Title
LIMITS ON CHARM-CHANGING NEUTRAL CURRENTS

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Publication Date
1979-05-01
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May 1979

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48
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I. INTRODUCTION

The experimental suppression to orders $G$ and $G^2$ of strangeness-changing neutral currents in $K_L \rightarrow \mu^+\mu^-$, $K_L \rightarrow K_S$, etc., is theoretically understood as a consequence of the "natural" conservation of flavor by neutral currents.\(^1\) Within this framework, a charm-changing neutral current is also unexpected. The present experimental limits on neutrino-production of charm and on charm semileptonic decay by neutral currents, as well as on $D^0 - \bar{D}^0$ mixing (decays into "wrong" sign kaons or leptons) indicate indeed qualitatively the absence of this current. The purpose of this note is to make a quantitative study - which lacks in the literature - of this question.

To be general, let us add an arbitrary $V,A,|\Delta C| = 1$, $\Delta Q = 0$ current

$$J'_{\mu} = \gamma_\mu \left( \frac{1-\gamma_5}{2} \right) u + \gamma_\mu \left( \frac{1+\gamma_5}{2} \right) u + h.c.$$  \hspace{1cm} (1)

to the standard model neutral current\(^2\)

$$J_{\mu} = \gamma_\mu \left( \frac{1-\gamma_5}{2} \right) v - \bar{e} \gamma_\mu \left( \frac{1+\gamma_5}{2} \right) e + 2\sin^2 \theta_W \bar{\nu}_\mu e + \ldots$$

The new term will then give rise to $|\Delta C| = 2$ and $|\Delta C| = 1$ pieces in the current x current lagrangian $\mathcal{L} j_{\mu}^0 j_{\mu}^{10}$:

$$\mathcal{D} |\Delta C|=2 = \mathcal{L} \left[ \left( \frac{-\gamma_5}{2} \right) u + \gamma_\mu \left( \frac{1+\gamma_5}{2} \right) u \right]^2 + h.c.$$  \hspace{1cm} (2)

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* This work was supported in part by the High Energy Physics Division of the U. S. Department of Energy under contract No. W-7405-ENG-48.

** On leave of absence of LPTHE, Orsay, France.
\[ L_{|\Delta C|=1} = \sqrt{2} G \left[ \bar{\psi}_e \chi_{\mu} \left( \frac{1-\gamma_5}{2} \right) e + 2 \sin^2 \theta_W \bar{\psi}_\mu e + \bar{\psi}_\mu \gamma_{\mu} \left( \frac{1+\gamma_5}{2} \right) e \right] + \bar{\psi}_\mu \gamma_{\mu} \left( \frac{1+\gamma_5}{2} \right) e + h.c. \]  

(3)

This \(|\Delta C| = 2\) piece would contribute to \(D^0 - \bar{D}^0\) mixing, while the \(|\Delta C| = 1\) piece to neutrino-production \( (\nu_\mu + \bar{\nu}_\mu + c) \) and semileptonic decay \( (c + u + e^+e^-) \) of charm by the neutral current.

In Section II we will deduce an upper limit on the mass difference between the two CP eigenstates from \(D^0\) decay into "wrong" sign kaons or leptons. In Section III we shall derive a constraint on \(g_L\) and \(g_R\) from the bound on \(\delta m\). We will pay special attention to the QCD corrections to \(\delta m\). Further constraints on \(g_L\) and \(g_R\) will be obtained in Section IV from experimental limits on the processes \(\nu_\mu + N \leftrightarrow \nu_\mu + c\) and on charm + e^+e^- + Hadrons.

Some details of the calculations will be given in the Appendices.

