Plasma Compensation Effects with Relativistic Electron Beams

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PLASMA COMPENSATION EFFECTS WITH RELATIVISTIC ELECTRON BEAMS*

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Abstract

A sufficiently dense plasma can neutralize the current of a high energy lepton beam propagating through it. We have studied an e⁺e⁻ linear collider design with this plasma compensation and found that high luminosities can be obtained without going to nanometer beam sizes currently being discussed. We have also studied the consequence of compensation on B-factory design. One severe limitation on such plasma based device which has not been thoroughly examined is the background due to the interaction of the high energy beams with the plasma ion nuclei.

Introduction

When two ultra relativistic bunches of particles collide, the genuine high energy interactions are mixed with spurious radiative effects which have their origin either in the Coulomb scattering of individual particles, the ordinary bremsstrahlung, or in the deflection by the bunch collective field, the so-called beamstrahlung. Of the two effects, beamstrahlung is stronger and it is also the only one which can be eliminated by a compensation technique. Several methods can be invoked to cancel the collective field of a bunch of particles: collision of co-moving electron - positron beams [1], ordinary plasma compensation [2] and a variant which consists of replacing the plasma ions by positrons [3]. None of them is simple to implement and the adverse effects of beamstrahlung have to be carefully analyzed before adopting a compensation scheme.

It must first be recognized that the collective beam-beam effect has the advantage of increasing the luminosity because the fields are focussing and the beams collapse to such an extent that the cross section area is significantly reduced, at least for perfectly aligned beams. However, it is precisely during the pinching phase that a great amount of radiation is emitted producing a net loss of particle energy and spreading of the particle energy distribution. The remedy to this situation has consisted of colliding very flat beams and choosing the bunch parameters in such a way that the peak of radiated energy is larger than the particle energy (quantum regime). It later turned out that the quantum regime was not much better than the classical regime because then photons are replaced by electron - positron pairs [4]. Moreover, the feasibility of the collision of ultra flat beams of nanometer thickness is questionable in that it requires great mechanical tolerances and very small emittances.

Because of these considerations, there is interest in beam compensation for which the beams can be round and larger thus alleviating alignment tolerance problems. The radiation is strongly diminished at the cost of a reduction of luminosity enhancement and of an extra system, the plasma section, which has its own constraints. Without prejudging what a final solution to the colliders problems may be, we have analyzed plasma based
compensation from a beam dynamics point of view, knowing that limitations may be imposed by the interaction of the beam with the nuclei of the plasma ions. It turns out that the results of analytical calculations and of simulations are sufficiently encouraging as to motivate an experimental test.

Principles of plasma compensation

The plasma is a fully ionized column of gas, say hydrogen, of transverse dimension larger than the beam cross section and of thickness about equal to the interaction length (half the bunch length). The compensation process is symmetric for the electron and positron beams because the plasma density is much higher than the beam density.

When the positrons impinge on the plasma, the plasma electrons are accelerated by the induced axial electric field component of the positron beam. However, the plasma electrons are much slower than the positrons and the magnetic compensation can only occur if their number exceeds the number of positrons by a large amount. The electron bunch charge is cancelled by the ions of the plasma and the current is cancelled by the plasma electrons which have not been expelled and flow against the beam direction. In other words, the plasma electrons always drift in the sense of the positron bunch.

The analytical treatment given in this paper deals essentially with the acceleration of the plasma electrons under the effect of the magnetic induction produced by the high energy bunch. Such a treatment assumes that the plasma responds quickly to the solicitation of the driving fields, the conditions for its validity are discussed in Reference 2. The basic requirement concerns the skin depth of the return current which must be much smaller than the beam radius. This condition which is necessary for beam compensation reduces bunch pinching and thus is the basic difference with the case of a plasma lens [5].

Analytic treatment

The high energy bunch is assumed, for the sake of simplicity, to be round and to have a uniform density. Its azimuthal magnetic field is at time t and at a point of abscissa z

\[ B_b(a,z,t) = \frac{\mu_0 J_b}{2} a H(t-z/c) \]

where a is the radius of a field line, c the bunch velocity equal to the light velocity, H the step-function which is zero for t smaller than z/c and unity elsewhere and J_b current density
\[ j_b = \frac{N_b e c}{2 \sigma_z \pi R^2} \]  \hspace{1cm} (2)

with \( N_b \), the number of particles, \( 2\sigma_z \) the bunch length and \( R \) the bunch radius. The electric field generates the mechanism of charge compensation as it does in a plasma lens. The magnetic field produces the current compensation which is the subject of the present discussion. The presence of a step function in the expression of the field implies, as we shall see, an infinitely short response from the plasma, which is not physically correct. However, the use of a more elaborate time dependence [2] does not change the final result significantly and the validity of a model based on impulsive functions is thus justified.

