Essays on the Specification of New Keynesian
Dynamic Stochastic General Equilibrium Model

A dissertation submitted in partial satisfaction of
the requirements for the degree Doctor of Philosophy
in
Economics

by

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2007
The dissertation of Yong-Gook Jung is approved, and it is acceptable in quality and form for publication on microfilm:

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Chair

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2007
To my beloved wife Heejin and daughter Seungwon.
TABLE OF CONTENTS

Signature Page ................................................................. iii
Dedication ........................................................................ iv
Table of Contents ............................................................... v
List of Figures ..................................................................... vii
List of Tables ....................................................................... viii
Acknowledgments ................................................................ ix
Vita .................................................................................... x
Abstract .............................................................................. xi

1 Investment Lags and Macroeconomic Dynamics ............... 1
  1.1. Introduction ............................................................... 1
  1.2. Model ........................................................................ 5
    1.2.A. Investment Specifications ....................................... 5
    1.2.B. Behavior of Economic Agents ................................ 9
  1.3. Estimation Method and Data ....................................... 13
  1.4. Empirical Result ....................................................... 15
    1.4.A. Maximum Likelihood Estimates .......................... 15
    1.4.B. Fit of the Models ................................................. 20
  1.5. Conclusion .............................................................. 28

2 Indexation Scheme in Calvo-Style Sticky Price Model ...... 30
  2.1. Introduction ............................................................... 30
  2.2. The Model .................................................................. 32
    2.2.A. The Finished Goods-Producing Firm ................. 32
    2.2.B. The Intermediate Goods-Producing Firm ........... 33
    2.2.C. Representative Household ................................. 35
    2.2.D. The Monetary Authority ................................. 37
  2.3. Estimation Method and Data ...................................... 37
  2.4. Empirical Results ..................................................... 39
  2.5. Conclusion .............................................................. 45

3 Can a Labor Hoarding Friction Explain the Delayed Effect of Monetary Policy? ........................................ 48
  3.1. Introduction ............................................................... 48
  3.2. Model ........................................................................ 49
    3.2.A. Representative Household ................................. 50
    3.2.B. The Finished Goods-Producing Firm ................. 52
    3.2.C. The Intermediate Goods-Producing Firm ........... 53
3.2.D. The Monetary Authority ........................................ 54
3.3. Results ................................................................. 55
  3.3.A. Parameter values ............................................... 55
  3.3.B. Dynamic responses to a monetary shock ................. 56
3.4. Conclusion ........................................................... 58
References ................................................................. 60
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1</td>
<td>Value-Weighted Average Completion Pattern, 1961-1991</td>
<td>3</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Implied Completion Patterns</td>
<td>20</td>
</tr>
<tr>
<td>Figure 1.3</td>
<td>Akaike and Schwarz Information Criteria</td>
<td>22</td>
</tr>
<tr>
<td>Figure 1.4</td>
<td>Impulse Responses (Model Ire)</td>
<td>25</td>
</tr>
<tr>
<td>Figure 1.5</td>
<td>Impulse Responses (Model KP6)</td>
<td>25</td>
</tr>
<tr>
<td>Figure 1.6</td>
<td>Impulse Responses (Model Cas6)</td>
<td>26</td>
</tr>
<tr>
<td>Figure 1.7</td>
<td>Impulse Responses (Model CEE)</td>
<td>26</td>
</tr>
<tr>
<td>Figure 1.8</td>
<td>Impulse Responses (Model Wen6)</td>
<td>27</td>
</tr>
<tr>
<td>Figure 1.9</td>
<td>Impulse Response of Investment Projects</td>
<td>27</td>
</tr>
<tr>
<td>Figure 2.1</td>
<td>Distribution of Completion Rates</td>
<td>41</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Impulse Responses (Benchmark Model)</td>
<td>46</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>Impulse Responses (Alternative Model)</td>
<td>46</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>Impulse Responses (Baseline Model)</td>
<td>57</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Impulse Responses (Extended Model)</td>
<td>58</td>
</tr>
</tbody>
</table>


| Table 1.1 | Imposed Parameters | 16 |
| Table 1.2 | Estimated Parameters | 17 |
| Table 1.3 | Maximum Likelihood Estimation Result (1) | 18 |
| Table 1.4 | Maximum Likelihood Estimation Result (2) | 19 |
| Table 1.5 | Log Likelihoods | 21 |
| Table 1.6 | Vuong Test | 23 |
| Table 1.7 | Clarke Test | 24 |
| Table 2.1 | Imposed Parameters | 40 |
| Table 2.2 | Estimated Parameters | 42 |
| Table 2.3 | Maximum Likelihood Estimation Result | 43 |
| Table 2.4 | Akaike and Schwarz Criterion | 44 |
| Table 2.5 | Vuong Test | 44 |
| Table 2.6 | Clarke Test | 44 |
| Table 2.7 | Forecast Accuracy (RMSE): 1999:Q1-2006:Q2 | 45 |
| Table 3.1 | Parameter values in a baseline model | 56 |
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ABSTRACT OF THE DISSERTATION

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The New Keynesian dynamic stochastic general equilibrium model has become one of the standard approach to monetary policy analysis and macroeconomic forecasting. Therefore, many researchers are trying to come up with a better specification of the model. My dissertation goes in line with those efforts. The first chapter focuses on the specification of investment, the second on the firm’s pricing, and the third labor market.

The first chapter addresses the importance of rigorous investment specification. Dynamic stochastic general equilibrium (DSGE) models models have a difficult time accounting for the slow response of investment spending to economic shocks that are generally found empirically. Four different specifications of investment dynamics are examined: (1) time-to-build as modeled by Kydland and Prescott (1982), (2) time-to-build as modeled by Casares (2006), (3) time-to-build as modeled by Wen (1998), and (4) investment adjustment costs as modeled by Christiano et al. (2005). Maximum likelihood estimation results indicate that the model with the investment lag specification of Casares (2006) fits the data significantly better than the other models.

The second chapter examines the indexation schemes in Calvo-style sticky price model. While sticky price DSGE models with the dynamic backward indexation are widely used for monetary policy analysis, statistical plausibility of the
indexation scheme has not yet been verified. The maximum likelihood estimation result does not find much support for the DSGE model with the backward indexation. The fit of the DSGE model with the dynamic backward indexation is no better than that with the static indexation. Also, the forecast of the model with the backward indexation is less accurate than that with the static indexation.

The third chapter shows that when the workweek of capital and efforts are allowed to vary, the employment lag itself cannot generate a hump-shaped response of output to a monetary shock. The reason for this is that despite the fact that the size of employment is predetermined, firms can rely on other low adjustment cost margins such as the workweek of capital and efforts to meet the increase in demand due to a positive monetary shock.
Investment Lags and Macroeconomic Dynamics

1.1 Introduction

Since Kydland and Prescott (1982) and Long and Plosser (1983) introduced real business cycle (RBC) modeling, dynamic stochastic general equilibrium (DSGE) models have been actively developed in every field of macroeconomics, making them stand as one of the past decade’s most significant achievements in macroeconomics. A DSGE model is useful for policy analysis and economic forecasting when the model is not rejected by data and generates a more accurate forecast than other models. However, the misspecification problem is quite prevalent in the earlier generation of DSGE models, since the state-space representation of a typical DSGE model implies a huge number of restrictions on the likelihood function. Those models are in many cases rejected against less restrictive specifications such as vector autoregressions (VAR). For example, Altug (1989) estimates a standard RBC model similar to that of Kydland and Prescott (1982) by maximum likelihood method, and concludes that the model cannot rationalize the joint behavior of per capita hours with the other macroeconomic time series. Watson (1993) measures the fit of a standard RBC model using the spectra of the model and data, and finds that the spectra are different from each other for periods typi-
cally associated with business cycle. Leeper and Sims (1994) estimate a monetary DSGE model by maximum likelihood method and find that their model fits the data worse than unrestricted VAR in terms of log likelihood.

Therefore, a variety of richer specifications in the DSGE framework have been suggested for the purpose of developing a model that is fully able to account for all of the dynamic correlations that one sees in an unrestricted VAR representation of the observed variables. To allow a DSGE model to imply that systematic monetary policy can make a substantial difference for the real economy, researchers have explicitly incorporated nominal rigidities, which become a distinguishing feature of New Keynesian macroeconomics. For price and wage stickiness, staggered price setting, as suggested by Calvo (1983), or explicit costs of nominal price adjustment, as suggested by Rotemberg (1982), are incorporated to the model. External habit formation in consumption is incorporated to improve the short-run dynamic behavior of consumption and output. Variable capital utilization is used to generate inertia in inflation and persistence in output with moderate wage and price stickiness. Smets and Wouters (2003) demonstrate a breakthrough of subsequent efforts. They develop a large-scale monetary DSGE model in the New Keynesian tradition based on work by Christiano et al. (2005), and estimate it on Euro-area data. One of the remarkable empirical results is that posterior odds favor their DSGE model relative to VARs estimated with a fairly diffuse prior.

This paper claims that we need to specify investment more carefully to improve the fit of DSGE models to data. In particular, investment lags, which are delays between the time an investment decision is made and the time project is completed, need to be accounted for in the DSGE framework. One can unambiguously verify that real-world investment projects involve spending that takes place over a considerable length of time. Since Mayer’s (1960) finding that 15 months are required on average to complete a typical construction project, many studies have confirmed the existence of gestation lags.\footnote{Almon (1968) estimates that there is a seven quarter lag from the time of appropriation to investment expenditure. McRae (1989) notes that investments in a power generating plant take more than 6 years to complete. Wheaton (1987) reports that the lag between issuing construction permit and the completion of an office building is between 18 and 24 months. Kling and McCue} Montgomery’s (1995) descriptive
study conveys very useful information about the aggregate investment delay. He calculates the value-weighted mean construction periods and completion rates for U.S. private nonresidential structures using survey data from U.S. Department of Commerce. Figure 1.1 shows that the distribution of value-completed is quite asymmetric, and the completion rate in the initial stage of a project is relatively small; less than 5% of the total value is completed during the first quarter of the project, and less than 12% during the second quarter. Those completion patterns are quite invariant over the eight different periods that the surveys are conducted. A standard investment specification omits investment lags, following the

![Figure 1.1: Value-Weighted Average Completion Pattern, 1961-1991](image.png)

(1987) estimate a five variable vector autoregression with office building construction, money supply, nominal interest rate, output and aggregate price time series. They show that the dynamic response of office construction to an innovation in the nominal interest rate peaks after 17 months.

The data are from surveys of over 52,000 private construction projects built between 1961 and 1991. The value-weighted construction periods for nonresidential structures average between five and six quarters over the period. While other studies give information on average construction period, the Commerce surveys also shed light on the distribution of value-weighted completion over time.

This finding coincides with Christiano and Todd’s (1996) notion of “time-to-plan” that little resources are used in the period an investment project is initiated.
findings of Rouwenhorst (1991) and Cogley and Nason (1995) that time-to-build in investment produces insignificant effect on output fluctuations, and includes a quadratic capital adjustment cost, since it provides a rigorous foundation for Tobin’s Q theory as shown by Abel (1979) and Hayashi (1982). Many DSGE models incorporate this standard investment specification, including Kim (2000), Ireland (2001), Dib (2003), Merz and Yashiv (2002), Bouakez et al. (2005), Keen (2005), and Christiensen and Dib (2006).

There has been only limited New Keynesian DSGE investigation of investment lags. Edge (2000) constructed a sticky-price monetary business cycle model with time-to-build in investment to account for liquidity effect. Recently, Casares (2006) shows that a model with time-to-build and capital adjustment cost specification successfully generates a delay in the peak responses of output and inflation to a monetary policy shock. This paper differs from that of Edge (2000) and Casares (2006) in two aspects. First, the fits of models with different investment specification are formally evaluated in terms of log likelihood. Second, the models with different investment specification are estimated, rather than calibrated.

Four different specifications of investment dynamics are examined in this paper: (1) time-to-build as modeled by Kydland and Prescott (1982), (2) time-to-build as modeled by Casares (2006), (3) time-to-build as modeled by Wen (1998), and (4) investment adjustment costs as modeled by Christiano et al. (2005). Kydland and Prescott (1982), Casares (2006), and Wen (1998) assume that a unit of investment started in period $t$ yields productive capital with a lag of $J$ periods. While Kydland and Prescott (1982) and Casares (2006) assume that the amount of capital stock available in period $t + J$ is determined by a commitment in period $t$, Wen (1998) assumes that the amount could be changed during the process of production and the unfinished investment project could even be discarded. According to Lucca (2005), the investment adjustment cost is equivalent to time-to-build model up to a first order approximation. This specification is becoming

\footnote{A few examples include Christiano and Todd (1996) and Christiano and Vigfusson (2003), which employ the investment lags of Kydland and Prescott (1982) to find they improve the fit of RBC model to data.}
more popular in the DSGE modeling because it generates a hump-shaped impulse response of the investment.

As a monetary DSGE model with a standard investment specification, Peter Ireland’s (2001) model is selected as a baseline. The baseline model and the models with investment lag specifications are estimated by maximum likelihood to evaluate the fit to the data.

