

UC Irvine

UC Irvine Previously Published Works

Title

Extension of the Nagayama Triangle for Visualization of Party Strengths

Permalink

<https://escholarship.org/uc/item/06b4n2pj>

Journal

Party Politics, 10(3)

ISSN

1354-0688

Author

Taagepera, Rein

Publication Date

2004-05-01

DOI

10.1177/1354068804042460

Peer reviewed

EXTENSION OF THE NAGAYAMA TRIANGLE FOR VISUALIZATION OF PARTY STRENGTHS

Rein Taagepera

ABSTRACT

The Nagayama triangle represents the conceptually allowed area when the vote or seat shares of the second-running contestant are graphed against the shares of the top contestant. This research note points to various uses of this tool in the study of Duvergerian processes.

KEY WORDS ■ Duverger's law ■ vote and seat shares of parties

In his study of the workings of Duverger's law in Italy, Reed (2001) uses to good effect a graphic approach devised by Nagayama (1997) for the study of candidate strengths in single-member districts (SMDs). The vote shares of the second-running contestant (s_2) are plotted against the vote shares of the top contestant (s_1). The total of the two shares cannot exceed 100 percent, nor can the second-largest share exceed the largest – as shown by thick lines in Figure 1. These two constraints force the data points to lie within a triangle that Reed (2001) calls the Nagayama triangle. Its left side denotes perfect parity of the two top contestants, while its right side denotes the dominance of the strongest contestant over a single opponent. At the peak, the two contestants have equal strength, and there are no others. The left corner area of the triangle corresponds to the presence of multiple contestants.

Reed (2001) applies this way of graphing to votes received by candidates in individual SMDs in the first Italian elections to use SMDs (as part of a mixed system): 1994 and 1996. The learning process by the voters is visualized with singular clarity. In 1994, the cloud of data points hugs the middle of the left side of the Nagayama triangle, reflecting appreciable multi-candidate competition. In 1996, the cloud has shifted to the peak of the triangle, reflecting two-candidate competition with minor third-candidate

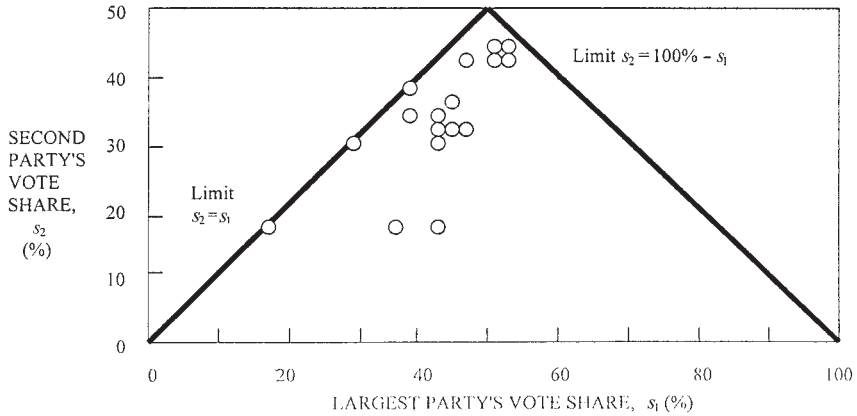


Figure 1. Votes for second-largest parties in 17 single-member district elections: closeness to two-party balance

input – well in line with Duvergerian expectations, once the Duverger psychological effect has had time to act.

Grofman et al. (2004) carry out an extensive theoretical study of the properties of the Nagayama triangle, dividing it into eight segments that reflect the relative strengths of first-, second- and third-ranking parties. They also compare this displaying pattern to simplex (equilateral triangle) representation. As empirical illustration, they apply both, like Reed (2001), to individual Italian SMDs, using the 1994, 1996 and 2001 elections.

The present research note has much more limited objectives. Two further possibilities for applying the Nagayama triangle are pointed out: to data other than votes in individual SMDs, and to contestants other than the second-largest party.

The Nagayama triangle can be usefully applied not only to candidates in individual SMDs but also to nationwide vote shares for entire parties – and to their seat shares. In retrospect, this idea may look obvious, yet all previous uses of the Nagayama triangle (Grofman et al., 2004; Nagayama, 1997; Reed, 2001) have dealt with candidates in individual SMDs. And the nationwide pattern differs from those in individual districts.

Figure 1 shows the pattern of nationwide votes for the 17 SMD elections listed in Mackie and Rose (1997). The countries involved are Australia, Canada, France, New Zealand, the UK and the US, each with two to four elections from 1987 to early 1996. The vote shares tend to hug the left side of the triangle, expressing a fair two-party balance, in line with Duvergerian expectations. Two exceptional points in the low centre (Canada 1993 and France 1988) reflect a much smaller second party, and the point near $s_1 = 20$ percent (France 1993) reflects near-equality of many more than two parties. (France of course has a two-round system not covered by Duverger's law.)

