Essays in Energy and Innovation

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy

in

Economics

by

Oleksiy Viktorovych Mnyshenko

Committee in charge:

Professor Graham Elliott, Chair
Professor Roger Bohn
Professor Richard Carson
Professor Mark Jacobsen
Professor Shirley Meng
Professor Hyoduk Shin

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This Dissertation of Oleksiy Viktorovych Mnyshenko is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2015
DEDICATION

To my lovely wife Gabriella, and my parents Olena and Viktor.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signature Page</td>
<td>iii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>v</td>
</tr>
<tr>
<td>List of Figures</td>
<td>viii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>xiii</td>
</tr>
<tr>
<td>Vita</td>
<td>xiv</td>
</tr>
<tr>
<td>Abstract of the Dissertation</td>
<td>xv</td>
</tr>
<tr>
<td>1 Product Innovation Investments and Supply Chain Contract Leadership for Broader Market Coverage</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Model</td>
<td>6</td>
</tr>
<tr>
<td>1.3.1 Decision Timing</td>
<td>10</td>
</tr>
<tr>
<td>1.3.2 Solution Concept: Equilibrium prices, quantities, and their dependence on stock of product quality</td>
<td>10</td>
</tr>
<tr>
<td>1.3.3 Equilibrium Product Quality Expansion</td>
<td>13</td>
</tr>
<tr>
<td>1.4 Contract Leadership Identity and Handovers</td>
<td>17</td>
</tr>
<tr>
<td>1.4.1 Normative Contract Leadership Handovers</td>
<td>17</td>
</tr>
<tr>
<td>1.4.2 Tierwise Rational Handovers</td>
<td>21</td>
</tr>
<tr>
<td>1.5 Product Lifecycle and Contract Leadership Transfers for Broader Market Coverage</td>
<td>24</td>
</tr>
<tr>
<td>1.6 Full Dynamics Extensions</td>
<td>29</td>
</tr>
<tr>
<td>1.7 Conclusion and future research</td>
<td>30</td>
</tr>
<tr>
<td>Appendix for Chapter 1</td>
<td>33</td>
</tr>
<tr>
<td>References for Chapter 1</td>
<td>52</td>
</tr>
<tr>
<td>2 Predictive Accuracy Comparison of Electricity Price Forecasting Methods for Energy Storage Valuation and Dispatch</td>
<td>55</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>55</td>
</tr>
<tr>
<td>2.2 Loss Function</td>
<td>57</td>
</tr>
<tr>
<td>2.3 Data and Approach</td>
<td>59</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Price and quantity choice under Tier 0 contract leadership</td>
<td>11</td>
</tr>
<tr>
<td>1.2</td>
<td>Stock versus flow of product quality</td>
<td>14</td>
</tr>
<tr>
<td>1.3</td>
<td>Maximal investor regions under Proposition 1</td>
<td>16</td>
</tr>
<tr>
<td>1.4</td>
<td>Misalignment penalty</td>
<td>19</td>
</tr>
<tr>
<td>1.5</td>
<td>Threshold $\gamma$ values for optimal contract leadership transfers</td>
<td>20</td>
</tr>
<tr>
<td>1.6</td>
<td>Voluntary leadership handover from Tier 0 to Tier 1</td>
<td>22</td>
</tr>
<tr>
<td>1.7</td>
<td>Components of profit expressions</td>
<td>23</td>
</tr>
<tr>
<td>1.8</td>
<td>Transfers reducing the leadership handover gap</td>
<td>24</td>
</tr>
<tr>
<td>1.9</td>
<td>Leadership transfers under decreasing production costs</td>
<td>26</td>
</tr>
<tr>
<td>1.10</td>
<td>Mechanism for market coverage-maximizing downstream leadership handovers</td>
<td>27</td>
</tr>
<tr>
<td>1.11</td>
<td>Three-period full dynamics</td>
<td>30</td>
</tr>
<tr>
<td>1.12</td>
<td>Threshold for the contract leadership transfer between Tier 0 and Tier 1</td>
<td>43</td>
</tr>
<tr>
<td>1.13</td>
<td>Individual Rationality of Tier 0 to Tier 1 handover</td>
<td>44</td>
</tr>
<tr>
<td>1.14</td>
<td>Reference points for Proposition 3 proof</td>
<td>46</td>
</tr>
<tr>
<td>1.15</td>
<td>Two-period full dynamics for Tier 0 to Tier 1 handover</td>
<td>49</td>
</tr>
<tr>
<td>1.16</td>
<td>Joint Investment</td>
<td>50</td>
</tr>
<tr>
<td>2.1</td>
<td>Optimal charge/discharge commitment for a single day energy time-shift</td>
<td>59</td>
</tr>
<tr>
<td>2.2</td>
<td>Segmented by hour RMSE for best performing specifications</td>
<td>67</td>
</tr>
<tr>
<td>2.3</td>
<td>Theoretical versus application-based loss functions</td>
<td>69</td>
</tr>
<tr>
<td>2.4</td>
<td>Global autocorrelation and partial autocorrelation of hourly DAM LMPs</td>
<td>109</td>
</tr>
<tr>
<td>2.5</td>
<td>Global correlation of hourly DAM LMPs with lags of actual and forecasted load</td>
<td>111</td>
</tr>
<tr>
<td>2.6</td>
<td>Global autocorrelation and partial autocorrelation of actual and forecasted load</td>
<td>111</td>
</tr>
<tr>
<td>2.7</td>
<td>Autocorrelation and partial autocorrelation of DAM LMPs segmented by hour</td>
<td>112</td>
</tr>
<tr>
<td>2.8</td>
<td>BIC and AIC values for ARMA lags specifications from Table (2.43)</td>
<td>112</td>
</tr>
<tr>
<td>3.1</td>
<td>Energy Storage Model</td>
<td>120</td>
</tr>
<tr>
<td>3.2</td>
<td>Decomposition of charge efficiency parameter, $(\gamma_c)$</td>
<td>121</td>
</tr>
<tr>
<td>3.3</td>
<td>Dispatched versus Procured Regulation for 7/11/2010</td>
<td>124</td>
</tr>
<tr>
<td>3.4</td>
<td>Decomposition of Regulation Down (RD) dispatch likelihood for 7/11/2010</td>
<td>124</td>
</tr>
<tr>
<td>3.5</td>
<td>Decomposition of Regulation Up (RU) dispatch likelihood for 7/11/2010</td>
<td>125</td>
</tr>
</tbody>
</table>
Figure 3.6: Actual regulation dispatch likelihoods for 7/11/2010 . . . . . . . . . . . . . 125
Figure 3.7: Actual regulation dispatch likelihoods for 7/14/2010 . . . . . . . . . . . . . 125
Figure 3.8: Actual regulation dispatch likelihoods for 7/17/2010 . . . . . . . . . . . . . 126
Figure 3.9: Market timelines . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 128
Figure 3.10: Day-ahead and real-time commitment and information horizons . . . . . . . . 129
Figure 3.11: 38-hour optimization horizon for $10^h$ hour of a trading day . . . . . . . . 130
Figure 3.12: 37-hour optimization window for $j = 11$ . . . . . . . . . . . . . . . . . . . 135
Figure 3.13: 14-hour optimization window for $j = 33$ . . . . . . . . . . . . . . . . . . . 135
Figure 3.14: Forecasts needed to support DART . . . . . . . . . . . . . . . . . . . . . . . . . . . 136
Figure 3.15: One-day energy and regulation dispatch, 5/4/2013 . . . . . . . . . . . . . . . . 142
Figure 3.16: Average Energy and Net Regulation Bids . . . . . . . . . . . . . . . . . . . . . . 144
LIST OF TABLES

Table 1.1: All possible scenarios of contract leadership and investor ........................................ 14
Table 1.2: The pairs \((\gamma, K_2)\) along the product lifecycle ......................................................... 25

Table 2.1: Specification of ESM models for day-ahead LMP forecasts ........................................ 70
Table 2.2: RMSE and MAE forecast performance of ESM models .................................................. 70
Table 2.3: Normalized Total and Average Daily Energy Time-Shift Revenues for ESM LMP forecasts .............................................................................................................. 71
Table 2.4: Pairwise Predictive Accuracy Tests within ESM family of models with respect to AE and SE loss given expanding and rolling estimation windows .......................... 72
Table 2.5: Pairwise Predictive Accuracy Tests within ESM family of models with respect to Energy Time-Shift Revenues given expanding and rolling estimation windows ............. 73
Table 2.6: Specification of ARMAX models for day-ahead LMP forecasts .................................... 74
Table 2.7: RMSE and MAE forecast Performance of ARMAX models .......................................... 75
Table 2.8: Normalized Total and Average Daily Energy Time-Shift Revenues for ARMAX LMP forecasts .............................................................................................................. 76
Table 2.9: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given expanding estimation window ............................................ 77
Table 2.10: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given expanding estimation window ........................... 78
Table 2.11: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 12-month rolling estimation window .................................... 79
Table 2.12: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 12-month rolling estimation window ............... 80
Table 2.13: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 9-month rolling estimation window ........................................ 81
Table 2.14: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 9-month rolling estimation window ....................... 82
Table 2.15: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 6-month rolling estimation window ........................................ 83
Table 2.16: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 6-month rolling estimation window ....................... 84
Table 2.17: Specification of Feed-Forward Neural Net (FFNN) Models for day-ahead LMP forecasts 85
Table 2.18: RMSE and MAE forecast performance of FFNN models 86
Table 2.19: Normalized Total and Average Daily Energy Time-Shift Revenues for FFNN LMP forecasts 87
Table 2.20: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given expanding estimation window 88
Table 2.21: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given expanding estimation window 89
Table 2.22: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 12-month rolling estimation window 90
Table 2.23: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 12-month rolling estimation window 91
Table 2.24: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 9-month rolling estimation window 92
Table 2.25: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 9-month rolling estimation window 93
Table 2.26: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 6-month rolling estimation window 94
Table 2.27: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 6-month rolling estimation window 95
Table 2.28: Specification of Locally Weighted ARX (LARX) Models for day-ahead LMP forecasts 96
Table 2.29: RMSE and MAE forecast performance of Locally Weighted ARX (LARX) models 97
Table 2.30: Normalized Total and Average Daily Energy Time-Shift Revenues for LARX LMP forecasts 98
Table 2.31: Pairwise Predictive Accuracy Tests within LARX family of models given expanding estimation window 99
Table 2.32: Pairwise Predictive Accuracy Tests within LARX family of models given 12-month rolling estimation window 100
Table 2.33: Pairwise Predictive Accuracy Tests within LARX family of models given 9-month rolling estimation window 101
Table 2.34: Pairwise Predictive Accuracy Tests within LARX family of models given 6-month rolling estimation window 102
Table 2.35: Specification of Cubic Splines (CS) Models for day-ahead LMP forecasts .......... 103
Table 2.36: RMSE and MAE forecast performance of Cubic Splines (CS) models ............... 103
Table 2.37: Normalized Total and Average Daily Energy Time-Shift Revenues for energy storage with CS day-ahead LMP forecasts and different energy to power ratios ......................... 104
Table 2.38: Pairwise Predictive Accuracy Tests within CS family of models .................. 105
Table 2.39: Pairwise Predictive Accuracy Tests across top performers (in RMSE sense) from each family of methods given expanding estimation window ............................................. 106
Table 2.40: Pairwise Predictive Accuracy Tests across top performers (in Revenue sense) from each family of methods given expanding estimation window ............................................. 107
Table 2.41: Iterative AR lag removal and out-of-sample performance evaluation for armax1 model specification .......................................................... 110
Table 2.42: Segmented-by-hour out-of-sample performance of various specifications of AR with a constant model .......................................................... 113
Table 2.43: Iterative removal of ARMA lags to increasing Bayesian and Akaike information criteria 114

Table 3.1: DAM LMPs Summary .................................................. 122
Table 3.2: RTM LMPs Summary .................................................. 123
Table 3.3: LMP differences across DAM and RTM ............................................. 123
Table 3.4: Energy Time-Shift via day-ahead energy market ............................................. 139
Table 3.5: Energy Time-Shift via DAM and RTM ............................................. 140
Table 3.6: DAM Energy and Regulation Revenues, symmetric regulation dispatch and $\gamma = 1$ ............................................. 145
Table 3.7: DAM Energy and Regulation Revenues, asymmetric regulation dispatch, $\gamma = 1$ ............................................. 146
Table 3.8: DAM Energy and Regulation Revenues with intra-day varying regulation dispatch, $\gamma = 1147$
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2011 Bachelor of Arts with Honors, The University of Texas at Austin

2011 Bachelor of Science in Mathematics with Honors, The University of Texas at Austin

2011-2014 Teaching Assistant, University of California, San Diego

2013 Master of Arts in Economics, University of California, San Diego

2014 Candidate of Philosophy in Economics, University of California, San Diego

2015 Research Assistant, CHARGES (Cycling Hardware to Analyze and Ready Grid-Scale Electricity Storage) ARPA-E funded project, University of California, San Diego team

2015 Doctor of Philosophy in Economics, University of California, San Diego
ABSTRACT OF THE DISSERTATION

Essays in Energy and Innovation

by

Oleksiy Viktorovych Mnyshenko

Doctor of Philosophy in Economics

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Professor Graham Elliott, Chair

This dissertation consists of three chapters. Chapter 1 is my joint work with Professor Vish Krishnan and Professor Hyoduk Shin on Product Innovation Investments and Supply Chain Contract Leadership for Broader Market Coverage. Chapters 2 and 3 are my joint work with Professor Graham Elliott on electricity price forecasts for energy storage valuation and dispatch. Chapter 2 performs a detailed comparison of predictive accuracy of various price forecasting methods using theoretical loss functions as well revenues from energy storage participation in energy time-shift in a day-ahead energy market. Chapter 3 relies on best performing price forecasting methods identified in Chapter 2 to perform valuation of energy storage used for individual and “stacked” applications across different pricing locations within deregulated electricity market operated by California Independent System Operator (CAISO).
Chapter 1

Product Innovation Investments and Supply Chain Contract Leadership for Broader Market Coverage

Abstract

Many new technological and product innovations, including some life-saving ones, traditionally traverse a sequentially downward path of gradually lowering costs and prices, which limits their initial availability and affordability to the lower-end of the market. In this paper, we focus on the question of how to achieve broader market coverage for innovative products. We unearth a new innovation investment degree of freedom in a multi-tiered supply chain that offers firms the ability to expand market coverage. We show that deliberately choosing which firm in a multi-tiered supply chain invests in product quality improvement and acts as a leader by initiating contract offers can have a significant impact on the market coverage of the product. Aligning product innovation investor decision with price-quantity decision-making leads to greater total supply chain profits and market coverage. We go beyond a normative prescription by identifying conditions under which tier-wise rational leadership transfers occur and discuss how to align individually rational schedule of leadership transfers with an optimal schedule that results in higher market coverage. These findings have subtle, but important, implications for firms launching innovative products and aspiring to expand market coverage while continuing to maximize profits. Specifically, to obtain broader market coverage for its innovations, innovating firms in a supply chain should finely tune the level of innovation investment, identity of the investor, and contract leadership. These results offer opportunities for firms to expand market coverage while maximizing profits.
1.1 Introduction

Technological advances, based on Research & Development (R&D), lead to new product and process innovations that elevate the standard of living of societies and individuals over time. However, innovations traditionally traverse a sequential path of gradually lowering costs and prices making them available/affordable to the high-end customers first and to the broader market only over a period of time (Battacharya et al. 2003). Even such important life-saving innovations such as passenger airbags and seat belts tended to be exclusive to luxury models and unavailable/unaffordable to the mass market for over a decade. For example, Mercedes-Benz first introduced airbags in its high-end S-Class models in the early 1980’s, but it took a decade or more for the lower-end of the automotive market to avail of and benefit from the safety features of the airbag technology.

As innovating organizations face intensifying competition, a major opportunity exists to create social surplus as well as profits and market value by expanding the coverage of innovative products. Such an approach of making new higher-performance quality products available and affordable to a broader swath of the market (as opposed to being limited to a narrow segment of high-end customers) constitute a form of inclusive innovation (which can also be achieved by other approaches such as frugal innovation that have become a topic of great interest in the industry). However, such inclusiveness has been difficult to achieve in the past due to a combination of higher costs (of development and production) and a desire by firms to extract the surplus generated by higher-end customers. This is the case for large established vertically-integrated monopolistic firms (with proprietary technology) that pursued development, production, and distribution in-house and whose profit-maximizing solution to introducing new higher-quality products is to cater to a narrow higher-end niche of the market (Moorthy and Png 1992).

Increasingly, however, more firms are focusing on specialized tasks (such as production or distribution) and turning to suppliers and partners outside of the firm for component and sub-system development. This is particularly the case in R&D and new product development, which do not show a clear empirical evidence of scale benefits Cockburn and Henderson (2001). Consequently, many large firms have turned to suppliers for their new product candidates, resulting in the emergence of multi-lateral supply chains that offer the benefits of specialization but may suffer from coordination and agency issues/costs that must be carefully managed. These coordination and agency issues can further restrict market coverage and cause products to become even more exclusive (Villas-Boas 1998). However, new internet-based search and networking technologies have made customers much more aware of and clamor for innovative technological developments; therefore, it is becoming important for manufacturers to be more inclusive in such a democratizing market
We propose and formalize the notion that a deliberate choice of product innovation investments by supply chain partners offers an opportunity to broaden market coverage and be more inclusive. This is particularly the case for complex products (such as next-generation automobiles and aircraft) whose performance quality improvement entails significant and non-linear development and production costs. Increasingly, software and hardware are integrated in cutting-edge products, such as those using wireless sensors and associated software to create smart products. Such products exhibit a cost structure with both fixed development costs and variable production costs that must be covered using investments to reach a certain concrete level of product quality.

One of our study companies that markets electric cars to US consumers offers a good illustration of the underlying issues considered in the paper. Electric cars present a cleaner emission-free alternative to conventional internal combustion engine-based automobiles, but they must achieve product innovation/performance quality improvement to overcome customer range anxiety arising from running out of battery power midway through a journey without access to a charging station. High quality electric cars that have a relatively long range are also quite expensive and are not affordable for broader segments of the market. A key limiting factor is the vehicle’s battery, which is designed and manufactured by an upstream Tier 2 supplier and then assembled into an electrical subsystem by a Tier 1 supplier before being integrated and distributed by our focal Tier 0 firm into a fully-functional electric vehicle. Our study company is acutely focused on maximizing the product’s attractiveness as well as the available market for its products by making it more inclusive. The analysis presented in the paper offers such firms a new degree of freedom that not only entails higher-quality products and profits but also broader market coverage for such innovations. We now begin with a discussion of the literature related to our work.

1.2 Literature Review

There exists a substantial body of literature in the Economics and Management domains on the investments needed for innovation/R&D and the impact they engender in terms of social welfare and industry/firm profits. More than 50 years of Economics literature on R&D investments was initially reviewed by Griliches (1979) and most recently by Hall et al. (2009) – these papers vividly depict the challenges of measuring the returns to R&D and also show that investments are associated with strongly positive firm (private) and social returns. Nevertheless, these papers do not detail who should make these investments in a network of firms to achieve the best social/economic outcomes. The issues regarding the organization of R&D activity and contractual arrangements (such as the allocation of property rights) have also been studied by Economics
researchers starting with the work of Aghion and Tirole (1994). Grossman and Hart (1986) develop a theory of costly contracts to examine benefits of integration/control when it is difficult to write and enforce complete contracts. Hart and Moore (1990) further examine distortion worker and manager incentives under different levels of integration. Stein (2002) examines extent of vertical integration on capital allocation to competing investment projects. Although they provide a good foundation for study of costs and incentives under different ownership structures, these papers do not deal with the issue of achieving broader (inclusive) market coverage for innovative products in a supply chain context.

Recently, product innovation has been a topic of active research interest in the Management Science/Operations Management literature by Krishnan and Ulrich (2001), but most of the literature has tended to be single-firm-centric focusing on project scheduling and management. There is a small stream of work on the interactions between product and supply chain design decisions, (Ulrich and Ellison 2005, Grahovac and Parker 2003). However, this literature focuses only on how a single firm should make decisions involving its suppliers, rather than the interaction between the decisions of firms. Closer to our paper is the work of Bhaskaran and Krishnan (2009) who study the joint-development of products in a bi-lateral supply chain context; however, they focus on contractual arrangements between firms that extend beyond revenue sharing to include the sharing of development cost and work. In contrast, this paper focuses on understanding the linkage between the source of innovation investments and the ensuing market coverage in a trilateral supply chain.

Innovative products exhibiting strong integration of hardware and software components require the careful modeling of both variable and fixed costs. For example, one of our study companies marketing electric vehicles incurs both the development cost of attaining technical capacity to deliver vehicles with a specific battery range as well as the production cost associated with manufacturing and customer support. The effects of development cost in isolation have been studied extensively in the vertical product differentiation literature in Economics - specifically, Shaked and Sutton (1982) and Bonanno (1986) examine environments in which investment in quality is primarily associated with development costs. Alternatively, increased product quality may result in production costs that are convex in product quality as modeled by the classic papers of Mussa and Rosen (1978). We consider a setting where a product’s development and production costs are both significant - modeling the non-linear form of both development and production costs is important and non-trivial. That is, it requires more than combining the two strands of literature and offers additional insights into how investor and leadership positions may have to change as costs decrease during the innovation process.

There exists a related stream of literature on products introduced sequentially to the market while experiencing rapid quality improvements, referred to as rapid sequential innovation (Dhebar 1994, Kornish
2001, Ramachandran and Krishnan 2008). However, this literature is primarily concerned with the purchase timing dilemmas of consumers when products improve in discounted terms, and the steps a monopolistic firm may take through pricing and product design to address such consumer concerns. This stream does not consider the context of a supply chain and does not focus specifically on broadening the market coverage.

Suppliers that invest in component technological innovation often wrestle with the issue of other supply chain participants not making mutually aligned decisions; as a result, they may under-invest in component technologies - downstream firms may sometimes decide to make the investment to foster innovation. Many papers in marketing and supply chain operations have studied the interaction between vertical firms and have proposed mechanisms to deal with price-quantity coordination problems (Jeuland and Shugan 1983, Lee and Staelin 1997, Cvsa and Gilbert 2002). Similar models have been used to analyze the effect of innovation by one of the firms on its channel partners. Gupta and Loulou (1998) study how interactions between firms in a channel affect innovation. Gilbert and Cvsa (2003) analyze the effect of strategic commitment to price by a supplier to stimulate downstream innovation in a supply chain. However, this stream of literature deals primarily with prices and quantities and ignores the decision-making about product quality, which constitutes the core of product innovation. Furthermore, complex contracts do not seem to be widely embraced by the industry due to the cost of implementing them (Arrow 1984). We address the misalignment/coordination problems in a way that keeps contracts simple - similar to the approach taken by Jerath (2007) for aligning marketing and operations efforts within a firm. The optimality of a contract leader position in a supply network has been examined by Majumder and Srinivasan (2008), who generalize the notion of double-marginalization from Spengler (1950) to show that contract leadership affects total supply chain profits. This literature provides us with a convenient solution approach to establish the equilibrium price and quantity resulting from a sequence of wholesale price contracts. However, our paper focuses on the identity of an investor, the choice of product quality, and the resulting profit and market coverage for complex products.

In this paper, we examine deliberate transfers of contract leadership as an additional degree of freedom that allows supply chains to adjust their leadership configuration at different stages of a product’s lifecycle to achieve broader market coverage and higher total supply chain profits. A similar notion of profit improving dynamic adjustments in the individual firm’s strategy and supply chain structure is examined by Druelhl et al. (2009), identifying optimal time-pacing strategies for new product development, and Xiao and Xu (2012), using sequential royalty revisions to realign incentives between the innovator and the marketer of a new product. Furthermore, Rhee et al. (2012) study the patterns of high-end encroachment in which new products gradually become available to a broader share of the market driven by cost reductions and technology improvements.
There is also an emerging stream of literature studying the impact of the supply chain and organization structure on equilibrium outcomes in static environments. Bimpikis et al. (2014) use a supply network perspective to show the adverse effects of multi-sourcing in mitigating the aggregate disruption risk. Similar to the normative prescription of optimal contract leadership offered in our paper, Girotra et al. (2010) identify optimal organizational structure for the generation of new product ideas and Roels et al. (2010) study optimal contract types for delivering collaborative services. Our work, however, is focused on broader coverage for innovative products. We now turn to the discussion of the modeling framework before presenting and discussing the key results.

1.3 Model

We consider a three-tier supply chain that involves the development, production and distribution of an innovative product. In the three-tier supply chain, basic components are supplied from an upstream Tier 2 to an intermediate assembler, Tier 1, followed by final integration, marketing and distribution at downstream Tier 0. For example, in the electric car case mentioned above, the battery supplier represents the Tier 2 firm; the electric subsystem assembler would be a Tier 1 firm, and the manufacturer/OEM would be considered as a Tier 0 firm, which sells products directly to consumers whose total market size is $N$. Our stylized model and analysis methodology can be easily extended to more tiers and supply networks, and key qualitative insights are preserved under supply networks of arbitrary size and complexity in monopolistic settings with deterministic demand.

The quality of the product to consumers is a function of its marketing, assembly and the quality of the components. For instance, in the case of an innovative product such as an electric car, the quality of a product to consumers is a function of its component performance (such as battery range), subsystem (drive train) capability, and the finished product quality and marketing (ease of use, safety, reliability, and design attractiveness). Each of these product features is associated with value added at one or multiple tier(s).

While our model can be extended to the case in which Tiers 1 and 0 qualities also affect the quality of the end product through the use of an appropriate (e.g., multiplicative or additive) quality functions, we aim to represent a simple, yet consistent, model with the motivating case of electric vehicles (EVs) and other industries, especially technology-driven industries, by focusing on Tier 2 quality; for example, one of the key hurdles for adoption of EVs under the current technology is consumer’s range anxiety related to the quality of batteries, which is the quality of components produced at Tier 2. In addition, in the electronics industry, including personal computers and cell phones, one of the key components that determines the end product quality is the performance of central processing units or chips produced by a component supplier, who would
be also a Tier 2 firm; in other words, we focus on the case in which a critical product feature limiting market penetration is associated with the quality of components supplied by an upstream tier. Our approach is similar to that of Altug and van Ryzin (2013), who also consider a problem in which consumer’s willingness to pay is modeled as a function of the supplier’s component quality, and the manufacturer/assembler does not contribute significant additional value in excess of what is derived from components themselves. Furthermore, our insights and results remain valid if a constraining product characteristic is associated with a different tier within the supply chain, as long as there exists a single primary bottleneck technology limiting the product’s market penetration.

On the consumer demand side, we follow the traditional vertical differentiation model of quality evaluation (see, e.g., Mussa and Rosen 1978, Moorthy and Png 1992); specifically, given the product quality $\Theta$, each consumer’s type denoted as $\alpha$ is uniformly distributed on $[0, 1]$, such that when a consumer of type $\alpha$ purchases a product with quality $\Theta$ at price $p$, her net utility is $U = \alpha \Theta^\beta - p$ with $\beta \leq 1$, which captures the saturation or decreasing returns to quality. A consumer’s reservation utility when she purchased none is normalized to zero. Consequently, a product of quality $\Theta$ with price $p$ is purchased by all consumer types with non-negative net utility, $\alpha \geq \underline{\alpha} = \frac{p}{\Theta^\beta}$. Here, $\underline{\alpha}$ corresponds to the marginal consumer who derives zero utility. Thus, depending on the product quality $\Theta$, such a product exhibits market coverage $\rho(\Theta) = 1 - \underline{\alpha}$, and the total market demand becomes $N \cdot \rho(\Theta)$. Our focus on the case in which a key bottleneck technology is at the components manufactured at Tier 2 leads us to the following assumption:

**Assumption 1** The product quality $\Theta$ is primarily associated with the performance of the Tier 2 component and the qualities of Tier 1 (assembly) and Tier 0 (marketing) are fixed.

We further formally define an equilibrium outcome that constitutes inclusive innovation.

**Definition 1** Consider two different cases of market coverage, Case A and Case B. Case A represents more inclusive innovation than Case B if the marginal consumer type for Case A is lower than that for Case B, i.e., the market coverage for Case A is higher than that of Case B.

On the cost side, we consider both the production cost and the development cost of innovation. First, we assume the following form of production costs:

1Market coverage is a natural measure for the inclusiveness of an innovative product. An alternative measure for social efficiency would be the consumer surplus $CS(\Theta)$, which can be written as $CS(\Theta) = \Theta^\beta \int_0^{\Theta} (\alpha - \underline{\alpha}) d\alpha = \frac{1}{2} \Theta^\beta \rho^2(\Theta)$. Unlike the market coverage, $CS(\Theta)$ captures the sum of the utilities of covered consumer types $\alpha \in [\underline{\alpha}, 1]$ in excess of the marginal type’s utility $\Theta^\beta \underline{\alpha}$. Given the product development investment (or, equivalently, $\Theta$), more inclusive innovation leads to a larger consumer surplus. Furthermore, since our focus is on inclusive innovation, market coverage directly captures the inclusiveness and, hence, is a more relevant metric for our purposes.
**Assumption 2** For Tier 2 being a critical determinant of product quality, the production cost of delivering \( q_2 \) units of components with quality \( \Theta \) is \( C_2(q_2, \Theta) = K_2 \Theta^{\delta_2} q_2^{2} \) with \( \delta_2 > 1 \). For Tiers 0 and 1, the production costs are \( C_0(q_0) = K_0 q_0^{1} \) and \( C_1(q_1) = K_1 q_1^{1} \), respectively, for producing \( q_i \) units for \( i = 0, 1 \).

For Tier 2 that is associated with the key bottleneck technology, its production cost is increasing and convex in quality \( \Theta \) as similarly modeled by Mussa and Rosen (1978) and Gal-Or (1983). Convexity in the quality parameter \( (\delta_2 > 1) \) captures the diseconomies associated with the production of an increasingly higher quality product resulting from the use of more expensive raw materials or skilled labor. Moreover, consistent with the Operations/Supply Chain literature we also focus on the production costs that are quadratic in quantity to incorporate capacity/resource constraints. We avoid a general case of production costs convex in quantity for a cleaner exposition and ability to highlight convexity in quality instead. We do not consider capacity investment in our model. As mentioned in Tunca and Wu (2009), the convexity of production costs in quantity reflects the increasing marginal cost of production as the producer uses the cheapest options first and move to more expensive ones later. Furthermore, this cost structure is common because it arises from convex in quantity production technology sets as demonstrated in Mas-Colell et al. (1995). The variable production cost will be concave in quantities if there is economies of scale in production. However, our model is not able to accommodate concavity in quantity and we acknowledge this as one of the limitations.

Next, for the development costs, a firm in the supply chain may invests in development and innovation to increase the existing stock of product quality \( \Theta \) to a new level \( \hat{\Theta} \) such that \( \hat{\Theta} \geq \Theta \). We assume the following form of the development cost:

**Assumption 3** The development cost to increase the existing stock of product quality from \( \Theta \) to \( \hat{\Theta} \) is of the form \( D(\Theta, \hat{\Theta}) = \gamma(\hat{\Theta}^{\delta_D} - \Theta^{\delta_D}) \) with \( \delta_D > 1 \).

Throughout this paper, we consider product development and innovation investment as effort/investment to increase the existing stock of product quality. In our development cost model, the marginal cost of expanding stock of product quality \( \Theta \) to a new level \( \hat{\Theta} \) is increasing both in the initial quality level \( \Theta \) and incremental improvement \( (\hat{\Theta} - \Theta) \), i.e. \( \delta_D > 1 \), which is consistent with the literature (e.g., Jones and Mendelson 2011). The development cost can be also considered to be direct investment into a Tier 2 supplier to improve the corresponding component quality. Recognize that investor’s problem can be equivalently formulated with respect to the monetary magnitude of investment, but for the purposes of cleaner exposition of results we deliberately let \( \hat{\Theta} \) be investor’s choice variable.

One of our research questions is who, or which tier firm, should invest in product development and innovation, i.e., in the expansion of the existing stock quality \( \Theta \). In order to examine this normative question,
we study three cases in which each Tier $i$ for $i \in \{0, 1, 2\}$ is an exclusive investor in product development within a supply chain to improve the quality of the product. Because we aim to establish the simplest link between the investor and the contract leader without the further complications to the model, we mostly focus on the case of a single dominant investor rather than joint investments by multiple firms, which has been already explored in previous literature including Bhaskaran and Krishnan (2009). Moreover, in practice, investment in new product development involves substantial fixed financing/transaction costs, which may curtail joint investments and lead to a dominant investment by a single firm within a supply chain consistent with our model. However, we do discuss what happens when firms jointly invest in product quality improvement in Appendix to Chapter 1 and illustrate that our key results remain valid in this extension. Further, in Lemma 2 we consider the environment in which development costs are shared while a single tier chooses an optimal stock of product quality level.

To examine how the identity and amount of innovation investment vary with the improvement in the product development and production costs, we adopt the product lifecycle perspective. The product lifecycle is divided into two phases, which is consistent with the empirically demonstrated pattern of innovation established in Utterback and Abernathy (1975) - in which early in the product lifecycle, the industry is characterized by a rapid rate of product change and development cost reduction (performance optimization phase), followed by a second stage of process innovation leading to variable-cost reduction as the product matures. In our context, we translate this as follows: the initial product innovation phase is associated with fast-decreasing development costs (parameter $g$), followed by process innovation characterized by an industry-wide decrease in production costs (captured by a reduction in component production cost parameter $K_2$). Large values of both $g$ and $K_2$ are indicative of the initial stage of product innovation, in which both development and production costs are high. However, the industry-wide focus on the improvement of product quality results in the reduction of development costs, i.e., $g$ is decreasing as the industry progresses through the product innovation phase. When the product enters maturity in the process innovation stage, the decreasing $K_2$ captures an improvement in the process productivity associated with the lowering production cost of the process innovation phase. Note that adoption of product lifecycle perspective only provides a convenient framework for discussion of observed equilibrium outcomes as exogenous parameters $g$ and $K_2$ vary. Our model can accommodate any path of development and production cost parameters.
1.3.1 Decision Timing

Due to amount of time it requires to attain technical ability to produce a good of quality \( \hat{\Theta} \), firms at first undertake strategic decision to use wholesale price contracts to determine price and quantity, once ability to deliver quality \( \hat{\Theta} \) is reached. The selection of contract type is followed by short term tactical choice of equilibrium price and quantity.