The findings of this study could be summarized as follows: First, investment lag specification alone could not improve the fit of the model. In particular, Kydland and Prescott’s (1982) specification of investment lag without capital adjustment cost generates strange dynamics of investment. Second, if the investment lag is specified along with capital adjustment cost, the fit of the model improves significantly. The model with investment lag specification of Casares (2006) shows a better fit in terms of log likelihood, information criteria, and shape of impulse response functions.

The paper proceeds as follows. Section 1.2 presents five different investment specifications and other features of the models. Section 1.3 illustrates the estimation strategies and data. Section 1.4 presents empirical results, which include the estimated parameters and standard errors, fit of the models, and impulse response functions. Section 1.5 concludes.

1.2 Model

1.2.A Investment Specifications

Model Ire (Baseline model)

A standard investment specification assumes that it takes one period of time to finish an investment project. This investment lag is sometimes called a “delivery lag.” The law of motion for the capital stock is

\[ K_{t+1} = (1 - \delta) K_t + I_t. \]  

(1.1)
where $K$ denotes capital stock, $I$ real investment, and $\delta$ depreciation rate. In order to transform $I_t$ units of finished good into $I_t$ unit of productive capital, an adjustment cost must be paid:

$$\frac{\phi_K}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t$$

(1.2)

where $\phi_K$ denotes parameter for capital adjustment cost. This adjustment cost is often described as the disruption of production or cost of learning by the installation of new machinery.

Model KP6

Kydland and Prescott (1982) assume that a unit of investment in period $t$ yields productive capital with a lag of $J$ periods. The average investment lag, $J$, is set at six quarters following the empirical findings of Montgomery (1995). Let $S_{6,t}$ denotes the number of new projects initiated at time $t$ and $S_{1,t}$ denotes the investment projects one period from completion in period $t$, then the law of motion that describes the evolution of the incomplete investment projects is given by

$$S_{j-1,t+1} = S_{j,t}$$

(1.3)

for $j = 2, 3, \cdots, 6$. It is also assumed that $\omega_j$ ($0 \leq \omega_j \leq 1$, $j = 1, 2, \cdots, 6$) of resources are expended in each period for the respective incomplete projects. Thus the total investment outlays in period $t$ are given by

$$I_t = \omega_1 S_{1,t} + \omega_2 S_{2,t} + \cdots + \omega_5 S_{5,t} + \omega_6 S_{6,t},$$

(1.4)

where $\sum_{j=1}^{6} \omega_j = 1$. Those $\omega_j$’s could be interpreted as the completion patterns of an investment project. For instance, $\omega_6$ is the completion rate for the first period of a new project and $\omega_1$ that for the last period of the project. As described by Montgomery (1995), the completion pattern is quite invariant over time. Therefore, we could regard $\omega_j$’s as structural parameters and estimate them.

The amount of capital production in period $t + 6$ is decided in period $t$ and a started investment project should be finished since an investment decision is
not reversible. This model does not include any form of capital adjustment cost. With these assumptions, the law of motion for the capital stock becomes

$$K_{t+6} = (1 - \delta) K_{t+5} + S_{6,t}. \quad (1.5)$$

**Model Cas6**

Casares (2006) modifies Kydland and Prescott’s (1982) setup using the adjustment cost structure specified by Woodford (2003). The costs to adjust capital stock for period $t + j$ is given by $i \left( \frac{K_{t+j}}{K_{t+j-1}} \right) K_{t+j-1}$. Six quarters of average investment lag is assumed here as well. Since it takes six periods to install a new capital, the investment outlays are the sum of the adjustment costs for the different investment projects.

$$I_t = \omega_1 i \left( \frac{K_{t+1}}{K_t} \right) K_t + \omega_2 i \left( \frac{K_{t+2}}{K_{t+1}} \right) K_{t+1} + \cdots + \omega_6 i \left( \frac{K_{t+6}}{K_{t+5}} \right) K_{t+5}$$

where $\sum_{j=1}^{6} \omega_j = 1$. $\omega_j$’s are interpreted as the completion patterns of an investment project as Model KP6. Since the model is going to be log-linearized around the steady state, the exact form of the function $i(\cdot)$ is not required. In steady state solution of the model, this function satisfies $i(1) = \delta$, $i'(1) = 1$, and

$\begin{align*}
\Theta \omega_6 \dot{\lambda}_t &= -\Theta [\omega_5 - \omega_6 (1 - \delta)] \beta E_t \dot{\lambda}_{t+1} - \Theta [\omega_4 - \omega_5 (1 - \delta)] \beta^2 E_t \dot{\lambda}_{t+2} \\
&- \Theta [\omega_3 - \omega_4 (1 - \delta)] \beta^3 E_t \dot{\lambda}_{t+3} - \Theta [\omega_2 - \omega_3 (1 - \delta)] \beta^4 E_t \dot{\lambda}_{t+4} \\
&- \Theta [\omega_1 - \omega_2 (1 - \delta)] \beta^5 E_t \dot{\lambda}_{t+5} + [\Theta \omega_1 (1 - \delta) \beta^6 + 1] E_t \dot{\lambda}_{t+6} \\
&+ E_t \dot{q}_{t+6} + \left( \frac{\phi_k}{q} \right) \dot{k}_{t+5} - \left( \frac{\phi_k}{q} \right) (1 + \beta) E_t \dot{k}_{t+6} + \left( \frac{\phi_k}{q} \right) E_t \dot{k}_{t+7}
\end{align*}$

where $\dot{\lambda}$, $\dot{q}$, and $\dot{k}$ denote the percent deviations from steady state value of marginal utility of consumption, rental rate of capital, and capital stock respectively. $\Theta$, $\beta$, and $q$ are some positive numbers. Since both $\dot{k}_{t+5}$ and $\dot{\lambda}_t$ have positive coefficients, an increase in $\dot{k}_{t+5}$ implies an increase in $\dot{\lambda}_t$, which means a decline in output. This negative correlation between $\dot{k}_{t+5}$ and $\dot{\lambda}_t$ results in a very small estimate of $\omega_5$ which represents the completion rate of $k_{t+5}$ in period $t$.
\( i''(1) = \kappa \). This indicates that in the steady state, the rate of investment spending required to maintain the capital stock is equal to \( \delta \) times the steady state capital stock. It also implies that near the steady state, a marginal unit of investment spending increases capital stock by an equal amount. Finally, the parameter \( \kappa \) (\( \kappa > 0 \)) determines the degree of the adjustment costs. The main difference between Model KP6 and Model Cas6 is the existence of positive capital adjustment cost in every stage of a capital building project.

**Model Wen6**

Model Wen6 employs Wen’s (1998) modified time-to-build specification. He proposes this specification because the time-to-build specification of Kydland and Prescott (1982) fails to generate seven years of investment cycles, which is observed in US investment/output data. The specification allows for a change in the amount of completed capital during the process of production. Let \( G \) denote the amount of “finished but not installed” capital as determined by a Cobb-Douglas function

\[
G_t = (I_t)^{\tau_1} (I_{t-1})^{\tau_2} (I_{t-2})^{\tau_3} \cdots (I_{t-5})^{\tau_6}
\]

where \( I_{t-j} \) is the investment spending in period \( t-j \) and \( 0 \leq \tau_j \leq 1 \) is a technical parameter that specifies the intertemporal resource share. It is also assumed that the average investment lag is six quarters and \( \sum_{j=1}^{6} \tau_j = 1 \). Those \( \tau_j \)'s could be interpreted as the completion patterns of an investment project as well.

Unlike Model KP6, a shock could change the whole scale of investment spending in period \( t \), changing the level of \( K_{t+1}, K_{t+2}, \ldots, K_{t+6} \). The law of motion for the capital stock becomes

\[
K_{t+1} = (1 - \delta) K_t + G_t.
\]

It is possible to abandon an unfinished investment project, but projects are more likely to be continued as the construction stage advances, since the costs of past investment spending on that project are sunk. An implication is that a multi-period commitment on successive investment spending is implicitly required when
rational investment decisions are made at the initial construction period. This implies that optimal investment must be serially correlated.

Model CEE

Christiano et al. (2005) use a different type of investment adjustment cost specification in DSGE model. The law of motion of capital stock is

\[ K_{t+1} = (1 - \delta) K_t + F(I_t, I_{t-1}), \]

where \( F(I_t, I_{t-1}) \) is defined as [1 - \( S \left( \frac{I_t}{I_{t-1}} \right) \)] \( I_t \). Investment adjustment costs, \( S \left( \frac{I_t}{I_{t-1}} \right) I_t \), should be paid if the current period investment expenditure is more or less than last period. The exact form of the function \( S(\cdot) \) is not required as well. \( S(1) = S'(1) = 0 \) and \( \varkappa \equiv S''(1) > 0 \) implying \( F_1(I, I) = 1, F_2(I, I) = 0 \) in a non-stochastic steady state. Therefore, a unit of investment spending increases capital stock the same amount in the steady state. The parameter \( \varkappa \) (\( \varkappa > 0 \)) governs the degree of the adjustment costs. Christiano et al. (2005) use this form to accommodate the finding that investment exhibits a hump-shaped response to a monetary policy shock.

1.2.B Behavior of Economic Agents

The economy is composed of an infinitely-lived representative household, a representative finished goods-producing firm, intermediate goods-producing firms indexed by \( i \in [0, 1] \), and a monetary authority. The household owns capital, rents it to intermediate-good producing firms, and collects profit from the firms. There is

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6 The capital adjustment cost used by CEE is different from the standard form of capital adjustment cost, and they succeed in generating a hump-shaped impulse response to a monetary policy shock.

7 In general, the capital adjustment cost is a function of \( I_t/K_t \).

8 However it is hard to explain the nature of capital adjustment cost intuitively with this specification. For example, if a firm spends the same amount of (perhaps huge) money for investments during the period \( t \), \( t + 1 \), and \( t + 2 \) (\( I_t = I_{t+1} = I_{t+2} \)), then the capital adjustment costs for period \( t + 1 \) and \( t + 2 \) are zero. This does not square with the explanation on the source of capital adjustment cost, which are (i) disruption of production by installing new capital, (ii) cost of learning, (iii) irreversibility of project owing to the lack of secondary market for capital goods, and so on.
no population growth and the population size is normalized to one. The monetary authority follows a Taylor type of interest rate rule.

**Representative Household**

The representative household carries $M_{t-1}$ units of money, $B_{t-1}$ bonds, and $K_t$ units of capital into period $t$. At the beginning of the period, the household receives a lump-sum nominal transfer $T_t$ from the monetary authority. Next, the household’s bonds mature, providing $B_{t-1}$ additional units of money. The household uses some of its money to purchase $B_t$ new bonds at nominal cost $B_t/r_t$; hence, $r_t$ denotes the gross nominal interest rate between period $t$ and $t + 1$.

During period $t$, the household supplies $h_t(i)$ units of labor to the firm at the nominal wage rate $W_t$ and $K_t(i)$ units of capital at the nominal rental rate $Q_t$. Thus, the total nominal factor payments that the household receives during period $t$ are $W_t h_t + Q_t K_t$. In addition, the household receives nominal profits $D_t(i)$ from each firm $i \in [0, 1]$. The household uses its funds to purchase output at the nominal price $P_t$ from the representative final goods-producing firm, which it divides among consumption $C_t$ and investment $I_t$.

The household faces a budget constraint given by

$$\frac{M_{t-1} + B_{t-1} + T_t + W_t h_t + Q_t K_t + D_t}{P_t} \geq C_t + I_t + \frac{B_t}{r_t} \frac{M_t}{P_t} + \frac{\phi_K}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t$$

for all $t = 0, 1, 2, \ldots$. $\phi_K$ is set at zero in Model KP6, Cas6, Wen6, and CEE, because no capital adjustment cost is assumed in Model KP6 and Wen6, and the adjustment cost is specified as loss of capital during the production of it in Model Cas6 and CEE.

The household’s preferences are described by the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t u \left( C_t, \frac{M_t}{P_t}, h_t \right),$$

where $0 < \beta < 1$ is a discount factor. Specifically, the utility function of representative household is given by

$$u \left( C_t, \frac{M_t}{P_t}, h_t \right) = a_t \left( \frac{\gamma}{\gamma - 1} \right) \ln \left[ \frac{C_t^{\gamma-1}}{\gamma} + b_t \left( \frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right] + \eta \ln (1 - h_t)$$
The preference shocks $a_t$ and $b_t$ follow the autoregressive processes.

$$\ln (a_t) = \rho_a \ln (a_{t-1}) + \varepsilon_{at} \quad (1.9)$$

$$\ln (b_t) = (1 - \rho_b) \ln (b) + \rho_b \ln (b_{t-1}) + \varepsilon_{bt} \quad (1.10)$$

where $0 < \rho_a < 1$, $0 < \rho_b < 1$ and $b > 0$. The zero-mean, serially uncorrelated innovations $\varepsilon_{at}$ and $\varepsilon_{bt}$ are normally distributed with standard deviations $\sigma_a$ and $\sigma_b$ respectively.

