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Array Thinning by Using Antennas in a Fabry–Perot Cavity for Gain Enhancement

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Abstract—A Fabry–Perot cavity (FPC) between a ground plane and a partially reflective surface (PRS) is used here to design array antennas with large distance between the radiating elements. This configuration provides some advantages: i) a reduction of the number of array elements to achieve high directivity; ii) large space between contiguous elements that may host a bulky feeding network as required for dual polarization or active antennas; iii) small coupling and easy feeding network designs because of the smaller number of elements with larger inter-element distance. We show that when designing the FPC antenna a frequency shift of the gain maximum may occur, especially in this sparse array configuration. We also show the existence of preferred distances between elements that controls both the directivity and the side lobe level, and how the presence of the FPC and the relaxed requirement of the interelement distance result in a lower interelement coupling. The presented dual polarized antenna comprises two interleaved 2 × 2 arrays placed in a 2-layer FPC, and exhibits a 19 dBi gain and 30 dB of isolation between the two ports over an operating bandwidth of approximately 5.7%, i.e., typical for patch antennas.

Index Terms—Antenna arrays, electromagnetic bandgap materials, enhanced directivity, Fabry–Perot cavities (FPC), leaky wave antennas.

I. INTRODUCTION

THE enhancement of directivity by using leaky wave antennas has been a subject of study in the last 40–50 years [1]–[5] and various geometries have been proposed. In the last twenty years attention has been brought also to planar structures made of dielectric layers [6]–[18], [34] called 2-D leaky wave antennas. The enhanced directivity is based on the excitation by a single radiator of a leaky wave along the antenna geometry, that is a complex modal solution allowed to propagate in the structure. In [9]–[18], [34] it has been shown that planar antennas with leaky waves with small propagation constant can produce narrow beams at broadside. Various configurations have been considered to enhance the directivity at broadside, such as single or multi dielectric layers to form a cavity resonator often called Fabry–Perot cavity (FPC). This structure is mainly formed by a half wavelength chamber between a ground plane and a partially reflective surface (PRS) located on top of the cavity. The leaky wave is trapped between the ground plane and the above PRS. In the original studies the top PRS was formed by alternating high-density and low-density quarter wavelength dielectric layers [9]–[12] or with a periodic screen [13]. In the past years the PRS has been often made of planar periodic structures, made of a thin sheet of periodic slots, dipoles or other more complex geometries [13]–[15]. In [16] it has been shown that the phenomenology and design criteria of a PRS made of multiple layer is equivalent to that made of a shunt admittance that is a good model to represent a thin periodic active screen such as a frequency selective surface. An in-depth analysis of the fundamental properties of enhanced broadside radiation is carried out in [18], [34] though the main idea was established already at the time of [9].

More recently, the superstrates that constitute the PRS of the FPC, made of multiple layers of homogeneous dielectric material, have been regarded as electromagnetic band gap (EBG) superstrates [19], [20]. Such superstrates have also been realized by more complex EBG materials that are periodic along the longitudinal and transverse (parallel to the antenna plane) directions [21]–[26]. EBG materials have also been used as substrates and radomes [27], [28].

Antennas with enhanced directivity have been proposed in [29], [30] where the homogeneous FPC covered by a PRS is substituted by a metamaterial with low effective permittivity or by an EBG material with a particular dispersion diagram. However the basic principle that produces the enhanced directivity is still based on the excitation of leaky waves as it has been shown in [2], [31], [32].

Although in line of principle the source directivity could be unlimitedly enhanced, a strong directivity results in a very small antenna bandwidth and a very critical design. As shown in this paper, this trade-off between directivity enhancement and bandwidth suggests to use a sparse array inside the FPC that is designed to produce a moderately enhanced directivity for each array elements. Such a configuration allows to preserve the needed operating bandwidth and achieve the requested directivity by using sparse arrays of sources. Since each element in the FPC is individually quite directive, a larger spacing between elements is permitted still avoiding grating lobes. The thinning of the array elements results in a simpler structure with fewer elements, a simpler feeding network and a lower coupling between elements as it is shown in this paper. In Sections II–V we investigate the abovementioned issues showing by some examples how the use of an FPC allows to decimate the radiators in a standard spaced array without degrading the pattern characteristics, especially focusing on coupling, side lobes, and gain effects. In Section II we analyze the frequency shift of the gain maximum with respect to the design frequency when using both ideal radiators and patch antennas. This shift should be accounted for when designing sparse arrays. In Section V we
explicitly analyze the performance of the sparse array versus interelement distance for a selected geometry. Furthermore, since a large space is available between elements, an interlaced array can be arranged to achieve dual polarization facilities. In Section VI we present a design example which comprises two interlaced array providing a dual polarized antenna with high decoupling between the two ports.

