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Contestable Leaderships: Party Discipline and Vote Buying in Legislatures.*

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Abstract

This paper examines the institutional determinants of discipline in legislative parties: what tools enable a party leader to induce her party to support an unpopular position? The model formalizes the tradeoff between resources at the leader’s discretion, and the incumbent’s need to maintain a minimum level of support to continue leading. I show that contrary to conventional wisdom, promises of future partisan benefits (such as nominations to party lists) are insufficient to grant significant power to the party leader. Unless a majority of the party agrees (ex ante) with the incumbent’s preferred position, future resources are valuable to the incumbent only if she also distributes benefits on the spot, or if she is protected by a supermajority requirement for removal. Moreover, I show that there is a complementarity between benefits distributed on the spot and the value of promises of future benefits, and that the multiplier effect of current resources is higher the more vulnerable the leader is to internal threats. As a result, more vulnerable leaders will allocate more current resources to buy members of their own party instead of the opposition.

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"At times, party leaders may seem more like generals guiding their disciplined troops into the lobbies. ... At other times, however, ... the leader keeps the party together, but basically by herding people together while letting the party go where it wants. Of course, legislative settings are hardly all or nothing. 'Generals' and 'shepherds' may be pure types, but ... we are more likely to find party leaders shuttling between these types." Bowler, Farrel and Katz (1999)

1 Introduction

One of the central questions in the study of representative democracy is how partisan organizations shape decision-making in legislatures. At the core of this matter is the balance of power between party leaders and rank-and-file party legislators (backbenchers, or PBs). Under what conditions will a party leader be able to induce her party to support an unpopular position? Conversely, when will party leaders have to yield to the views of a majority of the party?

In this paper, I provide a simple model to tackle these questions. In particular, I re-examine within the model the conventional wisdom in political science that leadership's promises of future benefits (such as nomination to party lists) are the key instruments through which a party leader can induce backbenchers to support the party line, possibly even against their preferences.¹ This ability of the party leader to change backbenchers’ voting behavior away from their ideal voting pattern is typically referred to as party discipline (see Krehbiel, 1993; Cox and McCubbins, 1993; Tsebelis, 1995).

The model in this paper formalizes the tradeoff between resources at the leader’s dis-

¹"The assumption here is simply that nomination control is a key determinant of an agent’s unity because leaders who possess this power should be able to discipline their followers.” (Morgenstern 2004); "The nature of the nominating procedure determines the nature of the party; he who can make the nominations is the owner of the party” (Schattschneider, 1942; p.64). For similar arguments, see, among many others, Ames (2002), Bowler et al (1999), and Sanchez de Dios (1999). The role of nomination power in the literature is only matched by that of the vote of confidence in parliamentary systems (see Diermeier and Feddersen (1998) for a formal statement of this argument).
cretion, and the inherently contestable nature of the leadership in political parties. On the one hand, the party leader is endowed with two types of resources with which to influence legislators’ voting behavior: (i) pork, which consists of current payments that can be distributed to both PBs and opposition legislators, and (ii) electoral benefits, which consist of promises of future partisan benefits that can only be distributed to PBs. On the other hand, the incumbent’s control of the leadership is always a potentially precarious construction: the leader needs to maintain a minimum level of support to continue leading (Panebianco 1988, Calvert, 1987).

Since promises of future benefits can only be delivered if the incumbent leader retains the command of the party, a collective action problem between backbenchers opposing the incumbent emerges. Backbenchers risk losing much by opposing a leader who (they believe) has a firm support inside the party, but might be willing to do so if they believe that others will do so too. In other words, in this environment the value of the incumbent’s promises is not exogenously given, but endogenously determined by the extent of support to the incumbent among backbenchers.

To consider this problem formally, I model the internal constraints faced by the incumbent party leader as the partisan equivalent of a confidence vote procedure. The party leader is overthrown - and her promises of electoral benefits abandoned in favor of a reward to the supporters of the new establishment - whenever her advocated position does not gather the support of a minimum proportion $\mu$ ($\mu \leq 1/2$) of party backbenchers in the legislature.\(^2\) Since $\mu < 1/2$ means that the removal of the leader requires the defection of more than a majority of PBs, I refer to this case as a supermajority requirement for removal.

I analyze the equilibrium outcomes in this environment under an assumption of incomplete information about PBs’ preferences. Although it is common knowledge that back-

\(^2\)I rule out the case $\mu > 1/2$, as it would imply that a challenger gathering the support of a minority of the party would be able to overthrow the incumbent from office.
benchers want to vote for policies which are close to their constituency’s preferred position, these ideal policies are assumed to be the legislators’ private information, and correlated with each other. Specifically, the ideal policy of PB i is composed of a common and an idiosyncratic unobservable components. As a result, backbenchers are uncertain about the distribution of fellow party members in the policy space, but can use the information contained in their constituencies’ preferred position to enhance their estimate.

While under an assumption of common knowledge of PBs preferences radically different behavioral patterns can be sustained as equilibria by self fulfilling beliefs, relaxing this assumption allows us to pin down a unique equilibrium, and thus leads to a much more productive analysis. I show, in particular, that electoral benefits can be useless for the incumbent leader; i.e., contrary to the conventional wisdom, nomination power can be completely ineffective in providing discipline in legislative parties.

Specifically, proposition 2 shows that if a majority of the party disagrees (ex ante) with the incumbent’s preferred position, electoral benefits are useless to the incumbent leader unless she also distributes benefits on the spot, or she is protected by a supermajority requirement for removal. This illustrates the central insight of the paper. Promises of future benefits will alter voting behavior only if party members believe that the incumbent leader has a strong hold to the reins of power. Understanding the role of different instruments in achieving discipline thus requires understanding their contribution to the formation of these expectations among backbenchers.

In this track, I show that there is a complementarity between the allocation of pork to party members and the value of electoral benefits. Keeping PBs’ beliefs about the actions of fellow party members fixed, an increase of one dollar in the allocation of pork to party members increases the net value of the incumbent’s offer by the same amount. Beliefs about the actions of other PBs will not remain fixed, however, as the revised offer will induce PBs

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3For a discussion of the methodological aspects underpinning this result, see Morris and Shin (2001, 2003), and Frankel, Morris and Pauzner (2003).
to anticipate a higher support to the party line among party members, and thus a higher probability of the incumbent’s survival, leading ultimately to a higher expected value of her promises.

As an immediate consequence of this complementarity, we have the following result. If endowed with sufficiently large amount of current resources (pork), the incumbent can make the electoral benefits valuable, even when ex ante a majority of the party opposes the party line. Moreover, in this case the incumbent needs in fact to buy the party in order to generate discipline. In the absence of a supermajority requirement for removal of the leader, then, the influence of backbenchers is not lost, but only reshaped in terms of a lower bound of payments that needs to be allocated to party members for party resources to be in play.

This raises the question of how the allocation of pork between party and non-party members is affected by the availability of future partisan benefits. While pork can be used to attain the support of opposition legislators, this allocation has an opportunity cost: buying the opposition implies weakening the support inside the party. In fact, our previous analysis implies that the magnitude of this opportunity cost will be determined by the strength of the complementarity between pork and electoral benefits. Proposition 4 exploits the fact that the multiplier effect of current resources is higher the more exposed the incumbent is to internal threats, to conclude that more vulnerable leaders will allocate a higher proportion of pork to buy members of their own party vis a vis the opposition.

The remainder of the paper is organized as follows. The basic model is presented in Section 2. Section 3 provides a characterization of voting equilibria, which constitutes the basis of the substantive study in Section 4. Section 5 extends the model to include an endogenous determination of the challenge to the incumbent leader. I show here that the basic model is a stylized description of this extended framework, assuming that policy alternatives are not “too close” in the policy space. Section 6 relates the framework and results with the literature, and Section 7 concludes.
2 The Basic Model

There are three types of agents in the model: (i) a party leader, (ii) a continuum of party backbenchers (PBs), with total size 1 and (iii) a continuum of size $\beta < 1$ of opposition legislators. PBs and opposition legislators integrate a legislature, which chooses between two given policy alternatives $q$ and $x$ in $\mathbb{R}$, $q < x$, by simple majority voting.

Legislators’ Preferences & Information.

PBs’ payoffs are determined by (i) ”monetary” benefits they can extract from the party leadership, and (ii) the distance between their constituents ”ideal policy” $\theta_i$ and the policy they voted for in Congress, $x_i \in \{q, x\}$.

In particular, monetary transfers enter linearly into their utility function, and policy preferences of PB $i$ are represented by a utility function $u(|x_i - \theta_i|)$.

It will be convenient to define - taking the pair $(q, x)$ of policy alternatives as given - the function $v(\theta_i) \equiv u(|q - \theta_i|) - u(|x - \theta_i|)$. The value $v(\theta_i)$ denotes the net gain of voting for $q$ instead of $x$ for PB $i$, with ideal policy $\theta_i$. Note that by construction $v(\theta_i) = 0$ at $\theta_i = \frac{x+q}{2}$, and that $|v(\cdot)|$ is symmetric around this point. Moreover, I will assume throughout that $v(\cdot)$ is a continuous function satisfying the following condition:

Assumption (A1). There exists $\alpha > 0$ such that for all $(\theta_i, \theta_i')$ with $\theta_i' > \theta_i$,

$$v(\theta_i) - v(\theta_i') \geq \alpha (\theta_i' - \theta_i)$$

Assumption A1 implies, in particular, that $v(\cdot)$ is strictly decreasing and that $|v(\cdot)|$ is convex, making $v(\cdot)$ unbounded above and below.

The ideal policy of each PB, $\theta_i$, is private information, but correlated with that of the other PBs. Specifically, I assume that the ideal policy of PB $i$ is given by $\theta_i = \theta + \varepsilon_i$.

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4This formulation intends to capture the essential tradeoff for legislators between pleasing their constituencies and the party leadership, two ”masters” with (generically) different objectives.

5The assumed properties of $v$ obviously translate into certain requirements that our primitive function $u$ must satisfy. In particular, for $v$ to be decreasing, it is necessary that $u$ is a decreasing, concave function.
where the common component $\theta$ is drawn from a $N(\theta_0, \eta^2)$ distribution, the idiosyncratic component $\varepsilon_i$ is i.i.d., and drawn from a $N(0, \sigma^2)$ distribution, and both $\theta$ and $\varepsilon_i$ are unobservable.\(^6\)

Opposition legislators have policy preferences $u(\cdot)$ identical to those of PBs. Although their ideal policies are private information, I assume that they are distributed according to a known c.d.f. $G(\cdot)$. This implies, in particular, that the proportion of opposition legislators with ideal policy below some number $z$ is public information.

\section*{Party Leadership & Payments}

The party leader cares about the policy outcome: the leader obtains net benefit $w > 0$ from the policy outcome being $x$ instead of $q$. The leader is endowed with two types of resources with which to influence legislators’ voting behavior: (i) pork, which consists of current payments that can be distributed to both PBs ($r$) and opposition legislators ($r_o$), and (ii) electoral benefits ($e$), which consist of promises of future partisan benefits that can only be distributed to PBs. As the notation suggests, I will restrict to payments that are symmetric among legislators of the same party. Moreover, I will only allow payments to an individual to be conditional on his actions, thus precluding more complex mechanisms that could possibly depend on aggregate voting patterns.