The Finished Goods-Producing Firm

The representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$ to produce $Y_t$ units of the finished good using the technology

$$\left[ \int_0^1 Y_t(i)^{\frac{\alpha}{\alpha-1}} di \right]^{\frac{\alpha-1}{\alpha}} \geq Y_t \quad (1.11)$$

Given that the price of Intermediate good $i$ is $P_t(i)$, the finished good sells at the nominal price $P_t$; the finished goods-producing firm chooses $Y_t$ and $Y_t(i)$ to maximize its profits,

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di \quad (1.12)$$

subject to the technology constraint (3.4).

The Intermediate Goods-Producing Firm

The representative intermediate goods-producing firm hires $h_t(i)$ units of labor and $K_t(i)$ units of capital from the representative household during period $t$ in order to produce $Y_t(i)$ units of intermediate good $i$ according to the constant returns to scale technology described by

$$K_t(i)^{\alpha} [z_t h_t(i)]^{1-\alpha} \geq Y_t(i) \quad (1.13)$$

The technology shock $z_t$ follows the autoregressive process

$$\ln (z_t) = (1 - \rho_z) \ln (z) + \rho_z \ln (z_{t-1}) + \varepsilon_{zt} \quad (1.14)$$
where $0 < \rho_z < 1$, $z > 0$ and the zero-mean, serially uncorrelated innovations $\varepsilon_{zt}$ are normally distributed with standard deviations $\sigma_z$.

The intermediate goods-producing firm sells its output in a monopolistically competitive market. In addition, each intermediate goods-producing firm faces costs of adjusting its nominal price, measured in terms of the finished good and given by

$$\frac{\phi_p}{2} \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

where $\pi$ is the steady-state rate of inflation. These costs of price adjustment make the firm’s problem dynamic; it chooses $h_t(i), K_t(i),$ and $P_t(i)$ to maximize its total market value, equal to

$$E \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[ \frac{D_t(i)}{P_t} \right]$$

(1.15)

where $\beta^t \Lambda_t$ represents the marginal utility to the representative household provided by an additional dollar of profits during period $t$ and where

$$\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t h_t(i) + Q_t K_t(i)}{P_t} - \frac{\phi_p}{2} \left[ \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

(1.16)

subject to the constraint

$$K_t(i)^{\alpha} \left[ z_t h_t(i) \right]^{1-\alpha} \geq \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t$$

(1.17)

for all $t = 0, 1, 2, \cdots$.

The Monetary Authority

The monetary authority conducts monetary policy by adjusting the nominal interest rate $r_t$, in response to deviations of output $Y_t$, inflation $\pi_t = (P_t/P_{t-1})$, and money growth $\mu_t = (M_t/M_{t-1})$ from their steady state values $Y, \pi$, and $\mu$ according to the rule

$$\ln \left( \frac{r_t}{r} \right) = \rho_y \ln \left( \frac{Y_t}{Y} \right) + \rho_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \rho_\mu \ln \left( \frac{\mu_t}{\mu} \right) + \varepsilon_{rt}$$

(1.18)

where $\varepsilon_{rt}$ is zero mean, serially uncorrelated, and normally distributed with standard deviation $\sigma_r$.

9Original specification of the monetary policy rule is as follows:

$$\ln \left( \frac{r_t}{r} \right) = \rho_r \ln \left( \frac{r_{t-1}}{r} \right) + \rho_y \ln \left( \frac{Y_t}{Y} \right) + \rho_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \rho_\mu \ln \left( \frac{\mu_t}{\mu} \right) + \varepsilon_{rt}$$
1.3 Estimation Method and Data

Matching-moment methods or likelihood-based alternatives could be considered as possible estimation procedures. Typical matching-moments methods are generalized methods of moments (GMM), simulated method of moments (SMM), and indirect inference. In particular, indirect inference minimizing the difference between the empirical and theoretical impulse responses to a monetary policy shock has recently become popular in DSGE model estimation, for example, Rotemberg and Woodford (1999) and Christiano et al. (2005). Classical maximum likelihood estimation (MLE) and Bayesian inference represent the likelihood based methods. Because matching-moments procedures accomplish estimation and testing on the basis of a pre-selected collection of moments, they entail a loss of efficiency that can sometimes be problematic in working with small sample sizes. Also, resulting inferences can potentially be sensitive to the particular collection of moments chosen for the analysis. The indirect inference using the impulse response function of vector autoregression requires a justification for the structural VAR identification assumption, which is always controversial. Under the classical maximum likelihood approach, we could fully utilize the information from the time series data which is of interest to researchers, as pointed out by Sargent (1989). Also, it provides a full characterization of the data generating process, and allows for proper specification testing and forecasting. The Kalman filter allows us to deal with unobserved or poorly measured predetermined variables (like the stock of capital) and yields the optimal solution to the problem of predicting and updating state-space models. Hansen and Sargent (2005) show that the maximum likelihood estimator obtained by applying the Kalman filter to the state-space representation of DSGE models is consistent and asymptotically efficient. Therefore, the models are estimated with the maximum likelihood method. Initially, maximum likelihood estimation was a challenging task, but recent improvements in numerical algorithms and computing power have begun to make estimation and

\[ \rho_r \ln (r_{t-1}/r) \] is removed from the specification because the estimate of \( \rho_r \) is very close to zero.

When the first-order conditions, identity equations, and exogenous shock equations are log-linearized around their steady state, one can write the system of equations in matrix form:

\[
\begin{bmatrix}
    k_{t+1} \\
    d_{t+1} \\
    x_{t+1}
\end{bmatrix} =
\begin{bmatrix}
    A & E_t & 0 \\
\end{bmatrix}
\begin{bmatrix}
    k_t \\
    d_t \\
    x_t
\end{bmatrix} +
\begin{bmatrix}
    B \\
    0
\end{bmatrix}
\begin{bmatrix}
    k_t \\
    d_t \\
    x_t
\end{bmatrix} +
C
\begin{bmatrix}
    x_t \\
    x_{t+1}
\end{bmatrix} =
P
\begin{bmatrix}
    x_t \\
    x_{t+1}
\end{bmatrix} +
\epsilon_{t+1}
\]

where \(k_t, d_t, x_t,\) and \(\epsilon_t\) are a vector of predetermined variables, non-predetermined variables, shock variables, and innovations in shocks, respectively.\(^{10}\) \(A, B, C,\) and \(P\) are coefficient matrices.

Ireland (2001) uses the method of Blanchard and Kahn (1980) to solve the model; I instead use the generalized Schur (or QZ) decomposition method because the matrix \(A\) is not invertible, when investment lags are assumed. Moreover, generalized Schur decomposition method is known to be more general and numerically more stable than that of Blanchard and Kahn (1980).\(^{11}\) The solution, if there is a unique equilibrium, takes the following form:

\[
\begin{bmatrix}
    k_{t+1} \\
    x_{t+1}
\end{bmatrix} =
\Pi
\begin{bmatrix}
    k_t \\
    x_t
\end{bmatrix} + W \epsilon_{t+1} \tag{1.19}
\]

\[
d_t =
U
\begin{bmatrix}
    k_t \\
    x_t
\end{bmatrix} \tag{1.20}
\]

where the elements of \(\Pi\) and \(U\) are nonlinear functions of the structural parameters of the model. Since the model takes the form of a state-space model, driven by

\(^{10}\)For example, the system of log-linearized equation from the benchmark model can be written in terms of the following vector variables: \(k_t = [\hat{k}_t, \hat{m}_{t-1}, \hat{r}_{t-1}, \hat{\pi}_{t-1}]', d_t = [\hat{y}_t, \hat{c}_t, \hat{\beta}_t, \hat{\mu}_t, \hat{\xi}_t, \hat{\lambda}_t, \hat{\xi}_t]', x_t = [\hat{a}_t, \hat{\beta}_t, \hat{\xi}_t, \epsilon_{rt}]',\) and \(\epsilon_t = [\epsilon_{at}, \epsilon_{bt}, \epsilon_{zt}, \epsilon_{rt}]'.\)

\(^{11}\)See Anderson et al. (1995) and Klein (2000) for more discussion of the QZ decomposition. The QZ decomposition exists more generally, including when \(A\) is singular.
four innovations in $\varepsilon_t$, maximum likelihood estimates of the parameters embedded in $\Pi$ and $U$ can be obtained as described by Hamilton (1994). Calculating the standard errors requires two steps, numerically evaluating the matrix of second derivatives of the log-likelihood function and then inverting that matrix having elements of varying magnitudes, both of which may introduce approximation error into the statistics. Hence, these standard errors, though useful, do need to be interpreted with a bit of caution.

The models are estimated using quarterly U.S. data on output, real money balances, inflation, and a nominal interest rate. The data are obtained from the database of Federal Reserve Bank of St. Louis (FRED©). Output is measured by real GDP, while real balances are measured by dividing M2 money stock by the GDP deflator. Inflation is measured by the change in the GDP deflator, and nominal interest rate by the rate on three-month Treasury bills. All series, except for the interest rate, are seasonally adjusted; the series for output and real balances are expressed in per-capita terms dividing by the civilian, non-institutional population, age 16 and above. Since the variables in the model are expressed in percentage deviations from the steady state, the output and real money balances are logged and detrended linearly. The sample is 1979:Q3 to 2006:Q2. Following Ireland (2001), the starting period corresponds to the beginning of Paul Volker’s tenure as Chairman of the Federal Reserve, when a fundamental change in U.S. monetary policy is widely believed to have occurred.\footnote{Ireland (2001) separates the sample by pre-Volker and post-Volker periods and estimates the model with each sample of data to test a structural change in U.S. economy.}

1.4 Empirical Result

1.4.A Maximum Likelihood Estimates

Before estimating the models, some of the parameters need to be imposed prior to estimation because of weak identification or insufficient information in the data. The imposed parameters and their values could be found at Table 1.1 as well.
Table 1.1: Imposed Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Weight on leisure</td>
<td>1.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Quarterly depreciation rate</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital</td>
<td>0.36</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The degree of market power</td>
<td>6</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Price adjustment cost parameter</td>
<td>58</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>Capital adjustment cost parameter in Model KP6</td>
<td>10</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Capital adjustment cost parameter in Model Cas6</td>
<td>16</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Capital adjustment cost parameter in Model CEE</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Consumer’s weight on leisure is set at $\eta = 1.5$ so as to make the steady state value of hours worked ($h$) be around 0.3, which implies that approximately one third of the time is devoted to market activity. The depreciation rate is set at $\delta = 0.0125$ to have 5 percent annual depreciation rate. Capital’s share in production is set at $\alpha = 0.36$. The degree of market power of a firm is set $\theta = 6$ so that the steady state markup of the intermediate-good producing firm is around 1.2, the figure used by Rotemberg and Woodford (1992). The price adjustment cost parameter is set at $\phi_p = 58$ which corresponds to a setting where around 25 percent of the intermediate goods-producing firms re-optimize their price in Calvo pricing. Following Ireland (2001), the capital adjustment cost parameter is set at $\phi_k = 10$ in Model Ire because $\phi_k$ higher than 10 results in unrealistically large $\sigma_z$, while lower than 10 does not help generate liquidity effect. The capital adjustment cost parameter for Model Cas6 is set at $\kappa = 16$ following Casares (2006). The investment adjustment cost parameter in Model CEE is set at $\zeta = 0.91$ because Christiano et al. (2005) estimated it at the value for a model without habit persistence in consumption.

This leaves the parameters $\beta, \gamma, \rho_a, \sigma_a, b, \rho_b, \sigma_b, z, \rho_z, \sigma_z, \pi, \rho_y, \rho_p, \rho_m, \omega_1, \ldots, \omega_6, \tau_1, \ldots, \tau_6$ to be determined. Those parameters are described in Table 1.2. Estimated parameters and standard errors are provided in Table 1.3 to Table 1.4. Estimated common structural parameters seem reasonable. The discount
factor ($\beta$) is estimated to be around 0.99, which is close to typical calibration practices. The estimated $\gamma$ implies that the interest elasticity of money demand ranges between -0.15 and -0.06, while $\rho_b$ does between 0.97 and 0.99, showing that the money demand shock tend to be quite persistent. The preference shock is quite persistent as well considering $\rho_a$ is estimated to be between 0.85 and 0.99. The persistence of the technology shock ($\rho_z$) is estimated more than 0.90, which is close to the one used in real business cycle literature.

