Platoon Collision Dynamics And Emergency Maneuvering I: Reduced Order Modeling Of A Platoon For Dynamical Analysis
Platoon Collision Dynamics and Emergency Maneuvering I: Reduced Order Modeling of a Platoon for Dynamical Analysis

Benson H. Tongue
Yean-Tzong Yang
Matthew T. White

PATH Research Report
UCB-ITS-PRRQI-15

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California, Business and Transportation Agency, Department of Transportation, and the United States Department of Transportation, Federal Highway Administration.

The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California. This report does not constitute a standard, specification, or regulation.

August 1991

ISSN 1055-1425
This paper has been mechanically scanned. Some errors may have been inadvertently introduced.
PATH Goal Statement

The research described in this report is part of the Program on Advanced Technology for the Highway (PATH). PATH research is being conducted at the Institute of Transportation Studies at the University of California at Berkeley, to develop more effective highways. The aim of PATH is to increase the capacity of the most used highways, to decrease traffic congestion, and to improve safety and air quality. PATH is a cooperative venture of the automobile and electronic industries, universities, and local, state, and federal governments.
Abstract

Platoon Collision Dynamics and Emergency Maneuvering I: Reduced Order Modeling of a Platoon for Dynamical Analysis

Benson H. Tongue, Yean-Tzong Yang, and Matthew T. White

PATH Project

August 1991

The purpose of this three-year project is to develop an operational model of vehicle platoon dynamics under emergency conditions and to evaluate the platoon's dynamical behavior under non-nominal, or emergency, conditions. New platoon protocols and/or controller modifications will be formulated after analyzing the platoon response data in order to minimize damage to the platoon and the vehicles' occupants.

During the first year of this research, a comprehensive review of previous work dealing with the dynamics and control of platoons of vehicles was undertaken. Following this, a nonlinear reduced order model (ROM) was developed from an accurate, and high order, model. Regression analysis, based upon the least-squares algorithm, was applied to the response of a full order model in order to determine the reduced order vehicle model.

Preliminary results have shown that the reduced order model provides an accurate response match with the original model at a substantial computational savings. A platoon model has been developed and coded and preliminary simulations of platoon dynamics have been performed in which system parameters such as sampling time, desired headway, vehicle spacings and road grades were varied.

This progress report discusses the results of the modeling phase of this work. A detailed literature review is included as Appendix B. A second report, dealing with the platoon dynamical simulations, is currently in preparation.
1. Introduction

The topic of platoon dynamics and longitudinal control has been studied extensively from the late 1960's into the 1970's, and has attracted researchers' attentions again since the mid 1980's. Much of the recent emphasis has to do with augmenting the capacity of existing roadways and increasing the safety of automobile travel. By far, the bulk of the effort has been directed towards the development and analysis of different control strategies for the platoons, both for longitudinal and lateral motion. Less effort has been expended upon the study of platoon dynamics, especially during non-nominal, or emergency, conditions. In addition, most of the dynamics work that has been done has considered a linear vehicle model. Recent research [38,39,46] has shown that strong nonlinearities are unavoidable in a given vehicle's dynamic response. It is known [19,21] that nonlinearities within a platoon can induce unstable wave propagation in the event of commanded speed reductions. Thus, the actual behavior of a realistic chain of vehicles, especially one experiencing a non-nominal operation such as an internal collision, is likely to be both complicated and unpredictable from a linear standpoint.

The main goal of this project in the first year of research was to construct an accurate reduced order model that reproduces the responses of realistic, full order models but that requires substantially less computation time. The reason for this is that full order models take a very long time to run on even powerful computers and extensive simulations are necessary in order to gain insight into platoon behavior during non-nominal operations. Based on Hedrick's full order vehicle model [46], a reduced order model has been obtained by the least-squares technique. Other vehicle specifications, such as the bumpers, aerodynamical forces, including the effect of drafting, rolling resistance force, etc., have also been implemented in this model.

Desoer's control law [37], which considers the lead car dynamics, has been implemented as a preliminary controller for platoon response determinations. Initially, a platoon of identical vehicles has been used in the numerical studies.

Attention in the coming years will be focused on quantifying the degree of disturbance propagation within a platoon, the effect of mild and strong collisions and the effect of vehicles entering or leaving the platoon. Since the aerodynamic forces on the vehicle will vary substantially during entry and exit as well as during changes in headway due to the drafting effect, it is expected that the platoon dynamical response will be much more complex than that shown under uniform aerodynamic assumptions, the type of assumption that has been made in all previous studies. Recommendations will be suggested towards appropriate modifications that should be made in the platoon decision protocols and/or the control algorithms in order to minimize damage in the event of a collision.
Nomenclature

\( C_a \quad = \quad \text{aerodynamical coefficient} \)
\( c_{a1}, c_{a2} \quad = \quad \text{acceleration gain coefficients of the controller} \)
\( c_f \quad = \quad \text{damping coefficient of the front bumper} \)
\( c_r \quad = \quad \text{damping coefficient of the rear bumper} \)
\( c_{v1}, c_{v2} \quad = \quad \text{velocity gain coefficients of the controller} \)
\( c_{x1}, c_{x2} \quad = \quad \text{displacement gain coefficients of the controller} \)
\( D_t \quad = \quad \text{duration for changing throttle angle, sec} \)
\( d_t \quad = \quad \text{sampling time, sec} \)
\( F_a \quad = \quad \text{aerodynamical force, N} \)
\( F_b \quad = \quad \text{braking force, N} \)
\( F_{b,\text{max}} \quad = \quad \text{maximum braking force, N} \)
\( F_g \quad = \quad \text{gravitational force, N} \)
\( F_r \quad = \quad \text{rolling resistance force, N} \)
\( F_{tf} \quad = \quad \text{traction force, N} \)
\( F_{tfm} \quad = \quad \text{maximum traction force, N} \)
\( f_r \quad = \quad \text{rolling resistance coefficient} \)
\( G_0 \quad = \quad \text{road roughness coefficient} \)
\( g \quad = \quad \text{gravitational acceleration, m/sec}^2 \)
\( H \quad = \quad \text{height of the front vehicle, m} \)
\( k_f \quad = \quad \text{spring constant of the front bumper, kN/m} \)
\( k_r \quad = \quad \text{spring constant of the rear bumper, kN/m} \)
\( k_{v1}, k_{v2} \quad = \quad \text{gain coefficients of the controller for the lead car dynamics} \)
\( M_i \quad = \quad \text{vehicle mass, kg} \)
\( m \quad = \quad \text{steady state slope of the traction force, N/sec} \)
\( t \quad = \quad \text{time, sec} \)
\( v \quad = \quad \text{velocity, km/hr} \)
\( v_i \quad = \quad \text{initial velocity for throttle-changing operation, km/hr} \)
\( \dot{x} \quad = \quad \text{acceleration of vehicle i, m/sec}^2 \)
\( \alpha \quad = \quad \text{braking percentage} \)
\( \Delta \quad = \quad \text{headway spacing, m} \)
\( \Delta_{\text{des}} \quad = \quad \text{desired headway spacing, m} \)
\( \Delta_e \quad = \quad \text{headway spacing error, m} \)
\( \Delta_i \quad = \quad \text{headway spacing of the i-th vehicle, m} \)
\( \lambda \quad = \quad \text{road grade} \)
\( \phi \quad = \quad \text{throttle angle, deg} \)
\( \dot{\phi} \quad = \quad \text{throttle rate, deg/sec} \)
\( \phi_f \quad = \quad \text{final throttle angle for throttle-changing operation, deg} \)
\( \phi_i \) = initial throttle angle for throttle-changing operation, deg
\( \tau_1 \) = first time constant of the ROM, sec
\( \tau_2 \) = second time constant of the ROM, sec
\( \tau_3 \) = third time constant of the ROM, sec
\( \tau_4 \) = fourth time constant of the ROM, sec
\( \tau_b \) = time constant of the braking system, sec
2. Current Work

The longitudinal dynamics of a vehicle can be simply expressed by Newton’s second law:

\[ \ddot{x}_i = \frac{1}{M_i} \left( F_{tf} - F_b - F_r - F_a - F_g \right) \]  

(1)

where \( \ddot{x}_i \) is the acceleration of vehicle \( i \),
\( M_i \) is the vehicle mass,
\( F_{tf} \) is the traction force,
\( F_b \) is the braking force,
\( F_r \) is the rolling resistance force,
\( F_a \) is the aerodynamic force,
\( F_g \) is the gravitational force caused by the road grade.

