Search for Supersymmetry in the one-Lepton Channel at the LHC

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by

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To my parents
ABSTRACT OF THE DISSERTATION

Search for Supersymmetry in the one-Lepton Channel at the LHC

by

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Doctor of Philosophy, Graduate Program in Physics
University of California, Riverside, June 2013
Dr. John Ellison, Chairperson

A search for supersymmetry was performed in events with one electron or muon, jets, and missing transverse momentum. The dataset was the first 5.0 fb$^{-1}$ of $\sqrt{s} = 7$ TeV proton-proton events from the Large Hadron Collider, as measured by the CMS detector in 2011. A data-driven method was used to estimate the background at high values of the transverse mass, where the suppression of the Standard Model backgrounds allows for relatively loose cuts on the jet activity. No excess above the expected background was observed, so a limit was set in the $m_0 - m_{1/2}$ plane of the CMSSM at $\tan\beta = 10$, $A_0 = 0$ and $\text{sgn}(\mu) = +1$. 

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Chapter 1

Introduction

The Standard Model (SM) of particle physics is the most successful attempt yet at describing the fundamental units of matter and the ways in which that matter interacts. The SM is not, however, designed to address gravity, dark energy, indirect observations of dark matter, and the observed nonzero masses of the neutrinos. There are also some unanswered theoretical questions about the reason for a 3-family structure of the fermions and the hierarchy in masses between the 3 generations, the disparity between the strengths of the weak, electromagnetic, and strong forces, and the Higgs mass hierarchy problem that concerns the large difference between the electroweak scale and the Planck scale.

By assuming a symmetry between bosons and fermions, supersymmetry directly address the Higgs mass hierarchy problem. It also lead to an unexpected unification between the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge coupling constants. When supersymmetry includes the symmetry of R-parity, which could itself be a consequence of a larger gauge symmetry that has unified couplings, the theory produces a dark matter candidate.

This dissertation first describes the SM and supersymmetry, then reports on a search for supersymmetric particles in proton-proton collisions. These collisions were produced at
Figure 1.1: A possible supersymmetry production mechanism in the one-lepton channel. Here, gluons from the proton-proton collision combine into a high energy gluon that produces a pair of quark superpartners (squarks), resulting in a decay chain with charginos ($\chi^\pm$), neutralinos ($\chi^0$), jets, and a lepton and accompanying neutrino from a $W$ boson decay. The $W$ might be real or virtual, depending on the model.

a center-of-mass energy of $\sqrt{s} = 7$ TeV by the Large Hadron Collider (LHC) and measured by the CMS experiment at CERN. Events containing one lepton (electron or muon) are analyzed using the transverse mass variable $M_T$ in order to separate the SM background from the signature expected from the Minimal Supergravity model. An example of a supersymmetric production mechanism in the one-lepton channel is shown in Fig. 1.1.

1.1 Outline of the Standard Model

There are two types of fundamental fermions in the Standard Model: leptons and quarks. Quarks carry both color charge and electric charge, so they interact via all of the SM forces: the strong, weak, and electromagnetic forces. There are 6 quarks in total, which are arranged into 3 families (also called generations) of doublets, as tabulated in Fig. 1.2. The first generation contains the two lightest quarks, called the up ($u$) and down ($d$) quarks. These carry fractional amounts of the standard unit of electric charge, and differ in charge
Figure 1.2: Fundamental fermions, gauge bosons, and Higgs boson of the Standard Model. There is an antimatter partner for each of the fermions, making a total of 12 quarks/antiquarks and 12 leptons/antileptons.

by 1 unit \((+2/3)e\) and \((-1/2)e\) for \(u\) and \(d\) respectively). The second generation is made up of the more massive charm and strange quarks, and the yet more massive 3rd generation quarks are the top and bottom quarks. Leptons do not have color charge, so they interact via the weak and electromagnetic forces only. Each generation has a massive charged lepton and a nearly massless uncharged neutrino.

Every fermion listed in Fig. 1.2 has an antimatter partner, which has the same mass and opposite charge.

Interactions between the spin-1/2 fermions are mediated by spin-1 gauge bosons. The eight electrically neutral, massless gluons carry color charge and represent the gauge bosons of the quantum chromodynamic (QCD) \(SU(3)_C\) symmetry. The massless photon \(\gamma\) and
the massive $W^\pm$ and $Z$ bosons are the mediators of the electroweak interactions. In the mechanism of electroweak symmetry breaking, the massless gauge bosons of the $SU(2)_{\text{Left}}$ (weak isospin) and $U(1)_{\gamma}$ (hypercharge) symmetries combine to form the massive $W^\pm$ and $Z$ bosons due to their interaction with the spin-0 Higgs field; the Higgs field also provides mass to all the massive fermions in the theory.

### 1.2 Development of the Standard Model

The quark model was devised as a way to explain the zoo of protons, neutrons, pions, and strangeness-carrying particles that were experimentally known at the time. Beginning with the discovery of the neutron, it was known that its mass and strong interactions were very similar to that of the proton. Therefore it was natural to try to arrange the proton and neutron into an “isospin” doublet, analagous to a spin doublet; the total isospin is labeled $I$, and the 3rd component of isospin is $I_3$. In this picture, the nucleon is viewed as a state that is a mixture of the proton ($|I = 1/2, I_3 = 1/2\rangle$) and neutron ($|I = 1/2, I_3 = -1/2\rangle$) basis states.

The discovery of three spin-0 pions in cosmic rays ($\pi^\pm$ and $\pi^0$) [1], which are nearly degenerate in mass and strongly interacting, invited the placement of the pions into an $I = 1$ (triplet) isospin representation of $SU(2)$.

A new conserved quantum number called strangeness was invoked to describe the production of strange particles, as in

\[ \pi^- p \rightarrow K^0 \Lambda^0 , \quad (1.1) \]

which were always produced in pairs. These were called “strange” particles because they
Table 1.1: Quantum numbers of the up, down and strange quarks.

<table>
<thead>
<tr>
<th>Spin</th>
<th>B</th>
<th>Q</th>
<th>$I_3$</th>
<th>S</th>
<th>Y</th>
</tr>
</thead>
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<tr>
<td>u</td>
<td>1/2</td>
<td>1/3</td>
<td>2/3</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>1/2</td>
<td>1/3</td>
<td>-1/3</td>
<td>-1/2</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>1/2</td>
<td>1/3</td>
<td>-1/3</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

were very long-lived compared to particles that decay via strong interactions (they decayed via the weak force). These particles were assigned $S(\Lambda^0) = -1$ and $S(K^0) = 1$.

Following the discovery of strangeness, the Gell-Mann-Nishijima formula \[2\] highlighted an interesting relationship between the electric charge, the 3rd component of isospin, baryon number, and strangeness:

$$Q = I_3 + \frac{B + S}{2} = I_3 + \frac{Y}{2} \quad (1.2)$$

where $Y = B + S$ was called the hypercharge.

In Fig. 1.3, we see that patterns emerge when particles of the same spin and baryon number are plotted on a graph of $Y$ vs. $I_3$. The spin-3/2 $\Omega^-$ shown at the bottom of Fig. 1.3(d) was predicted before its discovery \[3\], based on the anticipation of the pattern being completed. This discovery solidified acceptance of the quark model, which proposed that all of these patterns of bound states could be produced as products of up, down, and strange quarks with the quantum numbers given in Table 1.1 \[4\][5].

In an early study of a kind of supersymmetry in the context of hadronic physics, Miyazawa studied a multiplet containing both baryons and mesons; these were in the same multiplet in the sense it was a representation that was invariant under the simultaneous interchange of isospin and ordinary spin and of hypercharge and baryon number \[6\].
Figure 1.3: $Y$ vs. $I_3$ for (a) spin-0 mesons, (b) spin-1 mesons, (c) spin-1/2 baryons, (d) spin-3/2 baryons. The $N$ baryons are also known as delta baryons with the symbol $\Delta$.

In the quark model, the 3-quark flavor symmetry $SU(3)_f$ contains the $u$, $d$, and $s$ quarks,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad (1.3)$$

By forming various products with this $uds$ representation of $SU(3)_f$, along with its complex conjugate representation of anti-quarks, all of the known mesons ($3 \otimes \bar{3} = 8 \oplus 1$) and baryons ($3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$) at the time could be found. This symmetry was useful in illucidating the quark model, but unlike the $SU(2)_f$ symmetry of just the $u$ and $d$ quarks, it was not even an approximate symmetry because the $s$ quark is much more massive than the $u$ and $d$ quarks.

An exact $SU(3)$ symmetry appeared with the discovery of the color gauge symmetry
SU(3)\(_C\) associated with quantum chromodynamics (QCD). Prior to the discovery of QCD, the existence of the \(\Delta^{++}\) should have been impossible, because the fully spin-up \(|\frac{3}{2}, \frac{3}{2}\rangle\) state \(uuu\) appears to be totally symmetric, despite this being a spin-3/2 particle. This would violate the Pauli exclusion principle, and therefore hinted at the existence of a new quantum number which could further differentiate the quarks, which was determined to be the color quantum number of QCD \([7]\). Other evidence for a color quantum number was the number of colors inferred from the total cross-section of \(e^+e^-\) to hadrons, the fact that mesons were always observed to be \(qq\) bound states and never \(qq\), and the existence of baryons which were allowed to have three quarks \(qqq\) or anti-quarks \(\bar{q}\bar{q}\bar{q}\), but not other combinations. The \(SU(3)_C\) gauge symmetry applied to spin-1/2 quarks implies the QCD Lagrangian

\[
\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (\gamma^\mu D_\mu)_{ij} - m \delta_{ij} \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \tag{1.4}
\]

where

\[
G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu \tag{1.5}
\]

This Lagrangian contains \(3^2 - 1 = 8\) gluons, quark-gluon interactions, and 3- and 4-gluon interactions.

The weak force interactions were initially understood as 4-point interactions by Fermi \([8]\). A typical example is the beta decay of a neutron, where the neutron decays to a proton, electron and anti-neutrino. In the Wu Experiment \([9]\) and processes like \(\pi^+ \rightarrow \mu^+ \nu_L\) \([10]\), it was observed that the weak interaction maximally violates parity conservation: the \(W\) boson only couples to left-handed (negative helicity) particles and right-handed (positive helicity) anti-particles. Helicity is the dot product between a particle’s spin and momentum. In \(\pi^-\) decay, it was observed that the muon is always left-handed. Mathematically, the decay could
be formulated as having a vector minus axial vector \((V - A)\) interaction: 
\[
V - A = \gamma^\mu - \gamma^\mu \gamma^5
\]
is a projection operator that converts a state which has left- and right-handed components into a state with only the left-handed component.

The 4-point interaction picture was eventually replaced by a theory of electroweak interactions mediated by \(W^\pm\) bosons, the \(Z\) boson, and the photon. These gauge bosons and their many interactions can be understood from the theory of electroweak symmetry breaking (to be illustrated in Sec. 2.2), which describes the breaking of \(SU(2)_L \times U(1)_Y\) to the electromagnetic symmetry \(U(1)_{em}\) by the Higgs mechanism.
Chapter 2

Supersymmetry Theory and Phenomenology

In this chapter, a more top-down picture of the Standard Model is outlined. The Minimally Supersymmetric Standard Model (MSSM) is motivated and constructed, and the phenomenology of Minimal Supergravity is described.

2.1 Local Gauge Symmetry

In this section we give a brief outline of the gauge theory of Quantum Electrodynamics (QED) and the Noether procedure, which illustrates how a spin-1/2 particle charged under the U(1) symmetry is related to a spin-1 gauge boson. A free Dirac field Lagrangian can be written as

\[ \mathcal{L} = -\bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi} \psi. \]  

(2.1)

This Lagrangian involving a spin-1/2 field is invariant under a U(1) transformation in global phase (\( \psi \rightarrow \psi' = e^{iq\xi} \psi \)), but not under local changes in phase (\( \xi(x) \)), due to the derivative
term:

$$\partial_\mu \psi \rightarrow (\partial_\mu \psi(x))' = \partial_\mu (\psi(x))$$

$$= e^{iq\xi(x)} (\partial_\mu \psi(x) + iq\psi(x)\partial_\mu \xi(x))$$

We seek a modified derivative $D_\mu$ that transforms according to

$$D_\mu \psi(x) \rightarrow (D_\mu \psi(x))' = e^{iq\xi(x)} (D_\mu \psi(x)) .$$

Recognizing from Eq. (2.2) that $D_\mu$ must contain some compensating term to remove the phase dependence, we define

$$D_\mu \psi(x) = (\partial_\mu - iqA_\mu(x))\psi(x) .$$

Therefore,

$$D_\mu \psi \rightarrow (D_\mu \psi)' = (\partial_\mu \psi)' - iq(A_\mu \psi)'$$

$$= e^{iq\xi(x)} (\partial_\mu + iq\partial_\mu \xi \psi - iqA_\mu' \psi))$$

For this to satisfy Eq. (2.3) the transformation rule for $A_\mu$ must be

$$A_\mu \rightarrow A_\mu' = A_\mu + \partial_\mu \xi .$$

We call $A_\mu(x)$ the gauge field of QED. The spin-1 particle associated with this field is the photon.

New covariant quantities can be created by combining previous covariant quantities,
as in the antisymmetric product

\[ [D_\mu, D_\nu]\psi = D_\mu(D_\nu\psi) - D_\nu(D_\mu\psi) \]

\[
= -iqF_{\mu\nu}\psi
\]

where

\[ F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \]

is called the field strength tensor. Under the gauge transformation Eq. (2.6), one can show that \( F_{\mu\nu} \) is also gauge invariant. If any gauge invariant and Lorentz invariant term related to our quantum field can enter the Lagrangian, then the field strength tensor can be included in the full Lagrangian:

\[
\mathcal{L} = -\bar{\psi}iD\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\]

\[
= -\bar{\psi}\slashed{D}\psi - m\bar{\psi}\psi + iqA_\mu\bar{\psi}\gamma^\mu\psi - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2
\]

The requirement of local U(1) phase invariance of the complex quantum field \( \psi \) thus necessitates the introduction of a gauge field \( A_\mu \), which interacts with currents of the matter field (the current is \( j^\mu = \bar{\psi}\gamma^\mu\psi \)). The form of the interactions is the same as that between charged currents and the vector potential in the Lagrangian formulation of classical electrodynamics.

This procedure of deriving the locally invariant theory from the global theory is called the Noether procedure [11], particularly when it is written in terms of the action rather than the Lagrangian. Under a local phase change \( (\xi(x)) \), the free Dirac field action changes by

\[
\delta S_0 = \int d^4x \, \bar{\psi}\gamma^\mu\psi \partial_\mu\xi = \int d^4x \, j^\mu \partial_\mu\xi
\]
where

\[ j^\mu = \bar{\psi} \gamma^\mu \psi \]  

(2.11)

The action is made invariant by adding a gauge field (that transforms according to Eq. (2.6)) that couples to the current

\[ S = S_0 - \int d^4x \, j^\mu A_\mu = \int d^4x \, i\bar{\psi} \gamma^\mu (\partial_\mu + iA_\mu) \psi. \]  

(2.12)

### 2.2 Gauge Theory in the Standard Model

The full fermionic particle content of the Standard Model can be written as 3 copies (families) of left-handed Weyl fields in the representation \((1, 2, -\frac{1}{2}) \oplus (1, 1, +1) \oplus (3, 2, +\frac{1}{6}) \oplus (3, 1, -\frac{2}{3}) \oplus (3, 1, +\frac{1}{3})\) (the lepton fields are \(L, \bar{e}\), and the quark fields are \(Q, \bar{u}, \bar{d}\)), and a complex scalar Higgs field in the representation \((1, 2, -\frac{1}{2})\) [12]. The lepton and quark terms could alternatively be written as having left-handed doublets and right-handed singlets: \(L_L, e_R, Q_L, u_R, \) and \(d_R\). The gauge bosons will be a consequence of the local gauge invariance of the fermions under the color, weak isospin, and hypercharge gauge symmetries \(SU(3)_C \times SU(2)_L \times U(1)_Y\). The Lagrangian includes all possible interactions that preserve gauge and Lorentz invariance, and have mass dimension of four or less. The quantum action, and thus probabilities, calculated under the theory can be made manifestly invariant under rotations in color, weak isospin, and hypercharge if the Lagrangian is constructed from gauge covariant derivatives. The Higgs part of the full Lagrangian is

\[ \mathcal{L} = -\frac{1}{2} (D^\mu \varphi)_i (D_\mu \varphi)^i - V(\varphi) \]  

(2.13)
where $\varphi$ is the Higgs field and $V$ is the Higgs potential (which is constructed from gauge invariant products of $\varphi$). Local gauge invariance is achieved by using the covariant derivative,

$$
(D_\mu \varphi)_i = \partial_\mu \varphi_i - i[g_2 A^a_\mu T^a + g_1 B_\mu Y]_i \bar{\varphi} \varphi_j,
$$

(2.14)

where the SU(2) generators are $T^a = \frac{1}{2} \sigma^a$, and the hypercharge operator for the Higgs field is $Y = (-\frac{1}{2}) I$. The signs of constants in the potential term can be chosen such that the Higgs field has a potential with a nonzero vacuum expectation value (VEV),

$$
V(\varphi) = \frac{1}{4} \lambda \left( \varphi^\dagger \varphi - \frac{1}{2} v^2 \right)^2,
$$

(2.15)

This is called the “Mexican hat potential”. A nonzero VEV means that the potential $V(\varphi)$ is minimized for some nonzero expectation value of the Higgs field. The VEV can be placed in the first component and made real, as in

$$
\langle 0 | \varphi(x) | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}
$$