Steady state inflation rate is estimated around 0.8 percent which could be interpreted as around 3 percent of inflation per annum. The parameters in

\[ \left( \frac{b_t C_t}{M_t/P_t} \right)^{\gamma} = 1 - \frac{1}{r_t} \]

Let $R_t = r_t - 1$ denote the net nominal interest rate, and using the approximation

\[ \frac{1}{r_t} = \frac{1}{1 + R_t} = f(R_t) \approx f(0) + f'(0) R_t = 1 - R_t, \]

Then (a) can be written as

\[ \ln \left( \frac{M_t}{P_t} \right) \approx \ln (C_t) - \gamma \ln (R_t) + \ln (b_t) \]

which confirms that $b_t$ acts as a serially correlated shock to money demand and $\gamma$ as the interest rate sensitivity of money demand.
Table 1.3: Maximum Likelihood Estimation Result (1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model Ire</th>
<th>Model KP6</th>
<th>Model Cas6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9918 (0.0008)*</td>
<td>0.9917 (0.0010)</td>
<td>0.9924 (0.0009)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1545 (0.0090)</td>
<td>0.1229 (0.0337)</td>
<td>0.0638 (0.0094)</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.8568 (0.0029)</td>
<td>0.9919 (0.0027)</td>
<td>0.8610 (0.0182)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.0376 (0.0003)</td>
<td>0.0294 (0.0028)</td>
<td>0.0243 (0.0003)</td>
</tr>
<tr>
<td>$b$</td>
<td>1.5259 (0.0922)</td>
<td>1.7531 (0.1961)</td>
<td>2.2247 (0.1290)</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9725 (0.0144)</td>
<td>0.9931 (0.0116)</td>
<td>0.9804 (0.0126)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.0105 (0.0007)</td>
<td>0.0091 (0.0013)</td>
<td>0.0105 (0.0009)</td>
</tr>
<tr>
<td>$z$</td>
<td>3768.57 (16.80)</td>
<td>4331.91 (344.16)</td>
<td>3715.55 (274.19)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.9275 (0.0024)</td>
<td>0.9951 (0.0072)</td>
<td>0.9548 (0.0263)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0095 (0.0004)</td>
<td>0.0065 (0.0028)</td>
<td>0.0081 (0.0009)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.0085 (0.0001)</td>
<td>1.0181 (0.0033)</td>
<td>1.0082 (0.0004)</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.1077 (0.0008)</td>
<td>0.0060 (0.0022)</td>
<td>0.1275 (0.0281)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>1.3223 (0.0177)</td>
<td>0.6394 (0.0550)</td>
<td>1.0981 (0.0710)</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.6984 (0.0107)</td>
<td>0.3873 (0.0547)</td>
<td>0.7525 (0.0953)</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.0069 (0.0001)</td>
<td>0.0060 (0.0004)</td>
<td>0.0070 (0.0007)</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>–</td>
<td>0.1917 (0.0242)</td>
<td>0.1774 (0.1176)</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>–</td>
<td>0.1917 (0.0376)</td>
<td>0.1774 (0.1453)</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>–</td>
<td>0.1917 (0.0223)</td>
<td>0.1774 (0.1537)</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>–</td>
<td>0.1417 (0.0116)</td>
<td>0.1628 (0.1751)</td>
</tr>
<tr>
<td>$\omega_5$</td>
<td>–</td>
<td>0.1417 (0.0165)</td>
<td>0.1275 (0.0314)</td>
</tr>
<tr>
<td>$\omega_6$</td>
<td>–</td>
<td>0.1417 (0.0247)</td>
<td>0.1774 (0.2231)</td>
</tr>
</tbody>
</table>

* Standard errors in parentheses.

The monetary policy rule are quite reasonable regarding the parameter on output change ($\rho_y$) is less than one and that on inflation ($\rho_p$) is greater than one, which is supported by empirical literature on Taylor type monetary policy rule such as Orphanides (2004).

Estimated parameters in time-to-build models indicate that the completion patterns are almost uniform as assumed by Kydland and Prescott (1982) and
Table 1.4: Maximum Likelihood Estimation Result (2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model CEE</th>
<th>Model Wen6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9917 (0.0008)*</td>
<td>0.9990 (0.0012)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0859 (0.0110)</td>
<td>0.0560 (0.0078)</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.8536 (0.0201)</td>
<td>0.8817 (0.0169)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.0263 (0.0019)</td>
<td>0.0385 (0.0042)</td>
</tr>
<tr>
<td>$b$</td>
<td>2.0683 (0.1502)</td>
<td>2.4236 (0.1490)</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9866 (0.0116)</td>
<td>0.9813 (0.0137)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.0119 (0.0010)</td>
<td>0.0124 (0.0012)</td>
</tr>
<tr>
<td>$z$</td>
<td>3771.36 (136.02)</td>
<td>2594.09 (183.09)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.9498 (0.0349)</td>
<td>0.9724 (0.0163)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0069 (0.0011)</td>
<td>0.0068 (0.0009)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.0090 (0.0005)</td>
<td>1.0067 (0.0004)</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.1444 (0.0321)</td>
<td>0.1617 (0.0345)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>1.0682 (0.0714)</td>
<td>1.2008 (0.0940)</td>
</tr>
<tr>
<td>$\rho_\rho$</td>
<td>0.7498 (0.0965)</td>
<td>0.8416 (0.1139)</td>
</tr>
<tr>
<td>$\sigma_\rho$</td>
<td>0.0070 (0.0007)</td>
<td>0.0078 (0.0010)</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>–</td>
<td>0.1917 (0.1160)</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>–</td>
<td>0.1417 (0.1458)</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>–</td>
<td>0.1417 (0.1004)</td>
</tr>
<tr>
<td>$\tau_4$</td>
<td>–</td>
<td>0.1417 (0.1036)</td>
</tr>
<tr>
<td>$\tau_5$</td>
<td>–</td>
<td>0.1917 (0.1453)</td>
</tr>
<tr>
<td>$\tau_6$</td>
<td>–</td>
<td>0.1917 (0.1183)</td>
</tr>
</tbody>
</table>

* Standard errors in parentheses.

Casares (2006). As plotted in figure 1.2, estimated parameters Model KP6, Cas6, and Wen6 imply quite even completion rate for each stage of investment project.\textsuperscript{14}

\textsuperscript{14}When Model KP6 and Wen6 are specified with a quadratic capital adjustment cost, the model structure drives very small estimates of $\omega_5$ and $\tau_5$, implying that a very small portion of the project is completed in the second period of an investment project.
1.4.B Fit of the Models

Estimated log likelihoods and the number of estimated parameters are presented at table 1.5. Model Cas6 marks the highest log likelihood value and Model Ire follows it. However, the log likelihoods cannot be readily compared with one another since the five models are not nested each other. Therefore, fits of the models to the data are to be evaluated by information criteria, tests for non-nested models, and the shape of impulse response functions. Figure 1.3 shows the calculated AIC (Akaike information criteria) and SC (Schwarz information criteria) of each model.\(^{15}\) Recalling that a bigger absolute value implies a better fit, Model Cas6 leads the other model in terms of AIC. Model Ire, Model CEE, Model Wen 6, and Model KP6 follows it. The relative fit in terms of SC is similar.

\(^{15}\)AIC = \(-2 \log L + 2K\) and \(SC = -2 \log L + K \left(\log N\right)\), where \(K\) is the number of estimated parameters and \(N\) is the number of sample size.
Table 1.5: Log Likelihoods

<table>
<thead>
<tr>
<th>Model</th>
<th>log Likelihood</th>
<th>Number of Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Ire</td>
<td>1735.30</td>
<td>15</td>
</tr>
<tr>
<td>Model KP6</td>
<td>1673.48</td>
<td>21</td>
</tr>
<tr>
<td>Model Cas6</td>
<td>1744.68</td>
<td>21</td>
</tr>
<tr>
<td>Model CEE</td>
<td>1724.84</td>
<td>15</td>
</tr>
<tr>
<td>Model Wen6</td>
<td>1717.56</td>
<td>21</td>
</tr>
</tbody>
</table>

With a small margin Model Ire beats Model Cas6 but the other models have the same ranks.

The problem with the model selection approach is that it produces a deterministic outcome, defined by the ranking of the values of the criterion, and it does not take into account the probabilistic nature of that result. As Vuong (1989) points out, differences in the criterion values may not be statistically significant; yet the deterministic model selection approach would consider one model superior to another, while in fact they may be considered as being statistically equivalent.

To have a statistical implication of the estimated log likelihood, Vuong (1989) test could be readily implemented. Vuong’s test statistic, a corrected likelihood ratio divided by a standard error, indicate that there are statistically significant differences in the fit of the models as presented in Table 1.6. It is clear from the pairwise Vuong test that Model Cas6 fits the data significantly better than Model Ire, KP6, CEE, and Wen6 at the significance level of ten percent or less. However, Model KP6 and Model Wen6 do not beat Model Ire. This indicates that the time-to-build specification alone do not improve the fit of the model.

Clarke’s (2007) test could be easily implemented as well. Clarke statistic

\[ \text{Clarke statistic} = \sqrt{\frac{T}{\hat{\omega}_T}} \]

is given \( \hat{L}_R = L_f - L_g - \frac{\sqrt{T}}{2} \log \left( \frac{T-q}{p} \right) \) and \( \hat{\omega}_T = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left[ \log \frac{f(Y_t|x_t, \hat{\beta})}{g(Y_t|x_t, \hat{\gamma})} \right]^2 - \frac{1}{T} \sum_{t=1}^{T} \log \frac{f(Y_t|x_t, \hat{\beta})}{g(Y_t|x_t, \hat{\gamma})}^2} \) where \( L \) denotes log likelihood, \( T \) sample size, \( \hat{\beta} \) the estimated \( (p \times 1) \) parameter vector in Model \( f \), \( \hat{\gamma} \) the estimated \( (q \times 1) \) parameter vector in Model \( g \), \( Y_t \) observation at period \( t \), \( x_t \) exogenous variable at period \( t \), \( f \left( Y_t|x_t, \hat{\beta} \right) \) the likelihood at period \( t \) in Model \( f \), and \( g \left( Y_t|x_t, \hat{\gamma} \right) \) the likelihood at period \( t \) in Model \( g \). Vuong \( \sim N(0,1) \)
counts the positive number of $d_t = \log \frac{f(Y_t|x_t, \hat{\beta})}{g(Y_t|x_t, \hat{\gamma})}$, for $t = 1, \cdots, T$.\footnote{Clarke is calculated by $\sum_{t=1}^{T} I_{(0, +\infty)} (d_t)$, where $d_t = \log \frac{f(Y_t|x_t, \hat{\beta})}{g(Y_t|x_t, \hat{\gamma})}$, for $t = 1, 2, \cdots, T$.}

Pairwise Clarke test result supports the result of Vuong test. As presented at Table 1.7, Model Cas6 fits the data better than Model Ire, Model KP6, Model CEE, and Model Wen6 at the one percent significance level. The result indicates that Model Ire fits the data better than Model KP6, Model CEE, and Model Wen6 confirming the result of Vuong test.

Calculated impulse response functions to a monetary policy shock help to understand the relative fit of the models. Impulse responses of selected variables – output, consumption, investment, capital stock, capital rental rate, work hours, inflation, money, and the interest rate – to a 100 basis point of contractionary shock in monetary policy in period $t = 1$ are plotted at Figure 1.4 - Figure 1.8.
on the response of investment, Model Cas6 shows a hump-shaped response to a monetary policy shock, while Model Ire, Model KP6, and Model Wen6 do not.\textsuperscript{18} As predicted earlier, Model Wen6 generates very persistent impulse responses of investment suggesting a strong serial correlation in investment. Model KP6 generates cyclical ups and downs in investment with cycle. This strange response is already addressed by Rouwenhorst (1991) and Wen (1998). The cyclical response of investment in Model KP6 could be described well by Figure 1.9. When a contractionary monetary policy shock hits the economy in period $t$, then future profitability declines, so the number of new investment project ($\hat{s}_6$) declines. Since there are no capital adjustment cost in Model KP6, the number drops nearly 40 percent in the impact period. In period $t+1$, as the shock on interest rate almost fades away, new investment project increases substantially to make up the a decrease of capital stock in $t+6$. Since the number of investment project rolls over for six quarters as specified in Equation (1.3), $\hat{s}_5$, $\hat{s}_4$, $\cdots$, $\hat{s}_1$ responds identically

\textsuperscript{18}This hump-shaped impulse response of investment is one of the “stylized facts” according to vector autoregression studies, such as Bernanke and Gertler (1995), Christiano et al. (1996), and Bernanke and Mihov (1998).
with periods of delay. Investment is a weighted sum of $\hat{s}_j$ for $j = 1, \cdots, 6$, so the cyclical impulse response of investment results in. This unrealistic investment dynamics contributes the relatively bad fit of Model KP6. Wen (1998) proposes different investment lag specification noting that Kydland and Prescott’s (1982) time-to-build model generates this short cycle which does not squares with the seven years of investment cycle observed in data.

Impulse response of other variables do not differ much since the estimated
Figure 1.4: Impulse Responses (Model Ire)

Figure 1.5: Impulse Responses (Model KP6)
Figure 1.6: Impulse Responses (Model Cas6)

Figure 1.7: Impulse Responses (Model CEE)
Figure 1.8: Impulse Responses (Model Wen6)

Figure 1.9: Impulse Response of Investment Projects
parameters are quite stable. Output, consumption, work hours, and inflation responds sharply in the impact period and returns to their steady state level within two years. Interest rate and money stock respond to the opposite direction, describing liquidity effect well. None of the five model generates a hump-shaped impulse response of output since habit persistence in consumption is not assumed. This suggests that specifying investment lag is not enough to generate a stylized hump-shaped response of output to a monetary policy shock, since the steady state share of investment in output is only around 0.2, too small to have a major effect on the response of output.\textsuperscript{19}

\subsection{1.5 Conclusion}

Over the past decades, a variety of richer specifications in the DSGE framework is suggested for the purpose of developing a model that is able to account for the dynamic correlations that one sees in an unrestricted VAR representation of the observed variables. Recently, Smets and Wouters (2003) illustrate the success of those efforts. However, many DSGE models offer few details on investment lags.