II. FPC DESIGN: SINGLE ANTENNA UNDER AN EBG SUPERSTRATE

The FPC in Fig. 1 has the advantage of increasing the directivity of a single antenna located inside it [9]–[18], [34]. The antenna excites a leaky mode that produces the broadside pattern, as rigorously demonstrated in [9]–[12], [18], [34]. In those papers the multilayered structure is analyzed with the help of a transmission line (TL) as shown in Fig. 2, which is an effective tool for this kind of structures. Here we use that formalism and we provide some design criteria and numerical examples to show the gain enhancement. As an example, the radiation resistance by the factor $\varepsilon_r^n$

$$\mathcal{Z}_{\text{up}} = \frac{Z_0}{\varepsilon_r^n}$$

where $\mathcal{Z}_{\text{up}}$ is the impedance evaluated at $z = d - \Delta$ looking toward increasing $z$ (Fig. 2). Since we are interested in broadside radiation, all impedances, defined only for plane propagation, are evaluated assuming a plane wave traveling along the direction $z$, with wavenumber perpendicular to the $x$, $y$ plane, e.g., $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. From (1), a low impedance $\mathcal{Z}_{\text{up}}$ is obtained either by a high dielectric permittivity $\varepsilon_r$ or by a significant number of layers $n$. A low value of $\mathcal{Z}_{\text{up}}$, compared to $Z_0$, serves to create a PRS at $z = d - \Delta$ to form an FPC in the region $0 < z < d - \Delta$. In [9]–[18], [34] studies have been carried out for cavities without the $h_s$-thick dielectric layer (used here to support a patch), and the FPC was assumed to have a thickness of $d = \lambda_0/2$. This produces $\mathcal{Z}_{\text{up}} = 0$ at $z = d$, that almost matches the low value of $\mathcal{Z}_{\text{up}}$ in (1). In this paper instead, the presence of the $h_s$-thick dielectric layer with relative dielectric constant $\varepsilon_s$ modifies the height of the FPC and a correction factor $\Delta$ must be introduced to maintain the operating frequency equal to $f_0$. The correction factor depends on the height $h_s$ of the dielectric layer and on its relative dielectric constant $\varepsilon_s$, and the absence of the dielectric layer (either $h_s = 0$ or $\varepsilon_s = 1$) implies $\Delta = 0$. When the $h_s$-thick dielectric layer is present (Fig. 1), the condition $\mathcal{Z}_{\text{up}} = 0$ is encountered at a distance $z = d - \Delta$, with

$$\Delta = \frac{\lambda_0}{2\pi} \arctan \left[ \frac{1}{\sqrt{\varepsilon_s}} \tan \left( \frac{2\pi}{\lambda_0} \varepsilon_s h_s \right) \right] - h_s.$$
The details of its derivation are reported in Appendix A. For small substrate heights $h_s$, the correction factor in (2) can be approximated as

$$\Delta \approx \frac{h_s^2}{3} \left(\frac{2\pi}{\lambda_0}\right)^2 (\varepsilon_s - 1). \quad (3)$$

In our particular case $h_s = 0.762 \text{ mm}$, $\varepsilon_s = 2.5$ and $\lambda_0 = 21.4 \text{ mm}$, (for $f_0 = 14 \text{ GHz}$), the correction term becomes $\Delta = 0.78 \text{ mm}$, therefore the air gap between the patch substrate and the first layer at $z = d - \Delta$ has a thickness $d - \Delta - h_s = 9.93 \text{ mm}$. Note that the size of the FPC is determined here by $d - \Delta$, and not by the specific arrangement of the above PRS. In the case of inductive or capacitive PRS the height of the FPC depends on its reactance [16], [17].

The radiating rectangular conducting patch has dimensions $\ell = 6.2 \text{ mm} \times w = 10 \text{ mm}$. The probe is located at $s = 1.3 \text{ mm}$ from the edge of the patch, to match the $50 \Omega$ impedance of the coaxial line. Such feeding point location was designed in the case with $n = 0$ (no layer), but it provides an acceptable matching in all the considered cases $n = 1, 2, 3$ examined next.

