per capita income, which is taken as given by each agent. The parameters η and ϕ govern the level and slope of the tax schedule, respectively. When $\phi > 0$, the tax rate τ_t rises with the household's taxable income Y_t. When $\phi = 0$, all households face the constant tax rate $1-\eta$ regardless of their taxable income.

With regard to the progressivity features of the above taxation scheme, we first note that the marginal tax rate τ_{mt}, defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} \equiv \frac{\partial(\tau_t Y_t)}{\partial Y_t} = 1 - \eta(1-\phi)\left(\frac{Y^*}{Y_t}\right)^{\phi}. \tag{7}$$

In addition, our analysis is restricted to environments with $0 < \tau_t, \tau_{mt} < 1$ such that (i) the government does not have access to lump-sum taxes or transfers, (ii) the government cannot confiscate all productive resources, and (iii) households have incentive to supply factor services to the firm's production process. In the model's steady state, the preceding considerations imply that $0 < \eta < 1$ and that $\frac{\eta-1}{\eta} < \phi < 1$, where $\frac{\eta-1}{\eta} < 0$.

Next, in order to satisfy the second-order conditions of the representative agent's dynamic optimization problem, its budget constraint (5) needs to be jointly concave in the state and control variables, *i.e.* K_t, C_t and H_t. We find that this

[6] The specification with $\gamma = 0$ draws on the formulation of indivisible labor as in Hansen (1985) and Rogerson (1988).

[7] Ni (1995) considers the linear and Cobb-Douglas specifications of *effective* consumption, which specifies how private and public consumptions are combined into a composite good that enters the CRRA-variety utility function $U(C_t, G_t)$. When effective consumption is postulated as a linear function, the Edgeworth complementarity between C_t and G_t implies that $U(\cdot)$ is decreasing in government expenditures. This violates a standard assumption on the household preferences, and generates more unstable point estimates compared to those under the Cobb-Douglas form of effective consumption. Moreover, estimation results based on the generalized CES formulation of effective consumption show that the Cobb-Douglas specification is more appropriate than the linear alternative. Based on these findings, we adopt the CRRA Cobb-Douglas preference formulation (Eq. (4)) in our analysis.

requirement, together with $\eta \in (0,1)$ and $\phi < 1$, yields a more binding lower bound on the tax-slope parameter $\phi \geqslant 0$. Given these restrictions on η and ϕ, it is straightforward to show that when $\phi > 0$, the marginal tax rate is higher than the average tax rate given by (6). In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the level $1 - \eta$, and the tax schedule is said to be "flat".

We postulate that agents take into account the way in which the tax schedule affects their earnings when they decide how much to consume, invest and work over their lifetimes. Therefore, it is the marginal tax rate of income τ_{mt} that governs the household's economic decisions. The first-order conditions for the representative agent with respect to the indicated variables and the associated transversality condition (TVC) are

$$C_t: \ \theta_1 C_t^{\theta_1(1-\sigma)-1} G_t^{\theta_2(1-\sigma)} = \lambda_t, \tag{8}$$

$$H_t: \ \frac{AH_t^{\gamma}}{\lambda_t} = \underbrace{\eta(1-\phi)\left(\frac{Y^*}{Y_t}\right)^{\phi}}_{(1-\tau_{mt})} w_t, \tag{9}$$

$$K_t: \ -\frac{\dot{\lambda}_t}{\lambda_t} = \underbrace{\eta(1-\phi)\left(\frac{Y^*}{Y_t}\right)^{\phi}}_{(1-\tau_{mt})} r_t - (\rho + \delta), \tag{10}$$

$$TVC: \ \lim_{t\to\infty} e^{-\rho t}\lambda_t K_t = 0, \tag{11}$$

where $\lambda_t > 0$ is the Lagrange multiplier on the budget constraint (5), (9) equates the slope of the household's indifference curve to the after-tax real wage, (10) is the modified consumption Euler equation that takes into account the effect of public expenditures on the marginal utility of private consumption, and (11) is the transversality condition. Notice that under the restrictions on η and ϕ specified above, Eqs. (8)–(10) are not only necessary, but also sufficient conditions for the unique global maximum of the household's dynamic optimization problem.