II. \(D^0 - \bar{D}^0\) OSCILLATIONS

Let us first consider the pattern of possible decay amplitudes for a \(D^0\) (\(c\bar{u}\)) in the framework of the standard model,

\(c \rightarrow s + u + d \sim \cos^2 \theta_c\)  
\(c \rightarrow s + u + \bar{d} \sim \cos \theta_c \sin \theta_c\)  
\(c \rightarrow d + u + \bar{s} \sim \cos \theta_c \sin \theta_c\)  
\(c \rightarrow d + u + \bar{s} \sim \sin^2 \theta_c\)  
\(c \rightarrow s + \bar{c} + \nu_h \sim \cos \theta_c\)  
\(c \rightarrow d + \bar{c} + \nu_h \sim \sin \theta_c\)  

\(4a, 4b, 4c, 4d, 4e, 4f\)

where we have omitted the quark spectator, \(\bar{u}\). We expect then, from a \(D^0\), a single negative (positive) strangeness kaon with amplitude \(\cos^2 \theta_c \) (\(\sin^2 \theta_c\)) and, if any, a positive charged lepton. The opposite is true for the \(\bar{D}^0\). One has however to take into account the possibility of mixing between the \(D^0\) and the \(\bar{D}^0\). Indeed, a state, which at \(t = 0\) is a \(D^0\), becomes, at time \(t\)

\[ \frac{1}{2} \exp \left[ -i m_+ t - \frac{\lambda_+ t}{2} \right] + \exp \left[ -i m_- t - \frac{\lambda_- t}{2} \right] \]  

where \(m_\pm\) and \(\frac{\lambda_\pm}{2}\) are the masses and the inverse lifetimes of the two CP eigenstates \((D^0 \pm \bar{D}^0)/\sqrt{2}\), and one has neglected CP violation.

From Eq. (5) it is easy to deduce\(^3\) the ratio of probabilities of getting a positive or a negative electron starting from a \(D^0\):

\[ \frac{N(D^0 + e^- + \ldots)}{N(D^0 + e^+ + \ldots)} = \frac{\delta m^2 + \delta \lambda^2/4}{2 \lambda^2 + \delta m^2 - \delta \lambda^2/4} \]  

(6)

where \(\lambda = \frac{\lambda_+ + \lambda_-}{2}\), \(\delta \lambda = \lambda_+ - \lambda_-\), \(\delta m = m_+ - m_-\).

To compute the probability of finding "wrong" sign (positive) kaons one has to keep into account the possibility of direct decay of a \(D^0\) into a state with positive strangeness, with amplitude proportional to \(\sin^2 \theta_c\). (4d). Let us consider the decay (4a) of a \(D^0\) into a particular state with a single strange
particle $K^-$, $|i\rangle = |K^- + \ldots\rangle$ with amplitude $\alpha_i$, proportional to $\cos^2 \theta_c$. From the decay (4d) we see that $\bar{D}^0$ can also decay into the same mode with an amplitude $\beta_i$, proportional to $\sin^2 \theta_c$.

The $C$-transformed of this decay is the direct decay of $D^0$ into a "wrong" sign kaon. It is easy to obtain the probability of getting a $K^-$ or a $K^+$ as the single strange particle starting from a $D^0$. In Appendix I we generalize the formulas of Ref. (3) to take into account the $D^0 - \bar{D}^0$ oscillations as well as the direct decays into "wrong" sign kaons. We get:

$$N(D^0 \rightarrow K^- + \text{pions}) = \sum_i \left( \frac{\lambda}{2\lambda_\perp (\lambda^2 + \delta m^2)} \right) \left( 2\lambda^2 + \delta m^2 - \frac{\delta \lambda^2}{4} \right) |\alpha_i|^2 + \left( \frac{\delta \lambda^2}{4} + \delta m^2 \right) |\beta_i|^2$$

$$+ \left( \frac{\delta \lambda^2}{4} + \delta m^2 \right) \text{Re}(\alpha_i^* \beta_i) - \frac{\delta m}{\lambda^2 + \delta m^2} \text{Im}(\alpha_i^* \beta_i)$$