At a given instance of the product lifecycle characterized by a pair of parameters \((\gamma, K_2)\) and the available stock of product quality \( \Theta \). The quantity \( q \) and price \( p \) in a supply chain are determined in two stages. In the first stage, a Tier \( i \) investor for \( i \in \{0, 1, 2\} \) determines her product development investment level to expand product quality to a new level \( \hat{\Theta} \). Once stock of product quality has been increased to \( \hat{\Theta} \), in the second stage, firms within a supply chain contract with each other, which then yields the equilibrium price and quantity. We assume that once the stock of product quality is expanded the previous generation of products becomes obsolete and hence supply chain solves for new equilibrium price and quantity. Specifically, for the contracts between tiers in a supply chain, we consider a simple widely-used wholesale pricing, and study who should initiate such a contract, i.e., who should be the contract leader.

We analyze this problem via backward induction, starting with the second stage in which the equilibrium price and quantity are determined given the product quality, followed by innovation/quality choices in the first stage (product development stage); therefore, we proceed with the investigation of equilibrium price and quantity choice in Section 1.3.2 followed by Section 1.3.3 focusing on investment in innovation and the resulting product quality expansion.

1.3.2 Solution Concept: Equilibrium prices, quantities, and their dependence on stock of product quality

Once investor has chosen her contribution to the stock of product quality, firms in a supply chain sequentially contract through a simple wholesale price agreement at stage 2, which determines the equilibrium price and quantity depending on product quality level \( \Theta \). In terms of the sequence of contracts, or who initiates the contracts, there are three cases, i.e., each tier \( l \) for \( l \in \{0, 1, 2\} \) can initiate the contract. We call the supply chain tier that initiates the contract a contract leader. First, consider the case in which Tier 0 is a contract leader, as illustrated in Figure 1.1. In this case of Tier 0 contract leadership, Tier 0 initiates the wholesale price

\(^2\)Price and Quantity can be determined for any stock of quality \( \Theta \). Hence, in this section general \( \Theta \) is used instead of \( \hat{\Theta} \).
contract and offers the wholesale price $\omega_1$ to Tier 1. Subsequently, Tier 1 then offers the wholesale price $\omega_2$ to the Tier 2 supplier. Based on this wholesale price $\omega_2$, the Tier 2 supplier then determines how much to sell to Tier 1 at this wholesale price, i.e., Tier 2 decides on the quantity $q_2$. Considering this maximum quantity $q_2$ bought from Tier 2 and the wholesale price $\omega_1$ offered by Tier 0, Tier 1 now determines how much to sell to Tier 0, i.e., decides $q_1$. Finally, taking the maximum quantity $q_1$ procured from Tier 1, Tier 0 now determines the consumer price $p$ and how much to sell to the consumers, i.e., $q_0$. Based on this consumer price $p$ and the available stock of quality $\Theta$ resulting from product quality expansion, the consumer market demand becomes $N \cdot (1 - p\Theta^{-\beta})$. The sales quantity of Tier 0 ($q_0$) is constrained by both the quantity supplied by Tier 1, $q_1$, and the consumer market demand $N \cdot (1 - p\Theta^{-\beta})$.

Equations (1.3.1) provide tierwise profit expressions, $\Pi_2$, $\Pi_1$, and $\Pi_0$, for Tier 2, Tier 1 and Tier 0, respectively, for an increased stock of product quality $\Theta$ in a supply chain with Tier 0 contract leadership, as explained above. For example, Tier 1 obtains the revenue of $\omega_1q_1$ by selling $q_1$ units at the unit wholesale price $\omega_1$ to Tier 0, and it pays $\omega_2q_2$ to Tier 2 to purchase $q_2$ units of the component at unit price $\omega_2$. Furthermore, Tier 1 incurs an assembly/production cost of $K_1q_1^2$, and its selling quantity $q_1$ to Tier 0 is constrained by $q_2$.

$$
\Pi_2(q_2; \omega_2) = \omega_2q_2 - K_2\Theta q_2^2,
$$

$$
\Pi_1(q_1, \omega_2; \omega_1) = \omega_1q_1 - \omega_2q_2 - K_1q_1^2 \quad \text{s.t. } q_1 \leq q_2,
$$

$$
\Pi_0(p, q_0, \omega_1) = pq_0 - \omega_1q_1 - K_0q_0^2 \quad \text{s.t. } q_0 \leq \min \left\{ q_1, N(1 - p\Theta^{-\beta}) \right\}.
$$

Within the second stage, given the resulting product quality $\Theta$, we analyze the equilibrium quantities and prices backwards, following the marginalization operation presented in Majumder and Srinivasan (2008). In this case, first, Tier 2 receiving a price offer $\omega_2$ optimally responds with $q_2^*(\omega_2)$ by maximizing $\Pi_2$. When Tier 1 offers $\omega_2$ to Tier 2, Tier 1 takes this optimal response $q_2^*(\omega_2)$ into consideration, and Tier 1 offers $\omega_2$ so that $q_2^*(\omega_2) = q_1$. Technically, $q_2^*(\omega_2)$ constitutes an inverse factor demand for Tier 1, and Tier 1 firm replaces $\omega_2$ in its profit function $\Pi_1$ using the inverse function of $q_2^*(\omega_2) = q_1$. In addition to the binding constraint $q_1 = q_2$ under optimality, Tier 1’s profit function can be written as $\Pi_1(q_1; \omega_1) = \omega_1q_1 - C_1(q_1, \Theta)$.
where $\tilde{C}_1(q_1, \theta) = (K_1 + 2K_2\Theta^\delta_k)q_1^2$. Note that Tier 1’s profit $\tilde{\Pi}_1(q_1; \omega_l)$ takes a form similar to Tier 2’s profit with a different modified production cost function. Applying the same procedure to Tier 0, $\Pi_0(p, q_0)$ can be written as

$$\tilde{\Pi}_0(p, q_0) = pq_0 - \tilde{C}_0(q_0, \theta) \quad \text{s.t.} \quad q_0 \leq N(1 - p\Theta^{-\beta}),$$

(1.3.2)

where $\tilde{C}_0(q_0, \theta) = (K_0 + 2K_1 + 4K_2\Theta^\delta_k)q_0^2$. Again, $\tilde{\Pi}_0(p, q_0)$ takes a form similar to $\Pi_2(q_2; \omega_2)$. For Tier 0’s case, it sets the consumer price $p$ in addition to $q_0$. However, under optimality, it sets the consumer price $p$ at the level at which its constraint is binding. To summarize, the tierwise profit expressions in (1.3.1) can be reduced to a single contract leader’s problem in (1.3.2) by iterating the marginalization operation presented in Majumder and Srinivasan (2008). Once the problem has been reduced to a single contract leader’s optimization problem, we maximize (1.3.2) with respect to $p$ and $q_0$. After obtaining the optimal $p^*$ and $q_0^*$, we subsequently arrive at the equilibrium prices and quantities $\omega_l$, $q_1$, $\omega_2$ and $q_2$, depending on the expanded stock of product quality $\Theta$.

In the case of Tier 1 leadership, Tier 1 offers wholesale prices $\omega_l$ to Tier 0 and $\omega_2$ to Tier 2. Tier 0’s profit expression is $\Pi_0(q_0; \omega_l) = pq_0 - \omega_lq_0 - K_0q_0^2$ and hence the resulting inverse factor demand faced by Tier 1 when offering $\omega_l$ to Tier 0 is $\omega_l(q) = p - 2K_0q$. Note that Tier 0’s only choice variable here is $q_0$ and price $p$ is treated as a constant. Further, for Tier 2 profit expression is $\Pi_2(q_2; \omega_2) = q_2\omega_2 - K_2\Theta^\delta_kq_2^2$. Therefore, Tier 1 faces the following inverse factor demand when offering $\omega_2$ to Tier 2, $\omega_2(q) = 2K_2\Theta^\delta_kq$. Tier 1’s profit expression is $\Pi_1(p, q_1, \omega_l, \omega_2) = \omega_lq_0 - \omega_2q_1 - K_1q_1^2$, subject to supplied component quantity constraints $q_0 \leq q_1$, $q_1 \leq q_2$ that optimally bind $q_0 = q_1 = q_2$ and market size constraint $q_0 \leq N(1 - p\Theta^{-\beta})$. Incorporating inverse factor demands faced by Tier 1 yields the following modified profit expression $\tilde{\Pi}_1(p, q) = pq - (2K_0 + 2K_2\Theta^\delta_k + K_1)q^2$. The resulting equilibrium price and quantity are presented in Lemma 1 for $l = 1$. The details of the analysis of the case of Tier 2 contract leadership is presented in the Appendix to Chapter 1.

Lemma 1 establishes equilibrium prices and quantities in a supply chain with Tier $l$ being a contract leader for $l \in \{0, 1, 2\}$.

**Lemma 1** A supply chain led by Tier $l$ in equilibrium delivers $Q_l(\Theta)$ units at price $P_l(\Theta)$, depending on the available stock of product quality $\Theta$ with $P_l(\Theta) = (2 + 2\Theta^{-\beta}\Phi_l(\Theta)N)^{-1}(\Theta^\delta_k + 2\Phi_l(\Theta)N)$ and $Q_l(\Theta) = N(2 + 2\Theta^{-\beta}\Phi_l(\Theta)N)^{-1}$, where $\Phi_l(\Theta) = 2^lK_0 + 2^{1-l}K_1 + 2^{2-l}K_2\Theta^\delta_k$.

From Lemma 1, the product quality and contract leader location impact the equilibrium price and quantity outcomes. The dependence of equilibrium outcomes, $(Q_l(\Theta), P_l(\Theta))$, on leader location is driven by
the misalignment penalty $\Phi_l(\Theta)$ that can be thought of as the severity of double marginalization associated with Tier $l$’s contract leadership. The misalignment penalty $\Phi_l(\Theta)$ is the sum of the contract leader’s direct production cost coefficients, i.e. $K_0$ for $l = 0$, and $K_2\Theta^{0_2}$ for $l = 2$, and the coefficients on the production costs of the other firms in the supply chain, weighted by the distance from the contract leader, i.e. $2K_1 + 4K_2\Theta^{0_2}$ for $l = 0$, and $4K_0 + 2K_1$ for $l = 2$. As a result, as $\Phi_l(\Theta)$ increases, the effective production cost increases, which decreases the equilibrium production quantity $Q_l(\Theta)$. Furthermore, leadership configurations with higher misalignment penalty values exhibit lower market coverage, since the market coverage corresponding to the contract leadership by Tier $l$, $\rho_l(\Theta)$, is a ratio of the equilibrium quantity sold in the market to the total market size, $Q(\Theta)_{N}$.

The notion of quality-driven misalignment is related to the difference in production costs between a single vertically integrated firm and a multi-tiered supply chain incurring additional agency costs. Hence, supply chains, regardless of the leader location $l \in \{0, 1, 2\}$, exhibit higher effective production costs relative to the vertically integrated case. However, some leader locations yield lower misalignment penalties resulting in lower effective production costs. The intrinsic magnitude of production cost coefficients $\{K_0, K_1, K_2\}$ and the product quality $\Theta$ from the development/investment decision determine the leader location with the lowest misalignment penalty. Thus, the supply chain leader located close to tiers with high production costs lowers $\Phi_l(\Theta)$ by reducing the effect of double marginalization captured by $2|j-l|$, where $|j-l|$ is the distance between Tier $j$ and the contract leader $l$. Note that Tier 2’s production cost is increasing in $\Theta$; hence, the contract leader location minimizing $\Phi_l(\Theta)$ gravitates upstream to Tier 2 as the product quality improves. This dependence of contract leader position on product quality has significant implications (as discussed in the subsequent section) and comes out of a model that jointly considers qualities, quantities, and prices.

### 1.3.3 Equilibrium Product Quality Expansion

Next, we consider the investment decisions in product development and innovation to improve the quality. From Lemma 1, there are three different supply chain leadership scenarios, depending on who the contract leader is. For each leadership scenario, there are three distinct possibilities depending on who invests in quality. As illustrated in Table 1.1 in a supply chain comprising three tiers, there are nine scenarios that differ by the identity of the investor and the identity of the contract leader.

**Definition 2** Let $\hat{\Theta}_{jl}$ denote new equilibrium product quality level given the existing stock of quality $\Theta$. 


Table 1.1: All possible scenarios of contract leadership and investor leadership

<table>
<thead>
<tr>
<th>Contract Leader</th>
<th>( l = 0 )</th>
<th>( l = 1 )</th>
<th>( l = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investor ( i = 0 )</td>
<td>( \Theta_0</td>
<td>0 )</td>
<td>( \Theta_0</td>
</tr>
<tr>
<td>( i = 1 )</td>
<td>( \Theta_1</td>
<td>0 )</td>
<td>( \Theta_1</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>( \Theta_2</td>
<td>0 )</td>
<td>( \Theta_2</td>
</tr>
</tbody>
</table>

Salting from Tier 0 being an investor in innovation under Tier 1’s contract leadership, i.e.,

\[
\hat{\Theta}_l = \arg\max_{\Theta \geq 0} \left\{ \Pi_l \left( \hat{P}_l(\hat{\Theta}), \hat{Q}_l(\hat{\Theta}) \right) - \gamma \left( \hat{\Theta}_D^0 - \Theta^D_0 \right) \right\}.
\]

In our model marginal revenue and marginal development cost expressions are independent of initial stock of quality \( \Theta \) which allows us to approach every constrained maximization problem with respect \( \hat{\Theta} \) such that \( \hat{\Theta} \geq \Theta \), as unconstrained problem with respect to \( \hat{\Theta} \), i.e. \( \hat{\Theta} \geq 0 \), and then evaluate whether optimal stock of quality is greater than previous quality level which would result in quality expansion, or weakly lower leading to no investment. For example, in Figure 1.2 if initial stock of quality is \( \Theta^{\text{low}} \) and new optimal quality level is \( \hat{\Theta} \) then we observe expansion by \( \theta = \hat{\Theta} - \Theta^{\text{low}} \). Alternatively, if \( \Theta \leq \Theta^{\text{high}} \) then stock of product quality remains constant.

For example, \( \hat{\Theta}_{i|0} \) represents new expanded product quality when Tier 0 is the contract leader and Tier 1 is the investor in innovation to improve the product quality. By substituting the equilibrium price and quantity from Lemma 1 into (1.3.1) and subtracting the development cost incurred by Tier \( i \) firm, we obtain tierwise profit expressions including the development cost, denoted as \( \hat{\Pi}_i(\hat{\Theta}_{i|0} | \Theta) \) for \( i \in \{0, 1, 2\} \), given in (1.3.3) for the case of Tier 0’s contract leadership, i.e. \( l = 0 \).
Based on this result, we now focus our attention on three investment scenarios along the diagonal in Table 1.3.3: 

$$\Gamma_2(\hat{\Theta}_2) = \max_{\Theta > \Theta} \left\{ \left( K_2 \hat{\Theta}_2 \right) Q_0^2(\hat{\Theta}) - \gamma \left( \hat{\Theta}_2 - \Theta \right) \right\},$$

$$\Gamma_1(\hat{\Theta}_1) = \max_{\Theta > \Theta} \left\{ \left( 2K_2 \hat{\Theta}_2 + K_1 \right) Q_0^2(\hat{\Theta}) - \gamma \left( \hat{\Theta}_2 - \Theta \right) \right\},$$

$$\Gamma_0(\hat{\Theta}_0) = \max_{\Theta > \Theta} \left\{ \left( 4K_2 \hat{\Theta}_2 + 2K_1 + K_0 + \hat{\Theta}_0 N^{-1} \right) Q_0^2(\hat{\Theta}) - \gamma \left( \hat{\Theta}_2 - \Theta \right) \right\}. \quad (1.3.3)$$

Similar profit expressions for the remaining cases in which Tier 1 (or Tier 2) is a contract leader are provided in the Appendix. We now investigate the question of who should invest in product development and innovation to improve the product quality, given contract leadership:

**Proposition 1** (a) For any initial stock of product quality $\Theta \leq \left( \frac{\beta}{\delta_0 - \beta} \cdot \frac{2^{1/2}K_1 + 2\beta K_0}{2^{1/2}K_2} \right)^{1/2}$, if the production costs are moderately convex in quality, specifically $\delta_0 \in (1, 2\beta]$, and the development costs is not too low, i.e. $\gamma \geq \gamma_l$, Tier 1 contract leader invests the most in innovation, yielding the highest product quality expansion, total supply chain profits and market coverage.

(b) If convexity in quality is sufficiently high, i.e. $\delta_0 \in \left( \delta_l, \delta_q \right]$, where $\delta_l > 2\beta$, initial stock of quality is constrained between $\Theta_l$ and $\Theta_q$, and market size and product costs are such that $(K_0 + 2K_1)N^2K_2 > K'$ the Tier 2 optimally invests the most in product quality expansion for any Tier 1 leadership $l = 0, 1, 2$.

Proposition 1 establishes that the contract leader has the most interest in increasing the existing stock of product quality, which also generates the highest total supply chain profits,

$$\Pi_l(\hat{\Theta}_{l|l}) = \max_{i \in \{0, 1, 2\}} \left\{ \sum_{j=0}^{2} \Gamma_j(\hat{\Theta}_{j|l}) \right\} - \gamma \left( \hat{\Theta}_{l|l} - \Theta \right).$$

The supply chain contract leader gains the largest stake in the total supply chain profit compared to the other parties within the supply chain; hence, he has the greatest incentive to invest in product development and innovation to improve the product quality and the supply chain profits. Specifically, in the case of Tier 0 contract leadership, Tier 0’s marginal revenue from investing in product quality is strictly greater than the marginal revenues faced by non-leader tiers investing in quality. As a result, from the supply chain perspective, part (a) of Proposition 1 suggests that it is desirable for the contract leader to be the investor in product development.

What is even more interesting is the mechanism that brings about the most inclusive innovation, i.e., the alignment of the contract leader to be the investor in product development leads to the broadest market coverage as well as largest supply chain profits. This finding suggests that by aligning the investor to be the contract leader, one can achieve more inclusive innovation without compromising the supply chain profits.

Based on this result, we now focus our attention on three investment scenarios along the diagonal in Table 1.3.3:

3The specific expressions for $\gamma_l, K', \delta_l, \delta_q, \Theta_l, \Theta_q$ and $\hat{\Theta}_l$ for $l = 0, 1, 2$ are provided in the Appendix.
1.1, in which Tier $i$ investor is also a supply chain contract leader.

Figure 1.3 presents a numerical example where each region in the $(\delta, \Theta)$-space is labeled with the entity of the maximal investor conditional on Tier 0 leadership. One can observe that parameter region associated with contract leader Tier 0 investing the most in product quality expansion is significantly larger than parameter regions for which non-leader Tiers 1 and 2 are investing the most. Further, we identify the region where all of the tiers optimally choose not to expand existing stock of quality due to marginal revenues becoming negative for all tiers. Recall that profit expressions in (1.3.3) incorporate production costs and hence for sufficiently high values of $\delta$ and $\Theta$ in the top-right corner of Figure 1.3 result in no investment, when marginal revenues become negative. For example in the case of Tier 0 leader’s marginal revenue in the expression (1.3.4), $Q_2^2(\Theta)^{-1} > 0$ for all $\delta > 1$ and $\Theta > 0$. However, term $G(\Theta, \delta)$ in the brackets is such that $\lim_{\Theta \rightarrow \infty} G(\Theta, \delta) < 0$ for $\delta > 2\beta$, and $\lim_{\delta \rightarrow \infty} G(\Theta, \delta) < 0$ for $\Theta > 1$, when

$$MR_{00}(\Theta) = Q_0^2(\Theta)\Theta^{-1} \left[ (2\beta - \delta)4K_2\Theta^{\delta} + \beta\Theta^{\beta}N^{-1} + 2\beta(K_0 + 2K_1) \right].$$

Note that the analysis methodology presented in this section is not restricted to a simple three tier supply chain. It can easily be extended to a complex multi-tier supply chain or supply networks (with the above result making the search for the investor linear in the size of the network). Furthermore, even under the complex supply network case, the contract leader will claim the largest share of the total supply chain profits, which then leads to our result: it is optimal for the supply chain contract leader to be the investor in product...
development for the greatest profits and market coverage.

1.4 Contract Leadership Identity and Handovers

Since we have established that the contract leader should be also an investor in product development, a natural question is then who should be the contract leader (or equivalently, the investor)? In other words, given that the off-diagonal cases in Table 1.1 are suboptimal for inclusive innovation as well as the supply chain profit perspective, among those three diagonal cases in which the contract leader is also the investor, which case leads to more inclusive innovation or greater supply chain profits? Section 4.1 answers this normative question, i.e., who should be the contract leader in a supply chain for more inclusive innovation and/or for greater supply chain profits? This analysis reveals that the identity of the contract leader depends on key development and product cost parameter settings and the contract leadership must be handed over to other tiers as the cost parameters change along the product lifecycle. Next, in Section 4.2, we then ask a question about voluntary leadership handovers; specifically, do the firms have incentives to hand over the contract leadership and/or to take the leadership, and if so, when? How do these voluntary handovers compare with the normative prescription in Section 4.1? Finally, we numerically investigate the case in which all firms can jointly invest to improve the component product quality in Section 4.3, and demonstrate that our key insights are preserved.

1.4.1 Normative Contract Leadership Handovers

In this section, we assume a normative perspective and answer the question of who should be the contract leader for (i) supply chain profit maximization and (ii) more inclusive innovation (broader market coverage), depending on the development cost $\gamma$ and the key component production cost $K_2$. Equation (1.4.1) below provides a simplified expression of the contract leader’s profit maximization problem:

$$\hat{\Pi}_{ll} = \max_{\hat{\Theta} \in \Theta} \left\{ \frac{1}{4} \hat{\Theta}^{2\beta} \left( \Phi_l(\hat{\Theta}) + (\hat{\Theta})^\beta N^{-1} \right)^{-1} - \gamma \left( \hat{\Theta}^{\delta \beta} - \Theta^{\delta \beta} \right) \right\}. \quad (1.4.1)$$

Recognize that the contract leader’s revenue is decreasing in the component production cost $K_2$ as captured by the inverse dependence on the misalignment penalty $\Phi_l(\Theta)$. Furthermore, the rate at which higher values of $K_2$ or higher product quality levels reduce the contract leader’s revenue is determined by the distance between the leader $l$ and Tier 2. Intuitively, dependence on the distance between the contract leader and the investment target provides variation in the entity of optimal contract leader throughout a product’s lifecycle.
where reduction in the development cost leads to higher product quality followed by a decreasing component production cost.$^4$

**Proposition 2**

(a) For moderately convex production costs at Tier 2, i.e. $\delta_0 \in (1, 2\beta]$, Tiers 0 and 1 production costs satisfying $\frac{K_0}{K_1} \in (1, 4)$, and conditional on initial stock of quality $Q \geq 0$, supply chains with downstream contract leadership yield highest market coverage and product quality expansion outcomes under high development cost levels, i.e. high values of $\gamma$. For low levels of development cost supply chains with an upstream contract leader yield the highest market coverage and product quality expansion.

(b) The points of supply chain leadership reorganization yielding the highest path of product quality expansion, i.e. pivotal development cost levels $\tilde{g}_{i+1}^\rho$ for $i = 0, 1$, occur earlier in a product lifecycle relative to leadership reorganization points yielding the highest path of market coverage, i.e. $\tilde{g}_{i+1}^\rho > \tilde{g}_{i+1}^\rho$ for $i = 0, 1$.

(c) Supply chains with high upstream production costs, i.e. high values of $K_2$, exhibit earlier contract leadership reorganization points with respect to both highest path of product quality expansion and market coverage, i.e. pivotal development cost levels $\tilde{g}_{i+1}^{\text{objective}} > \tilde{g}_{i+1}^{\text{objective}}$ for $i = 0, 1$ and objective $\in \{\theta, \rho\}$, where tilde indicates handover points for supply chains with high values of $K_2$.

Part (a) of Proposition 2 shows that as $\gamma$ decreases, the supply chain leadership should be deliberately transferred from the downstream to the upstream tier, i.e., from Tier 0, to Tier 1, and then to Tier 2, to generate the highest investment and product quality, which then leads to the largest supply chain profit as well as market coverage. Decreasing development costs in the emerging stage of a product’s lifecycle yields rapid product innovation and, hence, increased product performance quality $\Theta$. However, Tier 2 must bear the increase in the production cost associated with higher product quality.

$^4$The condition $\frac{K_0}{K_1} \in (1, 4)$ is a sufficient condition for the proof. We numerically illustrate that this result can hold in more general cases (see, e.g., Figures 5 and 6).
Figure 1.4: Misalignment penalty

**Note:** The following parameter values were used to produce figure above, $(K_0, K_1, K_2) = (3, 1, 1)$, $\delta_0, \delta_2 = 2$ and $N, \beta = 1$.

Figure 1.4 depicts the misalignment penalty $\Phi_l(\Theta)$ increasing in $\Theta$ for tiers $l \in \{0, 1, 2\}$. Hence, as product quality $\Theta$ increases, the leader location minimizing the misalignment penalty gravitates upstream. From Figure 1.4, one can identify the existence of three distinct regions where the misalignment penalty $\Phi_l(\Theta)$ is the smallest under each corresponding leader location, $l = 0, 1, 2$. Any alteration in the relative levels of the misalignment penalty drives the shifts in the position of the contract leader yielding the highest total profits and market coverage.
Figure 1.5: Threshold $\gamma$ values for optimal contract leadership transfers

Note: Leadership transfers in this figure are designed to follow the highest product quality expansion path (and the total supply chain profit) in panel (a) and for inclusive innovation in panel (b). Parameter values are $(K_0, K_1, K_2) = (3, 1, 1)$, $\delta_0 = 2$, $\delta_2 = 2$, $N = 0.1$ and $\beta = 1$.

Finally, in part (c) of Proposition 2, we establish the fact that products with higher upstream production costs, as indicated by large values of $K_2$, exhibit leadership shifts at relatively higher levels of $\gamma$ for product quality and market coverage. In other words, part (c) demonstrates that contract leadership transfer
points shift to lower values of $\gamma$ as the upstream production cost, $K_2$, is decreasing, which implies that given a fixed $\gamma$, as $K_2$ decreases, downstream leadership transfers may become desirable from both the supply chain profit perspective and the inclusive innovation perspective.

Proposition 2 provides us with two mechanisms that drive leadership transfers and, hence, alteration of the entity of the optimal investor. For example, decreasing levels of $\gamma$ result in leadership transfers in an upstream direction. A reduction in $K_2$ results in a shift in leadership transfer points in the direction of decreasing $\gamma$, which then leads to the reassignment of some constant development cost level in a downstream direction. This dependence of the entity of the optimal investor on the underlying choice of model parameters, $(\gamma, K_2)$, leads us to examine comparative statics in the context of product and process innovation in Section 1.5.

1.4.2 Tierwise Rational Handovers

Thus far, we have taken a normative stance on who should be the contract leader in order to achieve the largest supply chain profit and/or to attain the broader market coverage, i.e., more inclusive innovation. The natural question is then whether each firm within a supply chain prefers (finds it individually rational) to transfer the contract leadership or assume the leadership. In this section, we question each tier’s individual incentive to transfer the contract leadership. We demonstrate the existence of upstream voluntary contract leadership handovers during the product innovation phase, where we keep track of product quality accumulation and consider dynamics in a myopic sense where static equilibrium is found in every investment period conditional on $\Theta$.

**Proposition 3** Voluntary contract leadership handovers in an upstream direction, i.e. from Tier 0 to Tier 1, and Tier 1 to Tier 2, during the product innovation phase, i.e. as $\gamma$ decreases, exist as both the production costs and consumer’s utility approach linearity in product quality.

Figure 1.6 illustrates the voluntary handover of contract leadership from Tier 0 to Tier 1. Specifically, if $\gamma > \hat{\gamma}$ (in region (E)), Tier 0 prefers to be the contract leader investing in quality improvement, i.e., his profit is higher under his contract leadership than under the other tier’s contract leadership. In addition, in this case, the other tiers also prefer Tier 0 to be the contract leader. Moreover, in this region (E), Tier 0 contract leadership also maximizes the total supply chain profits and leads to more inclusive innovation. If $\gamma$ decreases below $\gamma_{01}^{IR}$ (in region (A)), then Tier 0 is willing to hand over the contract leadership to Tier 1, and Tier 1 is
also willing to take over the contract leadership and to be the investor. Furthermore, in this case, Tier 2 is also
better off under Tier 1’s contract leadership than under Tier 0’s contract leadership. In addition, a voluntary
handover of leadership in this region (A) occurs past the optimal handover point yielding the broadest market
coverage, $\gamma_{01}^R \geq \gamma_{01}^R$, so in this region (A), Tier 1 contract leadership yields the broadest market coverage as
well as the largest total supply chain profits.

Figure 1.6: Voluntary leadership handover from Tier 0 to Tier 1

Note: Lines with “x” marks represent Tier 1’s profits, and lines without marks represent Tier 0’s profits.
Solid lines represent profits under Tier 0 leadership and dotted lines represent profits under Tier 1
leadership. Parameter values are $(K_0, K_1, K_2) = (6, 1, 1)$, $\delta_0 = 1.1$, $\beta = 1$, $N = 1$, and $\delta_D = 2$.

Denote Tier $j$’s profit under Tier $l$’s contract leadership as $\Pi_j^l$. Then, Tier $l$ leader investing in product
quality attains profits $\Pi_l^l(\gamma) = \left( \Phi_l(\gamma) + \Theta^{\beta}_{ll}(\gamma)N^{-1} \right) Q_2^l(\gamma)$, which can be understood as the common
revenue component $Q_2^l(\gamma)$ multiplied by the corresponding weight denoted as $g_{ll}(\gamma) = \Phi_l(\gamma) + \Theta^{\beta}_{ll}(\gamma)N^{-1}$.

Once Tier 0 hands over leadership to Tier 1, his profit becomes $\Pi_0^1(\gamma) = K_0 Q_2^1(\gamma)$, which can be also under-
stood as the multiplication of the common revenue component, $Q_2^1(\gamma)$ by the weight $g_{01}(\gamma) = K_0$. However,
now Tier 0 does not incur the product development cost, $\gamma(\Theta^{\beta}_{00} - \Theta^{\beta}_{00})$. Similarly, under Tier 0’s leadership,
Tier 1’s profit is, $\Pi_1^0(\gamma) = (K_1 + 2K_2 \Theta^{\beta}_{00}(\gamma))Q_2^0(\gamma)$, which is again the multiplication of the weight
$g_{10}(\gamma) = K_1 + 2K_2 \Theta^{\beta}_{00}(\gamma)$ and the common revenue component $Q_2^0(\gamma)$. Figure 1.7 illustrates the behavior of the constituents of the profit expressions in Figure 1.6. Observe that at $\hat{\gamma}$ when Tier 0 would like to
hand over leadership to Tier 1, the common revenue component under Tier 0’s leadership is still larger than
in the case of Tier 1’s leadership, \( Q_0^2(\hat{\gamma}) \geq Q_1^2(\hat{\gamma}) \). Similarly, the weights associated with Tier 0 are such that \( K_0 = g_{0|1}(\hat{\gamma}) = g_{0|0}(\hat{\gamma}) \), indicating that leadership should not be handed over. However, the benefit of handover for Tier 0 originates from avoiding the development cost, \( \hat{\gamma} \left( \hat{\Theta}_0^D - \Theta_0^D \right) \). Moreover, Tier 1 will accept leadership only at \( g_{IR} \), when the revenue component \( Q_1^2(\hat{\gamma}) \) and the weighting factor, \( g_{1|1}(\hat{\gamma}) \) are sufficiently large to offset the cost of development incurred by the leader, \( \hat{\gamma} \left( \hat{\Theta}_1^D - \Theta_1^D \right) \), as illustrated in Figure 1.7.

We demonstrated that if \( \gamma \) is either higher than \( \hat{\gamma} \) in region (E) in Figure 1.6 or lower than \( \gamma_{IR} \) in region (A), firms are willing to hand over the leadership. What happens at \( \gamma \in (\gamma_{IR}, \hat{\gamma}) \) in regions (B), (C) and (D) in Figure 1.6? In these regions, the negative impact of the product development cost dominates the benefit of contract leadership. As a result, Tier 0 prefers to hand over the contract leadership to Tier 1. However, Tier 1 also prefers Tier 0 to remain the contract leader. That is, in these intermediate regions, no firm wants to be the leader. From the normative perspective that we investigated in Section 4.1, in region (B), it is better for Tier 1 to be the contract leader, whereas in region (D), Tier 0 leadership is optimal from both the supply chain perspective and the market coverage/inclusive innovation perspective. In region (C), it is
better for Tier 1 to be the contract leader from the supply chain profit perspective, but it is better for Tier 0 to be the leader from the market coverage/inclusive innovation perspective.

![Figure 1.8: Transfers reducing the leadership handover gap](image)

Note: Parameter values are the same as those in Figure 1.6.

What can firms within a supply chain or policy makers do in the intermediate ranges, i.e., in regions (B), (C) and (D)? The policy maker may provide R&D tax credits for innovation investment to Tier 0 in regions (C) and (D) and to Tier 1 in region (B) to encourage more inclusive innovation with the deliberate contract leadership. From the supply chain perspective, Tier 1 may subsidize Tier 0’s investment in \( \gamma \in (\gamma_{01}^{T}, \hat{\gamma}) \) as illustrated in Figure 8; note that in this region, Tier 1’s gain from Tier 0 leadership compared to her own contract leadership is higher than Tier 0’s loss from his own leadership, so Tier 1 has a financial incentive to subsidize Tier 0. Similarly, in \( \gamma \in (\hat{\gamma}, \gamma_{01}^{T}) \), Tier 0 has an incentive to subsidize Tier 1’s contract leadership. In summary, either a tax credit from the policy maker’s perspective or a development investment subsidy within a supply chain can help lead to the more desirable contract leadership pattern that yields higher coverage.

### 1.5 Product Lifecycle and Contract Leadership Transfers for Broader Market Coverage

We now consider how the quality investment, market coverage, and leadership results of the previous section depend on the stage of a product’s lifecycle. It is widely recognized that most innovative products go through two distinct phases of product and process innovation (Utterback and Abernathy 1975). The lifecycle usually commences with a product innovation phase of rapidly-rising development productivity/decreasing development cost. However, as performance improvement/development productivity reaches
maturity, opportunities for gains are associated with potential reductions in the production cost. Specifically, at the maturity stage, process innovation rather than continued product innovation dominates, and production costs decrease as a result of industry-wide improvement in manufacturing technology. In this section, we examine comparative statics in the context of these two phases of the product lifecycle.