I found that investment lag specification alone could not improve the fit of the model. The model with Kydland and Prescott (1982) type of investment lag, that with Wen (1998) type of investment lag, and that with Christiano et al. (2005) type of investment adjustment cost do not achieve a better fit than the model with a standard quadratic capital adjustment specification. In particular, Kydland and Prescott’s (1982) specification of investment lag without any form of capital adjustment cost generates strange and volatile dynamics of investment which do not help to improve the fit of DSGE model. This finding confirms the findings of Rouwenhorst (1991) and Cogley and Nason (1995).

However, this paper shows that if we specify the investment lag along

\textsuperscript{19}However, Casares (2006) claims that the a variety of capital stock which has different investment lag could generate a hump-shaped response of output to a monetary policy shock even without assuming habit persistence in consumption.
with capital adjustment cost, we could significantly improve fit of the model. My estimation results indicate that the model with investment lag and capital adjustment costs fit the data significantly better than the standard model. The model with investment lag specification of Casares (2006) shows a better fit in terms of log likelihood, information criteria, and shape of impulse response functions.
Indexation Scheme in Calvo-Style Sticky Price Model

2.1 Introduction

The shortcomings of the New Keynesian Phillips curve with Calvo (1983) type of sticky price assumptions have been addressed by several studies. For example, Fuhrer and Moore (1995) find that the New Keynesian Phillips curve fails to explain inflation persistence in US inflation data. Collard and Dellas (2006) show that the original version of the New Keynesian dynamic stochastic general equilibrium (DSGE) model fails to generate a delayed and gradual response of inflation to a monetary policy shock in spite of various real rigidities, such as habit persistence, variable capital utilization, predetermined spending, and capital adjustment cost.

The modifications to improve the inflation dynamics implied by the New Keynesian DSGE model could be classified into two groups. One is sticky prices with the dynamic backward indexation, represented by Christiano et al. (2005) and Smets and Wouters (2003). According to the backward indexation scheme, firms that cannot re-optimize their price simply index to lagged aggregate inflation. The second is the sticky information model proposed by Mankiw and Reis (2002), according to which information about macroeconomic conditions spreads slowly.
Contrary to the sticky price model, prices in this model are adjusted always, but decisions on prices are not always based on the latest information.

In spite of ongoing controversies, sticky price models with the dynamic backward indexation are widely used for the analysis of monetary policy transmission mechanism since they are relatively easy to work with and generates a plausible inflation dynamics. However, the statistical plausibility of the dynamic backward indexation has not yet been verified. Eichenbaum and Fisher (2004) show that it is difficult to discriminate between the statistical fit of Calvo models with the dynamic backward indexation and Calvo models modified to allow for some rule of thumb firms. Eichenbaum and Fisher (2005) note that the New Keynesian DSGE model with the backward indexation scheme fails to generate plausible degree of inertia in price setting behavior at firm level if it is based on a standard assumption of a constant elasticity of demand and homogeneous capital.\(^1\) De Walque et al. (2006) find that the DSGE model performs well even when the parameter of the backward indexation is close to zero.

This paper tries to compare the statistical fit of the New Keynesian DSGE model with different indexation schemes according to log likelihood. If the dynamic backward indexation scheme improves the statistical fit to the likelihood function significantly, then it would be reasonable to say that the specification should be a standard one for policy analysis and economic forecasting.

The basic specification of the model is similar to Ireland (2001) as modified by Jung (2007). The model allows for real rigidities such as habit persistence in consumption and the investment lag specification of Casares (2006). Jung (2007) shows that the investment lag specification improves the fit of DSGE model more than alternative investment specifications.\(^2\) The benchmark model includes the static indexation to steady state inflation rate, which is assumed by Yun (1996). An alternative model is specified with the dynamic backward indexation scheme

\(^1\)If it is allowed that the elasticity of demand increases in a firm’s price and a firm specific capital stock, then the model implies a degree of inertia in price re-optimization that is much more plausible than the model with standard assumption.

\(^2\)Those other specifications include time-to-build as modeled by Kydland and Prescott (1982), time-to-build as modeled by Wen (1998), quadratic capital adjustment cost, and investment adjustment cost as Christiano et al. (2005).
The findings of this paper indicate that the dynamic backward indexation scheme does not improve the fit of DSGE model significantly. The log likelihood of the model with the backward indexation is in fact slightly lower than that with static indexation and the parameters are estimated in favor of the model with the static indexation. Also, the forecast accuracy of the model with the backward indexation is worse than that with the static indexation.

The paper is organized as follows. Section 2.2 presents the features of the model. Section 2.3 illustrates the estimation strategies and data. Section 2.4 presents empirical results, which include the estimated parameters and standard errors, fit of the models, and impulse response functions. Section 2.5 concludes.

2.2 The Model

2.2.A The Finished Goods-Producing Firm

The representative finished goods-producing firm uses \( y_t(i) \) units of each intermediate good \( i \in [0, 1] \) to produce \( y_t \) units of the finished good using the technology

\[
\left[ \int_0^1 y_t(i) \theta \, di \right] \theta^{-} \geq y_t \quad (2.1)
\]

Given that the price of Intermediate good \( i \) is \( p_t(i) \), the finished good sells at the nominal price \( p_t \); the finished goods-producing firm chooses \( y_t \) and \( y_t(i) \) to maximize its profits,

\[
p_t y_t - \int_0^1 p_t(i) y_t(i) \, di \quad (2.2)
\]

subject to the technology constraint (3.4).

The first order conditions for this problem are (3.4) with equality and

\[
y_t(i) = \left[ p_t(i) / p_t \right]^{-\theta} y_t \quad (2.3)
\]

for all \( i \in [0, 1] \) and \( t = 0, 1, 2, \ldots \). Because the finished good market is perfectly competitive the representative firm earns zero profits in equilibrium. Along with
(3.5) and (2.3), this zero-profit condition determines \( p_t \) as a Dixit-Stiglitz type of price index

\[
p_t = \left[ \int_0^1 p_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}
\]

(2.4)

for all \( t = 0, 1, 2, \ldots \).

### 2.2.B The Intermediate Goods-Producing Firm

The representative intermediate goods-producing firm hires \( h_t(i) \) units of labor and \( k_t(i) \) units of capital from the representative household during period \( t \) in order to produce \( y_t(i) \) units of intermediate good \( i \) according to the constant returns to scale technology described by

\[
k_t(i)^\alpha [z_t h_t(i)]^{1-\alpha} \geq y_t(i)
\]

(2.5)

The technology shock \( z_t \) follows the autoregressive process

\[
\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}.
\]

(2.6)

where \( 0 < \rho_z < 1, z > 0 \) and the zero-mean, serially uncorrelated innovations \( \varepsilon_{zt} \) are normally distributed with standard deviations \( \sigma_z \).

Assuming each firm operates at competitive input markets, it chooses input level by minimizing its total cost

\[
\min_{\{k_t(i), h_t(i)\}} \frac{W_t h_t(i) + Q_t k_t(i)}{p_t}
\]

subject to (2.5). Cost minimization implies

\[
\frac{Q_t k_t(i)}{W_t h_t(i)} = \frac{\alpha}{1 - \alpha}
\]

(2.7)

The condition results in the following expression for real total cost

\[
TC_t(i) = s_t y_t(i)
\]

where real marginal cost is given by

\[
s_t = \left[ \frac{(1 - \alpha)^{\alpha-1}}{\alpha^\alpha} \right]\frac{z_t^{\alpha-1} Q_t^\alpha W_t^{1-\alpha}}{p_t}
\]

(2.8)
Following Calvo (1983), firms set their prices for a stochastic number of periods. In every period, a firm gets the chance to adjust its price (an event occurring with probability \( \xi \)) or it does not. If it does not get the chance, it is assumed to set prices according to

\[ p_t(i) = \tau_t p_{t-1}(i) \] (2.9)

The indexation factor \( \tau_t = \pi \) in the benchmark case and \( \tau_t = \pi_{t-1} \) in the alternative case, where inflation \( \pi_t = p_t/p_{t-1} \) and \( \pi \) is its steady state value.

A firm that sets its price optimally chooses a price \( p^*_t(i) \) in order to maximize

\[
\max_{\{p^*_t(i)\}} E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left( 1 - \xi \right)^j (p^*_t(i) \Xi_{t,j} - p_{t+j}s_{t+j}) y_{t+j}(i)
\]

subject to the total demand it faces

\[ y_{t+j}(i) = \left[ \frac{p^*_t(i) \Xi_{t,j}}{p_{t+j}} \right]^{\frac{1}{\theta - 1}} y_{t+j} \]

and where

\[ \Xi_{t,j} = \begin{cases} 
\tau_{t+1} \cdots \tau_{t+j} & \text{for } j \geq 1 \\
1 & \text{for } j = 0
\end{cases} \]

Here, \( \beta \) denotes discount factor and \( \lambda \) the marginal value of income to household. This determines the price setting equation

\[ p^*_t(i) = \frac{\theta \sum_{j=0}^{\infty} E_t/\beta^j \lambda_{t+j} (1 - \xi)^j (p^*_{t+j}\Xi_{t,j} - \theta s_{t+j}) y_{t+j}}{(\theta - 1) \sum_{j=0}^{\infty} E_t/\beta^j \lambda_{t+j} (1 - \xi)^j p^*_{t+j} \Xi_{t,j}^{1-\theta} y_{t+j}} \] (2.10)

Note all optimizing firms would choose the same price, \( p^*_t(i) = p^*_t \) and a fraction \( \xi \) of firms adjust. The aggregate price index (2.4) could be rewritten as

\[ p_t = \left[ \xi p^*_{t} + (1 - \xi) (\tau_t p_{t-1})^{1-\theta} \right]^{\frac{1}{\theta - 1}} \] (2.11)

It follows that the New Keynesian Phillips curves are given in benchmark case by

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta (1 - \xi)) \xi}{(1 - \xi)} \hat{s}_t \] (2.12)
and in the alternative case by
\[
\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{(1 - \beta (1 - \xi)) \xi}{(1 + \beta) (1 - \xi)} \hat{s}_t
\]  
(2.13)
if the equation (2.10) and (2.11) are log-linearized around the steady state and arranged. Recall that a hat over a variable indicates the percent deviation from the steady state.

Equation (2.12) and (2.13) could be written in a general form as follows:
\[
\hat{\pi}_t = \frac{1}{1 + \beta - (1 - \chi) \beta} \left[ \chi \hat{\pi}_{t-1} + \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta (1 - \xi)) \xi}{1 - \xi} \hat{s}_t \right].
\]  
(2.14)
If \( \chi = 0 \), then equation (2.14) represents the benchmark case and otherwise if \( \chi = 1 \), the alternative case.

### 2.2.C Representative Household

The representative household carries \( M_{t-1} \) units of money, \( B_{t-1} \) bonds, and \( k_t \) units of capital into period \( t \). At the beginning of the period, the household receives a lump-sum nominal transfer \( T_t \) from the monetary authority. Next, the household’s bonds mature, providing \( B_{t-1} \) additional units of money. The household uses some of its money to purchase \( B_t \) new bonds at nominal cost \( B_t/r_t \); hence, \( r_t \) denotes the gross nominal interest rate between period \( t \) and \( t+1 \).

During period \( t \), the household supplies \( h_t \) units of labor to the firm at the nominal wage rate \( W_t \) and \( k_t \) units of capital at the nominal rental rate \( Q_t \). Thus, the total nominal factor payments that the household receives during period \( t \) are \( W_t h_t + Q_t k_t \). In addition, the household receives nominal profits \( D_t \). The household uses its funds to purchase output at the nominal price \( p_t \) from the representative final goods-producing firm, which it divides among consumption \( c_t \) and investment \( i_t \).

The household faces a budget constraint given by
\[
\frac{M_{t-1} + B_{t-1} + T_t + W_t h_t + Q_t k_t + D_t}{p_t} \geq c_t + i_t + \frac{B_t/r_t + M_t}{p_t}
\]  
(2.15)
for all \( t = 0, 1, 2, \cdots \).
Suppose that the utility flow of the representative household in period $t$ depends not only on its real consumption in that period $c_t$, but also the level of consumption in the previous period. The household’s preferences are described by the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t u \left( c_t, \frac{M_t}{p_t}, h_t \right),$$

where $0 < \beta < 1$ is a discount factor. Specifically, the utility function of representative household augmented with habit persistence is given by

$$u \left( c_t, \frac{M_t}{p_t}, h_t \right) = a_t \left( \frac{\gamma}{\gamma - 1} \right) \ln \left[ \left( c_t - \zeta c_{t-1} \right)^{\frac{\gamma - 1}{\gamma}} + b_t^\gamma \left( \frac{M_t}{p_t} \right)^{\frac{\gamma - 1}{\gamma}} \right] + \eta \ln \left( 1 - h_t \right)$$

where $0 \leq \zeta \leq 1$ measures the degree of habit persistence. In the special case where $\zeta = 0$, there is no habit persistence and the utility function becomes time-separable one. The preference shocks $a_t$ and $b_t$ follow the autoregressive processes.

$$\ln (a_t) = \rho_a \ln (a_{t-1}) + \varepsilon_{at}$$  \hspace{1cm} (2.16)

$$\ln (b_t) = (1 - \rho_b) \ln (b) + \rho_b \ln (b_{t-1}) + \varepsilon_{bt}$$  \hspace{1cm} (2.17)

where $0 < \rho_a < 1$, $0 < \rho_b < 1$ and $b > 0$. The zero-mean, serially uncorrelated innovations $\varepsilon_{at}$ and $\varepsilon_{bt}$ are normally distributed with standard deviations $\sigma_a$ and $\sigma_b$ respectively.