(7)

$$N(D^0 \rightarrow K^+ + \text{pions}) = \sum_i \left( \frac{\lambda}{2\lambda_\perp (\lambda^2 + \delta m^2)} \right) \left( 2\lambda^2 + \delta m^2 - \frac{\delta \lambda^2}{4} \right) |\beta_i|^2 + \left( \frac{\delta \lambda^2}{4} + \delta m^2 \right) |\alpha_i|^2$$

$$+ \left( \frac{\delta \lambda^2}{4} + \delta m^2 \right) \text{Re}(\alpha_i^* \beta_i) + \frac{\delta m}{\lambda^2 + \delta m^2} \text{Im}(\alpha_i^* \beta_i)$$

Let us now discuss the magnitude of the different terms in these formulae. Consider first the standard GIM model. $\beta_i$ is proportional to $G \sin^2 \theta_c$. The width difference $\delta \lambda + \sum_n \langle D^0 | H_w | n \rangle \langle n | H_w | D^0 \rangle$ is also Cabibbo suppressed, $G^2 \sin^2 \theta_c$, since the possible intermediate states come from the modes (4b) and (4c). $\delta m$ is also very small in the standard model. We give here the expression just for completeness:

$$(\delta m)_{\text{GIM}} = \frac{2}{3} \frac{G}{f_D m_D} \frac{\alpha}{2\pi} \left( \frac{m_s^2 - m_d^2}{M_w^2 \sin^2 \theta_W} \right) \sin^2 \theta_c \cos^2 \theta_c.$$

Therefore, in the GIM model, the probability of getting a negative charged lepton or $S = +1$ state from the decay of a $D^0$ is expected to be of order $\sin^4 \theta_c$.

If we introduce the new $|\Delta C| = 2$ piece (2), we would have a contribution to $\delta m$ at order $G$. Neglecting in (6) and (7) the terms $\delta \lambda^2$, $\delta \lambda \beta_i$, $|\beta_i|^2$ (all of order $\sin^4 \theta_c$), we obtain:

$$\frac{N(D^0 \rightarrow e^- + \ldots)}{N(D^0 \rightarrow e^+ + \ldots)} \approx \frac{\delta m^2}{2\lambda^2 + \delta m^2}$$

(8)

$$\frac{N(D^0 \rightarrow K^+ + \text{pions})}{N(D^0 \rightarrow K^- + \text{pions})} \approx \frac{\delta m^2}{2\lambda^2 + \delta m^2}$$

(9)

Although we have assumed CP-invariance and Born approximation for the weak interaction, the last term in (9) (which is potentially important since $\lambda$ is not Cabibbo suppressed) could be non-zero due to final state strong interactions. Therefore, we will keep it.
Let us now look for the experimental limits on $D^0 - D^0$ mixing. The best limit is given by the reaction $e^+e^- \to e^+e^+ + \ldots$:

$$\frac{N(e^+e^+) + N(e^-e^-)}{N(e^+e^+) + N(e^-e^-)} < 5\%$$  

(3.72 GeV < $E_{cm}$ < 4.14 GeV)

This implies, from Eq. (8)

$$\frac{\delta m^2}{2\lambda^2 + \delta m^2} < 0.025 .$$  

(10)

This upper limit is far beyond the prediction of the standard model $\delta m^2/\lambda^2 \sim \sin^4 \theta_c$. The theoretical estimates of the $D$ lifetime range from $0.7 \times 10^{-12}$ sec (Ref. 6) to $1.2 \times 10^{-12}$ sec (Ref. 7).

The present experimental limits are $0.50 \times 10^{-12}$ sec < $\tau(D)$ < 1.1 \times 10^{-12} sec. From the value $0.75 \times 10^{-12}$ sec., we get

$$|\delta m| < 0.25 \times 10^{12} \text{ sec}^{-1} .$$  

(11)

From (10) and (11) we can bound the r.h.s. of Eq. (9), if we make the reasonable assumption $|\beta_1| = \tan^2 \theta_c |\alpha_1|$. We predict:

$$\frac{N(D^0 + K^+ + \text{pions})}{N(D^0 + K^- + \text{pions})} < 3.8\%$$

This limit is consistent with the experimental upper limits:

(a) Ref. 9. $e^+e^- \to D^+ + \ldots$, $D^+ \to K^+ \text{ or } K^-$.

$$N(\text{wrong sign kaons}) < 16\% \text{ (90}\% \text{ CL)}$$

$$N(\text{all } D^+ \text{ events}) \text{ (} E_{cm} > 5 \text{ GeV})$$

(b) Ref. 10. $e^+e^- \to D^0K^+ + \ldots$

$$N(\text{wrong sign kaons}) < 18\% \text{ (90}\% \text{ CL)}$$

$$N(\text{all } D^0 \text{ events}) \text{ (} E_{cm} = 4.028 \text{ GeV})$$

III. LIMITS ON THE COUPLINGS FROM $D^0 - D^0$ MIXING.

Let us now compute $\delta m$ from the effective interaction (2). Without considering QCD corrections and with the vacuum insertion as the only intermediate state, we get

$$\delta m = \frac{G_F}{\sqrt{2}} \frac{m}{D^0 D^\ast} \left| (g_L - g_R)^2 + \frac{1}{2} \left[ g_L^2 + g_R^2 + 4g_L g_R \left( \frac{m_D}{m_0 + m_0} \right)^2 \right] \right| .$$  

(13)

There are two graphs (Fig. 1): annihilation and scattering ($Z^0$ exchanged respectively in the $s$- or $t$-channels), corresponding respectively to the two terms in the bracket. The factor $1/3$ comes from color, since to insert the vacuum we must perform a Fierz transformation of the scattering term. Moreover, the Lorentz forms $(V-A) \cdot (V-A)$ and $(V+A) \cdot (V+A)$ are invariant under Fierz, but $(V-A) \cdot (V+A)$ gives $2(P-S) \cdot (P-S)$. This explains the factor 4, the sign in front of it, and the mass ratios in the last term, since to evaluate the matrix elements of the pseudoscalar density $U$ we use
\[ q_u < 0|A_\mu|D^0> = (m_u + m_u) c < 0|P|D^0> \].

Note that only the axial piece contributes to the annihilation term. But, due to the Fierz transformation that we must perform, the scattering term gives also a vector contribution. \( f_D \) is defined according to:

\[ \langle 0|\bar{\psi}_\mu Y_\mu u|D^0>, \]

Let us now consider the QCD corrections. To evaluate all leading powers of \( \alpha_s \log(M^2/\Lambda_c^2) \) we use the renormalization group method. Since we have assumed a general \( V, A \) interaction, the anomalous dimension matrix is not diagonal, unlike the case of non-leptonic decays through the usual \( (V-A) \cdot (V-A) \) interaction.

We give details of the calculations in the Appendix II. Adopting the obvious notation \( O_1 = (V-A)^2 \), \( O_2 = (V+A)^2 \), \( O_3 = (V-A) \cdot (V+A) \), \( O_4 = (P-S) \cdot (P+S) \), the operators affected by a multiplicative renormalization

\[ U_1 \rightarrow U_1 \left[ 1 + \frac{25}{3} \frac{\alpha_s(m_c^2)}{4\pi} \log \frac{m^2}{m_c^2} \right] \]

are the following combinations:

\[ U_1 = O_1 \text{ with } \gamma_1 = -2 \]
\[ U_2 = O_2 \text{ with } \gamma_2 = -2 \]
\[ U_3 = O_3 - \frac{2}{3} O_4 \text{ with } \gamma_3 = -1 \]
\[ U_4 = O_4 \text{ with } \gamma_4 = +8 \]