2.3. Government

The government sets the tax rate τ_t according to (6), and balances its budget at each point in time. Hence, its instantaneous budget constraint is given by

$$G_t = \tau_t Y_t, \tag{12}$$

where government spending on goods and services G_t in turn contributes to the household's utilities. With the government, the aggregate resource constraint for the economy is

$$C_t + \dot{K}_t + \delta K_t + G_t = Y_t. \tag{13}$$

3. Macroeconomic (in)stability

To facilitate the analysis of our model's local stability properties, we make the following logarithmic transformation of variables: $k_t \equiv \log(K_t)$ and $c_t \equiv \log(C_t)$. It is straightforward to show that our model exhibits a unique interior steady state given by

$$k^* = \frac{\log\left\{\frac{\left[\varDelta(1-\eta)^{\theta_2(1-\sigma)}(x_2/x_1)^{\theta_1(1-\sigma)-1}\right]^{1-\alpha}}{x_1^{\alpha+\gamma+\sigma(1-\alpha)}}\right\}}{(1-\alpha)(\gamma+\sigma)}, \tag{14}$$

and

$$c^* = \frac{\log\left\{\frac{\left[\varDelta(1-\eta)^{\theta_2(1-\sigma)}(x_2/x_1)^{\gamma-\theta_2(1-\sigma)}\right]^{1-\alpha}}{x_1^{\alpha(1+\gamma)}}\right\}}{(1-\alpha)(\gamma+\sigma)}, \tag{15}$$

where[8]

$$\varDelta \equiv \frac{\eta\theta_1(1-\alpha)(1-\phi)}{A} > 0, \quad x_1 \equiv \frac{\rho+\delta}{\alpha\eta(1-\phi)} > 0 \quad \textit{and} \quad x_2 \equiv \frac{\rho + [1-\alpha(1-\phi)]\delta}{\alpha(1-\phi)} > 0.$$

The remaining endogenous variables at the economy's steady state can then be derived accordingly.

Next, in the neighborhood of this steady state, our model's equilibrium conditions can be approximated by the following log-linearized dynamical system:

[8] $[1-\alpha(1-\phi)] > 0$ is ensured by the lower bound of $\phi \geqslant 0$ together with $0 < \alpha < 1$.

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} k_t - k^* \\ c_t - c^* \end{bmatrix}, \quad k_0 \text{ given}, \tag{16}$$

where

$$\mathbf{J}_{11} = \eta\left[\frac{\alpha(1+\gamma)(1-\phi)}{\Psi} - 1\right]x_1 + x_2,$$

$$\mathbf{J}_{12} = \frac{\eta(\alpha-1)(1-\phi)[1-\theta_1(1-\sigma)]}{\Psi}x_1 - x_2,$$

$$\mathbf{J}_{21} = \frac{1}{\Pi}\left\{\alpha\eta(1-\phi)\left[\frac{\alpha(1+\gamma)(1-\phi)}{\Psi} - 1\right]x_1 + \frac{\alpha\theta_2(1+\gamma)(1-\sigma)[1-\eta(1-\phi)]}{(1-\eta)\Psi}\mathbf{J}_{11}\right\},$$

$$\mathbf{J}_{22} = \frac{1}{\Pi}\left\{\frac{\alpha\eta(\alpha-1)(1-\phi)^2[1-\theta_1(1-\sigma)]}{\Psi}x_1 + \frac{\alpha\theta_2(1+\gamma)(1-\sigma)[1-\eta(1-\phi)]}{(1-\eta)\Psi}\mathbf{J}_{12}\right\},$$

together with

$$\Psi = \frac{\theta_2(1-\alpha)(\sigma-1)[1-\eta(1-\phi)]}{1-\eta} + 1 + \gamma - (1-\alpha)(1-\phi), \tag{17}$$

and

$$\Pi = \frac{[1-\theta_1(1-\sigma)][1+\gamma-(1-\alpha)(1-\phi)]}{\Psi}. \tag{18}$$