(2.16)
The mass term is part of the kinetic term $-(D^\mu \varphi)^\dagger D_\mu \varphi$:

$$
\mathcal{L}_{\text{mass}} = -\frac{1}{8} v^2 \begin{pmatrix} 1, & 0 \end{pmatrix} \begin{pmatrix} g_2 A_\mu^3 - g_1 B_\mu & g_2 (A^1_\mu - i_\mu^2) \\ g_2 (A^1_\mu + i A^2_\mu) & -g_2 A_\mu^3 - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\frac{1}{8} g_2^2 v^2 \begin{pmatrix} 1, & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{c_W} Z_\mu & \sqrt{2} W_+^\mu \\ \sqrt{2} W_-^\mu & -\frac{1}{c_W} A_\mu \end{pmatrix}^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}
$$

$$
= -(G_2 v/2)^2 W_+^\mu W_\mu^- - \frac{1}{2} (g_2 v/2 c_W)^2 Z_\mu^\mu Z_\mu
$$

$$
= -M_W^2 W_+^\mu W_\mu^- - \frac{1}{2} M_Z^2 Z_\mu^\mu Z_\mu
$$

where

$$
\theta_W \equiv \tan^{-1}(g_1/g_2),
$$

$$
W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (A^1_\mu \mp i A^2_\mu),
$$

$$
Z_\mu \equiv c_W A_\mu^3 - s_W B_\mu,
$$

$$
A_\mu \equiv s_W A_\mu^3 + c_W B_\mu,
$$

$$
M_W = g_2 v/2,
$$

$$
M_Z = M_W / \cos \theta_W.
$$

Note that the SU(2) and U(1) gauge fields have mixed to form the massive $W_\mu^\pm$ and $Z_\mu$ gauge bosons, and the massless photon field $A_\mu$. Three of the Higgs field degrees of freedom have gone into providing longitudinal polarization components to the SU(2) × U(1) gauge fields, while the remaining degree of freedom is a real-valued field that expands $\varphi(x)$ about
Figure 2.1: For a certain choice of the Higgs potential shape, the ground state corresponds to a field with a nonzero vacuum expectation value. This constant VEV provides a mass term to the massive gauge bosons and massive fermions uniformly in spacetime. Fluctuations in the radial direction correspond to the dynamical field of the massive spin-zero Higgs boson. Figure from [13].

the ground state:

\[
\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}.
\]  

(2.24)

The Higgs potential is then

\[
V(\varphi) = \frac{1}{4} \lambda v^2 H^2 + \frac{1}{4} \lambda v H^3 + \frac{1}{16} \lambda H^4
\]  

(2.25)

and the Higgs boson mass is \( m_H = \sqrt{\frac{1}{2} \lambda v^2} \). The kinetic terms for the gauge fields are

\[
\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_a^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}.
\]  

(2.26)
When this is written in terms of a new covariant derivative
\[ D_\mu \equiv \partial_\mu - ig_2 A_\mu^2 \]
\[ = \partial_\mu - ig_2 (s_W A_\mu + c_W Z_\mu) \]
\[ = \partial_\mu - ie (A_\mu + \cot \theta W Z_\mu), \tag{2.27} \]

we see that the manifest SU(2)_L × U(1)_Y symmetry has been broken and replaced by the electromagnetic symmetry U(1)_{em} (where the unified coupling constant is \( e \equiv g_2 \sin \theta_W \) and the gauge fields are now the photon and the Z boson). More details of the process of electroweak symmetry breaking can be found in the original papers by Englert, Brouth, Higgs, Guralnik, Hagen, and Kibble [14][15][16]. The many electroweak interactions built into Eq. (2.26) become re-expressed as

\[ L_{WZA} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} - D^{\mu} W^{-\nu} D_\mu W^{+}_\nu + D^{\mu} W^{-\nu} D_\nu W^{+}_\mu + ie (F^{\mu\nu} + \cot \theta_W Z^{\mu\nu}) W^{+}_\mu W^{-}_\nu \]
\[ - \frac{1}{2} (e^2 / \sin^2 \theta_W) (W^{+\mu})(W^{+\mu} W^{+\nu} W^{+\nu} - W^{+\mu} W^{+\nu} W^{+\nu}) \]
\[ - (M_W^2 W^{+\mu} W^{+\mu} + \frac{1}{2} M_Z^2 Z^{\mu} Z_{\mu}) (1 + v^{-1} H)^2 \]
\[ - \frac{1}{2} \partial^\mu H \partial_\mu H - \frac{1}{2} m_H^2 H^2 - \frac{1}{2} v^{-1} H^3 - \frac{1}{8} m_H^3 v^{-2} H^4 \tag{2.28} \]

where \( Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu \) and the electromagnetic coupling constant \( e = g_2 \sin \theta_W \) has been identified.

The covariant derivatives of the lepton fields are

\[ (D_\mu L)_i = \partial_\mu L_i - ig_2 T^a_i (T^a)_j L_j - ig_1 (-\frac{1}{2}) B_\mu L_i \]
\[ (D_\mu \bar{e})_i = \partial_\mu \bar{e} - ig_1 (+1) B_\mu \bar{e} \tag{2.29} \]
The only possible gauge invariant mass term (i.e., term that is a SU(2) singlet and has zero hypercharge) that can be made from the available fields is a Yukawa interaction term between the Higgs field and the two leptonic fields.

\[ \mathcal{L}_{\text{Yuk}} = -y e^{ij} \varphi_i L_j \bar{e} + \text{h.c.} \]  

(2.30)

The covariant derivatives of the quarks are

\[ (D_\mu Q)_{\alpha i} = \partial_\mu Q_{\alpha i} - ig_3 A^a_\mu (T^a_3)_{\alpha}^{\beta} Q_{\beta i} - ig_2 A^a_\mu (T^a_2)_i^{\beta j} Q_{\beta j} - ig_1 (\pm \frac{1}{6}) B_\mu Q_{\alpha i} \]

\[ (D_\mu \bar{u})^\alpha = \partial_\mu \bar{u}^\alpha - ig_3 A^a_\mu (T^a_3)_{\beta}^{\alpha} \bar{u}^\beta - ig_1 (-\frac{2}{3}) B_\mu \bar{u}^\alpha \]  

(2.31)

\[ (D_\mu \bar{d})^\alpha = \partial_\mu \bar{d}^\alpha - ig_3 A^a_\mu (T^a_3)_{\beta}^{\alpha} \bar{d}^\beta - ig_1 (-\frac{1}{3}) B_\mu \bar{d}^\alpha \]

The Yukawa couplings in this case are

\[ \mathcal{L}_{\text{Yuk}} = -y' e^{ij} \varphi_j Q_{\alpha j} \bar{d}^\alpha - y'' \varphi^i Q_{\alpha i} \bar{u}^\alpha + \text{h.c.} \]  

(2.32)

It was necessary to use a factor of \( \varphi^1 \), which, we will see, is not possible in supersymmetry (SUSY). In SUSY a second Higgs doublet (with opposite hypercharge) is added to give mass to the up-type quarks.

### 2.3 Motivation Behind Supersymmetric Theories

In this section, supersymmetry is motivated first and foremost by the electroweak hierarchy problem which concerns the considerable fine tuning of the energy scale of electroweak symmetry breaking. It is also motivated by gauge coupling constant unification, an analogy with the positron partner to the electron, and as a source of dark matter.
2.3.1 The Hierarchy Problem

The Higgs boson has Yukawa interactions with the fermion fields, the largest of which is with the top quark, resulting in quantum corrections to the Higgs mass term of the form

\[
\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2} \left[ -2M_{UV}^2 + 6m_f^2 \ln(M_{UV}/m_f) + \ldots \right]
\]  

(2.33)

The Standard Model is an inherently incomplete theory, because this loop term would diverge to infinity if it were not cut off at some high energy scale \(M_{UV}\), which can be taken to be at the Planck scale at \(10^{19}\) GeV. This scale is where gravity becomes important at short distances. This ultraviolet-divergent term is seemingly at odds with the observed Higgs boson mass of \(125 \pm 0.4\) (stat) \(\pm 0.5\) (sys) GeV/c\(^2\) [17][18], because the bare (inherent) mass of the Higgs boson would have to cancel out the contribution from the loop term to about one part in \(10^{17}\). It is possible that this precise “fine-tuning” is how the observed Higgs mass is realized in nature, but it is also possible that some undiscovered new physics stabilizes the difference between the electroweak scale and the cutoff scale.

In SUSY, a hypothetical scalar particle interacting with the Higgs with exactly the same Yukawa coupling as the top quark would contribute a term with the opposite sign, as in

\[
\Delta m_H^2 = \frac{\lambda_s^2}{8\pi^2} \left[ +M_{UV}^2 - 2m_s^2 \ln(M_{UV}/m_s) + \ldots \right]
\]  

(2.34)

thereby exactly cancelling the quadratic ultraviolet divergence and solving the hierarchy problem. A much smaller logarithmic correction would remain, leaving fine-tuning at the 1% level remaining if superpartners exist near the TeV scale.
2.3.2 Analogy with bare electron mass

Prior to C.D. Anderson’s discovery of the positron [19], there was a theoretical problem between the self-energy of the electron and the measured size of the electron. The electron self energy is due to the Coulomb energy,

\[ E_{\text{self}} = \frac{3}{5} \frac{1}{3\pi\epsilon_0} \frac{e^2}{r_e}. \] (2.35)

In Murayama’s analogy [20] between the electron bare mass \((m_e)^0\) and the Higgs bare mass, the observed electron energy is expressed as

\[ m_e c^2 = (m_e)^0 c^2 + E_{\text{self}}. \] (2.36)

This picture has no significant fine tuning assuming a classical electron radius of \(5\) fm, but the electron has been found to be structureless down to \(10^{-3}\) fm. This can be resolved with a negative bare electron mass that is fine tuned at the 0.1% level. However, the existence of the
Figure 2.3: Significant cancellation occurs when the positron diagram is added.

Positron allows for a substantial cancellation in the self energy due to the combined processes of photon re-absorption and vacuum fluctuation. The vacuum fluctuation effectively smears out the bare charge over a large volume. The self energy is reduced to a small logarithmic divergence, which is only a 10% correction to the bare mass.

### 2.3.3 Dark Matter

Dark matter was first inferred from the rotational properties of galaxies. Rubin et al. surveyed 21 galaxies to study the velocity as a function of distance from the center of the galaxy [21][22]. They found that the velocity rises rapidly at small radii, then increases very slowly toward larger radii. This implies a very uniform mass density, which is in conflict with the observed visible mass density, but can be explained by the theory that approximately 50% of the mass was in a dark matter halo.

In the Bullet Cluster of galaxies shown in Fig. 2.4, gravitational lensing shows that the gravitational mass of the system is clearly separated from the post-collision X-ray-emmitting baryonic mass. In addition to showing a separation between the gravitational mass and visible mass, the event suggests that the dark matter was collisionless, which is a property shared by many models of galaxy rotation curves.

When R-Parity conservation is added to supersymmetry, the electrically neutral and
colorless neutralino becomes a good candidate for dark matter in models where it is the Lightest Supersymmetric Particle (LSP). Even though the neutralino LSP may have a mass on the order of 100 GeV, R-Parity would prevent the LSP from decaying to any lighter SM particles.

2.3.4 Gauge Coupling Unification

When Renormalization Group (RG) equations are used to find the values of the gauge coupling constants at very high interaction energies, we find that the three forces in the Standard Model do not converge at any point, as shown in Fig. 2.5. In the Minimally Supersymmetric Standard Model (MSSM), on the other hand, the coupling constants unify to the same value at approximately $M_U = 2 \times 10^{16}$ GeV. Coupling constant unification is an unexpected nice feature of supersymmetry.

In theories where the three gauge interactions are due to a larger gauge symmetry, we
call it a Grand Unified Theory (GUT). In the SO(10) GUT, R-parity occurs as a natural symmetry between baryons and leptons [24]. Thus, R-parity may be related to the gauge coupling unification evident in supersymmetry.

Assuming that the coupling constants unify at some high energy scale, and then using RG equations to relate the couplings at the electroweak scale, we find that (26)

\[
(54 + 3n_H) \sin^2 \theta_W(m_Z) = 9 + \frac{3}{2} n_H + (30 - n_H) \frac{\alpha_{EM}(m_Z)}{\alpha_3(m_Z)}
\]

(2.37)

where \( n_H \) is the number of Higgs doublets in supersymmetry. For \( n_H = 2 \), \( \alpha_{EM}(m_Z)^{-1} = 127.944 \pm 0.014 \), and \( \alpha_3(m_Z) = 0.1184 \pm 0.0007 \), we obtain

\[
\sin^2 \theta_W(m_Z) = 0.2308 \pm 0.0014
\]

(2.38)
while the experimental value is

\[ \sin^2 \theta_W(m_Z) = 0.2312 \pm 0.00015 \]  

(2.39)

This agreement is considered indirect evidence in favor of supersymmetry.

### 2.4 The Extended Poincaré group

The Poincaré group is the group of translations plus Lorentz transformations \[27, 28\],

\[
\begin{align*}
\left[ P^\lambda, P^\mu \right] &= 0, \\
\left[ M^{\mu\nu}, P^\lambda \right] &= i(\eta^{\nu\lambda} P^\mu - \eta^{\mu\lambda} P^\nu), \\
\left[ M^{\mu\nu}, M^{\rho\sigma} \right] &= i(\eta^{\nu\rho} M^{\mu\sigma} + \eta^{\mu\sigma} M^{\nu\rho} - \eta^{\mu\rho} M^{\nu\sigma} - \eta^{\nu\sigma} M^{\mu\rho}).
\end{align*}
\]  

(2.40)  
(2.41)  
(2.42)

The generator of space-time translations, \( P^\mu \), is the energy-momentum four-vector \( P^\mu = (E, p_x, p_y, p_z) \), while \( M^{\mu\nu} \) has as components the angular momentum generators \( (J^i = \frac{1}{2} \varepsilon^{ijk} M^{jk}) \) and boosts \( (K^i = M^{i0}) \). A global symmetry between bosons and fermions can be achieved by the addition of a spin generator \( Q_\alpha \) to the Poincaré group. The spin generator is a global left-handed Weyl spinor, so it has the basic action \( Q|\text{Boson} \rangle = |\text{Fermion} \rangle \) and \( Q|\text{Fermion} \rangle = |\text{Boson} \rangle \). We already know that it commutes with the momentum generator \( P^\mu \) (particle spin is unchanged by translations): \[
\left[ P^\mu, Q_\alpha \right] = 0.
\]  

(2.43)
There is a right-handed version of $Q_L$ that can be obtained with a boost. In the Weyl basis, a Majorana fermion made from these spinors is

$$\Psi = \begin{pmatrix} Q_L \\ \bar{Q}_R \end{pmatrix} = \begin{pmatrix} Q_\alpha \\ \bar{Q}^{\dot{\alpha}} \end{pmatrix}$$ (2.44)

In this notation, handedness is differentiated by the position of the index and the presence of a bar and dot.

### 2.4.1 Aside: spinor notation

A pair of (left, right)-handed states is conventionally represented as $(2s + 1, 2s' + 1)$. A product of two left-handed spin-1/2 states is the sum of a spin-0 antisymmetric state (e.g., $|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle$) and a spin-1 symmetric state (e.g., $|\uparrow \uparrow\rangle$ and $|\downarrow \downarrow\rangle$), which is represented as $(2, 1) \otimes (2, 1) = (1, 1)_A \oplus (3, 1)_S$, so that a field with two left-handed indices can be decomposed as

$$C_{\alpha\beta}(x) = \epsilon_{\alpha\beta}D(x) + G_{\alpha\beta}(x)$$ (2.45)

where $D(x)$ is a scalar field, $\epsilon_{\alpha\beta}$ is an antisymmetric set of constants (defined as $\epsilon^{12} = \epsilon_{21} = 1$ and $\epsilon^{21} = \epsilon_{12} = -1$, and the field $G(x)$ is symmetric in the indices. The $D(x)$ term is Lorentz invariant, with $D(x)$ a Lorentz scalar, and $\epsilon_{\alpha\beta}$ an “invariant symbol” which is invariant under a Lorentz transformation of both indices,

$$L^\rho_\alpha(\Lambda)L^\sigma_\beta(\Lambda)\epsilon_{\rho\sigma} = \epsilon_{\alpha\beta}.$$ (2.46)
This is analogous to the invariant symbol $g_{\mu\nu}$, which satisfies

$$\Lambda^\rho_\mu \Lambda^\sigma_\nu g_{\rho\sigma} = g_{\mu\nu}. \quad (2.47)$$

The Lorentz transformation operators in Eq. (2.46) can be written in terms of a generator that operates on either left- or right-handed fields. One finds that $\epsilon_{\alpha\beta}$ acts as a raising or lowering operator on spinor indices, which, when combined with complex conjugation, can transform left-handed spinors into right-handed spinors and vice versa:

$$\chi_{\alpha} = \epsilon_{\alpha\beta} \chi^{\beta} = \epsilon_{\alpha\beta} (\bar{\chi}^{\dot{\beta}})^* \quad (2.48)$$

We define the four-vector $A_\mu$ to accompany $A^\mu$ in such a way that $A \cdot A = A^\mu A_\mu$ is Lorentz invariant. The same is true for these Lorentz invariant products of spinors:

\[
\begin{align*}
\chi \psi &\equiv \chi^\alpha \psi_\alpha = \epsilon^{\alpha\beta} \chi_\beta \epsilon_{\alpha\sigma} \psi^\sigma = -\epsilon^{\beta\alpha} \epsilon_{\sigma\alpha} \chi_\beta \psi^\sigma = -\delta^\beta_\sigma \chi_\beta \psi^\sigma = \delta^\beta_\sigma \psi^\sigma \chi_\beta = \psi \chi \\
\bar{\chi} \bar{\psi} &\equiv \bar{\chi}_\dot{\alpha} \bar{\psi}^{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\chi}^{\dot{\beta}} \epsilon^{\dot{\sigma}\dot{\alpha}} \bar{\psi}^{\dot{\sigma}} = -\epsilon_{\dot{\beta}\dot{\alpha}} \epsilon^{\dot{\sigma}\dot{\alpha}} \bar{\chi}^{\dot{\beta}} \bar{\psi}^{\dot{\sigma}} = -\delta^{\dot{\sigma}\dot{\beta}} \bar{\chi}^{\dot{\beta}} \bar{\psi}^{\dot{\sigma}} = \delta^{\dot{\sigma}\dot{\beta}} \bar{\psi}^{\dot{\sigma}} \bar{\chi}^{\dot{\beta}} = \bar{\psi} \bar{\chi}
\end{align*}
\] (2.49) (2.50)

### 2.4.2 The relationship between Q, P and M

Under infinitesimal Lorentz transformations (rotations and boosts, with parameters $\omega_{\mu\nu}$), the generator $Q$ transforms as

\[
Q'_\alpha = S(\Lambda)_{\beta}^\alpha Q_\beta = (1 + \frac{i}{2} \omega_{\mu\nu} \sigma^{\mu\nu})_{\beta}^\alpha Q_\beta = U(\Lambda)^\dagger Q^\alpha U(\Lambda)
\]

\[
= Q_\alpha + \frac{i}{2} \omega_{\mu\nu} [M^{\mu\nu}, Q_\alpha]
\] (2.51)
where

\[ \frac{1}{2}(\sigma^{\mu\nu})^\alpha_\beta = \frac{1}{4} (\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)^\alpha_\beta \]

\[ \frac{1}{2}(\bar{\sigma}^{\mu\nu})^\dot{\alpha}_{\dot{\beta}} = \frac{1}{4} (\bar{\sigma}^\mu \sigma^\nu - \sigma^\nu \bar{\sigma}^\mu)^{\dot{\alpha}}_{\dot{\beta}} \]

so that

\[ [M^{\mu\nu}, Q_\alpha] = -i(\sigma^{\mu\nu})^\alpha_\beta Q_\beta \quad (2.53) \]

\[ [M^{\mu\nu}, \bar{Q}^{\dot{\alpha}}] = -i(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \bar{Q}^{\dot{\beta}} . \]

We note that the anti-commutators between two fermionic operators, \{Q_\alpha, Q_\beta\} and \{Q_\alpha, \bar{Q}^{\dot{\beta}}\}, must be bosonic, i.e., they can be a linear combination of \( P^\mu \) and \( M^{\mu\nu} \). The Weyl spinors commute with \( P^\mu \) but not with \( M^{\mu\nu} \). The only possibility is then (see details in [26])

\[ \{Q_\alpha, Q_\beta\} = 0 = \{\bar{Q}_\dot{\alpha}, \bar{Q}_\dot{\beta}\} , \quad (2.54) \]

\[ \{Q_\alpha, \bar{Q}_\dot{\beta}\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P^\mu . \quad (2.55) \]

The undotted index and dotted index on the left-hand-side of Eq. (2.55), and the single Lorentz index on \( P^\mu \) means that \( P^\mu \) should be associated with the invariant symbol \( \sigma^{\mu}_{\alpha\dot{\beta}} \); the normalization is a convention. These two anticommutators complete the extended Poincaré algebra.