Figure 1 shows this vehicle model, where “\( \Delta_i \)” is the headway spacing of the \( i \)-th vehicle from the preceding vehicle, (‘) denotes the time derivative, and \( \phi \) and \( \alpha \) are the applied throttle angle and braking percentage, respectively.

Clearly, \( F_b, F_r, F_a, \) and \( F_g \) are external forces. \( F_b \) depends on the braking system and tire performance. \( F_r \) is a function of road roughness, vehicle speed, and tire properties. \( F_a \) is the flow-induced force, which depends on the vehicle profile, velocity, spacing between vehicles, and so on. \( F_g \) is the gravitational force caused by the longitudinal grades of highways. These four terms have nothing to do with the engine dynamics, transmission system, and drivetrain system, which all possess some degree of nonlinearity. The only term that is dominated by the internal dynamics is \( F_{tf} \). Thus the present effort is focused on the \( F_{tf} \) term in equation (1). The fully-developed vehicle model we applied is Hedrick’s model [46]. This model, which is briefly presented in Appendix A, includes the engine dynamics, transmission system, and drivetrain. It is important to keep in mind that the modeling to follow is only concerned with the traction force. There are no external forces such as aerodynamic or tire damping. These are added later when complete platoon simulations are run.

Figures 2 through 5 show the response of Hedrick’s model with respect to different inputs. For these simulations, the throttle opens (Figures 2 through 4) or closes (Figure 5) at \( t = 2 \) sec, and the input pattern is a ramp function with a rising duration \( Dt \). As shown in these three plots, the system exhibits a significant reaction time delay. Figure 2 illustrates the variation of the traction force for varying forward velocities with \( \phi \) and \( Dt \) held constant. For the case of constant \( \nu_i \) and \( \phi \), Figure 3, different reaction time delays, corresponding to distinct \( Dt \) are seen. For the case of the same \( Dt \) and \( \nu_i \), Figure 4, the reaction time delay is different as \( \phi \) varies. Moreover, the traction force is higher for a higher \( \phi \), but the variation is not linear with the throttle value. As displayed
in Figure 5, the system’s response during throttle closing possesses similar characteristics to those seen during throttle opening.

As seen from the responses of the full order model, the relationships among input parameters are nonlinear and quite complicated. The traction force $F_{tf}$ corresponding to different throttle angles $\phi$, initial velocity $v_i$, and reaction time duration $Dt$, exhibits various reaction time delays, maximum traction, and steady-state performance. Based on the analysis of the full order model, a nonlinear numerical reduced order model scheme will be outlined. The idea is to bypass the vehicle’s complicated internal dynamics and direct the effort to the response of the overall system.

2.1 Curve-fit results of Reduced Order Modeling

Based on an inspection of the characteristics of the full order model, six performance indices, $\tau_1, \tau_2, \tau_3, \tau_4, F_{tjm}$, and $m$, have been chosen to capture the qualitative behavior of the nonlinear model. As illustrated in Figure 6, $F_{tjm}$ is the maximum traction force, $m$ is the rate of change of the traction force for constant throttle positions, $\tau_1$ is the first time constant, $\tau_2$ is the second time constant, $\tau_3$ is the third time constant, and $\tau_4$ is the fourth time constant. The variables $\tau_1$ and $\tau_3$ correspond to the reaction time delay. The time durations of the system responses are $\tau_2$ and $\tau_4$. The independent variables are the initial velocity $v_i$, reaction time duration $Dt$, and the throttle angle $\phi$.

2.1.1 First time constant, $\tau_1$

As shown in Figure 7, the data are scattered around a straight line in a log-log plot. The value of $\tau_1$ decreases as $\dot{\phi}$ increases. The initial velocity does not play an important role on the reaction time delay. The value of $\tau_1$ can be approximated as follows

$$\tau_1(\dot{\phi}) = 2.0905 \dot{\phi}^{-0.7033}$$  \hspace{1cm} (2)

Note that $\tau_1 = 0$ for $\dot{\phi} \to \infty$, i.e. for a step input function.

2.1.2 Second time constant, $\tau_2$

Figure 8 illustrates the variation of maximum traction with respect to throttle angle for various vehicle velocities. As can be seen $v_i$ is not a critical factor. Clearly, $\tau_2$ varies a good deal with $\phi_f$. The curve-fit formula is

$$\tau_2(\phi_i, \phi_f, Dt) = c_1(\phi_i, \phi_f) Dt + c_0(\phi_i, \phi_f)$$  \hspace{1cm} (3)
where

\[ c_1(\phi_i, \phi_f) = a_2(\phi_f)\phi_i^2 + a_1(\phi_f)\phi_i + a_0(\phi_f) \]
\[ c_0(\phi_i, \phi_f) = b_2(\phi_f)\phi_i^2 + b_1(\phi_f)\phi_i + b_0(\phi_f) \]
\[ a_2(\phi_f) = .1236 \times 10^{-6}\phi_f^3 -.0190 \times 10^{-3}\phi_f^2 + .9307 \times 10^{-3}\phi_f -.0147 \]
\[ a_1(\phi_f) = -4.9847 \times 10^{-6}\phi_f^3 + .7543 \times 10^{-3}\phi_f^2 -.0355\phi_f + .5290 \]
\[ a_0(\phi_f) = .0431 \times 10^{-3}\phi_f^3 - 6.6078 \times 10^{-3}\phi_f^2 + .3123\phi_f - 3.9746 \]
\[ b_2(\phi_f) = -.0309 \times 10^{-6}\phi_f^3 + 4.6716 \times 10^{-6}\phi_f^2 -.2153 \times 10^{-3}\phi_f + 2.9635 \times 10^{-3} \]
\[ b_1(\phi_f) = 2.1909 \times 10^{-6}\phi_f^3 -.3257 \times 10^{-3}\phi_f^2 + .0152\phi_f - .2257 \]
\[ b_0(\phi_f) = -.0120 \times 10^{-3}\phi_f^3 + 1.8282 \times 10^{-3}\phi_f^2 -.0851\phi_f + 1.6124 \]

\[ \text{and } \phi_i \text{ and } \phi_f \text{ are the initial and final throttle angles, respectively.} \]

The third and the fourth time constants may be derived in a similar manner. It should be noted that the assumed polynomial representation for the identifications was obtained both by visual inspection of the computer simulation results and by reference to a handbook [53] that dealt specifically with the question of nonlinear curve fitting.

2.1.3 Third time constant, \( \tau_3 \)

\[ \tau_3(\phi_i, \phi_f, Dt) = c_1(\phi_i, \phi_f)Dt + c_0(\phi_f) \]  \hspace{1cm} (4)

where

\[ c_1(\phi_i, \phi_f) = a_2(\phi_i)\phi_f^2 + a_1(\phi_i)\phi_f + a_0(\phi_i) \]
\[ c_0(\phi_f) = -.0163 \times 10^{-3}\phi_f^2 + \mathbf{1.1531} \times 10^{-3}\phi_f + .0501 \]
\[ a_2(\phi_i) = -.0779 \times 10^{-6}\phi_i^2 + \mathbf{5.2538} \times 10^{-6}\phi_i + .1508 \times 10^{-3} \]
\[ a_1(\phi_i) = 4.6947 \times 10^{-6}\phi_i^2 -.3556 \times 10^{-3}\phi_i + 3.9027 \times 10^{-3} \]
\[ a_0(\phi_i) = \mathbf{5.8772} \times 10^{-3}\phi_i -.1152 \]