Writing the original Lagrangian in terms of \( U_1 \) and using (14) we get an effective Lagrangian:

\[ \mathcal{L}_{\text{eff}}^{\Delta C} = \frac{G_f^2}{2} \left\{ \gamma_1 \mu \left[ \bar{\psi}_\mu (1 - \gamma_5) u \right]^2 + \gamma_2 \nu \left[ \bar{\psi}_\nu (1 + \gamma_5) u \right]^2 + \right\}

\[ + 2 \delta L_{\text{eff}} \left[ \gamma_2 \left[ \bar{\psi}_\nu (1 - \gamma_5) u \right] \left[ \bar{\psi}_\mu (1 + \gamma_5) u \right] - \frac{2}{3} \left[ \bar{\psi}_\nu (1 - \gamma_5) u \right] \right] \cdot \left[ \bar{\psi}_\mu (1 + \gamma_5) u \right] \right\} + \text{h.c.} \]

where \( f_1 \cdot f_2 \) and \( f_3 \) are strong interaction factors, related to the quantity \( f_+ \) which appears in non-leptonic hyperon decays,

\[ f_1 = f_+, f_2 = f_+^{1/2}, f_3 = f_+^4 \]

with

\[ f_+ = \left[ 1 + \frac{25}{3} \frac{\alpha_s(m_c^2)}{4\pi} \log \frac{m^2}{m_c^2} \right] \frac{1}{25} \]

From (15) we get

\[ \delta m = \frac{G_f^2}{2} m_D f_D^2 \left\{ \gamma_1 \mu \left[ \bar{\psi}_\mu (1 - \gamma_5) u \right]^2 + 2 \delta L_{\text{eff}} \right\} \cdot \left[ \bar{\psi}_\nu (1 - \gamma_5) u \right] \left[ \bar{\psi}_\mu (1 + \gamma_5) u \right] \right\} \]

which of course reduces to (13) in the limit \( \alpha_s \rightarrow 0 \).

For \( f_D \) we adopt the value \( f_D = 180 \text{ MeV} \). We discuss this choice in Appendix III. Taking \( m_D = m_c + m_u \), we get the domain from (11),

\[ -0.3 \times 10^{-6} < A_2^2 + B_2^2 < 0.3 \times 10^{-6} \]
where \( A = \frac{4f_1}{3} \), \( B = 2(5f_3^2 - 8f_2^2)/9 \). This domain is very sensitive to the value of the QCD corrections within the expected theoretical range. For \( f_+ < 0.75 \) – as is the case \( f_+ = 0.68 \) from Ref. 6 (\( g_\mu = 0.7 \)), plotted in Figure 2 - this domain lies between two hyperbolas. For \( f_+ = 0.75 \), these become straight lines and only the vector part is bounded. When \( f_+ > 0.75 \), the domain closes to an ellipse, as shown in the figure for the case without QCD corrections \( (f_+ = 1) \). Strictly speaking, large \( g_L \), \( g_R \) couplings are not excluded. Due to this critical dependence on the value of \( f_+ \), \( D^0 - B^0 \) mixing cannot exclude large (dominantly axial-vector) couplings.

However, if we assume a pure left-handed \((c,u)\) current (as was the case for the \((s,d)\) current in the Cabibbo theory before the GIM mechanism), we get the severe limit \( (f_+ > 0.6) \):

\[
|g_L| < 0.6 \times 10^{-3}
\]  

(19)

**IV. LIMITS FROM OTHER PROCESSES**

Further constraints on the couplings in (1) can be obtained from limits on neutrino production and semileptonic decay of charm by neutral currents. The relevant interaction is given by (3).

We get, in the valence quark parton model:

\[
R = \frac{\frac{1}{2} \frac{g_L^2}{g_L^2 + g_R^2}}{\frac{1}{2} \frac{g_L^2 + g_R^2}{\sin^2 \theta_W + \sin^2 \theta_W}} \quad (20)
\]

\[
= \frac{1}{4} \frac{g_L^2 + g_R^2}{\sin^2 \theta_W + \sin^2 \theta_W}
\]

where \( N \) is an isoscalar target, and \( C(X) \) denote final states with (without) charm.

