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Permanent Magnets for Production and Use of High Energy Particle Beams

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In the last few years, permanent magnet systems have begun to play a dominant role in the generation of synchrotron radiation and the operation of free electron lasers. Similarly, permanent magnets can lead to significant improvements of accelerators and systems that use them. The general conditions will be discussed under which one can expect benefits from permanent magnets, and a number of specific applications will be described in detail.

1) **Magnetic Properties of Some Permanent Magnet Materials**

All permanent magnets (PM) described in this paper use anisotropic material whose magnetic properties are adequately described by

\[ B_{\parallel} = B_r + \mu_0 \mu_{\parallel} H_{\parallel} ; B_{\parallel} > 0 \]  
\[ B_{\perp} = \mu_0 \mu_{\perp} H_{\perp} \]  

In these equations, \( B_{\parallel} \) and \( B_{\perp} \) refer to the direction parallel and perpendicular to the preferred direction of the material, the so-called easy axis. Depending on the application, it is necessary that eq. (1) holds well into the second quadrant (and for some strong magnets without the use of iron even in the third quadrant) of the B-H coordinates. It is further assumed that \( \mu_{\parallel} -1 \) and \( \mu_{\perp} -1 \) is smaller than .1 (\( \leq 0.05 \) for Rare Earth Cobalt). The materials that satisfy these conditions are Rare Earth Cobalt (\( B_r = 0.8 - 1 \text{T} \)), Neodymium Iron Boron.
(Br = 1 - 1.2T), and some of the ferrites (Br = .2 - .35T). Even though the first two mentioned materials are the most powerful PM materials, strength is not the only reason for preferring these kinds of materials for the design of magnets for very demanding applications: The above mentioned properties allow, as a very good approximation, the application of linear superposition of the effects of different blocks of PM material (as long as there is not strongly saturated iron present). The resulting simple theory gives a very good understanding of the properties of systems composed of these materials, and good designs follow rather easily from that good understanding.

It is easy to show that the magnetic field produced by a uniformly magnetized block of PM material with the above described properties is, in very good approximation, the same as the field produced by either currents or charges on the surface of the block. For that reason, we refer to this class of material to current sheet equivalent material or charge sheet equivalent material (CSEM).

2) Generic Advantages of Permanent Magnets

Before describing some specific devices, it is useful to discuss the general circumstances under which the use of permanent magnets is indicated, and what the preferable PM materials are.

When one scales an electromagnet in all dimensions while keeping the magnetic field at equivalent locations fixed, it is easy to see that the current density in the coils scales inversely proportional to the linear dimensions L of the magnet. Since superconductors have an upper limit for the current density j that can be carried, and dissipative coils have an upper limit for j due to the need to remove the dissipated power, j needs to be reduced below that prescribed by simple scaling when L reaches a certain small value that depends, of course, on many details of the magnet design. When j is reduced, the field in a magnet that does not use iron obviously is also reduced, even if the total Ampere turns are maintained by increasing the coil size. The same is also true for a magnet using iron, since an increase of the coil size invariably leads to a field reduction due to increased saturation of the iron. PM, on the other hand, can be scaled to any size without any loss in field strength. For the same reasons, small PM systems will be much more compact than electromagnets of equal, or even lesser, performance. From this follows that when it is necessary that a magnetically significant dimension of a magnet is very small, a permanent magnet will always produce higher fields than an electromagnet. This means that
with permanent magnets one can reach regions of parameter space that are not accessible with any other technology. The critical size below which the PM out-performs the electromagnet depends of course on a great many details of both the desired field strength and configuration as well as the properties of the readily available PM materials. In the region of the parameter space that is accessible to both technologies, the choice of one technology over the other will be made on the grounds of cost, reliability, or convenience, (main specifics: power supplies, power needed to run the system, equipment associated with cooling) and in this arena permanent magnet systems are often also preferable, but in general less so the larger the smallest magnetically relevant dimension becomes.