2.1.4 Fourth time constant, \( \tau_4 \)

\[ \tau_4(\phi_i, \phi_f, Dt) = c_1(\phi_i, \phi_f)Dt + c_0(\phi_i, \phi_f) \]  \hspace{1cm} (5)
where

\[
c_1(\phi_i, \phi_f) = a_2(\phi_i)\phi_f^2 + a_1(\phi_i)\phi_f + a_0(\phi_i)
\]
\[
c_0(\phi_f) = b_1(\phi_i)\phi_f + b_0(\phi_i)
\]
\[
a_2(\phi_i) = .2809 \times 10^{-6} \phi_i^2 - .0277 \times 10^{-3} \phi_i + .3220 \times 10^{-3}
\]
\[
a_1(\phi_i) = -9.5922 \times 10^{-6} \phi_i^2 + .9394 \times 10^{-3} \phi_i - .0196
\]
\[
a_0(\phi_i) = -8.6516 \times 10^{-6} \phi_i^2 - 5.1943 \times 10^{-3} \phi_i + 1.0761
\]
\[
b_1(\phi_i) = .4870 \times 10^{-6} \phi_i^2 - .0291 \times 10^{-3} \phi_i - 5.2394 \times 10^{-3}
\]
\[
b_0(\phi_i) = -.0162 \times 10^{-3} \phi_i^2 + 1.9726 \times 10^{-3} \phi_i + 1.820
\]

2.1.5 Maximum traction force, \(F_{tfm}\)

Figure 9 shows the regression analysis results for the maximum traction force. It is interesting to note that \(F_{tfm}\) increases nonlinearly with \(\phi_f\). For a smaller value of \(\phi_f, F_{tfm}\) changes more severely. Moreover, for a fixed \(\phi_f, F_{tfm}\) drops while \(v_i\) increases. The expression for \(F_{tfm}\) can be denoted by a function of throttle angle and initial velocity. The equation is

\[
F_{tfm}(\phi_f, v_i) = a(1 - e^{-\beta \phi_f})^\gamma
\]

where

\[
\alpha = 1. \times 10^3(-.0053 v_i + 2.7404)
\]
\[
\beta = 1. \times 10^{-3}(.0613 v_i + 101.9315)
\]
\[
\gamma = 1. \times 10^{-3}(18.8640 v_i + 855.0600)
\]

2.1.6 Steady-state slope of the traction force, \(m\)

The results for \(m\) are shown in Figure 10. As \(\phi_f\) increases, \(m\) drops to a minimum value when \(\phi_f\) is within the range of 20 to 40 degrees, and then maintains almost a constant value for higher \(\phi_f\) values. In addition, \(m\) decreases with \(v_i\) for a fixed \(\phi_f\). The results can be expressed as

\[
m(\phi_f, v_i) = (\alpha + \beta \phi_f)^{\phi_f} + \delta
\]

where

\[
a! = .8274 v_i - 4.2020
\]
\[ \beta = 1.9124 \times 10^{-3}v_i^2 - 0.4021v_i + 16.3670 \]
\[ \gamma = -0.6554 \times 10^{-3}v_i + 0.9620 \]
\[ \delta = 0.1484 \times 10^{-3}v_i^3 - 0.0432v_i^2 + 4.1972v_i - 158.3749 \]

### 2.1.7 Fidelity of the ROM

After completing the regression analysis on the full order model, it is necessary to compare the results obtained from the ROM and the original full order model. Two representative cases are presented in Figures 11 and 12 to demonstrate the fidelity of the ROM. As is clear from these figures, the reduced order model does an excellent job of approximating the full order model’s response. Table 1, shown below, illustrates that the reduced order model is more than an order of magnitude faster than the full order model. Thus the goal of attaining high accuracy with reduced computation times would seem to have been achieved. As mentioned previously, this is very important in view of the large number of simulations that must be run on the nonlinear platoon model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Order (No. of state variables)</th>
<th>CPU time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedrick’s</td>
<td>9</td>
<td>3.1</td>
</tr>
<tr>
<td>ROM</td>
<td>1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### 2.2 Other specifications of the ROM

To complete the ROM, subsystems must be implemented to model the braking dynamics, bumper structure, rolling resistant force, aerodynamical force, and gravitational force.

#### 2.2.1 Braking force, \( F_b \)

Basically, the braking force depends on applied braking percentage and properties of the braking system. The performance of different braking systems may vary significantly. However, the output characteristics of braking force are similar. The braking mechanism used in the ROM is adopted from Hedrick’s report [38]. The expression of braking force is

\[ F_b = \alpha F_{b,\text{max}} (1 - e^{-\frac{t}{\tau_b}}) \]  

\[ (8) \]
where \( \alpha \) is the braking percentage,
\[ F_{b,\text{max}} \] is the maximum braking force,
\( t \) is the braking time,
and \( \tau_b \) is the time constant of the braking system.

### 2.2.2 Bumper mechanism

The front and rear bumpers are modeled as parallel combinations of a linear damper and a linear compression spring. This mechanism will come into action as two adjacent vehicles experience a mild mutual collision. In the event of a major collision, the vehicles will be modeled as a single body. The switching instant from a minor collision to a major one is defined at the onset of 5 inch bumper travel. Figure 2.2.1 shows this simplified bumper model on the vehicle. The spring constants of the front and rear bumpers are \( k_f \) and \( k_r \), respectively. The variables \( c_f \) and \( c_r \) represent the damping coefficients of the bumper. Obviously this model is a highly simplified one and will be extended in later work.

### 2.2.3 Rolling resistance force, \( F_r \)

The reaction between tires and road surface is a complicated one. The friction force depends on the tire properties, road roughness, vehicle velocity, wheel rolling speed, and so on. However, to simplify the rolling resistance mechanism, the data contained in ASTM STP884 [47] has been adapted for the ROM. The formula can be expressed as follows:

\[
F_r = Mg f_r \cos(\lambda)
\]

\[
f_r = (0.4864 \times 10^{-3} G_0 - 0.0103 \times 10^{-6}) v^3 +
(0.0952 G_0 + 1.1425 \times 10^{-5}) v^2 +
(7.0982 G_0 - 0.0310 \times 10^{-3}) v + 0.01
\]

where \( M \) is the vehicle mass,
\( g \) is the gravitational acceleration,
\( f_r \) is rolling resistance coefficient,
\( \lambda \) is the road grade,
\( G_0 \) is the road roughness coefficient,
and \( 0.4050 \times 10^{-6} \leq G_0 \leq 6.400 \times 10^{-6} \), for highways.

Figure 14 shows the original ASTM data.

### 2.2.4 Aerodynamical force, \( F_a \)
Although the flow field around individual vehicles has been studied extensively by many researchers [48-52], the behavior of the wake region between two vehicles is still not well understood. The analysis of such a situation is complicated by not only the irregular profiles of different vehicles, but also by the variation of intervehicle spacing with time. Since the effect of adjacent vehicles is believed to be extremely significant with regard to the applied aerodynamic forces and existing information seems to be non-existent, a heuristic approximation has been used. Basically, the model chosen possesses a drag coefficient that varies linearly with inter-vehicle separation. When far apart, the aerodynamic forces are taken to scale with the square of velocity, the usual aerodynamic drag assumption. When very close, the force is taken to be equal to zero; the preceding vehicle is essentially shielding the following one from the wind. And between these extremes, the drag coefficient varies in the manner described. Thus the drag force can be represented as

\[
F_a = \begin{cases} 
0 & \text{if } A \leq 0.5H \\
A(\Delta - 0.5H)C_a v^2 & \text{if } 0.5H < A \leq 3H \\
C_a v^2 & \text{if } 3H < \Delta
\end{cases}
\]

where \(C_a\) is the aerodynamical coefficient,
\(H\) is the height of the front vehicle,
\(v\) is the vehicle velocity,
and \(A\) is the headway spacing between vehicles.

### 2.2.5 Gravitational force, \(F_g\)

\(F_g\) is the force caused by the longitudinal road slope. Depending on the vehicle weights and the road grades, \(F_g\) will induce a constant force during the grade section. The gravitational force can be expressed as

\[
F_g = Mg \sin(\lambda)
\]

where \(\lambda\) is the grade of the road.

### 2.3 Platoon model

Based on the vehicle model developed above, a platoon model has been established. An outline of this model is presented in Figure 15, where \(x_i\) is the position of vehicle \(i\), and \(\Delta_i\) is the spacing between vehicles \(i-1\) and \(i\).

### 3. Conclusions
This progress report has documented the development of an efficient numerical vehicle model that can be easily applied to the study of platoon dynamics. This reduced order model has shown the capability of accurately capturing the dynamical characteristics of the original full order model with an order of magnitude less computation time. All other relevant forces and systems (aerodynamic forces, tire forces, bumpers, etc.) have been included in the overall platoon model. A future report, now in preparation, will document the details of controller design and the results of platoon simulations for a wide variety of nominal and non-nominal operating conditions.