In Fig. 3(a) and (b) the far field normalized radiation patterns in the E- and H-planes at the operating frequency $f_0 = 14 \text{ GHz}$ are shown for an increasing number of superstrate layers. The maximum remains at broadside $(\theta = 0)$ and the main lobe narrower by increasing the number of layers, showing the increase of directivity.

In Fig. 4, the accepted gain in the boresight direction $(\theta = 0)$

$$G_{\text{acc}} = \frac{S(\theta = 0)4\pi r^2}{P_{\text{acc}}} = \frac{\text{EIRP}(\theta = 0)}{P_{\text{acc}}} \quad (4)$$

is plotted versus frequency for various number $n$ of layers. In (4), $S(\theta = 0) = |E(r, \theta = 0)|^2/(2\pi\eta)$ denotes the power density radiated by the antenna at boresight, at a far field distance $r$, EIRP is the effective isotropic radiated power (see Appendix B for the calculation of the power density at broadside $(\theta = 0)$ for the case of an elementary magnetic and electric dipole), and $P_{\text{acc}} = (1 - |\Gamma|^2)P_{\text{in}}$ is the power delivered to (accepted by) the patch via the probe, with $P_{\text{in}}$ denoting the power of the incident travelling wave in the coaxial line and $\Gamma$ the antenna input reflection coefficient. We look at the accepted gain because it is not affected by the antenna impedance mismatch, and it is thus mostly related to the properties of the FPC. Fig. 4 also shows how the bandwidth decreases when increasing the number of layers because the PRS at $z = d - \Delta$ becomes more reflective and the quality factor $Q$ of the FPC increases. Therefore high directivity can only be achieved by reducing the bandwidth as already indicated in [16]–[18], [34] for an ideal dipole excitation. The accepted gain 3 dB-bandwidth is shown in Table I for

<table>
<thead>
<tr>
<th>No. of layers</th>
<th>Max gain</th>
<th>Gain enhancement</th>
<th>Max gain $f$</th>
<th>frequency shift</th>
<th>Gain 3dB bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.4dB</td>
<td>+3.5dB</td>
<td>13.70 GHz</td>
<td>-300 MHz</td>
<td>2.2%</td>
</tr>
<tr>
<td>2</td>
<td>14.1dB</td>
<td>+7.2dB</td>
<td>13.94 GHz</td>
<td>-60 MHz</td>
<td>0.4%</td>
</tr>
<tr>
<td>3</td>
<td>17.7dB</td>
<td>+10.8dB</td>
<td>13.97 GHz</td>
<td>-30 MHz</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Fig. 3. Far field normalized radiation pattern for a patch in an FPC with various number $n$ of layers, (a) E-plane and (b) H-plane cuts.

Fig. 4. Broadside accepted gain versus frequency, for a patch inside an FPC with various number $n$ of layers.
Estimate of the Antenna Size

In all the above simulations the FPC antenna has an infinite transverse extent as well as in those in the next Sections. In practical implementations the whole structure (size of the dielectric layers and ground plane) shall be truncated at a radial distance $L$ from the source where the field is negligible compared to that at the center of the antenna. Since the magnitude of the field $E(\rho)$ on the aperture is mainly produced by two fundamental TE and TM leaky waves with almost equal attenuation constant $\alpha$.
[9–12] (the approximation $\phi^{TE} \approx \phi^{TM}$ is valid for highly directive antennas), it decays along the radial transverse direction as $E(\rho) \approx E_0 \exp(-\alpha \rho)$, where $\rho$ is the radial distance from the source, and $E_0$ is the leaky wave value at $\rho = 0$ (other wave species may be important near the source). The value $\alpha$ depends on the antenna design (number of layers and dielectric constants and the antenna size $2L$) is determined by requiring that the field is attenuated by a factor $E(L)/E_0 = \exp(-\alpha L)$ at its edges. Therefore, if one desires the field at the edges of the antenna to be $-X$ dB with respect to its center, the dimension $L$ should be estimated as $L = X/(\alpha 20 \log_{10} 0.1) \approx X/(8.686 \alpha)$. For instance, for a desired $-30$ dB edge illumination, $L \approx 3.45/\alpha$.