Table 1.2 provides an environment in which product innovation is associated with a period of decreasing development cost \( g \), i.e., the first phase of the lifecycle with gradually decreasing levels of development cost, \( g_H > g_M > g_L \), for some fixed and initially high-scale upstream production costs, \( K_{2H}^H \). Afterward, a period of process innovation follows, which is modeled as a decrease in the scale of upstream production costs \( K_2 \), while the scale/value of development costs remains constant. That is, once the magnitude of development costs reaches its lower bound \( g_L \) at \( t = t_3 \) at the maturity stage, process innovation proceeds with a decreasing sequence of upstream production costs, \( K_2^H > K_2^M > K_2^L \). The notion of a product lifecycle here is independent of individual firms and instead is an industry-wide phenomenon. Hence, \( t_j \) represents an instance in a specific industry characterized by a pair of cost parameters \( (g(t_j), K_2(t_j)) \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( t_1 )</th>
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<tr>
<td>( \gamma )</td>
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<td>( K_{2} )</td>
<td>( K_{2}^H )</td>
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Note: \( t = t_1, t_2, t_3 \) correspond to the product innovation phase, which is followed by process innovation phase in \( t = t_4, t_5 \). Parameters values satisfy \( t_1 < t_2 < t_3 < t_4 < t_5 \), \( g_L < \gamma_M < \gamma_H \) and \( K_{2}^L < K_{2}^M < K_{2}^H \).

In the results of Proposition 2, we see how the contract leadership offering the highest market coverage responds to decreasing product development costs, \( \gamma \), and the scale of upstream production costs, \( K_2 \). Thus, the examination of changes during a product lifecycle amounts to the application of Proposition 2 in the context of product and process innovation.

During product innovation phase that is associated with a reduction in \( \gamma \), the contract leader location resulting in both highest product quality and market coverage shifts upstream as the magnitude of development cost \( \gamma \) decreases. Formally, the supply chain maximizes its profits and, hence, product quality, \( \Theta \), during product innovation by deliberately transferring leadership from Tier 0 to Tier 1 when the level of development cost shifts from the high, \( \{\gamma_{01}^\theta, \infty\} \), to the medium, \( \{\gamma_{01}^\theta, \gamma_{01}^\theta + \Delta^\theta\} \) cost regime. Finally, once the development cost reaches \( \gamma_L < \gamma_{01}^\theta \), the contract leadership should be further transferred to Tier 2. A similar argument applies to the case of market coverage when the contract leadership transfers occur at \( \{\gamma_{01}^r, \gamma_{12}^r\} \). Thus, during the product innovation phase, it is optimal to gradually transfer leadership upstream and make the component
suppliers play a leading role in making contract and investment decisions. This strategy parallels the experience of the personal computer industry in the 1990s when the leadership was transferred from downstream players like Apple and HP to upstream component suppliers such as Intel and Microsoft.

Now, consider the process innovation phase to see what happens to the contract leadership. The analysis of process innovation corresponds to the case of decreasing levels of $K_2$, given a fixed low $\gamma$. Part (c) of Proposition 2 states that supply chains with relatively lower upstream production costs exhibit optimal leadership handover points that are associated with relatively lower development cost levels. Therefore, as the process innovation phase unravels, the optimal supply chain leadership with respect to both product quality and market coverage is shifted downward. In other words, during the process innovation phase, the supply chain is better off with the leadership being gradually transferred downstream for maximum market coverage. The downstream leadership shift captured by our model is similar to the return to the leadership of downstream players like Apple in the computer supply chain during the 2000-2010.

Figure 1.9: Leadership transfers under decreasing production costs

Note: Parameter values are $K_0 = 3$, $K_1 = 2$, $\delta_0 = 1.7$, $\beta = 0.9$, $N = 1$ and $\delta_D = 2$.

We first illustrate the shifts in leaderships using the following parameter values before formally
proving the existence of these shifts. Panels (a) and (b) of Figure 1.9 identify optimal product qualities, $\Theta_{\delta l}(\gamma)$, via intersections of the marginal revenue of investment, $MR_{\delta l}(\Theta)$, and the marginal development cost, $MC(\Theta) = \gamma \Theta$, for $\delta_D = 2$. Suppose that the development cost level is set to $\hat{\gamma} = 0.0183$. Panel (a) depicts the case of a high upstream production cost regime, $K_2 = 6$. In this regime, the optimal contract leader to achieve the largest supply chain profit is Tier 2 from $\hat{\gamma} = 0.0183 < \gamma_{12}^{\Theta}$. Panel (b) of Figure 1.9 considers a lower production cost regime, $K_2 = 1$ - where at the same development cost level $\hat{\gamma}$, the contract leader should be deliberately reassigned/transferred to Tier 1 in order to achieve the largest supply chain profit, since $\hat{\gamma} \in (\gamma_{12}^{\Theta}, \gamma_{01}^{\Theta})$ in this case. Thus, as the production cost parameter $K_2$ decreases, the contract leadership should be transferred in downstream direction, i.e., from Tier 2 to Tier 1.

A different way to illustrate a change in assigning the optimal leader location in a decreasing scale of production cost $K_2$ is provided in Figure 1.10. This figure enhances the understanding of the implications of the shift in handover points to a later time in the product lifecycle as a result of decreasing $K_2$, given a low fixed development cost level $\gamma$. The diagram in Figure 1.10 combines panels (a) and (b) of Figure 1.9 in its inner and outer quarter-circles, respectively. The division into sectors in the inner quarter-circle of the diagram corresponds to the development cost $\gamma$ ranges under the high production cost regime in panel (a) of Figure 1.9. The outer quarter-circle indicates a downward shift of optimal leadership handover points resulting from lower production costs, $K_2 = 1$, as captured in panel (b) of Figure 1.9. Thus, movement along the marginal cost curve from the inner to the outer quarter-circle represents stages of process innovation, where despite a fixed development cost level $\hat{\gamma} = 0.0183$, it is optimal to hand over leadership from Tier 2 to Tier 1.

Figure 1.10: Mechanism for market coverage-maximizing downstream leadership handovers
We present this sequence of smooth leadership handovers exhibiting a complete reversal toward downstream in the process innovation phase in Table 1.2, and Proposition 4 formally establishes the existence of the parameter values \( \{\gamma_H, \gamma_M, \gamma_L\} \) and \( \{K^H_2, K^M_2, K^L_2\} \) mentioned in Table 1.2.

**Proposition 4** *In supply chains exhibiting an initial asymmetry of production costs and process innovation resulting in a reduction of the scale of upstream production costs, i.e. \( K_2 \), there exists a sequence of deliberate leadership handovers, such that during product innovation phase (decrease in \( \gamma \)), the contract leadership is shifted upstream from Tier 0 to Tier 1 and then to Tier 2. This sequence is reversed during the process innovation phase in which the entry of an optimal leader with respect to both product quality and market coverage shifts downstream from Tier 2, to Tier 1, and then Tier 0.*

Examination of the comparative statics in a simple trilateral-supply chain yields a rich set of possible leadership handover sequences, depending on the choice of decreasing development costs and component production costs. Considering product and process innovation allows us to construct a leadership handover sequence that exhibits reversal/oscillation in the entity of the optimal leader with respect to both product quality (or total supply chain profit) and market coverage. Thus, this framework provides useful insights into the nature of the optimal leader location in the industries consisting of supply chains without a preexisting dominant contract leadership. Note that both in the initial stage of product innovation with a high development cost \( \gamma \) and the final stage of process innovation with a low production cost \( K_2 \), it is desirable for a Tier 0 firm to be the contract leader and the direct investor in a Tier 2 supplier, skipping the Tier 1 firm. It is interesting to notice that in our sequential contract stage, firms in the adjacent tiers contract with each other; however, in terms of investment, when a Tier 0 firm directly invests in a Tier 2 supplier skipping the Tier 1 assembler, it leads to higher supply chain profit as well as broader market coverage. As mentioned, these results mirror the development in industries such as the personal computer (PC) and cellphone industries. During the early stage product innovation phase of the PC industry in the 1990s, the leadership was gradually transferred from downstream players like IBM, Apple and HP to upstream component suppliers like Intel and Microsoft. Later, during the process innovation phase (a period of product maturity), we are seeing a return to the leadership of downstream players such as Apple and Lenovo. As the industry goes through a new S-curve of innovation, this oscillatory pattern may repeat itself explaining the swinging leadership and fortunes of companies during the lifecycle.
1.6 Full Dynamics Extensions

In Proposition 1-4 we consider dynamics in a myopic sense where stock of quality accumulation is taken into account but only static equilibrium is obtained in every investment period conditional on available stock of quality $\Theta$. In Proposition 5 we consider fully dynamic two period setting where revenues net development cost are shared and each investor solves a two-period inter-temporal optimization problem. In a pairwise manner we show that product quality expansion path under two-period leadership sequences with an upstream handover, i.e. Tier 0 to Tier 1, dominates product quality expansion path under static leadership, i.e. Tier 0 for two periods, when scale of development costs is decreasing. Similar result is shown for handover from Tier 1 to Tier 2, dominates a two-period Tier 1 leadership.

**Proposition 5** In a two-period setting if total supply chain revenues net production costs are shared equally among forward looking tiers and investor bears the entirety of development cost, (5.a) there exist development cost levels $(\gamma_1, \gamma_2)$ such that product quality, market coverage and total supply chain profit outcomes under leadership reorganization sequences in an upstream direction dominate corresponding outcomes under static leadership, i.e. Tier 0 and Tier 1 handover dominates two-period Tier 0 leadership and Tier 1 to Tier 2 handover dominates two-period Tier 1 leadership. Further, (5.b) there exist upstream production cost coefficients $(K_1^1, K_2^2)$, such that in a two-period setting with a constant development cost level $\gamma$ product quality, market coverage and supply chain profit outcomes under leadership handover sequences in a downstream direction dominate outcomes with static leadership, i.e. Tier 2 to Tier 1 handover dominates two-period Tier 2 leadership, and Tier 1 to Tier 0 handover dominates two-period Tier 1 leadership.

**Lemma 2** Proposition 5 holds in a weaker case when development costs are shared in addition to production costs.

We also numerically examine a three-period example in the Figure 1.11, where given a monotonically decreasing path of $\gamma$ from $\gamma_1 = 0.7$ to $\gamma_3 = 0.125$, product quality expansion path and market coverage path under a leadership sequence, $0 \rightarrow 1 \rightarrow 2$, dominates the ones under static leadership sequences, $l \rightarrow l \rightarrow l$ for $l = 0, 1, 2$. 
Figure 1.11: Three-period full dynamics

Note: Parameter values are $\beta = 0.95$, $\delta_q = 1.7$, inter temporal discount factor is 0.9, revenues net production costs are shared equally, investor in product quality expansion bears the entirety of development cost.

1.7 Conclusion and future research

In this paper, we have focused on the central question of how new innovative products can be made available and affordable to a broader market, making such innovations more inclusive. As discussed in the Introduction section, even important life-saving innovations such as automotive airbags have not been very inclusive in the past, missing the opportunity to improve both social welfare and the innovating firm’s financial performance. Our key finding is that the locus of development investment decision making in a multi-tiered supply chain offers a new hitherto undiscovered degree of freedom to streamline decision making and improve the market coverage of innovative products.

In deriving the results, we considered products with substantial development and production costs, as is the case with new knowledge-intensive products - such as in life sciences and even software-enabled hard goods (for instance, next generation automobiles, smart phones, tablets, and machine tools). Unlike prior work in new product development modeling (which deals primarily with product quality) or supply chain management (where the focus is on quantity), we are able to jointly consider decisions made about product quality, price and quantity. Specifically, the choice of product quality $Q$ is modeled as an investment in innovation. The equilibrium price and quantity are established via a sequence of wholesale price offers using a backward-induction solution approach. Using the standard framework that all consumers who receive positive net utility purchase the product, we are able to show that deliberately choosing which firm in a multi-tiered supply chain invests in innovation and development to improve product quality $\Theta$ can have a significant impact on the market coverage of the product. This result is derived gradually, beginning with the notion of
In conducting the analysis, we adopted the standard product lifecycle perspective; the early stages of product lifecycle are characterized by product innovation when the development productivity improves (the cost parameter $\gamma$ decreases) substantially, whereas in the later stages in which the product enters maturity, process innovation dominates and the production cost $K_2$ decreases. We are able to formally show that (1) the supply chain leader invests the most relative to other tiers, leading to the largest total supply chain profits (for a substantial range of production costs), and (2) the supply chain leader should be deliberately transferred from the downstream to the upstream tier, i.e., from Tier 0, to Tier 1, and then to Tier 2, as the development cost $\gamma$ decreases. In addition, we are able to construct a sequence of deliberate leadership handovers, such that during the product innovation stage, the leadership is optimally shifted upstream from Tier 0 to Tier 1 and then to Tier 2 whereas during the process innovation stage, the leadership should be shifted back downstream from Tier 2, to Tier 1, and then Tier 0 to optimize product quality and market coverage. Deeper analysis in Section 4.2 allows us to go beyond a normative prescription by identifying conditions under which tierwise rational leadership transfers occur and discuss how to align individually rational schedule of leadership transfers with an optimal schedule that results in the highest market coverage. We then translate these findings to the context of a product lifecycle in Section 5 and discover the interesting property of reversal/oscillation in the identity of the optimal leader for maximal market coverage.

Our results have important and subtle implications for firms such as the electric car maker (OEM) we discussed earlier in the paper. Specifically, the analysis presented in the paper suggests that to obtain broader market coverage for its innovations (a stated goal of the company), the OEM should initially drive investments in core technology (battery) innovation while gradually allocating a greater role to its suppliers as the product development becomes easier and costs decrease. Once the product is mature and the improvements have reduced production costs, it makes sense for the OEM to take greater control of the decision making with respect to innovation investments. By doing so, the firm is able to launch innovative products and ensure that more of the marketplace is able to enjoy them. As a result, the aggressive reduction of the development and production costs is naturally translated into more profitable, affordable, and higher-quality products. Our approach offers firms additional degrees of freedom, available in the context of easy-to-implement wholesale-price contracts, providing an alternative to more complex non-linear contracts in mitigating misalignment between vertically integrated and decentralized supply chain environments.

To keep a sharp focus on inclusive innovation and to manage complexity, we formulated a stylized analytical model with its own limitations. First, consistent with the electric car and electronics/computer industry examples discussed in the paper, we considered the innovation investment in a core
component/technology, such as the electric car battery or the microprocessor, developed by a Tier 2 supplier. In other industries, the primary quality-enhancing investment can be associated with other tiers of the supply chain, e.g., the final product quality can be affected primarily by the design of the end product, which is related to the investment in Tier 0 manufacturer. However, our analysis remains valid as long as there exists a single primary investment target in a supply chain, regardless of its location, which determines the final product quality. Second, we investigated in this paper the case of a three-tier serial supply chain investing in an advanced/monopolistic technology. In reality, a multi-tiered supply chain can be a supply network with a complex network relationship and, potentially, other competing supply chains. We focused our analysis on the simple linear supply chain case to derive primary first-order insights on inclusive innovation, and our analysis can be a building block to examine more complicated cases including supply chain competition, such as that studied in Corbett and Karmarkar (2001) and Carr and Karmarkar (2005).

In closing, we believe that this paper represents an important first step on an issue of growing importance in today’s digitally connected market environment, namely how innovative products can be made more inclusive or broadly affordable to a range of customers in a manner consistent with the profits of firms - by aligning actions and tapping previously unexplored degrees of freedom about investor identity in the industry supply chain.

Chapter 1, in full, has been submitted for publication of the material as it may appear in Management Science, Product Innovation Investments and Supply Chain Contract Leadership for Broader Market Coverage, 2015. Chapter 1 was co-authored with Professor Vish Krishnan and Professor Hyoduk Shin.
Appendix to Chapter 1

**Proof of Lemma 1**: Equilibrium price and quantity are obtained by reducing tierwise profit expressions into a single contract leader’s profit expression by iteratively applying marginalization operation presented in Majumder and Srinivasan (2008). First, in the case of Tier 0 contract leadership, the leaf node Tier 2’s profit expression can be written as

$$\Pi_2(q_2; \omega_2) = \omega_2 q_2 - \Theta^{\delta_0} K_2 q_2^2. \quad (1.7.1)$$

Tier 2 optimally responds to wholesale price offer of $\omega_2$ by maximizing $\Pi_2$ above with respect to $q_2$ which leads to $q_2(\omega_2) = \omega_2 (2K_2 \Theta^{\delta_0})^{-1}$. Now, from Tier 1’s perspective, taking Tier 2’s optimal response $q_2(\omega_2)$ into account, Tier 1 faces the inverse factor demand, which is $\omega_2(q_2) = 2K_2 \Theta^{\delta_0} q_2$. Furthermore, the quantity constraint $q_1 \leq q_2$ in Eq. (1.3.1) optimally binds. Substituting $\omega_2(q_2)$ and the binding quantity constraint $q_1 = q_2$ into $\Pi_1(q_1, \omega_2; \omega_1)$ in Eq. (1.3.1), we obtain Tier 1’s resulting profit expression as follows:

$$\Pi_1(q_1; \omega_1) = \omega_1 q_1 - \left(K_1 + 2\Theta^{\delta_0} K_2 \right) q_1^2. \quad (1.7.2)$$

Similarly, Tier 0 faces the inverse factor demand of the form $\omega_1(q_1) = (2K_1 + 2^2 \Theta^{\delta_0} K_2) q_1$. Substituting $\omega_1(q_1)$ into $\Pi_0(p, q_0, \omega_1)$ and using the optimally binding quantity constraint $q_0 = q_1$, we have

$$\Pi_0(p, q) = pq - \Phi_0(\Theta) q^{\delta_0} \ \text{s.t.} \ \mathcal{N}(1 - \frac{p}{\Theta^{\delta_0}}) \geq q, \quad (1.7.3)$$

where $\Phi_0(\Theta) = K_0 + 2K_1 + 2^2 \Theta^{\delta_0} K_2$. By optimizing (1.7.3) over $p$ and $q$, it follows that

$$R_0(\Theta) = \left(2 + 2\Theta^{-\beta} \Phi_0(\Theta) N\right)^{-1} \left(\Theta^\beta + 2\Phi_0(\Theta) N\right),$$
$$Q_0(\Theta) = \mathcal{N} \left(2 + 2\Theta^{-\beta} \Phi_0(\Theta) N\right)^{-1}, \quad (1.7.4)$$

which completes the proof for the case of Tier 0 contract leadership. For the other remaining cases, by following the similar steps, we obtain the equilibrium outcomes as stated in Lemma 1. □

**Claim 1**

If $R(\hat{\Theta})$ is a supply chain revenue associated with new expanded product quality $\hat{\Theta}$ such that $\hat{\Theta} \geq \Theta$, i.e. new stock of quality weakly exceeds the previous stock of quality $\Theta$, and development cost of raising product quality level from $\Theta$ to $\hat{\Theta}$ is $C(\hat{\Theta}, \Theta) = \gamma \cdot \max \{\hat{\Theta}^{\delta_0} - \Theta^{\delta_0}, 0\}$, then
arg\max_{\hat{\Theta}} \{ R(\hat{\Theta}) - C(\hat{\Theta}) \} = \max_{\hat{\Theta} \geq \Theta} \left\{ \arg\max_{\hat{\Theta}} \{ R(\hat{\Theta}) - C(\hat{\Theta}, 0) \}, \Theta \right\} \tag{1.7.5}

Proof: On the left-hand side of (1.7.5) \( C(\hat{\Theta}, \Theta) = \gamma \hat{\Theta} \delta_\Theta \) since maximization is constrained to \( \hat{\Theta} \geq \Theta \). Further, on the right-hand side of (1.7.5) \( C(\hat{\Theta}, 0) = \gamma \hat{\Theta} \delta_\Theta \) since \( \hat{\Theta} \geq 0 \). Therefore, maximization problems with respect to \( \hat{\Theta} \) on both sides of (1.7.5) have the same objective function. However, on the right-hand side the maximand of unconstrained problem, \( \hat{\Theta}^* \), may be less than initial stock of quality, i.e. \( \hat{\Theta}^* \leq \Theta \), which is corrected for by an additional \( \max \{ \cdot \} \) operator such that \( \max \{ \hat{\Theta}^*, \Theta \} \geq \Theta \). Hence, expression (1.7.5) holds.

\[ \square \]

Remark 1

Claim 1 establishes basis for a convenient proof technique for results in this paper that are dependent on the initial stock of quality \( \Theta \). If marginal revenue and marginal cost expressions depend only on \( \hat{\Theta} \) then it suffices to examine optimal product quality expansion outcomes conditional on \( \Theta = 0 \), i.e. unconstrained optimization \( \hat{\Theta} \geq 0 \), which can be revised as indicated in (1.7.5) for \( \Theta > 0 \). Figure (1.2) provides further intuition.

Proof of Proposition 1: Technically, for part (1.a) we show that for any initial stock of quality \( \Theta \leq \left( \frac{\beta}{\delta_\Theta - \beta} \right) \frac{\gamma}{\alpha} \left( \frac{\tau^2}{2^{1-i} K_1 + \tau^2 K_2} \right) \delta_\Theta \) and for all \( \delta_\Theta \in (1, 2\beta] \), and all \( \gamma > \max \{ \gamma_j \} \) where \( \gamma_j \) is defined below conditional on leader location \( l \in \{0, 1, 2\} \), the new stock of quality \( \hat{\Theta}_{il} = \Theta + \theta_{il} \) is such that

\[
\hat{\Theta}_{il} = \max_{i \in \{0, 1, 2\}} \{ \hat{\Theta}_{il} \} \text{ and } \Pi_i^T (\hat{\Theta}_{il}) = \max_{i \in \{0, 1, 2\}} \left\{ \Pi_i^T \left( \hat{\Theta}_{il} \right) \right\}.
\]

First, consider the case of Tier 2 contract leadership. Using the equilibrium price and quantity outcomes from Lemma 1, we obtain the resulting profit expressions for each firm as follows,

\[
\Pi_{i|2}(\hat{\Theta}) = \max_{\hat{\Theta}, s.t. \hat{\Theta} \geq \Theta} \left\{ \begin{array}{ll}
Q_2^i(\hat{\Theta}) & \text{if } i = 0 \\
2K_0 + K_1 & \text{if } i = 1 \\
\Phi_2(\hat{\Theta}) + \hat{\Theta} \delta N^{-1} & \text{if } i = 2
\end{array} \right\} - \gamma \left( \hat{\Theta} \delta_\Theta - \Theta \delta_\Theta \right), \tag{1.7.6}
\]

where for notational convenience subscript \( i \mid l \) on a new stock of quality \( \hat{\Theta} \) is suppressed and maximization
leads to the highest total supply chain profits. This is achieved under the leadership of Tier 2, as it invests relative to the supply chain optimal investment level. In addition, the marginal revenue for Tier 2 is given by \( MR_{2|2}(\hat{\Theta}) \). Denote the total marginal revenue of the supply chain under Tier 2’s leadership as \( MR_{2|2}(\hat{\Theta}) \), which holds for \( MR_{2|2}(\hat{\Theta}) > 0 \) for all \( \hat{\Theta} \geq 0 \) and \( MR_{2|2}(\hat{\Theta}) \geq 0 \) for \( i = 0, 1 \), where \( MR_{i|2}(\hat{\Theta}) \) corresponds to the marginal revenue for Tier \( i \) investor; specifically, they can be written as

\[
MR_{0|2}(\hat{\Theta}) = Q_2^2(\hat{\Theta}) \left( N^{-1} \hat{\Theta}^\beta + \Phi_2(\hat{\Theta}) \right)^{-1} \hat{\Theta}^{-1} \left( \beta (2K_1 + 4K_0) - (\delta_0 - \beta)K_2\hat{\Theta}^\delta \right) 2K_0, \\
MR_{1|2}(\hat{\Theta}) = Q_2^3(\hat{\Theta}) \left( N^{-1} \hat{\Theta}^\beta + \Phi_2(\hat{\Theta}) \right)^{-1} \hat{\Theta}^{-1} \left( \beta (2K_1 + 4K_0) - (\delta_0 - \beta)K_2\hat{\Theta}^\delta \right) (4K_0 + 2K_1), \\
MR_{2|2}(\hat{\Theta}) = Q_2^4(\hat{\Theta}) \hat{\Theta}^{-1} \left( (2\beta - \delta_0)K_2\hat{\Theta}^\delta + \beta \hat{\Theta}^\delta N^{-1} + 2\beta (4K_0 + 2K_1) \right).
\]

Note that \( MR_{2|2}(\hat{\Theta}) > MR_{0|2}(\hat{\Theta}) \) is simplified to

\[
(\delta_0 - 2\beta)K_2^2\hat{\Theta}^{2\delta} + ((2\delta_0 - 14\beta)K_0 + (2\delta_0 - 8\beta)K_1)K_2\hat{\Theta}^{\delta} + (\delta_0 - 3\beta)K_2N^{-1}\hat{\Theta}^{\delta} + \beta \hat{\Theta}^{\delta}N^{-2} + \beta (2K_1 + 4K_0)(4K_1 + 6K_0) + 3\beta (2K_1 + 4K_0)N^{-1}\hat{\Theta} \]

(1.7.8)

which holds for \( \delta_0 \leq \beta \cdot \min \left\{ 2, 3, 4 + \frac{6K_0}{2K_1 + 2K_0} \right\} = 2\beta \). Furthermore, \( MR_{2|2}(\hat{\Theta}) > MR_{1|2}(\hat{\Theta}) \) is equivalent to

\[
(\delta_0 - 2\beta)K_2^2\hat{\Theta}^{2\delta} + (\delta_0 - 3\beta)K_2N^{-1}\hat{\Theta}^{\delta} + \beta \hat{\Theta}^{\delta}N^{-2} + \beta (2K_1 + 4K_0)^2 + 3\beta (4K_0 + 2K_1)N^{-1}\hat{\Theta}^{\beta} + 3\beta (4K_0 + 2K_1)K_2\hat{\Theta}^{\delta} \]

(1.7.9)

which also holds for \( \delta_0 \leq \beta \cdot \min \{ 2, 3 \} = 2\beta \). From examining expression for Tier 2’s marginal revenue in (1.7.7), \( MR_{2|2}(\hat{\Theta}) \geq 0 \) for \( \delta_0 \leq 2\beta \). As a result, \( \hat{\Theta}_{2|2} = \max_{i \in \{0, 1, 2\}} \{ \hat{\Theta}_{i|2} \} \) follows.

Next, we show that a contract leader’s investment in product quality yields highest total supply chain profits. Denote the total marginal revenue of the supply chain under Tier 2’s leadership as \( MR_{T|2}(\hat{\Theta}) = MR_{0|2}(\hat{\Theta}) + MR_{1|2}(\hat{\Theta}) + MR_{2|2}(\hat{\Theta}) \). By comparing \( MR_{T|2}(\hat{\Theta}) \) and \( MR_{2|2}(\hat{\Theta}) \) and using algebra, we obtain \( MR_{T|2}(\hat{\Theta}) \geq MR_{2|2}(\hat{\Theta}) \) for \( \hat{\Theta} \leq \left( \frac{\beta}{\delta_0 - \beta} \frac{2K_1 + 4K_0}{K_2} \right)^{\frac{1}{\theta_2}} \). Furthermore, it follows that for all \( \gamma \geq \gamma_2 \), \( \hat{\Theta}_{2|2} \leq \left( \frac{\beta}{\delta_0 - \beta} \frac{2K_1 + 4K_0}{K_2} \right)^{\frac{1}{\theta_2}} \) holds. Consequently, it implies that for all \( \gamma \geq \gamma_2 \), \( \hat{\Theta}_{2|2} \leq \hat{\Theta}_{T|2} \), i.e., Tier 2 leader under-invests relative to the supply chain optimal investment level. In addition, \( \Pi_2^T(\hat{\Theta}) \) is quasi-concave. Hence, \( \Pi_2^T(\hat{\Theta}) \) is increasing in \( \hat{\Theta} \) for \( \hat{\Theta} \leq \hat{\Theta}_{T|2} \). Therefore, Tier 2 contract leader’s investment in product quality leads to the highest total supply chain profits.
It remains to show that contract leader’s choice of product quality also yields highest market coverage. Technically we show that for \( \gamma > \gamma'_l \) as specified below \( \rho_i(\hat{\Theta}_{ij}(\gamma)) > \rho_i(\hat{\Theta}_{ij}(\gamma)) i \neq l, l \in \{0,1,2\} \). Since we have already proved that \( \hat{\Theta}_{ij}(\gamma) = \max_{i \in \{0,1,2\}} \left\{ \hat{\Theta}_{ij}(\gamma) \right\} \), it remains to show that that if \( \gamma \geq \gamma'_l \) then \[ \frac{\partial \rho_i(\Theta)}{\partial \Theta} \Big|_{\Theta = \hat{\Theta}_{ij}(\gamma)} \geq 0. \] By construction \( \gamma'_l \) is such that \( \hat{\Theta}_{ij}(\gamma) \leq \left( \frac{\beta}{\delta_0 - \beta} \right) \left( \frac{2^{1-l}K_{ij}^{2} \delta_0 - \beta}{2^{1-l}K_{ij}^{2}} \right) \frac{1}{\gamma'_l} = \arg\max \left\{ \rho_i(\hat{\Theta}) \right\} \) for \( \gamma \geq \gamma'_l \). Therefore, given sufficiently high levels of development cost \( \gamma \geq \gamma'_l \) for \( l \in \{0,1,2\} \), contract leader’s choice of product quality also yields highest market coverage in addition to highest product quality.

For the remaining cases of Tier 0 and Tier 1 contract leadership, we follow similar steps to establish that for all \( \delta_0 \in (1,2\beta) \) and all \( \gamma \geq \gamma'_l \) for \( l \in \{0,1,2\} \), a contract leader’s investment in product quality yields the highest product quality expansion and the highest total supply chain profits given initial stock of quality \( \Theta = 0 \). To generalize above proof for any \( \Theta > 0 \) we recognize that

For part (1.b) we show that when \( \delta_0 \geq \delta_0^l \) and \( \Theta^l < \Theta < \Theta^l \) Tier 2 associated with critical product quality invests the most in product quality expansion relative to Tier 1 contract leader, i.e. \( \hat{\Theta}_{2ij} > \hat{\Theta}_{1ij} \) for \( i,l = 0,1, \) where \( l = 2 \) is a trivial case. Similarly to the proof of part (1.a), we identify sufficient values of \( \delta_0^l, \Theta^l \) and \( \Theta^l \) such that \( MR_{2ij}(\hat{\Theta}) \leq MR_{0ij}(\hat{\Theta}) \) and \( MR_{2ij}(\hat{\Theta}) \geq 0 \) for \( \forall \Theta_i < \Theta < \Theta^l, \delta_0 > \delta_0^l \) and \( i,l = 0,1 \).

For the case of Tier 0 leadership inequality \( MR_{00}(\hat{\Theta}) \leq MR_{20}(\hat{\Theta}) \) is simplified to

\[
(5\delta_0 - 2\beta)2K_{1}\hat{\Theta}^{\delta_0} + (5\delta_0 - 14\beta)2N(K_{0} + 2K_{1})K_{2}\hat{\Theta}^{\delta_0 - \beta} + (\delta_0 - 2\beta)24NK_{2}^{2}\hat{\Theta}^{2\delta_0 - \beta} >
6\beta(K_{0} + 2K_{1}) + 4\beta N\hat{\Theta}^{-\beta}(K_{0} + 2K_{1})^{2} + 2\beta\hat{\Theta}^{\beta}N^{-1}.
\]

(1.7.10)

Given an equal number of left- and right-hand-side terms in (1.7.10), we pair terms in presented order to yield three inequalities that are further combined into a single sufficient condition,

\[
\Theta > \Theta^0 = \max \left\{ \left( \frac{3\beta (K_{0} + 2K_{1})}{(5\delta_0 - 2\beta)K_{2}} \right)^{1/\gamma'_l}, \left( \frac{2\beta (K_{0} + 2K_{1})}{(5\delta_0 - 14\beta)K_{2}} \right)^{1/\gamma'_l}, \left( \frac{\beta}{12(\delta_0 - 2\beta)NK_{2}^{2}} \right)^{1/\gamma'_l} \right\}.
\]

(1.7.11)

It follows that for \( \delta_0 \geq \delta_0^0 = 3.6\beta \) and \( (K_{0} + 2K_{1})N\gamma K_{2}^{2} > K_{0}^{l} = \frac{5}{48} \), \( \Theta^0 = \left( \frac{3\beta (K_{0} + 2K_{1})}{(5\delta_0 - 12\beta)K_{2}} \right)^{1/\gamma'_l} \). An upper bound \( \Theta^0 \) is associated with constraint \( MR_{20} > 0 \) that can be simplified to

\[
((2\beta + \delta_0)K_{0} + (4\beta + 2\delta_0)K_{1})K_{2}\hat{\Theta}^{\delta_0} + (8\beta - 4\delta_0)K_{2}^{2}\hat{\Theta}^{2\delta_0} + \delta_0 K_{2}N^{-1}\hat{\Theta}^{\delta_0 - \beta} > 0
\]

(1.7.12)

Inequality (1.7.12) holds for \( \Theta < \Theta^0 = \left( \frac{(2\beta + \delta_0)K_{0} + (4\beta + 2\delta_0)K_{1}}{(\delta_0 - 2\beta)48K_{2}} \right)^{1/\gamma'_l} \). Comparing values of upper and lower bounds on initial stock of quality yields that \( \Theta^0 < \Theta^0 \) for \( \delta_0 > 2.8\beta \). Now consider conditions under which Tier 2
invests more than Tier 1, $MR_{2|0} > MR_{1|0}$, by following similar steps we find that the following restriction on $\Theta$ must hold.

$$\Theta > \max \left\{ \left( \frac{2\beta (K_0 + 2K_1)K_1}{(6\delta_0 - 12\beta)K_1 - (\delta_0 + 2\beta)K_0)K_2} \right)^{\frac{1}{\delta_0}}, \left( \frac{\delta_0 N^{-1}}{(\delta_0 - 2\beta)4K_2} \right)^{\frac{1}{\delta_0 - \beta}} \right\}$$

, where $\delta_0 > \beta \left( 2 + \frac{4K_0}{6K_1 - K_0} \right)$. It follows that $\Theta^0 > \left( \frac{\delta_0 N^{-1}}{(\delta_0 - 2\beta)4K_2} \right)^{\frac{1}{\delta_0 - \beta}}$ for $(K_0 + 2K_1)K_2 > \frac{\delta_0}{K_1}$ and $\delta_0 \in [3.6, 7.8]$. Therefore, when $(K_0 + 2K_1)K_2 > \max \left\{ \frac{\delta_0 (\delta_0 - 2\beta)}{4\beta (6\delta_0 - 12\beta)K_0 K_2}, \delta_0 \right\}$, $\delta_0 \in [3.6, 7.8]$ and $\Theta \in \left( \frac{3\beta (K_0 + 2K_1)K_1}{(5\delta_0 - 12\beta)K_2} \right)^{\frac{1}{\delta_0}}, \left( \frac{3\beta (K_0 + 2K_1)K_1}{(5\delta_0 - 12\beta)K_2} \right)^{\frac{1}{\delta_0 - \beta}}$ Tier 2 invests more than Tier 0 leader.