References


Figure 1. Vehicle model
$v_i = 85 \text{ km/hr}, \phi = 50 \text{ deg}$

**Figure 2.** Full Order Response to Throttle
$Dt = .50 \text{ sec, } \phi = 50 \text{ deg}$

Figure 3. Full Order Response to Throttle
$Dt = .50 \text{ sec}, v_i = 85 \text{ km/hr}$

**Figure 4.** Full Order Response to Throttle
\[ v_i = 80 \text{ km/hr}, \phi = 60 - 10 \text{ deg} \]

Figure 5. Full Order Response to Throttle
Figure 6. Reduced Order Model (ROM)
Figure 7. $\tau_1$ versus Throttle Rate
Figure 8. $\tau_2$ versus Throttle Angle
Figure 9. Maximum Traction Force versus Throttle Angle
Figure 10. Steady State Slope, $m$, versus Throttle Angle
**Figure 11.** Comparison of Full Order and Reduced Order Model
Figure 12. Comparison of Full Order and Reduced Order Model
Figure 13. Bumper Model
Figure 14. Rolling resistance coefficient versus Velocity
Figure 15. Platoon Model
APPENDIX A: Full Order Model

A nine state variable nonlinear vehicle model has been developed by Hedrick, McMahon, and Cho [38, 39, 46]. This full order model includes an internal combustion engine, engine transmission dynamics, and a drivetrain system. The following is the outline of this full order model.

A.1 Engine Model

Four engine state variables have been used. They are the pressure in the intake manifold, $P_m$, the exhaust gas recirculation rate, $m_{egro}$, the engine speed, $\omega_e$, and the mass flow rate of fuel entering the combustion chamber, $\dot{m}_{fi}$. The state equations are as following.

\[
\dot{P}_m = ((\frac{T_m}{\omega_e}) - 0.08873 \omega_e \eta_{vol})P_m + (\frac{RT_m}{V_m})(\dot{m}_{ai} + \dot{m}_{egri}) 
\]

\[
\dot{m}_{egro} = 0.08873 \omega_e \eta_{vol}(\dot{m}_{egri} - \dot{m}_{egro}) 
\]

\[
J_e \dot{\omega}_e = T_i - T_f - T_p 
\]

\[
\tau_f \dot{\dot{m}}_{fi} + \dot{m}_{fi} = \dot{m}_{fc} 
\]

and

\[
\dot{m}_{ai} = \text{MAX. TC . PRI} 
\]

\[
\eta_{vol} = \eta_{vol}(M_a, \omega_e) 
\]

\[
M_a = \frac{M_{air}P_m V_m}{RT_m} 
\]

\[
T_i = c_i \dot{m}_{ao}(t - \Delta t_{it})AFl(t - \Delta t_{it})SI(t - \Delta t_{it}) \omega_e(t - \Delta t_{st}) 
\]

where $T_m$ is the manifold temperature,

$\eta_{vol}$ is the volumetric efficiency,

$R$ is the universal gas constant for air,

$\dot{m}_{ai}$ is the mass rate of air entering the intake manifold,

$m_{egri}$ is the exhaust gas recirculation rate into the intake manifold,

$\tau_f$ is the fuel delivery time constant,

$\dot{m}_{fc}$ is the command fuel rate,

$J_e$ is the effective inertia of the engine and torque converter,

$T_i$ is the indicated engine torque,

$T_f$ is the engine friction torque,
\[ T_p \] is the torque converter pump torque,
\[ \text{MAX} \] is the maximum flow rate corresponding to a fully open throttle valve,
\[ TC \] is the normalized throttle characteristic,
\[ PRI \] is the normalized pressure influence,
\[ M_a \] is the mass of air in the intake manifold,
\[ c_t \] is the engine torque constant,
\[ \dot{m}_{ao} \] is the mass flow rate of air entering the combustion chamber,
\[ \Delta_{st} \] is the intake to torque production delay,
\[ \Delta_{st} \] is the spark to torque production delay.

**A.2 Transmission model**

The transmission system is modelled as a combination of a torque converter and a planetary gear train. The transmission model has two state variables. They are the angular velocity of the turbine and the angular velocity of the reaction carrier, \( \omega_t \) and \( \omega_cr \), respectively. The state equations are given by

\[
\begin{align*}
J_{tg} \dot{\omega}_t &= T_t - R_g R_d T_s \\
\dot{\omega}_{cr} &= \dot{\omega}_t R_g
\end{align*}
\]

where \( J_{tg} \) is the rotational inertia,
\( T_t \) is the turbine torque,
\( T_s \) is the axle shaft torque,
\( R_g \) is the gear ratio,
\( R_d \) is the final drive gear ratio.

**A.3 Drivetrain model**

Three state variables have been used in the drivetrain model. They are the angular velocity of the wheel, \( \omega_{wf} \), the axle shaft torque, \( T_s \), and the vehicle’s velocity, \( v \). The state equations are as follows:

\[
\begin{align*}
\dot{T}_s &= K_s ( R_g R_d \omega_t - \omega_{wf} ) \\
J_{wf} \omega_{wf} &= T_s - h_f F_{tf} - T_{rf} - T_{bf} \\
J_{wr} \omega_{wr} &= h_r F_{tr} - T_{rr} - T_{br} \\
M \dot{v} &= F_{tf} - F_{tr} - C_a v^2
\end{align*}
\]

where \( K_s \) is the combined stiffness of the left and right axle shafts,
\( \omega_{wf} \) is the angular velocity of the front wheel,
\( \omega_{wr} \) is the angular velocity of the rear wheel,
$J_{wf}$ is the front wheel inertia,
$J_{wr}$ is the rear wheel inertia,
$h_f$ is the static ground to axle height of the front wheel,
$h_r$ is the static ground to axle height of the rear wheel,
$F_{tf}$ is the tractive/braking force the front wheel,
$F_{tr}$ is the tractive/braking force the rear wheel,
$T_{rf}$ is the constant rolling resistance of the front wheel,
$T_{rr}$ is the constant rolling resistance of the rear wheel,
$T_{bf}$ is the front brake torque,
$T_{br}$ is the rear brake torque,
$M$ is the vehicle mass,
$C_a$ is the aerodynamical coefficient.
APPENDIX B: Literature Review

The motivation for automated highway systems (AHS) is essentially twofold, greater capacity on existing roadways and increased safety. To implement such a system, a variety of problems must be solved, including physical implementation, popular acceptance, political concerns, navigation and routing, and lateral and longitudinal control.

We are interested primarily in longitudinal control. First, a description and analysis of the three basic longitudinal control approaches is presented. Next, a review of specific research efforts is included, with an emphasis on those relating to PATH. International research areas, a brief summary, and an annotated bibliography conclude this survey.

LONGITUDINAL CONTROL APPROACHES

Block Control

Block control is currently in use in many public transit systems. Guideways are divided into imaginary segments, or blocks. Occupancy information for each block is relayed to a central computer that controls the flow of vehicles. Because of its inflexibility, block control is not a popular method for AHS's, but it has been suggested for use in emergency situations because of its simplicity and reliability.

Point-Follower Control

This control scheme also relies mainly upon roadside computers. Vehicles transmit state information to the computers and receive command signals to follow fictitious moving points along the highway. No intervehicle communication is required.

Vehicle-Follower Control

In this system, vehicles receive state information from the preceding vehicles and use this information to control their own velocity and position. If no leading vehicle is near, a preset control velocity is used.

Analysis

The spacing or headway for a vehicle-follower control system greatly affects the system performance. Headways generally fall into one of three categories: constant separation, constant time-headway, and constant safety factor.

Constant separation means that the distance between a leader-follower pair remains constant
regardless of speed. This sort of headway has been found to result in serious instability problems in some systems (2,5,21) but has been used successfully in others by including additional state information (43,60).

For constant time headway, the distance between vehicles increases with speed so that the time required for the follower to travel the separation distance is the same for all speeds. This implies constant throughput. With a constant safety factor, the spacing is linearly proportional to the distance required to come to a dead stop from the operating speed. The constant of proportionality (K) is called the safety factor (45). Thus, K=1 implies the so-called ‘brick wall’ stop, i.e. if the leader were to stop instantaneously, the follower would stop without collision.

Garrard (45) ranks the headways in order of preference as constant safety factor, constant time, and constant separation. For further discussion on headway analysis, see also (5,53,54).