In the case of absence of the patch dielectric layer (in other words, when $\Delta = 0$), the attenuation constant $\alpha$ can be estimated by using the formula [11, Eq. (40)] that in our case is simplified to $\alpha \approx k_0 \varepsilon_r^{-1/2} \sqrt{2/(\eta_0 F)}$, with $\eta_0 F = \pi [\sqrt{\varepsilon_r (2 \varepsilon_r - 1)} + 1] / (\varepsilon_r - 1) \sqrt{\varepsilon_r}$ and $k_0$ the free space wavenumber. Supposing an FPC design with two cover layers ($n = 2$) with dielectric constant $\varepsilon_r = 2.5$, the formula [11, Eq. (40)] predicts $\alpha \approx 0.18 k_0 = 53.3$ m$^{-1}$ at 14 GHz, and thus $30$ dB field reduction is achieved at $L \approx 65$ mm from the center, where the structure can be truncated without affecting the field. A more rigorous numerical analysis where the attenuation constant is determined numerically by imposing $Z_{up} + Z_{up} = 0$ (taking into account of the thin patch substrate with $h_s = 0.762$ mm and $\varepsilon_s = 2.5$) leads to $\alpha = 0.21 k_0 = 61.57$ m$^{-1}$, and thus to a distance $L \approx 56$ mm.

III. ANALYSIS VARYING THE DIELECTRIC CONSTANT OF THE LAYERS

As stated above, the increase of the gain is due to the increase of the reflectivity of the FPC superstrate at $z = d - \Delta$. This is achieved by reducing the equivalent wave impedance $Z_{up}$ in (1) at $z = d - \Delta$ by increasing either the dielectric constant $\varepsilon_r$ of the superstrate layers or the number $n$ of layers. In Fig. 8 we show the accepted gain versus frequency for an FPC structure with either (a) two or (b) three layers, for various dielectric constants $\varepsilon_r$ available among “Arlon AD” substrates. The dimensions in terms of wavelengths are the same as those in Fig. 1(a); namely, the thickness of the $n = 2, 3$ dielectric layers is $t = \lambda/4$. 

Fig. 6. Normalized broadside EIRP versus frequency, for an elementary magnetic dipole on the ground plane, at (a) $z' = 0$ and for an elementary electric dipole at patch level (b) $z' = h_s$ inside an FPC with various number $n$ of layers. EIRP is normalized with respect to the $n = 0$ (no layer) case. In (b), the results from Ansoft designer are in agreement with (10).

Fig. 7. Broadside accepted gain versus frequency, for an elementary magnetic dipole on the ground plane, at (a) $z' = 0$ and for an elementary electric dipole at patch level (b) $z' = h_s$ inside an FPC with various number $n$ of layers.
with \( \lambda = \lambda_0/\sqrt{\varepsilon_r} \) at the operating frequency \( f_0 = 14 \text{ GHz} \), and it thus depends on the material. When the reflectivity of the FPC superstrate at \( z = d - \Delta \) is increased by choosing larger \( \varepsilon_r \), also the quality factor \( Q \) of the FPC is increased, thus resulting in a reduced bandwidth. Note that again the maximum gain (when the size of the cavity is fixed) occurs at frequencies slightly lower than \( f_0 = 14 \text{ GHz} \), but it approaches \( f_0 \) when increasing the dielectric constant of the layers. As said above this can be explained by noticing that \( Z_{\text{up}} \) decreases when \( \varepsilon_r \) increases, therefore better approximating the resonant condition \( Z_{\text{up}} + Z_{\text{cl}} = 0 \).

### IV. ARRAY OF ANTENNAS UNDER AN EBG SUPERSTRATE

Since the above described gain enhancement is obtained to the detriment of the bandwidth, it is not possible to achieve high directivity and large bandwidth at the same time with this type of antennas [16]–[18], [34]. Therefore it may be convenient to use a limited number of layers (say two) to keep a reasonably large bandwidth and to increase the directivity by having an array of a few radiating elements. Such a sparse array configuration, with element radiating in the FPC with \( n = 2 \) and \( \varepsilon_r = 2.5 \) is compared in the following with a standard dense array configuration with no superstrate. Namely, a \( D = 0.8\lambda_0 \approx 17 \text{ mm} \) spacing is used for the standard dense array with no FPC and a double spacing \( D = 1.6\lambda_0 \approx 34 \text{ mm} \) is assumed for the sparse array in the FPC. As sketched in Fig. 9 an equivalence in terms of radiation properties can be established between a single patch in the FPC and a standard 2 \times 2 array of patches (i.e., with no FPC), as confirmed in Fig. 10 where the accepted gain is plotted versus frequency for both structures. A similar gain level implies a similar effective radiating area. One can see that in Fig. 10, the 2 \times 2 array (with no FPC) has approximately the same gain of the single patch (1 \times 1) inside the FPC. Analogously, the 4 \times 4 array (with no FPC) has approximately the same gain of the 2 \times 2 array inside the FPC. Note that, as expected, the gain-bandwidth of the array with no FPC is larger.