Similarly, for Tier 1 leadership, $MR_{2|1}(\Theta) > MR_{1|1}(\Theta)$ is simplified to inequality (1.7.13) and a lower bound on stock of quality $\Theta^1$ is presented in (1.7.14).

$$\Theta > \Theta^1 = \max \left\{ \left( \frac{\beta (K_0 + K_1)}{\delta_0 - 2\beta}K_2 \right)^{\frac{1}{\delta_0}}, \left( \frac{\beta (K_0 + K_1)}{3(\delta_0 - 2\beta)K_2} \right)^{\frac{1}{\delta_0}}, \left( \frac{\beta}{2(\delta_0 - 2\beta)K_2 N^{-1}} \right)^{\frac{1}{\delta_0 - \beta}} \right\} \text{ (1.7.14)}$$

It follows that for $\delta_0 > \delta_0^1 = 2\beta$ and $(K_0 + K_1)K_2 N^{-1} > K_1^1 = \frac{1}{2}$, $\Theta^1 = \left( \frac{\beta (K_0 + K_1)}{\delta_0 - 2\beta}K_2 \right)^{\frac{1}{\delta_0}}$. From comparison of values of upper and lower bounds it follows that $\Theta^1 < \Theta^0$ for $\delta_0 > 0$, which is always true given that $\delta_0 \geq 1$. The conditions to dominate Tier 0 by Tier 2 under Tier 1 leadership is $\Theta > \left( \frac{2K_0 \beta (K_0 + K_1)}{(6\delta_0 K_0 + (2\beta + \delta_0)K_1)K_2} \right)^{\frac{1}{\delta_0}}, \left( \frac{2K_0 \beta (K_0 + K_1)}{(6\delta_0 K_0 + (2\beta + \delta_0)K_1)K_2} \right)^{\frac{1}{\delta_0}} < \Theta^1$ which holds without any additional parameter restrictions, since $K_0^1 > -\frac{4\delta_0 + 4\beta}{2\beta + \delta_0}$. Similarly for Tier 1 let’s examine the case where Tier 2 yields highest supply chain profits, $MR_{3|1} > -MR_{0|1}$. This conditions is further simplified below to:

$$\left( (2\beta - \delta_0)2K_2 \Theta^0 + \beta \Theta^0 N^{-1} + 2\beta (K_0 + K_1) \right) (\Theta^0 N^{-1} + (2K_0 + K_1) + 2K_2 \Theta^0) > -\beta 2K_0 (K_0 + K_1) - 4K_0 (\beta - \delta_0)K_2 \Theta^0$$
Proof of Proposition 2: Technically, we prove that for all \( \delta_0 \in (1, 2\beta) \) and \( \frac{K_0}{K_1} \in (1, 4) \), there exist \( \{\gamma_0^m, \gamma_1^m\} \) where \( m \in p \) for market coverage, and \( m = \Theta \) for path of product quality expansion, such that \( \gamma_0^m > \gamma_1^m > \gamma \) and the following holds: Given a partition \( \{\Gamma_0^m, \Gamma_1^m, \Gamma_1^0\} = \{\gamma_0^m, \gamma_1^m, \gamma_0^1\} \) for \( m \in \{\Theta, p\} \).

(a.1) If \( \gamma \in \Gamma_k^p \) then \( \hat{\Theta}_{kl}(\gamma) = \max_{l \in \{0, 1, 2\}} \{\hat{\Theta}_{kl}(\gamma)\} \) for \( k \in \{0, 1, 2\} \);

(a.2) If \( \gamma \in \Gamma_k^\Theta \) then \( \rho_k(\gamma) = \max_{l \in \{0, 1, 2\}} \{\rho_l(\gamma)\} \) for \( k \in \{0, 1, 2\} \);

(b) \( \gamma_0^1 \geq \gamma_0^2 \); and \( \gamma_1^0 \geq \gamma_1^2 \);

(c) \( \frac{\partial \rho_k}{\partial \Theta}(\gamma) > 0 \);

(d) \( \forall K_2 > 0 \exists \Delta K_2 \) such that \( \gamma_1^1(K_2 + \Delta K_2) > \gamma_1^1(K_2) \).

First, for part (a.1), we prove the following claim.

There exists a unique value \( \Theta_{01}^{MR} \) in \( \{0, \Theta_{01}^\Theta\} \), where \( \Theta_{01}^\Theta = \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_0}} \), that solves \( MR_{01}(\Theta) = MR_{11}(\Theta) \). In addition, there exists a unique value \( \Theta_{12}^{MR} \) in \( \{\Theta_{01}^\Theta, \Theta_{12}^\Theta\} \), where \( \Theta_{12}^\Theta = \left(\frac{K_1 + 2K_0}{K_2}\right)^{\frac{1}{\delta_0}} \), that solves \( MR_{11}(\Theta) = MR_{21}(\Theta) \).

Proof: This claim examines behavior of marginal revenue objects only with respect to various levels of product quality \( \Theta \), and hence choice variable \( \hat{\Theta} \), the expansion of quality relative to \( \Theta \), does not appear.

The marginal revenue for a contract leader can be written as

\[
MR_{kl}(\Theta) = Q_l^1(\Theta)\Theta^{-1} \left(2\beta - \delta_0\right)^2 K_2 \Theta^{\delta_0} + \beta \Theta^{\delta_0} N^{-1} + 2\beta \left(2K_0 + 2^{l-1}K_1\right),
\]

(1.7.15)

where \( Q_l^1(\Theta) \) is given in Lemma 1. From (1.7.15), recognize that Tier 1’s marginal revenue exhibits a particular form, \( MR_{kl}(\Theta) = Q_l^1(\Theta)g_l(\Theta) \). Therefore, \( \Theta_{01}^{MR} \) solves \( \frac{g_1(\Theta_0^1)}{g_0(\Theta_0^1)} = \frac{g_1(\Theta_{01}^{MR})}{g_0(\Theta_{01}^{MR})} \). Furthermore, note that \( \frac{g_0(\Theta_0^1)}{g_1(\Theta_1^0)} = 1 \) holds, and

\[
g_1(\Theta_0^1) - 1 = \left(\frac{K_0 - K_1}{2^{\beta - \delta_0}} - 1\right) 2K_0 + 2K_1 + \frac{\beta}{\delta_0} \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_0}} N^{-1} > 0,
\]

(1.7.16)

i.e., \( g_1(\Theta_0^1) > g_0(\Theta_0^1) \) also holds. In addition,

\[
\frac{Q_1^1(0)}{Q_1^1(\Theta_0^1)} = \left(2 - \frac{3K_1}{2K_1 + K_0}\right)^2 > \left(2 - \frac{3K_1}{2K_1 + K_0}\right) = \frac{g_1(0)}{g_0(0)}.
\]

(1.7.17)
Next, we show that the difference of two objects, i.e., \( \frac{Q_i}{Q_1} - \frac{Q_2}{Q_2} \), is decreasing in \( \Theta \) for \( \Theta \in (0, \Theta_{01}^O) \), \( \delta_\theta \in (1,2\beta) \) and \( 4K_1 > K_0 > K_1 \). Expression for \( \frac{\partial}{\partial \Theta} \left( \frac{Q_i}{Q_1} - \frac{Q_2}{Q_2} \right) \) can be written as

\[
A_0(\Theta) \frac{\frac{\delta_\theta}{N}}{4K_2 \Theta^\delta_\theta + \beta - 1} + A_1(\Theta) \frac{12 \delta_\theta K_0 K_2 \Theta^\delta_\theta - 1}{(\Theta^\beta N - 1 + \Phi_0(\Theta))^2} + A_2(\Theta) \frac{\frac{\beta}{N}}{(K_0 - K_1) \Theta^\beta - 1} \tag{1.7.18}
\]

where

\[
A_0(\Theta) = \frac{1 - \alpha}{4} V - W, \quad A_1(\Theta) = \frac{1 - \alpha}{2} V - W, \quad A_2(\Theta) = \frac{1}{4} V - W,
\]

\[
V(\Theta) = \frac{(\Theta^\beta N - 1 + \Phi_0(\Theta))^2}{(\frac{1}{2} \Theta^\beta N - 1 + (1 - \alpha)4K_2 \Theta^\delta_\theta + 2K_1 + K_0)^2}, \tag{1.7.19}
\]

\[
W(\Theta) = \frac{\Theta^\beta N - 1 + \Phi_1(\Theta)}{\Theta^\beta N - 1 + \Phi_0(\Theta)},
\]

and \( \alpha = \frac{\delta_\theta}{2\beta} \). From (1.7.18), it follows that a sufficient condition for \( \frac{\partial}{\partial \Theta} \left( \frac{Q_i}{Q_1} - \frac{Q_2}{Q_2} \right) < 0 \) is

\[
A_1(\Theta) = \max \{ A_i(\Theta) \} < 0 \quad \text{for} \quad i = 0, 1, 2. \quad \text{Furthermore, this sufficient condition } A_1(\Theta) < 0 \text{ can be simplified to}
\]

\[
\left( \Theta^\beta N - 1 + \Phi_0(\Theta) \right)^3 < \left( \Theta^\beta N - 1 + \Phi_1(\Theta) + H(\Theta) \right)^2 \left( \Theta^\beta N - 1 + \Phi_1(\Theta) \right), \tag{1.7.20}
\]

where

\[
H(\Theta) = \left( 2^{\frac{1}{2}} \alpha - 1 \right) \Theta^\beta N - 1 + \left( 2^{\frac{1}{2}} \alpha - 1 - 2 \right) K_1 \Theta^\delta_\theta + \left( 2^{\frac{1}{2}} \alpha - 1 \right) K_1 + \left( 2^{\frac{1}{2}} \alpha - 2 \right) K_0 \tag{1.7.21}
\]

and \( \tilde{\alpha} = (1 - \alpha)^{-\frac{1}{2}} \). Recognize that \( H(\Theta) > 0 \) for \( K_0 < 4K_1 \) and \( \Theta < \Theta_{01}^O \). Furthermore, since \( \Phi_0(\Theta) < \Phi_1(\Theta) \) for \( \Theta < \Theta_{01}^O \), the sufficient condition \( A_1(\Theta) < 0 \) holds, i.e., \( \frac{\partial}{\partial \Theta} \left( \frac{Q_i}{Q_1} - \frac{Q_2}{Q_2} \right) < 0 \). Consequently, there exists the unique value \( \Theta_{01}^{MR} \) in \((0, \Theta_{01}^O)\) that solves \( M_{R_{01}}(\Theta) = M_{R_{11}}(\Theta) \). Similarly, the unique existence of \( \Theta_{12}^{MR} \) follows, which completes the proof of the claim.

\[
\square
\]

Now, denote \( \Theta_0^{MR} = M_{R_{11}} \left( \Theta_{01}^{MR} \right) \delta_D^{-1} \left( \Theta_{01}^{MR} \right)^{1-\delta_D} \) and \( \Theta_{12}^{MR} = M_{R_{12}} \left( \Theta_{12}^{MR} \right) \delta_D^{-1} \left( \Theta_{12}^{MR} \right)^{1-\delta_D} \). From the claim above, we have \( \Theta_{01}^{MR} > \Theta_{12}^{MR} \). Furthermore, if initial stock of quality \( \Theta < \Theta_{01}^{MR} \) product quality level, \( \Theta_{01}^{MR} \), is indeed optimally chosen by Tier 0 and 1 if marginal cost of product development crosses \( M_{R_{11}}(\Theta) \) from below at \( \Theta_{01}^{MR} \). Marginal cost of development \( \chi_{01}^{MR} \delta_D^{-1} \) for \( \delta_D > 2 \) is increasing by construction.
in $\hat{\Theta}$ and independent of $\Theta$. Tier 1 leader’s marginal revenue is increasing for $\Theta \leq \left( \frac{\beta(K_1+2K_0)}{\delta^0 - \beta} \right)^{\frac{1}{\delta^0}}$, such that $\Theta^{|MR}|_{01} \leq \Theta^{|MR}|_{01} < \left( \frac{\beta(K_1+2K_0)}{\delta^0 - \beta} \right)^{\frac{1}{\delta^0}}$. Hence, $MR_{|1|}(\Theta)$ and marginal cost of development are both increasing on $[0, \Theta^{|MR}|_{01}]$. Moreover, $\lim_{\Theta \to \infty} MR_{|1|}(\Theta) = 0$ and $\lim_{\Theta \to \infty} \frac{\partial MR_{|1|}(\Theta)}{\partial \Theta} = \infty$. Thus, there exists $\hat{\epsilon} > 0$, such that for all $\epsilon < \hat{\epsilon}$, $MR_{|1|}(\epsilon) > \Theta^{|MR}|_{01} \delta \epsilon^{\delta-1}$. Therefore, $\Theta^{|MR}|_{01} \delta \epsilon^{\delta-1}$ crosses $MR_{|1|}(\Theta)$ from below, since marginal cost of development contains point $(\Theta^{|MR}|_{01}, MR_{|1|}(\Theta^{|MR}|_{01}))$. By sub-modularity of profit function for both Tiers 0 and 1, $\Pi_l(\Theta, \gamma)$ for $l = 0, 1$, it follows that optimal product quality is decreasing in $\gamma$, i.e., $\frac{\partial \Pi_l(\gamma)}{\partial \Theta} < 0$. Consequently, $\forall \gamma > \Theta^{|MR}|_{01}, \Theta \leq \Theta^{|MR}|_{01}$ for $l = 0, 1$. Further, given some $\gamma > \Theta^{|MR}|_{01}$, Tier 1 optimally chooses $\hat{\Theta}_{|1|}(\gamma)$ such that $MR_{|1|}(\hat{\Theta}_{|1|}(\gamma)) = \gamma \delta \hat{\epsilon} (\hat{\Theta}_{|1|}(\gamma) \hat{\epsilon}^{\delta-1}$ and $MR_{|0|}(\hat{\Theta}_{|1|}(\gamma)) > MR_{|1|}(\hat{\Theta}_{|1|}(\gamma))$, resulting in $\hat{\Theta}_{|0|}(\gamma) > \hat{\Theta}_{|1|}(\gamma)$.

For the proof of (a.2), we first establish the existence of $\{\hat{\Theta}_{|1|}(\gamma)|_{11}, \hat{\Theta}_{|1|}(\gamma)|_{12}\}$ such that $\rho_j(\gamma|_{j,j+1}) = \rho_{j+1}(\gamma|_{j,j+1})$ for $j = 0, 1$. Figure 1.12 evaluating $MR_{|j|}(\Theta)$ and $\rho_j(\Theta)$ on $\Theta \in [0, 1]$ provides graphical illustration for the following proof: Recognize that $\rho_0(\gamma|_{01}) > \rho_1(\gamma|_{01})$ since at $\Theta^{|MR}|_{01}$ both Tiers 0 and 1 optimally choose $\Theta^{|MR}|_{01}$, where $\Theta^{|MR}|_{01} \leq \Theta^{|MR}|_{01}$ and $\rho_0(\Theta) \geq \rho_1(\Theta)$ for $\Theta \leq \Theta^{|MR}|_{01}$. Construct $\frac{\hat{\Theta}_{|01}}{\Theta^{|MR}|_{01}}$ such that $\rho_0(\hat{\Theta}_{|01}) = \rho_1(\hat{\Theta}_{|01})$, and let $\hat{\Theta}_{|01} = \Theta_{|1|}(\hat{\Theta}_{|01})$. Since $MR_{|1|}(\Theta) > MR_{|0|}(\Theta)$ for $\Theta > \Theta^{|MR}|_{01}$, it follows that $\frac{\hat{\Theta}_{|01}}{\Theta^{|MR}|_{01}} < \Theta^{|MR}|_{01}$. Hence, Tier 1 leadership yields the same coverage at lower development cost. Consider repeating this exercise of identifying value of $\gamma$ at which market coverage under Tier 1’s leadership, $\rho_1(\gamma)$, is equal to $\rho_0(\gamma)$ for values of $\gamma$ increasing from $\gamma_{01,0}$ to $\gamma_{01,1}$, where $\Theta_{01}(\gamma_{01,0}) = \Theta^{|MR}|_{01}$. Similarly, define $\gamma_{01,1}$ such that $\Theta_{1|1}(\gamma_{01,1}) = \Theta^{|MR}|_{01}$. Recognize that $\frac{\hat{\Theta}_{|01}}{\Theta^{|MR}|_{01}} > \gamma_{01,0}$ since $MR_{|1|}(\Theta) > MR_{|0|}(\Theta)$ for $\Theta > \Theta^{|MR}|_{01}$. In this case, Tier 1’s leadership yields the same market coverage, $\rho_0(\gamma_{01,0})$, as observed under Tier 0’s leadership. Mapping from values of $\gamma \in [\gamma_{01,0}, \gamma_{01,1}]$ to development cost $\gamma$ such that $\rho_1(\gamma) = \rho_0(\gamma)$ is continuous, since $MR_{|1|}(\Theta)$ is continuous and $\max \{\rho_0(\Theta)\} = \max \{\rho_1(\Theta)\}$ as indicated in (1.7.22), where $2K_1 + K_0 < K_1 + 2K_0$ since $K_0 > K_1$ is necessary for existence of handover between Tier 0 and 1.

$$\max_{\Theta > 0} \{\rho_1(\Theta)\} = N \left( 1 + 2\delta \left( 2^{\frac{1}{\beta^0}} - 1 \right) K_2 \right)^{\frac{1}{\delta^0}} \left( \frac{2^{\frac{1}{\beta^0}} - 1}{\delta^0 - \beta} \right)^{\frac{\delta^0 - \beta}{\delta^0}} N^{-1} \cdot (1.7.22)$$

By construction of this coverage under Tier 0’s leadership is evaluated at increasing values of $\gamma$ indicated by 45-degree line in panel (d) of Figure 1.12. Corresponding sequence of $\hat{\gamma}$ for Tier 1 yielding coverage $\rho_1(\gamma)$ for $\gamma > \Theta^{|MR}|_{01}$. If $\lim_{\Theta \to \infty} \max_{\Theta > 0} \{\rho_0(\Theta)\} < \Theta^{|MR}|_{01}$, then sequence of $\hat{\gamma}$ is at first increasing for $\gamma \in [\gamma_{01,0}, \gamma_{01,1}]$ and then decreasing for $\gamma \in [\gamma_{01,1}, \gamma^{|MR}|_{01}]$. Where
\( \Theta_{00}(\gamma_{00}^{\text{max}}) = \arg \max_{\Theta > 0} \{ \rho_0(\Theta) \} \). In both cases there exists a unique \( \gamma_{01}^0 \) such that \( \rho_0(\gamma_{01}^0) = \rho_1(\gamma_{01}^0) \). Similarly, one establishes existence of \( \gamma_{12}^0 \).

The proof of part (b) follows from the proof of parts (a.1) and (a.2) where \( \gamma_{01}^\beta > \gamma_{01}^0 \) and \( \gamma_{12}^\beta > \gamma_{12}^0 \).

For the proof of part (c), first, note that

\[
\frac{\partial}{\partial K_2} \left( \frac{g_1(\Theta)}{g_0(\Theta)} \right) = - \frac{(6 K_0 + \Theta \beta N^{-1})(2 \beta - \delta_0) 2 \Theta \delta_0}{\left( \left( 2 - \frac{\delta_0}{\beta} \right) 4 K_2 \Theta \delta_0 + \Theta \beta N^{-1} + 2 (K_0 + 2 K_1) \right)^2}. \tag{1.7.23}
\]

Furthermore, we also obtain

\[
\frac{\partial}{\partial K_2} \left( \frac{Q_0}{Q_1} \right) < - \frac{2 ((\delta_0 - \beta) 2 K_2 \Theta \delta_0 + \beta (K_0 - K_1)) \Theta \beta - 1}{\left( \Theta \beta N^{-1} + \Phi_0(\Theta) \right)^2} + (12 (\beta + (\delta_0 - \beta) K_0) K_2 \Theta^{-1} + K_0 (\delta_0 - 2 \beta)) 12 \Theta \delta_0 > 0, \tag{1.7.24}
\]

In addition, the following inequality,

\[
(2(\delta_0 - \beta) K_2 \Theta^{-1} + \delta_0 - 2 \beta) 2 N^{-1} \Theta \delta_0 + \beta (K_0 - K_1) N^{-1} \Theta \beta^{-1} + + (12 (\beta + (\delta_0 - \beta) K_0) K_2 \Theta^{-1} + K_0 (\delta_0 - 2 \beta)) 12 \Theta \delta_0 > 0, \tag{1.7.25}
\]

we obtain that the numerator of right-hand side expression in (1.7.24) is strictly greater than the numerator of (1.7.23). Moreover, the denominator of right-hand side expression in (1.7.24) is strictly smaller than the denominator of (1.7.23). As a result, \( \frac{Q_0}{Q_1} \) decreases with respect to \( K_2 \) at a higher rate relative to \( \frac{g_1(\Theta)}{g_0(\Theta)} \). Consequently, it follows that \( \frac{\partial \Theta_{01}^{HR}(K_2)}{\partial K_2} < 0 \), which in turn leads to \( \frac{\partial \gamma_{01}^\beta(K_2)}{\partial K_2} > 0 \).

For the proof of part (d), first, recall, that we established existence of \( \gamma_{01}^0 \) on an interval \( (\gamma_{01}^{01,1}, \gamma_{01}^{01}) \), where \( \Theta_{01}^{01} = \Theta_{11}(\gamma_{01}^{01,1}) \) and \( \Theta_{01}^{HR} = \Theta_{11}(\gamma_{01}^{01}) \). Since \( \Theta_{01}^{01} = \left( \frac{K_0 - K_1}{2 K_2} \right) \theta_{01} \), it follows that \( \frac{\partial \Theta_{01}^{HR}(K_2)}{\partial K_2} < 0 \) which implies that \( \frac{\partial \gamma_{01}^0(K_2)}{\partial K_2} > 0 \). Hence, \( \gamma_{01}^0 \) is interior to the interval where both endpoints are strictly increasing in \( K_2 \). For every \( K_2 \) we construct \( \Delta K_2 > 0 \) such that \( \gamma_{01}^0(K_2 + \Delta K_2) = \gamma_{01}^0(K_2) \), which guarantees that \( \gamma_{01}^0(K_2 + \Delta K_2) > \gamma_{01}^0(K_2) \). Similar analysis applies to the case of handover between Tiers 1 and 2.

Figure 1.12 illustrates the threshold of the \( \gamma \) value between Tier 0 and Tier 1 that generates the same market coverage. As illustrated in panel (c), at \( \gamma = \gamma_{01}^0 \), if Tier 0 is the contract leader, he makes an effort into attaining the product quality of \( \Theta_0 \), whereas if Tier 1 is the contract leader, she puts effort into achieving the product quality of \( \Theta_1 \). Moreover, at those quality levels of \( \Theta_0 \) and \( \Theta_1 \) under the corresponding contract leadership, the resulting market coverages are the same as those depicted in the lower part of panel (c). If \( \gamma > \gamma_{01}^0 \), Tier 0 contract leadership leads to broader market coverage; otherwise, Tier 1 contract leadership
results in more coverage. Panel (d) illustrates how to find this threshold of $\gamma$; in this panel, the x-axis is the $\gamma$ value for Tier 0 contract leadership and the y-axis is the corresponding $\gamma$ value for Tier 1 contract leadership that leads to the same market coverage. For example, if $\gamma = \gamma_{01}^Q$, i.e., at the maximum $\gamma$ in panel (d), as depicted in panel (a), the product quality becomes $\Theta_{01}^{MR}$ under Tier 0 contract leadership. In order to attain the same market coverage under Tier 1 contract leadership as illustrated in the lower part of panel (a), the product quality should be $\hat{\Theta}$, which is achieved at $\gamma = \gamma_{01}^Q (< \gamma_{01}^Q)$ under Tier 1 contract leadership. In contrast, the opposite holds at the minimum $\gamma = \gamma_{01}^Q > \gamma_{01}^Q$ in panel (d), i.e., as depicted in panel (b), at $\gamma = \gamma_{01}^Q$, the product quality becomes $\Theta_{01}^Q$ under Tier 0 contract leadership. The same market coverage is achieved at $\gamma = \gamma_{01}^Q (> \gamma_{01}^Q)$ under Tier 1 contract leadership. Finally, we can prove that as in panel (d), there exists the unique $\gamma = \gamma_{01}^Q \in (\gamma_{01}^Q, \gamma_{01}^Q)$, such that Tier 0 or Tier 1 contract leadership yields the same market coverage.

In addition, as stated in part (b) of Proposition 2 and illustrated in panel (d), this threshold $\gamma = \gamma_{01}^Q$, that results in the same market coverage, is smaller than the threshold $\gamma = \gamma_{01}^Q$, which leads to the same product qualities as well as the same total supply chain profits for Tier 0 contract leadership and Tier 1 contract leadership.

**Proof of Proposition 3:** Technically, we prove that $\exists \gamma_{j, j+1}^{JR}$ such that $\Pi_{j, j+1}(\gamma_{j, j+1}^{JR}) > \Pi_{j, j+1}(\gamma_{j, j+1}^{JR})$ for $j = 0, 1$ and $\gamma_{01}^{JR} > \gamma_{12}^{JR}$. First, consider individual rationality constraints that must be satisfied for handover from Tier 0 to Tier 1 to occur. As indicated in (1.7.26), incumbent leader’s profit must be higher under Tier 1’s leadership and Tier 1’s profit, when in a position of a leader, must exceed the one attained under Tier 0’s leadership. Hence, we proceed to identify the highest, i.e., the first from product lifecycle perspective, value of development cost, $\gamma_{01}^{JR}$, at which conditions (IR$_{01}^0$) and (IR$_{01}^1$) are both satisfied, where

$$
\begin{align*}
\text{(IR}^0_{01}) & : \quad \Pi_{0|0}(\gamma) < \Pi_{0|1}(\gamma), \\
\text{(IR}^1_{01}) & : \quad \Pi_{1|0}(\gamma) < \Pi_{1|1}(\gamma).
\end{align*}
$$

(1.7.26)

Given profit expressions in (1.7.6) individual rationality constraints necessary for leadership handover from Tier 0 to 1 are reduced to inequalities in (1.7.27), where product qualities optimally selected by Tier 0 and 1 leaders are $\Theta_0 = \Theta_{0|0}(\gamma)$ and $\Theta_1 = \Theta_{1|1}(\gamma)$ for some $\gamma$

$$
\begin{align*}
\text{(IR}^0_{01}) & : \quad G_0(\Theta_0)Q_0^1(\Theta_0) < K_0Q_1^2(\Theta_1), \\
\text{(IR}^1_{01}) & : \quad (K_1 + 2K_2\Theta_0^0)Q_0^1(\Theta_0) < G_1(\Theta_1)Q_1^2(\Theta_1),
\end{align*}
$$

(1.7.27)

where
Figure 1.12: Threshold for the contract leadership transfer between Tier 0 and Tier 1

**Note:** Transfers illustrated above were designed to attain broader market coverage in part (a) of Proposition 2. Parameter values are \((K_0, K_1, K_2) = (10, 2, 1), \delta_0 = 1.7, \beta = 0.9, \) and \(N = 1.\) The figures above are for the initial investment stage where stock of quality is zero, \(\Theta = 0.\)
Figure 1.13: Individual Rationality of Tier 0 to Tier 1 handover

Note: Parameter values are \((K_0, K_1, K_2) = (6, 1, 2), \delta_0 = 1.01, \beta = 0.99, \delta_0 = 2, \text{ and } N = 1.\)
\[ G_l(\Theta_l) = \left( 1 - \frac{2\beta - \delta_0}{\delta_D} \right) 2^{[2 - j]} K_2 \Theta_l^\delta_0 + \left( 1 - \frac{\beta}{\delta_D} \right) \Theta_l^\beta N^{-1} + \left( 1 - \frac{2\beta - \delta_0}{\delta_D} \right) 2^{[0 - j]} K_0 + 2^{[1 - j]} K_1 \]  

for \( l = 0, 1, 2 \).

Panels (b) and (c) of Figure 1.13 contain plots of objects on each side of \((IR_0^0)\) and \((IR_0^1)\) inequalities correspondingly. Recognize that right-hand sides of inequalities in (1.7.27) are functions of Tier 1 leader’s choice of product quality, \( \Theta_1 \), while left-hand sides depend only on \( \Theta_0 \). This distinction between dependence on \( \Theta_0 \) and \( \Theta_1 \) is graphically indicated by the use of solid and dashed lines correspondingly. Further, panel (a) of Figure 1.13 identifies optimal product quality level \( \Theta_{0|1}(\gamma) \) via intersection of marginal revenue of investment \( MR_{0|1}(\Theta) \) and marginal cost of development, \( \gamma \delta_D \Theta^\delta_0 - 1 \) for \( l = 0, 1 \).

To establish existence of \( \gamma_{01}^{\text{offer}} = \max \{ \gamma \mid \Pi_{0|0}(\gamma) < \Pi_{0|1}(\gamma) \} \) at which Tier 0 incumbent leader would prefer Tier 1 to become a contract leader instead, we prove the claim below:

If \( \frac{\delta_2 - \beta}{2N} \left( \frac{K_0 - K_1}{2K_2} \right) \frac{\partial}{\partial \Theta} = (3\beta - \delta_0 - \delta_D) K_0 + \delta_0 K_1 \) and \( \beta \geq \frac{\delta_0}{4} \), then \( \Theta_0 = \left( \frac{K_0 - K_1}{2K_2} \right) \frac{\partial}{\partial \Theta} \) such that \( G_0(\Theta_0)Q_0^2(\Theta_0) = K_0Q_1^2(\Theta_0) \).

**Proof:** In Claim 1.3 we have established existence of \( \Theta_{0|1}^{MR}(\Theta_0) \). Given \( \Theta_{0|1}^{MR}(\Theta_0) \), we show that \( \frac{K_0}{G_0(\Theta_0)} \) uniquely crosses \( \frac{Q_0^2(\Theta_0)}{Q_1^2(\Theta_0)} \) at \( \Theta_0 = \left( \frac{K_0 - K_1}{2K_2} \right) \frac{\partial}{\partial \Theta} \) when \( \frac{\delta_0 - \beta}{\delta_D} \left( \frac{K_0}{G_0(\Theta_0)} \right) < 0 \), \( \frac{K_0}{G_0(\Theta_0)} > \frac{Q_0^2(\Theta_0)}{Q_1^2(\Theta_0)} \) and \( \lim_{\Theta \to \infty} \frac{K_0}{G_0(\Theta_0)} < \lim_{\Theta \to \infty} \frac{Q_0^2(\Theta_0)}{Q_1^2(\Theta_0)} \) as indicated in (1.7.29) and (1.7.30) for \( \beta \geq \frac{\delta_0}{4} \). Further, \( \frac{\delta_0 - \beta}{\delta_D} \left( \frac{K_0}{G_0(\Theta_0)} \right) < 0 \) holds for all \( \Theta \) since \( G_0(\Theta) \) is strictly increasing in \( \Theta \), from (1.7.28). Figure 1.14 provides graphical intuition for the proof of this claim.

\[ \frac{K_0}{G_0(\Theta)} = \frac{K_0}{\left( 1 - \frac{2\beta - \delta_0}{\delta_D} \right) (K_0 + 2K_1)} \geq \frac{1}{3 \left( 1 - \frac{2\beta - \delta_0}{\delta_D} \right)} \geq 1 \geq \left( 2 - \frac{3K_1}{2K_1 + K_0} \right)^2 = \frac{Q_0^2(\Theta)}{Q_1^2(\Theta)} \]  

(1.7.29)

\[ \lim_{\Theta \to \infty} \frac{K_0}{G_0(\Theta)} = 0 < \frac{1}{4} = \lim_{\Theta \to \infty} \left( \frac{1 + 2K_2 \Theta^\delta_0 - \Theta^\beta (K_1 + K_0)}{1 + 4K_2 \Theta^\delta_0 - \Theta^\beta (2K_1 + K_0)} \right)^2 = \lim_{\Theta \to \infty} \frac{Q_0^2(\Theta)}{Q_1^2(\Theta)}. \]  

(1.7.30)

Existence of \( \gamma_{01}^{\text{offer}} \) is established by performing the following exercise. For each \( \gamma \in (\gamma_{01}, \gamma_{01}) \), we identify value \( \tilde{\gamma} \) such that \( K_0Q_0^2(\tilde{\gamma}) = G_0(\gamma)Q_0^2(\gamma) \), where \( \gamma_{01} = \lim_{\Theta \to \infty} \frac{\delta_0}{\delta_D} \left( \frac{K_0}{G_0(\Theta)} \right) = \frac{N^2}{2K_1 + K_0} \). Value of \( \gamma_{01}^{\text{offer}} \) is such that \( \Theta_{0|0}(\gamma_{01}) = \Theta_{1|1}(\gamma_{01}) = \Theta_{0|1}^{MR}(\gamma_{01}) \), which was established in proof of Proposition 2 where \( \Theta_{0|1}^{MR} \) exists on \( \left( 0, \left( \frac{K_0 - K_1}{2K_2} \right) \frac{\partial}{\partial \Theta} \right) \). From Claim 1.3 it follows that \( \Theta_{0|1}^{MR} \leq \tilde{\Theta}_0 \). Further, \( G_0(\gamma_{01})Q_0^2(\gamma_{01}) < K_0Q_1^2(\gamma_{01}) \) and
Figure 1.14: Reference points for Proposition 3 proof

Note: Parameter values are the same as those in Figure 13.

...
We proceed to show existence of \( \gamma^{\text{accept}}_{01} \) on \((\tilde{\gamma}, \tilde{\gamma}_0)\) where \( \hat{\Theta}_1 = \Theta_{01}(\tilde{\gamma}) \). For each \( \gamma \in (\tilde{\gamma}, \tilde{\gamma}_0) \) we follow a similar steps of identifying value of \( \dot{\gamma} \) such that \( \left( K_1 + 2K_2\Theta_{01}(\gamma) \right) Q_0^2(\gamma) = G(\gamma)Q_1^2(\gamma) \). As was established in Claim 1.3, \( \hat{\Theta}_1 \geq \left( \frac{K_0-K_1}{2K_2} \right)^{\beta} \) and hence \( \dot{\gamma} > \tilde{\gamma} \). Similarly, Tier 0 leader chooses not to invest and hence \( \left( K_1 + 2K_2\Theta_{01}(\gamma) \right) Q_0^2(\gamma) = G(\gamma)Q_1^2(\gamma) = 0 \). However, \( \dot{\gamma} \) at which Tier 1 leader chooses not to invest is \( \tilde{\gamma}_1 < \tilde{\gamma}_0 \) as discussed in Claim 1.3. Therefore, by IVT there exists \( \gamma^{\text{accept}}_{01} \) at which Tier 1 is ready to accept leadership from Tier 0. Recognize that \( \lim_{\delta_0 \to 1+} (\hat{\Theta}_1) = \hat{\Theta}_0 \). Therefore, \( \gamma^{R}_{01} = \max \left\{ \gamma^{\text{offer}}_{01}, \gamma^{\text{accept}}_{01} \right\} \). Similar analysis applies to the case of handover between Tier 1 and Tier 2.