Following Casares (2006) and Jung (2007), a modification of Kydland and Prescott’s (1982) time-to-build in investment is assumed. Jung (2007) finds that this form of investment delay improves the fit of DSGE model more than the other specifications, such as prototypical time-to-build specification of Kydland and Prescott (1982), time-to-build specification of Wen (1998), quadratic capital adjustment cost, or investment adjustment cost.\(^3\) The costs to adjust capital stock for period $t + j$ is given by $i \left( \frac{K_{t+j}}{K_{t+j-1}} \right) K_{t+j-1}$. Six quarters of average investment lag is assumed here following Montgomery (1995). Since it takes six periods to install a new capital, the investment outlays are the sum of the adjustment costs for the different investment projects.

\(^3\)Investment adjustment cost specification as in Christiano et al. (2005) is widely used in DSGE model since it could successfully generate a hump shaped dynamic response of investment.
$$I_t = \omega_1 i \left( \frac{K_{t+1}}{K_t} \right) K_t + \omega_2 i \left( \frac{K_{t+2}}{K_{t+1}} \right) K_{t+1} + \cdots + \omega_6 i \left( \frac{K_{t+6}}{K_{t+5}} \right) K_{t+5}$$

where $\sum_{j=1}^{6} \omega_j = 1$. Here, $\omega_i (i = 1, 2, \cdots, 6)$ could be interpreted as a completion rate for the $(7-i)th$ stage of an investment project. In steady state solution of the model, this function satisfies $i'(1) = \delta$, $i''(1) = 1$, and $i''(1) = \kappa$. This indicates that in the steady state, the rate of investment spending required to maintain the capital stock is equal to $\delta$ times the steady state capital stock. It also implies that near the steady state, a marginal unit of investment spending increases capital stock by an equal amount. Finally, the parameter $\kappa (\kappa > 0)$ determines the degree of the adjustment costs.

2.2.D The Monetary Authority

The monetary authority conducts monetary policy by adjusting the nominal interest rate $r_t$, in response to deviations of output $y_t$, inflation $\pi_t$, and money growth $\mu_t = (M_t/M_{t-1})$ from their steady state values $y, \pi,$ and $\mu$ according to the rule

$$\ln \left( \frac{r_t}{r} \right) = \rho_y \ln \left( \frac{y_t}{y} \right) + \rho_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \rho_\mu \ln \left( \frac{\mu_t}{\mu} \right) + \varepsilon_{rt} \quad (2.18)$$

where $\varepsilon_{rt}$ is zero mean, serially uncorrelated, and normally distributed with standard deviation $\sigma_r$.

2.3 Estimation Method and Data

The models are estimated by classical maximum likelihood estimation (MLE) because we could fully utilize the information from the time series data which is of interest to researchers. Also, it provides a full characterization of the data generating process, and allows for proper specification testing and forecasting. The Kalman filter allows us to deal with unobserved or poorly measured predetermined variables (like the stock of capital) and yields the optimal solution.
to the problem of predicting and updating state-space models. Hansen and Sargent (2005) show that the maximum likelihood estimator obtained by applying the Kalman filter to the state-space representation of DSGE models is consistent and asymptotically efficient.

When the first-order conditions, identity equations, and exogenous shock equations are log-linearized around their steady state, one can write the system of equations in matrix form:

\[
\begin{align*}
AE_t \begin{bmatrix} k_{t+1} \\ d_{t+1} \\ x_{t+1} \end{bmatrix} &= B \begin{bmatrix} k_t \\ d_t \\ x_t \end{bmatrix} + Cx_t \\
 x_{t+1} &= Px_t + \varepsilon_{t+1}
\end{align*}
\]

where \( k_t, d_t, x_t, \) and \( \varepsilon_t \) are a vector of predetermined variables, non-predetermined variables, shock variables, and innovations in shocks, respectively. \( A, B, C, \) and \( P \) are coefficient matrices.

To solve the model, I use the generalized Schur (or QZ) decomposition method because the matrix \( A \) is not invertible. The solution, if there is a unique equilibrium, takes the following form:

\[
\begin{align*}
\begin{bmatrix} k_{t+1} \\ x_{t+1} \end{bmatrix} &= \Pi \begin{bmatrix} k_t \\ x_t \end{bmatrix} + W\varepsilon_{t+1} \\
 d_t &= U \begin{bmatrix} k_t \\ x_t \end{bmatrix}
\end{align*}
\] (2.19) (2.20)

where the elements of \( \Pi \) and \( U \) are nonlinear functions of the structural parameters of the model. Since the model takes the form of a state-space model, driven by four innovations in \( \varepsilon_t \), maximum likelihood estimates of the parameters embedded in \( \Pi \) and \( U \) can be obtained as described by Hamilton (1994). Calculating the standard errors requires two steps, numerically evaluating the matrix of second derivatives of the log-likelihood function and then inverting that matrix having elements of varying magnitudes.

The models are estimated using quarterly U.S. data on output, real money balances, inflation, and a nominal interest rate. The data are obtained from the
database of Federal Reserve Bank of St. Louis (FRED®). Output is measured by real GDP, while real balances are measured by dividing M2 money stock by the GDP deflator. Inflation is measured by the change in the GDP deflator, and nominal interest rate by the rate on three-month Treasury bills. All series, except for the interest rate, are seasonally adjusted; the series for output and real balances are expressed in per-capita terms dividing by the civilian, non-institutional population, age 16 and above. Since the variables in the model are expressed in percentage deviations from the steady state, the output and real money balances are logged and detrended linearly. The sample is 1979:Q3 to 2006:Q2. Following Ireland (2001), the starting period corresponds to the beginning of Paul Volker’s tenure as Chairman of the Federal Reserve, when a fundamental change in U.S. monetary policy is widely believed to have occurred.4

2.4 Empirical Results

To overcome weak identification or insufficient information in the data, some of the parameters are imposed before estimation as summarized in Table 2.1. Consumer’s weight on leisure is set at $\eta = 1.5$ so as to make the steady state value of hours worked ($h$) be around 0.3, which implies that approximately thirty percent of the time is devoted to market activity. The depreciation rate is set at $\delta = 0.0125$ to have 5 percent annual depreciation rate. Capital’s share in production is set at $\alpha = 0.36$. The degree of market power of a firm is set $\theta = 6$ so that the steady state markup of the intermediate-good producing firm is around 1.2, the figure used by Rotemberg and Woodford (1992). The capital adjustment cost parameter is set at $\kappa = 16$ following Casares (2006). The completion rate for each stage of an investment project is imposed using the Montgomery’s (1995) calculation of the value-weighted mean construction periods and completion rates for U.S. private nonresidential structures. The study indicates that it takes six quarters on average to finish an investment project. His distribution of value-weighted

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4Ireland (2001) separates the sample by pre-Volker and post-Volker periods and estimates the model with each sample of data to test a structural change in U.S. economy.
Table 2.1: Imposed Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>η</td>
<td>Weight on leisure</td>
<td>1.5</td>
</tr>
<tr>
<td>δ</td>
<td>Quarterly depreciation rate</td>
<td>0.0125</td>
</tr>
<tr>
<td>α</td>
<td>Share of capital</td>
<td>0.36</td>
</tr>
<tr>
<td>θ</td>
<td>The degree of market power</td>
<td>6</td>
</tr>
<tr>
<td>κ</td>
<td>Capital adjustment cost parameter</td>
<td>16</td>
</tr>
<tr>
<td>ω₁</td>
<td>Completion rate of an investment project in period six</td>
<td>0.0244</td>
</tr>
<tr>
<td>ω₂</td>
<td>Completion rate of an investment project in period five</td>
<td>0.0427</td>
</tr>
<tr>
<td>ω₃</td>
<td>Completion rate of an investment project in period four</td>
<td>0.1086</td>
</tr>
<tr>
<td>ω₄</td>
<td>Completion rate of an investment project in period three</td>
<td>0.2102</td>
</tr>
<tr>
<td>ω₅</td>
<td>Completion rate of an investment project in period two</td>
<td>0.3314</td>
</tr>
<tr>
<td>ω₆</td>
<td>Completion rate of an investment project in period one</td>
<td>0.2826</td>
</tr>
</tbody>
</table>

Completion rate over forty eight months is recalculated assuming six quarters of time-to-build. The recalculated distribution is plotted at figure 2.1.

This leaves eighteen parameters $\beta, \gamma, \rho_a, \sigma_a, b, \rho_b, \sigma_b, z, \rho_z, \sigma_z, \pi, \rho_y, \rho_p, \rho_m, \sigma_r, \zeta, \xi,$ and $\chi$ to be determined. Those parameters are described in Table 2.2. The benchmark model imposes the parameter of indexation scheme, $\chi = 0$ so that it represent the the static indexation case. The alternative model leaves $\chi$ free with the restriction of $\chi \in [0, 1]$. If $\chi$ is estimated close to one with substantial accuracy, then we could conclude that the dynamic backward indexation is supported by data. On the other hand, if the estimate of $\chi$ is close to zero, then it indicates that data favors the static indexation scheme.

The point estimates and standard errors are provided in Table 2.3. Estimated common structural parameters seem reasonable. The discount factor ($\beta$) is estimated to be around 0.99, which is close to typical calibration practices. The estimated $\gamma$ implies that the interest elasticity of money demand is around -0.19, while $\rho_b$ does around 0.94, showing that the money demand shock tend to be quite persistent. The preference shock is quite persistent as well considering $\rho_a$ is estimated to be around 0.90. The persistence of the technology shock ($\rho_z$) is estimated
Quarter after Start
Percent of Total Value Completed

Figure 2.1: Distribution of Completion Rates

around 0.89, which is close to the one used in real business cycle literature. Steady state inflation rate is estimated around 0.8 percent which could be interpreted as around 3 percent of inflation per annum. The parameters in the monetary policy rule are reasonable regarding the parameter on output change (\(\rho_y\)) is less than that on inflation. The habit persistence parameter (\(\zeta\)) is estimated to be around 0.53 in both models. Regarding the standard errors the habit persistence parameter is significantly different from zero. The magnitude is smaller than the values reported by Fuhrer (2000); 0.63, and Christiano et al. (2005); 0.73.

The key result is that the indexation scheme parameter \(\chi\) is estimated to be very close to zero (0.003) with the alternative model. Noting relatively large standard error (0.027), it is hard to say \(\chi\) is far from zero. This result indicates that data do not support the dynamic backward indexation scheme. Another notable result is that the estimated degree of inertia in price optimization implied by the model is implausibly large. The probability of price optimization (\(\xi\)) is estimated at 0.07 with both models. This corresponds that firms re-optimize prices every three and half years, an interval of time that seems implausibly long. This result is similar to the point estimates of Eichenbaum and Fisher (2005) with
Table 2.2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Interest elasticity of money demand</td>
</tr>
<tr>
<td>$\rho_a, \rho_b, \text{ and } \rho_z$</td>
<td>Persistence of shocks</td>
</tr>
<tr>
<td>$\sigma_a, \sigma_b, \text{ and } \sigma_z$</td>
<td>Standard deviation of shocks</td>
</tr>
<tr>
<td>$b$ and $z$</td>
<td>Average value of shocks $b_t$ and $z_t$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady state value of inflation</td>
</tr>
<tr>
<td>$\rho_y, \rho_p, \text{ and } \rho_\mu$</td>
<td>Parameters for monetary policy rule</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>Standard deviation of monetary policy shocks</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Degree of habit persistence</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Probability of price optimization</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Indexation scheme</td>
</tr>
</tbody>
</table>

standard assumption.\(^5\) Their estimation result indicates that it takes nine quarters on average to reoptimize price. Taken at face value, the estimation result indicates that sticky price model fits the data well statistically but not in an economic sense.