Pork payments are conditional offers: a PB receives $r$ when voting in favor of $x$, and zero otherwise. Similarly, an opposition legislator receives $r_o$ when voting in favor of $x$, and zero otherwise. The party leader chooses $r$ and $r_o$ subject to the (ex ante) budget constraint $r_o\beta + r \leq R$, where $R$ denotes the total amount of pork resources available to the leader. Residuals from unaccepted offers are kept by the incumbent leader.

Unlike pork - the allocation of which is final and irreversible - conditional promises of electoral benefits can only be delivered if the incumbent leader survives internal challenges.\(^6\)

\(^6\)Note then that a PB is uncertain about the distribution of his fellow party members in the policy space: a democrat from California observes the preferences of his constituency, but can not perfectly separate what part is due to them being Californian and what part is due to them being democrats. Note, however, that a PB will use his private information to estimate where other party legislators lie in the policy space.
to her authority. Specifically, I assume that the party leader can choose between two alternative procedures, which I call a partisan and a non-partisan vote.

In a non-partisan vote the incumbent commits to distribute $e$ to every PB irrespective of his vote. Electoral benefits thus play no role in influencing the voting behavior of PBs. Moreover, in the basic model, I assume that this unconditional allocation of electoral benefits is never challenged. The net payoff of voting for $x$ for PB $i$ in a non-partisan vote is then given by $\Pi_{np}(\theta_i) = r - v(\theta_i)$

In a party vote, instead, the incumbent commits to distribute $e$ only to PBs voting for $x$, and zero to others. I will assume, however, that the conditional allocation of electoral benefits implicit in the party vote will always trigger a challenge to the party leader.\footnote{Section 5 extends the model allowing an endogenous determination of the challenge. There we show that the incumbent won’t be challenged (i) in a non-partisan vote or (ii) in a party vote if $x$ is sufficiently close to $q$ ($x < \tilde{x}$ for some $\tilde{x}$), but is challenged whenever $x > \tilde{x}$. The basic model is thus a reduced form of the complete model, assuming that policy alternatives are not "too close".}

A challenge consists of an alternative conditional distribution of electoral benefits: if a challenge is successful, PBs voting for $q$ receive electoral benefits $e$, and those voting for $x$ receive zero.\footnote{As with pork, due to unaccepted offers, in a party vote there won’t be ex post budget balance of electoral benefits. The remainder can be assumed to be distributed to party members who are not currently in Congress, kept in the party safe box, or burned.} A challenge is successful if the incumbent’s advocated position does not gather sufficient support by PBs in the legislature; i.e., if the mass of PBs in the incumbent’s coalition, denoted by $\Gamma$, does not reach a minimum threshold $\mu$ ($\mu \leq 1/2$). To summarize, the net monetary payoff for a PB voting for $x$ is $e$ if the incumbent survives the challenge (if $\Gamma < \mu$), and $-e$ if the incumbent is overthrown. The net expected payoff of voting for $x$ for PB $i$ in a party vote is then $\Pi_p(\theta_i) = r + e \left[ 1 - 2 \Pr(\Gamma < \mu|\theta_i) \right] - v(\theta_i)$

**Strategies and Equilibrium**

Taking advantage of my minimalist representation of opposition legislators, I will exclude them from the set of players, and instead consider their best responses as part of the environment. Specifically, since the pork resource constraint $r_o \beta + r \leq R$ will hold
with equality at the optimum, we substitute \( r_o = \frac{R-r}{\beta} \), and treat the main party leader’s allocation decision simply as a choice of a pork offer to party members \( r \in [0, R] \). Given any such offer \( r \), the mass of legislators in the opposition voting for \( x \) is then given by:

\[
1 - G \left( v^{-1} \left( \frac{R-r}{\beta} \right) \right) \beta.
\]

The players in the modified game are therefore PBs and the incumbent party leader.

The timeline consists of three stages. In Stage 1, nature chooses a realization of the unobservable random variables \( \theta \) and \( \{\varepsilon_i\} \), and each PB \( i \) privately observes his ideal policy \( \theta_i = \theta + \varepsilon_i \). The party leader receives no such private signal. In Stage 2, the party leader decides (i) whether to make the vote a non-partisan vote or a party vote, and (ii) an allocation of pork to PBs. In Stage 3, legislators vote between the alternatives \( x \) and \( q \).

A strategy for the incumbent leader is therefore a choice of a couple \((a_I, r)\), where \( a_I \in \{np, p\} \) and \( r \in [0, R] \). The incumbent’s choice of \( a_I \) induce, respectively, a non-partisan-voting game and a party-voting game among PBs. A strategy for a PB \( i \) can therefore be described by a pair of functions \( \kappa_{np} \) and \( \kappa_{p} \) mapping the set of types \( \Theta \) and possible pork allocations to party members \([0, R] \) to \( \{q, x\} \). The resulting \( \kappa_{np} (\theta_i; r) \) and \( \kappa_{p} (\theta_i; r) \) are therefore the votes of a PB \( i \) with ideal policy \( \theta_i \) in the non-partisan-voting and party-voting games, given an offer of pork \( r \) to party members.

An equilibrium is a strategy profile \(((a_I, r), \{\kappa_{np} (\cdot; r), \kappa_{p} (\cdot; r)\}_i)\) such that (i) \((a_I, r)\) is feasible and sequentially rational and that (ii) \( \{\kappa_{np} (\cdot; r)\}_i \) and \( \{\kappa_{p} (\cdot; r)\}_i \) constitute, respectively, a BNE of the non-partisan-voting and party-voting games.

### 3 The Fundamentals: Voting

This section considers voting equilibria, and is thus the basis of the substantive study in section 4. After characterizing equilibria in non-partisan voting (Remark 1), I turn to the core of the section: the analysis of party votes. I show that if the distribution of PBs’ preferences is common knowledge, radically different behavioral patterns can be sustained as equilibria of party votes by self-fulfilling beliefs (Remark 2). Relaxing this assumption
allows us to pin down a unique equilibrium, which I characterize in Proposition 1.

Consider first non-partisan voting. Note that the net payoff of voting for \( x \) for a PB \( i \) is here given by \( \Pi_{np}(\theta_i) = r - v(\theta_i) \), and is therefore independent of the actions of other players (this is a decision problem). Letting \( \delta_{np}(r) \equiv v^{-1}(r) \), we then have

\[ \text{Remark 1 (Non-Partisan Voting)} \]

In a non-partisan voting equilibrium, \( \pi_i^{np}(\theta_i; r) = x \) for all \( i \) such that \( \theta_i > \delta_{np}(r) \) and \( \pi_i^{np}(\theta_i; r) = q \) for all \( i \) such that \( \theta_i < \delta_{np}(r) \).

The situation is qualitatively different in a party vote. In a party vote, only PBs with "extreme" policy preferences are impervious to the actions of fellow party members. The decision of "centrist" individuals, instead, is determined by their beliefs about what others will do. For these individuals, supporting the incumbent’s party line is optimal only if doing so allows them to capture a sufficiently high level of expected party payments. The net expected value of the incumbent’s offer for individual \( i \) depends, in turn, on whether the incumbent leader will be able to retain the command of the party, and thus on \( i \)’s beliefs about the proportion of PBs supporting the incumbent’s party line. If \( i \) believes that more than \( \mu \) PBs will stick with the incumbent leader, he will want to do so as well; if he believes that at least \( 1 - \mu \) PBs will defect, he will "defect" too.

In particular, if the distribution of party members’ preferences is common knowledge, and the proportion of "extremists" is not high enough to determine the outcome of the incumbent’s survival from the outset, radically different behavioral patterns can be sustained as equilibria by self-fulfilling beliefs.\(^9\)

\[ \text{Remark 2} \]

Let \( \theta_i \equiv v^{-1}(r + e) \) and \( \theta_i \equiv v^{-1}(r - e) \). Suppose that \( \theta \) is common knowledge, and that \( \theta_i < \theta + \sigma \Phi^{-1}(1 - \mu) < \theta_i \). Then the following strategy profiles are BNE of the party voting game:

\(^9\)When \( \theta + \sigma \Phi^{-1}(1 - \mu) < \theta_i \), strategy profile (1) in the remark constitutes the unique BNE of the party vote game. Similarly, when \( \theta + \sigma \Phi^{-1}(1 - \mu) > \theta_i \), strategy profile (2) is the unique BNE.
Proof of Remark 2. Consider first strategy profile (1). Since \( \theta_i \sim N(\theta, \sigma^2) \), the proportion of PBs voting for \( x \) is then given by \( 1 - \Phi\left( \frac{\theta_i - \theta}{\sigma} \right) \), where \( \Phi(\cdot) \) is the c.d.f. of the standard normal. The incumbent survives the challenge (with certainty) if

\[
1 - \Phi\left( \frac{\theta_i - \theta}{\sigma} \right) > \mu \iff \theta > \theta_i - \sigma \Phi^{-1}(1 - \mu)
\]

Since this is true by hypothesis, the expected net payoff of voting for \( q \) for PB \( i \) is given by \( v(\theta_i) - r - e \). Then optimality implies \( \kappa_{i}^{p}(\theta_i; r) = q \) if \( \theta_i < v^{-1}(r + e) \equiv \theta_i \) and \( \kappa_{i}^{p}(\theta_i; r) = x \) if \( \theta_i > \theta_i \). Similarly, consider strategy profile (2). The proportion of PBs voting for \( x \) is then given by \( 1 - \Phi\left( \frac{\theta_i - \theta}{\sigma} \right) \). The incumbent leader will fall for sure if \( 1 - \Phi\left( \frac{\theta_i - \theta}{\sigma} \right) < \mu \iff \theta < \theta_i - \sigma \Phi^{-1}(1 - \mu) \), which again is true by hypothesis. The expected net payoff of voting for \( x \) for PB \( i \) is then given by \( r - e - v(\theta_i) \), and optimality implies \( \kappa_{i}^{p}(\theta_i; r) = x \) \( \forall i : \theta_i > \theta_i \) and \( \kappa_{i}^{p}(\theta_i; r) = q \) \( \forall i : \theta_i < \theta_i \).

3.1 Uniqueness of Equilibrium in Party Votes

The assumption that the distribution of party members’ preferences is common knowledge among PBs, however, is not desirable per se. Moreover, as recent developments in the global games literature show, relaxing this assumption allows us to pin down a unique equilibrium (see Morris and Shin (1998, 2001 and 2003), and Frankel, Morris and Pauzner (2003)). The basic results are summarized in proposition 1: when PBs are uncertain about the central tendency of the party (i) there exists a symmetric equilibrium in which PBs employ switching strategies with a cutpoint \( \delta_p \in (\theta_i, \theta_i) \). Moreover, (ii) this equilibrium is unique provided that the uncertainty about the central tendency of the party (as parameterized by \( \eta \)) is high enough. The cutpoint \( \delta_p \), which completely characterizes this equilibrium, is
pinned down by the net expected value attached by the critical player with ideal policy \( \delta_p \) to the promises of electoral benefits made by the incumbent leader.

Consider a symmetric strategy profile in which PBs employ switching strategies with an arbitrary cutpoint \( \delta \). Denote by \( \Pi(\theta; \delta) \) the net expected benefit of supporting \( x \) for a PB with ideal policy \( \theta \) given this strategy profile. Similarly, denote by \( \Gamma(\theta, \delta) \) the proportion of PBs voting for \( x \) according to this strategy profile given a particular realization of \( \theta \).