**Proof of Proposition 4:** Technically, we show that there exists a range of upstream production cost coefficients, \([\tilde{K}_2, \sigma K_2] \) for \( \sigma > 1 \) and some relatively low development cost coefficient \( \tilde{\gamma} \) such that \( \Theta_{01}(\tilde{\gamma}, \sigma K_2) = \max_{\gamma \in [0,1,2]} \{ \Theta_{01}(\tilde{\gamma}, \sigma K_2) \} \) and \( \Theta_{01}(\tilde{\gamma}, K_2) = \max_{\gamma \in [0,1,2]} \{ \Theta_{01}(\tilde{\gamma}, K_2) \} \), where with abuse of notation we indicate dependence of optimal product quality \( \Theta_{01}(\tilde{\gamma}, \sigma) \) on both development and production cost coefficients. Let \( \hat{\gamma} = MR_{11}(\Theta_{01}) \delta_D^{-1} \left( \Theta_{01}^{MR} \right)^{1-\delta}_D \) under high upstream production cost regime \( K_2 = \sigma K_2 \) as indicated by subscript \( h \). Hence, one needs to find \( \sigma \) such that \( \hat{\gamma} > MR_{11}(\Theta_{01}) \delta_D^{-1} \left( \Theta_{01}^{MR} \right)^{1-\delta}_D \), where \( l \) indicates low production cost regime, \( K_2 = \tilde{K}_2 \). Recall that values of \( \Theta_{01}^{MR} \) and \( \Theta_{12}^{MR} \) are contained in the intervals \([0, \left( \frac{K_0-K_1}{2K_2} \right) \frac{1}{\gamma_0}] \) and \([\left( \frac{K_0-K_1}{2K_2} \right) \frac{1}{\gamma_0}, \left( \frac{K_1+2K_0}{\sigma K_2} \right) \frac{1}{\gamma_0}] \) correspondingly. Therefore, we construct below a sufficient condition for existence of reversal in leadership assignment by using right end-points of these intervals instead of \( \Theta_{01}^{MR} \) and \( \Theta_{12}^{MR} \).

\[
\left(3-\frac{\delta_0}{\beta}\right)K_0 + \frac{\delta_0}{\beta}K_1 + \frac{1}{2\gamma_0} \left( \frac{K_0-K_1}{2K_2} \right) \frac{1}{\gamma_0} \leq \left( \frac{1}{\gamma_0} \left( \frac{K_0-K_1}{2K_2} \right) \frac{1}{\gamma_0} + K_0 \right) \left( \frac{\sigma (K_0-K_1)}{2(K_1+4K_0)} \right)^{\delta_0-2\beta} .
\]

**Proof of Proposition 5:** Consider expressions below for total supply chain revenues, \( R_l(\Theta) \) for \( l = 0,1,2 \), where production but not development costs are taken into account

\[
R_0^l(\Theta) = Q_0^l(\Theta_l)(\Phi_0(\Theta_l)) + \Theta_0^l N^{-1} + K_1 + 3K_2\Theta_1^\delta ,
\]

\[
R_1^l(\Theta) = Q_1^l(\Theta_l)(\Phi_1(\Theta_l)) + \Theta_1^l N^{-1} + K_0 + K_2\Theta_1^\delta ,
\]

\[
R_2^l(\Theta) = Q_2^l(\Theta_l)(\Phi_2(\Theta_l)) + \Theta_2^l N^{-1} + 3K_0 + K_1 .
\]

Given one-shot revenue expressions above consider net present value of two-period profits, \( \Pi_{l_1 \rightarrow l_2}(\Theta_1, \Theta_2) \), where \( \Theta_1 \) is chosen by \( l_1 \) in the first period and \( \Theta_2 \) is optimally chosen by \( l_2 \) in the second period such that \( \Theta_2 > \Theta_1 \).
\[ \Pi_{l_1 \rightarrow l_2} = \frac{1}{3} \left[ R_{l_1} (\Theta_1) - \gamma_1 \Theta_1^2 \right] + \frac{\alpha}{3} \left[ R_{l_2} (\Theta_2) - (\gamma_2 \Theta_2^2 - \gamma_2 \Theta_1^2) \right]. \]

Net present value of revenues when investor bears the entirety of development cost is

\[ R_{l_1 \rightarrow l_2} = \frac{1}{3} R_{l_1} (\Theta_1) + \frac{\alpha}{3} R_{l_2} (\Theta_2). \]

If development cost is fully on the investor in innovation then

\[ \begin{align*} 
\Pi^I_{l_1 \rightarrow l_2} &= R_{l_1 \rightarrow l_2} - \gamma_1 \Theta_1^2 \\
\Pi^I_{l_2 \rightarrow l_1} &= R_{l_1 \rightarrow l_2} - \alpha (\gamma_2 \Theta_2^2 - \gamma_2 \Theta_1^2) 
\end{align*} \]

Each investor individual faces his own optimization. First investor does not have any constraints.

\[ \begin{align*} 
\frac{\partial \Pi^I_{l_1 \rightarrow l_2}}{\partial \Theta_1} &= \frac{1}{3} MR_{l_1} (\Theta_1) - 2 \gamma_1 \Theta_1 \\
\frac{\partial \Pi^I_{l_1 \rightarrow l_2}}{\partial \Theta_2} &= \frac{\alpha}{3} MR_{l_2} (\Theta_2) - 2 \alpha \gamma_2 \Theta_2 \\
\text{s.t.} \quad &\Theta_2 \geq \Theta_1 
\end{align*} \]

Rewriting net present value profits as shown below illustrates that \( \Pi_{l_1 \rightarrow l_2} \) is separable in \( \Theta_1 \) and \( \Theta_2 \)

\[ \Pi_{l_1 \rightarrow l_2} = \left[ \frac{1}{3} R_{l_1} (\Theta_1) - \frac{1}{3} (\gamma_1 - \alpha \gamma_2) \Theta_1^2 \right] + \left[ \frac{\alpha}{3} R_{l_2} (\Theta_2) - \frac{\alpha \gamma_2}{3} \Theta_2^2 \right]. \]

Instead of treating above maximization problem as a constrained maximization of \( \Pi_{l_1 \rightarrow l_2} \) with respect to \( \Theta_1 \) and \( \Theta_2 \) subject to \( \Theta_2 \geq \Theta_1 \) we consider an unconstrained problem first and then impose parameter constraints that guarantee that resulting \( \Theta_2 \geq \Theta_1 \). Taking partial derivates yields the following expression below

\[ \begin{align*} 
\frac{\partial \Pi_{l_1 \rightarrow l_2}}{\partial \Theta_1} &= \frac{1}{3} MR_{l_1} (\Theta_1) - \frac{2}{3} (\gamma_1 - \alpha \gamma_2) \Theta_1 \\
\frac{\partial \Pi_{l_1 \rightarrow l_2}}{\partial \Theta_2} &= \frac{\alpha}{3} MR_{l_2} (\Theta_2) - \frac{2 \alpha \gamma_2}{3} \Theta_2 
\end{align*} \]

Similar to graphical nature other proofs the optimal stock of quality is the intersection of marginal revenue and linear marginal development cost. Marginal Revenue expressions are listed below,
than established then it remains to enforce condition guaranteeing MR the most in a one shot setting. Similarly, to Prop 2 we at first establish existence of \( g \) 0-0, 1-1 and 2-2, it suffices to show that needs to establish that when marginal cost of development is

\[
MR_0(T) = Q_0^2 \Theta^{-1} \left[ 2 \left( \beta(K_0 + 2K_1) - 4(\delta_0 - \beta)K_2 \Theta^\delta \right) \left( 1 + \frac{K_1 + 3K_2 \Theta^\delta}{\Theta \Phi_0^{m-1} + \Phi_0} \right) + \left( \frac{\delta_0 K_2 \Theta^\delta + \beta \Theta^{2} N^{-1}}{\Phi_0^{m-1} + \Phi_0} \right) \right]
\]

\[
MR_1(T) = Q_1^2 \Theta^{-1} \left[ 2 \left( \beta(2K_0 + K_1) - 2(\delta_0 - \beta)K_2 \Theta^\delta \right) \left( 1 + \frac{K_1 + 3K_2 \Theta^\delta}{\Theta \Phi_1^{m-1} + \Phi_1} \right) + \left( \frac{\delta_0 K_2 \Theta^\delta + \beta \Theta^{2} N^{-1}}{\Phi_1^{m-1} + \Phi_1} \right) \right]
\]

\[
MR_2(T) = Q_2^2 \Theta^{-1} \left[ 2 \left( \beta(4K_0 + 2K_1) - (\delta_0 - \beta)K_2 \Theta^\delta \right) \left( 1 + \frac{3K_0 + K_1}{\Theta \Phi_2^{m-1} + \Phi_2} \right) + \left( \frac{\delta_0 K_2 \Theta^\delta - 1 + \beta \Theta^{-1} N^{-1}}{\Phi_2^{m-1} + \Phi_2} \right) \right]
\]

To show that there exist \( \gamma_1 \) and \( \gamma_2 \) such that product quality outcomes \( \Theta^\gamma \gamma_1 \) and \( \Theta^\gamma \gamma_2 \) associated with upstream leadership handovers in sequences 0-1 0-2 dominate those under static leadership sequences 0-0, 1-1 and 2-2, it suffices to show that needs to establish that when marginal cost of development is \( 2 \gamma \) \( (\gamma_1 - \alpha \gamma_2) \) Tier 0 invests the most, and when marginal cost of development is \( 2 \alpha \gamma_2 \) Tier 1 or Tier 2 invests the most in a one shot setting. Similarly, to Prop 2 we at first establish existence of \( \gamma_0 \) and \( \gamma_2 \) such that

\[
MR_0(T^*(\gamma_0))) = MR_0(T^*(\gamma_0))) \text{ and } MR_0(T^*(\gamma_1))) = MR_0(T^*(\gamma_2))) .
\]

From existence of \( \gamma_0 \) and \( \gamma_2 \) it follows that we can pick \( \gamma_1 \) and \( \gamma_2 \) such that \( 2(\gamma_1 - \alpha \gamma_2) > \gamma_0 \) and \( 2 \alpha \gamma_2 < \alpha \gamma_0 \) and \( 2 \alpha \gamma_2 > \alpha \gamma_2 \). These three inequalities are further simplified to \( 2 \gamma_1 > 2(1 + \alpha) \gamma_2 \) and \( 2 \gamma_2 > \gamma_2 \). Once existence of \( \gamma_1 \) and \( \gamma_2 \) are established then it remains to enforce condition guaranteeing \( \Theta \gamma_1 > \Theta \gamma_2 \). This insured by \( 2 \gamma_2 \) being lower than \( \gamma \) associated with intersection of \( 2 \gamma_0 \) and \( 2 \gamma_1 \).

\[ \square \]
B Analysis of Joint Investment

In this paper we have examined market coverage outcomes resulting from single investor scenarios in which Tier \( i \) investor under Tier \( l \) contract leadership is directly investing in product development, which yields final product quality and, hence, is the only investor. However, all tiers may invest jointly and the final product quality may become a function of these joint contributions. In order to understand whether our results remain valid under this joint investment case, we perform a numerical study of joint investment levels within an equilibrium framework where the Tier \( i \) choice of product quality under Tier \( l \) leadership, \( \tilde{Q}_i \mid l \) \( (\gamma) \), depends on other investors’ choices of product quality \( \tilde{Q}_{-i \mid l} \) \( (\gamma) \). Furthermore, we let final product quality be additive in individual investment levels, such that \( \tilde{Q}_\text{joint} \mid l = \sum_{i=0}^{2} \tilde{Q}_i \mid l \).

Panel (a) of Figure 1.16 plots the market coverage resulting from joint investments across different leader locations, \( \rho_l \left( \tilde{Q}_\text{joint} \mid l \right) \) for \( l = 0, 1, 2 \). Recognize that our primary results remain valid under this joint investment case; specifically, there exist \( \gamma_{12}^0, \gamma_{01}^0 \), development cost levels at which handing over leadership across tiers allows to maintain the broadest market coverage. Under high development costs \( (\gamma > \gamma_{12}^0) \), Tier 0 contract leadership yields the most inclusive innovation, and under intermediate development costs \( (\gamma \in (\gamma_{12}^0, \gamma_{01}^0)) \), Tier 1 contract leadership leads to the most inclusive innovation. Finally, under small \( \gamma \) \( (< \gamma_{12}^0) \), Tier 2 contract leadership is optimal from the market coverage perspective.

Panel (b) of Figure 1.16 examines tierwise investment in product development for \( (\gamma > \gamma_{10}^0) \), where market coverage from joint investment is highest under Tier 0 leadership. This figure indicates that for \( \gamma > \gamma_{10}^0 \) product quality contributions by non-leader Tiers 1 and 2 are negligible relative to Tier 0 leader’s investment in product development, \( \tilde{Q}_{00} \) \( (\gamma) \). This illustration aligns with the intuition provided by Proposition 1, where the contract leader invests the most, justifying our focus on the cases in which the contract leader is a single

**Figure 1.16: Joint Investment**

**Note:** Parameter values are \((K_0, K_1, K_2) = (0.3, 0.2, 0.1)\), \( \delta_0 = 2 \), \( \beta = 1 \), \( N = 1 \), and \( \delta_0 = 2 \).
dominant investor. Overall, this illustration extends the relevance of our normative recommendations about the contract leader’s location yielding the highest market coverage, i.e., inclusive innovation.
References for Chapter 1


Chapter 2

Predictive Accuracy Comparison of Electricity Price Forecasting Methods for Energy Storage Valuation and Dispatch

2.1 Introduction

The problem of forecasting wholesale prices for electricity has generated an extensive empirical literature. This is due both to its importance in practice, through the needs of market agents in what is becoming a more and more deregulated market, as well as due to the availability of a large publicly available dataset on which the latest forecasting method can be applied. Many forecasting methods have been suggested as appropriate approaches. Typically standard approaches to evaluating the quality of the forecasts, such as mean absolute loss, are employed rather than directing the evaluation towards actual uses of price forecasts in a realistic decision making setting. Regardless of the particular statistic employed for evaluation, the outcome of these exercises is a statistical average although it is rarely the case that these estimates are subjected to a formal statistical evaluation.

This paper makes contributions on four fronts. First, our primary contribution is that we employ a loss function that is directly relevant to one of the uses of wholesale electrical energy price forecasts. In
particular we consider the opportunity for intra-day demand shift of energy for a storage technology in a grid connected to the California wholesale electricity (i.e. is able to buy and sell in the day ahead market (DAM) run by the California ISO (CAISO). Forecasts are made in accordance with the time schedules required by the participation in the DAM, and the loss function is the revenue generated through forecasting the next days prices at hourly intervals then contracting for purchases and sales in the DAM. By directly evaluating forecasts on the application of those forecasts we are better able to evaluate which methods might be appropriate as well as develop an understanding of how well forecast methods capture potential revenues. We also report the standard methods for comparison. This is described more carefully in section 2.2.

Second, we examine a wide set of methods that have been employed in forecasting electricity prices and compare them on a level playing field. Many models have been suggested for electricity price forecasting. Comprehensive recent reviews are available in Aggarwal et al. (2009) and Weron (2014). Conejo et al. (2005) provides a comparison over a number of forecast methods based on standard measures of performance. We evaluate a selected set of methods that have proven to be popular in the literature using a common estimation and evaluation sample, thus comparing the methods on a level playing field. The models we examine are ARMAX methods and their variations, artificial neural nets, exponential smoothing, locally linear ARX models, and cubic splines.

Third, we evaluate carefully the results statistically, extending the standard analysis of the performance of methods. It is often the case in comparisons of methods that the mean absolute errors are calculated on a 'test' sample and compared - if the numbers are close then the methods are considered equivalent however if one method is seen to be a bit smaller it is declared to be better. Such a comparison is true for that test sample, however the calculations are better understood to be estimates of a sample mean (the sample mean absolute error) which as a statistical estimate has standard errors and formal statistical tests can be constructed for whether or not the difference between the performances of the methods is indeed statistically significant. Examining statistical significance is important for the usual reasons used in the rest of empirical science, a poorly estimated performance (one with large standard errors) might be lucky in a particular test sample (we might get an outlier draw that suggests it is a very good approach) when in another sample it might be far worse than other methods. Formal statistical evaluation of these statistical averages sheds light on whether or not we might expect better performance to extend to other time periods rather than being specific to the time period being evaluated. With this in mind we also choose the evaluation period to be large enough so that tests have enough power to differentiate methods that are large in terms of valuation of the storage technology.

Finally, it is critical for the valuation of grid connected energy storage devices that realistic valuations of this application are constructed (DOE (2011), see page7). Many studies use perfect foresight of
prices in their valuations (Eyer and Corey (2010)) or simple rules (Sioshansi et al. (2009)). We build on this work by carefully considering how different forecasting strategies affect these valuations.

2.2 Loss Function

In a deregulated market, short term (typically day ahead) forecasts of the electricity price are useful for a variety of reasons. First, energy providers in California must determine the bids they will submit for the day ahead market by 10am on the day prior to the provision of this energy. Of course their own actions - the bids they submit - impact the price. Forecasts will be useful but will be part of a complicated objective function that involves the form of the auction market for setting prices as well as the bidding schedules provided. Independent systems operators and other firms involved in the convergence bidding market will have their own objective functions. Owners of storage technologies will also have different objectives and valuations for a set of forecasts, based on how this storage technology is employed.

In the academic literature it is common to employ loss functions for evaluation that are not directly related to any of the actual uses of the forecasts in practice. These are typically decision theoretic based loss functions such as mean absolute error (MAE) or root mean square error (RMSE) or alternatively (or as well as) the first of these measures divided by the outcome such as mean average percentage loss (MAPE). It is not obvious that methods that perform well or better than another method based on these measures extends to actual uses of the forecasts in market situations. It is of interest then to understand the extent to which these measures are useful in determining good forecasting methods for realistic applications of the forecasts. In the empirical section below we examine the extent to which these metrics are useful for our application.

Our application values forecasts based on how useful they are for providing value through employing a storage device to engage in energy time shift using the DAM in the California wholesale energy market.

The DAM is an hourly market where by 10am of the trading day prior to the day the actual energy is delivered (settlement day) market participants must provide CAISO with their bids (price and quantity purchased or sold). CAISO matches supply and demand for energy for each hourly interval and sets the price as a function of the bids. A storage device can participate in this market by submitting bids for charging and discharging with quantities that are limited by the physical operation of the device. Our (simplified) model of the storage device limits the quantity available since the device must be charged before it can be discharged, so the state of charge (total energy available) must remain between a lower limit and the total potential charge of the storage device. The speed at which the device charges or discharges depends on the power rating of the device, so a power energy ratio of one means all the energy can be discharged (or
the device fully charged) in one hour, if this ratio is less than one then multiple hours are required to fully charge or discharge. We assume no efficiency losses in charging and discharging - accounting for such losses will adjust all of the numbers downwards slightly but will not change the main points of the study since all forecasting methods will be affected similarly. Revenues from use of a storage device in intra day energy time shift are available through contracting to buy energy when forecasts suggest energy will be cheap and reselling when the forecasts suggest energy will be more expensive.

Whilst the notion of 'buy low, sell high' is obvious, a detailed understanding of this procedure shows why the loss function might matter in choosing a forecast method as well as makes clear the trade-offs made in forecast errors in this applied revenue loss function. Figure (2.1) explains how revenue is generated given a sequence of prices for a representative day (6 February 2014). DAM prices are hourly as this is the standard contracting period for operation in the DAM in California. It shows the DAM prices (solid line) from 10am of the day contracts are written the day before trading until the end of the trading day. In addition we show the forecasts generated prior to these prices being observed for the trading day (dashed lines). If prices for the trading day were known, we could optimally switch between charging the battery for the hours when prices are low then discharging when prices are high. Since these prices are unknown at the time decisions on charging and discharging are made, the forecasts provide the signal. Charging and discharging times are chosen based on the expected revenue given the forecasts, and the realized revenue is the difference between the actual prices paid are calculated. For example consider the second charge-discharge cycle in Figure (2.1). The forecasts suggest a low point in the prices at the hour ending 16 and a high at the hour ending at 19. If we contract in the DAM to charge the battery over the period from 3-4pm and discharge the battery over the hour from 6-7pm we obtain a purchase price of just under 35 $/MWh and a sales price of over 45 $/MWh, generating a profit from time shift. Our predicted gain is lower than the realized gain from the market, but none-the-less the choice of times to charge and discharge look good ex post.

Good predictions under RMSE or MAE are predicted prices that are close always to the actual prices. This is of course very helpful, however in terms of maximizing the revenue from the storage device we do not necessarily need to be very close to the actual prices. What is required is that the forecasts are good predictors of the actual prices at times when the prices are at extremes (low or high) rather than being close at all times. Forecast methods with this quality will be more useful for forecasting revenues, whilst measures such as MAE and RMSE may indicate good predictors that are good primarily because they predict well at times where prices are not too variable and predict poorly when there is variation in the prices that can be exploited by the storage technology. Whether or not this is happening is an empirical question, which we examine carefully in Section 2.5.
Figure 2.1: Optimal charge/discharge commitment for a single day energy time-shift

**Note:** Energy to power ration (E/P) of the battery is set to 1. Horizontal axis spans the interval from 2013/1/19 10:00 to 2013/1/20 24:00.

We consider a few precise measures of revenue, all based around the calculations in the above example. First, we compute daily revenue in dollars from each days trades. Since ex post we know the true prices, we also use our algorithm with the actual prices in place of the forecasts which gives the potential daily revenue that could have been achieved had the forecasts been ‘perfect’. The ratio of achieved revenue to potential revenue (a value of one is best) is then averaged over the evaluation sample to provide normalized average revenue (NAR). In addition we take the total revenue achieved as a proportion of the total potential revenue and report this as normalized total revenue (NTR).

### 2.3 Data and Approach

Data was obtained from California ISOs OASIS web portal for the period from 1/1/2012 to 4/30/2015. Hourly Locational Marginal Prices (LMPs) in $/MWh from day-ahead market settlement (DAM) were downloaded for LAJOLLA_6_N001 pricing node as defined in CAISO full network model. Existing literature has predominantly focused on forecasts of marginal cost of energy (MCE) which is a component of LMP in addition to marginal cost of congestion (MCC) and marginal cost of loss (MCL). There are very few examples where LMP is an object of interest, Li et al. (2007) and Hong and Hsiao (2002). Need to forecast LMPs is further motivated by our economic loss function, i.e. revenues from energy time-shift, that require actual LMPs for settlement.

Hourly load data as well day-ahead forecasts were obtained for an aggregate CAISO transmission control area (TAC). When choosing between load data for an aggregate TAC instead of local San Diego Gas and Electric TAC we acknowledge the trade-offs between aggregate load which directly sets common to all
pricing nodes MCE price component via always binding energy balance constrain enforced by ISO versus local load levels that may capture variation in MCC component of LMP driven by congestion events at near transmission lines.

Predictive accuracy comparisons require parameter estimation to be performed with expanding and rolling windows. Data from 1/1/2012 - 12/31/2012 was used as initial estimation sample and forecast performance was evaluated using the remaining 20400 hourly observations from 1/1/2013 to 4/30/2015. Models were estimated with actual contemporaneous load levels and then forecasted day-ahead load levels were used to form price forecasts.

For participation in the DAM forecasts of the following (settlement) day need to be made by 10 a.m. of the current (trading) day for submission to CAISO. Hence only data available at this time are used to construct forecasts for the following day. We assume a bidding strategy that ensures that the bids are taken up.

2.3.1 Forecast Evaluation

For the evaluation sample we compute for each hour the absolute value of the forecast error and the square of the forecast error. Averages of these measures are the MAE and MSE respectively. The square root of the MSE is the RMSE. Since these are averages then we can also report the standard deviation. These are calculated using a robust measure of the standard error (Newey and West (1987)). For application we need to specify a maximal number of autocovariances to consider, this is set to four after examination of the serial correlation in the loss measures. Similar measures are constructed for the loss measures based on revenues.

With multiple forecast methods we can compare whether or not these averages indicate differences in the performance of the forecast methods. We construct such tests for pairwise comparisons using t tests for the differences in the loss function measure, using robust measures of the standard deviation of the differences in the losses as described in the previous paragraph. For the expanding window, these are DM tests (Diebold and Mariano (2012)) and for the rolling window the tests follow from Giacomini and White (2006). The former are t-tests, the latter are presented as the square of the t-test (which has an asymptotic $\chi^2$ distribution).

2.4 Price Forecasting Methods

As detailed in Weron (2014), the problem of forecasting electricity prices has resulted in an extraordinarily large variety of methods being proposed as candidates for forecasting models. The majority employ prior electricity prices as predictors, along with measures of demand (for example load forecasts or
measures of load) and weather variables. In terms of estimation approaches, variations on ARMAX models have often proved difficult to outperform. For examples of these approaches see Contreras et al. (2003), Cuaresma et al. (2004), Nogales et al. (2002) and Li et al. (2007). Often even simpler models capture enough of the variation in prices that they provide useful forecasts. The ‘naive’ or similar day method has been found to be difficult for more complicated methods to beat Nogales et al. (2002). As a simple model we include exponential smoothing models, which have been found to be useful in many problems.

Since these models are linear in nature, they may not capture nonlinearities in the data that may aid forecasting performance. Whilst specific nonlinear models have been suggested (for example threshold models in Weron and Misiorek (2006)), a far more popular approach in the absence of theory based information on the form of the nonlinearity has been to estimate more flexible models. The most popular approach has been the use of sieve methods such as artificial neural networks. Examples include Szkuta et al. (1999), Yamin et al. (2004), Catalao et al. (2007), amongst a large published literature. Since often these methods appear to provide fairly similar results as simpler approaches (Catalao et al. (2007), Conejo et al. (2005)), we also examine locally linear models which allow ARMAX models to vary from linearity but nest the linear model in the absence of any nonlinearities. Finally, we also employ cubic splines which is an alternative approach to allowing for nonlinearities.

Regardless of the statistical approach taken, building good forecasting models requires accounting for the ‘seasonal’ structure of the data. This can be broken into two aspects - the extremely pronounced intra-day seasonal variation as well as variations in the model over longer time periods. As it is very difficult to model intra-day seasonality and we have a very large number of observations, we follow the approach of estimating 24 different models, one for each hour of the next day (in load forecasting see Ramanathan (1997) and in price forecasting see Cuaresma et al. (2004)).

As stated in Mokrian and Stephen (2006) given day-ahead price forecast problem of optimal battery commitment is deterministic and can be solved using linear programing, unlike alternative dynamic and stochastic programing approaches to battery dispatch and economic valuation.

The following gives more precise detail on the forecast methods to be employed and evaluated.

**Model 1 - (ARMAX) Auto-Regressive Moving Average with Exogenous Variables**

The benchmark AR(3)MA(1)X model is presented in Eq. (2.4.1), where \( p_t \) is a day-ahead energy price at hour \( t \), \( p_{t-1} \), \( p_{t-24} \) and \( p_{t-168} \) are lags of price capturing autocorrelation structure presented in Figure (2.4), \( \alpha_t \) is a time-varying intercept as specified in Eq. (2.4.2), \( x_t \) is a contemporaneous load, and \( \varepsilon_{t-1} \) and \( \varepsilon_{t-2} \) are lagged error terms.
\[ p_t = \alpha_t + \beta_1 p_{t-1} + \beta_2 p_{t-24} + \beta_3 p_{t-168} + \gamma_1 x_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \varepsilon_t \quad (2.4.1) \]

\[ \alpha_t = \alpha_0 + \alpha_1 t + \sum_{j=1}^{23} \alpha_2 j 1 \{ t \mod (24) = j \} + \sum_{j=1}^{6} \alpha_3 j 1 \{ t \text{ in day } j \} + \sum_{j=1}^{11} \alpha_4 j 1 \{ t \text{ is in month } j \} \quad (2.4.2) \]

Model in Eq. (2.4.1) can be alternatively applied to 24 time series segmented by hour of the day. Segmented formulation of Model 1 is provided in Eqs. (2.4.3) and (2.4.4) for \( h = 1, 2, \ldots, 24 \), where time subscript \( t \) now iterates over days not hours, and only divisible-by-24 MA lags are retained. Autoregressive lags that are not divisible by 24 are hence considered an exogenous variable. Formulation of segmented ARMAX with lags of adjacent hours is referred to as “crossed” in Cuaresma et al. (2004). Intra-day seasonality term is removed from time-varying intercept \( \alpha_h^b \) since now it is captured by estimating 24 different models.

\[ p_t^h = \alpha_h^0 + \alpha_h^1 t + \beta_h^0 p_{t-1}^h + \beta_h^1 p_{t-24}^h + \beta_h^2 p_{t-168}^h + \gamma_h^1 x_t + \gamma_h^2 p_{t-1}^h + \phi_h^1 \varepsilon_{t-1} + \phi_h^2 \varepsilon_{t-2} + \varepsilon_t \quad (2.4.3) \]

\[ \alpha_h^0 = \alpha_h^0 + \alpha_h^1 t + \sum_{j=1}^{6} \alpha_h^2 j 1 \{ t \text{ in day } j \} + \sum_{j=1}^{11} \alpha_h^3 j 1 \{ t \text{ is in month } j \} \quad (2.4.4) \]

**Model 2 - (FFNN) Feed-Forward Neural Net**

Model 2 is an augmentation of ARX component of Model 1 with a non-linear component \( G([p_{t-1}, p_{t-24}, p_{t-168}, x_t]) \) as presented in global formulation in Eq.(2.4.5),

\[ p_t = \alpha_t + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ p_{t-24} \\ p_{t-168} \end{bmatrix} + \gamma x_t + G([p_{t-1}, p_{t-24}, p_{t-168}, x_t]) + \varepsilon_t, \quad (2.4.5) \]

where \( G(y) = \sum_{j=1}^{m} v_j g(\sum_{i=1}^{3} w_{ji} y_{t-1}) \), \( g(y) = (1 + e^{-y})^{-1} \) and parameter \( m \) indicates number of neurons/layers. Model 2 can be alternatively formulated in a segmented form using ARX component from Eq.(2.4.3).
Model 3 - (ES) Exponential Smoothing

For exposition of this class of models we adopt notation from Hyndman (2008). Four sub-classes of models are considered in Eqs.(7) - (10) as determined by form of trend and seasonality components. Each exponential smoothing model below is expressed as a single source of error state space model where given general formulation in Eq. (2.4.6), \( p_t \) is energy price at hour \( t \) and \( z_t \) is a state vector, and functions \( f(\cdot) \), \( \omega(\cdot) \), \( g(\cdot) \) are specific to exponential smoothing method framework. Additive instead of multiplicative error formulation is used to avoid numerical instability during estimation due to Locational Marginal Prices occasionally exhibiting near zero or negative values during off-peak hours.

\[
p_t = \omega(z_{t-1}) + \epsilon_t
\]

\[
z_t = f(z_{t-1}) + g(z_{t-1}) \epsilon_t
\]

Models 3.1 and 3.2 in Eqs. (2.4.7) and (2.4.8) do not have a seasonal component and hence will be used only in segmented form for \( h = 1, \ldots, 24 \). Model 3.1 is a simple exponential smoothing method, where a state variable is a smooth level \( l_t \). Model 3.2 in addition to a smooth level contains additive trend component \( b_t \).

\[
p^h_t = l^h_t + \epsilon^h_t
\]

\[
l^h_t = l^h_{t-1} + \alpha^h \epsilon^h_t
\]

\[
p^h_t = \begin{bmatrix} 1 & \phi^h \\ \end{bmatrix} \begin{bmatrix} l^h_{t-1} \\ b^h_{t-1} \end{bmatrix} + \epsilon^h_t
\]

\[
\begin{bmatrix} l^h_t \\ b^h_t \end{bmatrix} = \begin{bmatrix} 1 & \phi^h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l^h_{t-1} \\ b^h_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha^h \\ \beta^h \end{bmatrix} \epsilon^h_t
\]

Model 3.3 in Eq. (2.4.9) contains an additive seasonal state variable \( s_{t-24} \) capturing intra-day seasonality. Segmented formulation of Eq. (2.4.8) will instead use weekly seasonality that becomes dominant for each individual hour.

\[
p_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ s_{t-1-24} \end{bmatrix} + \epsilon_t
\]

\[
\begin{bmatrix} l_t \\ s_{t-24} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ s_{t-1-24} \end{bmatrix} + \begin{bmatrix} \alpha \\ \gamma \end{bmatrix} \epsilon_t
\]
Model 3.4 in Eq. (2.4.10) captures both trend and seasonality and hence can be applied in either global or segmented form.

\[ p_t = \begin{bmatrix} 1 & \phi & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ b_{t-1} \\ s_{t-24} \end{bmatrix} + \epsilon_t \]

\[ \begin{bmatrix} l_t \\ b_t \\ s_{t-24} \end{bmatrix} = \begin{bmatrix} 1 & \phi & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_{t-1} \\ b_{t-1} \\ s_{t-24-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \epsilon_t \]  

(2.4.10)

Formulations of exponential smoothing models with exogenous variables were considered but due to poor forecasting performance we omit exposition of such models in this section.

**Model 4 - (LARX) Locally Weighted ARX**

Model 4 allows us to address potential nonlinearities of prices by placing more weight on previous data that is close to values of predictor variables for a given forecast. Hence, given vector of predictor variables \( z_t \) for \( t = 1, \ldots, T \) we produce h-step-ahead forecast at time \( T \) by minimizing weighted sum of squared error in Eq. (2.4.11) where \( K(\cdot) \) is a Gaussian product kernel, and estimated \( \hat{\beta}_0 \) by construct is a forecast at time \( T \).

\[ (\hat{\beta}_0, \hat{\beta}_1) = \arg \min_{\beta_0, \beta_1} \left\{ \sum_{t=1}^{T} (p_{t+h} - \beta_0 - \beta_1(z_t - z_T))^2 K(z_t - z_T) \right\} \]  

(2.4.11)

\[ K(z_t - z_T) = |B|^{-1} \exp \left\{ \frac{1}{2} (z_t - z_T)' B^{-1} (z_t - z_T) \right\} \]  

(2.4.12)

In Table (2.28) we consider various specifications of \( z_t \) containing either load only and lagged prices. If we let \( \tilde{Z} = [(z_1 - z_T) \ldots (z_T - z_T)] \) then bandwidth matrix can be constructed as \( B = \text{diag}(\text{std}(\tilde{Z}))T^{\frac{1}{5}}. \)

**Model 5 - (CS) Cubic Splines**

Model 5 similarly to FFNN augments linear ARX component with a nonlinear term as presented in Eq. (2.4.13). However, here we consider only non-linearity in load \( x_t \), where empirical distribution of previously observed load levels is partitioned by percentiles \( c_i \) for \( i = 1, \ldots, K \).
\[ p_t = \alpha_t + \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ p_{t-24} \\ p_{t-168} \end{bmatrix} + \gamma_1 x_t + \gamma_2 x_t^2 + \gamma_3 x_t^3 + \sum_{i=1}^{K} \gamma_{i+1} (x_t - c_i)^3 \mathbb{1}(x_t \geq c_i) + \epsilon_t \] (2.4.13)

In Table (2.35) we examine different specifications for number and location of knots. If value of predicted load used to form price forecast is smaller than \( c_1 \) or greater than \( c_K \), linear instead of cubic extrapolation is used where slope of the line is the first derivative of estimated polynomial evaluated at an edge knot.