From the experimental limit \( R < 2\% \)

we get, with \( \sin^2 \theta_W = 0.25 \),

\[
g_L^2 + \frac{1}{3} g_R^2 < 0.025 \quad \text{(21)}
\]

In a similar way we can compute

\[
\frac{\Gamma(c + u + e^- \bar{e})}{\Gamma(c + s + e^+ \nu_e) + \Gamma(c + d + e^+ \nu_e)} = \frac{1}{4} \frac{g_L^2 + g_R^2}{\Gamma(\text{Charm} + e^+ \bar{e}^- + \text{hadrons})}
\]

(22)

(23)

which, within the parton model, is expected to be equal to the ratio:

\[
\frac{\Gamma(\text{Charm} + e^+ \bar{e}^- + \text{hadrons})}{\Gamma(\text{Charm} + e^+ \nu_e + \text{hadrons})} < 2\%.
\]

We get the much less strict bound:

\[
g_L^2 + g_R^2 < 0.16.
\]

As an illustration we plot in Fig. 3 domain (21) together with the region allowed by \( D^0 - B^0 \) mixing due to uncertainties in the QCD corrections when \( f_+ > 0.6 \).
If the \((c,u)\) current is assumed to be pure left-handed, the limit from \(D^0 - \bar{D}^0\) mixing (19) predicts:

\[
\frac{\sigma(\nu_\mu \rightarrow \nu_\mu)}{\sigma(\nu_\mu \rightarrow \nu_X)} < 3 \times 10^{-7}
\]

\[
\frac{\sigma(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)}{\sigma(\bar{\nu}_\mu \rightarrow \bar{\nu}_X)} < 2 \times 10^{-7}
\]

\[
\Gamma(\text{Charm} + e^+e^- + \text{hadrons}) < 3 \times 10^{-8}
\]

V. CONCLUSION

We have obtained bounds on the strengths of a charm-changing neutral current \(g_L(c,u) + g_L(c,u)\). Due to the critical dependence on the QCD corrections, \(D^0 - \bar{D}^0\) cannot exclude large couplings, and the experimental limits on neutrino-production of charm by neutral currents are necessary to further constrain the general current. However, assuming a pure left-handed coupling, \(D^0 - \bar{D}^0\) mixing gives the severe bound \(\vert g_L \vert < 0.6 \times 10^{-3}\).

APPENDIX I.

We deduce here the formulae (7), which take into account \(D^0 - \bar{D}^0\) oscillations as well as direct \(D^0(\bar{D}^0)\) decays into "wrong" sign kaons \(K^+(K^-)\). This is a simple generalization of the formalism of reference 3.

A \(D^0\) produced at \(t = 0\) evolves as

\[
\begin{align*}
\psi(t) &= a(t)|D^0\rangle + b(t)|\bar{D}^0\rangle + \sum_i c_i(t)|X_i^\pm\rangle + \sum_i d_i(t)|X_i^0\rangle + \\
&\quad + |\text{Other decay products}\rangle,
\end{align*}
\]

where \(|X_i^\pm\rangle\) is a decay product containing a \(K^\pm\), and \(|X_i^0\rangle\) is a decay containing a \(K^0\), CP transformed one of another. The amplitudes \(a(t), b(t)\) are given by

\[
\begin{align*}
\{a(t)\} &= \frac{1}{2} \left[ e^{-\lambda^+_t t/2} e^{\text{im}_t t/2} \right] \\
\{b(t)\} &= \frac{1}{2} \left[ e^{-\lambda^-_t t/2} e^{\text{im}_t t/2} \right]
\end{align*}
\]