Note that the pattern for nationwide SMD plurality elections differs from that for individual districts, as graphed by Reed (2001) and Grofman et al. (2004). In relatively mature SMD plurality elections (Italy 1996 and 2001), only two major contestants tend to arise in each district, leading to data points at the peak of the triangle or even tilting toward its right side. But since the party affiliation of the top two contestants may differ from district to district, vote shares are more fractionalized nationwide than in individual districts, expressed as a shift toward the left corner of the triangle.

Nationwide, the approach can be extended from vote shares to seat shares, something that would be pointless for individual SMDs, where 100 percent of the single seat always goes to one candidate. Figure 2 shows the seat shares corresponding to the vote shares in Figure 1. Rather than hugging the left side of the triangle, the seat shares tend to hug the right side, illustrating the development of a one-party dominance through the workings of the Duverger mechanical effect.¹

The degree of shift, when going from vote shares to seat shares, can be visualized by graphing both together. This is illustrated in Figure 4, which shows three very different systems: France (two rounds), the UK (plurality with relatively strong third parties) and the US (an unusually pure two-party constellation).² In plurality systems the arrows are fairly close to the conceptual maximum, showing a two-stage overall pattern. Movement up along the left side reflects Duvergerian enhancement of dominance by two top parties on a fairly equal basis. The subsequent movement down the right side expresses increasing single-party dominance at the expense of the second-largest party. In SMDs with two rounds (France), two very different outcomes are possible: enhancement of the two top parties (as in plurality),

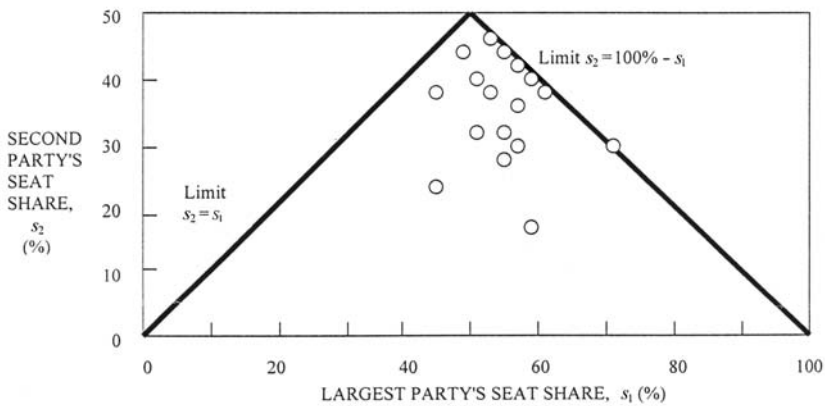


Figure 2. Seats for second-largest parties in 17 single-member district elections: closeness to one-party dominance

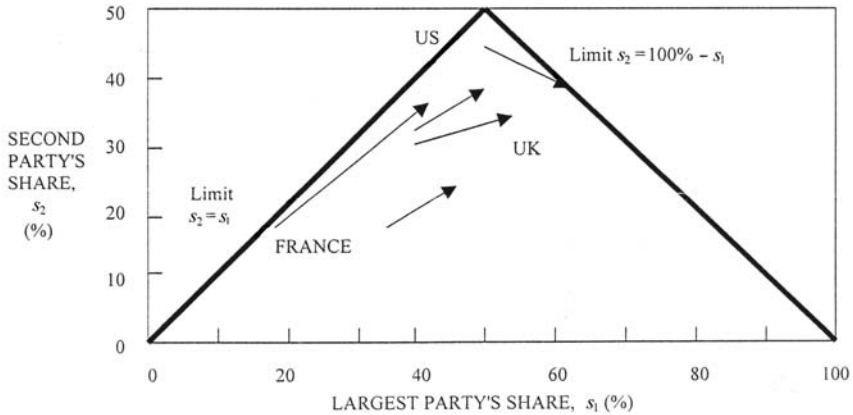


Figure 3. Shift from second-largest party's votes to seats in three single-member district countries: moving right, close to the logical maximum

or preservation of a multiparty constellation, with little Duvergerian enhancement of the two top parties.

Analogous graphs could be constructed for proportional representation (PR) and mixed systems as well. However, the vote and seat patterns do not differ appreciably in most of these cases.

We now come to the second idea in this note: applying the Nagayama triangle to contestants other than the second-largest party. The original Nagayama triangle expresses the logical constraints on the share of the second-largest contestant. What are the constraints on the third-largest contestant, and so on? The share of the i th largest contestant (s_i) is subject to the following constraints. It can be at most as large as the largest (implying equality of the first i shares): $s_i = s_1$. When s_i is graphed against s_1 , this constraint yields the same line for contestants at all ranks. Furthermore, s_i can be at most as large as the complement of the largest share (100 percent - s_1) divided equally among the next $i - 1$ contestants: $s_i = (100 \text{ percent} - s_1)/(i - 1)$. Those lines cross the line $s_i = s_1$ at the point where the first i contestants all have equal shares. Together, the two constraint lines define different Nagayama triangles for the third-ranking contestants, and so on. For the third-largest contestant, these limits are shown as thick lines in Figure 4.