It follows that the determinant and trace of the model's Jacobian matrix **J** are

$$Det = \frac{\alpha\eta(1-\phi)\Omega}{[1-\theta_1(1-\sigma)][(1-\alpha)(1-\phi)-(1+\gamma)]}x_1x_2, \tag{19}$$

where

$$\Omega = \sigma(1-\alpha)(1-\phi) + \gamma[1-\alpha(1-\phi)] + \phi\left[1 - \frac{\theta_2(1-\alpha)(1-\sigma)}{1-\eta}\right]; \tag{20}$$

and

$$Tr = \rho + \frac{\theta_2(1-\sigma)(1+\gamma)[1-\eta(1-\phi)]\{\rho + [1-\alpha(1-\phi)]\delta\}}{(1-\eta)(1-\phi)[1-\theta_1(1-\sigma)][(1-\alpha)(1-\phi)-(1+\gamma)]}. \tag{21}$$

The model's local stability property is determined by comparing the eigenvalues of **J** that have negative real parts with the number of initial conditions in the dynamical system (16), which is one because c_t is a non-predetermined jump variable. As a result, the economy displays saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of **J** are of opposite signs ($Det < 0$). If both eigenvalues have negative real parts ($Det > 0$ and $Tr < 0$), then the steady state is a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots. When both eigenvalues have positive real parts ($Det > 0$ and $Tr > 0$), the steady state becomes a completely unstable source whereby any trajectory that diverges away from it may settle down to a limit cycle or to some more complicated attracting sets.

3.1. When $0 < \sigma < 1$ and $0 < \phi < 1$

In this case, C_t and G_t enter the household utility (4) as Edgeworth complements, and the tax schedule (6) is progressive. Since $0 < \alpha, \phi < 1$ and $\gamma \geqslant 0$, the term $[(1-\alpha)(1-\phi)-(1+\gamma)]$ in the denominator of (19) is negative. This finding, together with $0 < \eta, \theta_1, \sigma < 1$ and $x_1, x_2 > 0$, implies that the model's Jacobian matrix **J** possesses a positive determinant when Ω given by (20) is negative, *i.e.*

$$\theta_2 > \theta_2^{Det} \equiv \frac{(1-\eta)\{\phi + \sigma(1-\alpha)(1-\phi) + \gamma[1-\alpha(1-\phi)]\}}{\phi(1-\alpha)(1-\sigma)}, \tag{22}$$

where $\theta_2 \in (0,1)$ and θ_2^{Det} denotes the level of government-spending preference externality at which $\Omega = Det = 0$.

Proposition 1. *Under (i) utility complementarity between private and public consumption expenditures and (ii) progressive income taxation, the necessary and sufficient condition for our model economy to exhibit equilibrium indeterminacy and belief-driven business cycles is given by (22).*

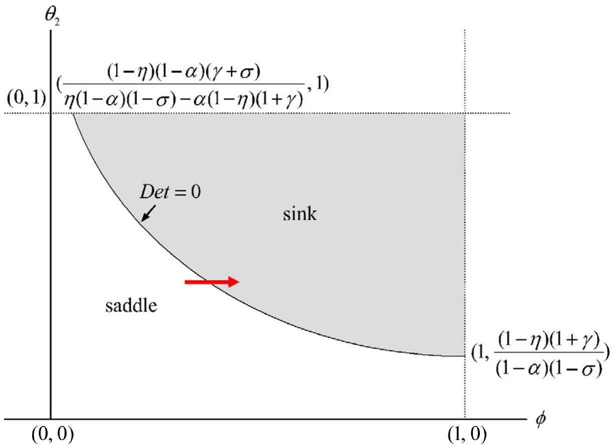

Fig. 1. When $0 < \sigma < 1$ and $0 < \phi < 1$.