There are several basic requirements that must be met by any vehicle control system (43). Controllers must be stable for each individual vehicle as well as asymptotically stable, that is disturbances must be attenuated as they are passed from the lead vehicle to following vehicles. Acceleration and jerk limits must be employed for both passenger comfort and to account for engine limitations. Standard limits are 0.2 g and 0.3 g/s, respectively (60). This obviously adds a nonlinear effect and can be destabilizing (2,45).

To increase capacity, very short headways must be used. Currently, freeways operate at safety factors of about K=0.3 (15,44) and on the verge of instability (33). At such headways, collisions are sure to occur. However, if managed correctly in an AHS, very little damage will result since the severity of an accident depends upon the relative velocity between vehicles. Merging situations are especially troublesome for this case (24,61).

In some ways, point-follower control is superior to vehicle-follower. Communication problems are reduced, since vehicles communicate only with the central computer and not with each other. Ex/entrainment maneuvers are simplified, if the headways are not small, since the computer may easily create appropriate point spacing on all parts of the roadway. It should be noted, however, that this would not be possible for small headways. Asymptotic stability is guaranteed because each vehicle is independently controlled (38). The computer has information concerning the states of all vehicles in its range, thus in emergency situations, communication delays to following vehicles may be avoided (25).

Despite these benefits, PATH advocates vehicle-following (64). This is mainly because vehicle-following facilitates the formation of tightly-spaced platoons, leading to increased capacity. In addition, modern communication technology has reduced many of the communication problems between vehicles.

PREVIOUS RESEARCH

Optimal Control

Levine and Athans (34) were perhaps the first to apply optimal control theory to the longitudinal
problem. They employed a simple dynamic model and a quadratic cost function that included relative position, relative velocity, and acceleration changes. Wilkie (35) extended this work with an analytical method for determining the weighting factors. Chu (37) proposed a unified approach and studied the benefits of increased state information. Garrard and Kornhauser (38,39,55) improved the dynamic model and considered merging and emergency operations and the effects of data-sampling.

Classical / State Feedback Control

There have been numerous efforts in this area. Fenton (43) suggested a phase-plane dependent controller that allowed constant separation headway. Fenton and Chu (28) used a PI controller with compensator on a vehicle model that included nonlinear tire-slip. Chiu, Stupp, and Brown (1) employed a variable-gain state feedback controller, as did Shladover (60) for platoons. Frank, Liu, and Lang (41) added feedfoward control to a nested PID controller. Other researchers in this area include: Olson and Garrard (16), Garrard and Caudill (5), Rouse and Hoberock (3,4), and Hauksdottir and Fenton (42).

In general, most previous efforts have used simplified linear models with little empirical validation, despite arguments that such approximations are inadequate. This is especially true for the propulsion system (16,28,69). Powertrain systems are generally modeled as first-order time delays, with values roughly corresponding to electric motors. Automobile time lags are generally longer and vary with speed, which makes them more difficult to control (38,40,60).

PATH Research

There are two main groups working on longitudinal control, one under Hedrick and one under Desoer. Hedrick’s work has been primarily in modeling for control. With Cho (47), they developed an eight-state, nonlinear powertrain model from physical principles. The emphasis was on transient effects, including gear shifting. Empirical and simulation comparisons were made to show the validity of the model. This work has continued with McMahon (6,69). Two surface sliding control was added, based on a technique described in (46). Robustness studies were performed for disturbances and parameter variations.

Desoer’s research with Sheikholeslam has employed control using relative position, velocity, and acceleration with respect to the immediate leader as well as relative velocity and acceleration with respect to the platoon leader. In (65,66), a linearized model with a first-order engine lag was used. In (67,68), the propulsion model was enhanced to include a velocity dependent time lag. In addition, effects of nonidentical vehicles, mass perturbations, communication delays, and measurement noise were simulated.

International Research

Research has been underway in both Europe and Japan with regard to automated highway sys-
terns, with most of the longitudinal control work having been concentrated in Europe under project Prometheus. Most of the readily available literature concentrates on public transportation systems and navigation such as Japan’s AMTICS (50) and Great Britain’s Autoguide (49). Prometheus, Europe’s major traffic research effort, is putting a great deal of emphasis on driver warning systems and control assistance, i.e. supplementing the driver’s inherent knowledge and response capabilities while leaving the control in the driver’s hands. Gillan (48) downplays the importance of ‘road trains’ or platoons due to the technical difficulties involved (48).

The closest area of research to the AHS idea involves specialized personal rapid transit (PRT) vehicles. These systems are generally electrically powered and run on specially-designed guideways. MacKinnon reviewed these systems in (13). Japan’s CVS and France’s Aramis are two of the best-known efforts (61). Unfortunately, most of the technical information is proprietary and therefore unavailable. Ishii (52) does provide some information on the CVS, which would have employed point-following and a two-target tracking scheme, but was never implemented.

PATH sponsored two fact-finding trips in 1987, Kanafani and Parsons traveled to Germany and Ross visited Japan (31). Both trip reports conclude that the research emphasis in Europe and Japan is on navigation and information systems, with some work being done on lateral guidance of public transportation vehicles.

SUMMARY

Longitudinal control is just one aspect of the AHS problem. There are three basic control approaches: block, point-following, and vehicle-following. PATH advocates the use of vehicle-following because it allows for the formation of platoons, which increases highway capacity more than other approaches. Most of the previous work on longitudinal control has been performed using simplified linear dynamical models with little empirical verification. Finally, it seems that Japan and Europe are not interested in the AHS longitudinal control problem, preferring to direct their efforts toward navigation and improved public transportation.
ANNOTATED BIBLIOGRAPHY

References


This paper examines the requirements and constraints imposed on a longitudinal control system at constant headway from 0.4 to 3.0 seconds. A linear second-order model is used to design a constant gain controller. Two extreme nominal overtaking maneuvers are then analyzed to show that the response of the system under constant gain control is unsatisfactory for certain extreme transient maneuvers as they lead to unacceptable initial controller errors. A fourth- order model was then used to design a time-variable-gain control system.

An exponential function was chosen to obtain satisfactory vehicle response characteristics. Simulations were carried out to show that this controller is capable of performing extreme overtaking maneuvers without resulting in unacceptably large initial errors after maneuver completion.


The authors investigated three different spacing policies: constant separation, constant time- headway and constant safety factor. A simple vehicle model was used with only the acceleration nonlinearity taken into account. They established the steady-state stability using a linear approximation of the transfer function between consecutive vehicles. It was shown that for the constant separation policy it is impossible to obtain steady state stability. They also discussed the effect of nonlinear drivetrain characteristics on the performance of the system by making use of a modified describing function technique. The transfer functions for different velocities were obtained and the jump phenomenon was shown to exist in the nonlinear system. Transient stability was investigated using Liapunov theory but it was pointed out that for nonlinear models computer simulation must be used.


The longitudinal control of vehicles under following-law control when grouped into controlled platoons with wayside controllers (see (3)) is analyzed. Nonlinearities such as limits on jerk, and powerplant dynamics are included in the vehicle model used to study emergency operations. The gains are adjusted on a trial and error basis to obtain acceptable response in computer
simulations. A set of gains is found to give satisfactory results for a specific set of vehicle parameters. The effects of parameter uncertainty and errors in velocity measurements are not included in the analysis.


The control of a platoon of vehicles where the lead vehicle of a platoon is controlled by wayside controllers is analyzed. The wayside controllers are proposed to ensure that sufficient stopping distances are maintained between platoons by making use of “modified block control”. Nonlinear models are used to analyze the response of the system for different disturbances. Only the lead vehicle’s response is presented here and the reader is referred to [3] for the complete analysis.


The three different spacing policies: constant separation, constant time-headway and constant safety-factor are implemented in the longitudinal control of a platoon of vehicles. A simple nonlinear model and a PID control law is used to implement the three different policies. The models are used in computer simulation to adjust gains obtained from linear analysis and the response to a number of different conditions (inter alia emergency braking) are discussed. It is concluded that the constant-safety-factor spacing policy is superior to the other two. The constant gain controller provides satisfactory response for most nominal operations as well as emergency braking. However for severe merging maneuvers additional damping is needed and it is proposed that nonlinear controllers might improve dynamic response during off-nominal operations.