The anticommutator of Eq. (2.55) is interesting, because it states that an internal symmetry is related to a spacetime Poincaré symmetry, i.e., that two spin transformations
generate a translation in spacetime. Under the Haag-Lopuszanski-Sohnius theorem, this addition of two spin-1/2 generators to the Poincaré algebra results in the largest possible symmetry group consistent with a 4-dimensional quantum field theory [29]. Also, one can show that Eq. (2.55) implies that for each representation of the supersymmetry algebra, there are an equal number of fermionic and bosonic degrees of freedom, which is what we needed to cancel the ultraviolet divergences underlying the hierarchy problem.

2.5 Supersymmetric Lagrangians

It is possible to construct a supersymmetric Lagrangian using separate scalar and Weyl fields, which transform between each other in a way that is consistent with the extended Poincaré algebra, but it turns out to be more convenient to use a Superfield approach, which was developed by Abdus Salam and John Strathdee [30]. A superfield is a field in the superspace \((x_\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})\), where normal space is paired with a fermionic dimension. One can show [31] that a supersymmetry transformation on a superfield (parameterized by anti-commuting Grassman parameters \(\xi\) and \(\bar{\xi}\)) and a spacetime translation,

\[
S(x^\mu, \theta, \bar{\theta}) \to \exp(i(Q\xi + \bar{Q}\bar{\xi} - a^\mu P_\mu))S(x^\mu, \theta, \bar{\theta})
\] (2.56)

is generated by

\[
P_\mu = i\partial_\mu
\]

\[
iQ_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu
\]

\[
i\bar{Q}\dot{\alpha} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu
\] (2.57)
A general superfield could be written as an expansion in \( \theta \) and \( \bar{\theta} \), with component fields that depend only on spacetime, such as

\[
S(x, \theta, \bar{\theta}) = f(x) + \theta \psi(x) + \bar{\theta} \bar{\xi}(x) + \theta \theta m(x) \\
+ \bar{\theta} \bar{\theta} n(x) + \theta \sigma^\mu \bar{\theta} v_\mu(x) + \theta \bar{\theta} \theta \rho(x) + \theta \theta \bar{\theta} \bar{\theta} d(x)
\]  

(2.58)

but this representation is reducible, with a large number of degrees of freedom. There are smaller representations, one of which is called the left-handed chiral superfield \( \Phi \) (which is defined to have a vanishing supercovariant derivative \( \bar{D}_\alpha \Phi(x, \theta, \bar{\theta}) = 0 \) where \( \bar{D}_\alpha \equiv -\partial_\alpha - i \theta^\alpha \sigma^\mu_{\alpha \alpha} \partial_\mu \)), which is

\[
\Phi(y, \theta) = \varphi(y) + \theta \chi(y) + \frac{1}{2} \theta \theta F(y)  \\
\Phi(x^\mu, \theta, \bar{\theta}) = \varphi(x) - \frac{i}{2} \theta \sigma^\mu \bar{\theta} \partial_\mu \varphi(x) - \frac{1}{16} \theta \theta \bar{\theta} \bar{\theta} \Box \varphi(x) \\
+ \theta \chi(x) - \frac{i}{2} \theta \sigma^\mu \bar{\theta} \theta \partial_\mu \chi(x) + \frac{1}{2} \theta \theta F(x)
\]  

(2.59, 2.60)

where \( \varphi \) and \( F \) are complex scalar fields and \( \chi \) is a left-handed Weyl field. The left-chiral superfield \( \Phi \) is written most compactly as a function of the variable of

\[
y^\mu = x^\mu - \frac{i}{2} \theta \sigma^\mu \bar{\theta}.
\]  

(2.61)

There is also the vector superfield representation (for which \( V^\dagger = V \))

\[
V(x, \theta, \bar{\theta}) = C(x) + \theta \xi(x) + \bar{\theta} \bar{\xi}(x) + \theta \theta M(x) + \bar{\theta} \bar{\theta} \bar{M}(x) \\
+ \theta \sigma^\mu \bar{\theta} v_\mu(x) + \theta \theta \bar{\theta} \bar{\theta} \lambda(x) + \theta \theta \bar{\theta} \bar{\theta} \lambda(x) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D(x).
\]  

(2.62)

When a supersymmetry transformation is applied to \( \Phi \), namely \( \delta \Phi = i(\xi Q + \bar{\xi} \bar{Q}) \Phi \),
it can be shown that the $F(x)$ component field in (2.60) varies by a total divergence only, which is a necessary property for an invariant action. Considering the products of chiral superfields $\Phi_i \Phi_j$, $\Phi_i \Phi_j \Phi_k$, and $\Phi_i \Phi_j$, we note that the so-called F-terms and D-terms

\[
\Phi_i \Phi_j \big|_F \equiv \Phi_i \Phi_j \big|_{\text{coeff. of } \theta \theta/2} \tag{2.63}
\]

\[
\Phi_i \Phi_j \Phi_k \big|_F \equiv \Phi_i \Phi_j \Phi_k \big|_{\text{coeff. of } \theta \theta/2} \tag{2.64}
\]

and

\[
\Phi_i \Phi_j \big|_D \equiv \Phi_i \Phi_j \big|_{\text{coeff. of } \bar{\theta} \theta \theta/4} \tag{2.65}
\]

also vary by a total divergence only. A supersymmetric Lagrangian can thus have the form

\[
\mathcal{L} = \Sigma \Phi_i \Phi_i \big|_D + (\mathcal{W}(\Phi))_F + \text{h.c.} \tag{2.66}
\]

where the superpotential $\mathcal{W}$ is

\[
\mathcal{W}(\Phi_1, \Phi_2, ..., \Phi_n) = \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \tag{2.67}
\]

with $m_{ij}$ and $\lambda_{ijk}$ real and symmetric in the indices. Products of more than 3 chiral superfields are not allowed, because the $F$-term would have dimension 5 or higher. It is worth noting that any time we only need the $F$ term of a product of chiral superfields, we can set $y = x$. The explicit calculation of (2.63), (2.64), and (2.65) yields the final Lagrangian for a non-gauge theory:

\[
\mathcal{L} = \partial_\mu \partial_\mu \Phi_i - m_{ij} \psi_j \bar{\psi}_i - \lambda_{ijk} \varphi_j \varphi_j \psi_k - (\frac{1}{2} m_{ij} \psi_j \bar{\psi}_i + \lambda_{ijk} \psi_i \psi_j \varphi_k + \text{h.c.}) \tag{2.68}
\]
where the equation of motion for $F$ in the potential term $V$ was used: $V = F_i^\dagger F_i = |F_i|^2 = |m_{ij} + \lambda_{ijk} \varphi_j \varphi_k|^2$. It is possible to use the equation of motion because there are no derivatives on $F$ in the path integral. The two Yukawa interactions between fermions and their bosonic superpartners are drawn in Fig. 2.6.

![Figure 2.6: Yukawa interactions and the $\varphi^4$ interaction in a non-gauge theory.](image)

A simple model employing Eq. (2.68) with 2 massless chiral superfields $\Phi_x$ and $\Phi_y$ (whose bosonic components are analogous to the Higgs boson), and one massive chiral superfield $\Phi_z$ (analogous to a top quark) is enough to demonstrate that radiative corrections to the massless fields are not induced by $m_z$ [31].

2.5.1 Supersymmetric Gauge Theories

A local $U(1)$ supersymmetric gauge transformation

$$\Phi \rightarrow e^{2i\sigma^\Lambda} \Phi$$

(2.69)

has a phase $\Lambda = \Lambda(x)$ that is a function of space-time. Also, for $\Phi$ to remain a chiral superfield, $\Lambda$ should also be a chiral superfield, having the usual form

$$\Lambda = \varphi_\Lambda - \frac{i}{2} \theta \sigma^\mu \overline{\theta} \partial_\mu \varphi_\Lambda - \frac{1}{16} \theta \theta \overline{\theta} \overline{\theta} \varphi_\Lambda$$

(2.70)

$$+ \theta \chi_\Lambda - \frac{i}{2} \theta \sigma^\mu \overline{\theta} \partial_\mu \chi_\Lambda + \frac{1}{2} F_\Lambda \theta \theta$$

30
but with each term having dimension 0 since $\Lambda$ is in the exponent. The kinetic term then transforms as

$$\Phi^\dagger \Phi \rightarrow e^{2iq(\Lambda - \Lambda^\dagger)} \Phi^\dagger \Phi \quad (2.71)$$

but this term is not gauge invariant. We thus replace it with

$$\Phi^\dagger e^{2iq\mathcal{V}} \Phi \quad (2.72)$$

and require $\mathcal{V}$ to undergo a super gauge transformation

$$\mathcal{V} \rightarrow \mathcal{V} - i(\Lambda - \Lambda^\dagger) \quad (2.73)$$

The factor $i(\Lambda - \Lambda^\dagger)$ is real, so we can take the gauge superfield $\mathcal{V}$ to be real ($\mathcal{V}^\dagger = \mathcal{V}$).

There is a lot of gauge freedom in the way the component fields transform, and many of them can be set to zero in the Wess-Zumino gauge $[32]$. In this gauge, the vector superfield of (2.62) reduces to

$$\mathcal{V} = \frac{1}{2} \theta \sigma^\mu \bar{\theta} A_\mu + \frac{1}{2\sqrt{2}} \theta \theta \bar{\theta} \bar{\lambda} + \frac{1}{2\sqrt{2}} \bar{\theta} \bar{\theta} \theta \lambda - \frac{1}{8} \theta \theta \bar{\theta} \bar{\theta} D \quad (2.74)$$

where the component fields $\lambda$ (which is a spin-1/2 partner of $A_\mu$) and $D$ (an auxiliary field) are gauge-invariant. The only field that undergoes a gauge transformation is

$$A_\mu \rightarrow A_\mu - 2 \partial_\mu [\text{Re} (\varphi_\Lambda)] \quad (2.75)$$
The only supersymmetry invariant term in the kinetic term (2.72) is the D-term,

\[ \left[ \Phi^\dagger e^{2q\nu} \Phi \right]_D \] (2.76)

for which the D-terms in the complete Taylor series are

\[ \Phi^\dagger e^{2q\nu} \Phi \bigg|_D = \Phi^\dagger \Phi \bigg|_D + 2q\Phi^\dagger \Phi \nu \bigg|_D + 2q^2\Phi^\dagger \Phi \nu^2 \bigg|_D \] (2.77)

In terms of the covariant derivative

\[ D_\mu \equiv \partial_\mu + iqA_\mu , \] (2.78)

the end result is

\[ \Phi^\dagger e^{2q\nu} \Phi \bigg|_D = (D_\mu \varphi)^\dagger(D^\mu \varphi) + i\overline{\chi}\sigma^\mu D_\mu \chi + F F^\dagger - q\varphi^\dagger \varphi D \]
\[ - (\sqrt{2}q\chi\lambda \varphi^\dagger + h.c.) . \] (2.79)

The superfield kinetic term has the usual fermion kinetic term \( i\overline{\chi}\sigma^\mu D_\mu \chi \), as well as the kinetic term for the spin-0 partner field. There are also two potential terms made from the auxiliary \( F \) and \( D \) fields, and a fermion-gaugino-scalar interaction. The \( F \) and \( D \) fields are

\[ F_i^\dagger = - \frac{\partial W}{\partial \varphi_i} \] (2.80)
\[ D = - \sum_i q_i \overline{\varphi_i^\dagger} \varphi_i \] (2.81)
The field-strength superfield is

$$F_a \equiv \bar{D} \bar{D} D_a \mathcal{V}$$ \hspace{2cm} (2.82)$$

which, in addition to being a spinor, is actually a left-chiral superfield (it satisfies $\bar{D}_b F_a = 0$). This field-strength is gauge invariant with respect to the underlying vector superfield transformation of (2.73). The Lorentz scalar suitable for a supersymmetric Lagrangian is

$$\frac{1}{4} \left( F^a F_a \right) \bigg|_F = \frac{1}{2} D^2 + i \bar{\lambda} \sigma^\mu \lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$ \hspace{2cm} (2.83)$$

A non-abelian gauge theory using the group $SU(N)$ has a matrix-valued gauge superfield, in which case we decompose it into a linear combination of the generators of the group,

$$\mathcal{V} \equiv \mathcal{V}^i T^i_F,$$ \hspace{2cm} (2.84)$$

and the gauge field terms are

$$\mathcal{L} = \frac{1}{2} \text{Tr}(F^a F_a) \big|_F + \Phi^\dagger e^{2g} \Phi \big|_D.$$ \hspace{2cm} (2.85)$$

### 2.5.2 Minimally Supersymmetric Standard Model

The set of quantum numbers for the left-chiral superfields that can replicate the Standard Model fermion content is given in Table 2.1. The subscript $i$ is the generation index.
<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_u$</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$H_d$</td>
<td>1</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>3</td>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td>$U_i$</td>
<td>3</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$D_i$</td>
<td>3</td>
<td>1</td>
<td>-4/3</td>
</tr>
<tr>
<td>$L_i$</td>
<td>1</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>$E_i$</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2.1: Superfields necessary to replicate the Standard Model fermions.

The $U(1)$, $SU(2)$, and $SU(3)$ gauge vector superfields are

\[ B = \frac{1}{2} \sigma^{\mu} \bar{\theta} B_{\mu} + \frac{1}{2} \sqrt{2} \theta \bar{\theta} \lambda_D Y - \frac{1}{8} \theta \theta \bar{\theta} \bar{\theta} D_Y \]  

(2.86)

\[ W^i = \frac{1}{2} \sigma^{\mu} \bar{\theta} W_{\mu}^i + \frac{1}{2} \sqrt{2} \theta \bar{\theta} \lambda_L^i - \frac{1}{8} \theta \theta \bar{\theta} \bar{\theta} D_L^i \]  

(2.87)

\[ G^a = \frac{1}{2} \sigma^{\mu} \bar{\theta} G_{\mu}^a + \frac{1}{2} \sqrt{2} \theta \bar{\theta} \lambda_C^a - \frac{1}{8} \theta \theta \bar{\theta} \bar{\theta} D_C^a \]  

(2.88)

Immediately evident are the spin-1/2 Binos, Winos, and Gluinos that partner with the spin-1 $B_{\mu}$, $W_{\mu}^i$ and gluon $G_{\mu}^a$ fields.

There must be two Higgs fields in order for the up-type quarks to acquire mass. Noting that the only possible terms in the superpotential are the gauge-invariant products of 2 or 3 left-chiral superfields, we find

\[ W = y_{ij}^u U_i Q_j H_u - y_{ij}^d D_i Q_j H_d - y_{ij}^e L_i E_j H_d + \mu H_u H_d \]  

(2.89)

Some other possible interactions, which violate lepton number and baryon number conservation, we choose to rule out via an application of R-parity conservation. R-parity is defined as

\[ R = (-1)^{3B+L+2S} = (-1)^{3(B-L)+2S} \]  

(2.90)
We get $R = 1$ for all Standard Model quarks and leptons, and the Higgs scalars, while all their superpartners have $R = -1$. Conservation of R-parity means that in a collider, superpartners can only be pair-produced. Also, the lightest supersymmetric particle must be absolutely stable. If it is electrically neutral and colorless, it is a candidate for dark matter. Without R-parity, there would be rapid proton decay and flavor-changing neutral currents.

### 2.6 The MSSM and the Hidden Sector

If supersymmetry is realized in nature, it must be a broken symmetry because the superpartners must be very massive for them to have escaped detection. It should be “softly” broken, which is the type of breaking where the quadratic divergences are cancelled exactly while allowing the small logarithmic corrections. Analogous to the situation with the Higgs, a vacuum expectation value of the potentials of the $F$ or $D$ fields in (2.68) and (2.74) can break supersymmetry, but these scenarious are ruled out phenomenologically. There are also phenomenological problems with adding various new tree-level couplings. We therefore expect that the supersymmetry breaking occurs in some “hidden sector” of particles that have only very small couplings to the particles in the visible sector of chiral superfields in the MSSM. Small interactions between the hidden and visible sectors communicate the supersymmetry breaking in the hidden sector into the visible sector.

We can parameterize our ignorance of the details of the higher energy theory by saying that it can potentially induce gaugino masses, scalar mass-squared terms, $\varphi^3$ couplings, and
tadpole couplings:

\[
L_{\text{soft}} = -\left( \frac{1}{2} M_\alpha \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} b^{ij} \varphi_i \varphi_j + t^i \varphi_j \right) + h.c. - (m^2)^i_j \varphi^j \varphi_i \quad (2.91)
\]

The result of this \( L_{\text{soft}} \) is softly broken supersymmetry, because it does not result in any quadratic divergences \[33\].