Fig. 11(a) shows that at \( f_0 = 14 \text{ GHz} \) the radiation patterns of the 2 \times 2 array (with no FPC) is similar to the one produced by the single patch in the FPC. The main beam is almost coincident in both the E and the H planes. The same radiation pattern equivalence is established in Fig. 11(b), between the 4 \times 4 array (with no FPC) and the 2 \times 2 array in the FPC. Hence the use of the FPC that enhances the gain is adopted to decimate...
the radiating elements in an array antenna, thus simplifying their arrangement and the design of their feeding network. In the presented example the decimation ratio is 1:4, however the decimation ratio versus operating bandwidth trade off can be properly fixed by adjusting the FPC gain enhancement effect. Since for resonating antennas, like patches (with no FPC), the frequency range is limited by the impedance bandwidth that is narrower than the pattern bandwidth, a reduction of the pattern bandwidth due to the FPC does not affect the operating bandwidth until the pattern bandwidth becomes smaller than the impedance one. Hence the proposed array thinning does not affect the operating bandwidth up to a certain ratio of thinning.

In what follows, we investigate the radiation properties of the proposed sparse array configuration inside the FPC, by showing how the interelement coupling and radiation features, like gain and sidelobe level, depend on the interelement distance.

V. ANALYSIS OF A 2 × 2 ARRAY CHARACTERISTICS VERSUS INTERELEMENT DISTANCE

We first analyze the radiation gain and side lobe level (SLL) of a 2 × 2 array of patches versus the interelement distance. Such quantities are plotted in Fig. 12 for the various configurations, namely without FPC (no layer) and inside FPCs with $n = 1, 2$ and $3$ dielectric layers. As expected, the gain increases by increasing the interelement distance in all the FPC arrangement and tends to reach a limit value corresponding to 6 dB over the single element gain shown in Fig. 4. Indeed, for large-enough interelement distances, the coupling between the elements becomes negligible, so that the power radiated by the array is 4 times the power radiated individually by each element. In the broadside direction, where the element contributions add coherently, the field radiated by the array is 4 times the field radiated by each element and consequently, the broadside power density of the array is 16 times larger than that of each element; hence the array gain is approximately 4 times, i.e., 6 dB more, than that of a single element. The $2 \times 2$ array radiation pattern presents side lobes corresponding to the array factor grating lobes modulated by the single-element radiation pattern. As a matter of facts, when $D/\lambda_0$ approaches unity, the array factor predicts grating lobes at $\theta = \pm 90^\circ$ and the SLL increases for all configurations. Further increasing the interelement distance, the array factor first grating lobes occur at $\theta = \pm \theta_{GLL}$ with $\theta_{GLL} = \sin^{-1}(\lambda_0/D)$. By choosing $D/\lambda_0$ such that $\theta_{GLL}$ occurs at the element pattern minimum (it is visible in Fig. 3 in both planes, in the range $40^\circ < \theta < 60^\circ$ for the various cases considered), the array factor grating lobes are particularly re-

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Fig. 11. E- and H-plane normalized radiation patterns for a $2 \times 2$ (a) and $4 \times 4$ (b) array of patches without superstrate (continuous line) and for a single patch, i.e., (a) $1 \times 1$, and (b) $2 \times 2$ array of patches radiating inside an FPC with $n = 2$ layers (dashed line).

Fig. 12. (a) Accepted gain and (b) SLL versus interelement distance at $f_0 = 14$ GHz, for a $2 \times 2$ patch array with no superstrate and inside an FPC with $n = 1, 2, 3$ layers.
produced by the element factor hence, in turn, the SLL presents a minimum. Finally, when $\theta_{GL}$ enters into the element pattern main lobe (it is shown in Fig. 3), the grating lobes approach the broadside direction and their level grows so that the SLL tends to 0 dB. In summary, the array interelement distance can be chosen in order to maximize the gain, still preserving an acceptable SLL. Focusing now on the 2 layers FPC, it is noticed that for $D = 1.6\lambda_0$ the gain is maximized and the SLL is near the local minimum at $D = \lambda_0 / \sin(45^\circ) = 1.4\lambda_0$ (the element pattern in Fig. 3 has a minimum at $\theta \approx 45^\circ$), thus exhibiting a level comparable with that of a standard (without FPC) uniformly illuminated array [cfr. Fig. 11(b)].