The authors developed a fully nonlinear vehicle model using simple physical principles. The model makes use of nine state variables and include the internal combustion engine, transmission dynamics and tire friction characteristics. This model was then used in the design of a nonlinear controller making use of a two surface sliding control technique for the longitudinal control in a platoon of vehicles. Simulation results for a platoon of two vehicles are presented indicating the feasibility of implementing this nonlinear controller. Tracking errors were investigated in the presence of modelling errors and a disturbance input to show the robustness of the controller.

The following articles all appeared in:
The research currently underway to develop an autonomous vehicle command and control system are presented. This is basically an overview and very little technical information is given.

The new technologies of roadway electrification and automation are discussed to show how their implementation can alleviate the severe transportation/environmental/energy problems in major metropolitan areas. The necessity of a complete system solution is stressed and coordination in the development and implementation of the different technologies will lead to an ultimate solution.

This article describes a communication system that can be used by vehicles under automatic control to exchange information.

Similar to (9) but using fixed beacons to communicate to vehicles as they pass.

The political aspects of automating urban freeways are discussed. A five stage implementation procedure is proposed from voluntary on-board navigation and guidance devices to full automation of all lanes. Further issues such as capacity, air quality and noise, safety and liability, costs, benefits and equity, privacy, and public-private and local-state cooperation are also discussed.

The acceptance of new technology by the public is discussed. Prior technological revolutions that have changed society significantly are examined. The feasibility to involve private business and the cooperation with the public sector are investigated. Incentives to involve public and private sectors are proposed.
All the above articles are overviews of the different aspects involved in the implementation of an Automated Highway System. Technical, economical as well as political considerations are discussed.


High capacity personal rapid transit (HCPRT) is a system concept which utilizes small vehicles at very short headway on exclusive guideway networks. This paper explores the effect of basic parameters such as vehicle length, reaction time, maximum jerk etc. on potential minimum operating headway. The results are then discussed in context of five different programs in high-capacity PRT systems: Cabtrack (UK), Aerospace (USA), Cabintaxi (FRG), Controlled Vehicle System (Japan) and Matra “Aramis,” (France).


The design and implementation of a lateral controller is discussed. A linear 2-DOF model incorporating the effect of roll-steer is used to describe the lateral dynamics of the system. The parameters of the model were obtained from dynamic and static testing as well as manufacturer’s data. This model was used to design single and multiple loop controllers. The systems were implemented in an actual test vehicle and acceptable results were obtained. The predicted performance corresponded very closely to the actual measured response. Single loop configurations were found to have adequate performance up to 80 mph. No emergency situations were considered and the tests only consisted of high-speed tracking in straight and curved sections of a freeway.


The interaction between a vehicle’s longitudinal control and the constant headway spacing policy is discussed. Different controller types such as point-following and car-following modes are developed and analyzed on the bases of safety, cost and capacity. The most important parameters were identified through a sensitivity analysis and subsequently used in the analyses. They are: maximum deceleration of lead vehicle, maximum deceleration of second vehicle, time delay of controller, and a velocity deviation threshold. A number of conditions under which safe operation is possible were identified.

A design approach for the design of a longitudinal control system for automated transit vehicles is proposed. Two model-following controller designs are discussed. Preliminary second and third order controllers are designed making use of a linearized model. Nonlinear effects such as jerk and propulsion limitations are included and the describing function technique is used to analyze the stability of the system. In the case of nonlinearities for which describing functions can not be found, numerical integration is used to obtain the corresponding results.

Computer simulations were also carried out to verify the theoretical results and the occurrence of a limit cycle. It was shown that it is possible to design longitudinal controllers with excellent behavior during station keeping operations. Additional controllers are necessary for transitional modes of operation. It is also shown that controllers generating jerk commands become unstable at relatively long headway while controllers issuing acceleration commands can become unstable at very short headway.


The influence of communication and digital computation on the longitudinal control of vehicles in an automated transit system is discussed. Two cases are considered, a dumb-vehicle model where all the computation is carried out by the wayside computers, and a smart-vehicle model where the computation is carried out onboard the vehicle. A simple nonlinear model of the vehicle is linearized to carry out the analysis. Such factors as the word length, data rates and allowable transmission time lags are investigated. Computer simulations are used to evaluate the response of the fully nonlinear system. A set of values is obtained for the smart-vehicle model that will satisfy all the requirements for nominal and emergency control.


The lateral dynamic performance of automatically guided single and dual steered axle vehicles are investigated using a 3-DOF nonlinear model which includes yaw, sideslip and roll. Three different controllers, an independent controller, yaw-controlled rear steering and a zero-sideslip controller, are implemented and the response analyzed for a number of different vehicle maneuvers including curve and station entry.

All three controllers showed improved performance compared to a single-axle controller. It was also found that all the ride quality requirements were met by the controllers. It seems as if the zero-sideslip controller was preferred due to reduced tracking error and acceleration in curve and station entry while having performance on straight guideway sections similar to that of the single-axle controller.
The European road traffic situation is discussed with reference to safe driving systems, communication, and guidance and traffic control. The PROMETHEUS (PROgraM for a European Traffic with Highest Efficiency and Unprecedented Safety) project is also presented with all the different modules: Pro-Car - Development of computer-assisted systems to support the driver. Pro-Net - Development of communication networks between vehicles. Pro-Road - Development of information systems on road conditions. Pro-Chip - Development of micro-electronic components. Pro-Art - Development of fundamental artificial intelligence systems. Pro-Corn - Development of structures and standards for data exchange between vehicles and their environment. Pro-Gen - Development of scenarios simulating the road traffic of the future that are suitable for analysis and evaluation of the proposed systems.

An overview of the scope and aims of Project Trilby at General Motors are given by the authors. The importance of taking the human factor into consideration during this development is stressed by regarding the driver as a dynamic subsystem and not merely as a source of manual input. The incorporation of artificial intelligence is proposed to take care of driver preferences in the automatic control process of the automobile. In “The Adaptive Vehicle” all the subsystems must be completely integrated to perform their various tasks to the complete satisfaction of the driver over a wide range of environmental conditions. A three-level hierarchical control structure is proposed in order to reduce the complexity of the optimal control problem to solve.

very interesting article on the modelling of vehicle strings. All the analysis are carried out on a linearized model. The authors considered a car as a servo-mechanism with a lag to compensate for the driver’s reaction time, the braking system and the inertia of the vehicle. A variety of analyses are carried out to investigate the stability of the system. Models range from proportional control, constant spacing policy to the California Motor Vehicle Code (allow spacing of one car length for every ten miles per hour) and Laplace transforms are used to determine stability. It was shown that constant spacing control is always unstable but that relative velocity control can be stable under most circumstances. A number of experiments were also carried out and it was shown that the most applicable model is that of relative velocity control.

This article is a follow-on to (21) where the emphasis is now put on the stability of the string. Again Laplace transforms are used to solve the resulting differential equations, but the results are further examined by numerically integrating the ODE’s on a digital computer. A more complex control law is also investigated where not only the state of the first vehicle, but also that of the lead is taken into account. A case where the states of the vehicle in front and the one behind are used is also described. A statistical criterion is derived for stability by examining the statistics of the n'th car in a line assuming that of the first car is known. ‘Acceleration noise’ is proposed as a parameter that might be employed to characterize the car- driver-road complex under various conditions.


Yet another survey with many references (all pre 1970). The topics that are covered are: longitudinal control, vehicle- spacing detection, vehicle communication systems, lateral control, automatic merging, propulsion, decision-making capabilities, automatic vehicle checkouts and evolutionary developments.


This paper attempts to evaluate the severity and cost of each collision for a variety of accident scenarios. The approach includes a specification of the collision model, a selection of an accident severity measure, a consideration of a corresponding cost function and a sensitivity analysis. A quantitative measure of the effects of key parameters on accident severity is specified. Three different platoon-accident scenarios are then investigated to demonstrate the effectiveness of the methods.


Simple vehicle dynamics and control was employed to model an automated vehicle system with vehicle-borne or wayside controllers. An accident severity measure was defined and is then subsequently used in computer simulations to compare the effect of different parameters in the case of the sudden failure of a vehicle and the case where a stalled vehicle on the roadway is not detected. The only significant parameter was found to be the deceleration of the vehicles. The wayside controller proved to be far superior to the vehicle-borne system. An attempt was also made to compare the results of automobile traffic and an automated system with wayside controllers. It was also found that a considerable improvement in safety could be obtained if vehicles could be kept in motion at high speeds after a collision.