3) Magnetic lenses and related magnets for high energy accelerators and storage rings

3.1 Reasons for Use of Magnetic Lenses.

When charged particle beams travel very large distances (in some storage rings, beams circulate for hours with velocities close to that of the velocity c of light) the particles obviously need to experience an average force directed toward some axis that represents the ideal trajectory. Without this focusing force, the particles would very soon hit the vacuum chamber and be lost for the intended use. This force can be produced either by application of a transverse electric field $E$ or by a transverse magnetic field $B$ that produces a force equal to that of the equivalent electric field $E_{eq} = vB$, with $v$ representing the velocity of the particle. If $v > E_m/B_m$, where $E_m$ and $B_m$ are maximum values of $E$ and $B$ that can be produced with a reasonable effort, the magnetic focusing force is larger than that achievable with electric fields. Assuming $B_m = 1T$ and (for electrode distances of the order of 1 cm) $E_m = 10^5 \text{ Vcm}^{-1}$, the critical value for $\beta = v/c$ is of the order .03. For this reason, magnetic focusing is used exclusively for focusing of high energy charged particle beams.

Most accelerator magnets are considerably longer than the radius of their useful field aperture. For that as well as some more sophisticated reasons, the properties of most interest are the two dimensional (2D) aspects of the magnetic fields produced by these magnets, and we will discuss only these aspects here.

Nearly all accelerator magnets are multipole magnets whose 2D fields can be derived from a scalar potential of
the form
\[ V = \text{Const. } r^n \cos(n\phi); \ n = \text{integer}. \] (3)

Among these magnets, those with \( n = 2 \) (quadrupoles) are used most often, closely followed by those with \( n = 1 \). Corrective magnets with \( n = 3; 4 \) are also used, but in much smaller numbers.

3.2) Magnetic Lens Properties

Examination of the particle motion caused by a quadrupole lens shows that while the force in one transverse direction is focusing, it is defocusing in the transverse direction perpendicular to that direction. By using a properly separated pair of lenses with opposite excitations, one gets a net focusing force for this alternate gradient focusing system. This is clear from Fig. 1 when one takes into account that the focusing/defocusing force in each lens is proportional to the distance of the trajectory from the axis.

3.3) Pure CSEM Multipoles.

To design an iron-less magnet that produces a potential according to eq. (3), validity of linear superposition of fields from different pieces of the CSEM makes the question "what is the optimum orientation of the easy axis as function of \( r \) and \( \phi \) to contribute most to a potential described by eq. (3)?" (See Fig. 2) a reasonable question. The answer to that question (see ref (1)) is

\[ a(r,\phi) = (n + 1)\phi \] (4)

Unfortunately, it would be exceedingly difficult to make material according to that prescription, but the next best thing, namely approximating eq. (4) by segmentation, gives nearly the same performance. Fig. 3 shows schematically a quadrupole designed this way, with the arrows inside the trapezoidal blocks indicating the easy axis orientation in the blocks. The field at the aperture radius of such a magnet is given by (ref. 2)

\[ B(r_1) = B_\text{r} \sum_{n=1}^{n} (\cos \frac{n\pi}{M}) \sum_{n=1}^{n} (\sin \frac{n\pi}{M})(1 - \frac{r_1}{r_2})^n; n > 1 \] (5a)

\[ B(r_1) = B_\text{r} \frac{\sin \frac{2\pi}{M}}{2\pi/M} \ln \frac{r_2}{r_1}; n = 1 \] (5b)

with \( r_1, r_2 \) representing the inside and outside radius of the magnet, and \( M \) = number of blocks in a magnet. In
deriving eq. (5), it has been assumed that \( \mu = \mu_l = 1 \)
in eqs (1) and (2), and that they are valid over the
whole range of values of \( B, H \) occurring in the magnet,
particularly down to \( \mu_0 H = -B(r_1) \). Notice, in
particular, that for \( n = 2 \), \( B(r_1) \) can approach \( 1.5 B_r \),
requiring that the \( B \) vs \( \mu_0 H \) curve is a straight
line well into the third quadrant. Fortunately, many
manufacturers produce materials that satisfy this
requirement.