### 2.5 Results and Evaluation

#### 2.5.1 Results

As we have discussed in section 2.4, within each family there are a number of models that we consider. For each of these models we can use either a global approach (use all of the hourly data for each day to construct estimates and forecasts) or segmented approach (construct different models for each hourly period and then recombine the forecasts into a single set of forecasts for the DAM price). Finally, for each of these methods we can construct out of sample forecasts using an expanding window (using all past observations to construct the forecast) or use rolling window regressions (which use either the past 6, 9 or 12 months of data). And as we have noted we have a number of loss functions. Sections 2.5.2 to 2.5.6 present the results for each of these variations organized first by the model family and then loss function. Tables 2.39-2.40 in section 2.5.7 present comparisons across the best performers of each of the families, with Table 2.39 using the best performers in terms of RMSE/MAE and Table 2.40 using the best performers based on revenue. Finally, a select set of tables showing the statistical significance of pairwise comparisons within families are also presented in Sections 2.5.2 to 2.5.6.

#### 2.5.1.1 Segmented versus Global forecasting methods

Within day 'seasonality' is an issue for both load and price forecasting at intra-day frequencies. For each of the families, we have examined both approaches to forecasting the DAM. For the ARMAX family, we find that the segmented approach appears to be favored over all measures of loss. However, the difference
between the segmented and global approaches is not large. For the expanding window the best segmented model yields an RMSE of 8.44 and the best global model an RMSE of 8.60, a difference of 16 cents. Similar results obtain for the rolling window estimates. For the neural net (FFNN) approaches we see a split result with the segmented models doing better for the traditional measures of loss but the global method better for maximizing revenue. Again the differences are modest, especially when compared to results between families. For the ESM, LARX and CS approaches we find that global methods do better everywhere. Hence we obtain very mixed results for choosing between these methods, with both appearing competitive especially amongst the top performing forecast methods.

2.5.1.2 How do the methods compare with traditional measures of accuracy?

Looking at the traditional measures of accuracy such as RMSE and MAE, there is a clear difference in the performances both within and across families of forecasting methods. Since there is a strong choice between families, we will first contrast the performance of the best performers in each family across families. The first noticeable result, one that is in line with former studies, is that the ARMAX family and FFNN family dominate other methods. Exponential smoothing was included as a simple baseline method, since in actual implementation simpler methods are preferred as models need to be re-estimated often. The locally weighted autorregressive methods and cubic spline methods provide a different form of accounting for potential nonlinearity, although neither appear to capture nonlinear effects as well as the FFNN models do.

The differences between the ARMAX and FFNN families appears relatively small, so it is not surprising that some energy forecasting papers have found one of these methods or its variants (such as transfer models as a variant of ARMAX) to be the best in their studies.

The above results are for the averages over all of the evaluation observations. For the best performers under RMSE overall, Figure (2.2) breaks up their average performance to show an hour by hour comparison. It shows that the rankings are consistent across all of the hours, so aggregating to a single number does not hide variations in performance. It also shows the closeness of the armax15 and ffnn9 methods in terms of RMSE. Finally, it does show that prices are much harder to predict for all methods at the peak usage times (during the business day).

2.5.1.3 Do traditional methods of accuracy extend to maximizing revenue?

The loss function matters for which forecasting model would be chosen as the best performer. First, consider choosing models within model groups. For the ARMAX family of models, in terms of RMSE and
MAE the best model is armax15. This is true for the expanding window and 12 month window, for the shorter rolling windows armax5 does a little better in terms of MAE (and for the 6 months rolling window, armax14 is best). In terms of revenue however, armax15 is never the best (although for the 12 month rolling window with E/P=6 it is close although for this storage technology there are very small differences between the models). The differences can be large economically, for example under the expanding window choosing armax15 over armax6 results in a loss in revenue over the year of $1,571 ((0.813 – 0.774) \cdot $40,302). A similar result arises from the family of neural network models. In terms of RMSE and MAE, the best performer is ffnn9 or sometimes ffnn10 (for the 12 months rolling and the 6 months rolling MSE, differences between these two models are small). However in terms of revenue generated from the forecasts, the best performer overall is ffnn6 (for the 9-month rolling window ffnn1 is better). Interestingly, this means that in terms of RMSE and MAE, where fitting everywhere is important, it is better to use the segmented approach whereas for revenue generation it is better to use the global approach. Again, the economic difference can be large (report number here). For the remaining methods, which were (as we see below in more detail) inferior to both the ARMAX and neural net approaches, the best performers within each group tended to be best across both the groups of loss functions.

Figure 2.2: Segmented by hour RMSE for best performing specifications

Turning to the results across loss functions, for the best performers across loss functions we see the same effect as for within the ARMAX and FFNN models. Comparing the best of these models in terms of RMSE and MAE, we see that armax15 has a smaller RMSE and MAE than ffnn9, resulting in a ranking of ARMAX over FFNN models. In terms of the best performers over revenues, consider results for E/P=1. Here in terms of NAR (results are the same for NTR) we have that the best performer amongst the ARMAX group is armax6, which captures 83.9% of potential revenues. The best FFNN model is ffnn6, which captures 85.4% of potential revenue, reversing the ranking between ARMAX and FFNN forecasting methods. Note
that if we compared armax15 with ffnn9 in terms of revenue (the best performers under RMSE) we get a similar reversal in ranking (80.8% for armax15 of potential revenue against 82.9% for ffnn9).

These results make clear that the loss function matters for the ranking of the methods and the choice of forecasting method. The RMSE and MAE loss functions are not unhelpful - we do not get reversals in the ranking between very poor methods and very good ones. However they do not indicate the correct ranking amongst good forecasting methods.

2.5.1.4 Do results change when considering the statistical properties of the estimates?

Whilst there is a clear ranking between the loss function evaluations, these evaluations are sample means (averages of loss over the evaluation data) that have statistical uncertainty. We only have confidence in the ranking if indeed the differences between these sample means is large relative to the sampling uncertainty over the location of the mean in the data. To this end, we test pairwise combinations of methods to test whether or not the differences in loss are statistically significant. The relatively large sample sizes (20400 hours of evaluation data for the RMSE/MAE and 850 days of data for the revenue) mean that power of these tests will be high, although much higher for the traditional loss functions than the revenue measures. However we have ensured that sample sizes are large enough in this study to differentiate models that are different from a valuation perspective, so power is large enough to distinguish models unless the actual differences in revenues are small enough to be mostly uninteresting.

In Table 2.39 the best performers (based on RMSE) are compared for the MAE and RMSE methods. In all cases but one (the comparison of the armax15 and ffnn9 with the MAE loss function) the p-values are less than or equal to 0.01. In the only case where there is less evidence that the armax15 model is better than the ffnn9 model the test still rejects equivalence at the 5% level (but not 10% level) even though the difference between the two methods is a mere ten cents per MWh on average.

Turning to the best performers based on revenue, again p-values are all zero apart from one case for each energy power ratio. For the comparison between the armax6 and the ffnn6 we have a p-value of 0.03 at both energy power ratios, so we still reject that the two methods are equivalent at the 5% level. Hence the reversal we found between the measures of loss for the traditional measures and the revenue measures holds up statistically.

In terms of the other comparisons made above, we noted that in Section 2.5.3 that the differences in RMSE between the best segmented and best global models for the ARMAX family did not appear to be large. However statistically the differences between these models is significant at the 5% level for each of the comparisons between the best segmented and best global model. For the FFNN models, we found that the
best model under RMSE was segmented but under revenue considerations was a global model. Again, these differences are statistically significant at the 5% level.

2.5.1.5 What is the relevant economic value of the storage technology?

The best forecast methods capture a little over 80% of the potential revenue available were the future prices known at the time the operation of the storage device in the market is decided. This amounts to $36.72 in revenue per day of operation when the energy to power ratio is one. The standard error on this estimate is 59 cents, so the 95% confidence interval is of the order of ±$1.18 and hence the estimate is quite precise.

2.5.2 Exponential Smoothing Methods (ESM)

Entries N, A and Ad in Trend column of Table 2.1 indicate absent, additive, and additive with a drift specifications. Similarly, N and A(S) in Seasonality column indicate absent and additive with a dominant seasonality S specifications. Models without a seasonality component were considered only in segmented form since intra-day season-
ality can be still captured unlike global form of such specifications. All models are additive in errors. Models with load included as a regressor could not be reliably estimated by maximizing $T \log(\sum_{t=1}^{T} \tilde{e}_t)$ since search space for regression coefficient is unconstrained unlike parameters $\alpha, \beta, \gamma, \phi$ in Eqs.(2.4.7)-(2.4.10) constrained to (0, 1) interval. Segmented AA(7) and AdA(7) models were considered but not reported due to very poor performance independent of loss function.

Table 2.1: Specification of ESM models for day-ahead LMP forecasts

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</tr>
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<td>esm5</td>
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<td>N</td>
</tr>
<tr>
<td>esm7</td>
<td>g</td>
<td>Ad</td>
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Table 2.2: RMSE and MAE forecast performance of ESM models

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### Table 2.3: Normalized Total and Average Daily Energy Time-Shift Revenues for ESM LMP forecasts

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Table 2.4: Pairwise Predictive Accuracy Tests within ESM family of models with respect to AE and SE loss given expanding and rolling estimation windows

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Note: Table above reports t-statics in case of expanding estimation window and $\chi^2_1$ in case of rolling estimation window with corresponding p-values reported in parentheses below. P-values greater than 5% are indicated with bold font. Upper- and lower-triangular sections of each block of statistics corresponds to pairwise comparisons with absolute and squared error loss accordingly.
Table 2.5: Pairwise Predictive Accuracy Tests within ESM family of models with respect to Energy Time-Shift Revenues given expanding and rolling estimation windows

<table>
<thead>
<tr>
<th>esm#</th>
<th>1</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td></td>
<td>(E/P=1)/(E/P=6), Expanding Estimation Window</td>
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<td></td>
<td></td>
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<td></td>
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<tr>
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<td>-2.1</td>
<td>12.8</td>
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</tr>
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<td>-3.6</td>
<td>40.1</td>
<td>13.9</td>
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Revenue for (E/P=1)/(E/P=6), 12-month Rolling Estimation Window

| esm1 | 1528.2 | 25.7 | 0.8  | 2139 | 434.1 | 2145 |     |
| esm2 | 1086   | 1187 | 1138 | 55.7 | 357.9 | 58.1 |     |
| esm3 | 2.3    | 1421 | 38.6 | 1554 | 239.8 | 1551 |     |
| esm4 | 14.6   | 1123 | 6.5  | 1335 | 489.9 | 1344 |     |
| esm5 | 1395   | 48.3 | 1813 | 1453 | 844.9 | 1.3  |     |
| esm6 | 97.6   | 876.7 | 81.9 | 30.7 | 1288 | 852.8 |     |
| esm7 | 1398   | 49.6 | 1820 | 1444 | 1.3   | 1281 |     |

Revenue for (E/P=1)/(E/P=6), 9-month Rolling Estimation Window

| esm1 | 1038 | 44.7 | 1.4  | 1982 | 429.1 | 1987 |     |
| esm2 | 1015 | 894.4 | 1424 | 13.3 | 225   | 14.1 |     |
| esm3 | 4.6   | 1162 | 61.2 | 1333 | 205   | 1338 |     |
| esm4 | 0.0   | 722.4 | 1.5  | 1510 | 400.5 | 1518 |     |
| esm5 | 1240  | 1.3  | 1410 | 796.2 | 762.7 | 5.4  |     |
| esm6 | 132.1 | 660  | 92.8 | 57.6 | 943.9 | 771.9 |     |
| esm7 | 1240  | 1.3  | 1411 | 796.2 | 943.8 |     |     |

Revenue for (E/P=1)/(E/P=6), 6-month Rolling Estimation Window

| esm1 | 1286 | 40.3 | 0.6  | 1524 | 364   | 1527.4 |     |
| esm2 | 1165 | 778.8 | 999.7 | 0.3  | 289.9  | 0.2  |     |
| esm3 | 0.5   | 1290 | 50.2  | 1186 | 172.9  | 1181 |     |
| esm4 | 4.1   | 873.1 | 7.8   | 1330 | 363.2  | 1328 |     |
| esm5 | 918.7 | 42   | 1059 | 704.9 | 395   | 2    |     |
| esm6 | 45    | 809.1 | 51.2  | 63   | 592.2  | 393.2 |     |
| esm7 | 922.3 | 42.6 | 1060 | 703.7 | 0.2   | 594.3 |     |

Note: Table above reports t-statics in case of expanding estimation window and $z^2_{i}$ in case of rolling estimation window with corresponding p-values reported in parentheses below. P-values greater than 5% are indicated with bold font. Upper- and lower-triangular sections of each block of statistics corresponds to pairwise comparisons with absolute and squared error loss accordingly.
2.5.3 Autoregressive-moving average models with exogenous variables (ARMAX)

ARMAX models were applied in either segmented-by-hour (s) or global (g) manner as indicated by “s/g” column in Table 2.6. TVI (time-varying intercept) includes linear trend, and hour, weekday and month indicators. Month indicators are dropped from TVI in case of 9 and 6 month rolling window estimation. ARX parameters are estimated on a daily basis via OLS. ARMAX parameters are estimated on a weekly basis via log-likelihood maximization. Choice of AR and MA lags is based on preliminary model specification tests in Tables 2.41 for global AR Lags, Table 2.42 for hour-specific AR Lags, and Figure 2.8 for joint ARMA Lag selection. AR and MA lags are defined in a global sense and for segmented model applications can be only multiples of 24.

<table>
<thead>
<tr>
<th>Name</th>
<th>s/g</th>
<th>AR Lags</th>
<th>MA Lags</th>
<th>TVI</th>
<th>X</th>
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<td>g</td>
<td>1, 2, 23, 24, 25, 144, 168</td>
<td>1 load</td>
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<td>24, 168</td>
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<td>1</td>
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<td>s</td>
<td>24, 168</td>
<td>1 load</td>
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<td>1 load</td>
<td>load</td>
<td>lags of adjacent hours</td>
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<td>24</td>
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<td>1, 2, 23, 24, 25</td>
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<td>1, 24</td>
<td></td>
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</tr>
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<td>g</td>
<td>1, 24, 25</td>
<td>1, 24</td>
<td>load</td>
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<td>24, 48</td>
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<td>s</td>
<td>24, 168</td>
<td>24</td>
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<td>s</td>
<td>24, 168</td>
<td>24</td>
<td>load</td>
<td>lags of adjacent hours</td>
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<tr>
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Table 2.7: RMSE and MAE forecast Performance of ARMAX models

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<td>MAE</td>
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Energy storage device is optimally committed to charge/discharge in a day-ahead energy market given forecasted LMPs at LAJOLLA_6_N001 for 850 days, 01/01/2013 and 04/30/2015. All efficiency parameters are set to 1, state of charge is constrained between 0 and 1, and there are no restrictions on number of intra-day charge/discharge cycles. Let $\overline{R}_d$ be maximum energy time-shift revenue on day $d$ under perfect foresight scenario. If $R_f^d$ are revenues associated with price forecast $f$ then $NAR = \frac{1}{850} \sum_{d=1}^{850} \left( \frac{R_f^d}{\overline{R}_d} \right)$. Further, NTR is a ratio of total realized revenues given some price forecast to total revenues under perfect foresight scenario.
<table>
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<td>E/P=1 NTR NAR</td>
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<td>148.806 0.884</td>
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<td>armax8</td>
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<td>149.098 0.893</td>
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<td>149.919 0.894</td>
<td>149.417 0.893</td>
<td>148.015 0.886</td>
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Table 2.9: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given expanding estimation window

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Note: Lower - and upper-triangular sections report t-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.10: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given expanding estimation window

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Note: Lower - and upper-triangular sections report t-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.11: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 12-month rolling estimation window

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Note: Lower- and upper-triangular sections report $X^2$ statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.12: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 12-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.13: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 9-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.14: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 9-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.15: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to AE and SE loss given 6-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.16: Pairwise Predictive Accuracy Tests within ARMAX family of models with respect to Energy Time-Shift Revenues given 6-month rolling estimation window

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Note: Lower - and upper-triangular sections report $Z^2$-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
## 2.5.4 Feed-Forward Neural Nets (FFNN)

In Table 2.17 globalARX linear base is \( p_t(1 - \alpha_24 L^{24} - \alpha_{48} L^{48} - \alpha_{168} L^{168}) = TVI + l_t + \epsilon_t \), where \( l_{[t]} \) is contemporaneous load and TVI is specified in Eq. (2.4.2). SegmentedARX linear base is \( p_t^h(1 - \alpha_1^h L - \alpha_7^h L^7) = TVI^h + l_t^h + \epsilon_t^h \) for \( h = 1, 2, \ldots , 24 \) and \( TVI^h \) from Eq. (2.4.4). Number of layers/neurons within neural net is provided in column “m”.

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Table 2.18: RMSE and MAE forecast performance of FFNN models

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Table 2.19: Normalized Total and Average Daily Energy Time-Shift Revenues for FFNN LMP forecasts

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Table 2.20: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given expanding estimation window

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Note: Lower - and upper-triangular sections report t-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.21: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given expanding estimation window

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| ffn2  | -1 | -0.5| 0.8 | 3.8 | -5.5| -4.3| -20.4| -2.5| -3.5| -4  | -8.3| -4.7| -5.3| -4.6| -6.3|-
| ffn3  | -1.7| -2 | 0.9 | 3.8 | -5.5| -4.2| -20.4| -2.5| -3.5| -4  | -8.3| -4.7| -5.3| -4.6| -6.3|-
| ffn4  | -1.9| -1.5| -1  | 3.6 | -5.6| -4.3| -20.5| -2.6| -3.6| -4  | -8.4| -4.8| -5.4| -4.7| -6.4|-
| ffn5  | 2.3 | 2.4 | 2.5 | 2.8 | -7.2| -5.7| -20.7| -4.1| -5  | -5.7| -9.6| -6.1| -6.6| -6.1| -7.6|-
| ffn6  | -5.5| -5.5| -5.4| -5.3| -6.5| 3.1 | -14.4| 1  | -0.1| -0.5| -5.1| -1.6| -1.7| -1.4| -3  |-
| ffn7  | -5.5| -5.4| -5.3| -5.8| -0.6| -19 | -1 | -2 | -2.5| -7.2| -3.6| -4 | -3.3| -5.2|-
| ffn8  | -33.9| -33.8| -33.8| -33.8| -34| -33.6| -31.6| 13 | 11.3| 10.6| 6.4 | 10.2| 9.9 | 9.4 | 8.2  |-
| ffn9  | -7.6| -7.6| -7.5| -7.5| -8.7| -2.4| -1.7| 31.1| -2.9| -0.6| -7.4| -2.2| -2.2| -2.5| -4.9|-
| ffn10 | -5.9| -5.9| -5.8| -5.7| -7.1| -1.5| -0.8| 29.9| -2.6| 1.2 | -6.4| -1.6| -1.9| -1.8| -3.8|-
| ffn11 | -10.1| -10| -10| -10| -10.9| -6.4| -5.6| 31.6| -7.47| -5.8| 54  | 4.2 | 5.3 | 3.4 |-
| ffn12 | -6.8| -6.7| -6.7| -6.6| -7.9| -3.6| -2.6| 32.1| -4.2| -1.2| -2.2| 3.2 | -0.3| 0.1 | 2.2 |-
| ffn13 | -8.1| -8.1| -8 | -8 | -8.9| -4.5| -3.5| 30.3| -5.5| -2.6| -3.5| 1.7 | -1.3| 0.4 | 1.8 |-
| ffn14 | -8.1| -8.1| -8 | -8 | -9.3| -3.8| -3.2| 29  | -4.4| -2.2| -3.8| 2.5 | -0.6| 0.7 | 2.9 |-
| ffn15 | -8.4| -8.3| -8.2| -9.6| -4.1| -3.5| 26.7| -4.6| -2  | -3.6| 2.5 | -0.6| 0.6 | 0  |-
| ffn16 | -8.4| -8.3| -8.2| -9.6| -4.1| -3.5| 26.7| -4.6| -2  | -3.6| 2.5 | -0.6| 0.6 | 0  |-

Note: Lower - and upper-triangular sections report t-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.22: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 12-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.23: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 12-month rolling estimation window

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Note: Lower - and upper-triangular sections report \(\chi^2\)-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.24: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 9-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.25: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 9-month rolling estimation window

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Note: Lower - and upper-triangular sections report \chi^2-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.26: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to AE and SE loss given 6-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with absolute and squared error loss accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.27: Pairwise Predictive Accuracy Tests within FFNN family of models with respect to Energy Time-Shift Revenues given 6-month rolling estimation window

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Note: Lower - and upper-triangular sections report $\chi^2$-statistics corresponding to pairwise comparisons with energy time shift revenues for batteries with energy to power ratio equal to 1 and 6 accordingly. P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
### 2.5.5 Locally Weighted ARX (LARX)

Table 2.28: Specification of Locally Weighted ARX (LARX) Models for day-ahead LMP forecasts

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Table 2.29: RMSE and MAE forecast performance of Locally Weighted ARX (LARX) models

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Table 2.30: Normalized Total and Average Daily Energy Time-Shift Revenues for LARX LMP forecasts
Table 2.31: Pairwise Predictive Accuracy Tests within LARX family of models given expanding estimation window

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Note: Lower - and upper-triangular sections for each block of results report t-statistics corresponding to pairwise comparisons with absolute versus squared error loss (top section) and revenues under energy to power ratio 1 versus 6 (bottom section). P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.32: Pairwise Predictive Accuracy Tests within LARX family of models given 12-month rolling estimation window

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**Note:** Lower - and upper-triangular sections for each block of results report $\chi^2$-statistics corresponding to pairwise comparisons with absolute versus squared error loss (top section) and revenues under energy to power ratio 1 versus 6 (bottom section). P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.33: Pairwise Predictive Accuracy Tests within LARX family of models given 9-month rolling estimation window

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**AESE, 9-month Rolling Estimation Window**

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**Revenue for E/P=1/E/P=6, 9-month Rolling Estimation Window**

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**Note:** Lower - and upper-triangular sections for each block of results report $\chi^2$-statistics corresponding to pairwise comparisons with absolute versus squared error loss (top section) and revenues under energy to power ratio 1 versus 6 (bottom section). P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.
Table 2.34: Pairwise Predictive Accuracy Tests within LARX family of models given 6-month rolling estimation window

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<td>79.9</td>
<td>17.2</td>
<td>30.2</td>
<td>460.1</td>
<td>1049</td>
<td>30.9</td>
<td>94.2</td>
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<th>Revenue for E/P=1/V=P=6, 6-month Rolling Estimation Window</th>
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<td>18.7</td>
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<td>larx9</td>
<td>77.7</td>
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<td>larx11</td>
<td>9.3</td>
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<tr>
<td>larx12</td>
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Note: Lower - and upper-triangular sections for each block of results report $\chi^2$-statistics corresponding to pairwise comparisons with absolute versus squared error loss (top section) and revenues under energy to power ratio 1 versus 6 (bottom section). P-values are reported in parentheses below. P-values greater than 5% are indicated with bold font.

2.5.6 Cubic Splines (CS)

In Table 2.35 below TVI = 1 indicates inclusion of time-varying intercept compatible with segmentation level and size of rolling estimation window. Segmented specifications with time-varying intercept, load levels and logs, and varying number of knots were considered but not reported due to very poor RMSE and MAE performance.
Table 2.35: Specification of Cubic Splines (CS) Models for day-ahead LMP forecasts

<table>
<thead>
<tr>
<th>Name</th>
<th>s/g</th>
<th>TVI AR Lags</th>
<th>knots (%)</th>
<th>Load</th>
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<td>1, 24, 48, 168</td>
<td>1, 20, 40, 60, 80, 90, 99</td>
<td>level</td>
</tr>
<tr>
<td>cs2</td>
<td>g</td>
<td>1</td>
<td>1, 24, 48, 168</td>
<td>1, 20, 40, 60, 80, 90, 99</td>
</tr>
<tr>
<td>cs3</td>
<td>g</td>
<td>1</td>
<td>1, 24, 48, 168</td>
<td>1, 10, 20, …, 80, 90, 99</td>
</tr>
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<td>cs4</td>
<td>g</td>
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<td>1, 24, 48, 168</td>
<td>1, 20, 40, 60, 80, 90, 99</td>
</tr>
<tr>
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<td>s</td>
<td>24, 168</td>
<td>1, 20, 40, 60, 80, 90, 99</td>
<td>level</td>
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In Table 2.36 simple global insanity filter (gIF) was applied by replacing forecast values smaller/greater than minimum/maximum price observed in a rolling 12-month window with yesterday’s price. gIF is global in a sense that a single maximum-minimum pair is used to perform an insanity check regardless of hour of the day, i.e. not segmented insanity filter sIF. Column ’gIF%’ indicates a percentage of forecasts affected by filter. Use of filtering is motivated by unreliable forecasts formed for forecasted load levels that are smaller greater then percentiles associated with first last knot.

Table 2.36: RMSE and MAE forecast performance of Cubic Splines (CS) models

<table>
<thead>
<tr>
<th>Name</th>
<th>Expanding Window</th>
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<th>Rolling Window</th>
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<td>MAE</td>
<td>gIF%</td>
<td></td>
<td>RMSE</td>
<td>MAE</td>
<td>gIF%</td>
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<tr>
<td></td>
<td>(9 months)</td>
<td>(9 months)</td>
<td>(9 months)</td>
<td></td>
<td>(6 months)</td>
<td>(6 months)</td>
<td>(6 months)</td>
<td></td>
</tr>
<tr>
<td>cs1</td>
<td>11.62 (0.109)</td>
<td>6.78 (0.090)</td>
<td>0.35 (0.000)</td>
<td></td>
<td>12.81 (0.32 )</td>
<td>7.22 (0.074)</td>
<td>2.32 (0.07)</td>
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</tr>
<tr>
<td></td>
<td>18.53 (0.117)</td>
<td>9.83 (0.11 )</td>
<td>0.35 (0.000)</td>
<td></td>
<td>18.57 (0.206)</td>
<td>9.77 (0.087)</td>
<td>1.94 (0.087)</td>
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<tr>
<td></td>
<td>12.17 (0.178)</td>
<td>6.8 (0.071)</td>
<td>0.33 (0.003)</td>
<td></td>
<td>17.19 (0.20)</td>
<td>9.16 (0.071)</td>
<td>1.56 (0.07)</td>
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<td></td>
<td>10.87 (0.175)</td>
<td>6.13 (0.065)</td>
<td>0.27 (0.005)</td>
<td></td>
<td>10.08 (0.20)</td>
<td>5.88 (0.071)</td>
<td>0.16 (0.054)</td>
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<tr>
<td></td>
<td>9.74 (0.175)</td>
<td>5.03 (0.054)</td>
<td>0.16 (0.004)</td>
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<td>9.74 (0.20)</td>
<td>6.03 (0.061)</td>
<td>0.16 (0.06)</td>
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<tr>
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<td>9.79 (0.157)</td>
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<td>0.27 (0.017)</td>
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<td>11.45 (0.20)</td>
<td>6.38 (0.071)</td>
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<td></td>
<td>18.57 (0.111)</td>
<td>9.77 (0.087)</td>
<td>1.94 (0.071)</td>
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<td>17.19 (0.20)</td>
<td>9.16 (0.071)</td>
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<td></td>
<td>10.87 (0.178)</td>
<td>6.13 (0.071)</td>
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<td>12.17 (0.20)</td>
<td>6.8 (0.071)</td>
<td>1.56 (0.071)</td>
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<td>9.74 (0.175)</td>
<td>5.03 (0.054)</td>
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<td>10.08 (0.20)</td>
<td>5.88 (0.071)</td>
<td>0.16 (0.054)</td>
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<td>0.33 (0.003)</td>
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<td>12.17 (0.20)</td>
<td>6.8 (0.071)</td>
<td>1.56 (0.07)</td>
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<tr>
<td></td>
<td>9.74 (0.175)</td>
<td>5.03 (0.054)</td>
<td>0.16 (0.004)</td>
<td></td>
<td>10.08 (0.20)</td>
<td>5.88 (0.071)</td>
<td>0.16 (0.054)</td>
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<tr>
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<td>9.22 (0.144)</td>
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<td>0.10 (0.017)</td>
<td></td>
<td>10.08 (0.20)</td>
<td>5.88 (0.071)</td>
<td>0.16 (0.054)</td>
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<tr>
<td></td>
<td>9.74 (0.175)</td>
<td>5.03 (0.054)</td>
<td>0.16 (0.004)</td>
<td></td>
<td>9.74 (0.20)</td>
<td>6.03 (0.061)</td>
<td>0.16 (0.06)</td>
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<tr>
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<td>22.93 (0.29)</td>
<td>16.34 (0.113)</td>
<td>0 (0.13)</td>
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<td></td>
<td>22.93 (0.29)</td>
<td>16.34 (0.113)</td>
<td>0 (0.13)</td>
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<td>22.93 (0.29)</td>
<td>16.34 (0.113)</td>
<td>0 (0.13)</td>
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<td>22.93 (0.29)</td>
<td>16.34 (0.113)</td>
<td>0 (0.13)</td>
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<td>22.93 (0.29)</td>
<td>16.34 (0.113)</td>
<td>0 (0.13)</td>
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</tbody>
</table>
Table 2.37: Normalized Total and Average Daily Energy Time-Shift Revenues for energy storage with CS day-ahead LMP forecasts and different energy to power ratios

| Model | Expanding Window | | | | Rolling Window | | | | | 12 month | 9 months | 6 months |
|-------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|       | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR | E/P=1 AR NAR AR NAR | E/P=6 AR NAR AR NAR |
| cs1   | 30.632 (0.497) | 0.679 (0.006) | 133.524 (2.152) | 0.805 (0.007) | 29.06 (0.151) | 129.019 (2.187) | 0.805 (0.007) | 29.06 (0.151) | 129.019 (2.187) | 0.805 (0.007) | 29.06 (0.151) | 129.019 (2.187) |
| cs2   | 34.084 (0.54 ) | 0.746 (0.006) | 144.514 (2.599) | 0.856 (0.007) | 32.711 (0.556) | 141.288 (2.407) | 0.837 (0.006) | 32.711 (0.556) | 141.288 (2.407) | 0.837 (0.006) | 32.711 (0.556) | 141.288 (2.407) |
| cs3   | 33.784 (0.549) | 0.74 (0.006)  | 143.977 (2.406) | 0.853 (0.007) | 32.691 (0.556) | 140.76 (2.407)  | 0.841 (0.006)  | 32.691 (0.556) | 140.76 (2.407)  | 0.841 (0.006)  | 32.691 (0.556) | 140.76 (2.407)  |
| cs4   | 34.035 (0.535) | 0.748 (0.006) | 144.372 (2.406) | 0.857 (0.007) | 33.204 (0.556) | 142.882 (2.407) | 0.853 (0.006) | 33.204 (0.556) | 142.882 (2.407) | 0.853 (0.006) | 33.204 (0.556) | 142.882 (2.407) |
| cs5   | 29.193 (0.609 )| 0.63 (0.012)  | 127.329 (2.726) | 0.76 (0.011)  | 1.148 (0.849)  | 35.273 (1.789)  | 0.213 (0.009)  | 1.148 (0.849)  | 35.273 (1.789)  | 0.213 (0.009)  | 1.148 (0.849)  | 35.273 (1.789)  |
Table 2.38: Pairwise Predictive Accuracy Tests within CS family of models

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<th>Revenues for (E/P=6)</th>
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<td>11.2</td>
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<td>0.7</td>
<td>-8.8</td>
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<td>-10.3</td>
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<td>-8.4</td>
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<td>-8.8</td>
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<td>8.4</td>
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<td>-7.9</td>
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**Note:** Table above reports t-statics in case of expanding estimation window and $\chi^2$ in case of rolling estimation window with corresponding p-values reported in parentheses below. P-values greater than 5% are indicated with bold font.
### 2.5.7 Predictive Accuracy Comparisons across families of methods

Table 2.39: Pairwise Predictive Accuracy Tests across top performers (in RMSE sense) from each family of methods given expanding estimation window

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of models were shown to do statistically better in terms of revenues relative to their segmented form. We
demonstrate statistical significance. Global formulations of these two top performing families of models dominate other methods with respect to both energy time-shift revenues and RMSE/MAE losses. Both global and segmented formulations were examined. We show that ARMAX and FFNN estimation and evaluation sample. Predictive accuracy comparisons were performed within and across families of models. Theoretical (MAE/RMSE) and application-specific (energy time-shift revenues) loss functions using common

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Table 2.40: Pairwise Predictive Accuracy Tests across top performers (in Revenue sense) from each family of methods given expanding estimation window

2.6 Conclusion

We have compared performance of a wide a set energy price forecasting methods with respect to theoretical (MAE/RMSE) and application-specific (energy time-shift revenues) loss functions using common estimation and evaluation sample. Predictive accuracy comparisons were performed within and across families of models. Both global and segmented formulations were examined. We show that ARMAX and FFNN families of models dominate other methods with respect to both energy time-shift revenues and RMSE/MAE performance in a statistically significant manner. Global formulations of these two top performing families of models were shown to do statistically better in terms of revenues relative to their segmented form. We
have conveyed strong intuition for the features of forecasts that are important for energy time shift application, i.e. ability to capture locations and not necessarily levels of intra-day minima and maxima. Further, in Chapter 3 one of top performing ARMAX specifications, armax6, will be used to produce 24-hour-ahead DAM and 1-hour ahead real-time market energy price forecasts to support valuation and dispatch of “stacked” applications for energy storage.