Proof. See Appendix A. for the proof that the inequality reported in (22) not only leads to a positive determinant, but also guarantees a negative trace, indicating the presence of two eigenvalues with negative real parts ($Det > 0$ and $Tr < 0$). □

The intuition for the above indeterminacy result can be understood as follows. Start the economy from its steady state, and consider a slight deviation caused by agents' optimistic anticipation about an expansion of future economic activities. Acting upon this belief, households will consume less and invest more today, which in turn lead to increases in future aggregate output (because of higher levels of capital and labor inputs in production), private consumption and income tax rate in that the fiscal policy rule is progressive. Through the government's balanced-budget constraint (12), the level of public spending also rises, which will then generate a further increase in future private consumption since C_t and G_t are Edgeworth complements in the household's utility function. For this alternative path to be justified as a self-fulfilling equilibrium, the after-tax return on investment $(1 - \tau_{mt})r_t$ must be monotonically increasing with respect to higher private consumption expenditures. Using Eqs. (1), (6), (7), (8), (9) and (12), it can be shown that the aforementioned requisite condition is satisfied as long as $\Omega < 0$ or $Det > 0$, *i.e.* the government-spending preference externality θ_2 is sufficiently strong to satisfy the inequality as in (22). Consequently, agent's initial rosy expectations about the economy's future are validated in equilibrium. If the degree of preference externality from public consumption is not high enough to meet condition (22), and thus $\Omega > 0$ or $Det < 0$, our model's steady state will be a locally determinate saddle point.

Fig. 1 depicts the combinations of ϕ (the tax progressivity) and θ_2 (the positive preference externality from government spending) that graphically characterize our model's local stability properties under $0 < \sigma,\ \phi < 1$. To examine the empirical plausibility for the associated "saddle" and "sink" regions, the capital share of national income, α, is chosen to be 0.3; the level parameter of the tax schedule, η, is set equal to 0.8 based on the mean value of Chen and Guo's (2013a) year-by-year point estimates; and the degree of utility complementarity between private and public consumptions, σ, is fixed at 0.3308, which is the lower bound that Ni (1995, Table 3, p. 603) reports from his generalized method of moments (GMM) estimation of a CRRA Cobb-Douglas preference formulation on "effective consumption" as in (4). Table 1 presents values of the θ_2-intercept (when $\phi = 1$) and the ϕ-intercept (when $\theta_2 = 1$) under several calibrations on the household's labor supply elasticity $\left(=\frac{1}{\gamma}\right)$ that have been adopted in the RBC-based indeterminacy literature: (i) $\gamma = 0$ (*i.e.* indivisible labor) á *la* Benhabib and Farmer (1994) and Farmer and Guo (1994); (ii) $\gamma = 0.25$ á *la* Guo and Harrison (2001); and (iii) $\gamma = 0.4545$ á *la* Harrison and Weder (2013).

For each parametric configuration under consideration, the downward-sloping and convex curve $Det = 0$ divides Fig. 1 into well-defined regions of "saddle" and "sink" with both intercepts $\in (0, 1)$.[9] Table 1 also implies that the area in the $\theta_2 - \phi$ space of Fig. 1 that exhibits macroeconomic instability will expand as the household's labor supply becomes more elastic (or when γ falls). The intuition for this finding is the same as in many previous indeterminacy studies within no-government RBC models. With more elastic labor supply, agents are more willing to move out leisure into hours worked, which in turn helps fulfill their initial optimism about the economy's future.

[9] Given the calibrated α, η and σ mentioned above, we find that the highest possible value of γ that ensures both the vertical and horizontal intercepts for the dividing locus $Det = 0$ to lie between zero and one is 1.3422, which results in a labor supply elasticity of 0.745. However, recent empirical work of Chetty et al. (2011, 2012) point out that modern macroeconomic calibrations often imply a larger labor supply elasticity than that supported by the micro-level evidence, and recommend an aggregate Frisch elasticity of 0.5 on the intensive margin for varying hours worked. Given this parameterization with $\gamma = 2$, the "sink" region in Fig. 1 will become empty. Appendix B. addresses this concern by showing that a slightly-modified version of our model economy, which allows for an empirically plausible level of positive productive externalities, continues to exhibit equilibrium indeterminacy when $\gamma > 1.3422$.