The field strength achievable with this type of
quadrupole exceeds that obtainable with a room temperature
electromagnet with iron poles. In addition, the pure CSEM
quadrupole is exceedingly compact. For that reason, this
type of quadrupole is very well suited for use as drift
tube quadrupole in fixed energy linear accelerators (ref. 3),
and in fixed energy beam lines.

3.4) Hybrid Multipoles.

While methods have been suggested to make quadrupole
systems of this type that have adjustable focusing
strength (ref. 4, 5), their implementation is sufficiently
difficult that, to the best knowledge of this author, no
systems of that kind have been used so far. However, a
class of multipole magnets has been developed that uses
iron together with CSEM that produces a strong field that
is easily adjustable (ref. 6, 7).

Fig. 4 shows a 2D cross section of the quadrupole
member of that multipole family. The dotted areas
represent iron, and the areas with arrows inside them
identify CSEM, with the arrow indicating the easy axis
orientation inside the block. The outer iron ring has
CSEM attached to it, and by rotating that ring with the
attached CSEM, one can change the field strength in the
aperture region without changing the field distribution.
While this hybrid quadrupole is not quite as compact as
the pure CSEM quadrupole, and the field strength at the
aperture radius is limited (because of iron saturation) to
a respectable value of about \( 1.2-1.4 \) T, the field
variability is obviously a great asset. In addition, the
field distribution in the aperture region of the hybrid
quadrupole is controlled by the iron, and is therefore
insensitive to homogeneity of magnetization of the CSEM
blocks. The volume integral of the magnetization over
each CSEM block is important for equal excitation of all
poles. However, an old theory of perturbation effects in
iron dominated magnets (ref. 8) has been amended to
include these effects, and block sorting procedures have
been developed to make the resulting field errors
insignificant. Two prototype hybrid quadrupoles have been
built (ref. 9). Each performed as expected.
It is clear from Fig. 4 that multipoles of any order (or even hybrid magnets that produce combinations of multipole fields) can be built with this basic design. While this includes dipoles, for dipoles magnets the design shown in Fig. 5 can be advantageous. Very compact fixed field dipole magnets of this basic design have been built by Field Effects Inc. and used successfully by Los Alamos National Laboratory as spectrometer magnets.


4.1) Pure CSEM Undulators/Wigglers for Production of Linearly Polarized Light

A very effective method to obtain spontaneous or stimulated electromagnetic radiation from high energy electrons is to expose them to strong static magnetic fields that alternate (spatially) rapidly. Fig. 6 shows, schematically, such a device, a pure CSEM undulator/wiggler (U/W). In Fig. 6, the electrons travel from the left to the right, and "wiggle" in the direction perpendicular to the paper plane. Since it is desirable under most circumstances that the electrons "see" fields that are independent of the coordinate perpendicular to the paper plane, we restrict again the discussion to the 2D aspects of the magnets. The 2D fields produced by the structure shown in Fig. 6 is dominated by (see ref. 4 for a more complete expression):

\[ B_x - i B_y = 2 B_r \cos(k(x + iy)) e^{-kh}(1 - e^{-kL}) \frac{\sin(\pi/M')}{\pi/M'} \]  

(6)

where:

- \( k = \frac{2\pi}{\lambda} \)
- \( M' \) = number of blocks/period in one array.

The device obviously has to be wide enough for this formula to be applicable, and formulas to assess these 3D effects can also be found in ref. 4. Variation of field strength in this device can be achieved by variation of the clear gap between the two arrays of CSEM blocks. It is worth noting that each of the two linear arrays of CSEM in Fig. 6 can be obtained by letting the radius of a multipole magnet with fixed radial thickness and period length go to infinity, and eq. (6) can be obtained by this process from the general multipole equation in ref. 1.