In terms of log likelihood, the benchmark model fits data better than the alternative model. Since the log likelihood of the benchmark model (1765.79) is slightly higher than that of the alternative (1765.72) and an additional parameter ($\chi$) is estimated in the alternative case, the null hypothesis that the fit of the benchmark model is as good as the alternative cannot be rejected on the likelihood ratio test. Table 2.4 shows that inference does not change in terms of Akaike and Schwarz information criteria. The magnitudes of AIC and SC from the benchmark model (-3498 and -3452) are bigger than those from the alternative model (-3495 and -3447). Moreover, Vuong (1989) test and Clarke (2007) test result indicate that the backward indexation does not improve the relative fit of the model. According to Vuong test (table 2.5), the fit of the benchmark model is better than that of the alternative at the one percent significance level. Result from Clarke test, given at table 2.6, also indicates that the benchmark model fits

\(^5\)A constant elasticity of demand and homogeneous capital
Table 2.3: Maximum Likelihood Estimation Result

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9906 (0.0017)*</td>
<td>0.9907 (0.0009)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1918 (0.0354)</td>
<td>0.1912 (0.0349)</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.9038 (0.0240)</td>
<td>0.9036 (0.0249)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.0431 (0.0067)</td>
<td>0.0430 (0.0043)</td>
</tr>
<tr>
<td>$b$</td>
<td>2.3889 (0.3081)</td>
<td>2.3955 (0.0388)</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9459 (0.0181)</td>
<td>0.9458 (0.0140)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.0207 (0.0034)</td>
<td>0.0207 (0.0046)</td>
</tr>
<tr>
<td>$z$</td>
<td>4180.35 (294.70)</td>
<td>4177.72 (137.75)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.8901 (0.0471)</td>
<td>0.8897 (0.0056)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0375 (0.0247)</td>
<td>0.0375 (0.0012)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.0081 (0.0005)</td>
<td>1.0081 (0.0002)</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.1658 (0.0467)</td>
<td>0.1658 (0.0142)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>0.8360 (0.1297)</td>
<td>0.8365 (0.0948)</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>1.2748 (0.2326)</td>
<td>1.2744 (0.3351)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0101 (0.0018)</td>
<td>0.0101 (0.0016)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5324 (0.0523)</td>
<td>0.5324 (0.0733)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.0706 (0.0223)</td>
<td>0.0708 (0.0007)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>– –</td>
<td>0.0032 (0.0274)</td>
</tr>
</tbody>
</table>

* Standard errors in parentheses.

Table 2.7 reports the out-of-sample forecast accuracy of benchmark model and alternative model. The table compares the root-mean-squared forecast errors (RMSEs) of four time series used for estimation.\(^6\) The forecast accuracy con-

\(^6\)To create the statistics shown in Table 2.7, each of the model is estimated with data from 1979:Q3 to 1998:Q4 and used to generate out-of-sample forecasts one through four quarters ahead. Next, the sample is extended to 1999:Q1, and additional forecasts are generated using the updated estimates. Continuing this way yields series of one-quarter-ahead forecasts running from 1999:Q1 through 2006:Q2, series of two-quarters ahead forecasts running from 1999:Q2 through 2006:Q2, series of three-quarters ahead forecasts running from 1999:Q3 through 2006:Q2, and
Table 2.4: Akaike and Schwarz Criterion

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>-3497.6</td>
<td>-3452.0</td>
</tr>
<tr>
<td>Alternative</td>
<td>-3495.4</td>
<td>-3447.2</td>
</tr>
</tbody>
</table>

Table 2.5: Vuong Test

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Vuong</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark=Alternative</td>
<td>3.5400</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

Table 2.6: Clarke Test

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Clarke</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark&gt;Alternative</td>
<td>37</td>
<td>0.9997</td>
</tr>
<tr>
<td>Alternative&gt;Benchmark</td>
<td>71</td>
<td>0.0069</td>
</tr>
</tbody>
</table>

firms the inference from the estimated log likelihood. Forecasts from benchmark model are more accurate for every forecast horizon of output, real money balance, inflation, and nominal interest rate.

Since $\chi$ is estimated very close to zero and the estimates of other parameters are similar, impulse responses from the two competing models are almost identical. Figure 2.2 and figure 2.3 plot impulse responses of key variables to a monetary policy shock which is a 100 basis point of contractionary shock in period $t = 1$. The benchmark model fails to generate delayed and gradual dynamics of inflation as plotted at figure 2.2. This result comes from the fact that the New Keynesian Phillips curve given by equation (2.12) lacks a term with $\hat{\pi}_{t-1}$. Although the New Keynesian Phillips curve from the alternative model includes the term, inflation fails to respond with a strong inertia because $\chi$ is estimated very small. The dynamics of aggregate demand are consistent with vector autoregression (VAR) studies. Consumption responds with hump-shape thanks to the habit persistence. Investment also responds with delay and persistence because

series of four-quarters-ahead forecasts running from 1999:Q4 through 2006:Q4, all of which can be compared to the actual data realized over those periods.
Table 2.7: Forecast Accuracy (RMSE): 1999:Q1-2006:Q2

<table>
<thead>
<tr>
<th></th>
<th>Quarters ahead</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>Benchmark</td>
<td>0.5754</td>
<td>1.1533</td>
<td>1.6742</td>
<td>2.0329</td>
</tr>
<tr>
<td></td>
<td>Alternative</td>
<td>0.7759</td>
<td>1.6530</td>
<td>2.4410</td>
<td>2.9505</td>
</tr>
<tr>
<td>Real Money Balance</td>
<td>Benchmark</td>
<td>1.3366</td>
<td>2.3162</td>
<td>3.1658</td>
<td>4.0016</td>
</tr>
<tr>
<td></td>
<td>Alternative</td>
<td>1.6692</td>
<td>2.9674</td>
<td>3.8364</td>
<td>4.8315</td>
</tr>
<tr>
<td>Inflation</td>
<td>Benchmark</td>
<td>0.1889</td>
<td>0.2174</td>
<td>0.2061</td>
<td>0.2232</td>
</tr>
<tr>
<td></td>
<td>Alternative</td>
<td>0.3934</td>
<td>0.5328</td>
<td>0.4779</td>
<td>0.5242</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>Benchmark</td>
<td>0.2881</td>
<td>0.4825</td>
<td>0.6499</td>
<td>0.7823</td>
</tr>
<tr>
<td></td>
<td>Alternative</td>
<td>0.2867</td>
<td>0.4808</td>
<td>0.6473</td>
<td>0.7802</td>
</tr>
</tbody>
</table>

investment lag is incorporated. Therefore, the two models generate hump-shaped dynamic responses of output which is one of the stylized facts from VAR studies, such as Bernanke and Gertler (1995), Christiano et al. (1996), and Bernanke and Mihov (1998).

Those empirical result implies that the benefit of the dynamic backward indexation in DSGE model needs to be reconsidered carefully. A recent empirical study of Dhyne (2005) further supports this concern. The study shows that empirical evidence is at odds with the dynamic backward indexation scheme since price changes are sizable compared to the inflation rate prevailing in each country in EU. Therefore the pricing specification in DSGE model should be improved so that it may fit macroeconomic data better and correspond with microeconomic evidence as well.

2.5 Conclusion

Sticky price model with the dynamic backward indexation is widely used for the analysis of monetary policy transmission mechanism due mainly to its merit of generating delayed and persistence inflation dynamics. However, statistical plausibility of the dynamic backward indexation has not yet been verified.
Figure 2.2: Impulse Responses (Benchmark Model)

Figure 2.3: Impulse Responses (Alternative Model)

The maximum likelihood estimation result does not support the claim that the DSGE model with the backward indexation is more suitable for macroe-
conomic policy analysis and economic forecasting. The log likelihood of the model with the backward indexation is estimated in fact slightly smaller than that of the model with the static indexation. Also, the forecast accuracy of the model with the backward indexation is worse than that with the static indexation. Regarding these findings and the empirical study of Dhyne (2005), the benefit of the dynamic backward indexation needs a reconsideration. Furthermore, it reminds us that we need to try to improve the price setting specification of DSGE model.
3

Can a Labor Hoarding Friction Explain the Delayed Effect of Monetary Policy?

3.1 Introduction

This paper investigates whether a labor hoarding friction can account for the delayed effect of monetary shocks that has been documented in empirical studies. Typically, after a positive money shock, output rises over several quarters and then declines. Walsh (2004) is the first to show that the dynamic stochastic general equilibrium (DSGE) model incorporating the aggregate labor searching friction with nominal price rigidity can generate a hump-shaped response of output to a monetary shock.\footnote{Alternatively, Christiano et al. (2005) allow for habit persistence in consumption, variable capital utilization, and investment adjustment costs as well as wage stickiness. They conclude that wage rigidity is the most important factor in explaining the hump-shaped response of output and inflation to monetary shocks.} However, he only considers the searching friction which prevents employments from being instantaneously adjusted. Firms can utilize other low adjustment cost margins to adjust their production besides the high adjustment cost margin such as employment. As shown in Bresnahan and Ramey (1994), the typical way firms can change their production without bearing high adjustment
costs is by varying the workweek of capital via overtime. Another low adjustment cost margin would be changing the effort level per worker. Hence, it seems fruitful to examine whether a labor searching friction can still lead to a hump-shaped response to a monetary shock in DSGE model when the workweek of capital and efforts are allowed to vary.

In incorporating both low adjustment cost margins (i.e., capital hours and effort) and a matching friction, we simplify the matching friction. Following Burnside and Eichenbaum (1996), we capture the labor hoarding friction by assuming that it is infinitely costly to make current-quarter adjustment on the employment instead of using a fully articulated labor searching model.

Despite the large built-in friction in adjusting employment, we find that the highest response of output to a monetary shock occurs on the impact period in the standard DSGE model allowing capital hours and effort to vary. This result suggests that the sluggish adjustment on the employment due to the matching friction might not be the main factor in explaining the delayed effect of monetary shocks.

The remainder of paper is organized as follows. Section 3.2 displays our baseline model economy. In section 3.3, we discuss our calibration procedures and present the main results. Section 3.4 discusses the possible extension that might lead employment friction to generate the hump-shaped response of output.

3.2 Model

Our baseline model modifies the standard new Keynesian model (Ireland (2001)) by integrating the variations in capital hours and efforts and a labor hoarding friction.

The economy consists of households, a central bank in charge of the conduct of monetary policy and two productive sectors: a competitive sector producing a final good and a monopolistic sector providing intermediate goods. These intermediate goods are the only input necessary for the production of the final good, which can be used for consumption or investment. Intermediate goods are
produced by combining capital services and labor inputs. Because it is infinitely costly to make current-quarter adjustment on the employment by assumption, firms in the intermediate good sector must choose the size of employment before observing monetary shocks. Even though firms in the intermediate goods sector cannot change employment in response to monetary shocks, they can adjust their production through varying the workweek of capital and effort instantaneously. However, there are costs associated with increasing the workweek of capital and effort. Firms must compensate workers for an increasing disutility associated with a longer nonstandard workweek and greater efforts. The specification governing the disutility due to the longer workweek and greater effort is based on that of Bils and Cho (1994). Obviously, compensating worker disutility for a longer non-standard workweek is not the only cost to increasing capital hours. An accelerated depreciation is thus considered as another important cost. As a robustness check, we extend our baseline model to incorporate these two costs to increasing capital hours as Kim (2006) did.

3.2.A Representative Household

The economy is populated by a continuum of identical households of unit measure. Their momentary utility function is given by

\[
u(C_t, M_t/P_t, N_{t-1}, h_t, e_t) = \left(\frac{\gamma}{\gamma - 1}\right) \ln \left[\frac{C_t^{\gamma-1}}{\gamma} + \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}\right] - V(N_{t-1}, h_t, e_t)
\]

(3.1)

where \(C_t, N_{t-1}, h_t, e_t, \text{ and } M_t/P_t\) are consumption, the number of workers, hours per worker, effort per hour of work and real balances. \(V(N_{t-1}, h_t, e_t)\) describes the disutility of providing labor services. Following Bils and Cho (1994), we specify that

\[
V(N_{t-1}, h_t, e_t) = \left[\theta_1 \frac{N_{t-1}^{1+\nu}}{1+\nu} + \theta_2 N_{t-1} \frac{h_t^{1+\chi}}{1+\chi} + N_{t-1} h_t \frac{e_t^{1+\zeta}}{1+\zeta}\right]
\]
The first component of \( V(N_{t-1}, h_t, e_t) \) represents the cost of sending \( N_{t-1} \) member of the households to work in a period \( t \), even if hours worked are arbitrarily small. It may be interpreted as costs for commuting or costs incurred due to having fewer people available for home production. The second component reflects the disutility of working \( h_t \) hours per period associated with reduced leisure and longer work during nonstandard hours. Finally, the third term reflect disutility from exerting effort.

Next, we describe the sources of funds that can be used to purchase consumption goods and assets. Households enter each period holding an \( M_{t-1} \) amount of money stock and amount \( B_{t-1} \) of a risk free discount bond. Households receives a lump-sum nominal transfer \( T_t \) from the monetary authority and an amount \( D_t \) corresponding to intermediate firms’ profits. Finally, households receive a (real) total wage bill by providing labor services from intermediate goods firms. We assume that the equilibrium wage bill is determined as Bils and Cho (1994) suggest: households present their employer with a wage bill that takes the form of \( V(N_{t-1}, h_t, e_t) \) and allow firms to freely choose the size of employment, hours per worker and effort. Hence, the equilibrium (real) total wage, \( W_t \), takes the following form:

\[
W_t = \left[ \theta_1 \frac{N_{t-1}^{1+\nu}}{1+\nu} + \theta_2 N_{t-1} \frac{h_t^{1+\chi}}{1+\chi} + N_{t-1} h_t e_t^{1+\zeta} \right]
\]

Households use their funds to purchase an amount \( C_t \) of the finished good at a nominal price \( P_t \). Households purchase \( B_t \) risk-free bonds at an unitary cost of \( 1/R_t \), where \( R_t \) is the gross nominal rate of return between period \( t \) and \( t + 1 \). The following relation, which represents households’ budget constraint, must hold at every period:

\[
C_t + \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - \frac{M_t}{P_t} = W_t + \frac{T_t}{P_t} + \frac{D_t}{P_t} \tag{3.2}
\]

This states that consumption expenditures plus asset accumulation must equal disposable income.