Table 1
Empirical plausibility of "Saddle" and "Sink" regions.

	θ_2-Intercept $\left(=\frac{(1-\eta)(1+\gamma)}{(1-\alpha)(1-\sigma)}\right)$	ϕ-Intercept $\left(=\frac{(1-\eta)(1-\alpha)(\gamma+\sigma)}{\eta(1-\alpha)(1-\sigma)-\alpha(1-\eta)(1+\gamma)}\right)$
$\gamma = 0$	0.4269	0.1471
$\gamma = 0.25$	0.5337	0.2713
$\gamma = 0.4545$	0.6210	0.3824

In sharp contrast to earlier research with wasteful government purchases of goods and services (*e.g.* Schmitt-Grohé and Uribe (1997), Guo and Lansing (1998), and Christiano and Harrison (1999)), the arrow in Fig. 1 illustrates that when $\frac{(1-\eta)(1+\gamma)}{(1-\alpha)(1-\sigma)} < \theta_2 < 1$, raising the tax progressivity ϕ *ceteris paribus* will eventually transform the steady state from a saddle point into a sink. It follows that unlike traditional Keynesian-type stabilization policies, a more progressive tax schedule may operate as an "automatic destabilizer" in our model economy by generating endogenous business cycle fluctuations, provided the level of public-spending preference externality is sufficiently high. As it turns out, this result also holds true in a one-sector RBC model with productive government spending á *la* Chen and Guo (2013a); or in a one-sector representative-agent model of endogenous growth with productive flow of public expenditures á *la* Chen and Guo (2013b).

3.2. When $\sigma > 1$ and $0 < \phi < 1$

In this case, C_t and G_t enter the household utility (4) as Edgeworth substitutes, and the tax schedule (6) is progressive. It is straightforward to show that the eigenvalues of the Jacobian matrix $\mathbf{J}$ display opposite signs ($Det < 0$), indicating the presence of saddle-path stability and equilibrium uniqueness. Therefore, when agents become optimistic and decide to raise their investment spending today, the mechanism described in the preceding subsection that makes for multiple equilibria, *i.e.* an increase in the equilibrium after-tax marginal product of capital, will not be realized in that higher public expenditures now lower the marginal utility of private consumption. This implies that given the initial capital stock k_0, the period-0 level of the household's private consumption c_0 is uniquely determined to place the model economy on the convergent path toward its steady state (k^*, c^*), and always stays there without any possibility of deviating transitional dynamics. As a result, equilibrium indeterminacy and belief-driven cyclical fluctuations can never occur in this setting, regardless of the strength of public-consumption preference externality.

3.3. Special cases

Our analysis also allows for a rich set of theoretical possibilities regarding the macroeconomic (in)stability effects of progressive or flat income taxation within a one-sector representative agent model and helps bring together some recent findings in the RBC-based indeterminacy literature. First, we recover the result of Guo and Harrison (2008, section 3.2.2), under utility-generating government spending ($\theta_2 > 0$) that is complementary to private consumption ($0 < \sigma < 1$), indivisible labor ($\gamma = 0$) and a flat tax schedule ($\phi = 0$). It is straightforward to show that within this specification, the model's Jacobian matrix has a positive determinant when

$$(\theta_1 + \theta_2)(1 - \sigma) > 1, \tag{23}$$

which turns out to be the necessary and sufficient condition for the Guo–Harrison economy to possess an indeterminate steady state.[10]

Next, it can be shown that our model exhibits saddle-path stability and equilibrium uniqueness either when there is no public-spending preference externality $\theta_2 = 0$, no matter whether the tax progressivity is positive or zero (as in the horizontal axis of Fig. 1); or when the fiscal policy rule is flat $\phi = 0$, regardless of private and public consumption expenditures being Edgeworth complements (as in the vertical axis of Fig. 1) or substitutes in the household utility. In both cases, the after-tax marginal product of capital will not rise in response to agents' belief-driven investment spurts, thus preventing their optimistic expectations from becoming self-fulfilling. It follows that as in a prototypical one-sector RBC model under laissez-faire, the economy does not display endogenous business cycles caused by changes in agents' animal spirits.