Next, in Fig. 13, the coupling scattering parameters between array elements in the different analyzed configurations are plotted versus interelement distance, at the operating frequency $f_0 = 14$ GHz. Element ports are tagged as shown in Fig. 1(b) so that $s_{12}$ and $s_{13}$ are relevant to side by side elements aligned on the H- and E-plane, respectively, whereas $s_{14}$ is relevant to corner by corner elements. When the patches are placed inside the FPC, for a given interelement distance, their coupling increases with the number of layers, i.e., with the reflectivity of the top wall that traps more field around the sources. However, the gain enhancement permits to increase the interelement distance so that the coupling between the elements is dramatically dropped. Furthermore, the interference between spatial and leaky waves inside the FPC reshapes the $s$-parameters versus interelement distance behavior creating nulls, i.e., distances for which the element coupling is very weak. In the $n = 2$ layers FPC case, the distance $D = 1.6\lambda_0$ is near to a deep null of the E-plane coupling parameter $s_{13}$, which is typically the stronger coupling contribution; indeed in standard array configurations, i.e., without the FPC, $s_{13}$ decreases like $D^{-4}$, whereas on the H-plane the coupling factor $s_{12}$ behaves like $D^{-2}$ due to the null in the space wave radiation pattern. Thereby the array configuration inside an FPC with a $n = 2$ layers and $D = 1.6\lambda_0$ presents a very low interelement coupling (less than $-25$ dB) and the active reflection coefficient substantially coincides with the isolated element reflection coefficient, thus drastically simplifying the feeding network design.

VI. DUAL POLARIZED INTERLEAVED SPARSE ARRAYS

On the basis of the previous investigations, as an illustrative example, a dual polarized antenna is presented here (see Fig. 14), which consists of two interleaved $2 \times 2$ sparse arrays designed and simulated by using the commercial MoM software Ansoft Designer. The multistrate arrangement is still as that in Fig. 1 and comprises the patch substrate with $h_s = 0.702$ mm and $\varepsilon_s = 2.5$ on whose top face patches and feeding lines are printed, and $n = 2$ superstrates with $\varepsilon_r = 2.5$. As explained at the end of Section II, with the help of Fig. 4 and Fig. 7, and...
In summary, realize 3 dB power when the feed, and are interlaced in the sense lines leads to a that matches the of the air gap between, and the maximum gain 2 arrays cause they are inside FPCs that do not have large probe ports are shown for varying frequency. Note the very low as shown in Fig. 15 where the scattering parameters at the two ports of the dual polarized antenna.

This is the case of sparse arrays with a significant bandwidth because they are inside FPCs that do not have large truncations effects could be minimized by following the reasoning at the end of Section II; namely the substrates and the ground plane can be truncated at a distance larger than $L = 56$ mm from the patches, to ensure a negligible edge truncation effect. This criterion leads to a overall antenna size of $174 \times 174$ mm$^2$. The patches of the two arrays are oriented orthogonally providing two orthogonally polarized arrays. Each array is fed by a corporate beam forming network printed on the same plane of patches. The microstrip width is $w_m = 0.6$ mm, corresponding to a characteristic impedance of $Z_c = 100 \Omega$ that matches the input impedance of the patches. Note that due to the low coupling, the patches active input impedance practically coincides with their single element input impedance. Quarter wavelength transformers ($w_m = 0.25$ mm, $Z_c = 140 \Omega$) realize 3 dB power splits; the junction of two $Z_c = 100 \Omega$ lines leads to a $Z_{in} = 50 \Omega$ input impedance at each probe feed. The antenna matching is provided over a quite wide band $f_{MATCH} = 800$ MHz = 5.7%, as shown in Fig. 15 where the scattering parameters at the two probe ports are shown for varying frequency. Note the very low coupling ($< -30$ dB) obtained between the two $2 \times 2$ arrays despite their overlapped arrangement. In Fig. 16, the antenna gain is plotted versus frequency when alternatively feeding port 1 or 2. Note that the actual gain $G_{actual} = S(\theta = 0)4\pi r^2/P_{in}$, is plotted here, that also accounts for antenna impedance mismatching because $P_{in}$ is the incident power, whilst in the previous figures we have used the accepted gain (4) that has $P_{osc} =$
(1 - |Î|²)P_in at the denominator. The two antennas exhibit very similar gain curves with a maximum gain of 19 dB at the center frequency f₀ = 14 GHz, and the -3 dB gain bandwidth is estimated to be B_Gain = 800 MHz = 5.7%. The antenna normalized radiation patterns are plotted in Fig. 17(a) and (b) when transmitting/receiving from port 1 and 2, respectively. The two ports provide analogous radiation patterns with orthogonal polarization, as expected. The co-polar radiation pattern is equivalent to that of a 4 x 4 array as demonstrated in Fig. 11(b) for individually probe-fed arrays; the cross-polar component is lower than -20 dB, and is mainly due to the feeding network. Higher polarization purity may be easily achieved by laying out the feeding network on a further substrate below the ground plane and feeding the patches by coupling slots etched in the ground plane. Such a solution however results in a more complicated structure.