In terms of the MSSM superfields,

\[
L_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + h.c. \right)
- \left( \tilde{u}_u Q H_u - \tilde{d}_d Q H_d - \tilde{e}_e L H_d + h.c. \right)
- \tilde{Q}^i m_Q^2 \tilde{Q} - \tilde{L}^i m_L^2 \tilde{L} - \tilde{u}_u m_u^2 \tilde{u}^\dagger - \tilde{d}_d m_d^2 \tilde{d}^\dagger - \tilde{e}_e m_e^2 \tilde{e}^\dagger
- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + h.c.) \quad (2.92)
\]

The gluino, wino, and bino are given mass terms. The second line corresponds to the \( \varphi^3 \) couplings, and the third and fourth line to the \( b^{ij} \) and \( (m^2)^i_j \) mass terms. The \( a_u, a_d, a_e, m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2 \) are all \( 3 \times 3 \) matrices in family space. In total, there are 105 new parameters to describe the masses, phases, and mixing angles in the MSSM Lagrangian.

Experimental limits on flavor-changing currents and CP-violation suggest we enforce the "universality" principle, in which scalar and gaugino masses are assumed to be uni-
versal at the high energy scale of the hidden sector, and with no mixing between flavors:

\begin{align}
    m_{\tilde{Q}}^2 &= m_{\tilde{Q}}^2 \mathbf{1} \\
    m_{\tilde{\ell}}^2 &= m_{\tilde{\ell}}^2 \mathbf{1} \\
    m_{\tilde{d}}^2 &= m_{\tilde{d}}^2 \mathbf{1} \\
    m_{\tilde{L}}^2 &= m_{\tilde{L}}^2 \mathbf{1} \\
    m_{\tilde{e}}^2 &= m_{\tilde{e}}^2 \mathbf{1} 
\end{align}

(2.93)

In a model-dependant way, renormalization-group evolution tells us what happens to these masses when they are evaluated at the Electroweak scale, as shown in Fig. 2.7.

Figure 2.7: Evolution of scalar and gaugino masses with boundary conditions set at $2.5 \times 10^{16}$ GeV.
2.7 Supergravity

Supersymmetry can be made into a theory of gravity by making the group parameters local rather than global. This means that the parameters $\xi$, $\bar{\xi}$, and $a^\mu$ of Eq. (2.56) become functions of spacetime. Allowing translations to be a function of spacetime makes it a theory of gravity; local supersymmetry is referred to as supergravity.

2.7.1 Pure Supergravity

Starting with a pure, globally supersymmetric on-shell gravity theory we have just a spin-2 graviton and its spin-3/2 partner the gravitino. After applying the Noether procedure, this theory can be extended to supergravity [34].

Written as a massless Majorana vector spin field $\Psi_\mu$, the spin-3/2 particle has the Rarita-Schwinger action [35]

$$S_{RS} = -\frac{1}{2} \int d^4 x \epsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \Psi_\sigma$$  \hspace{1cm} (2.94)

For a globally supersymmetric and on-shell theory, it is enough to start with the linearized Einstein action where the spin-2 field is a deviation $h_{\mu\nu}$ from flat spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$  \hspace{1cm} (2.95)

where $\kappa^2 = 8\pi G_N$ and $G_N$ is the Newtonian gravitational constant. The linearized Einstein action is

$$S^L_{\text{EINSTEIN}} = -\frac{1}{2} \int d^4 x (R^L_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R^L) h^{\mu\nu}$$  \hspace{1cm} (2.96)
where the Ricci tensor $R_{\mu\nu}^L$ is

$$R_{\mu\nu}^L = \frac{1}{2} \left( \frac{\partial^2 h_{\mu\nu}}{\partial x^\lambda \partial x^\lambda} + \frac{\partial^2 h_{\lambda\nu}}{\partial x^\mu \partial x^\lambda} + \frac{\partial^2 h_{\lambda\nu}}{\partial x^\nu \partial x^\lambda} + \frac{\partial^2 h_{\mu\nu}}{\partial x^\nu \partial x^\nu} \right) \quad (2.97)$$

and the curvature scalar $R^L$ is

$$R^L = \eta^{\mu\nu} R_{\mu\nu}^L \quad (2.98)$$

It can be shown that the combined action

$$S_{\text{GLOBAL}} = S_{\text{RS}} + S_{\text{EINSTEIN}}^L \quad (2.99)$$

can be closed under the supersymmetry algebra using on-shell relations for the fields, after defining suitable transformations between the spin-2 and spin-3/2 fields [26]. After the Noether procedure is applied to make the theory local, one can eventually obtain the locally supersymmetric action.

### 2.7.2 The gravitino coupling to the visible sector

In mSUGRA, supersymmetry breaking will be caused by a gravitino mass term. The gravitino will need to couple to the visible sector (of leptons, quarks, etc.) for this breaking to be communicated. To see how this happens, consider a free massless multiplet of a scalar and a spinor. In terms of a Majorana fermion, the Lagrangian is

$$\mathcal{L}_0 = \partial_{\mu} \varphi^* \partial^{\mu} \varphi + \frac{i}{2} \bar{\Psi} \gamma^{\mu} \partial_{\mu} \Psi \quad (2.100)$$
Writing $\varphi$ in terms of real fields $A$ and $B$ ($\varphi = \frac{1}{\sqrt{2}}(A + iB)$), we note that the Lagrangian has global supersymmetry invariance under

$$\delta A = \bar{\xi} \Psi$$

(2.101)

$$\delta B = i\bar{\xi} \gamma_5 \Psi$$

(2.102)

$$\delta \Psi = -i\gamma^\mu \partial_\mu (A + i\gamma_5 B) \xi$$

(2.103)

where $\xi$ is a Majorana spinor parameter (made of a Weyl spinor)

$$\xi = \begin{pmatrix} \xi_W \\ \bar{\xi}_W \end{pmatrix}$$

(2.104)

Under these transformations, the Lagrangian changes by a total divergence

$$\delta L_0 = \partial_\mu (\bar{\xi} j_\mu)$$

(2.105)

where

$$j^\mu = \bar{\Psi} (A - i \gamma_5 B) \gamma^\mu \Psi .$$

(2.106)

In the U(1) case, the phase parameter was real and so the gauge field had a single Lorentz index. In this case, the phase parameter is a Majorana spinor $\xi(x)$ and so the gauge field is a vector spinor $\Psi_\mu$ which we identify as the gravitino. The gravitino transformation law is already fixed from the pure supergravity solution alluded to in the last section; to leading order in $\kappa$ it is

$$\Psi_\mu \rightarrow \Psi_\mu + 2\kappa^{-1} \partial_\mu \xi$$

(2.107)
The Lagrangian can be made invariant to zeroth order in $\kappa$ if we add the term

$$L_1 = -\frac{\kappa}{2} \bar{\Psi}_j j^\mu,$$  \hspace{1cm} (2.108)

i.e., $\delta L_1$ will cancel $\delta L_0$. Adding additional terms to the Lagrangian can extend the invariance to order $\kappa^2$.

Supergravity can thus communicate supersymmetry breaking to the visible sector via a small coupling to the gravitino.

### 2.8 mSUGRA Phenomenology

The 5 free parameters in mSUGRA, defined at the GUT scale, are

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)$$ \hspace{1cm} (2.109)

The gaugino masses are primarily functions of $m_{1/2}$, while the sfermions are functions of both $m_0$ and $m_{1/2}$. The parameter $\tan \beta$ is the ratio of the two Higgs vevs, and sign($\mu$) is the sign of the constant in the $\mu \mathcal{H}_u \mathcal{H}_d$ coupling of Eq. (2.89) (the absolute value of $\mu$ is determined from the $Z$ mass).

At the electroweak scale, the gaugino mass parameters are

$$M_3 \equiv M_{\tilde{g}} \simeq 2.7 m_{1/2}$$
$$M_2(M_Z) \simeq 0.8 m_{1/2}$$ \hspace{1cm} (2.110)
$$M_1(M_Z) \simeq 0.4 m_{1/2}$$

The neutralinos $\tilde{\chi}_i^0$ ($i = 1 - 4$) are the 4 mass eigenstates of the combined $\tilde{W}^3$, $\tilde{B}$,
$\tilde{H}_1^0$, and $\tilde{H}_2^0$ fields. The charginos $\tilde{\chi}_i^\pm$ ($i = 1, 2$) are the mass eigenstates of the $\tilde{W}^\pm$ and $\tilde{H}^\pm$. The possible decay modes are

$$
\begin{align*}
\tilde{\chi}_i^0 &\rightarrow Z\tilde{\chi}_j^0, \ W^\pm\tilde{\chi}_j^\pm, \ h^0\tilde{\chi}_j^0, \ l\tilde{\nu}, \ [A^0\tilde{\chi}_j^0, \ H^0\tilde{\chi}_j^0, \ H^\pm\tilde{\chi}_j^\mp, \ q\bar{q}] \\
\tilde{\chi}_i^\pm &\rightarrow W^\pm\tilde{\chi}_j^0, \ Z\tilde{\chi}_1^\pm, \ h^0\tilde{\chi}_1^\pm, \ l\tilde{\nu}, \ [A^0\tilde{\chi}_j^\pm, \ H^0\tilde{\chi}_1^\pm, \ H^\pm\tilde{\chi}_j^\mp, \ q\bar{q}]
\end{align*}
$$

(2.111)

where the decays thought to be kinematically disfavored are in brackets. Three-body decays mediated by off-shell gauge bosons, Higgs scalars, sleptons, and squarks are

$$
\begin{align*}
\tilde{\chi}_i^0 &\rightarrow ff\tilde{\chi}_j^0, \ \tilde{\chi}_i^0 \rightarrow ff'\tilde{\chi}_j^\pm, \ \tilde{\chi}_i^\pm \rightarrow ff'\tilde{\chi}_j^0, \ \tilde{\chi}_2^\pm \rightarrow ff\tilde{\chi}_1^\pm
\end{align*}
$$

(2.112)

An example of the fermion-gaugino-scalar interaction of Eq. (2.79) is the squark decaying to a quark and gluino, $\tilde{q} \rightarrow q\tilde{g}$, when kinematically allowed. A squark can also decay to a quark and a chargino or neutralino,

$$
\tilde{q} \rightarrow q\chi_i^0, \ q'\chi_i^\pm
$$

(2.113)

Following the production of gluinos and squarks (for example, via the s-channel diagrams of Fig. 2.8), a lepton can be created in the decay chain of gluino or squark.

A supersymmetry event falls into the one lepton channel whenever the two decay chains result in a total of one lepton. Squark and gluino decay chains can individually result in anything from 0 to multiple leptons. One example of a one-lepton channel event would start with an s-channel production of a squark and gluino, (Fig. 2.8c), followed by the the decay chains of the gluino and squark shown in Fig. 2.9.

The Yukawa couplings of the first and second generation fermions are approximated
Figure 2.8: A sample of s-channel gluino and squark production diagrams.

Figure 2.9: (a) A gluino decay resulting in one lepton and (b) a squark decay resulting in a quark and a neutralino. A directly produced squark can of course also produce a lepton, as in the subdiagram of the gluino decay.
as zero when calculating the first and second generation sfermion masses, which are:

\[ m^{2}_{\tilde{u}_{L}} \simeq m_{0}^{2} + 5.0m_{1/2}^{2} + 0.35 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{d}_{L}} \simeq m_{0}^{2} + 5.0m_{1/2}^{2} - 0.42 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{u}_{R}} \simeq m_{0}^{2} + 4.5m_{1/2}^{2} + 0.15 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{d}_{R}} \simeq m_{0}^{2} + 4.4m_{1/2}^{2} - 0.07 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{e}_{L}} \simeq m_{0}^{2} + 0.49m_{1/2}^{2} - 0.27 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{e}_{R}} \simeq m_{0}^{2} + 0.49m_{1/2}^{2} + 0.50 \cos 2\beta M_{Z}^{2} \]
\[ m^{2}_{\tilde{\nu}} \simeq m_{0}^{2} + 0.49m_{1/2}^{2} + 0.50 \cos 2\beta M_{Z}^{2} \]

The third generation sfermions have more complicated expressions due to nonzero Yukawa couplings.

In Fig. 2.10, 3 different domains in the \( m_{1/2} \) vs. \( m_{0} \) plane are indicated. In region 1, gluinos are heavier than squarks, so typical decay chains are \( \tilde{g} \to \tilde{q} \tilde{q} \) and \( \tilde{q} \to q\chi \). In region 2, the gluino is lighter than some squarks and heavier than others; some possible decay chains are \( \tilde{q} \to \tilde{g}q, \tilde{g} \to \tilde{b}\tilde{b}, \tilde{b} \to b\chi \) (3rd generation fermions are expected to be the lightest to minimize fine-tuning). In region 3, gluinos are lighter than the squarks, leading to decay chains such as \( \tilde{g} \to \tilde{g}q, \tilde{g} \to q\bar{q}\chi \).

Due to R-parity conservation, in mSUGRA there will be a pair of massive supersymmetric particles in the initial state. These will typically be squarks and/or gluinos. Another key characteristic of the mSUGRA topology is the presence of at least two missing particles, due to the neutralino acting as the lightest supersymmetric particle.
Figure 2.10: Regions of the $m_{1/2}$ vs. $m_0$ plane, with production cross-sections for gluinos (in blue) and squark production (in red). [37]
Figure 2.11: The sparticle masses for the LM6 benchmark model point, from [38]. The LM6 model is at $m_0 = 85$, $m_{1/2} = 400$, $\tan \beta = 10$, $A_0 = 0$, and $\text{sgn}(\mu) = +1$.

### 2.9 Previous results

The CMS and ATLAS experiments set limits on the CMSSM parameter space using $36/\text{pb}$ of data from the 2010 proton-proton run of the LHC, which was enough to supercede limits set by the Tevatron. These limits are shown in Figs. 2.12 and 2.13.
Figure 2.12: A 95% CLs upper limit was set on the cross-section for all of the models in the $m_{1/2}$ vs. $m_0$ parameter space for $\tan \beta = 10$, $A_0 = 0$, and $\text{sgn}(\mu)=+1$ [39].

Figure 2.13: The ATLAS experiment used the combined 0- and 1-lepton channels to set a limit on $m_{1/2}$ vs. $m_0$ space at $\tan \beta = 3$, using 35 pb$^{-1}$ of p-p data [40].
Chapter 3

The Large Hadron Collider

The Large Hadron Collider is a 26.658883 km long superconducting proton-proton accelerator and collider. It was built in the same tunnel as the LEP experiment, between 45 m and 170 m below the surface. At 8 points around the ring, there are two high luminosity interaction regions at CMS and ATLAS, two low luminosity regions at ALICE and LHC-B, two beam cleaning stations, an RF acceleration unit, and the beam dump. The key LHC beam parameters are given in Table 3.1.

3.1 Magnets

The LHC makes extensive use of superconducting NbTi Rutherford cable magnets, cooled by superfluid Helium below 2 K. These can achieve fields above 8 T. Almost all LHC magnets use a "twin-bore" design, wherein both beam pipes share a common cold mass and cryostat. This complicates the design due to the strong magnetic and mechanically coupling between the two beam channels. In a typical dipole magnetic, the beam channel is surrounded by a inner layer (28 strands) and an outer layer (36 strands) of ribbon-shaped
<table>
<thead>
<tr>
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<tr>
<td><strong>Proton energy</strong></td>
<td>450 GeV</td>
<td>7000 GeV</td>
</tr>
<tr>
<td><strong>Relativistic gamma</strong></td>
<td>479.6</td>
<td>7461</td>
</tr>
<tr>
<td><strong>Number of particles per bunch</strong></td>
<td>$1.15 \times 10^{11}$</td>
<td></td>
</tr>
<tr>
<td><strong>Number of bunches</strong></td>
<td></td>
<td>2808</td>
</tr>
<tr>
<td><strong>Longitudinal emittance (4σ)</strong></td>
<td>[eVs] 1.0</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Transverse normalized emittance</strong></td>
<td>[µm rad] 3.5</td>
<td>3.75</td>
</tr>
<tr>
<td><strong>Circulating beam current</strong></td>
<td>[A] 0.582</td>
<td></td>
</tr>
<tr>
<td><strong>Stored energy per beam</strong></td>
<td>[MJ] 23.3</td>
<td>362</td>
</tr>
</tbody>
</table>

**Peak Luminosity Related Data**

<table>
<thead>
<tr>
<th></th>
<th>Injection</th>
<th>Collision</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RMS bunch length</strong></td>
<td>cm 11.24</td>
<td>7.55</td>
</tr>
<tr>
<td><strong>RMS beam size at IP5</strong></td>
<td>µm 375.2</td>
<td>16.7</td>
</tr>
<tr>
<td><strong>Geometric luminosity reduction factor F</strong></td>
<td>-</td>
<td>0.836</td>
</tr>
<tr>
<td><strong>Peak luminosity at IP5</strong></td>
<td>$[cm^{-2}s^{-1}]$ -</td>
<td>$1.0 \times 10^{34}$</td>
</tr>
<tr>
<td><strong>Peak luminosity per bunch crossing at IP5</strong></td>
<td>$[cm^{-2}s^{-1}]$ -</td>
<td>$3.56 \times 10^{30}$</td>
</tr>
</tbody>
</table>

Table 3.1: LHC beam parameters.

superconducting cable. The cross-sections of the cables are 1.9 mm x 15.10 mm and 1.48 mm x 15.10 mm, for the inner and outer layers respectively. These cables can produce a peak field of 8.33 T.

The LHC ring has 1232 main dipoles. Each contains a dipole cold mass, which holds both of the beam channels. The dipole cold masses are 16.5 m long, have a diameter of 570 mm, and a mass of 27.5 tonnes. They are curved at an angle of 5.1 mrad, meaning a radius of curvature of 2812 m. The integrated field and shape imperfections between different dipole magnets does not exceed 1 part in $10^4$.

The quadrapoles are also superconducting, except for some normal conducting quadrapoles at the beam cleaning insertions.

About 4800 multipole corrector magnets (sextupole, decapole, and octopole), both single and double aperture, are used to apply corrections to various beam parameters and for beam cleaning.
3.2 Injection Chain

The chain starts with a hydrogen bottle, which feeds a duoplasmatron proton source with 6 mL/min of Hydrogen. A cathode filament emits energetic electrons, which impact the hydrogen, creating a plasma which is then accelerated by charged grids, yielding a 500 mA ion beam of 90 KeV protons. A 1 m RF quadrapole boosts the energy to 750 keV. Linac2 is a 30 m accelerator that further increases the energy to 50 MeV at 150 mA.

Next is the Proton Synchrotron Booster (PSB) which is comprised of 4, 157 m circumference rings that accelerate the beams from 50 MeV to 1.4 GeV. The PSB uses dipoles for bending the beam, quadrapoles for beam shaping, and RF cavities to accelerate the beam. The PSB injects the accelerated beams into the Proton Synchrotron (PS).