4.2) Hybrid Undulators/Wigglers for Production of Linearly Polarized Light.

A disadvantage of the design shown in Fig. 6 is the sensitivity of the fields to quality control in the production of the CSEM blocks. Even though assigning
blocks to locations in the arrays according to measured magnetic properties (see for instance ref. 10, 11, 12) helps reduce these undesirable effects, the hybrid design shown in Fig. 7, using iron in addition to CSEM, is much less sensitive to material tolerances. In addition, the hybrid can produce stronger fields than the pure CSEM U/W. A 2D computer optimization of the peak field $B$ in the midplane of a hybrid U/W has been performed for CSEM with $B_r = .9T, \mu^u = \mu^\perp = 1, B^u = .2B_r$ in the bulk of the CSEM, and the saturation characteristics of Vanadium Permendur for a number of different values of the ratio of gap $g$ divided by U/W period $\lambda$ (ref. 13) within the range $.07 > g/\lambda < .7$. The results can be well represented by

$$B = 3.33 \exp \left(-\left(5.47 - 1.8g/\lambda\right)g/\lambda\right)$$  \hspace{1cm} (7)

Here, as with the pure CSEM U/W, 3D effects have to be taken into account and corrected to actually get the performance given by (7). These procedures have been developed, but not published yet.

4.3 Systems for the Production of Circularly Polarized Light

The U/W described above produce linearly polarized light in the forward direction. For the production of circularly polarized light one can use a helical U/W. Fig. 8 shows the 2D cross-section of a pure CSEM dipole. By making axially short segmented dipoles and rotating each short dipole by a fixed angle relative to the previous dipole, one can obtain the desired helical field. The expected performance of such a helical U/W is given in ref. 4. Another way to produce circularly polarized light with PM undulators is schematically shown in Fig. 9 (ref. 14): The two undulators produce two linearly polarized wave trains. By passing them through a monochromator of sufficiently narrow bandwidth, each wave train is lengthened, thus leading to significant overlap of the wave trains, and with it elliptically polarized light. By adjusting the electron trajectory length between the two undulators with the modulator, the phase shift between the two wave trains can be adjusted, and with it the polarization properties of the light emerging from the monochromator.

Conclusions

Permanent magnet systems have been developed that are used very successfully in accelerators and as sources of synchrotron radiation in electron storage rings. The main advantage of these systems is the possibility to obtain, under some circumstances, performance characteristics that
can not be obtained with any other technology. One unique advantage of pure CSEM devices not mentioned above is the possibility of "immersing" a pure CSEM magnet in the field of another magnet, with essentially linear superposition of the fields produced by the two magnets. Examples are, for instance, a pure CSEM U/W inside a quadrupole (that provides focusing for the particles), and a pure CSEM quadrupole inside a solenoid. Under other circumstances, the advantage may be merely one of economy or convenience. A recently published proposal (ref. 15) to build an electron storage ring entirely with permanent magnets is an indication that this technology will be used even more frequently in the future.
Figure Captions

1) Net Focusing by a Pair of Quadrupole Lenses
2) Optimization of Easy Axis Orientation.
3) Pure CSEM Segmented Quadrupole.
4) Adjustable Strength Hybrid Quadrupole.
5) Hybrid Dipole.
6) Pure CSEM Undulator.
7) Hybrid Wiggler/Undulator.
8) Pure CSEM Dipole.
9) System for Production of Elliptically Polarized Synchrotron Light.
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9) Private communications: One was built under the direction of J.T. Tanabe at U.C. LBL, and one under the direction of R.F. Holsinger at Field Effects Inc. for Los Alamos National Lab.
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Tuning Stud (Steel)
Backbone Plate (Steel)
CSEM Blocks (Easy Axis Direction Shown)
Pole (Vanadium Permendur)
gap

Figure 7

XBL 849-3879
Figure 8

CSEM Blocks

Easy Axis Direction

Aperture Field
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