\(m_\pm, \lambda_\pm\) are the masses and inverse lifetimes of the two CP-eigenstates. A state \(|X_i^\pm\rangle\) could come from the decay of a \(D^0\) through the process (4a) with amplitude \(\alpha_1\), or from the decay of a \(\bar{D}^0\) through (4d) with amplitude \(\beta_1\). The reverse is true for the state \(|X_i^0\rangle\). We have then the differential equations for the amplitudes of probability:

\[
\frac{d}{dt} c_1(t) = \alpha_1 a(t) + \beta_1 b(t) - i E_1 c_1(t)
\]

\[
\frac{d}{dt} d_1(t) = \alpha_1 b(t) + \beta_1 a(t) - i E_1 d_1(t)
\]
The solutions are, for \( t \to \infty \):

\[
\begin{pmatrix}
\delta_1 \\
\rho_1
\end{pmatrix} \to e^{-\beta E_t t} \left( \begin{pmatrix}
\frac{\alpha_1 + \beta_1}{2} \\
\frac{\alpha_1 + \beta_1}{2}
\end{pmatrix} \left( \frac{\lambda_1^2 + 1}{2} + i(m_1 - E_t) \right) + \left( \frac{\alpha_1 + \beta_1}{2} \right) \left( \frac{1}{2} + i(m_1 - E_t) \right) \right)
\]

The intensity of "wrong" \((K')\) or right \((K)\) sign kaons is then given by

\[
N (D^+ + K^+ + \text{pions}) = \sum_1 \int \left( \left| d_1 \right|^2 \right) \ dE_t.
\]

From these expressions, integrating over the Breit-Wigner's we get the formulae (7).

**APPENDIX II.**

We sketch here the calculation of the QCD corrections. We start from the Lagrangian (2). We have two types of interactions, \((V \pm A)^2\) and \((V - A) \cdot (V + A)\). Computing the lowest order graphs (Fig. 4) in the \( \alpha_s \log \) approximation, we get the variation of these operators. The \((V \pm A)^2\) operators become

\[
\left[ \bar{c} \gamma_\mu (1 \mp \gamma_5) u \right]^2 - \frac{2}{3} \beta \left[ \bar{c} \lambda^\alpha (1 \mp \gamma_5) u \right]^2
\]

and the \((V - A) \cdot (V + A)\)

\[
\left[ \bar{c} \gamma_\mu (1 - \gamma_5) u \right] \cdot \left[ \bar{c} \gamma_\mu (1 + \gamma_5) u \right] + \frac{3}{2} \beta \left[ \bar{c} \lambda^\alpha (1 + \gamma_5) u \right] \cdot \left[ \bar{c} \lambda^\alpha (1 + \gamma_5) u \right]
\]

where \( \beta = \frac{\alpha_s}{4 \pi} \log \left( \frac{m_W^2}{m_c^2} \right) \).

Of course, in this case the vertex correction and self energy graph cancel. This result can be expressed without the color matrices by performing a Fierz transformation:

\[
(q_4 a_3)(q_2 a_1) = \frac{1}{2}(q_4 q_3)(q_2 a_1) + \frac{1}{2}(q_4 q_3)(q_2 a_1)
\]

\((V \pm A)^2\) are Lorentz forms invariant under Fierz, but \((V - A) \cdot (V + A)\) gives \(2.(P + S) \cdot (P - S)\). We get then for the \((V \pm A)^2\) terms:

\[
\left[ \bar{c} \gamma_\mu (1 \mp \gamma_5) u \right]^2 (1 - 2\beta)
\]

and for the \((V - A) \cdot (V + A)\) piece:

\[
\left[ \bar{c} \gamma_\mu (1 - \gamma_5) u \right] \cdot \left[ \bar{c} \gamma_\mu (1 + \gamma_5) u \right] (1 - \beta) +
\]

\[
+ 68 \left[ \bar{c} (\gamma_5 - 1) u \right] \cdot \left[ \bar{c} (\gamma_5 - 1) u \right].
\]