The PS accelerates the protons from 1.4 GeV to 25 GeV. It also initiates the 25 ns bunch spacing, which is the shortest amount of time between any proton bunches in the
LHC. The rest of the beam gaps that comprise the filling scheme of the LHC are determined by the SPS filling scheme, the SPS and LHC injection kicker rise times, and the LHC beam dump kicker rise time, as shown in Fig. 3.2. When the PS cycle is complete, it has 72 proton bunches (plus 12 empty buckets for the kicker) separated by a bunch spacing of 24.97 ns, and can repeat the process every 3.6 s.

![Bunch Disposition in the LHC, SPS and PS](image)

Figure 3.2: The proton bunch filling scheme is determined by the required beam gaps and the partial filling of the SPS.

The PS transfers the beam to the Super Proton Synchrotron (SPS), which is a 6.9 km circumference ring that accelerates the beams to 450 GeV. The SPS originally delivered colliding proton-antiproton beams to the UA1 and UA2 experiments from 1981 to 1984 (resulting in the discoveries of the W and Z bosons), and currently provides 400 GeV proton beams to several active fixed target experiments, and produces a neutrino beam for the Gran Sasso laboratory, in addition to what it does for the LHC.

The SPS is limited to accelerating an intensity of about $4 \times 10^{13}$ protons per cycle, which limits it to 4 PS extractions per fill of the SPS. Only 3/11 or 4/11 of the SPS is filled at maximum (see Fig. 3.2). The SPS has transfer lines that finally inject the proton beams
into the LHC rings.

3.3 RF System

At Point 4 there is a 400 MHz RF superconducting cavity system called the RF Accelerating System (ACS). This is used to capture, accelerate and store the injected 450 GeV beam in the main LHC ring and accelerate it to nominal 7 TeV. Injection errors are damped using the same RF system. A 7 TeV beam requires a 16 MV voltage per beam. During beam acceleration, the RF system applies 275 kW to each beam.

The 200 MHz Capture System (ACN) is an additional RF system used to significantly reduce capture losses when the injected bunch emittance (beam width) is larger than expected. A transverse damping and feedback system (ADT) is used to dampen transverse injection errors, prevent coupled bunch instabilities, and to create transverse oscillations for the purpose of beam measurements.

3.4 Interaction Regions

In the interaction region, the beams are brought together into one beam pipe so that they can collide. Starting from the IP and moving out the to insertion point, the interaction region is set up as follows:

- A low-β triplet of quadrupoles is used for focusing. Each quadrupole focuses in one direction while defocusing in the other. The net effect of all the quadrupoles is to make a very small beam diameter at the interaction point.

- The D1 dipole magnet is a conventional single-bore magnet used for separating (and recombining) the beams at 1.38 T.
Figure 3.3: The beams are focused to very small dimensions at the interaction region.

- The D2 dipole is a 9.45 m long superconducting twin bore magnet. The beams travel in separate channels again.

- Finally, a sequence of four quadrupole magnets. The first is a wide-aperture magnet at 4.5 K, followed by 3 normal-aperture quadrupole magnets at 1.9 K.

3.5 Luminosity

The LHC provides collisions at a rate of

\[ N_{\text{event}} = L \sigma_{\text{event}} \]  

(3.1)

for a process of cross-section \( \sigma \) and machine luminosity \( L \), which is given by

\[ L = \frac{N_b^2 n_b f_{\text{rev}} \gamma_r}{4 \pi \epsilon_n \beta*} F. \]  

(3.2)
$N_b$ is the number of protons per bunch, $n_b$ is the number of bunches per beam, $f_{\text{rev}}$ is the revolution frequency, $\gamma_r$ is the relativistic gamma factor, $\epsilon_n$ is the normalized transverse beam emittance (a measure of the spatial RMS width and also the spread in momentum of the beam), $\beta^*$ is the beta function at the collision point (which is the beam width over the emittance), and $F$ is a geometric luminosity reduction factor due to the non-zero beam crossing angle at the interaction point. The LHC is designed to provide an ultimate center-of-mass energy of $\sqrt{s} = 14$ TeV and peak luminosity of $L = 10^{34}$ cm$^2$s$^{-1}$ in proton collision mode.
Chapter 4

The CMS Detector

4.1 Introduction

CMS is a general purpose detector designed to make high resolution measurements of proton-proton collisions at a high rate [41]. When the LHC operates at its design parameters, it will provide collisions at a center of mass energy of $\sqrt{s} = 14$ TeV at a luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$, which for an inelastic proton-proton cross-section of 100 mb will produce $10^9$ inelastic events/s. With a bunch crossing every 25 ns, this means that CMS must be able to handle about 20 inelastic collisions yielding 1000 charged particles per bunch crossing, superimposed over any interesting events. To handle this high rate, high pile-up environment, CMS uses detectors with high granularity and good time resolution, so that the bunch crossing for each particle can be identified. The subdetectors also have certain resolution requirements to meet the physics needs:

- The subdetector closest to the interaction point is the silicon pixel tracker and silicon strip tracker, which provide precise secondary vertex and charged track measurement.
• Next is the electromagnetic calorimeter (ECAL), which is a finely segmented collection of lead tungstate crystals (PbWO$_4$) for accurately measuring the energy deposited by photons and electrons. It provides coverage up to $|\eta| < 3.0$.

• The hadronic calorimeter (HCAL) is a brass/scintillator sampling calorimeter that measures the energy deposited within 7-10 hadronic interaction lengths $\lambda_I$ (10-15 $\lambda_I$ in the barrel) of material. This has coverage out to $|\eta| < 3.0$, with an iron-quartz fiber calorimeter extending coverage out to $|\eta| < 5.0$. In the barrel, additional calorimetry (the HO) after the solenoid extends the measurement to more interaction lengths.

• The superconducting solenoid after the HCAL provides a 3.8 T field for accurate momentum measurement in the silicon tracker, and the ferromagnetic field return yoke in the CMS endcap and outer barrel results in bending of muon tracks in the muon system.

• The muon tracking system uses drift tubes (DT) in the barrel and cathode strip chambers (CSC) in the endcap. Muons segments are measured in up to 4 stations as the particles move outward beyond CMS. The muon system is used for triggering purposes and a good momentum resolution is achieved by combining tracking information from the silicon tracker and muon system.

### 4.2 Particle Identification

The particle identification strategy of CMS is summarized in Fig. 4.2. The silicon tracker identifies displaced vertices (due to tau leptons, charm and bottom quarks, for example) and measures charged tracks. Electrons and photons are stopped in the ECAL;
Figure 4.1: Perspective view of CMS with subdetectors indicated.
electrons should be associated with a track that has the same energy as the ECAL deposit. Hadrons deposit most of their energy in the HCAL. Muons leave a silicon track, a small amount of energy in the calorimeters due to minimum ionizing energy loss, and finally a track in up to 4 segments in the muon tracking system that is interleaved with the field return yoke.

4.3 Superconducting magnet

The magnetic field of CMS is produced by a superconducting solenoid designed to achieve a field of 4 Tesla. The full field is available within a 6 m diameter, 12.5 m long cylinder that houses the tracker, ECAL, HCAL, and the first station of the muon CSC endcap. Outside the solenoid, the field is compressed and returned by a 10,000 tonne iron yoke that is interleaved with the barrel and endcap muon tracking chambers.

A high field was chosen so that CMS could achieve good momentum resolution on
muons and other charged particles. This also makes it possible to avoid difficult require-
m ents on the spatial resolution and alignment on the muon chambers, and allows for com-
 pact electromagnetic and hadronic calorimeters. The length of the magnet allows for good
 tracking efficiency up to a high pseudorapidity of $|\eta| = 2.4$.

To minimize multiple scattering, the coil is thin ($\Delta R/R \approx 0.1$). The magnet has a
 relatively high hoop strain ($\epsilon$) compared to previous experiments, but the stored energy
 divided by the hoop strain is very high compared to other magnets. This is achieved by
 using the conductor as part of the structural material.

<table>
<thead>
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<th>Value</th>
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<td>Magnetic length</td>
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</tr>
<tr>
<td>Cold bore diameter</td>
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</tr>
<tr>
<td>Central magnetic induction</td>
<td>3.8 T</td>
</tr>
<tr>
<td>Total Amerbere-turns</td>
<td>41.7 MA-turns</td>
</tr>
<tr>
<td>Nominal current</td>
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</tr>
<tr>
<td>Inductance</td>
<td>14.2 H</td>
</tr>
<tr>
<td>Stored energy</td>
<td>2.6 GJ</td>
</tr>
</tbody>
</table>

Table 4.1: Properties of superconducting solenoid magnet.

### 4.4 Iron yoke

The iron yoke is embedded in 6 endcap disks and 5 barrel wheels. Aside from returning
the field, the yoke serves the purpose of being the main structural support of the various
subdetectors. Each endcap and wheel has high-pressure air pads, and for the final approach,
special flat grease-pads, so that the components can be moved around. When the detector is
being closed, the wheels and disks are aligned at the aluminum-alloy z-stops and pre-stressed
with 100 t between each element prior to turning on the magnet. All of the detector elements
can be aligned within 2 mm with respect to the coil center and coil axis using a 70 point
Outer diameter of the iron flats | 14 m
---|---
Length of barrel | 13 m
Thickness of the iron layers in barrel | 300 and 630 mm
Mass of iron in barrel | 6000 t
Thickness of iron disks in endcaps | 250, 600 and 600 mm
Mass of iron in each endcap | 2000 t
Total mass of iron in return yoke | 10,000 t

Table 4.2: Parameters of the iron yoke.

Figure 4.3: The return of the solenoidal magnetic field in the ferromagnetic yoke. In the barrel and endcap muon tracking systems, most of the track bending takes place inside the iron.

reference system installed in the detector hall.

4.5 Silicon Tracker

The inner tracking system is designed to reconstruct primary and secondary vertices and make precise measurements of charged particle tracks. The tracker is an all-silicon design, which allows it to have high granularity, fast response time (for measuring interactions at every 25 ns bunch crossing), and radiation hardness. The tracker geometry is
illustrated in Fig. 4.4. In total there is about 200 m² of active silicon making it the largest silicon tracker ever made. The on-detector electronics requires cooling. All of this material results in a certain amount of multiple scattering, bremsstrahlung, photon conversion, and nuclear interactions that reflect compromises in the design. The material budget, in terms of radiation lengths and hadronic interaction lengths, as a function of $\eta$ is shown in Fig. 4.5.

The pixel detector is made of 1440 modules, covering an area of 1 m² with 66 million pixels. It is capable of 3D vertex reconstruction, which is necessary for identifying b and tau decays. The silicon strip tracker has a total of 9 million strips.

The transverse momentum resolution of the CMS tracker is very good, achieving a 1-2% resolution on $p_T=100$ GeV tracks up to $|\eta| \approx 1.6$. High $p_T$ tracks have an impact parameter resolution of just 10\(\mu\)m. The track reconstruction efficiency is about 99% for muons.
4.6 Electromagnetic calorimeter

The electromagnetic calorimeter (ECAL) is a homogenous, high granularity detector composed of 61,200 lead tungstate (PbWO$_4$) crystals in the barrel and 7324 crystals in the endcap. The density (8.28 g/cm$^3$) and short radiation length (and small molière radius) makes the detector fast, which makes bunch crossing identification easier. The material is radiation hard. The ECAL barrel (EB) covers $|\eta| < 1.479$, and the endcap (EC) extends coverage to $|\eta| < 3.0$.

A pre-shower detector is installed in front of the ECAL in the endcap to identify neutral pions, and to help distinguish them from prompt photons. Electrons and photons interact with a lead layer to initiate electromagnetic showers. The early part of the shower is measured with very good spatial resolution. The crystals are oriented in a nearly-projective geometry, with the pointing offset by 3$^\circ$ from the interaction point so that interpolation between crystals can improve spatial resolution. The overall geometry and grouping of the
crystals is shown in Fig. 4.7.

The energy resolution can be parameterized with a stochastic term, a noise term, and a constant term:

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{2.8\%}{\sqrt{E}}\right)^2 + \left(\frac{0.12}{E}\right)^2 + (0.30\%)^2$$

(4.1)

The performance of the ECAL can be illustrated with the energy resolution of electrons in $Z \rightarrow e^+e^-$ events as shown in Fig. 4.8.

4.7 Hadron Calorimeter

The CMS hadron calorimeter (HCAL) is used to measure the energy deposited by quark and gluon jets. Its also a key component in measuring the missing transverse energy, which can be due to Standard Model neutrinos and potentially new particles such as the neutralino.

The HCAL comes after the ECAL at $R = 1.77$ m and before the magnet at $R = 2.95$ m. In the barrel, the outer hadron calorimeter (HO) is a collection of scintillating tiles placed after the magnet to catch the tail end of large hadronic showers.
Figure 4.7: The ECAL is arranged into 5x5 supercrystals, modules, and supermodules. In each endcap, the crystals are arranged into two Dee structures.

Figure 4.8: The energy resolution of electrons in $Z \rightarrow e^+e^-$ using the 2011 dataset.
Figure 4.9: HCAL geometry.

The HCAL barrel (HB) extends to $|\eta| < 1.3$. The HB is a sampling calorimeter: it has alternating layers of metal and scintillator to create hadronic interactions and measure the energy deposited. The first layer is 40mm of steel, followed by 14 layers of brass, then a final back plate of steel. Between each layer is a scintillating tile; in all there are 70,000 tiles. Each tile has a wavelength shifting fiber (WLS) that absorbs incoming light and emits lower energy photons, which are passed to a clear fiber. The clear fiber transports the light to a hybrid photodiode (HPD) where it is amplified and read out.

The HCAL is arranged into $\eta$ towers, each of which has a size $(\Delta\eta, \Delta\phi) = (0.087, 0.087)$, which overlap with 5x5 arrays of ECAL crystals. The first 14 $\eta$ towers have a single longitudinal read-out, while the geometry in the overlap region results in 2 read-out segments in depth.

The hadron calorimeter endcap (HE) covers $1.3 < |\eta| < 3$, and provides hermetic coverage in the endcap and overlap region. It has about 10 interaction lengths of material.
Figure 4.10: WLS fibers in the scintillator tiles use phosphorescence to create low energy photons from the initial high energy photons. The metal absorber layers are in red.

The outer hadron calorimeter (HO), installed in the barrel $\eta$ region beyond the magnet, provides additional shower containment. The HO uses the solenoid coil and (in the central wheel) the first ring of iron yoke as additional absorbers for late starting showers and large showers.

The forward hadron calorimeter (HF) which extends coverage to $|\eta| < 5$ is subject to very high particle fluxes. It uses quartz fibers which produce Cherenkov light when charged particles are incident. These fibers are layered into the grooves of 5 mm thick steel plates.

The jet resolution performance of particle flow jets in two eta regions is indicated in Fig. 4.11.
4.8 The muon system

Muons are central to the design of CMS due to the precision with which the momentum can be measured compared to other prompt particles. Also important is how they can be identified with a high degree of confidence. This is a result of muons being minimum-ionizing particles, which allows muons of a high-enough energy to travel through the entire detector without losing much of their energy.

The muon system is arranged into a barrel section consisting of 5 wheels in the endcap, plus endcaps in a planar disk geometry. Resistive Plate Chambers (RPCs) are used in both the barrel and endcap to provide very precise timing information and unambiguous bunch crossing identification. In the barrel, where the flux and neutron-induced background is lower, Drift Tube (DT) chambers in rectangular cells are used. Depending on the chamber, they measure the bending in the r-φ plane or the z coordinate. In the endcap, trapezoidal Cathode Strip Chambers (CSCs) arranged in disks are designed to make a precise
The individual drift tubes contain an anode wire (+3600 V), electrode strips (+1800 V), and cathode strips (-1200 V) on the far walls, as shown in Fig. 4.13. In each drift tube, there is a long wire (about 2.4 m long in the r-φ case) in a gas mixture of 85% Ar and 15% CO₂. This gas is ionized by the muon, and the level of ionization is amplified by the strong electric field. The charges travel with a very constant drift velocity toward either the positive or negative electrodes. The resolution is about 250 µm per DT cell and 100 µm per chamber (for 8 hits).

The muon endcaps consist of 468 cathode strip chambers (CSCs). The cathode strip chambers are multiwire proportional chambers, with 6 planes of anode wires perpendicular to cathode strips. The wires make a coarse measurement of the radial coordinate and the cathode strips (which have a width constant in ∆φ) measure φ. Charges are deposited on
more than one cathode strip, allowing interpolation to get an accurate measurement of $\phi$.

A CSC chamber is shown in Fig. 4.14.

The resolution of the muon system is given in Fig. 4.15.

4.8.1 Resistance Plate Chambers

The resistance plate chambers (RPCs) are gaseous parallel-plate detectors with a very fast time response of 3 ns (much faster than the 25 ns between collisions). These are distributed throughout the muon system to help unambiguously identify the bunch crossing associated with each muon. The muon system trigger is based on the RPCs.

RPCs are double-gap modules (consisting of up and down gas gaps, shown in Fig. 4.17). The 2mm gaps are filled with a non-flammable mixture of $\text{C}_2\text{H}_2\text{F}_4$, 1-3 atm above atmospheric pressure. A high voltage (8.5 - 9.0 kV) results in an avalanche mode, where
Figure 4.14: (a) The radial strips and horizontal wires comprising one CSC plane. (b) Close-up view of avalanche with charge deposition on the wires and cathode strips.

Figure 4.15: A comparison between the muon $p_T$ resolution in the standalone muon system, the silicon tracker, and the combined fit.
Figure 4.16: RPC chambers on a muon endcap disk.

Figure 4.17: The CMS RPCs have a 2mm gas gap sandwiched between 2mm thick panels of bakelite semiconductor. The outside of the bakelite is coated in conductive graphite paint. The aluminum anode strips (separated from the graphite by an insulating film) lie between the gaps.
charge is picked up by read-out strips between the gaps.

4.9 Trigger

For a proton-proton beam crossing every 25 ns, the bunch crossing frequency is 40 MHz. To be able to process and store the large number of events, events are filtered by the trigger system, which acts as the first step in the event selection. The trigger operates at two levels: the Level-1 (L1) Trigger and the High-Level Trigger (HLT). The L1 Trigger is done at the hardware level, with programmable electronics (using field-programmable gate arrays) and subdetector-specific integrated circuits. The L1 trigger computes observables of interest using coarsely grained data while holding the full resolution data in pipelined memories. The L1 trigger architecture is shown in Fig. 4.18. The HLT is a software-based filter and is run by about one thousand processors. The HLT has the full detector data available and reconstructs the event objects in a way similar to that of the full offline reconstruction. The combined filter factor is about $10^5$ for the whole trigger system, with an output rate from L1 of up to 100 kHz (from 40 MHz), and an HLT output rate of 300 Hz (from 100 kHz).