The author, former director of the PATH program, gives an overview of the scope and type of research planned for the next 2-3 years. The activities include: lateral control, longitudinal control, non-contact electrification and navigation.


AVCCS is being developed as the major solution to current highway congestion. The objective is to automate vehicle operations while minimizing human involvement. Different degrees of control are proposed for the different street types encountered in one journey. The components of the system included in the discussion are: the measured wheel, the flux gate compass, the automatic vehicle location sensor system, the radar proximity detection system, the logical bit map and the command and control communications system.


The authors discuss the design and implementation of a longitudinal controller in an automobile. They use a simple nonlinear model of the vehicle to account for powerplant dynamics and wheel slip for the design of the controller. In the design of the controller the following aspects are taken into consideration: vehicle acceleration limits, ride comfort, tracking accuracy and external disturbances such as headwinds. No emergency maneuvers were incorporated in the design. The performance of the controller was investigated making use of numerical simulations. The designed controller was subsequently implemented in a full scale test. It was found that the controller performed satisfactorily under nominal conditions such as merging and constant speed operation. The response of the system under emergency braking was also found to be adequate.


A fixed reference system is proposed to eliminate the shockwave associated with moving reference systems. The model of the vehicle dynamics include only wind drag force, gravitational force and the propulsion force which are assumed to be linear. A PID controller is designed using first a continuous system approach and then refined making use of sampled data techniques because the vehicle only receives inputs when it passes the fixed reference posts. Computer simulations are also carried out to evaluate the system response to initial spacing errors and wind gusts. In section 2 vehicle control under emergency braking is considered. In this case
it is assumed that the posts provide information on the desired displacement, velocity and acceleration. Four PI controllers are designed and their responses analyzed through numerical simulations. It was found that oscillations of the wheel’s angular velocity resulted from the effect of the discrete nature of the input leading to longer stopping distances than for continuous input.


A survey of automobile navigation techniques including radio location, dead reckoning, map matching, and proximity beacons. In all cases the aim is to maximize the efficiency of the route and to determine the absolute position as opposed to sensing relative position to other vehicles. This article concludes with a description of where the U.S., Europe, and Japan are concentrating their research efforts.


This is a summary of Kanafani’s and Parson’s exploratory trip to Germany and Ross’ visit to Japan as part of the PATH program. Both Europe and Japan seem to be concentrating on navigation for automobiles and lateral guidance public transportation systems. Europe’s Prometheus project is highly secretive, while the Japanese IVS system under the Ministry of International Trade and Industry is somewhat more open. Neither Europe nor Japan seems to be actively pursuing longitudinal control, although Assoc. Prof. Yoshio Tsukio of Nagoya University made the interesting comment that if Japan were to pursue an automated highway system, it would be fully autonomous (i.e. without any roadside communication).


The authors construct a linear, time-invariant model of a human controller, which includes such effects on performance as visual screening, task interference, and workload. Neuromotor dynamics are modeled as a time lag. Experimental data was collected using actual human responses to disturbance inputs given a scalar controller and an error indicator.


This paper presents some early car-following theories. The vehicle dynamics are represented with differential difference equations including a time-lag and sensitivity. The model is evaluated with constant and varying sensitivities. Local and asymptotic stability are discussed. Some
empirical results are presented, with the comment that most drivers operate on the verge of instability. Relationships for the maximum flow are discussed for each sensitivity.


This paper appears to be the first of several to apply optimal control theory to the platoon problem. Specifically, an optimal linear regulator with a quadratic cost function was employed. The cost function penalizes errors in relative position and velocity, as well as changes in acceleration. This control approach leads to a linear, time-invariant, stable feedback control system. The optimization problem is solved only for the steady-state case by solving the Riccati equation on a digital computer.

Simulation results are presented for the three-car case. The vehicle model includes only propulsive force and nonlinear drag, which is linearized with a Taylor series expansion. The simulation was performed on an analog computer. The method presented employs information on the states of all three vehicles. The authors suggest that the first vehicle has little effect on the last vehicle in large strings, but they do not address asymptotic stability specifically.


This article presents a cell-following control theory that attempts to avoid the prohibitive amount of information needed for high-dimension controllers and the possible instabilities of car-following. A simple linearized model is used for the vehicle dynamics. A quadratic cost function that considers position, velocity, acceleration, and jerk is included in the optimal controller. An analytical sensitivity method is used to calculate the weighing factors. Simulation results for a merging maneuver are presented.


A small-signal, car-following model is developed from the vehicle’s longitudinal dynamics. Criteria for asymptotic stability are developed. Simulations and empirical trials were run for a two vehicle string initially at steady-state. For the empirical trials, an experimental car with electrohydraulic braking, acceleration, and steering was employed with a phantom lead car undergoing sinusoidal velocity variations. Six cases with varying control parameters were studied to show effects upon stability and car-spacing.

This work presents a unified approach to the optimal control of a string of vehicles. The vehicle dynamics are represented with a simple model that accounts for drag and input force. A rather thorough analysis of asymptotic stability is given based on the $z$-transform of the dynamic model. The quadratic cost function includes relative position, relative velocity, and acceleration (control force and jerk). Six control cases are discussed for varying amounts of feedback information. The general trend is that more information leads to better control, but with diminishing marginal returns.


A study of slot-following control. A dynamical model is derived that includes drag forces, propulsive forces, road grade, and mechanical resistance. The equations are non-dimensionalized and linearized about a mainline operating point. A quadratic performance index is employed for optimal control that includes position, velocity, and acceleration errors, and the rate of change of propulsive force (proportional to jerk). Observers are used in place of measuring acceleration error and jerk. Simulations for constant disturbances, merging, and emergency stops were run with the observers and measured acceleration and jerk. Sampled data effects were also simulated.


This article supports the results of (38) above and shows that the linearized equations are applicable even to emergency and merging situations.


This article presents a clear motivation for vehicle automation, although without much quantitative proof of predicted benefits. A discussion of the pros and cons of vehicle- and point-follower control systems is included. A vehicle model for simulation is described that includes propulsion with a first order lag, acceleration and jerk limits, sampling errors, and signal noise. A constant coefficient, vehicle-following control system is employed with knowledge of position, velocity, and acceleration of the immediately preceding vehicle and command acceleration and velocity of the platoon leader. Other effects are added to specific trials, such as delay of information to rear vehicles, an enhanced jerk limiter, rolling resistance, and square-law aerodynamic drag.

The first simulation uses a propulsion time delay corresponding to an electric motor. The results show that the controller is not suitable for emergency situations. Simulations with an increased time constant (roughly equivalent to an internal combustion engine) show a heavy
dependence of the control parameters on the propulsion system. The author notes that most automobile propulsion parameters vary with speed, leading to increased complexity.


This paper presents simulation results from a vehicle-follower control model. The non-linear vehicle model includes propulsion and sensing delays, power saturation, and dissimilar vehicle characteristics for members of the platoon. A nested control system is used, consisting of a PID regulator for disturbance rejection, proportional space-controller for entrainment, and a feedforward speed-synchronizer to change the command speed.


The authors develop a velocity-dependent, nonlinear vehicle model. It is noted that propulsion nonlinearities can have significant effects upon the outcome. The nonlinear effects include throttle and tire slip.

A cascade compensator (PI) was employed to deliver small tracking errors. An observer/controller was also used to deliver a velocity invariant response. Poles were placed through trial-and-error. Simulations were run for merging, step speed change, and an organizing maneuver. Robustness to headwinds and sensitivity to tire slip and throttle parameters were also examined.


A comparison of performance for automated and unautomated transportation is presented. A phase-plane dependent controller is suggested for a simple dynamic vehicle model, which allows for constant headway control. A limited discussion on asymptotic stability concludes the article.


One of the first comprehensive articles on the automated transportation problem, this paper describes basic control modes, probable capacity increases, and safety concerns.
This article presents a survey of block, vehicle-follower, and point-follower control. Included are analyses for constant separation, constant time, and constant safety factor headways.

This paper describes the basis for the sliding control procedure employed by McMahon and Hedrick in (6).