The limits on third and other finishers may seem to lack substantive interest, since the Duvergerian processes tend to focus attention on the two top contestants. Figure 4 shows, however, that unexpected results can emerge when using the Nagayama format. The third-largest parties are graphed for the 17 SMD elections previously used. However, here we add, using a different symbol, those PR and mixed elections from Mackie and Rose (1997) where the largest party share exceeds 35 percent. Duverger's

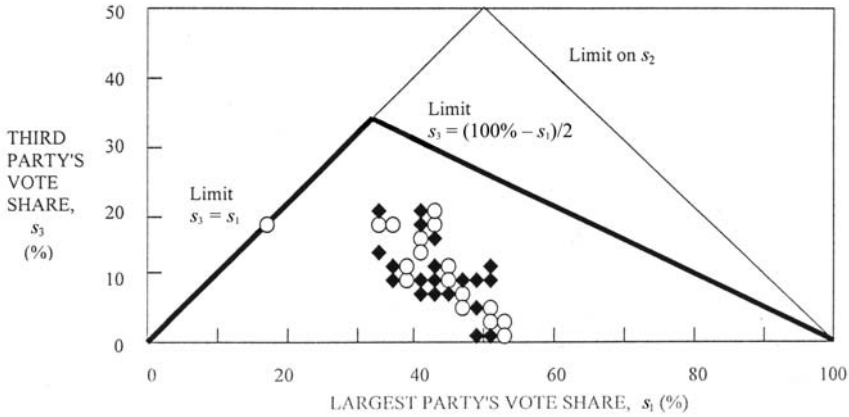


Figure 4. Vote shares of third-largest parties in single-member district systems (white circles) and PR systems (black diamonds). Superimposed diamonds in the centre of the cloud have been omitted

law might lead us to expect that, compared to PR systems, third parties would be reduced to smaller vote shares in SMD systems. Surprisingly, no visible difference emerges. For given largest party vote shares, third-party votes tend to be the same in PR and SMD systems. This is a counter-intuitive finding that begs an explanation.

Another contrast also arises. We observed (in Figures 1 to 3) that second-largest party shares tend to be close to their conceptual maximum size, hugging the upper sides of the Nagayama triangle. This is sometimes true for third-largest parties, as long as the largest party share remains below 40 percent. However, the third-largest parties plunge to less than one-half of the possible maximum when the largest share reaches 45 percent, be it SMD or PR.

The objective of this note was to point out various ways in which the Nagayama triangle can be a useful tool for analysing party systems. I have graphed a dataset of limited extent, yet random, since the selection was done not by me but by what was included in Mackie and Rose (1997). The tentative patterns found raise a number of questions. Do nationwide second party vote shares in SMD elections always tilt to the left side of the possible area of occurrence, while their seat shares tilt to the right side? Does the conversion of vote shares into seat shares always approach the pattern outlined by France, the UK and the US in Figure 3, hugging the upper part of the Nagayama triangle? Are the third-party vote shares under SMD and PR rules always similar, at a given share for the largest party?

Many further questions could be asked about the PR and SMD patterns, including fourth-ranking parties and beyond. To answer them, much more extensive data need to be processed. Here I have simply pointed out that

Nagayama (1997), Reed (2001) and Grofman et al. (2004) have opened up a potentially fertile new avenue for studying Duvergerian processes in a broad sense.

Notes

- 1 For Canada 1996, the lowest point in the seats graph corresponds to Bloc Quebecois, which in terms of votes was surpassed by the Reform and Conservative parties. Such flips do occur but are rare.
- 2 Because of crowding, only one typical US election is shown, i.e. 1990.

References

- Grofman, Bernard, Alessandro Chiamonte, Roberto D'Alimonte and Scott L. Feld (2004) 'Comparing and Contrasting the uses of Two Graphical Tools for Displaying Patterns of Multiparty Competition: Nagayama Diagrams and Simplex Representations', *Party Politics*, 10(3): 273–99.
- Mackie, Thomas T. and Richard Rose (1997) *A Decade of Election Results: Updating the International Almanac*. Glasgow: Centre for the Study of Public Policy, University of Strathclyde.
- Nagayama, Masao (1997) 'Shousenkyoku no kako to genzai' [The present and future of single-member districts]. Annual Conference of the Japan Political Science Association, Kyoto, September 4–6.
- Reed, Steven R. (2001) 'Duverger's Law Is Working in Italy', *Comparative Political Studies* 34: 312–27.

REIN TAAGEPERA is Research Professor at the University of California at Irvine, CA, USA and Tartu University, Estonia. He is senior author of *Seats and Votes* (Yale University Press, CT, 1989), and was the founding chair (2001–2) of a new political party, Res Publica, which won the 2003 Estonian parliamentary elections and formed a coalition cabinet.

ADDRESS: School of Social Sciences, University of California, Irvine, CA 92697, USA and Department of Political Science, Tartu University, 50090 Tartu, Estonia. [e-mail: rtaagepe@uci.edu]

Paper submitted 31 March 2003; accepted 15 October 2003.