VII. CONCLUSION

Highly directive antenna systems are obtained using sparse arrays under a partially reflective superstrate made of a few high permittivity layers to form a FPC. The use of a sparse array with a few layers (say two) permits to achieve a reasonable bandwidth of at least 5.7% and a significant gain at the same time. Sparse arrays are made of fewer elements than their dense counterpart radiating without FPC, permitting simplified beam forming networks and integration of other devices in the space between the elements. However, in the sparse array configuration the array elements radiate inside a cavity (the FPC) and one may be concerned about their mutual coupling. Interestingly, we have experienced low interelement couplings because of the larger than usual distances used in these sparse arrays, thus permitting a simple design procedure. A simple formula to take into account of the additional patch-substrate thickness is shown in (2), whose derivation is based on the imposed resonant condition (7). However, still a small shift of the operating frequency that yields the gain maximum, from the desired frequency, occurs in the design of sparse arrays because the FPC has not a very large quality factor for obtaining a significant bandwidth.

Though the final design in the present study consists of two interleaved 2 x 2 arrays, other designs with a larger number of elements could be made when higher directivity is desired.

In general, arrays with high directivity, thus requiring several radiating elements in free space, could be designed by using this kind of partially reflective superstrate with only 1/4 of the elements, at the cost of some bandwidth reduction. However, as pointed out in Section V, some gain benefit due to one or two layers is “free,” in the sense that usually the input matching bandwidth is narrower than the pattern bandwidth, and therefore we can increase the gain by using an FPC until the pattern bandwidth is reduced to be equal to the input matching bandwidth.

The main results of this paper, i.e., thinning of the array, dual polarized interleaved array application, design criteria, and a solid analysis of the performances for this class of FPCs comprised of dielectric layers, can be extended to similar geometries where the partially reflective surface is made of a frequency selective surface represented by a shunt admittance, as it has been shown in [16] for an ideal single source excitation.

APPENDIX A

RESONANT CONDITION

Due to the presence in the FPC of a dielectric substrate of permittivity ε₀ and thickness h₀, the height of the FPC has to be adjusted to match the resonance at the design frequency. Resorting to the modal TL model in Fig. 2, the resonance condition is fixed setting the first wavenumber eigenvalue (resonance) of the TL close to the longitudinal wavenumber kₜ = k_cosθ at the design frequency. Since we are interested to broadside radiation θ = 0 for which TE and TM impedance definitions in the TL become equal, the same resonance condition applies to both cases. For lossless non-radiating cavities, eigenvalues are real and it is possible to exactly match the wavenumber k at the design frequency. In our case, due to radiation eigenvalues are complex, thus a perfect match is not possible. Therefore, each observable quantity (modal voltage, current or power at a certain section) will present a (possibly) pronounced peak for frequencies near the resonance frequency but not a singularity. Since these different observable quantities (radiated power, gain, TL...
voltages and currents) exhibit different frequency dependencies, each one would have its maximum at a slightly different frequency. However, a high quality factor $Q$ of the FPC ensures very small frequency deviation of the maxima of the various observable quantities, and in practice every quantity presents the peak at the same frequency. Conversely, for FPCs with low $Q$, the frequency shifts of the various maxima could be significant and these definitions of “resonance” (based on the maxima locations) become ambiguous and dependent on the position of source. For these reasons in the following we adopt a more rigorous and standard definitions of resonance.