Since \( \theta_i \mid \theta \sim N(\theta, \sigma^2) \), then \( \Gamma(\theta, \delta) = 1 - \Phi(\frac{\delta - \theta}{\sigma}) \), where \( \Phi(\cdot) \) is the c.d.f. of the standard normal. Hence

\[
\Pi(\theta; \delta) = r + e \left[ 1 - 2 \Pr(\theta < \delta - \sigma \Phi^{-1}(1 - \mu) \mid \theta_i) \right] - v(\theta_i)
\]

By Bayes’ Law, \( \theta \mid \theta_i \sim N(\hat{\theta}(\theta_i), \hat{\eta}^2) \), where \( \hat{\theta}(\theta_i) \equiv \frac{\sigma^2 \theta_0 + \eta^2 \theta_i}{\sigma^2 + \eta^2} \) and \( \hat{\eta} \equiv \frac{\sigma \eta}{\sqrt{\sigma^2 + \eta^2}} \). I then define the function

\[
P(\delta, \theta_i) \equiv 1 - 2\Phi\left(\frac{\theta - \hat{\theta}(\theta_i)}{\hat{\eta}}\right)_{\theta = \delta - \sigma \Phi^{-1}(1 - \mu)}
\]

Intuitively, \( P(\delta, \theta_i) \) is the net expected value of a dollar of electoral benefits made conditional on supporting the incumbent leader’s party line for an individual with ideal policy \( \theta_i \), when every PB uses a switching strategy with cutoff point \( \delta \). Then:

\[
\Pi(\theta_i; \delta) = r + eP(\delta, \theta_i) - v(\theta_i)
\]

Denoting by \( p(\delta) \equiv P(\delta, \delta) \) the net expected value of a dollar of electoral benefits for the critical PB with ideal policy \( \delta \), and letting \( \pi(\delta) \equiv \Pi(\delta; \delta) \), we have

\[
\pi(\delta) = r + ep(\delta) - v(\delta)
\]

Lemma 3 in the appendix shows that (i) \( p(\cdot) \) is a decreasing function, and that (ii) \(|p'(\cdot)|\) is bounded above by a decreasing function of \( \eta \) which goes to zero as \( \eta \to \infty \).  

\[10\text{To grasp the intuition for this result, note that this is equivalent to saying that a more "right-winged"}


Since by A1 the slope of $v(\cdot)$ is bounded away from zero, this implies that for sufficiently high $\eta$, $\pi(\cdot)$ is an increasing function and $\pi(\delta) = 0$ at exactly one point.

[Figure 1]

Proposition 1 is then a rather straightforward application of similar results in the *global games* literature (see Morris and Shin (1998, 2001 and 2003), and Frankel, Morris and Pauzner (2003)):

**Proposition 1** Let $\delta_p \in \{\delta : \pi(\delta) = 0\} \neq \emptyset$. There exists a symmetric equilibrium of the party vote game in which $\pi^p_i(\theta_i, r) = x$ for all $i$ such that $\theta_i \geq \delta_p$ and $\pi^p_i(\theta_i, r) = q$ for all $i$ such that $\theta_i < \delta_p$. Moreover, there exists a $\eta_i$ such that whenever $\eta > \eta_i$, $\{\delta : \pi(\delta) = 0\}$ has a single element $\delta_p$, and this equilibrium is unique.

**Proof.** See Appendix A. ■

4 Party Discipline and Vote Buying

In this section, I turn to the substantive analysis leading to the main conclusions of the paper. In doing so, I assume throughout that the condition in proposition 1 is met. I start by making precise the definition of party discipline that I will employ in the remainder of the paper.

*critical* PL assigns a higher probability to the incumbent being overthrown. Note, then, that increasing $\delta$ (i) increases the cutoff point determining whether other PLs will support or challenge the incumbent (vote for $x$ or $q$), and (ii) changes the beliefs of the critical PL concerning the central tendency of the party. Since the cdf of $\theta$ conditional on $\theta_i$ is stochastically increasing in $\theta_i$, a more right-winged critical PL will consider less likely that the incumbent will be overthrown. This effect, however, is dampened by the prior beliefs. As a result, the increase in the cutoff predominates, producing the result. The same result follows from the same logic, since increasing $\eta$ diffuses the prior, and thus diminishes the "dampening" of the change in beliefs.
4.1 Party Discipline: A Definition

The informal definition of party discipline advanced in the introduction referred to the ability of party leaders to influence the voting behavior of PBs with party resources (resources that can only be distributed among party members; i.e., electoral benefits). This brief section has the double purpose of providing a rationale for this definition, and of making it more precise. The definition I will employ is as follows:

**Definition 1** Define party discipline, $d: [0, R] \rightarrow \mathbb{R}$, by

$$d(r) \equiv \inf\{\theta_i : \pi^{np}_i(\theta_i; r) = x\} - \inf\{\theta_i : \pi^p_i(\theta_i; r) = x\}$$

That is, given an allocation $r$ of pork to party members, I define party discipline as the difference between the ideal policy of the most left-winged PB supporting the incumbent’s party line in a non-partisan vote, and that of the most left-winged PB supporting the party line in a party vote. By Remark 1 and Proposition 1, then, it follows that:

**Remark 3** (i) $d(r) = \delta_{np} - \delta_p$, and (ii) $d(r) > 0 \iff p(\delta_p) > 0$

Point (ii) above simply notes that discipline is positive if and only if the critical PB $\delta_p$ assigns positive (net) value to the promises of electoral benefits of the incumbent leader.

This definition satisfies several appealing properties. First, a useful definition of party discipline must distinguish between the non-partisan and the partisan frameworks. Specifically, party discipline should not reflect unity in voting that is driven by the absence of conflict between PBs over their preferred alternative. Instead, party discipline must indicate the ability of the party, and in particular of the party leadership, to mold PBs’ behavior. This is in the spirit of Krehbiel (1993), Cox and McCubbins (1993), and Tsebelis (1995), and is now standard in the recent literature.\(^{11}\) The comparison of the partisan

\(^{11}\)Krehbiel (1993) makes the point sharply: ”[D]o legislators vote with fellow party members ”in spite of their disagreement about the policy in question, or ... because of their agreement about the policy in
and non-partisan thresholds $\delta_p$ and $\delta_{np}$ accomplishes this demand without being (directly) influenced by the distribution of preferences within the party (e.g., heterogeneity of PBs’ preferences, $\sigma$). The notion I introduce differs from what is the norm in the literature in the choice of the non-partisan framework to employ. In particular, this definition does not include changes in party members’ voting behavior that are achieved with resources that could have otherwise been destined to non-party members (i.e., pork). This view emphasizes that allocating pork to party members means having to buy their support, and is therefore not an indication of power within the organization.

4.2 Conditional Party Governance

I consider first the situation in which the incumbent leader is not protected by supermajority requirements for removal ($\mu = 1/2$), and no pork is allocated to party members ($r = 0$). I show that in this setting, credible promises of electoral benefits confer only limited strength to the party leader, and a result similar to Aldrich and Rohde’s conditional party governance emerges: The incumbent leader will use electoral benefits to support the party line only if the leadership’s incentives are aligned (ex ante) with those of the majority of the party. Recall that $\theta_0$ denotes the ideal policy of the ex ante party median. Then:

**Proposition 2** Let $R = 0$ and $\mu = 1/2$ be given. Then (i) party votes occur in equilibrium if and only if $v(\theta_0) < 0$ (i.e., iff $x_{\theta_0} \succeq q$), and (ii) in party votes, the ex ante median is in the incumbent’s coalition: $\delta_p \leq \theta_0$.

**Proof.** First note that the incumbent will call a party vote in equilibrium if and only if discipline is positive. Now, $d = \delta_{np} - \delta_p \geq 0 \iff p(\delta_p) \geq 0$. That is, discipline is positive if and only if the critical PB $\delta_p$ assigns net positive value to the incumbent’s promises of question ??" In the same vein, Cox and McCubbins (1993) argue that "[I]nvestigations of parties as floor voting coalitions ought to be conducted in terms of loyalty to party leaders and not, as has usually been done in the previous literature, in terms of general party cohesion". Similarly, Tsebelis (1995) differentiates discipline - "the ability of parties to eliminate dissent after a decision is made" - from cohesion - "the size of differences [in policy preferences] before the discussion" (italics in original).
electoral benefits. But with $\mu = 1/2$, $p(\delta_p) \geq 0 \Leftrightarrow \delta_p \leq \theta_0$, because

$$\Pr(\Gamma(\theta, \delta_p) < \mu | \theta_i = \delta_p) = \Pr(\theta < \delta_p | \theta_i = \delta_p) < 1/2 \Leftrightarrow \delta_p < \theta_0$$

That is, with $\mu = 1/2$, the critical PB $\delta_p$ assigns net positive value to the incumbent’s promises of electoral benefits if and only if the ex ante party median is in the incumbent’s coalition (iff $\delta_p < \theta_0$). Hence $d \geq 0 \Leftrightarrow \delta_p \leq \theta_0$. Now, with $r = 0$, $\delta_{np} = v^{-1}(0)$, and then $v(\delta_{np}) = 0$. Since $ep(\delta)$ is continuously decreasing, but everywhere flatter than $v(\delta)$, then $\delta_{np} \leq \theta_0 \Leftrightarrow \delta_p \leq \delta_{np} \Leftrightarrow d \geq 0$. Finally, $x_{\theta_0} \geq q \Leftrightarrow \delta_{np} = v^{-1}(0) \leq \theta_0$, implying that $x_{\theta_0} \geq q \Leftrightarrow d \geq 0$. ■

Note that this result holds independently of the level of electoral benefits available to the incumbent leader. Thus, proposition 2 shows in a crude way that even if credible per se, and significant in amount, promises of electoral benefits do not necessarily have influence over policy outcomes. This is specially so under the conditions assumed in the proposition. In this case, the incumbent will choose to allocate electoral benefits to PBs conditionally on their support of the incumbent’s party line only when the leadership’s policy preferences are aligned (ex ante) with those of the majority of the party. Furthermore, the power that electoral benefits confer to the leadership in this environment can be attributed entirely to the heterogeneity of policy preferences among party backbenchers.

**Proposition 3** Let $R = 0$ and $\mu = 1/2$ be given. In equilibrium, discipline in party votes decreases with the homogeneity of PBs’ preferences, and $\lim_{\sigma \to 0} d = 0$.

While the proof of this result is deferred until proposition 6 - which contains it as a special case - I provide here the basic logic behind this result. Recall that PBs use both

(i) public information about the central tendency of the party and (ii) the information

12 A full discussion of the result is included as Appendix B, which complements the more efficient but less revealing formal proof in Proposition 6.
contained in their own preferences to form beliefs about the distribution of fellow party members’ preferences (and thus ultimately about their actions). The need to anticipate the reaction of other party legislators is due to the basic coordination problem arising between legislators willing to oppose the incumbent’s mandate.

Central for any PB in this problem is comparing his preferences with those of other party members. When party members’ preferences are heterogeneous, only the ex ante median believes he is ”centrist”, attaching equal probability to any member having ideal policy higher or lower than his own. PBs with ideal policy $\theta_i < \theta_0$, instead, believe that a majority of the party is to the left of the ex ante median. The informational content of a PB’s ideal policy, in turn, increases with the homogeneity of the party. This implies, in particular, that PBs with ideal policy $\theta_i < \theta_0$ will attach a higher probability to the incumbent being overthrown (and thus a lower value to her promises of electoral benefits) the more homogeneous the party is.