An in-depth development of an eight-state, nonlinear powertrain model based on physical principles is presented. Empirical and simulation data are compared to show the validity of the model, with an emphasis on transient and gear-shifting effects.

This report presents the Prometheus system in Europe with an emphasis on electronic route guidance. ‘Road trains’ (i.e. platoons) are mentioned, but their importance is downplayed due to the technological difficulties involved.

An overview of the UK’s proposed Autoguide navigation system is given.

The Japanese navigation and on-line map system is described.

This paper focuses primarily on directional, as opposed to longitudinal control, with an emphasis on road design using a kinematic study of transition curves.
This study employs a simple dynamic model of an electrically powered vehicle with four modes: powering, coasting, electrical braking, and mechanical braking. Simulations were run for point-following control using PD control. Time delays were implemented to control jerk. The two-target tracking system is introduced briefly as a means to decrease position jerk.

Minimum safe headways are discussed for both point- and vehicle-follower control schemes. In addition, analyses are made for the no collision condition and for very short headways that allow for collisions given limited accident severity.

This study may be applied to inter-platoon spacing. It presents an analysis of factors affecting safe headways for the no collision condition with brick-wall and modified brick-wall stopping situations.

This is an extension of previous work (38,39). A point-follower controller with a quadratic performance index is combined with a Kalman filter. Simulations were performed with varying sampling interval lengths, wind conditions, and noise levels. Three vehicle operations were used, constant velocity, slot advance, and acceleration for merging from an off-line station.

This paper uses a kinematic analysis to examine very short headways for applications on PRT systems. No specific control systems are described.

An actual PRT system is described in detail, but without much technical information. It uses point-follower control at very low speed (10 mph).

An actual PRT system is described in detail, but without much technical information. It uses point-follower control at very low speed (10 mph).
This report proposes a 1 second ‘headway barrier’ for PRT systems after analysis of the state of the art of headway capabilities and a kinematic study of braking.

A point-follower controller is introduced for a vehicle model including dc-motor propulsion and aerodynamic drag. The kinematics of speed change and backward point skipping are discussed. The results show that constant time headway is required for constant throughput and that the necessary speed reduction results in a decrease in safety. The author suggests tachometer feedback and a PI controller with relative position data for best performance.

This paper describes a car-following controller with special emphasis on asymptotic stability and the effects of jerk limiting. A standard mass with time-delayed propulsion model (electric motor) is used with grade/gravitation and aerodynamic external forces. The simulations were performed at low speeds (30 mph).
A basic feedback controller is employed, although the coefficient values are varied to account for specific needs of steady-state and ex/entrainment. The coefficients were determined through pole placement.
An analysis of asymptotic stability shows that absolute velocity feedback is necessary for constant-spacing control. Calculations show that the vehicle response is much more strongly related to the lead velocity than to the relative velocity.
Finally, the destabilizing effects of jerk-limiting are discussed. Analyses are performed for sinusoidal and random inputs, resulting in jump resonances and limit cycles in both cases. The author suggests that these situations may be avoided by limiting the error values.

This paper provides a rather extensive analysis on the cost and safety benefits of AGT systems. It also includes an historical description of past efforts, including the Japanese CVS and the French Aramis systems.

This paper presents that design of lateral feedback and feedforward controllers designed to meet lateral tracking requirements by utilizing frequency-shaped linear quadratic (FSLQ) control
theory. This design method accounts for ride quality and permits the high-frequency robustness characteristics to be improved through a proper choice of weighting factors.


Employs an optimal cost function developed by Levine and Athans (34) and describes a merging algorithm based on set theory. On-ramp acceleration and off-ramp deceleration are not addressed specifically.


This is a general description of the PATH program, which supports a platoon approach with minimal crash effects due to very short headways, leading to double or triple present capacity. To implement platoons, the vehicle-follower is preferred over the point-follower. It also describes possible measuring and communication devices to aid the controller. Finally, brief descriptions of Hedrick’s vehicle model (6,47,69), Desoer’s controller (65-68), and lateral controllers (62,70) are given.


The vehicle model includes grade effects, linearized air resistance, constant mechanical drag, and a first order engine lag (0.2s). A state feedback controller is employed that includes relative position, relative velocity, relative acceleration, lead-car velocity and lead car-acceleration. Simulations were run for a platoon of four identical vehicles starting at an initial velocity of 40 mph and accelerating to 72 mph. The maximum values for jerk and acceleration were 0.3 g/s and 0.5 g, respectively. Both values are on the upper limit of what is commonly considered acceptable. The maximum position error was 0.22 m. Robustness to communication delays and measurement noise was investigated.


This work continues that of (65) by demonstrating the need for the derivatives of distance deviations for pole placement and the benefits of this information on transient response and allowable spacings.

The vehicle model described in (65) is enhanced by including a velocity dependent time lag in the engine dynamics. The model is then linearized using the exact linearization technique. Simulations were run for platoons of 4 to 16 vehicles with nonidentical characteristics. Robustness to mass perturbations, communication delays, and measurement noise was investigated.

This is essentially the same paper as (67), except that a push-button device is included in the controller model to compensate for mass perturbations.

A restatement of the conclusions reached in (6), this gives a more detailed description of their conclusions and recommendations for further study. The authors suggest that the vehicle propulsion dynamics are sufficiently nonlinear to warrant a nonlinear model for simulation, but that there are simplifications which may be made to their present model.

A discussion of some lateral control research within the PATH project.

A precursor to (46), this paper presents a highly mathematical development of sliding control. Parameter sensitivity and robustness to disturbance inputs are discussed. A continuous approximation of the discontinuous control law is applied to reduce chattering. Application to a two-link, flexible manipulator is shown.

A vehicle-follower controller is designed based upon the results of Levine and Athans (34) optimal control efforts and the kinematic constraints found by Chu, et al (1) due to the no-collision condition and the jerk and acceleration limits. Sub-optimal controllers to reduce input information and computation are evaluated by computer simulation. These simulations were run for an overtaking maneuver.

This paper demonstrates that acceleration and jerk limiting have destabilizing effects for vehicle controllers. External forces, braking, and propulsion dynamics are neglected, so the results are best interpreted from a qualitative standpoint.


The authors start with an overview of AGV system requirements. The no-collision condition is required, justified with the statement that less rigorous safety requirements do not significantly reduce throughput. A braking maneuver for a string of vehicles is investigated. Pure Moving Block (minimum headway equals instantaneous braking distance) and Moving Time Block (constant time separation) theories are employed. Maximum braking for lead vehicle results in unacceptable headways for the following vehicles. Thus, headways must be increased at the expense of throughput. Acceptable results are achieved by shaping the velocity profiles using jerk and acceleration.


An emergency obstacle avoidance maneuver was simulated for a four-wheel steering vehicle and compared to a front wheel steering vehicle. A nonlinear, bicycle model including a nonlinear tire model was employed. The 4WS vehicle employed a closed loop linear control law for rear wheel steering including a yaw rate and front wheel steering angle with parameter and speed dependent gains.


A one-surface, sliding mode controller is developed for a two state (air mass in intake manifold and engine speed) engine model. Robustness to parameter variations and disturbances are simulated with sensor delay. The appendix briefly describes sliding mode control theory.


The engine model described incorporates a carburetor, intake manifold, combustion, and dynamic effects. Analysis of the intake manifold includes fast (EGR and air/fuel) flow and slow (liquid fuel film) flow. Simulations for steady state were compared to actual engine data. Computer simulations for transient effects were then run for the model.

This paper develops a dynamic model of a torque converter that is suitable for control applications. The model is generated from physical principles and then simplified. A regression fit of actual data was then compared to a computer simulation of the model predictions.


This is a further extension to previous work (65-68). It uses the same dynamic model and exact linearization method as before, however the control law is changed due to the absence of information from the leading vehicle. Only information from the immediately preceding vehicle is available. The object of this study was to show that control may still be maintained in the absence of such information (e.g. an emergency failure.)


This work discusses the exact linearization method employed by Sheikholeslam and Dcsoer (67,68).


These pages discuss the exact linearization method in more detail than is included in (80).


This is a very general report outlining suggestions for implementation and safety analysis of the PATH project.


This thesis develops an automotive powertrain model based on physical principles and empirical data. It also develops the use of sliding mode control. Finally, simulation data is presented, along with conclusions and suggestions for further work.