Household’s preferences are given by the life-time utility function \( U_0 \).

\( ^2 \) The subscript \( t-1 \) is due to the assumption that the size of employment is predetermined.
This function represents the expectation of the discounted sum of monetary utility function conditional on the information set at date $t = 0$.

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, M_t/P_t, N_{t-1}, h_t, e_t)$$ (3.3)

where $\beta$ denotes households’ discount factor.

Household’s optimal behavior involves choosing a sequence $\{C_t, M_t, B_t\}$ that maximizes their life-time utility function (3.3) subject to the budget constraint (3.2).

### 3.2.B The Finished Goods-Producing Firm

The representative finished good-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$ to produce $Y_t$ units of the final good using the technology

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{1-\varepsilon}{\varepsilon}} di \right]^{\frac{\varepsilon}{1-\varepsilon}}$$ (3.4)

Given that the price of intermediate good $i$ is $P_t(i)$, the finished good sells at the nominal price $P_t$. The finished goods-producing firm chooses $Y_t$ and $Y_t(i)$ to maximize its profits,

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di$$ (3.5)

subject to the constraint imposed by (3.4). The first-order conditions for this problem imply that the optimal level of demand for a intermediate good $i$ is given by

$$Y_t(i) = \left[ P_t(i)/P_t \right]^{-\varepsilon} Y_t$$ (3.6)

Since the firm is operating in a competitive market, the zero-profit condition determines $P_t$ as a Dixit-Stiglitz aggregator given by

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}$$ (3.7)
3.2.C The Intermediate Goods-Producing Firm

There is a continuum of monopolistically competitive firms, each producing an intermediate good. The representative intermediate goods firm produces its output from effective units of labor, \( L_t(i) \) and effective units of capital, \( K'_t(i) \).

We allow three dimensions of effective labor units: employment, \( N_t \), hours per worker, \( h_t \) and effort per hour at work, \( e_t \). We assume that it is infinitely costly to make current-quarter adjustment on the employment. It implies that intermediate firms start at each period \( t \) with a predetermined size of employment. \( L_t(i) \) is therefore given by

\[
L_t(i) = h_t(i) e_t(i) N_{t-1}(i)
\]

Following Bils and Cho (1994), we assume that if a worker works longer hours or works at a more rapid physical pace, the utilization of the capital he operates will increase proportionately. \( K'_t \) is therefore given by

\[
K'_t(i) = h_t(i) e_t(i) K_{t-1}(i)
\]

where \( K_{t-1} \) denotes the capital stock at the end of period \( t - 1 \).

We assume that the technology available to intermediate goods firms exhibits a constant-returns-to-scale Cobb-Douglas production function. The output of a representative intermediate goods firm at period \( t \) is therefore given by

\[
Y_t(i) = A_t (K'_t)^\alpha (L_t)^{1-\alpha} = A_t h_t(i) e_t(i) [K_{t-1}(i)]^\alpha [N_{t-1}(i)]^{1-\alpha} \tag{3.8}
\]

The parameter \( \alpha \in (0,1) \) and \( A_t \) represents an aggregate productivity parameter which follows the autoregressive process

\[
\ln (A_t) = (1 - \rho_A) \ln (A) + \rho_A \ln (A_{t-1}) + \varepsilon_{A_t} \tag{3.9}
\]

where \( \varepsilon_{A_t} \) is a technology shock with standard deviations \( \sigma_A \).

A representative intermediate goods firm choose a sequence of \( \{K_t(i), I_t(i), N_t(i), h_t(i), e_t(i), P_t(i)\} \) that maximizes the discounted stream of expected nominal profits \( D_t \):
subject to the requirement that it satisfies the representative final goods firm’s demand (3.6) and the constraint imposed by production function (3.8). In (3.10), \( \lambda \) is the Lagrange multiplier on the budget constraint from the representative household’s problem.

Real profits of a typical intermediate goods firm at the beginning of any period \( t \), \( \frac{D_t(i)}{P_t} \), are defined as

\[
\frac{D_t(i)}{P_t} = \frac{P_t(i) Y_t(i)}{P_t} - \left( \theta_1 \frac{N_{t-1}^{1+\nu}(i)}{1+\nu} + \theta_2 N_{t-1} (i) \frac{h_{t}^{1+\chi}(i)}{1+\chi} + N_{t-1} (i) h_{t} (i) \frac{e_{t}^{1+\varsigma}(i)}{1+\varsigma} \right) \text{real total wage bill}
\]

\[- I_t(i) - AC_{k,t}(i) - AC_{p,t}(i) \]  

(3.11)

where

\[ I_t(i) = K_t(i) - (1-\delta)K_{t-1}(i) \]  

(3.12)

is investment in capital goods, with \( \delta \) being the rate of depreciation. The terms \( AC_{k,t} \), \( AC_{p,t} \) in (3.11) represent a capital adjustment cost and a cost of changing the nominal price of the goods it produces, measured in terms of the finished goods:

\[ AC_{k,t}(i) = \frac{\phi_k}{2} \left( \frac{I_t(i)}{K_{t-1}(i)} - \delta \right)^2 K_{t-1}(i) \]

(3.13)

\[ AC_{p,t}(i) = \frac{\phi_p}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 Y_t \]

(3.14)

where \( \pi \) is the steady-state rate of inflation.

### 3.2.D The Monetary Authority

At each period of time, the monetary authority supplies the money stock which is growing at a rate

\[ \mu_t = \frac{M_t}{M_{t-1}} \]  

(3.15)
It is assumed that the monetary authority follows an exogenous policy rule:

\[ \mu_t = (1 - \rho_\mu) \mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \quad (3.16) \]

where \( \rho_\mu \) is the persistence parameter and serially uncorrelated policy shock \( \varepsilon_{\mu,t} \) are normally distributed with mean zero and standard deviations \( \sigma_{\mu} \).

### 3.3 Results

#### 3.3.A Parameter values

Table 3.1 summarizes the values assigned to our baseline model’s structural parameters. Specifically, the discount factor (\( \beta \)) is set to 0.99 so that the steady-state real interest rate is 3%. Following the estimates of Ireland (2001), we set the elasticity of money demand to the nominal interest rate (\( \gamma \)) to 0.1184. The capital’s share on aggregate income (\( \alpha \)) is set to 0.338. Following Ireland (2001), we set the parameters of the price adjustment costs function (\( \phi_p \)) and capital adjustment costs (\( \phi_k \)) to be 77.1 and 10 respectively. The elasticity of intermediate goods (\( \varepsilon \)) is set to 6 so that the steady-state markup of the intermediate-good producing firms is 1.2. The parameter values dictating the responsiveness of the employment, workweek of capital (hours per worker) and efforts per hour are taken from Bils and Cho (1994). The values for \( \nu, \chi, \) and \( \varsigma \) are 1.57, 2 and 3 respectively.

The steady state value of the fraction of hours beyond a standard 40-hour workweek is set to 0.26, taken from Ramey and Shapiro (1998). This implies a 50.4-hour workweek of capital in the steady state. Normalizing a 40-hour workweek to unity, we set the steady state value of the workweek of capital, \( h \) to be 1.26. The steady state value of employment, \( N \) is set to ensure that the steady state ratio of total hours worked to the total time endowment of the household\(^3\) is 0.24. The resulting value of \( N \) is 0.56. Finally, the scale coefficients (\( \theta_1, \theta_2 \)) in the utility function are obtained from solving the equilibrium conditions satisfied in the steady-state.

\(^3\)The time endowment available to household is normalized to 2.63.
Table 3.1: Parameter values in a baseline model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal discount rate</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Elasticity intermediate good</td>
<td>$\epsilon$</td>
<td>6</td>
</tr>
<tr>
<td>Price adjustment costs</td>
<td>$\phi_p$</td>
<td>77.1</td>
</tr>
<tr>
<td>Capital adjustment costs</td>
<td>$\phi_k$</td>
<td>10</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.018</td>
</tr>
<tr>
<td>Elasticity money demand</td>
<td>$\gamma$</td>
<td>0.1184</td>
</tr>
<tr>
<td>Parameter governing the disutility of employment</td>
<td>$\nu$</td>
<td>1.57</td>
</tr>
<tr>
<td>Parameter governing the disutility of hours per worker</td>
<td>$\chi$</td>
<td>2</td>
</tr>
<tr>
<td>Parameter governing the disutility of efforts per worker</td>
<td>$\varsigma$</td>
<td>3</td>
</tr>
<tr>
<td>Steady-state inflation rate</td>
<td>$\pi$</td>
<td>1.016</td>
</tr>
<tr>
<td>Mean money growth rate</td>
<td>$\mu$</td>
<td>1.016</td>
</tr>
<tr>
<td>Steady-state participation rate</td>
<td>$n$</td>
<td>0.5</td>
</tr>
<tr>
<td>Steady-state hours per worker</td>
<td>$h$</td>
<td>1.2</td>
</tr>
<tr>
<td>The scale coefficient in the utility function</td>
<td>$\theta_1$</td>
<td>4.27</td>
</tr>
<tr>
<td>The scale coefficient in the utility function</td>
<td>$\theta_2$</td>
<td>4.46</td>
</tr>
<tr>
<td>Persistence money growth</td>
<td>$\rho_\mu$</td>
<td>0.5</td>
</tr>
<tr>
<td>Standard deviation money growth</td>
<td>$\sigma_\mu$</td>
<td>0.003</td>
</tr>
</tbody>
</table>

3.3.B Dynamic responses to a monetary shock

Figure 3.1 displays the response of key macroeconomic variables to a positive monetary growth shock. It shows that the employment friction is not able to generate the hump-shaped response of output to a monetary shock in a standard DSGE model. As is clearly shown in the response of hours per worker and effort, even though firms cannot change the size of employment due to the matching friction, they are able to meet the initial increase in demand induced by a positive monetary shock by raising the workweek of capital and effort.

In the subsequent period when the size of employment is allowed to change, firms substitute away from the workweek of capital and efforts toward employment. The reason for this is that the model is parameterized to incorporate the idea that employment is high adjustment-low marginal cost margin whereas capital hours and efforts are low adjustment-high marginal cost margin. The parameter governing the disutility of employment ($\nu = 1.57$) is smaller than those governing the disutility of hours per worker and efforts ($\chi = 2$, $\varsigma = 3$). Hence, when firms can change the size of employment, they find it less costly to adjust
Following Kim (2006), we now investigate whether incorporating another cost to increasing the workweek of capital would change the dynamic responses. Another important cost to increasing capital hours considered in the literature is an accelerated depreciation. To capture this depreciation cost, we modify the capital accumulation equation\(^4\).

\[
K_t = (1 - \delta(h_t))K_{t-1} + I_t
\]  

where \(\delta(h_t)\) denotes the depreciation rate. The rate of depreciation depends on the capital utilization rate, reflecting a ‘user-cost’. It is modeled as an increasing, convex function of capital utilization. The specific functional form of \(\delta(u_t)\) is

\[
\delta(h_t) = \frac{h_t^\omega}{\omega}, \quad \omega > 1
\]  

\(^4\)Note that for simplicity, efforts per hour are assumed to be constant.
Firms now have to bear two types of cost to extending capital hours: worker disutility for a longer workweek and an accelerated depreciation. To solve the model, we assume that the quarterly depreciation rate is 0.018 and the elasticity of depreciation ($\omega$) is set to 2, as suggested by Basu and Kimball (1997). The result presented below is not sensitive to the choice of $\omega$. Figure 3.2 displays the response of key macroeconomic variables to a positive monetary growth shock when both utility and depreciation costs are incorporated. It clearly shows that even when depreciation cost is added to our baseline model, the employment friction is not able to produce a hump-shaped response of output to a monetary shock.

### 3.4 Conclusion

This paper has shown that when the workweek of capital and efforts are allowed to vary, the employment lag itself cannot generate a hump-shaped response
of output to a monetary shock. The reason for this is that despite the fact that the size of employment is predetermined, firms can rely on other low adjustment cost margins such as the workweek of capital and efforts to meet the increase in demand due to a positive monetary shock.

However, it should be noted that our result here is subject to one caveat. We have not considered another important margin that firms can use to meet the increase in demand induced by a positive monetary shock: inventory. Introducing inventory into our model might lead to a hump-shaped response. To clarify this point, suppose that some fraction of the increased demand can be met by adjusting inventory. Firms then have a stronger incentive to defer the change in labor input until they adjust the size of employment rather than relying on the contemporaneous change in labor input (i.e., hours per worker and efforts per hour). As shown in section 3.3.B, it is because employment adjustment is less costly than hours per worker and efforts adjustments. This will lead to a smaller response of hours per worker and efforts per hour and a greater response of the size of employment compared to our baseline model. This mechanism will in turn help to generate a hump-shaped response of output. Hence, it seems to constitute an exciting avenue for future research to investigate whether incorporating inventory margin would lead the employment friction to generate the delayed effect of monetary policy shocks.

Acknowledgement

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References