Finally, when $\theta_2 = \phi = 0$, our model collapses to one with useless government purchases and a constant income tax rate, as in Guo and Harrison (2004). In this formulation, it is straightforward to show that its Jacobian's determinant is negative, thus the eigenvalues of the log-linearized dynamical system (16) are of opposite signs and local determinacy always prevails.

[10] Since $0 < \sigma < 1$ under utility complementarity between private and public consumptions, satisfying condition (23) requires that $\theta_1 + \theta_2 > 1$, which is not considered in the current paper. To our knowledge, there is no available empirical evidence that is based on $\theta_1 + \theta_2 \neq 1$.

4. Conclusion

This paper has explored the theoretical interrelations between a progressive tax schedule and equilibrium (in)determinacy in an otherwise standard one-sector real business cycle model with balanced budget and utility-generating government purchases of goods and services. Under utility complementarity between private and public consumption expenditures together with progressive income taxation, we analytically show that the economy possesses an indeterminate steady state if and only if the degree of government-spending preference externality is higher than a critical value. In contrast to a conventional automatic stabilizer, raising tax progressivity may destabilize this formulation of our model by generating endogenous belief-driven cyclical fluctuations. We also find that the economy always exhibits saddle-path stability and equilibrium uniqueness under utility substitutability between private and public consumptions together with progressive income taxation. Finally, the same stability/uniqueness result continues to hold when there is no preference externality from government purchases, regardless of the level of tax progressivity under consideration; or when the fiscal policy rule is flat, no matter whether private and public consumption expenditures are Edgeworth complements or substitutes in the household's utility function.

This paper can be extended in several directions. For example, it would be worthwhile to examine our model economy with national debt (*i.e.* non-balanced budget) *a la* Schmitt-Grohé and Uribe (1997, p. 990), or sustained endogenous growth *a la* Fernández et al. (2004), or consumption taxation *a la* Nourry et al. (2013). In addition, we can incorporate features that are commonly considered in the New-Keynesian literature, such as nominal price/wage rigidities and investment adjustment costs, among others. These possible extensions will allow us to study the robustness of this paper's theoretical results and policy implications, as well as further enhance our understanding of the dynamic (in)stability effects of progressive income taxation in representative-agent models with utility-generating government spending. We plan to pursue these research projects in the future.

Acknowledgements

We thank an anonymous referee, Been-Lon Chen, Hung-Ju Chen, Sharon Harrison, Chia-Hui Lu, Victor Ortego-Marti, Jia Pan, Richard Suen, Yi-Chan Tsai, Cheng Wang, Ping Wang, Chong-Kee Yip, Yan Zhang, and seminar participants at Chinese Economic Association in North America at the ASSA Meetings, National Taiwan University, Asian Meeting of the Econometric Society, Fudan University, and Shanghai University of Finance and Economics for helpful comments and suggestions. Part of this research was conducted while Guo was a visiting research fellow of economics at Academia Sinica, Taipei, Taiwan, whose hospitality is greatly appreciated. Of course, all remaining errors are our own.

Appendix A

Proof of Proposition. Using Eqs. (19) and (22), it is straightforward to show that (i) $\frac{\partial(\theta_2^{Det})}{\partial\phi} < 0$ and $\frac{\partial^2(\theta_2^{Det})}{\partial\phi^2} > 0$, therefore the locus of $Det = 0$ is negatively sloped and convex to the origin in Fig. 1 and (ii) $\frac{\partial(Det)}{\partial\theta_2} > 0$ thus the area above (below) the downward-sloping curve $Det = 0$ exhibits a positive (negative) determinant. Next, we find that the level of government-spending preference externality, denoted as $\theta_2^{Tr} \in (0,1)$, at which the Jacobian's trace (21) equals zero is given by