In the calorimeters, the L1 trigger forms trigger towers that compute the $E_T$ of each $0.087 \times 0.087$ HCAL unit and the $E_T$ of the underlying 5x5 ECAL cluster, and assigns the bunch crossing number to these deposits. The Regional Calorimeter Triggers determine the electron and photon candidates, transverse energy sums, minimum-ionizing particle bits and isolation bits (the latter information is passed to the Global Muon Trigger).

The L1 muon trigger is comprised of the DT, CSC and RPC triggers. The DT trigger finds track segments in the $\phi$ direction, and hit patterns in the $\eta$-projection, while the CSCs find 3D track segments. Each chamber type separately determines the bunch
crossing number. The Regional Muon Trigger reconstructs standalone muon system tracks, as does the RPC trigger, and the Global Muon Trigger combines all the sub-detectors.
Chapter 5

Monte Carlo Simulation

The Monte Carlo simulations and cross-sections used in this analysis are listed in Table 5.1. The MadGraph event generator [42] is used to simulate the $W$, $Z$, and $t\bar{t} + jets$ samples with the final hadronization performed by Pythia [43]. The QCD and diboson (WW, WZ, ZZ) samples are generated entirely in Pythia. Single top quark production is simulated by powheg followed by Pythia. The supersymmetric signal model LM6 and the 2D scan of the mSUGRA $m_0 - m_{1/2}$ plane are simulated in Pythia. The CMS detector response is modeled in Geant4 [44].
<table>
<thead>
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<th>Dataset</th>
<th>σ (pb)</th>
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<tbody>
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<td>9.361 \times 10^3</td>
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</table>

Table 5.1: Monte Carlo samples used in this analysis.
Chapter 6

Event Reconstruction and Selection

6.1 Discriminating variables

The key variables that discriminate between the Standard Model (SM) background and Supersymmetry in this analysis are the scalar sum of the transverse jet momenta ($H_T$), the missing transverse energy ($E_T$), the number of jets ($N_{jet}$), and the transverse mass ($M_T$).

6.1.1 Number of jets

The cross-section for W and Z production falls quickly as the number of quark and gluon jets increases, since each additional strong coupling vertex multiplies the interaction probability by a factor of the strong coupling constant $\alpha_S$.

In decays of pair-produced top quarks, there are always at least 3 jets when one of the W bosons decays leptonically. Each top quark decays to a W boson and $b$-quark. In
the one-lepton channel, one $W$ decays to a pair of quarks which hadronize to form jets, while the other $W$ decays to a lepton and a neutrino. Initial state radiation (ISR) and final state radiation (FSR) will often add additional jets. Therefore two $b$-jets and at least one additional jet are associated with $t\bar{t} \rightarrow l^\pm + jets$.

Top quark pairs can also decay to two leptons, in which case there are at least two $b$-jets but not necessarily any additional jets.

We require the presence of at least 4 reconstructed jets (each with $p_T > 40$ GeV/c), which results in a relatively efficient rejection of $W$ and $Z$ bosons, and a partial suppression of the $t\bar{t}$ dilepton background.

### 6.1.2 Scalar jet momentum sum

$H_T$ is defined as the scalar sum of the transverse jet momenta:

$$H_T \equiv \sum p_T^{jet} \quad (6.1)$$

Jets included in the sum must be above the $p_T > 40$ GeV threshold.

### 6.1.3 Missing transverse momentum

The missing transverse momentum is denoted by the symbol $E_T$. It is calculated from the negative vector sum of the momenta of all the particle flow candidates.

$$E_T = \left| - \sum p_T \right| \quad (6.2)$$
6.1.4 Transverse Mass

In the decay of a $W$ boson to a lepton and a neutrino, the invariant mass of the $l + \nu$ system can be written as

$$M^2 = (p_l + p_{\nu})^2 = (E_l + E_{\nu})^2 - (\vec{p}_l + \vec{p}_{\nu})^2$$

$$= m_l^2 + m_{\nu}^2 + 2(E_l E_{\nu} - \vec{p}_l \cdot \vec{p}_{\nu})$$

(6.3)

The energies of the lepton and neutrino are

$$E_l = \sqrt{m_l^2 + p_{x,l}^2 + p_{y,l}^2 + p_{z,l}^2}$$

$$E_{\nu} = \sqrt{m_{\nu}^2 + p_{x,\nu}^2 + p_{y,\nu}^2 + p_{z,\nu}^2}$$

(6.4)

The transverse mass $M_T$ is defined as the invariant mass without the $z$ components of the energy or momentum:

$$M_T^2 \equiv m_l^2 + m_{\nu}^2 + 2(E_{T,l}E_{T,\nu} - p_{T,l}p_{T,\nu} \cos \phi)$$

(6.5)

The angle $\phi$ is the azimuthal angle between the lepton and neutrino (i.e., the angle between them in the $x - y$ plane; this plane is transverse to the beam line). Compared to the $W$, the lepton and neutrino are approximately massless, so we use

$$M_T^2 = 2E_{T,l}E_{T,\nu}(1 - \cos \phi)$$

(6.6)
Experimentally, there is no way to directly measure the neutrino $p_T$ but we can infer it from the missing transverse momentum. $M_T$ is taken to be

$$M_T^2 = 2p_{T,l}E_T(1 - \cos \phi)$$ \hspace{1cm} (6.7)

The lack of $z$ components means that $M_T$ obeys the inequality

$$M_T \leq M_W$$ \hspace{1cm} (6.8)

In the one-lepton channel, this inequality allows us to strongly reject background events that include semileptonic decays of $W$ bosons if we require $M_T > M_W$. In events with one prompt lepton (due to a $W$), it’s still possible to have $M_T > 80.4$ GeV/$c^2$ for the following reasons:

- The $W$ boson mass follows a Breit-Wigner distribution with width 2.14 GeV.
- Mismeasurement of lepton $p_T$.
- Mismeasurement of $E_T$.
- Additional neutrino(s) from b and c quark decays, and low momentum neutrinos from jets.

When top quarks decay to two leptons and two neutrinos, the transverse mass is not bounded by $M_W$. Likewise, in R-Parity conserving Supersymmetry, the leptons and neutrinos can be due to particles other than the $W$ boson and there are at least two missing particles.

In the MC simulation, shown in Fig. 6.1 we see that mis-measurement of the lepton transverse momentum does not result in a large change to the $M_T$ shape. As shown in
Fig. 6.2, we see a small change in $M_T$ shape when we use all neutrinos instead of the single correct neutrino, and a much larger change when we use the reconstructed missing transverse momentum $\not{E}_T$. As a shorthand, we say that the difference between the reconstructed $\not{E}_T$ and the true neutrino momentum is the *artificial* $\not{E}_T$.

The largest resolution effect on the transverse mass tail is expected to be from the reconstructed missing transverse momentum, so we model this component of $M_T$ using a data-driven method.
Figure 6.1: $M_T$ for a lepton momentum which is either generated (in black) or reconstructed (in green). The missing transverse momentum is taken to be the true simulated neutrino momentum. Events with exactly one leptonic decay, $H_T > 500$ GeV, $N_{jet} \geq 4$, and $E_T > 150$ GeV/c are selected.

6.2 Transverse mass in conjunction with other variables

At high values of $E_T$, a large fraction of the missing momentum tends to be due to the prompt neutrino rather than the artificial $E_T$. This results in a reduction in the tail of the $M_T$ distribution.
Figure 6.2: $M_T$ computed from either the generated neutrino momentum (in black), the true $E_T$ from all missing particles (in green), or the reconstructed $E_T$ (in red). The true lepton momentum is used. Events with exactly one leptonic decay, $H_T > 500$ GeV, $N_{jet} \geq 4$, and $E_T > 150$ GeV/c are selected.
Figure 6.3: $H_T$ vs. $M_T$ for the combined $\mu$ and $e$ channels.

Figure 6.4: $E_T$ vs. $M_T$ for the combined $\mu$ and $e$ channels.
Table 6.1: List of single electron and single muon high level triggers, and dilepton triggers used in this analysis.

<table>
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<th>Single electron triggers</th>
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<td>HLT_Ele10_CaloIdL_CaloIsoVL_TrkIdVL_TrkIsoVL_HT200_v*: v2-v4</td>
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<td>HLT_Ele15_CaloIdT_CaloIsoVL_TrkIdT_TrkIsoVL_HT250_PFMHT40_v*: v4-v5</td>
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<table>
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<th>Single muon triggers</th>
</tr>
</thead>
<tbody>
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<td>HLT_Mu8_HT200_v*: v1-v5</td>
</tr>
<tr>
<td>HLT_Mu15_HT200_v*: v2-v4</td>
</tr>
<tr>
<td>HLT_HT250_Mu15_PFMHT20_v*: v3, v5, v7</td>
</tr>
<tr>
<td>HLT_HT250_Mu15_PFMHT40_v4</td>
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</table>

<table>
<thead>
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<th>Dilepton triggers</th>
</tr>
</thead>
<tbody>
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<tr>
<td>HLT_DoubleMu3_HT150_v*</td>
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<td>HLT_DoubleMu5_Mass4_HT150_v*</td>
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<tr>
<td>HLT_DoubleMu5_Mass8_HT150_v*</td>
</tr>
<tr>
<td>HLT_DoubleEle8_CaloIdL_TrkIdVL_HT160_v*</td>
</tr>
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<td>HLT_DoubleEle8_CaloIdL_TrkIdVL_HT150_v*</td>
</tr>
<tr>
<td>HLT_DoubleEle8_CaloIdL_TrkIdVL_Mass8_HT150_v*</td>
</tr>
</tbody>
</table>

6.3 Triggers

The data are selected using triggers requiring a single lepton, $H_T$ and $E_T$. Table 6.1 lists the high-level triggers used. The offline selection cuts are above the efficiency turn-on curve for all of the triggers. In estimating the background due to dileptons, a dilepton control sample is needed, for which the dilepton+$H_T$ triggers paths are utilized in addition to the single lepton paths.

6.4 Leptons

Two categories of leptons are used: tight leptons and veto leptons. The tight leptons have high transverse momentum, tight requirements on ID variables, and are well isolated.
from other particles in the event. Veto leptons satisfy looser requirements, and by definition, tight leptons are removed from the list of veto leptons in each event. The event selection then requires that there is exactly one tight lepton and no veto leptons. The lepton isolation requirement, along with the cut on $E_T$, ensure that the QCD background is kept very low. The veto on loose leptons minimizes the contribution due to $Z$ and $t\bar{t}$ where two leptons are produced but one of them fails to satisfy the tight ID or isolation cuts.

For a muon to satisfy the tight ID and isolation, it must satisfy the following requirements.

- “GlobalMuonPromptTight” ID: a global track fit (starting in the muon system) has $\chi^2 < 10$
- Muon is a “Tracker Muon”: an extrapolated track from the silicon tracker matches to segments in the muon system
- $p_T \geq 20$ GeV
- $|\eta| < 2.1$
- Combined relative isolation (RelIso) < 0.1. RelIso is the sum of the energy deposits (within a cone of size $\Delta R = 0.3$) in the ECAL, HCAL, and tracker, divided by the $p_T$ of the muon. The muon track itself and energy deposited in ECAL/HCAL cells very close to the extrapolated muon track are not included in the sum.

$$RelIso \equiv \left( \sum_{\Delta R<0.3} E_{T}^{\text{had}} + \sum_{\Delta R<0.3} E_{T}^{\text{em}} + \sum_{\Delta R<0.3} p_{T}^{\text{trk}} \right) / p_{T}^{\mu} \quad (6.9)$$

- $|d_0| < 0.02$ cm
- $|d_Z| < 1$ cm
• At least 11 valid tracker hits.

• At least 2 muon segment matches to the global track.

• At least 1 pixel hit on inner track

• Small $p_T$ error on global track: $p_{TErr}/p_T^2 < 0.001$

• Agreement with Particle Flow muon: $|p_T^{reco} - p_T^{PF}| < 0.2$

The electrons requirements are designed to reject converted photons and ensure a good match between the silicon track and the supercluster in the ECAL.

• $p_T \geq 20$ GeV

• $|\eta| < 2.5$

• $|\eta| > 1.566$ and $|\eta| < 1.4442$ to avoid barrel/endcap transition region

• Satisfy VBTF 80% efficiency working point cuts (see Table 6.2). This means a close match in $\phi$ and $\eta$ between the silicon track and the ECAL supercluster ($\Delta \phi$ and $\Delta \eta$), a small HCAL to ECAL response ratio ($H/E$), and a small shower shape $\sigma_{\eta\eta}$.

• Variables useful in rejecting photons that converted to an $e^+e^-$ pair are the distance between the electron and the nearest candidate partner electron track (“Dist”), the difference in angle $\cot \theta$ between these tracks, and the number of hits in the electron track.

• $|d_0| < 0.02$ cm

• $|d_Z| < 1$ cm
Table 6.2: Maximum allowed values for electron identification variables under the 80% and 95% efficiency working points.

<table>
<thead>
<tr>
<th></th>
<th>VBTF80</th>
<th>VBTF95</th>
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<tr>
<td>Missing hits $\leq$</td>
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<td>1</td>
</tr>
<tr>
<td>Dist</td>
<td>0.02</td>
<td>N/A</td>
</tr>
<tr>
<td>$\Delta \cot \theta$</td>
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<tr>
<td>Barrel:</td>
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<tr>
<td>Combined isolation</td>
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</tr>
<tr>
<td>$\sigma_{\eta \eta}$</td>
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<td>0.01</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
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<td>0.8</td>
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<tr>
<td>$\Delta \eta$</td>
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<td>0.007</td>
</tr>
<tr>
<td>$H/E$</td>
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<td>0.15</td>
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<tr>
<td>Endcap:</td>
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<td></td>
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<td>Combined isolation</td>
<td>0.07</td>
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<tr>
<td>$\sigma_{\eta \eta}$</td>
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<tr>
<td>$\Delta \phi$</td>
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<td>$\Delta \eta$</td>
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</tr>
<tr>
<td>$H/E$</td>
<td>0.025</td>
<td>0.15</td>
</tr>
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</table>

An event is vetoed if it has additional leptons satisfying looser cuts. The loose muons are defined as follows.

- “GlobalMuonPromptTight” ID
- $p_T \geq 15$ GeV
- $|\eta| < 2.5$
- $RelIso < 0.15$
- $|d_0| < 0.1$ cm
- $|d_Z| < 1$ cm

Loose electrons have the same cuts as tight electrons, but need only satisfy $p_T > 15$ GeV and the 95% efficiency working point called VBTF95.
Leptons satisfying tight ID requirements are cross-cleaned against jets to prevent double counting. First, all jets within $\Delta R < 0.1$ of muons and $\Delta R < 0.3$ of electrons are removed. Then, muons within $\Delta R < 0.3$ of jets are removed.

For the leptons satisfying the veto definitions (i.e. loose leptons), no overlapping jets are removed. Loose muons within $\Delta R < 0.3$ of jets are removed.

The event selection requiring one tight lepton and no loose leptons is done after the cross-cleaning.

### 6.5 Jets

Jets are reconstructed with the Anti-$K_T$ algorithm with a cone size $\Delta R = 0.5$, with the L1FastTJet, L2Relative, and L3Absolute jet energy corrections. The L1FastJet correction subtracts out the diffuse energy from pile-up measured in that event. The L2Relative and L3Absolute corrections calibrate the jets to have a uniform energy response in $\eta$ and on an absolute scale. The jet quality cuts are as follows:

- $p_T > \text{GeV}/c$
- $|\eta| < 2.4$
- Neutral particle electromagnetic fraction $< 0.99$
- Charged particle electromagnetic fraction $< 0.99$
- Charged particle multiplicity $> 0$
- Charged hadronic energy fraction $> 0$
6.6 Sample after Preselection

The lepton $p_T$, $N_{jet}$, jet $p_T$, $H_T$, $E_T$, and $M_T$ are shown in Figs. (6.5)-(6.12). In each plot, the MC is re-normalized to the combined $e$ and $\mu$ preselected data yield.

Figure 6.5: Transverse momentum of the lepton.
Figure 6.6: $N_{jet}$ after preselection.

Figure 6.7: Leading jet $p_T$ after preselection.
Figure 6.8: 2nd jet $p_T$ after preselection.

Figure 6.9: 3rd jet $p_T$ after preselection.
Figure 6.10: $H_T$ after preselection.

Figure 6.11: $E_T$ after preselection.
Figure 6.12: $M_T$ after preselection.

Figure 6.13: $M_T$ in the signal region ($E_T > 300$, $H_T > 400$, $N_{jet} \geq 4$). Note that this is a data vs. MC comparison, not a comparison between the data and the full data-driven background estimation.
Chapter 7

Background Determination

7.1 Analysis strategy

Two types of methods being employed in CMS searches for new physics are (a) multivariate methods and (b) cut-based data-driven methods. In Supersymmetry, the $H_T$, $E_T$, number of jets, transverse mass, and assorted variables related to the event topology (such as the stransverse mass $M_{T2}$) have distributions that are typically distinct from those of the Standard Model background. A multivariate method maximizes the signal to background ratio $S/B$ by weighting each event according to the signal-like and background-like characteristics of the event variables. In this way, a signal event, that might ordinarily fail one of the cuts in a cut-based analysis, might still be classified as a signal-like event in a multivariate analysis. Multivariate methods that work in this way include Boosted Decision Trees [45] and Artificial Neural Networks [46].

The disadvantage to using multivariate methods is that they require a training sample (consisting of background and signal MC) that must be accurate in simulating the tails of the various distributions; many distinguishing features of SUSY might present themselves
as tails in distributions rather than as clear features such as resonance peaks. Prior to analyzing the collider data, there is no guarantee the MC will properly model the data. One example of a variable that might not be properly modeled by the MC is the missing transverse momentum, which depends on a correct modeling of every type of particle in the event from all of the subdetectors.

A data-driven analysis will combine information from MC and several collision data control samples to model the background in a region of interest (called the signal region). For example, by incorporating the $E_T$ distributions from a set of a QCD-dominated control samples, one can automatically incorporate most of the resolution effects that affect this variable in the real data. In this analysis, we incorporate data-driven background methods in a cut-based analysis to ensure robustness in analyzing the 2011 data.