Note, however, that we are not concerned with how any arbitrary PB forms its beliefs, but with how the critical PB $\delta_p$ does. But we know from Proposition 2 that when the incumbent can be overthrown by a simple majority of rebelling PBs, the critical PB $\delta_p$ assigns positive value to the incumbent’s promises of electoral benefits only if the ex ante median is in the incumbent’s coalition; i.e., only if $\delta_p < \theta_0$. The argument in the previous paragraph then implies that if discipline is positive, it must decrease with an increase in the homogeneity of PBs’ preferences.

Opposite to the case of heterogeneous preferences, where as I noted only the ex ante median believes he is ”centrist”, in the limit as $\sigma$ goes to zero every individual believes he is ”centrist” (as no weight is given to the ex ante median). But then for the critical PB, whose ideal policy coincides with the symmetric strategy’s cutoff point, electoral benefits offered by the incumbent must have no value. This means that electoral benefits will have no bite in equilibrium, and therefore discipline must vanish in equilibrium as $\sigma$ goes to zero.
4.3 Vote Buying

The analysis so far assumed that the incumbent could be overthrown by a simple majority of PBs, and that the incumbent could not use resources other than the partisan electoral benefits to sway legislators’ behavior. In this section, I relax these assumptions. I show that while both innovations have the unambiguous effect of increasing the leader’s power, they also have substantively different repercussions with respect to party backbenchers, the relation of the leader with the party, and the formation of legislative coalitions.

Being endowed with pork, the incumbent can now buy the support of legislators in the opposition. This, however, has an opportunity cost, as buying the opposition means weakening the support inside the party. The key to the results in this section is that this cost is magnified in a party vote as a result of a complementarity between the allocation of pork to party members and the value of electoral benefits. In a non-partisan vote - where PBs’ beliefs about the actions of fellow party members are irrelevant - decreasing the allocation of pork to the party by one dollar leads to an equivalent reduction in the value of the incumbent’s offer. In a party vote, instead, the value of the incumbent’s promises of electoral benefits is tied to the fate of the leader. But the reduction in the allocation of pork to party members will lead PBs to anticipate a lower aggregate support for the party line and, as a result, a higher probability of the incumbent being overthrown. This reduction in the allocation of pork to party members will thus lead to a depreciation of the value of the incumbent’s promises of electoral benefits, and hence to a more than proportional effect over the net value of the incumbent’s offer.

The first implication of this logic is in the impact of endowing the leader with pork resources upon what I have dubbed conditional party governance. In the context of the previous section I showed (Proposition 2) that party benefits were used to favor the party line only when - according to public information - the majority of the party preferred the party line to the legislative alternative. When the incumbent can influence legislators’ decisions with pork, however, party votes can exist in equilibrium even if $q > x$. Nonetheless,
in the absence of a supermajority requirement for removal of the leader, the influence of
backbenchers is not lost, but only reshaped in terms of a lower bound of payments that
needs to be allocated to party members for party resources to be in play. In particular,
the allocation of pork to party members has to be at least as large as to attain the sup-
port of the (ex ante) party median. The simple result follows, in effect, from the proof of
proposition 2, and is stated in the following remark.

**Remark 4** Let \( \mu = 1/2 \). If there is a party vote in equilibrium, \( r \geq -v(\theta_0) \)

Raising the bar for removal of the incumbent leader, instead, directly reduce the in-
fluence (and well-being) of backbenchers. Party discipline increases with the protection
to the incumbent \( 1 - \mu \), and therefore internal dissent is reduced even in the absence of
compensations. Indeed, for every \( \mu \in (0, 1/2] \) there is a \( r_{\text{min}}(\mu) \) such that \( r_p(\mu) \geq r_{\text{min}}(\mu) \)
for a party vote to be possible in equilibrium, and it can be easily verified that \( r_{\text{min}}(\mu) \) is
an increasing function, with maximum at \( r_{\text{min}}(1/2) = -v(\theta_0) \).

Furthermore, the next proposition shows that when party votes occur in equilibrium,
the incumbent will allocate less pork to buy opposition legislators the more contestable
the leadership position is. In essence, the result is due to the fact that increasing the
contestability of the leadership boosts the complementarity between pork and the value
of electoral benefits. In this situation, ”weak” leaders find more profitable buying their
own party, thus avoiding large depreciations of the value of the electoral benefits at their
disposal.

**Proposition 4** Suppose that the incumbent would make a party vote with \( \mu = \mu^0 \) and that
\( \mu^1 < \mu^0 \). Then \( r_p(\mu^0) \geq r_p(\mu^1) \). Moreover, if \( r_p(\mu^1) \in (r_{\text{min}}(\mu^1), R) \), then \( r_p(\mu^0) > r_p(\mu^1) \)

**Proof.** The first step is to characterize optimal allocations of pork to party members
under rule \( \mu, r_p(\mu) \). Let \( H(\cdot) \equiv [1 - G(v^{-1}(\cdot))] \). The mass of legislators in the opposition
voting for \( x \) given pork offer \( r_o \) is given by \( H(r_o) \). Note that \( H'(r_o) \geq 0 \) for all \( r_o \). Pork resource constraint is given by \( r_o \beta + r \leq R \). Since this will hold with equality in the optimum, I write \( r_o = \frac{R-r}{\beta} \). Conditional on \( \theta \), then, \( y = x \) iff \( H\left(\frac{R-r}{\beta}\right) \beta + \Gamma(\theta, \delta_p(r, \mu)) \geq \frac{(1+\beta)}{2} \). Since \( \Gamma(\theta, \delta_p(r, \mu)) = 1 - \Phi\left(\frac{\delta_p(r, \mu) - \theta}{\sigma}\right) \), this is

\[
\theta \geq \delta_p(r, \mu) - J(r)
\]

, where \( J(r) \equiv \sigma \Phi^{-1}\left(\frac{(1-\beta)}{2} + H\left(\frac{R-r}{\beta}\right) \beta\right) \). Then for the incumbent,

\[
\Pr(y = x) = 1 - \Phi\left(\frac{1}{\eta} [\delta_p(r, \mu) - \theta_0] - J(r)\right)
\]

An optimal allocation of pork for the incumbent \( r_p(\mu) \) maximizes \( \Pr(y = x) \). The FOC is:

\[
\left| \frac{\partial \delta_p(r_p(\mu), \mu)}{\partial r} \right| - J'(r_p(\mu)) \begin{cases} 
> 0 \text{ and } r_p(\mu) = R \\
= 0 \text{ and } r_p(\mu) \in (r_{\min}(\mu), R) \\
< 0 \text{ and } r_p(\mu) = r_{\min}(\mu)
\end{cases}
\]

(2)

The second and final step is to show that for all \( r \)

\[
\left| \frac{\partial \delta_p(r, \mu^0)}{\partial r} \right| > \left| \frac{\partial \delta_p(r, \mu^1)}{\partial r} \right|
\]

whenever \( \mu^0 > \mu^1 \)

(3)

, which implies that

\[
\left| \frac{\partial \delta_p(r_p(\mu^1), \mu^0)}{\partial r} \right| > \left| \frac{\partial \delta_p(r_p(\mu^1), \mu^1)}{\partial r} \right|
\]

whenever \( \mu^0 > \mu^1 \)

(4)

Then (4) together with (2) will imply that \( r_p(\mu^0) \geq r_p(\mu^1) \). Moreover, if \( r_p(\mu^1) \in (r_{\min}(\mu^1), R) \), so that \( \left| \frac{\partial \delta_p(r_p(\mu^1), \mu^1)}{\partial r} \right| = J'(r_p(\mu^1)) \), then \( \left| \frac{\partial \delta_p(r_p(\mu^1), \mu^0)}{\partial r} \right| > J'(r_p(\mu^1)) \), and hence \( r_p(\mu^0) > r_p(\mu^1) \).

Note that for all \( r, \mu \),

\[
\left| \frac{\partial \delta_p(r, \mu)}{\partial r} \right|^{-1} = \left| \frac{\partial v(\delta_p(r, \mu))}{\partial \delta} \right| - e \left| \frac{\partial p(\delta_p(r, \mu))}{\partial \delta} \right|
\]

so that (3) can be written as:

\[
e \left\{ \left| \frac{\partial p(\delta_p(r, \mu^0))}{\partial \delta} \right| - \left| \frac{\partial p(\delta_p(r, \mu^1))}{\partial \delta} \right| \right\} > \left| \frac{\partial v(\delta_p(r, \mu^0))}{\partial \delta} \right| - \left| \frac{\partial v(\delta_p(r, \mu^1))}{\partial \delta} \right|
\]

(5)
Note, next, that since in a party vote $\delta_p(r, \mu)$ is increasing in $\mu$, then $\delta_p(r, \mu^1) < \delta_p(r, \mu^0)$. Assumption (A1) then implies that

$$|v'(\delta_p(r, \mu^1))| \geq |v'(\delta_p(r, \mu^0))| \quad (6)$$

Also, since

$$\left| \frac{\partial p(\delta; \mu)}{\partial \delta} \right| = 2\phi\left( \frac{1}{\eta} \left[ \frac{\sigma^2}{\sigma^2 + \eta^2} (\delta - \theta_0) - \sigma \Phi^{-1}(1 - \mu) \right] \right) \frac{1}{\eta} \frac{1}{\sqrt{1 + \frac{\eta^2}{\sigma^2}}}$$

it can be verified that if $d > 0$ then $\frac{\partial}{\partial \mu} \left( \left| \frac{\partial p(\delta; \mu)}{\partial \delta} \right| \right) > 0$, so that

$$\left| \frac{\partial p(\delta_p(r, \mu^1); \mu^0)}{\partial \delta} \right| \geq \left| \frac{\partial p(\delta_p(r, \mu^0); \mu^1)}{\partial \delta} \right| \quad (7)$$

and that (ii) $\frac{\partial^2 p(\delta; \mu)}{\partial \delta^2} < 0$, so that $\delta_p(r, \mu^0) > \delta_p(r, \mu^1)$ implies that

$$\left| \frac{\partial p(\delta_p(r, \mu^0); \mu^1)}{\partial \delta} \right| > \left| \frac{\partial p(\delta_p(r, \mu^1); \mu^1)}{\partial \delta} \right| \quad (8)$$

Then (7) and (8) imply that

$$\left| \frac{\partial p(\delta_p(r, \mu^0); \mu^0)}{\partial \delta} \right| > \left| \frac{\partial p(\delta_p(r, \mu^1); \mu^1)}{\partial \delta} \right| \quad (9)$$

Then (6) and (9) imply that (5) holds.