$$\theta_2^{Tr} = \frac{\sigma\rho(1-\eta)(1-\phi)[1+\gamma-(1-\alpha)(1-\phi)]}{\mu_1(1-\sigma)}, \tag{A.1}$$

and that

$$\frac{\partial(\theta_2^{Tr})}{\partial\phi} = \frac{\theta_2^{Tr}(1+\gamma)(\delta\mu_2+\rho\mu_3)}{\mu_1(1-\phi)[1+\gamma-(1-\alpha)(1-\phi)]} \gtrless 0, \tag{A.2}$$

where

$$\mu_1 \equiv \delta(1+\gamma)[1-\alpha(1-\phi)][1-\eta(1-\phi)] + \rho\left[\phi(1+\gamma)+(1-\alpha)(1-\eta)(1-\phi)^2\right] > 0,$$

$$\mu_2 \equiv (1-\alpha)(1-\phi)[2-(\alpha+\eta)(1-\phi)] - (1+\gamma)\left[1-\alpha\eta(1-\phi)^2\right] \gtrless 0,$$

and

$$\mu_3 \equiv (1-\alpha)(1-\phi)[2-\eta(1-\phi)] - (1+\gamma) \gtrless 0.$$

Table 2
Indeterminacy under Increasing Returns.

γ	1.4	1.6	1.8	2	2.2
$\chi_{\min}$	0.0245	0.1101	0.1955	0.2808	0.3662

It follows that the exact shape and curvature for the locus of $Tr = 0$ cannot be analytically determined. However, using Eq. (21), it is straightforward to show that $\frac{\partial(Tr)}{\partial\theta_2} < 0$, hence points above (below) the nonlinear curve $Tr = 0$ possess a negative (positive) trace.

Finally, we prove that $\theta_2^{Det} > \theta_2^{Tr}$ under this parameterization with $0 < \sigma, \phi < 1$.[11] As a result, the locus of $Tr = 0$ (regardless of its shape and curvature) will lie *entirely* below the downward-sloping and convex curve $Det = 0$ depicted in Fig. 1. This implies that the region of $Det > 0$ is completely subsumed by that with $Tr < 0$. Therefore, condition (22) not only leads to a positive determinant, but also guarantees a negative trace, thus the steady state is a (locally indeterminate) sink. □

Appendix B

We incorporate positive productive externalities into the model economy described in Section 2. Specifically, the representative firm's production function now becomes

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} \left(\overline{K}_t^{\alpha} \overline{H}_t^{1-\alpha}\right)^{\chi}, \quad 0 < \alpha < 1 \text{ and } \quad \chi > 0, \tag{A.3}$$

where $\overline{K}_t$ and $\overline{H}_t$ denote the economy-wide average levels of capital and labor inputs that are taken as given by each individual firm, and χ measures the degree of productive externalities. In a symmetric equilibrium, all firms make the same decisions such that $K_t = \overline{K}_t$ and $H_t = \overline{H}_t$, for all t. Substituting this equilibrium condition into (A.3) yields the following social technology that displays aggregate increasing returns-to-scale:

$$Y_t = K_t^{\alpha(1+\chi)} H_t^{(1-\alpha)(1+\chi)}, \tag{A.4}$$

where $\chi < \frac{1}{\alpha} - 1$ to rule out the possibility of sustained economic growth. We then find that the determinant and trace of this modified model's Jacobian matrix are

$$Det = \frac{\alpha\eta(1-\phi)\Lambda}{[1-\theta_1(1-\sigma)][(1-\alpha)(1+\chi)(1-\phi)-(1+\gamma)]} x_1 x_2, \tag{A.5}$$

where x_1 and x_2 are given below Eq. (15), together with

$$\Lambda = (1-\phi)[\sigma(1-\alpha)(1+\chi)-\chi] + \gamma[1-\alpha(1+\chi)(1-\phi)] + \phi\left[1 - \frac{\theta_2(1-\alpha)(1+\chi)(1-\sigma)}{1-\eta}\right]; \tag{A.6}$$