The presence of $n$ dielectric layers spaced by air gaps realizes a $n$-fold impedance transformer, and at the design frequency $f = c/\lambda_0$, with $c$ the free space wavenumber, the impedance looking upward, at the cavity upper level $z = d - \Delta$, with $d = \lambda_0/2$, is (Fig. 1)

$$Z(d - \Delta) = \left(\frac{Z}{Z_0}\right)^{2n} Z_0 = \frac{Z_0}{\varepsilon_r^n}, \quad (5)$$

It is a resistance that can be made small by increasing either the layers permittivity $\varepsilon_r$, or their number $n$. Looking downward, at $z = d - \Delta$, the impedance is

$$Z(d - \Delta) = \frac{jZ_0 \tan[k_0(d - \Delta - h_s)]}{Z_0 - Z_s \tan[k_0(d - \Delta - h_s)]},$$

where we have taken into account for the presence of the patch substrate. Thus imposing the resonance condition (for real frequencies)

$$\tan \left(\frac{\pi}{2} Z(d - \Delta) + Z(d - \Delta)\right) = 0 \quad (7)$$

leads to the determination of the small correction factor of the cavity thickness

$$\Delta = \frac{1}{k_0} \arctan \left(\frac{1}{\sqrt{\varepsilon_r}} \tan(k_0 h_s)\right) - h_s. \quad (8)$$

that leads to (2). Note that for $\varepsilon_r = 1$, $\Delta = 0$ as expected.

**APPENDIX B**

**TL EXPRESSIONS FOR THE POWER AT BROADSIDE AND GAIN**

After simple treatment of the circuit in Fig. 2 with a TL formalism, the power density at broadside ($\theta = 0$) normalized with respect to the isotropic spreading [see text after (4)] is expressed as

$$\text{EIRP}(\theta = 0) = \frac{M_0^2}{2\pi \eta_0} |T|^2_{k_0 = 0} \quad (9)$$

with $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ the free space impedance and $k_0 = \omega / \sqrt{\mu_0 \varepsilon_0}$ the free space wavenumber, for the case of an elementary magnetic dipole $M_0$ at $z' = 0$, or

$$\text{EIRP}(\theta = 0) = \frac{k_0^2}{2\pi \eta_0} |M_0|^2 \left(\frac{v}{Z}\right)^2 T^2 T_{k_0 = 0} \quad (10)$$

for an elementary electric dipole $J_0$ at $z' = h_s$. In (9) and (10) for an elementary electric dipole $J_0$ at $z' = h_s$. In (9) and (10)

$$T = \frac{V(h)}{V'(z')} = \frac{Z_0}{Z_0 A + B}, \quad (11)$$

denotes the voltage transmission coefficient in the modal equivalent TL of Fig. 2, from the source point $z'$ to the top of the stack of dielectric layers at $z = h$, i.e., at the beginning of the final TL where free-space starts. The transmission coefficient $T$ is directly calculated in terms of the ABCD (transmission matrix) parameters relevant to the section $z' < z < h$ of the equivalent TL, and it depends on the longitudinal TL propagation constant $k_0 = \sqrt{k_0^2 + k_t^2}$, that in turns depends on the transverse wavenumber $k_t$, which is set to $k_t = 0$ in (9) and (10) for the broadside direction of observation $\theta = 0$. Furthermore, in (10) $Z$ is the total impedance (parallel of upper $\bar{Z}$ and lower $\bar{Z}$ impedances) at the generator section $z' = h_s$.

The gain is derived by the formula (4), where the accepted power $P_{\text{acc}}$ produced by the ideal electric and magnetic dipoles is determined by using a TL formalism. The final expressions are

$$P_{\text{acc}} = \frac{|M_0|^2}{8\pi} \int_0^{+\infty} \frac{k_0}{k_t} \text{Re} \left\{ Y' (k_t) = Y'' (k_t) \right\} dk_t \quad (12)$$

when excited by a magnetic dipole, and

$$P_{\text{acc}} = \frac{|J_0|^2}{8\pi} \int_0^{+\infty} \frac{k_0}{k_t} \text{Re} \left\{ \bar{Z}' (k_t) = \bar{Z}'' (k_t) \right\} dk_t \quad (13)$$

when excited by an electric dipole. Here, $\bar{Z}$ and $\bar{Z}$ are the total admittance or impedance evaluated at the source location $z' = 0$ or $z' = h_s$, when excited by the magnetic or the electric dipole, respectively. Primed and double primed symbols refer to transverse magnetic (TM) and transverse electric (TE) quantities, respectively. For the used formalism and notation see [33].

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**REFERENCES**


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