**4.4 Cohesion and Discipline Revisited**

In the context of Section 4.2 I showed that in equilibrium, discipline in party votes decreases with the homogeneity of PBs’ preferences. Proposition 6 revisits this result, allowing for arbitrary majority requirements for removal and allocations of pork to party members. The proposition shows that provided $\mu = 1/2$, the result does generalize to arbitrary $r \leq R$ as stated. When $\mu < 1/2$, instead, the main intuition described above breaks down, and this is no longer the case. The gist of the argument is that with $\mu < 1/2$, it is possible for the ex ante party median to be in the rebelling coalition, while still having positive
discipline. When this is the case, the same argument used in proposition 3 shows that an increase in the heterogeneity of preferences will now diminish discipline.\footnote{In addition to the discussion in Appendix B, the case of $\mu < 1/2$ adds an additional element to the analysis. This, however, reinforces the positive effect of heterogeneity on party discipline. For any given cutoff $\delta$, the minimum value of $\theta$ for which the incumbent would not be overthrown, $\delta - \sigma \Phi^{-1}(1 - \mu)$, is decreasing in the majority required to successfully overthrow the incumbent $1 - \mu$. This effect, furthermore, is proportional to the heterogeneity of PLs’ preferences; i.e., the more heterogeneous the party is, the more extreme $\theta$ has to be in order for a supermajority of the party to join in the challenge to the incumbent leader. (while an increase in $\sigma$ increases the probability of extreme events - see appendix B - this is outweighed by the direct effect of the change in the critical central tendency of the party described above).} Note, however, that this happens for relatively low levels of discipline, where (ex ante) a majority of the party opposes the party leader’s mandate.

**Proposition 5** Let $\mu = 1/2$ and $r \in [0, R]$ be given. In equilibrium, discipline in party votes decreases with the homogeneity of PBs’ preferences, and $\lim_{\sigma \to 0} d = 0$. With $\mu < 1/2$, however, this is not necessarily so, and $\lim_{\sigma \to 0} d > 0$.

**Proof.** Note first that

$$p(\delta) = P(\delta, \delta) = 1 - 2\Phi \left( \frac{1}{\eta} \left[ \frac{\sigma^2}{\sigma^2 + \eta^2} (\delta - \theta_0) - \sigma \Phi^{-1}(1 - \mu) \right] \right)$$

where

$$\frac{1}{\eta} \left[ \frac{\sigma^2}{\sigma^2 + \eta^2} (\delta - \theta_0) - \sigma \Phi^{-1}(1 - \mu) \right] = \left( \frac{\theta - \hat{\theta} (\theta_i = \delta)}{\eta} \right)_{\theta = \delta - \sigma \Phi^{-1}(1 - \mu)}$$

Thus

$$\frac{\partial p(\delta; \sigma)}{\partial \sigma} = -2 \phi(\cdot) \frac{1}{\sqrt{\sigma^2 + \eta^2}} \left[ \frac{\eta}{\sigma^2 + \eta^2} (\delta - \theta_0) - \frac{\sigma}{\eta} \Phi^{-1}(1 - \mu) \right]$$

so that $\frac{\partial p(\delta; \sigma)}{\partial \sigma} \geq 0$ if and only if:

$$\theta_0 \geq \delta_p - \sigma \Phi^{-1}(1 - \mu) \left( 1 + \frac{\sigma^2}{\eta^2} \right)$$

(10)

But if $p(\delta; \sigma)$ increases with $\sigma$ at $\delta_p (\sigma')$, then $\sigma'' > \sigma' \implies \delta_p (\sigma'') < \delta_p (\sigma')$. Hence more heterogeneity of PBs’ preferences must in this case increase discipline. Similarly, if $p(\delta; \sigma)$
decreases with $\sigma$ at $\delta_p$ ($\sigma'$), then more heterogeneity of PBs’ preferences must in this case reduce discipline. Now,

$$d \geq 0 \iff p(\delta_p) \geq 0 \iff \left( \frac{\theta - \hat{\theta}(\theta_i = \delta_p)}{\hat{\eta}} \right)_{\theta = \delta_p - \sigma \Phi^{-1}(1-\mu)} \leq 0$$

That is, $d \geq 0$ if and only if

$$\theta_0 \geq \delta_p - \left( 1 + \frac{\eta^2}{\sigma^2} \right) \sigma \Phi^{-1}(1-\mu)$$

Hence, in equilibrium, discipline in party votes necessarily increases with $\sigma$ if (10) is satisfied whenever (11) is. Since $\delta_p$ is a continuously decreasing function of $\theta_0$, bounded below by $\theta_i \equiv v^{-1}(r + e)$ and above by $\overline{\theta}_i \equiv v^{-1}(r - e)$, there is a unique $\theta^*_0$ solving (10) with equality, and a unique $\theta^{**}_0$ solving (11) with equality. If $\mu = 1/2$, these two inequalities collapse to $\theta_0 \geq \delta_p$. Therefore in equilibrium, discipline in party votes necessarily increases with $\sigma$. Moreover, $\delta_p = \theta_0 \iff p(\delta_p) = 0 \iff \theta_0 = v^{-1}(r)$, so that $\theta^*_0 = \theta^{**}_0 = v^{-1}(r)$. With $\mu < 1/2$, however, (10) is satisfied whenever (11) is only if $\sigma \geq \eta$.

The results for the limit as $\sigma \to 0$ follow from Lemma 4 in the appendix, which shows that

$$\lim_{\sigma \to 0} d = v^{-1}(r) - v^{-1}(r + e[1 - 2\mu])$$

\section*{5 Extension: Endogenous Challenge}

In the setting of the basic model, I assumed that challenges to the incumbent party leader occurred if and only if she made the vote a party vote. In this section I endogeneize the challenge. Given the lesser role of pork in this stage, I take an allocation $r$ as given, and focus instead on the characteristics of the policy alternative being supported by the incumbent leader.\footnote{In our formulation, pork allocations are unalterable, and therefore are not the prime determinants of challenges to the incumbent leader. The central elements, instead, are given by the policy alternatives being considered and the allocation of electoral benefits.} I show that under the assumptions in this section, (i) the incumbent
is only challenged in party votes. Moreover, I distinguish two sets of alternatives \( x \) possibly being supported by the incumbent leader in party votes: a set of "moderate" policies \( \{ x : q \leq x \leq \bar{x} \} \) and a set of "radical" policies \( \{ x : x \geq \bar{x} \} \). I show that (ii) the incumbent is not challenged in party votes for moderate policies, but always challenged in party votes for radical policies. The basic model is thus a stylized description of this extended framework. After reviewing the amendments I impose to the model, I provide a formal statement of these results, and note its implications for the uniqueness of equilibrium outcomes.

5.1 The Extended Model

I will consider the following "challenge technology". After the incumbent’s choice, PBs in a given set of potential challengers \( \Omega \) simultaneously decide whether they will propose or not a challenge to the incumbent leader. I assume that the preferences of potential challengers are common knowledge, that \( \{ \theta_i : i \in \Omega \} \) is compact and, let \( \omega \equiv \min \{ \theta_i : i \in \Omega \} \). A challenge occurs if some potential challenger \( i \in \Omega \) proposes a challenge. Denoting the challenge decision of individual \( i \in \Omega \) by \( c_i(\theta_i; x) \in \{0, 1\} \), and by \( c(x) \in \{0, 1\} \) the occurrence of a challenge, then \( c(x) = 1 \) whenever \( c_i(\theta_i; x) = 1 \) for some \( i \in \Omega \), and \( c(x) = 0 \) otherwise. Proposing the challenge is costless, and provides no special benefits (in the event the challenge is successful) vis a vis the remaining PBs opposing the incumbent leader.

I modify the definition of equilibrium to exclude equilibria containing weakly dominated strategies. I will also impose the following additional assumption about PBs’ preferences (replacing A1).\(^{15}\)

Assumption (A1'). For every \( x \), there exists \( \bar{\alpha} \geq \alpha > 0 \) s.t. for all \( (\theta_i, \theta'_i) \) with \( \theta'_i > \theta_i \),

\[
\bar{\alpha} (x - q) (\theta'_i - \theta_i) \geq v(\theta_i; x) - v(\theta'_i; x) \geq \alpha (x - q) (\theta'_i - \theta_i)
\]

\(^{15}\)Again, this satisfied by a quadratic utility function \( u(x_i; \theta_i) = -b (x_i - \theta_i)^2 \). Here \( \bar{\alpha} = \alpha = 2b \)
For a given $q$ and $x$, $(A1’)$ bounds the change in $v(\theta_i)$ above and below. It also requires the bounds $\bar{\alpha}(\theta' \theta_i)$ and $\alpha(\theta' \theta_i)$ to hold for any $x > q$ once corrected by the distance $x - q$.

### 5.2 Main Result, and Implications for Uniqueness of Equilibrium Outcomes

Proposition 1 showed that given any pair of policy alternatives $(q, x)$, the party vote game has a unique equilibrium provided there is sufficient uncertainty about the central tendency of the party. Specifically, keeping $q$ fixed, I showed that for any $x$ there is a $\eta(x)$ such that a party vote equilibrium is unique whenever $\eta > \eta(x)$. Under reasonable assumptions about preferences, however, $\eta(x)$ decreases with $|x - q|$, and $\lim_{x \to q} \eta(x) = \infty$. Thus for fixed $\eta$, there is an $x$ sufficiently close to $q$ such that $\eta < \eta(x)$, and the sufficient condition for uniqueness is not met.

Note, however, that while the absence of policy-driven conflict allows for multiple resolutions of a challenge should one occur, it also diminishes the benefit of mounting the challenge in the first place. Proposition 6 shows that if PBs are sufficiently uncertain about the distribution of fellow party members preferences, and challengers do not use weakly dominated strategies, challenges occur in equilibrium only for "radical" alternatives, and these always have a unique resolution.

**Proposition 6** There exists a $\eta$ such that for all $x > q$, whenever $\eta > \eta$: $c(x) = 1 \implies \eta > \eta(x)$. Moreover, for each $\eta > \eta$ there exists a $\bar{x}_\eta \in R$ such that $c(x) = 1 \iff x \geq \bar{x}_\eta$

**Proof.** The result is implied by Remark 6, Lemma 5 and Lemma 6 in Appendix A.

### 6 Relation with the Literature

Students of political parties unanimously agree in that parties are not "horizontal" organizations, but rather are characterized by having a hierarchical structure, in which leadership
posts can be clearly distinguished from the rank and file.\textsuperscript{16} The creation of a leader - which is also a characteristic of congressional parties - has been rationalized as an optimal institutional response, (implicitly) agreed upon by party members in a "constitutional stage", and designed to further the welfare of the collective. For Kiewit and McCubbins (1991), for example:

"[I]t is the delegation of authority to a central agent to lead or manage the organization that is the key to overcoming problems of collective action .... In the case of congressional parties, leaders can exploit the prominence of their position to identify a focal point, thus solving problems of coordination by rallying support around one of possibly many acceptable alternatives."

With the possible exception of small or regionally concentrated parties, however, congressional parties bundle together individuals with significantly heterogeneous policy preferences. Structuring collective action in parties thus also involves resolving, to one way or the other, diverging views among party members. As a result of this, the definition of who occupies the leadership, and what the "party line" is, expresses the resolution of power struggles inside the party:

"Power equilibria within the coalition can be altered at any moment .... A dominant coalition is therefore always a potentially precarious construction. It disintegrates due to the pressure of [minority elites] ... because of internal conflicts due to changes in its internal distribution of power." (Panebianco, 1988)

\textsuperscript{16}We must nonetheless take account of the established fact (established by a lot of empirical research of parties) that the principal power resources tend to be concentrated in the hands of small groups. Michels' oligarchy, Duverger's 'inner circle', Ostrogorski and Weber's 'ceasaristic-plebiscitarian dictatorship' are just a few examples which bring this phenomenon to mind." (Panebianco, 1988).
"The key determinant of the desirability of checks within the structure of party leadership is the degree of homogeneity in the policy preferences of the membership... when the party caucus is riven by serious policy disputes, there is more support for checks. Without them, one faction, upon gaining control of the machinery of leadership, might pursue policies that are anathema to another faction, thereby weakening or even splintering the party." Kiewit and McCubbins (1991)

A similar view in fact emerges in the works of Aldrich and Rohde (1998), Cox and McCubbins (1993), and Calvert (1987). Understanding the determinants of the power of congressional leaders over their "followers" is thus crucial to determine how preferences of party members are aggregated to produce partisan outcomes. In this area there is, however, much less theoretical agreement.