### 7.2 Single Lepton Background Determination

The background at high $M_T$ is estimated by applying a ratio to the number of events in the low $M_T$ sideband in data. The high-to-low $M_T$ ratio comes from the transverse mass shapes of the W and $t\bar{t}$ backgrounds, as modeled by Monte Carlo samples that have been modified by data-derived artificial $E_T$ templates.

The reconstructed $E_T$ is due to the real $E_T$ caused by the presence of a neutrino, and by artificial $E_T$ that is introduced by mismeasurement of all the other particles in the detector. As electrons and muons are generally well measured, the artificial $E_T$ is primarily due to jet mismeasurement.

Following the method of the CMS search supersymmetry in the channel with one lepton and missing transverse energy [47][48][49], we build templates of the artificial $E_T$ from collision data, by selecting events with zero leptons, and various amounts of jet activity.
The templates are binned in $H_T$ (with bin boundaries at 250, 300, 350, 400, 500, 600, and 800 GeV) and $N_{\text{jet}}$ (with bins for 3, 4, 5, and $N_{\text{jet}} \geq 6$). In data we trigger on the leading jet $p_T$. For each trigger, a different leading jet $p_T$ cut is used in order to stay above the turn on curve for that trigger. Also, the $p_T$ must be below a certain value so that highly prescaled triggers do not introduce large statistical fluctuations in areas that are statistically well covered by high $p_T$ triggers. The luminosity recorded for each trigger in the 2011 run range is determined from the `lumicalc2.py` script. In Fig. 7.1 we verify that the weighting factors for the various triggers are properly accounting for the L1 and HLT prescales over a very wide range. These events are used to fill $E_T$ templates as in Fig. 7.2 where the $E_T$ in QCD MC and Data are plotted.

We model the reconstructed $E_T$ by adding artificial $E_T$ to the simulated neutrino momentum in Monte Carlo events. The measured $E_T$ templates are utilized as probability distributions of the artificial $E_T$. The artificial $E_T$ is added at a random angle relative to the neutrino. In MC, the azimuthal angle of artificial $E_T$ has a small correlation with the neutrino angle. The angle between them, $|\Delta \phi|$, is shown in Fig. 7.3. The assumption that the artificial $E_T$ angle is uncorrelated with the neutrino angle results in a small systematic uncertainty.

To improve the uncertainty and the stability of the background estimate in the tail of the $M_T$ distribution, every simulated seed event samples the artificial $E_T$ template 100 times. The resampling necessitates the use of a bootstrapping method [50] to determine the uncertainty as follows. A list containing the event number and $M_T$ bin for each of the 100 samples is used to fill a histogram (one histogram for each $M_T$ bin) of the number of
Table 7.1: List of leading jet $p_T$ triggers used to measure the artificial $E_T$ templates.

<table>
<thead>
<tr>
<th>Trigger</th>
</tr>
</thead>
<tbody>
<tr>
<td>HLT_Jet30</td>
</tr>
<tr>
<td>HLT_Jet60</td>
</tr>
<tr>
<td>HLT_Jet80</td>
</tr>
<tr>
<td>HLT_Jet110</td>
</tr>
<tr>
<td>HLT_Jet150</td>
</tr>
<tr>
<td>HLT_Jet190</td>
</tr>
<tr>
<td>HLT_Jet240</td>
</tr>
<tr>
<td>HLT_Jet300</td>
</tr>
<tr>
<td>HLT_Jet370</td>
</tr>
</tbody>
</table>

Figure 7.1: Leading jet $p_T$ in MC and data.
Figure 7.2: A selection of $E_T$ templates.

(a) $N_{\text{jet}} = 4$, $400 < H_T < 500$.

(b) $N_{\text{jet}} = 4$, $H_T > 800$.

(c) $N_{\text{jet}} = 5$, $400 < H_T < 500$.

(d) $N_{\text{jet}} = 5$, $600 < H_T < 800$. 
events, with one entry per sample. In each $M_T$ bin, the background prediction is then given by the average number of events, as weighted by the number of samples (i.e. the mean of the histogram). (Fig. 7.4). The uncertainty is taken to be the RMS of the number of events. Thus, despite the many resamplings, the event count in each $M_T$ bin is based on the background prediction corresponding to the most samples, and the uncertainty is calculated from the spread in the number of independent simulated events.

7.2.1 Leptonically decaying taus

When $W \to \tau \nu \to l \nu \nu$, where the lepton is an electron or muon, both of these neutrinos are used in the modeled $E_T$. These events tend to have lower $M_T$.

7.2.2 Composition of the control region

The transverse mass is modeled separately for $W$ and $t\bar{t}$ backgrounds, resulting in two $M_T$ ratios $R_W = N_S(W)/N_C(W)$ and $R_{t\bar{t}} = N_S(t\bar{t})/N_C(t\bar{t})$. The total number of events

Figure 7.3: The angle between the artificial $E_T$ and the simulated neutrino, after all preselection cuts except for $E_T$ have been applied.
Figure 7.4: Number of samples vs. number of unique events.

$N_C$ in the low $M_T$ control region is then interpreted as being due to $f_W N_C$ events from W decays, and $(1 - f_W)N_C$ events due to $t\bar{t}$. While the $W$ boson fraction $f_W$ is taken from MC, the uncertainty on the relative $W$ and $t\bar{t}$ cross-sections is derived from data in section 8.3. The total background estimate in MC is then

$$N^{\text{pred}} = (f_W R_W + (1 - f_W) R_{t\bar{t}}) N_C$$ \hspace{1cm} (7.1)

The control and signal regions are defined by

- $N_C = N(30 < M_T < 120)$

- $N_S = N(M_T > 130)$

The control region has adequate statistics without having to use the lowest $M_T$ events in $0 < M_T < 30$. By avoiding this region, the chance of QCD making it into the selection is reduced even further. Also, there is some shape mismatch revealed in closure test at very
In Fig. 7.6 we do a closure test in a higher statistics, low \( \mathcal{E}_T \) validation region. We compare the \( M_T \) distribution from even-numbered simulated events in MC with the shape derived from the \( M_T \) model. The seed events in the \( M_T \) model are from odd-numbered MC events. In this closure test, artifical \( \mathcal{E}_T \) templates are taken from QCD MC.

In the high \( M_T \) signal region, the data-derived \( \mathcal{E}_T \) templates increase the ratio \( R_W \) by 8\%(\( e \)) and 9\%(\( \mu \)). However, for the \( t\bar{t} \) background which has very few modeled events at \( M_T > 130 \text{ GeV}/c^2 \), the ratio \( R_{t\bar{t}} \) increases by 150\%(\( e \)) and 300\%(\( \mu \)). On an absolute scale,
Figure 7.6: Closure test at (150 < \( \not{E}_T \) < 225) comparing the \( M_T \) shape of reconstructed MC events (in black) with modeled \( M_T \) (in red).

Table 7.4: Single lepton background prediction in the low \( \not{E}_T \) region (150 < \( \not{E}_T \) < 225).

<table>
<thead>
<tr>
<th></th>
<th>( R_W ) (Data)</th>
<th>( R_{t\bar{t}} ) (Data)</th>
<th>Bkg. est. (Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e )</td>
<td>0.039 ± 0.006 ± 0.010</td>
<td>0.014 ± 0.006 ± 0.008</td>
<td>3.2 ± 0.7 ± 1.9</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.034 ± 0.006 ± 0.009</td>
<td>0.011 ± 0.005 ± 0.007</td>
<td>4.5 ± 0.9 ± 1.6</td>
</tr>
</tbody>
</table>
(a) Muons in W+jets MC.

(b) Electrons in W+jets MC.

(c) Muons in $t\bar{t}$+jets MC.

(d) Electrons in $t\bar{t}$+jets MC.

Figure 7.7: Closure test in the signal region ($E_T > 300$) comparing the $M_T$ shape of reconstructed MC events (in black) with modeled $M_T$ (in red).

<table>
<thead>
<tr>
<th></th>
<th>$R_W$ (Data)</th>
<th>$R_{t\bar{t}}$ (Data)</th>
<th>Bkg. est. (Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>0.039 ± 0.012 ± 0.005</td>
<td>0.006 ± 0.012 ± 0.007</td>
<td>0.7 ± 0.2 ± 0.3</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.027 ± 0.008 ± 0.004</td>
<td>0.003 ± 0.009 ± 0.006</td>
<td>0.5 ± 0.16 ± 0.16</td>
</tr>
</tbody>
</table>

Table 7.5: Single lepton background prediction in signal region ($E_T > 300$).
however, the change in the $t\bar{t}$ background estimate is not dramatic due to the small size of the background compared to $W + jets$ and dilepton top quark background.

### 7.2.3 Contamination in the single lepton control region

The $M_T$ modeling is meant to find the high-to-low $M_T$ ratios for the $W$ and single-lepton $t\bar{t}$ backgrounds, so the low $M_T$ control region needs to be free of contaminating backgrounds that have a different $M_T$ shape. Contamination is chiefly from single top and dileptonic $t\bar{t}$. These backgrounds are small compared to the primary $W$ and $t\bar{t}$, so it is sufficient to subtract the expected MC yield from the control region. A 50% systematic error is assigned to the yield of the contaminating backgrounds when computing the uncertainty in the control sample $N_C$. Prior to subtraction, 13% of the high $E_T$, low $M_T$ control sample is contamination.

### 7.2.4 k-factor in data prediction

For the background prediction in the data, the k-factor $k = \text{true/predicted}$ is applied to the prediction in order to correct for the imperfect closure test in MC. The total single-lepton background prediction in data is then

$$N_{\text{pred}}^{} = k(f_W R_W + (1 - f_W) R_{\bar{t}t}) N_C$$

(7.2)

where the control yield $N_C$ is from data and the $M_T$ ratios are modeled from the $E_T$ templates measured in data.

The uncertainty on the correction factor $k$ is taken to be $(1 - k) \times 100\%$.  

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7.3 Dilepton Background Determination

The largest background at high $M_T$ is due to $t\bar{t}$ decaying to two leptons. In the case of $\mu^+\mu^-$, $e^+e^-$, and $e^\pm\mu^\mp$ the second lepton is either outside of the geometric detector acceptance or it escapes the loose lepton veto. The $\mu\tau$ and $e\tau$ backgrounds are substantial because there is no veto on the $\tau$.

In these events, the transverse mass has no upper bound because the missing momentum is due to 2 neutrinos. The artificial $E_T$ is thus less important in determining the size of the $M_T$ tail, so the method we use is largely based on determining the high-to-low $M_T$ ratio from Monte Carlo. Following the method for predicting the dilepton background described in the search for a single lepton, jets and $E_T$ described in [49], a control sample of dilepton events is used for predicting the $M_T$ ratio of dilepton backgrounds that pass the single lepton selection.

Lost and ignored leptons from $ee$, $\mu\mu$, and $e\mu$ are treated together. When a lepton is ignored, it evades the veto on the second lepton, but the lepton is taken into account in the $E_T$ calculation. Lost leptons are either outside of the detector acceptance or are not reconstructed, but these do not contribute much to the $E_T$. Therefore, the $M_T$ distribution from the dilepton control sample is used as the prediction for this background. The prediction is given by $\alpha N_l$, where $\alpha$ is the ratio in Monte Carlo of the true MC yield of dilepton events that pass the single lepton cuts with the number of events in the dilepton control sample, and $N_l$ is the number of events in the dilepton control sample at high $M_T$. The factor $\alpha$ is computed in the low $M_T$ control region $50 < M_T < 100$.

The background with a single lepton and a hadronic $\tau$ is treated differently due to the presence of a tau-jet and at least one additional neutrino compared to the dilepton
control sample. For each lepton, a random number $r_j$ is taken from a $p_T$-dependent tau-jet response function constructed from a $t\bar{t}$ Monte Carlo sample. Then, the lepton is replaced by a tau-jet with transverse momentum $r_j p_T^l$, and $(1-r_j) p_T^l$ is added vectorially to the $E_T$. The event then needs to have at least 3 jets and pass the $H_T$ cut. Each dilepton seed event is sampled one hundred times with different random numbers $r_j$ from the tau-jet response functions. The $M_T$ is computed from the modified $E_T$.

![Histograms](image)

(a) (b)

Figure 7.8: Tau response functions for (a) hadronic taus and (b) leptonically decaying taus.

The fourth category of dilepton background has one reconstructed lepton and one tau that decays to a lepton which is then ignored or lost. This background is treated in the same way the hadronic tau background, except the tau response function is built for taus that decay to a lepton. Each seed event in the dilepton control sample is sampled one thousand times with a random number $r_l$, and the event must pass the 4-jet cut. A reasonable background estimate is found when we assume that the lepton coming from the modeled tau is lost 100% of the time. The systematic error due this assumption is taken into account in a closure test.
For each of the dilepton backgrounds, a parameter $k = N_{\text{true}}/N_{\text{est}}$ is computed from a closure test in MC to quantify how closely the background prediction is the true background yield. Any deviation from a perfect closure test is taken as a systematic error. The total systematic error on the dilepton prediction is given by

$$ (1 - k) \times 100\% \oplus \Delta \alpha_{\text{stat}} \quad (7.3) $$

The statistical uncertainties of the control region yields for lost+ignored leptons, $l+\tau_h$, and $l + \tau_l$ are assumed to be 100% correlated with each other because many of the same dilepton events are used for the different cases.

### 7.3.1 Signal contamination

In the dilepton background prediction, signal contamination in the control regions can artificially increase the background prediction in the signal region. This can potentially decrease the significance of an observed excess of events, and it can weaken the expected limit on new physics because it reduces the difference between the background-only hypothesis and the signal+background hypothesis. We can increase the background yield by relaxing the $H_T$ and $E_T$ cut on $\alpha$ and the dilepton control region. A scale factor is used to translate the lower $H_T$ and $E_T$ to the signal region. This method thus reduces signal contamination as well as the statistical uncertainty of the background in exchange for an increased systematic uncertainty. For the dilepton background, $M_T$ varies much more slowly than $H_T$ and $E_T$.

The background estimate is then

$$ N_{\text{est}} = R_{H_T, E_T} \alpha N_{ll} \quad . \quad (7.4) $$
Table 7.6: Dilepton background in validation region.

In data, a factor $k = \frac{\text{True}}{\text{Predicted}}$ is also applied which corrects for the imperfect closure in MC. The ratio $\alpha$ and the dilepton count $N_{ll}$ are calculated at low $H_T$ and $E_T$.

The factor $R_{H_T,E_T}$ is computed from dilepton yields in MC:

$$R_{H_T,E_T} \equiv \frac{N_{H_T>450, E_T>300}}{N_{H_T>350, 225<E_T<300}} \quad (7.5)$$

Figure 7.9: $M_T$ of the background control region for (a) lepton + lost/ignored leptons, (b) $l+\tau_h$, (c) $l+\tau_l$. Shown super-imposed are opposite-sign events from the LM6 model point, which has a relatively large cross-section of 0.40 pb.
Figure 7.10: Background from $t\bar{t} \rightarrow (e/\mu) +$lost and ignored leptons. Comparisons between shapes of the true MC background (in black) with the prediction from the dilepton control sample (in red).

Figure 7.11: Closure test for $(e/\mu) +$ lost $\tau_h$. 
Figure 7.12: Closure test for $(e/\mu) + \text{lost } \tau_l$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Bkg. est. (MC)</th>
<th>Bkg. truth (MC)</th>
<th>k-factor</th>
<th>Bkg. est. (data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l+\text{lost/ig. } l$</td>
<td>$2.03 \pm 0.57$</td>
<td>$1.9 \pm 0.7$</td>
<td>$1.0 \pm 0.5$</td>
<td>$0.5 \pm 0.5$</td>
</tr>
<tr>
<td>$l + \tau_h$</td>
<td>$1.20 \pm 0.26$</td>
<td>$7.5 \pm 1.8$</td>
<td>$2.8 \pm 1.0$</td>
<td>$0.4 \pm 0.6$</td>
</tr>
<tr>
<td>$l + \tau_l$</td>
<td>$0.35 \pm 0.14$</td>
<td>$1.1 \pm 0.2$</td>
<td>$1.0 \pm 0.5$</td>
<td>$0.9 \pm 0.1$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$10.5 \pm 2.1$</strong></td>
<td><strong>$4.7 \pm 1.2$</strong></td>
<td><strong>$4.1 \pm 1.1 \pm 1.7$</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.7: Dilepton background in signal region $E_T > 300$. 
7.4 QCD background

The QCD background is due to low $p_T$ leptons coming from bottom quarks, charm quarks, and assorted hadrons from jets. This includes charged pions that decay in flight to a muon and neutrino, and “punchthrough” kaons that leave a track in part of the muon system. A photon from final-state radiation of a quark can convert to an $e^+e^-$ pair close to the beamline and be mis-identified as a prompt electron. The total QCD background however can be expected to be very small (as suggested by MC) due to the lepton id and isolation requirements, the $E_T$ cut and the $M_T$ cut. Nevertheless, a data-driven background estimation method was developed for the CMS search for a single lepton, jets and $E_T$ in case the QCD yield at high values of jet multiplicity, $H_T$, and $E_T$ was not modeled well by MC. The CMS $E_T$ analysis used the same object identification criteria and the same event reconstruction variables as the present analysis.

The QCD estimation relies on measuring the combined relative isolation ($RelIso$) distribution for QCD leptons in QCD-dominated sample data, and applying that $RelIso$ distribution to a control sample near the signal region. The shape of $RelIso$ from QCD leptons is obtained by inverting the cut on the impact parameter between the lepton and the beamspot $|d0BS|$, and by requiring $E_T < 100$. These cuts reduce the signal lepton contamination from $t\bar{t}$ and $W/Z$. Isolated leptons from $W$, Drell-Yan/Z, and $t\bar{t}$ dominate the sample at small $RelIso$, and present a small contamination of the QCD control region at large $RelIso$. In data, the MC-expected yield from $W/Z/t\bar{t}$ is subtracted from the high $RelIso$ control sample. The non-isolated region is $0.5 < RelIso < 1.5$ for muons while the isolated region is $RelIso < 0.1$ for muons and $RelIso < 0.07$ for electrons. Due to the low efficiency of the electron trigger for non-isolated electrons, the electron control region
Figure 7.13: Left: Muon RelIso distribution in stacked histogram of MC signals and QCD background at $150 < E_T < 250$. Right: RelIso in data, data with MC signal shapes

is $0.35 < \text{RelIso} < 0.70$.