The anomalous dimension matrix is thus non-diagonal. We need to compute the variation of \( \bar{c} (\gamma_5 + 1) u \) under lowest order gluon corrections. For this operator we get that the box diagrams (c) and (d) cancel in the leading log approximation, and the self energy and vertex graphs give the same operator:

\[
\left[ \bar{c} (\gamma_5 + 1) u \right] \cdot \left[ \bar{c} (\gamma_5 - 1) u \right] (1 + 88).
\]
This means that a general interaction of the form \( L = \sum_i a_i O_i \),
where \( O_1 = (V - A)^2 \), \( O_2 = (V + A)^2 \), \( O_3 = (V - A) \cdot (V + A) \),
\( O_4 = (P + S) \cdot (P - S) \), which can be represented as the vector \( \mathbf{L} = (L^1, L^2, L^3, L^4) \),
is changed by lowest order gluon corrections by an amount
\[
\delta \mathbf{L} = \frac{\alpha_s}{4\pi} \log \left( \frac{\mu^2}{\Lambda^2} \right) A \mathbf{L}
\]
where the matrix \( A \) is given by
\[
A = \begin{pmatrix}
-2 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 68
\end{pmatrix}
\]
This matrix has the eigenvalues \( \gamma_i = -2, -2, -1, 8 \), corresponding to the eigenvectors \( U_1 = O_1, U_2 = O_2, U_3 = O_3 - \frac{2}{3} O_4, U_4 = O_4 \).
Summing up all leading log orders, these operators will be affected by a multiplicative renormalization
\[
U_i = U_i \left( 1 + \frac{25}{3} \frac{\alpha_s(m_c^2)}{4\pi} \log \frac{\mu^2}{\Lambda^2} \right)^{\frac{3\gamma_i}{25}}
\]

**APPENDIX III.**

Let us discuss the value of \( f_D \) that we have adopted. In the non-relativistic limit, this quantity is given in terms of the wave function at the origin by the Van Royen and Weisskopf formula\(^{15}\)
\[
f_D = \frac{2|\psi_D(0)|}{\sqrt{m_D}}
\]
A measurement of \( f_D \) - very difficult due to the small BR expected for \( u\bar{u} \) - would give very interesting information on the wave function of a system made up of a very light quark and a heavy quark. This quantity will be indicative of the type of SU(4) -breaking forces. Here we cannot do better than try to get a value from indications on these forces from light mesons. The generalization to SU(4) of the Weisskopf-Van Royen "paradox"
\[
|\psi_\pi(0)|/|\psi_K(0)| \approx \frac{\sqrt{m_\pi}}{\sqrt{m_K}}
\]
which ensures \( f_\pi \approx f_K \) would give a value for \( f_D \) within the same range: \( f_D \approx 140 - 180 \text{ MeV} \). Another estimation could come from the empirical formula\(^{16}\) for the vector meson wave function at the origin from \( V + e^+e^- \),
\[
|\psi_V(0)|^2 = m_V^{1.99}
\]
Note that this formula applies to vector mesons in which \( q \) and \( \bar{q} \) have the same mass, and could be invalid for the pseudoscalar \( c \bar{u} \) system. Assuming however the same mass dependence, we have \( f_D = 180 \text{ MeV} \). Within the uncertainties of these estimations, this value - adopted in the text - seems to us reasonable.
ACKNOWLEDGEMENTS

It is a pleasure to thank Dr. A. Barbaro-Galtieri for proposing this subject to us. We are also indebted to Prof's. S. Coleman and S. Suzuki and to Dr. R. Shrock for very useful discussions. We thank the Theoretical Physics group of the Lawrence Berkeley Laboratory for the kind hospitality extended to us.

One of us (L. O.) was supported in part by the High Energy Physics Division of the U. S. Department of Energy.

REFERENCES

5. J. Kirkby et al., unpublished.
   73B (1978) 422.

FIGURE CAPTIONS

Fig. 1: Annihilation and scattering contributions to the \( D^0 \overline{D}^0 \) transition.

Fig. 2: Domain from the bound on \(| \delta m | \) as a function of the QCD corrections.

Fig. 3: Domain allowed by \( v_\mu N \rightarrow v_\mu C \) and from \( \delta m \) taking into account the uncertainties on the QCD corrections.

Fig. 4: Lowest order gluon corrections to the \( |\Delta C| = 2 \) interaction.
Figure 1
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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