At one extreme, exemplified by Michels' iron law of oligarchy (Michels, 1916), party leaders "are not checked by those who hold subsidiary positions within the organization" (Casinelli, 1953). In this view, parties "never operate 'democratically' - i.e., rule by the rank-and-file rather than by the leaders." (Schonfeld, 1981), and "the rank and file are manipulated into accepting policies with which they would not otherwise agree, and which are not in their interests, or at least are primarily in the interests of the leadership group." Hands (1971). At the opposite extreme of the spectrum, most studies of parties in the rational choice camp (inspired by, and mostly applied to, contemporary parties in the U.S.) conceived party leaders as agents of the rank and file. This being understood either in a strict principal-agent framework (see Fiorina and Shepsle (1989), or Kiewit and McCubbins,

\footnote{In the case of Calvert, the same notion appears with a different emphasis: "In general the leader's goals do not correspond exactly with an abstract notion of political welfare for the group, and in any event the leader's goals will probably differ from those of any individual follower. Thus a rational, utility-maximizing leader will pursue collective action for the group in such a way that his own goals are achieved." (Calvert, 1987)}

\footnote{It should be noted, however, that in both Aldrich and Rodhe's and Cox and McCubbins's view, the rank and file will not delegate the powers to the leadership unless their views are sufficiently homogeneous. When they are, instead, this delegation will occur, and the structure of the leader's incentives will make her "internalize the goals of the members, and therefore behave to a large extent in the party members' best interest". (Cox and McCubbins, 1993). We return to this point below.}
1991) or in a broader sense, as in Aldrich and Rohde (1998).

As I emphasized before, however, party leaders are never owners of the organization; power, instead, resides collectively in the ”principals” (the backbenchers). On the other side, with the exception of a truly ”constitutional” stage, incumbent leaders will not be neutral spectators of the decisions of the ”principals”. These alternative views can thus be taken to represent opposite understandings - motivated in part by the observation of different realities - about the degree of difficulty for the rank and file to effectively coordinate in opposing their leaders; i.e., in constituting an effective check to the leader’s power. While this coordination is precluded outright in the world of the iron law, it is assumed to work without frictions in the framework of Aldrich and Rohde.

The explicit consideration of this coordination problem is then essential to understand the limits of the incumbent’s power over legislators. This is, in fact, the approach of the paper. While in the past the assumption of common knowledge of preferences has precluded the fruitful analysis of this problem, the developments in the global games literature (Carlsson and van Damme (1993), Morris and Shin (1998, 2001 & 2003), and Frankel, Morris and Pauzner (2003)) allows me to study the properties of a unique equilibrium.

The different assumptions about how the coordination among backbenchers is resolved result in markedly different conclusions. In our framework, Proposition 3 shows that unless the incumbent is protected by a supermajority rule for removal, discipline in party votes can be entirely attributed to the heterogeneity of PBs’ preferences; i.e., increases with

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19 This observation - which is fairly evident for a vast number of countries - can also, according to Bowler et al (1999), be taken as a feature of U.S. parties: "While it may be true that there is an asymmetry between leaders and followers, given that the former have access to patronage and the ability to play divide and rule, whereas the latter must overcome problems of collective action and rivalry, leaders can still be disciplined by the rank and file. ... At times, party leaders may seem more like generals guiding their disciplined troops into the lobbies. Examples such as Margaret Thatcher or Newt Gingrich suggest a highly cohesive and willing body of legislators, willing to do or die. ... At other times, however, parties are not nearly so compliant. ... The leader keeps the party together, but basically by herding people together while letting the party go where it wants (e.g. Sam Rayburn as Speaker of the U.S. House; John Major as Conservative Party leader in Britain)."

20 As radically different behavioral patterns could be sustained as equilibria by self fulfilling beliefs.
heterogeneity of PBs’ preferences and vanishes in the limit as $\sigma$ goes to zero. As can be noted from the previous quote of Kiewit and McCubbins (1991), this is indeed the same conclusion obtained in the social choice framework. This is, however, based on a different mechanic. In their case, more heterogeneity allows an agenda setter broader discretion. My notion, instead, emphasizes that when preferences are private information, not only it is relevant the existence of opposition, but also that this becomes common knowledge among the group members.

Moreover, based on the frictionless coordination between backbenchers alluded to earlier, Aldrich and Rohde (1995) take this result one step further:

"If there is much diversity in preferences within a party, a substantial portion of the members will be reluctant to grant strong powers to the leadership, or to resist the vigorous exercise of existing powers, because of the realistic fear that they may be used to produce outcomes unsatisfactory to the members in question."

This paper emphasizes, instead, a markedly different timing and coordination of the collective (heterogeneous) principal. It is not the choice of a single PB, I argue, to "resist the vigorous exercise of existing powers". Moreover, except possibly in a truly constitutional stage, both resisting the exercise or removing existing powers will be a collective choice determined by the common knowledge of opposition to the incumbent. To sum up, although both views lead to the same conclusion regarding the effect of the heterogeneity of PBs’ preferences over the influence of the leader, the empirical implications are diametrically opposed. In Aldrich and Rohde’s and Cox and McCubbins (1993) view, the "party effect" will be present when there is substantial agreement among party members.

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21 To some extent, a similar distinction applies to a remark advanced by Calvert (1987), who although based on a non-cooperative game, does not model explicitly the "collective action" problem of opposing the leader: "The more heterogeneity there is among follower’s interests, the less valuable will be the ongoing collective action maintained by the leader, because each follower is required to give up more in order for the group to accomplish common goals. ... The more heterogeneity among followers, then, the greater the temptation for followers to disobey."
In the view advanced in this paper, instead, the party effect will be more important when
the party is more heterogeneous.

7 Conclusions

Legislative Parties can be conceived as teams. In fact, this representation seems adequate
when intra-party preferences are homogeneous, and inter-party preferences are heteroge-
neous. Party leaders here coordinate the actions of the members and enforce plans that
further the interest of the group. Large, ”catch-all” parties in modern democratic societies,
however, usually cluster individuals with significantly heterogeneous views. In this case,
conflict about the collective decisions emerge. Here the Leadership not only solves pure
coordination among members, but also embodies the resolution of power struggles inside
the party. Understanding the factors determining the extent of the leader’s power over
backbenchers thus becomes essential to understanding the functioning of legislatures.

According to the main views prevailing in American Politics, ”backbenchers rule”. When
internal dissent is high, they opt not to delegate power to a party leader. When
they are homogeneous, instead, they grant powers to a leader, who in turn internalizes the
objectives of the members. Opposite this view, in which coordination among the collective
is assumed to be smooth, the analysis following the line of Michels’ Iron Law of Oligarchy,
conceives party leaders as basically unchecked by the rank and file.

In this paper, I adopt an intermediate view, which in turn enables us to approach
the relation between leader and ”followers” in comparative perspective: ultimately, power
always resides in the ”principals”, but only collectively. Thus, how coordination among
opposing internal views is resolved, is essential to delimit the leader’s power. This is
specially relevant when resources that can not be delivered on the spot are used to influence
behavior in the present, as in the case of promises of electoral benefits.

The central message of this paper is that even if credible per se, promises of electoral
benefits (e.g., nominations) are insufficient to grant significant power to the party leader.
Instead, in order to anchor beliefs in his favor and make her promises valuable, the incumbent needs either provide benefits on the spot, or be protected by a supermajority requirement for removal.

In particular, when neither of these conditions is present, electoral benefits will be used to support the party line only if (ex ante) a majority of the party prefers it to the legislative alternative. When endowed with pork, instead, the incumbent can make the electoral benefits valuable, even when ex ante a majority of the party opposes the party line. This is due to the fact that the link between the value of the incumbent’s promises, and her ability to overcome contests to her authority, creates a complementarity between the allocation of pork to party members and the value of electoral benefits. Moreover, since the multiplier effect of pork allocated to party legislators is higher the more exposed the incumbent is to internal threats, weaker (less protected) leaders will allocate less pork to buy opposition legislators, and more to buy members of their own party.

To sum up, the paper provides novel empirical implications for the comparative analysis of parties and legislatures. Even after controlling for other factors, the effect of nomination power over party discipline will depend on (i) the structure of the legislative party’s institutions (ii) the heterogeneity of preferences among party backbenchers, and (iii) the leader’s capacity to allocate resources on the spot (pork). This might help reconcile the theories of party discipline with the observed variation in voting behavior across parties in the same country (and thus subject to the rules of the same electoral system) and in the same party across time. Moreover, the arguments presented in the paper provide an alternative view on why party leaders would ”buy” the votes of fellow party members. The model predicts a subtle relationship between the allocation of pork, the power of nomination, and party’s legislative institutions. More vulnerable leaders will enjoy - controlling for the distribution of pork - less power from nominations. It is precisely this type of leaders, however, who will also tend to allocate more pork to buy the support of members of their own party, increasing discipline as a result.
8 Appendix A

Proof of Proposition 1. The following definitions will be used here. For a given strategy profile of the party vote game \( \{ \kappa^p_i \} \), where each \( \kappa^p_i : \Theta \times [0, R] \to \{ q, x \} \), let \( \xi(z) \) denote the proportion of PBs for whom \( \kappa^p_i(z; r) = x \), let \( \Gamma(\theta; \xi) \) denote the proportion of PBs that would end up supporting \( x \) given a particular realization of \( \theta \) and an aggregate voting mapping \( \xi \), and let \( \Pi(\theta_i; \xi) \) denote the expected net benefit of supporting \( x \) for a PB with ideal policy \( \theta_i \), given \( \xi \).

Proposition 1 follows from three lemmas. In Lemma 1, I show that (i) \( \{ \delta : \pi(\delta) = 0 \} \neq \emptyset \), and that (ii) with \( \delta_p \in \{ \delta : \pi(\delta) = 0 \} \), there exists a symmetric equilibrium of the party vote game in which \( \kappa^p_i(\theta_i, r) = x \) for all \( i \) such that \( \theta_i \geq \delta_p \) and \( \kappa^p_i(\theta_i, r) = q \) for all \( i \) such that \( \theta_i < \delta_p \). In Lemma 2 I show that if \( \pi(\delta) \) is strictly increasing \( \{ \delta : \pi(\delta) = 0 \} \) has a single element \( \delta_p \), and this equilibrium is unique. The next step is thus to provide a sufficient condition for \( \pi(\delta) \) to be strictly increasing. Note that this happens iff \( ep'(\delta) > v'(\delta) \) for every \( \delta \), and that we know already that \( v(\cdot) \) is a strictly decreasing function. Lemma 3 shows that while \( p(\delta) \) is also a decreasing function, it can be made arbitrarily flat by reducing the precision of public information (by increasing \( \eta \)). Specifically, for any \( Q > 0 \), there exist a \( \overline{\eta}(Q) \) such that if \( \eta > \overline{\eta}(Q) \), then \( |p'(\delta)| < Q \). Then \( \pi(\delta) \) is strictly increasing if \( \eta > \overline{\eta} \left( \frac{1}{e} |v'(\delta)| \right) \), and we are done. \( \blacksquare \)

Lemma 1 \( \{ \delta : \pi(\delta) = 0 \} \neq \emptyset \). Let \( \delta_p \in \{ \delta : \pi(\delta) = 0 \} \). There exists a symmetric equilibrium of the party vote game in which \( \kappa^p_i(\theta_i, r) = x \) for all \( i \) such that \( \theta_i \geq \delta_p \) and \( \kappa^p_i(\theta_i, r) = q \) for all \( i \) such that \( \theta_i < \delta_p \).