Chapter 2, in part, is currently being prepared for submission for publication of the material under the working title, Predictive Accuracy Comparison of Electricity Price Forecasting methods for Energy Storage valuation and dispatch, 2015. Chapter 1 was co-authored with Professor Graham Elliott.
Appendix to Chapter 2

![Graphs showing autocorrelation and partial autocorrelation of hourly DAM LMPs](image)

Figure 2.4: Global autocorrelation and partial autocorrelation of hourly DAM LMPs

**Note:** Above figure is based on initial estimation sample, 1/1/2012 - 12/31/2012 and it suggests that the following AR and MA lags should be used in global ARMAX model specification: 1, 2, 22, 23, 24, 25, 48, 72, 96, 120, 144, 168. Table (2.6) reports only best performing specifications for each subclass of models within ARMAX class.
Table 2.41: Iterative AR lag removal and out-of-sample performance evaluation for armax1 model specification

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Note: The initial AR(12) model for day-ahead prices \( p_t(L) = \alpha_0 + \epsilon_t \) was reduced to AR(7) such that \( p_t(L) = \alpha_0 + \epsilon_t \).
Figure 2.5: Global correlation of hourly DAM LMPs with lags of actual and forecasted load

Note: Above figure is based on initial aggregate CAISO load sample, 1/1/2012-12/31/2012. Indicates that DAM LMPs correlation slightly stronger with forecasted rather than actual load levels, and the following lags of load should be included in ARMAX model specifications: 1, 24, 48, 72, 96, 120, 144, 168. Recall that electric load itself exhibits very strong correlation as indicated in Figure 2.4. Therefore, including only contemporaneous load may suffice.

Figure 2.6: Global autocorrelation and partial autocorrelation of actual and forecasted load

Note: Above figure is based on initial sample of aggregate CAISO load, 1/1/2012 - 12/31/2012.

From visual inspection of panels (a) and (d) in Figure 2.7 it follows that for off-peak hour models AR lags 1, 2, 3, 4, 5, 7 should be considered. Magnitude of autocorrelation for morning ramp hours, e.g. HE7 in Figure 2.7 (b) and (e), decreases more sharply relative to off-peak hours while strong weekly seasonality at lags 6, 7 and 8 is preserved. Hence, lags 1, 3, 6, 7 and 8 should be considered for morning ramp hours. On-peak hours as represented by HE15 in Figure 2.7 (c) and (f), exhibit strong autocorrelation at lags 1, 2 and 3 with relatively weaker weekly seasonality. Therefore, the following AR with a constant model specifications...
Figure 2.7: Autocorrelation and partial autocorrelation of DAM LMPs segmented by hour

Note: Hours ending 2, 7 and 15 are representative of autocorrelation exhibited by off-peak, morning ramp and on-peak hours accordingly.

Figure 2.8: BIC and AIC values for ARMA lags specifications from Table (2.43)

will be tested for each individual hour:

M1 AR lags: 1, 2, 3, 4, 5, 7;
M2 AR lags: 1, 6, 7, 8;
M3 AR lags: 1, 2, 3;
M4 AR lags: 1, 7;
M5 AR lags: 1, 2.
Table 2.42: Segmented-by-hour out-of-sample performance of various specifications of AR with a constant model

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Note: One can choose single specification of AR lags across all hours or tailor choice of AR lags to each hour. Retaining divisible by 24 AR lags from global time-series specification in Table() results in $p_h^h (1 - \alpha_1^h L - \alpha_2^h L^6 - \alpha_3^h L^9) = \alpha_0^h + \varepsilon^h$ for hours $h = 1, \ldots, 24$. Revenues are not reported when evaluating model performance on hour-by-hour basis since battery dispatch is optimized in 24-hour windows and not on an hourly basis.
Table 2.43: Iterative removal of ARMA lags to increasing Bayesian and Akaike information criteria

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References for Chapter 2


Chapter 3

Valuation of Energy Storage for Stacked Applications

3.1 Introduction

In this chapter we perform technology-neutral valuation exercise of energy storage devices used for individual and “stacked” applications by participating in wholesale energy and regulation markets administered by California Independent System Operator. We contribute to existing literature on energy storage modeling, dispatch and valuation by forecasting all price inputs, enforcing market specific dispatch and commitment rules, using more realistic assumptions on regulation dispatch and outlining procedures for efficiency parameter measurements. Valuation is performed across four different nodes exhibiting distinct behavior of the congestion component of locational marginal price. Choice of forecasting model for day-ahead price inputs is based on findings presented in Chapter 2, where day-ahead energy time-shift, i.e. one individual application, revenues were used as a loss function to perform predictive accuracy comparisons.

3.2 Energy Storage Applications

There exists an extensive range of studies outlining various applications for energy storage. Eyer and Corey (2010). These applications can be divided into two groups (1) wholesale grid connected, (2) islanded and behind-the-meter. In this chapter we focus on wholesale grid connected applications, where energy
storage is participating in markets for energy and regulation administered by independent system operator (ISO). In this chapter energy storage application refers to combination of markets that energy storage device is participating in. When battery is co-optimized across many different markets we refer to such application as “stacked.” This term has been widely used in the industry even though once “stacked” operational behavior exhibited by the battery, i.e. typical duty cycle, for any individual application may disappear due to co-optimization, hence undermining the visual intuition of “stacking” stand-alone application that preserve their operational features. This intuition may hold for the case where revenues are optimized in a discrete manner where a single application is chosen for a day based on pairwise comparison of expected revenues across many different applications. Such discrete strategy to application stacking is not considered in this chapter.

The analytical framework for constrained optimization of energy storage over a finite horizon has been outlined in Mokrian and Stephen (2006). Further, Byrne and Silva-Monroy (2012) consider co-optimization across day-ahead energy and regulation markets but valuation is performed under perfect foresight scenario, i.e. known future prices, and assumption of fixed intra-day real-time regulation dispatch likelihoods. Haley et al. (2014) consider more extensive stack of applications but with a goal of making a policy argument that new flexible capacity metrics are needed to make energy storage more competitive during annual capacity procurement process. To the best of our knowledge, none of the existing literature performs co-optimization across real-time and day-ahead markets and uses only forecasted price inputs for energy storage dispatch and valuation. However, we retain assumptions of sufficiently small market participant not affecting market clearing prices. We further assume that use of linear programming approach for deterministic dispatch problem conditional on accurately forecasted price inputs is sufficient to perform a valuation exercise. We do not consider alternative dynamic and stochastic programming approaches discussed in Mokrian and Stephen (2006).

3.2.1 Markets for Energy and Capacity

California Independent System Operator administers day-ahead and real-time markets to procure energy and ancillary services. The scheduling coordinators submit supply and demand bids into both markets on behalf of load and generation that they represent. An energy award in MWh for a given trading hour obliges a storage operator to inject or withdraw an awarded quantity of energy during TH by following ISO’s dispatch signal. There are five types of Ancillary Services procured in the market: Regulation Up, Regulation Down, Regulation Mileage Up, Regulation Mileage Down, Spinning Reserve and Non-Spinning Reserve. The regulation up/down award for a given trading hour requires an energy storage resource to be
capable of increasing/decreasing its power output as instructed by 4-second Automatic Generation Control (AGC) signal. The energy storage device can also provide spinning/non-spinning reserves since it is always synchronized (except outage events) and can be ramped to any power output within its operating range in less than 10 minutes. However, the use of energy storage device for a lower quality regulation, i.e. spinning and non-spinning reserves, is outside of application “stack” considered in this chapter. The resource providing Regulation Up/Down must meet the minimum continuous energy requirement, i.e. ability to maintain an awarded power level for 60 minutes in a day-ahead and 30 minutes in real-time market.

The payments for energy are settled at Locational Marginal Price (LMP) in $/MWh associated with a resource’s pricing node (PNode). Energy payments may be associated with either energy directly scheduled in day-ahead and real-time markets or regulation up/down dispatch requiring battery to sell/purchase energy at real-time LMPs. The revenues for awarded Up/Down Regulation capacity are settled at the Ancillary Service Marginal Prices (ASMPs) in $/MW specific to resource’s ancillary service region or sub-region, which are not mutually exclusive subsets of PNodes defined in a full network model.

In this chapter we consider energy storage participating in the following three combinations of markets:

1. Day-ahead energy market only,
2. Day-ahead and real-time energy markets,
3. Day-Ahead energy and day-ahead regulation markets.

Application 1 above in the industry is commonly referred to as energy time-shift, which describes the intuition for the optimal dispatch solution, “charge when price is low, retain energy and discharge when price is high.” The naming convention based on the operational profile is motivated by importance of application-specific duty cycle, i.e. sequence of charges/discharges and idle periods, used for battery testing. However, for valuation purposes we identify each application by source of revenue, i.e. markets entered. Design of representative duty cycles is outside of scope of this chapter. We include demand charge management into analytical formulation of application stack but we limit empirical evaluation to only wholesale applications.

### 3.3 Energy Storage Model

Figure (3.1) below illustrates an energy storage model where for any $t$ amount of recoverable stored energy $S_t$ in MWh is constrained between $\underline{S}$ and $\bar{S}$. Amount of energy charged and discharged in an hour is constrained by charge/discharge power ratings such that $Q_C^f \leq Q_C^e \leq \bar{Q}_C$ and $Q_D^f \leq Q_D^e \leq \bar{Q}_D$. Upper and lower
bounds on \( S_t \) are not necessarily an absolute 0 and chemistry-specific theoretical maximum since constraints may arise from reliability requirements established in a marketplace or need to control depth of discharge for purposes of extending number of available charge/discharge cycles. Even though any \([S, S]\) interval can be normalized to \([0, \bar{S}/S]\), the actual maximum energy capacity of the battery \( \bar{S} \) is used for market potential per MWh of energy capacity calculation typically compared to per MWh cost of manufacturing such battery.

Figure 3.1: Energy Storage Model

The change in stored energy \( S_t \) is governed by Eq.(3.3.1), where \( \gamma_S \in (0, 1) \) is a time-based efficiency capturing loss of energy due to self-discharge and \( \gamma_c \in (0, 1) \) is a charge efficiency, where only \( \gamma_c \) of each 1 MWh stored is recoverable.

\[
S_t = \gamma_S S_{t-1} + \gamma_c Q_c^t - Q_d^t
\]  

Figure (3.2) illustrates components of charge efficiency \( \gamma_c \) and provides a measurement procedure. In Figure (3.2) only \((1 - \gamma_a)\) of one unit of energy charged by a battery as recorded by meter “A” directly goes to a storage medium while \( \gamma_a \in (0, 1) \) is consumed by auxiliary components such as pumps and heaters especially relevant in the case of scaled flow batteries and chemistries requiring extreme thermal conditions for operation. Even though meter “B” recorded \((1 - \gamma_a)\) of energy flowing to storage medium only \( \delta(1 - \gamma_a) \) of energy is recoverable, where \( \delta \) is technology/chemistry-specific parameter. Further, share of discharged energy is consumed by auxiliary components resulting in \( \delta(1 - \gamma_a)^2 \) of recoverable energy.

It is important to note that a stylized measurement procedure assumes losses to auxiliary components are the same across charge and discharge. Further, we don not model dependence on charge/discharge power and time, i.e. capturing deterioration of storage medium. In addition to charge/discharge power ratings and energy capacity each battery has a distinct ramp rate measured in MW/min, typically assumed to be symmetric for charge/discharge modes. High ramp rate is one of the main advantages of electricity storage over conventional generators. Further, FERC order 755 resulted in the institutional improvements in "pay for
performance” such as CAISO’s flexible ramping product compensating resources offering flexible ramping capability. However, due to novelty of the market instruments commoditizing ramp rates and our primary focus on energy and regulation markets with sufficient historical data to perform optimal operation exercises, we use a simplifying assumption that storage device can instantaneously switch between different power levels.

3.4 Data and Approach

The California ISO provides public market results data via OASIS web-portal OASIS (2015). The acquired data spans period from 1/1/2012 to 12/31/2014 and four pricing nodes (PNodes): LAJOLLA_6_N001, RIOHONDO_6_N001, KIFER_6_N001 and VACADIX_1_N085. The primary node of interest is LAJOLLA_6_N001 at which energy imports/exports between University of California, San Diego (UCSD) and CAISO are settled. Specific contractual arrangements between energy serving entity and UCSD will not be discussed, and the choice of location is motivated by a possibility of future wholesale grid connected energy storage projects that may utilize results presented in Chapters 2 and 3. Further, LAJOLLA_6_N001 pnode is an element of SDGE demand load aggregation point with an average load share of 0.28% during on- and off-peak hours. The choice of other three nodes was motivated by congestion patterns in CAISO and location of existing energy storage pilot projects. It is known that congestion on high voltage transmission lines in Southern California often increases prices within Southern California Edison (SCE) and San Diego Gas & Electric (SDG&E) service areas, but decreases prices in the Pacific Gas and Electric (PGE) area. Congestion in Northern California often has the opposite effect. Hence, it was decided to select two nodes from PGE and one from SCE. The PNodes RIOHONDO_6_N001 and KIFER_6_N001 are relatively large load contributors in SCE and PGE areas. In a generating resource list as a part of CAISO’s full network
model one can find that Vaca Dixon sodium-sulfur battery with a 2-hour net-dependable capacity of 1.85MW is mapped to VACADIX_1_N085, and is a participating market resource as noted in Generating Capabilities List published 1/13/2015.

For each pricing node locational marginal prices, LMPs, were downloaded along with ASMPs (Ancillary Service Marginal Prices) for regulation up and down at AS_CAISO_EXP regulation region. AS_CAISO_EXP regulation region includes all resources internal to CAISO. Note that energy storage participating in regulation markets may also receive some additional payments from their local AS region. For example, resources located at LAJOLLA_6_N001 may receive capacity payments from AS_SP15_EXP region. In this chapter we focus on AS_CAISO_EXP as the only source of capacity payments, while variation in total regulation revenues across PNodes is driven by differences in DAM and RTM LMPs. Data on actual aggregate CAISO load and its day-ahead forecasts was downloaded to support LMP forecasts. Tables (3.1) and (3.2) provide summary of day-ahead and real-time locational marginal prices and its components across four nodes. Further, Table (3.3) performs comparison between DAM and RTM LMPs for each node which is important for forming intuition about successful operating strategies when co-optimizing storage across day-ahead and real-time energy markets or participating in DAM energy and regulation markets where battery is exposed to RTM LMPs via real-time regulation dispatch.

In addition to price and load data obtained from CAISO OASIS, we also rely on sample automatic generation control signal (AGC) for 7/11/2010 - 7/17/2010 that was released as a part of regulation energy management for non-generator resources stakeholder process. One-week sample of actual AGC signal allowed us to form better understanding of real-time regulation dispatch. Figure (3.3) depicts aggregate CAISO AGC signal for 7/11/2010 where instantaneous dispatched power in MW is indicated by black line.
### Table 3.2: RTM LMPs Summary

<table>
<thead>
<tr>
<th></th>
<th>LAJOLLA_6_N001</th>
<th>VACADIX_1_N085</th>
<th>RIOHONDO_6_N001</th>
<th>KIFER_6_N001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>µ</td>
<td>σ</td>
<td>µ</td>
<td>σ</td>
</tr>
<tr>
<td>LMP off</td>
<td>36.12 (12.71)</td>
<td>34.88 (11.2)</td>
<td>37.14 (17.09)</td>
<td>36.1 (11.54)</td>
</tr>
<tr>
<td>LMP on</td>
<td>49.24 (20.65)</td>
<td>43.64 (15.3)</td>
<td>48.25 (20.57)</td>
<td>47.22 (18.39)</td>
</tr>
<tr>
<td>ΔLMP&lt;sub&gt;RTM&lt;/sub&gt;</td>
<td><strong>13.11</strong> (20.98)</td>
<td><strong>8.76</strong> (14.24)</td>
<td><strong>11.11</strong> (22.16)</td>
<td><strong>11.12</strong> (17.75)</td>
</tr>
<tr>
<td>MCE off</td>
<td>-0.61 (7.06)</td>
<td>-0.8 (5.78)</td>
<td>0.7 (6.95)</td>
<td>-0.89 (6.21)</td>
</tr>
<tr>
<td>MCE on</td>
<td>1.46 (10.41)</td>
<td>-1.27 (8.1)</td>
<td>1.32 (8.61)</td>
<td>-0.42 (11.56)</td>
</tr>
<tr>
<td>ΔMCE&lt;sub&gt;RTM&lt;/sub&gt;</td>
<td><strong>2.07</strong> (11.42)</td>
<td><strong>-0.47</strong> (8.69)</td>
<td><strong>0.62</strong> (9.47)</td>
<td><strong>0.47</strong> (12.04)</td>
</tr>
<tr>
<td>MCC off</td>
<td>0.17 (0.69)</td>
<td>-0.69 (0.43)</td>
<td>-0.08 (0.34)</td>
<td>0.42 (0.47)</td>
</tr>
<tr>
<td>MCC on</td>
<td>0.78 (1.07)</td>
<td>-1.24 (0.92)</td>
<td>0.02 (0.6)</td>
<td>0.65 (0.81)</td>
</tr>
<tr>
<td>ΔMCC&lt;sub&gt;RTM&lt;/sub&gt;</td>
<td><strong>0.61</strong> (0.8)</td>
<td><strong>-0.55</strong> (0.74)</td>
<td><strong>0.1</strong> (0.44)</td>
<td><strong>0.23</strong> (0.6)</td>
</tr>
<tr>
<td>MCL off</td>
<td>0.11 (0.65)</td>
<td>-0.002 (0.38)</td>
<td>-0.08 (0.24)</td>
<td>0.04 (0.36)</td>
</tr>
<tr>
<td>MCL on</td>
<td>0.53 (1.05)</td>
<td>-0.21 (0.74)</td>
<td>-0.04 (0.47)</td>
<td>-0.17 (0.66)</td>
</tr>
</tbody>
</table>

### Table 3.3: LMP differences across DAM and RTM

<table>
<thead>
<tr>
<th></th>
<th>LAJOLLA_6_N001</th>
<th>VACADIX_1_N085</th>
<th>RIOHONDO_6_N001</th>
<th>KIFER_6_N001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>µ</td>
<td>σ</td>
<td>µ</td>
<td>σ</td>
</tr>
<tr>
<td>ΔLMP&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td>2.02 (11.19)</td>
<td>2.21 (9.38)</td>
<td>1.33 (16.14)</td>
<td>2.31 (9.83)</td>
</tr>
<tr>
<td>ΔLMP&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td><strong>3.09</strong> (18.25)</td>
<td><strong>2.81</strong> (12.8)</td>
<td><strong>2.56</strong> (19.06)</td>
<td><strong>1.07</strong> (16.07)</td>
</tr>
<tr>
<td>ΔMCE&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td>0.15 (7.08)</td>
<td>0.25 (5.82)</td>
<td>-0.39 (6.85)</td>
<td>0.41 (6.18)</td>
</tr>
<tr>
<td>ΔMCE&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td>0.12 (10.99)</td>
<td>-0.26 (8.03)</td>
<td>0.08 (8.39)</td>
<td>-0.85 (11.54)</td>
</tr>
<tr>
<td>ΔMCC&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td>0.11 (0.65)</td>
<td>-0.002 (0.38)</td>
<td>-0.08 (0.24)</td>
<td>0.04 (0.36)</td>
</tr>
<tr>
<td>ΔMCC&lt;sub&gt;DAM-RTM&lt;/sub&gt;</td>
<td>0.53 (1.05)</td>
<td>-0.21 (0.74)</td>
<td>-0.04 (0.47)</td>
<td>-0.17 (0.66)</td>
</tr>
</tbody>
</table>
and shaded blue and red areas between line and horizontal axis correspond to dispatched regulation down and up energy in MWh accordingly. The bounding regions above and below and horizontal axis correspond to total procured regulation up and down capacity. Hence, likelihood of regulation dispatch depends on whether AGC signal is below or above horizontal axis and the share of dispatched energy to total procured capacity for a particular hour.

![Dispatched versus Procured Aggregate CAISO Regulation, 7/11/2010](image)

Figure 3.3: Dispatched versus Procured Regulation for 7/11/2010

Figure (3.4) illustrates components of final regulation down (RD) dispatch likelihood, where for each hour we at first compute share of 4-sec intervals for which AGC signal is below zero (stem with a box) followed by a calculation of the ratio of mean dispatched energy to the height of the bounding box (stem with a cross), i.e. not all recipients of capacity payments will be called upon to regulate. Product of these two probabilities yields the final likelihood of dispatch (stem with a solid circle). The same procedure is followed in Figure (3.5) to compute regulation up dispatch likelihoods.

![Decomposition of Regulation Down (RD) dispatch likelihood for 7/11/2010](image)

Figure 3.4: Decomposition of Regulation Down (RD) dispatch likelihood for 7/11/2010
Figures (3.6)-(3.8) illustrate observed regulation dispatch likelihoods used in optimal operation exercises presented in Section 3.6. Use of actual observed dispatch likelihoods that vary within day instead of constant across day hypothetical values is an important contribution to understanding the variation in revenues associated with regulation markets and understanding of whether regulation dispatch settled at real-time prices amplifies or undermines revenues based on capacity payments alone.
3.5 Day-Ahead and Real-Time (DART) co-optimization

Assumption of known future prices is relaxed by forecasting clearing prices for day-ahead and real-time energy markets that are further used as inputs to our optimal commitment algorithm. However, we do retain an assumption that an electricity storage device under the consideration is sufficiently small not to affect clearing prices. Hence, under this price-taking assumption known and forecasted energy prices are independent of the quantities supplied/demanded by a given battery. In case of day-ahead regulation we contribute to existing literature by using actual real-time prices for regulation dispatch settlement, forecasted ASMPs, i.e. prices for regulation up/down capacity, and more realistic regulation dispatch likelihoods varying within day.

Existing literature has primarily focused on optimizing battery operation across day-ahead energy and ancillary services markets, where regulation up and down capacities are submitted jointly with energy commitments on the day-ahead basis. Optimizing charge/discharge schedules jointly over day-ahead and real-time markets requires one to account for differences in the commitment horizons and timelines for submission of energy bids and publication of clearing prices. Therefore, we at first discuss in detail CAISO’s energy market timelines and operational rules in Subsection (3.5.1) followed by formal exposition of the optimization approach in Subsection (3.5.3). We follow Byrne and Silva-Monroy (2012) for exposition of linear program formulation.

3.5.1 CAISO operational rules for electricity storage

The CAISO has two resource models that allow storage technologies to participate in the ancillary services and energy markets: pump storage and non-generator resource. Non-Generator Resource (NGR) can serve as both generation and load and can be dispatched to any operating level within their capacity range. Pump Storage resources consume energy when pumping water to higher elevation reservoirs, and...
then operate as generators when water is released back to lower reservoirs. The category of non-generator resources includes demand response (DR) and limited energy storage resources (LESRs). The assumptions on a hypothetical electricity storage device made in this paper fit well with LESR model that accommodates small electrochemical batteries. Consideration of pump storage would require more careful treatment of ramp rates since an assumption of instantaneous change in power output/input is not appropriate for such facilities. Further, examination of the spatial variation in the operational pattern and the value of the electricity storage across different pricing nodes is less relevant for pumped-hydro technology relying on unique geographic conditions.

The electricity storage participating in the CAISO’s markets as a NGR must satisfy participating load and generator agreements. A participating generator is able to sell and provide energy or ancillary services given a minimum power rating of 1 MW, i.e. $\bar{Q}_D, \bar{Q}_C \geq 0$. The NGRs have a continuous operating range, no startup, shutdown, minimum load or transition costs, which makes our simplified electricity storage model presented in Section 3.3 a good representation of actual operating environment. CAISO offers NGRs an option to participate in the Regulation Energy Management (REM), which awards resources four times the regulation capacity they can provide in 15 minutes with an option of purchasing offset energy in the real-time market. However, under REM option the electricity storage can participate only in the regulation market and discharge/charge decisions are subject to full CAISO control. Hence, we focus on an electricity storage device operating as a NGR without REM option in day-ahead and real-time energy markets.

### 3.5.2 CAISO day-ahead and real-time market timelines

In the DA market hourly energy supply (discharge) or demand (charge) quantities for 24 hours of the following trading day (TD+1) must be committed by 10:00 of the current trading day (TD) as shown in Figure 3.9. The DA prices for every hour of (TD+1) are published at 13:00 of TD. Real-time market occurs on the hourly basis requiring the electricity storage operator to commit discharge quantities at least 75 minutes prior to the start of the trading hour (TH) when physical delivery is due. The RT energy price for TH is published 45 min prior to the start of TH.

To operationalize enforcement of RT and DA market timelines within linear program presented in Subsection 3.5.3 let index $j = 1, \ldots, T$ indicate an hour within timeframe of our forecasting/evaluation exercise. For example, as described in Section 3.6 we optimize electricity storage operation and forecast DA and RT prices for 17520 hours in 2013 and 2014. Hence, $j = 1$ corresponds to 00:00-01:00 on Jan 1st 2013 and $j = 730 \cdot 24$ corresponds to 23:00-24:00 on Dec 31st 2014. At 10:00 on each trading day the
Figure 3.9: Market timelines

Note: Above market timelines are consistent with CAISO’s market design as presented in BPM (2015).

storage operator extends device’s commitment in the DA market by 24 hours. Therefore, by the end of hour $j$ commitment horizon in the DA market extends to $H_{da}^c(j)$ as indicated in Eq.(3.5.1). The DA market prices for every hour of the following day are posted at 13:00 of the current day. Hence, by the end of hour $j$ day-ahead information horizon extends to $H_{da}^i(j)$, as presented in Eq. (3.5.2). As demonstrated in Figure 3.10 day-ahead information horizon lags behind day-ahead commitment horizon during 10th, 11th, 12th and 13th hours of the day.

$$H_{da}^c(j) = \begin{cases} 
\lfloor \frac{j}{24} \rfloor & \text{if } (j \mod 24) \leq 9 \\
\lfloor \frac{j}{24} \rfloor + 1 & \text{if } (j \mod 24) \geq 10 \\
\lfloor \frac{j}{24} \rfloor + \lfloor \frac{j}{24} \rfloor + 1 & \text{if } (j \mod 24) = 0 \\
\lfloor \frac{j}{24} \rfloor + 24 & \text{if } (j \mod 24) = 24 
\end{cases}$$ (3.5.1)

$$H_{da}^i(j) = \begin{cases} 
\lfloor \frac{j}{24} \rfloor & \text{if } (j \mod 24) \leq 13 \\
\lfloor \frac{j}{24} \rfloor + 1 & \text{if } (j \mod 24) \geq 14 \\
\lfloor \frac{j}{24} \rfloor + 24 & \text{if } (j \mod 24) = 0 \\
\lfloor \frac{j}{24} \rfloor + 24 & \text{if } (j \mod 24) = 24 
\end{cases}$$ (3.5.2)

Also, by the end of hour $j$ the electricity storage operator must commit RT energy for hour $(j + 2)$ and the CAISO publishes RT price for $(j + 1)$. Therefore, RT information and commitment horizons are $H_{rt}^i(j) = j + 1$ and $H_{rt}^c(j) = j + 2$ correspondingly.

### 3.5.3 A Linear Program

The problem of maximizing profits from an energy storage device participating in day-ahead and real-time energy markets, day-ahead regulation market and demand charge management application over the course of $T$ time periods can be formulated as a series of linear constrained optimization problems indexed by $j$ as shown in Eq.(3.5.3) along with energy and capacity commitment mechanism enforcing specific market timelines discussed in subsection (3.5.2). In a trivial case where a battery is optimized on an hourly basis and $T = 24$ only one day-ahead commitment problem is considered. However, in the case of multi-year optimal
operation studies a series of finite overlapping horizon problems are considered where the maximum length of optimization window is constrained by price forecast horizon.

For every hour \( j = 1, \ldots, T \) a linear function \(-f_j x_j\) is minimized subject to inequality constraint \( A_j x_j \leq b_j \), equality constraint, \( A_j^{eq} x_j = b_j^{eq} \), and the range for the choice variable, \( lb_j \leq x_j \leq ub_j \). At first, we describe elements of the linear programs for \( j = 10, 11, 33 \) prior to generalizing the notation for an arbitrary \( j \).

\[
\begin{align*}
\min_{x_j} \{-f_j x_j\} & \quad \text{s.t.} \\
A_j x_j & \leq b_j \\
A_j^{eq} x_j & = b_j^{eq} \\
lb_j & \leq x_j \leq ub_j
\end{align*}
\]

(3.5.3)

Recall from subsection (3.5.2) that by 10:00 of each trading day, i.e., any \( j \) such that \( \text{mod}(j, 24) = 10 \), the energy storage operator must commit energy and regulation capacity to each hour of the following day in DA market, energy to 12\(^{th}\) hour, i.e., \( j = 12 \), in the RT market and discharge quantities for the following trading day to mitigate demand charges for associated load/customer. Therefore, linear program at \( j = 10 \) must yield optimal energy and capacity quantities for \( j = 11, 12, \ldots, 48 \), subject to already committed energy. Hence, the relevant optimization interval spans 38 hours. By utilizing timing function in Eq.(3.5.1), length of the optimization interval associated with \( j \th \) linear program is equal \( K_j = H_{da}'(j) - j \). Figure (3.11) illustrates the 38-hour optimization interval associated with \( j = 10 \), where expected net energy quantities committed to DA and RT energy markets are denoted by \( Q_j^{da} \) and \( Q_j^{rt} \) accordingly. Note that energy commitment associated with real-time regulation dispatch is accounted for as well based on expected likelihoods of dispatch. Recognize
that within the relevant optimization interval, \( Q_{11}^{da}, Q_{12}^{da}, \ldots, Q_{24}^{da} \), are already committed to DA market and \( Q_{11}^{rt} \) is committed to RT market. In Figure (3.11) committed net energy quantities are boxed.

**Note:** Boxed \( \hat{Q}^{da}_j \) and \( \hat{Q}_{j}^{rt} \) indicate already committed net energy quantities. Quantities \( Q_{j}^{da} \) and \( Q_{j}^{rt} \) will be reoptimized in the subsequent DART iterations prior to commitment.

Eq. (3.3.1) provides a general model for the path of energy stored given net energy charged/discharged in period \( t \). Eq.(3.5.4) decomposes energy charge/discharge by corresponding market or application, where \( q_{t}^{da}, q_{t}^{rt} \) and \( \gamma_{t}^{rd} \) indicate energy charged via day-ahead and real-time energy markets, and by providing regulation down service. When energy storage operator commits day-ahead \( q_{t}^{rd} \) MWh of battery capacity available for charge during hour \( t \) only \( \gamma_{t}^{rd} \) is actually charged in real-time. The components of net energy discharged are energy sales in day-ahead and real-time markets, \( \hat{q}_{t}^{da} \) and \( \hat{q}_{t}^{rt} \), share of committed regulation up capacity actually dispatched by ISO in real-time, i.e. \( \gamma_{t}^{da} \hat{q}_{t}^{da} \), and energy committed to demand charge management, \( \hat{q}_{t}^{dcm} \). Restricting demand charge management to discharges only forces energy charge needed to support peak-shaving to come from DAM energy market. Note that this is different from load shaping where customer’s load is increased during off-peak hours not affecting total monthly energy charge but reducing the demand charge. We force day-ahead commitment onto demand charge management discharges. Time subscripts on parameters \( \gamma_{t}^{da} \) and \( \gamma_{t}^{rd} \) allow one to accommodate different likelihoods of regulation dispatch depending on the hour of the day, e.g. regulation need and hence real-time dispatch is significantly lower during off-peak hours.

\[
S_t = \gamma_{t} S_{t-1} + \gamma_{t} \left( \hat{q}_{t}^{da} + \hat{q}_{t}^{rt} + \gamma_{t}^{rd} \hat{q}_{t}^{rd} \right) - \left( q_{t}^{da} + q_{t}^{rt} + \gamma_{t}^{da} q_{t}^{da} + q_{t}^{dcm} \right) \quad (3.5.4)
\]

Application/market-specific quantities in Eq.(3.5.4) are mapped to aggregate expected day-ahead and real-time energy, i.e. \( Q_{j}^{da} \) and \( Q_{j}^{rt} \), in Figure (3.11) as follows, \( Q_{j}^{da} = \hat{q}_{j}^{da} + q_{j}^{da} + \gamma_{j}^{rd} q_{j}^{rd} + \gamma_{j}^{da} q_{j}^{da} + q_{j}^{dcm} \) and \( Q_{j}^{rt} = \hat{q}_{j}^{rt} + q_{j}^{rt} \). We proceed to describe elements of the linear program associated with \( j = 10 \). The choice variable, \( x_{10} \), is a column vector in Eq. (3.5.5) consisting of \( 3 \cdot 38 \times 1 \) block of charge and \( 4 \cdot 38 \times 1 \)
One can recognize a pattern in Eqs. (3.5.7) such that of Eq. (3.5.4) given initial state of charge. To express state of charge constraint in a convenient matrix multiplication form at first consider two iterations constrained between 0 and 1 MW.