and

$$Tr = \rho + \frac{\theta_2(1-\sigma)(1+\gamma)(1+\chi)[1-\eta(1-\phi)]\{\rho + [1-\alpha(1-\phi)]\delta\}}{(1-\eta)(1-\phi)[1-\theta_1(1-\sigma)][(1-\alpha)(1+\chi)(1-\phi)-(1+\gamma)]} - \frac{\chi(1+\gamma)(\rho+\delta)}{(1-\alpha)(1+\chi)(1-\phi)-(1+\gamma)}. \tag{A.7}$$

In contrast to Section 3.1 that examines the benchmark specification with constant returns-to-scale in production ($\chi = 0$), we can no longer analytically derive the necessary and sufficient condition under which our modified model exhibits equilibrium indeterminacy for *all* feasible degrees of positive productive externalities. As a result, numerical examples are used to quantitatively explore the economy's local stability properties under aggregate increasing returns-to-scale. Based on the discussion in footnote 8, our baseline formulation does not display belief-driven cyclical fluctuations when the labor supply elasticity parameter γ is higher than 1.3422. Using the same calibrations of α, η and σ specified earlier, together with $\rho = 0.01$ and $\delta = 0.025$ that affect the Jacobian's trace (A.7), Table 2 presents the quantitative interrelations between some selected values of $\gamma \geqslant 1.4$ (including $\gamma = 2$ that Chetty et al. (2011, 2012) have recommended) versus the minimum level of productive externalities (denoted as $\chi_{\min}$) above which the modified model possesses an indeterminate steady state.

[11] In particular, the difference between θ_2^{Det} and θ_2^{Tr} is given by

$$\theta_2^{Det} - \theta_2^{Tr} = \frac{(1-\eta)(\rho\Delta_1 + \delta\Delta_2)}{\Delta_3},$$

where

$$\Delta_1 \equiv \sigma(1-\alpha)^2(1-\phi)^2[1-\eta(1-\phi)] + \left[\phi(1+\gamma) + (1-\alpha)(1-\eta)(1-\phi)^2\right]\{\phi + \gamma[1-\alpha(1-\phi)]\} > 0,$$
$$\Delta_2 \equiv (1+\gamma)[1-\alpha(1-\phi)][1-\eta(1-\phi)]\{\phi + \gamma[1-\alpha(1-\phi)] + \sigma(1-\alpha)(1-\phi)\} > 0, \text{ and}$$
$$\Delta_3 \equiv \phi(1-\alpha)(1-\sigma)\left\{\delta(1+\gamma)[1-\eta(1-\phi)][1-\alpha(1-\phi)] + \rho\left[\phi(1+\gamma) + (1-\alpha)(1-\eta)(1-\phi)^2\right]\right\} > 0.$$

Since $\eta, \delta \in (0,1)$ and $\rho > 0$, θ_2^{Det} is always higher than θ_2^{Tr}.

Next, we note that the largest estimate on the level of aggregate returns-to-scale in U.S. private business economy, after correcting reallocation of productive inputs across industries, obtained by Basu and Fernald (1997, Table 3, p. 268), is 1.03 (standard error = 0.18). It follows that for each value of γ considered in Table 2, the resulting returns-to-scale of the social technology $(= 1 + \chi)$ is empirically plausible *vis-à-vis*the upper bound of the 95% confidence interval associated with Basu and Fernald's point estimate. Table 2 also shows that keeping other parameter values the same, $\chi_{\min}$ and γ are positively related $\left(\frac{\partial \chi_{\min}}{\partial \gamma} > 0\right)$. As in the no-government environment of Benhabib and Farmer (1994) and Farmer and Guo (1994), the requisite degree of productive externalities that fulfills agents' anticipation of an expansion in future output *certis paribus* will increase under a less elastic labor supply (or when γ rises). In sum, this Appendix shows that equilibrium indeterminacy and endogenous business cycles may take place under empirically-relevant parameterizations in our model economy with positive productive externalities.

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