Proof. Our first task is to show that \( \{ \delta : \pi(\delta) = 0 \} \neq \emptyset \). Consider the points \( \overline{\theta}_i \equiv v^{-1}(r + e) \) and \( \underline{\theta}_i \equiv v^{-1}(r - e) \) that were defined in Remark 2. Note that the net payoff of voting for \( q \) for PB \( i \) in the event that the incumbent survives the challenge is given by \( v(\theta_i) - r - e \). Since the net payoff of voting for \( q \) for PB \( i \) is always at least
$v(\theta_i) - r - e$, then $\theta_i > \overline{\theta}_i \Rightarrow \Pi(\theta_i; \xi) > 0$ for any $\xi$. Similarly, since the net payoff of voting for $x$ for PB $i$ is always at least $r - e - v(\theta_i)$ (where the challenge is successful for sure), then $\theta_i < \underline{\theta}_i \Rightarrow \Pi(\theta_i; \xi) < 0$ for any $\xi$. It should be noted that the points $\overline{\theta}_i$ and $\underline{\theta}_i$ are well defined, since $v(\cdot)$ is continuously decreasing, and $\lim_{\theta_i \to -\infty} v(\theta_i) = \infty$, while $\lim_{\theta_i \to \infty} v(\theta_i) = -\infty$, by A1. Now, $\pi(\delta) \equiv \Pi(\delta, \delta) \equiv \Pi(\theta_i = \delta; \xi = 1_{\{\theta_i \geq \delta\}})$. Then the previous argument implies, in particular, that $\pi(\delta) > 0$ for $\delta > \overline{\theta}_i$, and $\pi(\delta) < 0$ for $\delta < \underline{\theta}_i$. Since $\pi(\delta)$ is continuous, this implies that $\{\delta : \pi(\delta) = 0\} \neq \emptyset$. Next, let $\delta_p \in \{\delta : \pi(\delta) = 0\}$. To show the existence of the symmetric equilibrium, it is now enough to show that $\Pi(\theta_i; 1_{\{\theta_i \geq \delta\}})$ is increasing in $\theta_i$. But it is easy to see from (1) that $P(\delta, \theta_i)$ is increasing in $\theta_i$. Since $v(\theta_i)$ is decreasing, the result follows. ■

**Lemma 2** Suppose that $\pi(\delta)$ is strictly increasing. Then $\{\delta : \pi(\delta) = 0\}$ has a single element $\delta_p$, and the equilibrium of Lemma 1 is unique.

**Proof** (Morris and Shin, 1998). If $\pi(\delta)$ is strictly increasing, there is a unique $\delta_p$ solving $\pi(\delta) = 0$. I show next that this in turn implies that the symmetric equilibrium with switching strategies at $\delta_p$ is the unique equilibrium. So consider any equilibrium of the game, and define the numbers

$$z \equiv \inf\{z\mid \xi(z) > 0\} \text{ and } \overline{z} \equiv \sup\{z\mid \xi(z) < 1\}$$

Note first that

$$\overline{z} \equiv \sup\{z\mid \xi(z) < 1\} \geq \sup\{z\mid 0 < \xi(z) < 1\} \geq \inf\{z\mid 0 < \xi(z) < 1\} \geq \inf\{z\mid \xi(z) > 0\} \equiv \underline{z} \quad (\text{IS})$$

Now, for any $z \in \{z\mid \xi(z) > 0\}$, there is some $i$ for which $x_i^p(z; r) = x$. This is only consistent with equilibrium behavior if the payoff to supporting $x$ (for mr $i$ and for anyone else, since they are all identical, ex ante) is at least as high as the payoff to supporting $q$ given ideal policy $z$; i.e., $\Pi(z, \xi) \geq 0$. By continuity, this is also true at $\underline{z}$; i.e.,

$$\Pi(\underline{z}, \xi) \geq 0 \quad (12)$$
Now consider the payoff $\Pi(\hat{z}, 1_{\{\theta_i \geq \zeta\}})$. It is clear that, for any $z$, $1_{\{\theta_i \geq \zeta\}}(z) \geq \xi(z)$. But - in general - whenever $\xi(z) \geq \xi'(z)$ for any $z$, then $\Pi(z, \xi) \geq \Pi(z, \xi')$. Hence $\Pi(z, 1_{\{\theta_i \geq \zeta\}}) \geq \Pi(z, \xi)$ for any $z$, and in particular

$$\pi(\hat{z}) \equiv \Pi(\hat{z}, 1_{\{\theta_i \geq \zeta\}}) \geq \Pi(\hat{z}, \xi) \tag{13}$$

Thus combining (12) and (13) I obtain

$$\pi(\hat{z}) \geq 0 \tag{14}$$

Now by hypothesis, $\pi(\delta)$ is increasing in $\delta$. Since $\delta_p$ is the unique value of $\delta$ which solves $\pi(\delta) = 0$, this means $\hat{z} \geq \delta_p$. A symmetric argument establishes that $\bar{z} \leq \delta_p$. Thus $\bar{z} \leq \delta_p \leq \hat{z}$. This together with (IS) implies that $\hat{z} = \delta_p = \bar{z}$. Thus in any equilibrium the $x$’s aggregate support mapping $\xi$, and thus the strategy of every PB, $x_i^p$, is given by $1_{\{\theta_i \geq \delta_p\}}$.

**Lemma 3** $p(\cdot)$ is a decreasing function of $\delta$. Furthermore, for any $Q > 0$, there exists a $\bar{\eta}(Q)$ such that if $\eta > \bar{\eta}(Q)$, then $|p'(\delta)| < Q$

**Proof.** Since

$$\left(\frac{\theta - \tilde{\theta}(\theta_i)}{\eta}\right)_{\theta=\delta-\sigma \Phi^{-1}(1-\mu)} = \frac{1}{\eta} \left[\frac{\sigma^2 (\delta - \theta_0) + \eta^2 (\delta - \theta_i)}{\sigma^2 + \eta^2} - \sigma \Phi^{-1}(1-\mu)\right]$$

, then

$$p(\delta) = P(\delta, \delta) = 1 - 2\Phi \left(\frac{1}{\eta} \left[\frac{\sigma^2}{\sigma^2 + \eta^2} (\delta - \theta_0) - \sigma \Phi^{-1}(1-\mu)\right]\right)$$

Hence

$$\frac{\partial p(\delta)}{\partial \delta} = -2\phi \left(\frac{1}{\eta} \left[\frac{\sigma^2}{\sigma^2 + \eta^2} (\delta - \theta_0) - \sigma \Phi^{-1}(1-\mu)\right]\right) \frac{1}{\eta \sqrt{1 + \frac{\eta^2}{\sigma^2}}}$$

That $p'(\delta) < 0$ follows immediately. And since $|p'(\delta)|$ is bounded above by $\frac{2}{\eta}$, $|p'(\delta)| < Q$ for $\eta > 2/Q = \bar{\eta}(Q)$. ■
Lemma 4 \( \lim_{\sigma \to 0} d = v^{-1}(r) - v^{-1}(r + e[1 - 2\mu]) \)

Proof.

\[ f(\sigma) = \Phi \left( \frac{\theta - \hat{\theta} (\theta_i = \delta)}{\eta} \right)_{\theta=\delta-\sigma \Phi^{-1}(1-\mu)} \]

\[ = \Phi \left( \frac{1}{\eta} \left( 1 + \frac{\eta^2}{\sigma^2} \right)^{-\frac{1}{2}} (\delta_p(\sigma) - \theta_0) - \left( 1 + \frac{\sigma^2}{\eta^2} \right)^{\frac{1}{2}} \Phi^{-1}(1 - \mu) \right) \]

Since \( f(\sigma) \) is continuous in an interval around 0,

\[ \lim_{\sigma \to 0} f(\sigma) = \Phi \left( \lim_{\sigma \to 0} \left[ \frac{1}{\eta} \left( 1 + \frac{\eta^2}{\sigma^2} \right)^{-\frac{1}{2}} (\delta_p(\sigma) - \theta_0) - \left( 1 + \frac{\sigma^2}{\eta^2} \right)^{\frac{1}{2}} \Phi^{-1}(1 - \mu) \right] \right) \]

Note that \( \lim_{\sigma \to 0} \left( 1 + \frac{\sigma^2}{\eta^2} \right)^{\frac{1}{2}} = 1 \), and \( \lim_{\sigma \to 0} \left( 1 + \frac{\eta^2}{\sigma^2} \right)^{-\frac{1}{2}} = 0 \). Since \( \delta_p(\sigma) \) is bounded (by \( \theta_i \) and \( \bar{\theta}_i \)), this implies that

\[ \lim_{\sigma \to 0} f(\sigma) = \Phi \left( -\Phi^{-1}(1 - \mu) \right) = \Phi \left( \Phi^{-1}(\mu) \right) = \mu \]

Now,

\[ v(\delta_p) \equiv r + e \left[ 1 - 2\Phi \left( \frac{\theta - \hat{\theta} (\theta_i = \delta)}{\eta} \right)_{\theta=\delta-\sigma \Phi^{-1}(1-\mu)} \right] \]

Therefore in the limit as \( \sigma \to 0 \), \( v(\delta_p) = r + e[1 - 2\mu] \), so that

\[ \lim_{\sigma \to 0} d = v^{-1}(r) - v^{-1}(r + e[1 - 2\mu]) \]

\[ \blacksquare \]

Remark 5 (i) Suppose that for all \( i \) in a given set \( \Omega_0 \), \( c_i(\theta; x) = 0 \) for \( \Omega = \{i\} \). Then in a equilibrium with no weakly dominated strategies, \( c(x) = 0 \) for \( \Omega = \Omega_0 \); (ii) Suppose that for a given set \( \Omega_0 \), there exists \( i \in \Omega_0 \) such that \( c_i(\theta; x) = 1 \) for \( \Omega = \{i\} \). Then in a equilibrium with no weakly dominated strategies \( c(x) = 1 \) for \( \Omega = \Omega_0 \)
Lemma 5  There exists a $\eta$ such that for all $x > q$, whenever $\eta > \eta$: $c(x) = 1 \Rightarrow \eta > \eta(x)$.