The variation of the plasma current density \( j_p \) with time is given by the equation:

\[ \frac{dj_p}{dt} = \frac{n_p e^2}{m} E \]  \hspace{1cm} (3)

The driving electric field \( E \) is associated with the sudden time variation of the magnetic field when the bunch arrives in \( z \):

\[ E = -\frac{\partial A_z}{\partial t} \]  \hspace{1cm} (4)

The axial component \( A_z \) of the vector potential consistent with a uniform charge density \( j \) is

\[ A_z(a,z,t) = \frac{\mu_0 j}{4} a^2 H(t - \frac{z}{c}) \]  \hspace{1cm} (5)

where \( j \) is the superposition of \( j_b \) and \( j_p \). The electric field \( E \) is then given by

\[ E(a,z,t) = -\frac{\mu_0 j}{4} a^2 \delta(t - \frac{z}{c}) \]  \hspace{1cm} (6)

After substitution of (6) into (3) and integration of (3) over time, the plasma density, assumed to be the same for all the values of \( a \), can be expressed by the ratio:
where $\delta$ is the plasma skin depth:

$$
\delta = \left( \frac{\mu_0 n_p e^2}{m} \right)^{\frac{1}{2}}
$$

One verifies, as expected, that the plasma skin depth has to be small with respect to the bunch radius in order to have a good compensation.

A more realistic approach consists of assuming a radial dependence of the current density. The azimuthal component of the magnetic field [2] is then

![Figure 1. Beam compensation versus R/δ.](image)

Figure 1. Beam compensation versus R/δ.
\[ B_\phi(R) = - \int_0^\infty I_1^0 \left( \frac{a_{\text{min}}}{\delta} \right) K_1^0 \left( \frac{a_{\text{max}}}{\delta} \right) \frac{\partial j_p}{\partial a} \, da \]  

where \( a_{\text{min}} \) and \( a_{\text{max}} \) are the maximum and minimum values of \( \{a,R\} \) respectively. The variations of \( B_\phi \) normalized to the uncompensated field \( B_{\phi0} \) are plotted as a function of \( R/\delta \) in Figure 1.

### Stability and Simulation

One concern about the plasma compensation scenario is that the plasma current which flows through the interacting point might be disrupted when the two compensated beams collide. In fact, this is not the case: the ion background exerts a much larger force insuring quasi-neutrality and forcing the plasma electron density to follow the beam density. The beam pinches slowly in the residual magnetic field and, as the beam density increases adiabatically, the plasma electron density decreases within the beam volume adiabatically. This behaviour is observed in simulations.

The analytical model assumes a collisionless regime for the plasma and does not take the transient phenomena into account. More realistic evaluations have been carried out by J.J. Stewart [2] and J.J. Su [6] using simulation codes. Numerical studies have confirmed the analytic results, namely that compensation does occur. Moreover, although beam pinching mitigates against the condition of current compensation, it is clear that, for our parameters, the collision is over before blowout can occur. The required plasma density may be higher than that given in Figure 1 by at most a factor 2.

### Parameters

As we have just seen, the beam compensation can never be perfect. In order to appreciate the potential applications of the method, we shall review two cases: a 1 TeV linear collider and a B factory.

#### 1. TeV Collider

Parameters of the TLC [7] have a beam which is very flat (\( \sigma_x/\sigma_y = 180 \)) and a semi-axis which is only 1.3 nm. With plasma compensation one can for the same luminosity, have a round beam and therefore a larger \( \sigma_y \). In Table 1 we present the relevant parameters for the TLC and a compensated collider. A straightforward compensation of the TLC would need an exceedingly high plasma density of \( 10^{24} \, \text{cm}^{-3} \). We therefore define new parameters consistent with a beam radius of 0.1 \( \mu \text{m} \) for a collider.
called PB-TLC (plasma based TLC). We assume a luminosity enhancement in the residual magnetic field of a factor 5. Notice that the compensated collider requires a much larger emittance than the TLC needs.

<table>
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<tr>
<th>Parameters</th>
<th>TLC</th>
<th>PB-TLC</th>
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</thead>
<tbody>
<tr>
<td>energy / restenergy</td>
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<td>10^6</td>
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<td>luminosity [10^{33} cm^{-2}s^{-1}]</td>
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<td>repetition rate [Hz]</td>
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<tr>
<td>σ_z [μm]</td>
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<tr>
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<td>relative energy loss</td>
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<tr>
<td>plasma density [cm^{-3}]</td>
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<td>6 \times 10^{21}</td>
</tr>
</tbody>
</table>

Table 1. Comparison between TLC and a plasma based linear collider (PB-TLC).

**ii. B-Factory.** For a B factory made of storage rings, the interest would consist of reducing the beam-beam tune shift Δv. The machine proposed in [8] promises a luminosity of 10^{34} cm^{-2}s^{-1} for Δv equal to 0.1 and a beam radius of 55 μm. It would certainly be safer to reduce Δv. If the beam-beam force was compensated within 80%, the plasma density would be 7.5 \times 10^{16} cm^{-3}. This is quite a small density and it is likely that the beam-plasma background will be acceptable.

**Conclusion**

Analytic and numerical work indicates that a plasma at the interaction point can reduce the beam collective field. We have exhibited typical collider and B-factory parameters for plasma based design, noting in particular the eased constraints on spot size and magnet alignment in the case of a linear collider and the reduction in beam-beam tune-shift for a B-factory. The background problems have not been addressed but the plasma physics aspects seem to merit an experimental study. It is relevant to note that the physics of plasma compensation is very similar to that of the plasma lens and the adiabatic compressor. We conclude that the theoretical work is complete and that time for experimental work has come.
References