For every hour $t$ constraints on the state of charge and net energy charged/discharged must hold, i.e. $S \leq S_t \leq \hat{S}, \ C^C \leq q^d_t + \gamma^r q^r_t \leq \hat{C}$ and $Q^D \leq q^d_t + q^r_t + \gamma^u q^u_t \leq \hat{Q}^D$, where constraint boundaries were defined in Section (3.3). We can further define upper and lower bounds on elements of vector $x_{10}$, such that $lb_{10} \leq x_{10} \leq ub_{10}$. Restriction on the values of choice variables, i.e. elements of $x_{10}$, ensures that resource is not dispatched beyond its power rating, which is possible when only net energy charge/discharge constrains are enforced. For example, energy operator participating only in day-ahead regulation market with expectation of 0.1 real-time regulation down dispatch may bid 10MW into the market which would produce expected charge of 1 MWh. However, regulation market is a capacity market where procured service is the ability to provide committed power rating. Therefore, expected charge of 1MWh of energy must be performed at 10MW power level for only 6 minutes, which is not feasible if battery’s charge/discharge power rating is constrained between 0 and 1 MW.

$$lb_{10} = \begin{bmatrix} C_{1 \times 38}^C & D_{1 \times 38}^D \end{bmatrix}^T, \quad ub_{10} = \begin{bmatrix} C_{1 \times 38}^C & D_{1 \times 38}^D \end{bmatrix}^T$$

(3.5.6)

To express state of charge constraint in a convenient matrix multiplication form at first consider two iterations of Eq.(3.5.4) given initial state of charge $S_{10}$.

$$S_{11} = S_{10} + \gamma (q^d_1 + q^r_1 + \gamma^u q^u_1) - (q^{dA}_1 + q^{RA}_1 + \gamma^u q^{RA}_1 + q^{Rdcm}_1)$$

$$S_{12} = S_{10} + \gamma (q^d_1 + q^r_1 + \gamma^u q^u_1) - (q^{dA}_1 + q^{RA}_1 + \gamma^u q^{RA}_1 + q^{Rdcm}_1) + \ldots$$

(3.5.7)

One can recognize a pattern in Eqs.(3.5.7) such that $\begin{bmatrix} S_{11}, \ S_{12}, \ldots, S_{48} \end{bmatrix}^T = A'_{10} x_{10} + S_{10} \hat{B}$ where $A'_{10} = \begin{bmatrix} \gamma \hat{A}, \ \gamma \hat{A}, \ \gamma \hat{A} R^d, \ -\hat{A}, \ -\hat{A}, \ -\hat{A} R^u, \ -\hat{A} \end{bmatrix}$.
Eqs. (3.5.8) to (3.5.10).

\[
\hat{A} = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
\gamma_s & 1 & 0 & \cdots & 0 \\
\gamma_s^2 & \gamma_s & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\gamma_s^{37} & \gamma_s^{36} & \gamma_s^{35} & \cdots & 1
\end{bmatrix} \in \mathbb{R}^{38 \times 38} \tag{3.5.8}
\]

\[
\hat{B} = \begin{bmatrix}
\gamma_s & \gamma_s^2 & \cdots & \gamma_s^{37} & \gamma_s^{38}
\end{bmatrix}
\tag{3.5.9}
\]

\[
R^d = \begin{bmatrix}
\gamma_{11}^d, & \cdots, & \gamma_{48}^d
\end{bmatrix} I_{38 \times 38} \quad R^u = \begin{bmatrix}
\gamma_{11}^u, & \cdots, & \gamma_{48}^u
\end{bmatrix} I_{38 \times 38} \tag{3.5.10}
\]

Inequality constraint \(A_{10} \leq b_{10}\) is formed by combining state of charge and net energy charge/discharge constraints as indicated in Eqs. (3.5.11) below,

\[
\begin{align*}
S_t \leq \bar{S}, \\
-S_t \leq -\bar{S}, \\
\left(\hat{q}_{\alpha t}^d + \hat{q}_{\alpha t}^r + \gamma_s^{\hat{q}^{\alpha d}} \hat{q}_{\alpha t}^d\right) & \leq \bar{Q}^C, \\
-\left(\hat{q}_{\alpha t}^d + \hat{q}_{\alpha t}^r + \gamma_s^{\hat{q}^{\alpha d}} \hat{q}_{\alpha t}^d\right) & \leq -\bar{Q}^C, \\
\left(\hat{q}_{\alpha t}^d + \hat{q}_{\alpha t}^r + \gamma_s^{\hat{q}^{\alpha u}} \hat{q}_{\alpha t}^u\right) & \leq \bar{Q}^D, \\
-\left(\hat{q}_{\alpha t}^d + \hat{q}_{\alpha t}^r + \gamma_s^{\hat{q}^{\alpha u}} \hat{q}_{\alpha t}^u\right) & \leq -\bar{Q}^D.
\end{align*} \tag{3.5.11}
\]

Expressions for \(A_{10}\) and \(b_{10}\) are provided in Eqs. (3.5.12) and (3.5.13) below, where identity and zero matrices are such that \(I, 0 \in \mathbb{R}^{38 \times 38}\).

\[
A_{10} = \begin{bmatrix}
\gamma_s \hat{A} & \gamma_s \hat{A} & \gamma_s \hat{A} R^d & -\hat{A} & -\hat{A} & -\hat{A} R^u & -\hat{A} \\
-\gamma_s \hat{A} & -\gamma_s \hat{A} & -\gamma_s \hat{A} R^d & \hat{A} & \hat{A} & R^u & \hat{A} \\
I & I & I & 0 & 0 & 0 & 0 \\
-I & -I & -I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & I & I & I \\
0 & 0 & 0 & -I & -I & -I & -I
\end{bmatrix} \tag{3.5.12}
\]

\[
b_{10} = \begin{bmatrix}
\gamma_s \hat{b}_{\alpha t}^d & \gamma_s \hat{b}_{\alpha t}^r & \gamma_s \hat{b}_{\alpha t}^{\hat{q}^{\alpha d}} & -\hat{b}_{\alpha t} & -\hat{b}_{\alpha t} & -\hat{b}_{\alpha t}^{\hat{q}^{\alpha u}} & -\hat{b}_{\alpha t}
\end{bmatrix} \tag{3.5.13}
\]
Equality constraint $A_{10}^{eq} = b_{10}^{eq}$ is associated with enforcement of already existing commitments and any binding institutional requirements. Hence, at $j = 10$ energy/capacity quantities are committed in the day-ahead market up to $j = 24$ and in the RT market up to $j = 11$. More generally, linear program at $j$ will observe DA commitments up to $H^{24}_{e}(j - 1)$ and RT commitments up to $j + 1$, which will result in $H^{24}_{e}(j - 1) - j$ equality constrains in DA and one equality constraint in the RT market. Further, CAISO operational rules do not allow purchases of energy in the RT market, hence forcing $q_{rt}^j = 0$ for $j = 11, \ldots, 48$. 

\[
A_{10}^{eq} = \begin{bmatrix}
I_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 14} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14} & 0_{14 \times 38} \\
0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 0_{14 \times 38} & 1_{14 \times 14}
\end{bmatrix} \begin{bmatrix}
(\bar{q}_{11}^{da} + \bar{q}_{11}^{ct})^- & \cdots & (\bar{q}_{24}^{da} + \bar{q}_{24}^{ct})^-
\end{bmatrix}^T \quad (3.5.13)
\]

\[
B_{10}^{eq} = \begin{bmatrix}
(\bar{q}_{11}^{da} + \bar{q}_{11}^{ct})^+ & \cdots & (\bar{q}_{24}^{da} + \bar{q}_{24}^{ct})^+
\end{bmatrix}^T
\]

\[
\begin{bmatrix}
0_{38 \times 1} \\
\bar{q}_{11}^{ct}, \ldots, \bar{q}_{24}^{ct} \\
\bar{q}_{11}^{da}, \ldots, \bar{q}_{24}^{da} \\
(\bar{q}_{11}^{ct} + \bar{q}_{11}^{ct})^+ \\
\bar{q}_{11}^{ct}, \ldots, \bar{q}_{24}^{ct} \\
\bar{q}_{11}^{da}, \ldots, \bar{q}_{24}^{da}
\end{bmatrix} \quad (3.5.15)
\]

We proceed with formulation of objective function in Eq.(3.5.16), where $P^{da}$ and $P^{rt}$ are day-ahead
and real-time energy prices in $/MWh, \( P_{t}^{rd} \) and \( P_{t}^{ru} \) are day-ahead regulation capacity payments in $/MW and \( P_{t}^{dcm} \) equal to savings from mitigating demand charge per MWh of energy committed. It is important to note one notational convention that simplifies presentation of above linear program, where units of \( \tilde{q}_{j}^{rd} \) and \( \tilde{q}_{j}^{ru} \) are MWh not MW, meaning that in an hourly settlement capacity market bidding total energy of \( \tilde{q} \) (MWh) that will be delivered at constant power for an hour is equivalent to a capacity bid \( \frac{\tilde{q}}{t \text{ hour}} \). Hence, definition of \( \tilde{q}_{j}^{rd} \) and \( \tilde{q}_{j}^{ru} \) as maximum quantities of energy that can be dispatched at a constant power still allows one to compute capacity payments from \( P_{t}^{rd} \tilde{q}_{j}^{rd} \) and \( P_{t}^{ru} \tilde{q}_{j}^{ru} \) without need for unit conversion. Further, \( \hat{C} \) and \( \check{C} \) are non-energy costs of discharging and charging in $/MWh, e.g. cost of labor and maintenance. Recall, that all energy related costs of performing charge and discharge by construction are internalized in value of charge efficiency \( \gamma \) as presented in Figure (3.2).

\[
\Pi_{0} = \sum_{t=1}^{T} \left[ -(P_{11}^{da} + \hat{C})q_{11}^{da} - (P_{t}^{rt} + \check{C})\tilde{q}_{t}^{rt} + (P_{t}^{rd} - \tilde{q}_{t}^{rd})(P_{t}^{rt} + \check{C})\tilde{q}_{t}^{rd} + \left( P_{t}^{da} - \hat{C} \right) \tilde{q}_{t}^{da} + (P_{t}^{rt} - \hat{C})\tilde{q}_{t}^{rt} + (P_{t}^{ru} + \check{C})(P_{t}^{rt} + \check{C})\tilde{q}_{t}^{ru} + \left( P_{t}^{dcm} + \check{C} \right) \tilde{q}_{t}^{dcm} \right] \tag{3.5.16}
\]

The linear function \( f_{10} \) such that \(-\Pi_{10} = -f_{10}x_{10}\) is presented in Eq.(3.5.17) below. Note that negative signs appear in front of prices used for settlement of charged quantities because in Eq.(3.5.6) all elements of \( x_{10} \) are constrained to a positive range independent of either charge or discharge.

\[
f_{10} = \begin{bmatrix}
-P_{11}^{da} + \check{C} & \ldots & -P_{48}^{da} + \check{C} \\
-P_{11}^{rt} + \hat{C} & \ldots & -P_{48}^{rt} + \hat{C} \\
-P_{11}^{rd} + \gamma_{11}^{d}(P_{11}^{rt} + \hat{C}) & \ldots & -P_{11}^{rd} + \gamma_{48}^{d}(P_{48}^{rt} + \hat{C}) \\
-P_{11}^{da} + \check{C} & \ldots & -P_{48}^{da} + \check{C} \\
-P_{11}^{rt} + \hat{C} & \ldots & -P_{48}^{rt} + \hat{C} \\
-P_{11}^{ru} + \check{C} & \ldots & -P_{48}^{ru} + \gamma_{48}^{u}(P_{48}^{rt} + \hat{C}) \\
-P_{11}^{dcm} + \check{C} & \ldots & -P_{48}^{dcm} + \check{C}
\end{bmatrix} \tag{3.5.17}
\]

Once optimal charge/discharge vector \( x_{10}^{*} \) is obtained,

\[
Q_{12}^{r} = \tilde{q}_{12}^{r} + \check{q}_{12}^{r} \quad \text{and} \quad \left\{ Q_{j}^{da} = \tilde{q}_{j}^{da} + \check{q}_{j}^{da} + \gamma_{j}^{d} q_{j}^{rd} + \gamma_{j}^{u} q_{j}^{ru} + \check{q}_{j}^{dcm} \mid j = 25, \ldots, 48 \right\}
\]

are committed to RT and DA energy markets, and \( \tilde{q}_{j}^{rd} \) and \( \check{q}_{j}^{ru} \) for \( j = 25, \ldots, 48 \) are committed capacities to regulation down and up markets. Figure (3.12) associated with linear program at \( j = 11 \) incorporates energy commitments produced at \( j = 10 \) as indicated by boxed net quantities.

For \( j = 11, 12, \ldots, 33 \) day-ahead commitment horizon remains constant at 48. Hence, the length of
the optimization window shrinks from the maximum of 38 hours for \( j \) such that \( (j \mod 24) = 10 \) to 14 for \( j \) such that \( (j \mod 24) = 9 \), as shown in Figures (3.11) and (3.13). Figure (3.10) provides good visual intuition of the real-time market commitment horizon “catching up” with a stepwise day-ahead commitment horizon.

This section provides analytical framework for a “stacked” set of applications including participation in day-ahead and real-time energy markets, day-ahead capacity market and demand charge management application. However, one can apply the same framework to subsets of these applications by reducing the size of the choice variable vector \( x \) and reformulating constraints appropriately. Commitment mechanism is simplified significantly if energy storage participates in day-ahead markets only since commitments are performed in 24-hour blocks without rolling real-time horizon. For an arbitrary hour \( j \) the length of the optimization window is \( K_j = H_{c}^{da}(j) - j \) and energy storage operator has already committed DA energy up to \( H_{c}^{da}(j - 1) \) resulting in \( n_{da} = H_{c}^{da}(j - 1) - j \) equality constraints. Therefore, one can use \( K_j \) and \( n_{da} \) to appropriately adjust dimensions of elements of the linear program presented above.

### 3.5.4 Forecasts to support valuation via DART

To operationalize DART co-optimization we need to forecast prices within relevant optimization horizon for \( j \)th linear program. Figure 3.14 illustrates key forecasting environments at \( j \) such that \( (j \mod 24) = 10 \), \( (j \mod 24) = 13 \) and \( (j \mod 24) = 9 \). At 10:00 of every trading day storage operator commits
charge/discharge schedule to DA market. For example, if storage operator expects on-peak RT prices to be higher than on-peak DA price, then the discharge of the energy purchased during off-peak hours of DA is postponed till RT market.

The middle panel in Figure 3.14 illustrates publication of DA prices for the following trading day by the end of 13th hour of the current day. Therefore, a good forecast of real-time prices would use this information to refine location of the discharge hour in the RT market, given that the storage operator committed to selling energy in RT market. The updates of the real-time prices are unnecessary if the battery is committed to discharging in the DA market.

The bottom panel of Figure 3.14 indicates the shortest possible forecast horizon for real-time prices during hour \( j \) such that \( (j \mod 24) = 9 \), i.e. hour before DA energy commitments are due. However, short forecast horizon would be beneficial only if energy storage is committed to discharge in the RT market. Therefore, top panel of Figure 3.14 represents most critical forecast environment where forecasted reduced-form event (DA peak price is greater or lower than a RT peak), determines relevance of any further RT forecast refinements until the next DA market commitment deadline.
3.6 Results

In the subsections below we present revenues resulting from optimal operation exercises for individual and stacked applications resulting from participation in day-ahead and real-time energy and day-ahead regulation markets. We further examine variation in revenues with respect to battery parameters such as energy to power ratio (E/P) and charge efficiency ($\gamma_c$).

3.6.1 Energy Time-Shift via DAM

We observe the highest profits on per-MWh basis at LAJOLLA_6_N001 node with total revenues decreasing with respect to energy to power ratio (E/P) and increasing with charge efficiency, $\gamma_c$. Further, E/P normalized mileage indicated in column ‘M’ of Table (3.4) is decreasing with higher E/P ratios naturally indicating shallower discharges. Drop in normalized mileage from 4.1 to 2.3 between E/P=1 batteries operated at LAJOLLA_6_N001 as we decrease charge efficiency from $\gamma_c = 1$ to $\gamma_c = 0.8$ is due to rescaling effect that lower $\gamma_c$ values have on discharge price such that discharge during morning ramp, i.e. first intra-day local maximum, becomes sub-optimal and discharge is instead postponed till evening on-peak hours. This suggests that successful battery chemistries with high charge efficiencies (round-trip efficiency in engineering literature) will also exhibit high resistance to material deterioration associated with frequent cycling, i.e. high values of M.

Ordering of PNodes with respect to magnitude of $\Delta LMP_{DAM}$ spread between on-peak and off-peak DAM prices from Table (3.1) is preserved for DAM energy time-shift revenues across all E/P ratios and discharge efficiencies. Therefore, one can use simple summary statistics $\Delta LMP_{DAM}$ to compare DAM energy time-shift revenue potentials across different nodes. As we can see from Table (3.1) dominance of energy time-shift revenues at LAJOLLA_6_N001 is driven by MCC component that is typically negative during off-peak and positive during on-peak hours, hence adding node-specific $\Delta MCC_{DAM} = 2.04$ S/MWh to $\Delta MCE_{DAM} = 11.11$ S/MWh available to all pricing nodes.

3.6.2 Energy Time-Shift via DAM and RTM

Unlike participation in DAM energy market only, co-optimization across day-ahead and real-time markets allows for pure financial arbitrage opportunities. Hence, in Table 3.5 below each Total Revenue value we report percentages of financial and real revenues. Real revenues are associated with transactions
that change state of charge. Similarly to energy time-shift via DAM only, total revenues on a per-MWh basis are decreasing in E/P ratio and increasing in charge efficiency, $\gamma_c$. Recognize that share of financial relative to real revenues is decreasing as E/P ratio is increasing, which is driven by a fixed number of pure financial arbitrage opportunities identified by day-ahead and real-time LMP forecasts. Both total revenues (TR) in $/MWh of battery capacity and average revenues (AR, not-normalized by E/P) in Table (3.5) exceed those in Table (3.4). However, participation in RTM energy market in addition to DAM results in extreme variance of AR as indicated by standard errors reported in parentheses below daily averages. Mileage normalized by E/P is decreasing for larger batteries indicating shallower discharges.

Given CAISO’s operational rules restricting placement of energy demand bids in RTM, we expect charge during off-peak DAM price and discharge during on-peak RTM price to be an operating strategy added to energy time-shift via DAM only. Therefore, in addition to $\Delta \text{LMP}_{\text{DAM}}$ we should consider $\Delta \text{LMP}_{\text{DAM-RTM}}$ from Table (3.3). Differences between on-peak RTM and on-peak DAM LMPs across all PNodes on average are positive indicating that energy time-shift via DAM only strategy will dominate. Unlike total revenues in Table (3.5) preserving ordering of PNodes by $\Delta \text{LMP}_{\text{DAM}}$, energy time-shift revenues from participation in both RTM and DAM exhibit slightly different ordering such that RIOHONDO_6_N001 with the smallest $\Delta \text{MCC}_{\text{DAM-RTM}}$ is dominated by revenues from VACADIX_1_N085 and KIFER_6_N001 with -0.85 and -0.26 $\Delta \text{MCC}_{\text{DAM-RTM}}$ values indicating that RTM congestion dominates day-ahead congestion levels hence allowing additional revenues to be extracted by charging in DAM off-peak and discharging in RTM on-peak hours. However, negative $\Delta \text{MCC}_{\text{DAM-RTM}}$ for VACADIX_1_N085 and KIFER_6_N011 appear to be not sufficiently large to dominate $\Delta \text{LMP}_{\text{DAM}} = 14.18$ at LAJOLLA_6_N001.

### 3.6.3 DAM Energy and Regulation

Revenues for energy storage devices participating in both energy and capacity/regulation markets exceed those under energy time-shift via DAM only, or DAM and RTM, across all assumptions on likelihoods of real-time regulation dispatch examined in this chapter. Note that here we focus on energy versus capacity sources of revenues. However, purely financial energy transactions are not restricted and hence $T\text{R}^{\text{em}}$ is a sum of revenues associated with both physical/real and financial transactions. Unlike energy time-shift via DAM and RTM, standard errors on average daily revenues are modest and state of charge mileage (M) is significantly smaller. However, one must be very careful to always acknowledge that these observations are subject to specific assumptions on likelihood of regulation dispatch and any real-time imbalances, i.e. differences between expected and realized regulation dispatch. Tables (3.6), (3.7) and (3.8) in stated order
Intra-day state of charge mileage normalized by \( E/P \), e.g., two possible SoC paths corresponding to \( M=2 \) are assumed, Forecast model parameters were further re-estimated daily with an expanding window. No self-discharge is assumed. Data from 1/1/2012 - 12/31/2012 was used as an initial parameter estimation sample for price forecasts. With \( \text{armax6} \) forecasting method from Table (2.6) used for 24-hour-ahead forecasts of DAM energy prices. Note: Above performance is based on a two-year optimal dispatch exercise from 1/1/2012 to 12/31/2014 with \( \gamma = 1 \). Total Revenues (TR) are reported on per MWh basis. Column "M" indicates average intra-day state of charge mileage normalized by \( E/P \), e.g., two possible SoC paths corresponding to \( M=2 \) are \( \bar{S} \rightarrow \bar{S} \rightarrow \bar{S} \) and \( \frac{1}{2} (\bar{S} + \bar{S}) \rightarrow \bar{S} \rightarrow \frac{1}{2} (\bar{S} + \bar{S}) \rightarrow \bar{S} \rightarrow \frac{1}{2} (\bar{S} + \bar{S}) \). No constraints are enforced on intra-day SoC mileage.

<table>
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<th>TR AR($) M</th>
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\[ \text{Note: Above performance is based on a two-year optimal dispatch exercise from 1/1/2012 to 12/31/2014 with armax6 forecasting method from Table (2.6) used for 24-hour-ahead forecasts of DAM energy prices. Data from 1/1/2012 - 12/31/2012 was used as an initial parameter estimation sample for price forecasts. Forecast model parameters were further re-estimated daily with an expanding window. No self-discharge is assumed, } \gamma = 1. \text{ Total Revenues (TR) are reported on per MWh basis. Column 'M' indicates average intra-day state of charge mileage normalized by } E/P, \text{ e.g., two possible SoC paths corresponding to } M=2 \text{ are } \bar{S} \rightarrow \bar{S} \rightarrow \bar{S} \text{ and } \frac{1}{2} (\bar{S} + \bar{S}) \rightarrow \bar{S} \rightarrow \frac{1}{2} (\bar{S} + \bar{S}) \rightarrow \bar{S} \rightarrow \frac{1}{2} (\bar{S} + \bar{S}). \text{ No constraints are enforced on intra-day SoC mileage.} \]
## Table 3.5: Energy Time-Shift via DAM and RTM

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| **Note:** Above results are based on a two-year optimal dispatch exercise from 1/1/2013-12/31/2014, with armax6 method from Table (2.6) used for 24-hour ahead DAM and hourly RTM price forecasts, and hour-ahead RTM forecasts, with daily parameter re-estimation and 1/1/2012-12/31/2012 used as an initial estimation sample. DAM and RTM energy prices are treated as two separate data generating processes and hence forecasted separately. Financial and physical components of total revenues are reported on per-MWh basis. Energy storage is constrained not to charge in RTM. Computation of intra-day E/P normalized mileage is the same as in Table (3.4).
examine the following assumptions on regulation dispatch probabilities:

1. Symmetric and fixed within and across days, $\gamma^u = \gamma^d$ for all hours $t$ within optimal operation exercise,

2. Asymmetric and fixed within and across days, $\gamma^u_{t_1} = \gamma^d_{t_2}$, $\gamma^u_{t_1} = \gamma^d_{t_2}$, $\gamma^u_{t_1} \neq \gamma^d_{t_2}$ $\forall t_1, t_2$,

3. Observed likelihoods for a specific day are replicated for each day within optimal operation exercise, $\gamma^u_{h_{t_1}} = \gamma^d_{h_{t_2}}$ and $\gamma^d_{h_{t_1}} = \gamma^u_{h_{t_2}}$ for any day $t_1$ and $t_2$, but $\gamma^u_{h_{t_1}} \neq \gamma^u_{h_{t_2}}$ and $\gamma^d_{h_{t_1}} \neq \gamma^d_{h_{t_2}}$ $\forall t$ and $h_{t_1}, h_{t_2} \in [1, 2, \ldots, 24]$,

where in none of the scenarios we consider real-time penalties if realized signal deviates from expected $\gamma^u$ and $\gamma^d$.

Figure (3.15) illustrates the reason for which “stacking” participation in energy and regulation markets dramatically increases expected revenues. Figure (3.15) describes battery’s optimal DAM energy and regulation bids for 5/4/2013 submitted by 10am on 5/3/2013 given forecasted DAM LMP at LAJOLLA_6_N001 pricing node and ASMPs for regulation at AS_CAISO_EXP regulation zone. Forecasted energy and capacity prices are indicated by dotted lines in panels (c) and (d) accordingly. The resulting optimal power bids are presented in panel (a) where battery symmetrically bids 1MW into both regulation up and down except for 0 MW bid at hour 19 and net regulation up bids at hours 5 and 6. Given assumption of symmetric $\gamma^u = \gamma^d = 0.1$ and no real-time imbalances the battery operator is receiving capacity payments for both regulation up (RU) and regulation down (RD) for hours 1-4, 7-18, 20-24. Forecasted capacity payments alone amount to $230.19 for a single day which exceeds an intraday DAM LMP spread of $63.79 in an ideal case of perfect foresight (forecasted difference between off-peak and on-peak LMP is $53.69). Note that battery still performs energy time-shift. At hour 5 battery is committed to charging 1 MWh in DAM and also is committed to 1 MW of upward regulation with $\gamma^u = 0.1$ resulting in net charge of 0.9 MWh. This is captured in panel (b) where SoC increases from 0 to 0.9MWh between hours of 5 and 6. Additional 0.1MWh are obtained by placing a combination of 0.112 MWh DAM energy charge bid and net regulation up bid of 0.12 MW. Again given that $\gamma^u = 0.1$, this transaction yields additional 0.112MW $h - 0.1(0.12MW h) = 0.1MW h$ of energy. Battery is discharged at hour 19, which is not forecasted intra-day maximum LMP. The reason for not discharging at hour 18 is due to combined RU and RD hour 18 capacity payment exceeding symmetric capacity payment for hour 19 by a quantity greater than a difference between energy time-shift revenues from discharging at 19th versus 19th hour. Hence, DAM energy discharge is postponed by 1 hour to the 2nd highest forecasted intra-day LMP.

In Table (3.6) revenues were computed from two-year optimal dispatch exercise 1/1/2013 - 12/31/2014, where 24-hour ahead forecasts of both node-specific DAM LMPs and DAM ASMPS for reg-
Figure 3.15: One-day energy and regulation dispatch, 5/4/2013

**Note:**
(a) hourly energy and regulation capacity bids; (b) path of state of charge in MWh consistent with bids in (a); (c) forecasted and actual day-ahead LMPs at LAJOLLA_6_N001 pricing node; (d) forecasted and actual ancillary service marginal prices (ASMPs) for upward and downward regulation reported for AS_CAISO_EXP regulation region. Assumed regulation dispatch probabilities are $\gamma^u = \gamma^d = 0.1$. 
ulation up and down at AS_CAISO_EXP were forecasted with armax6 from Table (2.6). Forecasts were produced with daily re-estimation and data from 1/1/2012-12/31/2012 used as initial estimation sample. Probabilities of regulation dispatch are symmetric and assumed to be constant across all hours within a two-year exercise. Energy charged/discharged based on regulation down/up dispatch was settled at node-specific hourly RTM LMPs. Actual probability of real-time regulation dispatch is assumed to be exactly equal to expected probabilities, $\gamma^u$ and $\gamma^d$, hence no additional real-time energy imbalance cost. In Table (3.6) revenues on per-MWh basis are increasing in $\gamma^u, \gamma^d$ subject to $\gamma^u = \gamma^d$. Figure (3.16) panel (b) provides average energy and net regulation bids by hour of the day for symmetric case of $\gamma^u = \gamma^d = 0.3$. Recognize that device performs energy time shift via energy market and by relying on regulation dispatch such that net regulation down during off-peak hours yields charge and net regulation up during on-peak hours results in discharge. Also recall from Table (3.3) that across all pricing nodes on average both on- and off-peak DAM LMPs exceed RTM LMPs, i.e. $\Delta LMP^\text{on}_{\text{DAM-RTM}}, \Delta LMP^\text{off}_{\text{DAM-RTM}} > 0$. Hence, increasing likelihoods of regulation dispatch in a symmetric manner allows battery to charge more at relatively lower RTM LMPs during off-peak hours and then discharge at either on-peak DAM LMPs that are on average higher than RTM LMPs or discharge in RTM on-peak hours exhibiting frequent price spikes despite our limited ability to forecast them.

In Table (3.7) the details of optimal operation exercise used to compute above revenues are identical to those discussed in the note to Table (3.6) except likelihoods of regulation dispatch are assumed to be asymmetric. Note that operating in environment with asymmetric values of $(\gamma^d, \gamma^u)$ is very different from choosing not to participate in either regulation up or down markets. Asymmetric likelihoods, e.g. $\gamma^d = 0$ and $\gamma^u = 0.3$, only indicate that battery will receive payments for regulation energy only when regulating up, while receiving capacity payments for both regulation up and down.

Figure (3.16) further assists us with understanding results in Table (3.7) where total revenues on per-MWh across all nodes and E/P ratios for $(\gamma^u = 0, \gamma^d > 0)$ dominate those under $(\gamma^u > 0, \gamma^d = 0)$. In panel (a) of Figure (3.16) $\gamma^d = 0.3$ and $\gamma^u = 0$ meaning that battery operator enjoys opportunities to charge at RTM LMPs during regulation down dispatch without being exposed to settlement at RTM LMPs during regulation up dispatch. Hence, scenario in panel (a) exactly exploits $\Delta LMP^\text{on}_{\text{DAM-RTM}}, \Delta LMP^\text{off}_{\text{DAM-RTM}} > 0$ documented in Table (3.3). In the opposite case presented in panel (c) of Figure (3.16) battery operator has an expected obligation to discharge at RTM LMPs during regulation up dispatch while having to always charge at DAM LMPs that are on average higher than RTM LMPs across all hours of the day.

Table (3.8) presents revenues for energy storage participating in both DAM energy and regulation markets where assumed probabilities of dispatch vary within each day but not across days in a two-year
optimal operation exercise. Three sub-blocks within Table (3.8) correspond to revenues associated with regulation dispatch probabilities observed on 7/11/2010, 7/14/2010 and 7/17/2010 and then repeated for each day of this exercise. Revenues in Table (3.8) are lower than those in Table (3.6) across all nodes at E/P, indicating that using slightly more realistic assumptions on regulation dispatch likelihoods significantly change expected revenues. The highest revenues are observed for 7/11/2010 dispatch probabilities followed by 7/14/2010 and 7/17/2010. Our discussion of effects of regulation dispatch asymmetries on observed revenues in Table (3.6) is helpful for understanding this ordering. Recall that having more hours with regulation down dispatch greater than regulation up, $\gamma^d > \gamma^u$, allows battery operator to rely on charge at on average lower RTM LMPs and then perform a simultaneous or time-shifted discharge at DAM LMPs. Simple count of hours for which $\gamma^d > \gamma^u$ across 7/11, 7/14 and 7/17/2010 yields 19, 18 and 17 accordingly. Even though these three regulation dispatch configurations differ only by 1 hour with an extra regulation down dominance it is sufficient to produce observed ordering of revenues in (3.8) since optimal operation exercise is performed over 17520 hours.
Table 3.6: DAM Energy and Regulation Revenues, symmetric regulation dispatch and $\gamma = 1$

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<th>AR(S)</th>
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$\gamma_h^n = \gamma_h^d = 0.2 \ h = 1, \ldots, 24$

$\gamma_h^n = \gamma_h^d = 0.3 \ h = 1, \ldots, 24$
Table 3.7: DAM Energy and Regulation Revenues, asymmetric regulation dispatch, $\chi = 1$

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$\gamma_m = 0, \gamma_m' = 0, h = 1, ... , 24$

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$\gamma_m = 0, \gamma_m' = 0, h = 1, ... , 24$

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<th>KIFER_6_N001</th>
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<td>TR_m</td>
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<td>18,138</td>
<td>254 (3.45)</td>
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$\gamma_m = 0, \gamma_m' = 0, h = 1, ... , 24$

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<td>21,848</td>
<td>249.8 (3.22)</td>
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</tbody>
</table>

$\gamma_m = 0, \gamma_m' = 0, h = 1, ... , 24$

<table>
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<tbody>
<tr>
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<td>TR_m</td>
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<td>TR_m</td>
</tr>
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<td>17,331</td>
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$\gamma_m = 0, \gamma_m' = 0, h = 1, ... , 24$

<table>
<thead>
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</thead>
<tbody>
<tr>
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<td>TR_m</td>
<td>TR_m'</td>
<td>AR($) M</td>
<td>TR_m</td>
</tr>
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<td>22,844</td>
<td>249 (3.77)</td>
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</tbody>
</table>
Table 3.8: DAM Energy and Regulation Revenues with intra-day varying regulation dispatch, $\chi = 1$

| E/P | TR\textsuperscript{ena} | TR\textsuperscript{enp} | AR($) M | TR\textsuperscript{ena} | TR\textsuperscript{enp} | AR($) M | TR\textsuperscript{ena} | TR\textsuperscript{enp} | AR($) M | TR\textsuperscript{ena} | TR\textsuperscript{enp} | AR($) M | TR\textsuperscript{ena} | TR\textsuperscript{enp} | AR($) M |
|-----|----------------|-----------------|---------|----------------|-----------------|---------|----------------|-----------------|---------|----------------|-----------------|---------|----------------|-----------------|---------|---------|
| 1   | 51,407         | 29,464          | 110.8   | 41,316         | 31,000          | 99.1    | 51,526         | 29,516          | 111     | 46,482         | 30,582          | 105.6   | 10.2          |
| 2   | 35,294         | 15,609          | 99.1    | 27,218         | 17,103          | 97.3    | 34,611         | 15,735          | 99.9    | 30,448         | 16,671          | 129.1   | 6.6           |
| 3   | 29,496         | 10,428          | 99.1    | 22,087         | 11,780          | 99.9    | 28,519         | 10,547          | 99.9    | 24,715         | 11,328          | 148.1   | 5.3           |
| 4   | 25,907         | 7,852           | 99.1    | 19,091         | 8,933           | 99.9    | 21,329         | 8,573           | 99.9    | 16,633         | 7,852           | 175.6   | 4.3           |
| 5   | 23,169         | 6,210           | 99.1    | 16,943         | 6,384           | 99.9    | 19,271         | 5,373           | 99.9    | 18,900         | 6,740           | 183.1   | 3.8           |
| 6   | 20,599         | 5,228           | 99.1    | 15,007         | 5,859           | 99.9    | 19,271         | 5,373           | 99.9    | 16,633         | 5,649           | 183.1   | 3.3           |

Note: Revenues above are from optimal operation exercise where forecasts of day-ahead LMPs and AS_CAISO_EXP ASMPs for regulation up and down were produced in a manner identical to Tables (3.6) and (3.7). However, instead of fixed symmetric or asymmetric likelihoods of dispatch we assume here that every day of two-year exercise exhibits regulation dispatch observed on 7/11, 7/14 and 7/17/2010 as illustrated in Figures (3.4) - (3.8). While likelihoods of regulation dispatch are still constant across days they vary within each day.
3.7 Conclusion

In this Chapter we have performed valuation of energy storage for three different applications where methods identified in Chapter 2 were used to produce day-ahead and hour-ahead forecasts of energy and regulation capacity payments. We found that co-optimization across day-ahead and real-time energy markets increases expected revenues relative to day-ahead market case alone, but at the cost of extreme variance of average daily revenues. Participation in day-ahead energy and regulation markets increases expected revenues dramatically on per-MWh basis due to capacity payments collected by battery from symmetrically bidding its capacity. We have further illustrated sensitivity of the revenues under DAM energy and regulation application to assumptions on likelihoods of real-time regulation dispatch. Use of more realistic assumptions on regulation dispatch did not deteriorate dominance of energy plus regulation application over pure energy applications with respect to revenues.

Chapter 3, in part is currently being prepared for submission for publication of the material under the working title, Valuation of Energy Storage for Stacked Applications, 2015. Chapter 3 was co-authored with Professor Graham Elliott.
References for Chapter 3