Proof. (1) Let $p_{ov}(\theta_i)$ denote the probability that a PB with ideal point $\theta_i$ assigns to the incumbent being overthrown in the event of a challenge. Then $i \in \Omega$ would challenge the incumbent if and only if:

$$p_{ov}(\theta_i) e + u(q;\theta_i) \geq \max \{ u(x;\theta_i) + (e + r); u(q;\theta_i) \}$$

That is, iff

$$p_{ov}(\theta_i) \geq \max \left\{ \frac{e + r - v_x(\theta_i)}{e}, 0 \right\}$$

(15)

(2) It is easy to see from here that if $p_{ov}(\theta_i) = 1$ for some $i$ (if $i$ believes that if the incumbent is challenged, she will be overthrown), then $i$ would challenge iff $\theta_i < v_x^{-1}(r)$. It follows from this that there for any belief about the resolution of a challenge $p_{ov}(\theta_i)$ the incumbent will not be challenged provided that $\omega = \min \{ \theta_i : i \in \Omega \} > v_x^{-1}(r)$.

(3) A sufficient condition for a unique voting equilibrium following a challenge is that $e |p'(\delta)| < |v'(\delta)|$ for every $\delta$. Since for every $\delta$ (i) $|p'(\delta)| < \frac{2}{\eta}$ and (ii) $|v'(\delta)| > \alpha(x - q)$ (by A1’), this occurs if

$$\frac{2}{\eta} e < \alpha(x - q)$$

Then there will always be a unique equilibrium if

$$(x - q) < \frac{2e}{\eta \alpha} \Rightarrow \omega > v_x^{-1}(r)$$

(16)

Note, moreover, that A1’ implies that $v_x^{-1}(r) < \delta_0 - \frac{r}{\alpha(x-q)}$. Hence (16) becomes:

$$(x - q) < \frac{2e}{\eta \alpha} \Rightarrow \omega > \delta_0 - \frac{r}{\alpha(x-q)}$$

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22Suppose that $p_{ov}(\theta_i) = 1$ for some $i$ (i.e., $i$ believes that if the incumbent is challenged, she will be overthrown). Then $i$ would challenge iff $\theta_i < v_x^{-1}(r)$. To see this, note that $e + r - v_x(\theta_i) = 0$ if $\theta_i < v_x^{-1}(e + r) \equiv \tilde{\theta}_i$, while $e + r - v_x(\theta_i) > 0$, and increases continuously with $\theta_i$ for $\theta_i \geq \tilde{\theta}_i$. Letting $\tilde{\theta}_c$ denote the value of $\theta_i$ that solves $1 = \max \left\{ \frac{e + r - v_x(\theta_i)}{e}, 0 \right\}$, it follows that (i) a PL would challenge iff $\theta_i < \tilde{\theta}_c$, and that (ii) $\tilde{\theta}_c > \tilde{\theta}_i$, so that $e + r - v_x(\tilde{\theta}_c) > 0$ and then $\tilde{\theta}_c = v_x^{-1}(r)$. 35
Writing $\delta_0$ as $q + \frac{(x-q)}{2}$, this will always be satisfied provided that:

$$q + \frac{e}{\eta \alpha} - \frac{1}{2 \alpha} \frac{r}{\eta} \alpha < \omega$$

Since the LHS is decreasing in $\eta$ and diverges to $-\infty$ as $\eta \to \infty$, for any given $\omega$ there is an $\eta$ such that whenever $\eta > \eta$, this inequality is satisfied. ■

**Lemma 6** For $\eta > \overline{\eta}$ there exists a $\bar{x}_\eta \in R$ such that $c(x) = 1 \iff x \geq \bar{x}_\eta$

**Proof.** Fix $\eta > \overline{\eta}$. By Lemma 5, $c(x) = 1 \Rightarrow \eta > \overline{\eta}(x)$. Then for a potential challenger with ideal point $\theta_i$, the probability that an active incumbent is overthrown is given by

$$\Pr \left( \theta < \delta - \sigma \Phi^{-1}(1 - \mu) | \theta_i \right) = \Phi \left( \frac{\theta - \tilde{\theta}(\theta_i)}{\overline{\eta}} \right)_{\theta = \delta - \sigma \Phi^{-1}(1 - \mu)}$$

Note that $\Pr(\theta < \delta - \sigma \Phi^{-1}(1 - \mu) | \theta_i)$ is a continuous, decreasing function of $\theta_i$, and that $\lim_{\theta_i \to -\infty} \Pr(\theta < \delta - \sigma \Phi^{-1}(1 - \mu) | \theta_i) = 1$, while $\lim_{\theta_i \to \infty} \Pr(\theta < \delta - \sigma \Phi^{-1}(1 - \mu) | \theta_i) = 0$. Then $c_i(\theta_i; x) = 1 \iff \theta_i < \theta_c$, where $\theta_c > \theta_i$, and is uniquely defined by:

$$r + e P(\delta_p, \theta_c) \equiv v_x(\theta_c) \quad (17)$$

Note that $\theta_c$ so determined is an increasing function of $x$, $\theta_c(x)$. This result can be obtained totally differentiating (17) noting that (1) since the LHS is bounded between $r$ and $r + e$, $v_x(\theta_c) > 0$ (every challenger prefers $q$ to $x$), (2) whenever $v_x(\theta_i) > 0$, $v_x(\theta_i) > v_x(\theta_i)$ for $x(x > x$ for individuals who prefer $q$ to $x$, increasing $x$ increases the payoff of voting for $q$), (3) $\delta_p$ is increasing in $x$, and therefore $P(\delta_p, \theta_c)$ is decreasing in $x$ (since the probability of a successful challenge increases with $x$). Also (4) $P(\delta_p, \theta_c)$ is increasing in $\theta_i$ and (5) $v_x(\theta_i)$ is decreasing in $\theta_i$.

For a given $x$, there will be a challenge if and only if $\theta_c(x) \geq \omega$. We know by the previous lemma that if $x < q + \frac{2e}{\eta \alpha}$, then $c(x) = 0$. Thus $\theta_c(x) < \omega$ for $x < q + \frac{2e}{\eta \alpha}$. Since
$\theta_c(\cdot)$ is an increasing function of $x$, $c(x) = 1 \Leftrightarrow x \geq \bar{x}$, where $\bar{x}$ is defined by $\theta_c(\bar{x}) \equiv \omega$; i.e., by

$$r + eP(\delta_p(\bar{x}), \omega) \equiv v_{\bar{x}}(\omega)$$

9 Appendix B

The discussion of proposition 3 in the text emphasizes the change in the weight that PBs give to their preferences vis a vis public information as a result of changes in the heterogeneity of the party. This note explains in some detail why this weight effect, while not reflecting the entire story, is dominant in producing the result.

As before, let $\delta$ denote an arbitrary cutoff for PBs’ strategies. Letting $\beta(\sigma) \equiv \hat{\eta}^{-1}$ and $k(\sigma) \equiv \frac{\sigma^2}{\sigma^2 + \eta^2}$, I write

$$\left(\frac{\delta - \hat{\theta}(\theta_i)}{\hat{\eta}}\right) = \beta(\sigma) [k(\sigma)(\delta - \theta_0) + (1 - k(\sigma))(\delta - \theta_i)]$$

The derivative of this expression with respect to $\sigma$ is:

$$\beta'(\sigma) [k(\sigma)(\delta - \theta_0) + (1 - k(\sigma))(\delta - \theta_i)] + \beta(\sigma)k'(\sigma)(\theta_i - \theta_0)$$

(18)

Note that the value that individual $\theta_i$ attaches to the incumbent’s promises of electoral benefits, $P(\delta, \theta_i)$, has an inverse relationship with the probability that this individual attaches to the incumbent being overthrown, and that this probability increases (decreases) with $\sigma$ if (18) is positive (negative). Using this expression, I can separate the total effect of increasing $\sigma$ on the probability that individual $\theta_i$ attaches to the incumbent being overthrown into two components.

First, there is a change in the precision of his estimation of the central tendency of the party. Given that $\mu = 1/2$, the incumbent is overthrown if the ex post median is not in the incumbent’s coalition; i.e., if $\theta < \delta$. If $\delta$ is big relative to the weighted
average \( k(\sigma)\theta_0 + (1 - k(\sigma)) \theta_i \) (with the weights \( k(\sigma) \) fixed), then this is a relatively "common" event. But a lower precision makes "common" events less likely (in opposition to "extreme" events). Thus, whenever \( \delta > k(\sigma)\theta_0 + (1 - k(\sigma)) \theta_i \), or equivalently \( k(\sigma)(\delta - \theta_0) + (1 - k(\sigma)) (\delta - \theta_i) > 0 \), this precision effect induces individual \( i \) to consider less likely that the incumbent will be overthrown.

Second, there is a change in the weight that \( i \) gives to his own preferences vis a vis the public information in his estimation of the central tendency of the party. As noted in the text, a higher \( \sigma \) means that individual \( i \) will attach more weight to the public information and less to his own preferences in estimating the central tendency of the party. \( \phi(\cdot) \beta(\sigma)k'(\sigma)(\theta_i - \theta_0) \) reflects the change in the probability that individual \( \theta_i \) attaches to the incumbent being overthrown brought by the change in weights between public and private information. Thus a higher \( \sigma \) will make individuals with ideal policies \( \theta_i > \theta_0 \) believe that the central tendency of the party is farther away from the policy supported by the incumbent (more to the left). Thus, such an individual will attach a higher probability to the incumbent being overthrown.

Note that for individual \( i \), with ideal policy \( \theta_i \), the "precision" term is clearly decreasing in \( \delta \), while the "weight" term is independent of \( \delta \). However, for the critical PB, with ideal policy \( \delta \), a higher \( \delta \) implies both a change in the cutoff and a change in his information, and both the precision and weight terms depend on the difference between \( \delta \) and \( \theta_0 \). For the critical PB, (18) becomes:

\[
\beta'(\sigma)k(\sigma)(\delta - \theta_0) + \beta(\sigma)k'(\sigma)(\delta - \theta_0)
\]  

The precision term is, as before, decreasing in \( \delta \). Even if the estimate of the central tendency of the party changes with the preferences of the individuals I consider (a more "right-winged" PB believes "the party" is more "right-winged"), this only happens with the weight given to their preferences vis a vis the public information, and thus is not strong enough to compensate the increase in the cutoff. Furthermore, the precision term is positive.
if $\delta < \theta_0$ and negative otherwise. However, for the critical PB the weight term is now increasing in $\delta$, and positive if $\delta > \theta_0$. \footnote{This means that according to the precision effect, a less cohesive party would led a "left-winged" critical PL to consider more likely that the incumbent will be overthrown (and thus the value that he would attach to the incumbent’s promises would decrease). According to the weight term, however, a less cohesive party would led a "left-winged" critical PL to consider less likely (and not more likely as above) that the incumbent will be overthrown (thus reducing the value of her promises).} Thus, both the precision and weight terms depend on the difference $\delta - \theta_0$, and (19) is positive when $\delta > \theta_0 \iff \beta'(\sigma)k(\sigma) + \beta(\sigma)k'(\sigma) > 0$, or equivalently, iff

$$\frac{k'(\sigma)}{k(\sigma)} > \frac{\beta'(\sigma)}{\beta(\sigma)} \quad \text{(20)}$$

; i.e., iff the rate of growth of the "weight" term outweighs the rate of growth of the "precision" term. Some algebra shows that (20) is indeed satisfied, and therefore that the weight terms dominates for the critical PB. Therefore (19) is positive iff $\delta > \theta_0$, the probability that individual $\delta$ attaches to the incumbent being overthrown increases with $\sigma$ iff $\delta > \theta_0$, and then $p(\delta) \equiv P(\delta, \delta)$ decreases with homogeneity iff $\delta < \theta_0$. 


10 Bibliography


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