When there is no $E_T$ cut, the $M_T$ cut used in the control sample ($N_C$) selection has an efficiency of $0.52 \pm 0.07$ for QCD, decreasing to $0.26 \pm 0.11$ for $E_T > 100$. At the higher $M_T$ cut of the signal region ($N_S$), an increasing $E_T$ cut is not so effective at reducing the $M_T$ cut efficiency, which goes from $0.06 \pm 0.07$ for $E_T > 0$, to $0.15 \pm 0.06$ for $E_T > 50$, to $0.14 \pm 0.10$ for $E_T > 100$. We use the $0.14 \pm 0.10$ efficiency value for the efficiency of the signal region $M_T$ selection. The yields from the CMS $E_T$ analysis are also adapted for the different $H_T$ selection.

The resulting small yields for the QCD background are given in Table 7.8

### 7.5 Single top quark background

Single top quark production occurs in the $s$, $t$, and $tw$ channels indicated in Fig. 7.14. This background has a top quark and at least one other quark, where the top quark decays
Table 7.8: QCD background prediction.

<table>
<thead>
<tr>
<th></th>
<th>muon prediction</th>
<th>electron prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$150 &lt; E_T &lt; 225$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_C$</td>
<td>$0.14 \pm 0.03 \pm 0.08$</td>
<td>$1.5 \pm 1.1 \pm 1.2$</td>
</tr>
<tr>
<td>$N_S$</td>
<td>$0.08 \pm 0.02 \pm 0.06$</td>
<td>$0.0 \pm 0.6 \pm 0.6$</td>
</tr>
<tr>
<td>$E_T &gt; 300$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_C$</td>
<td>$0.008 \pm 0.007 \pm 0.011$</td>
<td>$1.4 \pm 1.1 \pm 0.8$</td>
</tr>
<tr>
<td>$N_S$</td>
<td>$0.005 \pm 0.004 \pm 0.007$</td>
<td>$0.0 \pm 0.2 \pm 0.2$</td>
</tr>
</tbody>
</table>

to a $b$-quark and a $W$ boson that decays to a lepton and neutrino. This background is small due to the $|V_{tb}|$ coupling, and is estimated from the yield expected from MC.

Drell-Yan and $Z$ bosons produced at high jet multiplicity represent a small background. This is due to the small cross section, and the fact that $Z \rightarrow l^+l^-$ is suppressed by the veto on a second lepton. We estimate this background by taking the yield predicted by Monte Carlo and apply a 50% systematic uncertainty, which reflects the systematic uncertainty associated with using the MC yield as the prediction.

The diboson backgrounds $WW$, $WZ$, and $ZZ$ are also very small due to the single lepton selection and the cross-sections. A 50% systematic uncertainty is applied to the MC-predicted yield.

7.6 Vector and Dibosons

Drell-Yan and $Z$ bosons produced at high jet multiplicity represent a small background. This is due to the small cross section, and the fact that $Z \rightarrow l^+l^-$ is suppressed by the veto on a second lepton. We estimate this background by taking the yield predicted by Monte Carlo and apply a 50% systematic uncertainty, which reflects the systematic uncertainty associated with using the MC yield as the prediction.

The diboson backgrounds $WW$, $WZ$, and $ZZ$ are also very small due to the single lepton selection and the cross-sections. A 50% systematic uncertainty is applied to the MC-predicted yield.
Chapter 8

Systematic Error Analysis

8.1 Azimuthal angle of artificial MET

Prior to any $E_T$ cut, the single-lepton $M_T$ model assumes that there is a uniform distribution in the azimuthal angle $\Delta \phi$ between the artificial $E_T$ and the prompt neutrino. However, in MC there is a small non-uniformity in $\Delta \phi$. In the modeling, the artificial $E_T$ can be added to the event according to the non-uniform $\Delta \phi$ distribution, but this still does not result in a perfect agreement in the $\Delta \phi$ distributions after the $E_T$ cut is applied in the signal region. To compute the systematic error of the flat-$\phi$ model, we compute the variation in the $M_T$ ratio for uniform vs. non-uniform $\phi$.

In the signal region, the uncertainty is just 2% for muons and electrons for $W + jets$.

The non-uniform $\phi$ model does not account for the whole shape change that occurs when the $E_T$ cut is applied. The additional systematic uncertainty due to the difference between the flat $\phi$ model and the target MC distribution is taken into account in the closure test systematic.
Figure 8.1: Difference between artificial $E_T$ angle and simulated neutrino angle for the (a) flat $\phi$ model, (b) nonuniform $\phi$ model, and (c) target distribution.
For $W +\text{jets}$, the $\Delta \phi$ uncertainty is found to be just 2\% for electrons and muons.

![Comparison of modeled $M_T$ shape for flat $\phi$ model (black) and nonuniform $\phi$ model (red) for muons in $W +\text{jets}$ and $t\bar{t}$ at $E_T > 300$.](image)

**Figure 8.2**: Comparison of modeled $M_T$ shape for flat $\phi$ model (black) and nonuniform $\phi$ model (red) for muons in $W +\text{jets}$ and $t\bar{t}$ at $E_T > 300$.

### 8.2 Jet energy scale

To ascertain the sensitivity of the background prediction to variations in the jet energy scale, the jet energy (of all jets) is rescaled according to the $\eta$ and $p_T$-dependant energy scale uncertainties. This also effects the missing transverse momentum calculation. An upward and downward variation is applied. For the single lepton background, the uncertainty is taken to be the maximal variation in the $M_T$ ratios $R_W$ and $R_{t\bar{t}}$.

The uncertainty on $R_W$ is found to be just 0\% and 14\% on electrons and muons, respectively.

In the signal scan MC, the uncertainty is derived from the maximal variation in the signal yield at each model point, because the jet activity varies as a function of $m_0$ and
8.3 Systematic uncertainty on $W$ and top cross-sections

The different sizes of the $M_T$ ratios $R_W$ and $R_{t\bar{t}}$ means that the $W$ fraction $f_W$ and the uncertainty on the assumed $t\bar{t}$ and $W$+jets cross-sections needs to be properly estimated. Here, the accepted cross-sections and MC modeling of $W$+jets and $t\bar{t}$ are used to find $f_W$, while the uncertainty on those cross-sections at high jet multiplicity and $H_T$ is found in a data-driven method. These uncertainties were measured in the CMS $E_T$ analysis, which used identical object selection criteria, the same triggers, and very similar cuts (requiring 4 or more jets and $H_T > 500$).

The $W$+jets cross-section uncertainty is derived from a data/MC comparison of $Z$+jets events. Opposite sign leptons (dimuon and dielectron) are selected at $H_T > 400$ with 4 or more jets. Events are counted in three mass bins: 75 to 95, 80 to 100, and 95 to 105 GeV. The total uncertainty is found by summing in quadrature the statistical uncertainty, the difference in the data/MC ratio from 1, and the maximal difference in yield over the three bins. The uncertainty is 11% for muons and 13% for electrons.

The uncertainty on the $t\bar{t}$+jets cross-section is determined with the top-box $\chi^2$ variable, which measures how similar an event is to the single lepton $t\bar{t}$ topology. The $\chi^2$ variable is comprised of the reconstructed hadronic top mass $M_3$, the hadronic $W$ mass $M_2$, and the leptonic top mass $M_{3_{Lep}}$.

Under the assumption that the lepton and missing momentum are due to a $W$ boson with mass $M_W$, the $z$-component of the neutrino momentum can be determined up to a two-fold ambiguity. The value that, when a jet is added, results in the best reconstruction...
of the top mass is used. With the remaining jets, \( M_{2\text{had}} \) is the from the two jets closest to the \( W \) mass, and \( M_{3\text{had}} \) adds an additional jet that makes the 3 jets closest to the top mass. Then, the a \( \chi^2 \) value is computed as

\[
\chi^2 = \frac{(M_{3\text{had}} - M_{\text{top}})^2}{\sigma_{M_{3\text{had}}}} + \frac{(M_{2\text{had}} - M_{W})^2}{\sigma_{M_2}} + \frac{(M_{3\text{lep}} - M_{\text{top}})^2}{\sigma_{M_{3\text{lep}}}} \tag{8.1}
\]

The uncertainties in the denominators are determined from the distribution widths in MC, with truth matching applied to the reconstructed objects in \( t\bar{t} \).

Three different cuts are applied to the \( \chi^2 \) distributions, at \( \chi^2 < 3 \), \( \chi^2 < 4 \), and \( \chi^2 < 5 \). The total uncertainty is the from the sum in quadrature of the statistical uncertainty, the difference in the data/MC ratio from 1, and the maximal difference from the 3 different \( \chi^2 \) cuts. The total uncertainty is found to be 8% for the muon channel and 25% for the electron channel.
8.4 Pile-up

All of the MC samples were reweighted to the distribution of the number of vertices observed in the data. The artificial $E_T$ templates were taken from collision data and so automatically reflect any pile-up effects on the missing energy, which are expected to be small.

8.5 W boson polarization

The Monte Carlo simulations make certain assumptions about the polarization of the $W$ bosons in the $W$ rest frame in $W+$jets and in $t\bar{t}$. For $t\bar{t}$ events, in the $W^+$ rest frame, the angular distribution of the charged lepton is

$$
\frac{dN}{d\cos \theta^*_l} = f_{+1} \frac{3}{8} (1 + \cos \theta^*_l)^2 + f_{-1} \frac{3}{8} (1 - \cos \theta^*_l)^2 + f_0 \frac{3}{4} \sin^2 \theta^*_l
$$

(8.2)
where \( f_{+1}, f_{-1}, \) and \( f_0 \) are the polarization fractions of W boson helicity states \(+1, -1,\) and 0, respectively. The angle \( \cos \theta^*_l \) is measured with between the lepton direction in the W rest frame, and momentum vector of the W in the top quark rest frame. Taken together with the Lorentz boosts of the W and top quark, the charged lepton and neutrino \( p_T \) and \( \eta \) distributions are determined. Due to cuts on lepton \( p_T \) and \( E_T \), variations in the polarization distributions can also be expected to result in a higher order change to the transverse mass.

In the CMS \( E_T \) analysis, the polarization effect in \( tt \) was quantified by reweighting the \( \cos \theta^*_l \) distribution in the W test frame. The \( \lambda(W^+) = -1 \) and \( \lambda(W^+) = 0 \) distributions were varied in a correlated way, while keeping the the \( \lambda(W^+) = +1 \) component very small, as it is predicted to be in the SM. As can be seen in the decay of the \((J = 1/2)\) top quark, \( t \rightarrow bW^+ \), the \( V-A \) coupling results in the \( b \) quark being almost entirely in the \( \lambda(b) = -1/2 \) state, so conservation of angular momentum requires the \( \lambda(W^+) = +1 \) state.

Figure 8.6: Number of interactions in MC, before and after reweighting to the observed distribution in data.
to be very small.

The higher probability for $\lambda(W^+) = -1$ corresponds to the charged lepton being emitted preferentially in the backward direction. The angular distribution of the charged lepton is the same for $W^-$ decays, where the helicity of each particle is flipped.

The CMS $E_T$ analysis applied a 5% variation on the polarization fractions $\lambda(W^+)$ and $\lambda(W^+)$, which is about 10 times larger than the theoretical uncertainty.

![Figure 8.7: $\frac{dN}{d\cos\theta_l}$ and ±5% polarization variation for $t\bar{t}$.](image)

In $W$+jets events, all three polarization states are present with non-negligible amplitudes. Also, the $W^+$ and $W^-$ polarization states are slightly different (due to the proton-proton initial state), and depend on $W$ $p_T$, $W$ helicity, and $W$ rapidity.

The $\cos\theta_l^*$ distribution at generator level is binned in $W$ $p_T$, rapidity, and $W$ charge. The $p_T$ bins are at $100 < p_T < 300$ GeV/c, $300 < p_T < 500$ GeV/c, and $p_T > 500$ GeV/c. The rapidity bins are $0 < |y| < 1$, $1 < |y| < 2$, and $2 < |y| < 5$. 

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Three different 10% variations were applied to the angular distributions in each \( p_T \) and rapidity bin to see how they change the background prediction, based on the theoretical uncertainties in [51].

When applied to transverse mass, using the same uncertainties as the \( E_T \) analysis is a conservative overestimate because \( M_T \) is not as sensitive to variations in the longitudinal components of the lepton and neutrino momentum. For the \( E_T \) selection in this analysis, the error on the W background due to polarization is 8%.

The systematic uncertainties associated with the single lepton background prediction, along with the uncertainties on the V+jets, diboson, and single-top quark backgrounds are summarized in Table 8.9. The systematic uncertainty associated with the \( t\bar{t} \) dilepton background (due to \( \alpha_{stat} \) and the k-factor) was found to be 41%.

Figure 8.8: \( \frac{dN}{d\cos\theta} \) and ±5% polarization variation for \( W^+ \) and \( W^- \) in the bins \( 0 < |y| < 1 \) and \( 300 < p_T < 500 \).
Figure 8.9: Summary of systematic uncertainties.
Chapter 9

Results

The single and dilepton backgrounds, as determined by the data-driven methods, and the backgrounds determined by MC are summarized in Table 9.1. The total background was found to be \( 6.1 \pm 1.2^{\text{stat}} \pm 1.8^{\text{sys}} \), which should be compared to the observed yield of 3 events. This observed yield is fairly consistent with our Standard Model expectation of the background yield, so we set a limit on the Minimal Supergravity parameter space.

<table>
<thead>
<tr>
<th>Source</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single lepton</td>
<td>( 1.2 \pm 0.3^{\text{stat}} \pm 0.3^{\text{sys}} )</td>
</tr>
<tr>
<td>Dilepton</td>
<td>( 4.1 \pm 1.1^{\text{stat}} \pm 1.7^{\text{sys}} )</td>
</tr>
<tr>
<td>Single-top</td>
<td>( 0.3 \pm 0.1^{\text{stat}} \pm 0.15^{\text{sys}} )</td>
</tr>
<tr>
<td>Z</td>
<td>( 0.3 \pm 0.2^{\text{stat}} \pm 0.2^{\text{sys}} )</td>
</tr>
<tr>
<td>Diboson</td>
<td>( 0.2 \pm 0.1^{\text{stat}} \pm 0.09^{\text{sys}} )</td>
</tr>
<tr>
<td>QCD</td>
<td>( 0.0 \pm 0.2^{\text{stat}} \pm 0.2^{\text{sys}} )</td>
</tr>
<tr>
<td>Total (predicted)</td>
<td>( 6.1 \pm 1.2^{\text{stat}} \pm 1.8^{\text{sys}} )</td>
</tr>
<tr>
<td>Data (observed)</td>
<td>( 3 ) (2e, 1( \mu ))</td>
</tr>
</tbody>
</table>

Figure 9.1: MC expectations, data-driven predictions and observed yields in data in the \( M_T > 130 \text{ GeV}/c^2 \), \( \slashed{E}_T > 300 \text{ GeV}/c \), \( H_T > 450 \text{ GeV}/c \) signal region. The combined \( e + \mu \) yield is given in data and the separate \( (e, \mu) \) yields.

We use the \( CLs \) method \cite{52} \cite{53} \cite{54} calculated by the RooStatsCl95 package \cite{55} to determine which parts of the CMSSM parameter space are excluded at the 95\% confidence
level. The $CLs$ statistic can be thought of as a modified-frequentist probability. In one derivation, it begins with the Poisson probability of observing $s+b$ (signal plus background) events, and then conditions that probability with the assumption that the observed number of events is greater than or equal to the true number of background events. This protects the method from spurious exclusion of the signal in experiments that had a downward fluctuation of the background.

Fig. (9.2) illustrates the total production cross-section in mSUGRA, which decreases mostly as a function of $m_{1/2}$. $m_0$ and $m_{1/2}$ are each scanned in 20 GeV steps, up to about 1.9 TeV in $m_0$ and 800 GeV in $m_{1/2}$. The particle masses were computed in SoftSUSY [56], and the cross-sections were computed at next-to-leading-order (NLO) by Prospino [57]. Up to 10k events were generated at each point, which were then put through a Monte Carlo simulation of the detector and the reconstruction software.

Figure 9.2: CMSSM production cross-section in the $m_0$-$m_{1/2}$ plane, at $\tan \beta = 10$, $A_0 = 0$, $\text{sgn} \mu = +$. 

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The combined signal efficiency times acceptance of the detector and the selection cuts is shown in Fig. 9.3.

![Signal efficiency times acceptance in the $m_0-m_{1/2}$ plane at $H_T > 450, \slashed{E}_T > 300$.](image)

Figure 9.3: Signal efficiency times acceptance in the $m_0-m_{1/2}$ plane at $H_T > 450, \slashed{E}_T > 300$.

The signal model yields are assigned systematic uncertainties relating to jet energy scale variation (Fig. 9.4), QCD scale variation, and a 3% parton distribution function uncertainty uniform in $m_{1/2}$ and $m_0$. The integrated luminosity uncertainty is taken to be 6%.

The resulting $CL_s$ limit in the $m_0 - m_{1/2}$ parameter space is shown in Fig. 9.5. At small $m_0$, the parameter space below $m_{1/2} = 550$ is excluded in the observed limit. This decreases to a limit $m_{1/2} = 260$ at $m_0 = 1900$. The LM6 benchmark point ($m_0 = 85, m_{1/2} = 400$) described at Fig. 2.11 is excluded.

Gluinos with masses below approximately 600 GeV/$c^2$ are excluded everywhere at
Figure 9.4: Relative uncertainty due to jet energy scale variation in the $m_0$-$m_{1/2}$ plane at $H_T > 450$, $E_T > 300$.

Figure 9.5: Median expected limit, shown with $\pm 1\sigma$ uncertainty band, and observed limit.
the 95% C.L. At small $m_0$, gluinos with masses below 1.1 TeV/$c^2$ are excluded. Squarks with mean $u, d, s, c$ mass below about 1.1 TeV are also excluded everywhere.
Chapter 10

Conclusions

A search was performed on 5.0 fb$^{-1}$ of 2011 LHC collisions at a proton-proton center of mass energy of $\sqrt{s} = 7$ TeV. Data-driven methods were used to estimate the Standard Model background in the channel with one lepton, jets, missing transverse momentum, and high transverse mass. The event yield was found to be in agreement with Standard Model predictions, so limits were set on the $m_0 - m_{1/2}$ parameter space of the CMSSM. For $\tan \beta = 10$, $A_0 = 0$, and $\text{sgn}(\mu) > 0$, it was found that squark masses below 1.1 TeV/$c^2$ and gluino masses below 600 GeV/$c^2$ are excluded at the 95% confidence level.
Bibliography


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