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Semihydrodynamic model for ion separation in a fast pinch

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It has been found experimentally that if a mixture of hydrogen and argon is pinched, a separation between the two components occurs very early during the implosion phase; the light ions move faster to the center, leaving the heavier ions behind. A model is presented which describes the dynamics of the pinch and explains the separation. The collisional electron gas is treated as a fluid and the ions are treated as free particles accelerated by the collective electrostatic field set up by the flow. The set of equations describing such a flow is presented and some simple problems are worked out in order to show that a separation is possible. These problems include the ion sound wave, the general flow with impurities, and the shock wave structure.

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In a pinch, a fast rising current is driven through a column of plasma. Due to skin effect, this current flows mainly on the external boundaries. The self-magnetic field is thus excluded from the plasma and the pressure gradient induces an implosion. The process may be divided into three stages: (i) initial heating of the gas (up to ~ 2 eV) accompanied by ionization, (ii) a slow implosion, (iii) a very fast pinch followed by instabilities and disassembly of the plasma. In this letter we shall deal only with the second phase. We assume that ionization has already taken place and that the fast rising current starts the implosion.

It has been found in the puff-gas Z pinch experiment[1] that a mixture of two different species tends to separate early in the implosion stage. The light atoms move faster toward the center and if the atomic mass ratio is large enough a separation into two distinct annuli occurs. Such a separation was also observed indirectly in θ pinches.[2]

Two extreme models have been proposed to describe the pinch dynamics[3]; the snow plow and the sheath. The first applies to a fully collisional plasma (the simple snow plow model can easily be generalized to include the plasma pressure yielding a fully MHD description), while the second applies to a collisionless plasma. In both cases a mixture does not separate into its components during the implosion. Collisions prevent a rapid separation in the hydrodynamic flow, just as in the case of an ordinary flow in gases. In the sheath model both electrons and ions are totally reflected from the "magnetic piston." A strong electric field is formed in a narrow sheath ($\delta \sim c/\omega_p$) which carries the current. The sheath structure depends on the ion composition. However, no separation is possible since all species are totally reflected.

Due to the large mass ratio $M_{\rm ion}/m_e$, a situation may occur in which the plasma consists of a collisional high-temperature electron gas and collisionless low-temperature ion gas. Characteristic temperature for the initial stage of implosion is $kT \approx 2$ eV. If $n > 10^{14}$ cm^{-3} electrons may be considered collisional. Electron-ion relaxation time is long, especially for heavy ions. Thus ions may heat up very little during the implosion time. Their energy is then mostly directional. There is a density region ($10^{14} < n < 10^{17}$ cm^{-3}) for which electrons can be described as a hot collisional gas and ions as cold and collisionless.

The physical mechanism which gives rise to a separation can easily be described qualitatively. Electrons are pushed inward like a fluid by the external magnetic piston. (For simplicity we shall assume that the current flows in a very narrow boundary layer so that the magnetic pressure is zero inside. Its surface value supplies the necessary boundary condition.) A small charge separation is induced, resulting in an electric field within the plasma. Ions are dragged along this field assuring quasineutrality. However, different species are dragged along with different speeds: the heavier the ion is, the smaller is its speed.

The equations which describe such a model include continuity and equation of motion for each ion species in addition to mass, momentum, and energy conservation for the electrons. To these, the Poisson equation should be supplemented. For the ions the equations for conservation of particles and momentum are

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x}(N_i V_i) = 0\,, \tag{1}$$

$$\frac{\partial (M_i V_i)}{\partial t} + \frac{\partial}{\partial x}\left(e\varphi + \frac{1}{2} M_i V_i^2\right) = 0\,. \tag{2}$$

For simplicity we assume one-dimensional planar flow. N, V, and M stand for number density, speed, and mass of the ions and $i = 1,2$. φ is the electrostatic potential. We assume singly ionized atoms.

The set of equations can be completed by the three conservation laws for the electrons and the Poisson equation. A substantial simplification is possible by the quasineutrality assumption, namely,

$$n_e \simeq N_1 + N_2\,. \tag{3}$$

This is a very good approximation for a dense plasma, because a very small charge separation gives rise to a huge electrostatic field. Combining Eqs. (1) and (3) with the continuity equation for the electrons gives for a bounded plasma

$$n_e V_e = N_1 V_1 + N_2 V_2\,. \tag{4}$$

We also note that in the equation of motion for the electrons,

$$m_e \frac{dV_e}{dt} = -eE - \frac{1}{n_e}\frac{\partial p_e}{\partial x}\,, \tag{5}$$

the rhs is close to zero; i.e.,

$$\left| m_e \frac{dV_e}{dt} \right| \ll |eE| \approx \left| \frac{1}{n_e} \frac{\partial p_e}{\partial x} \right| ,$$

thus eE can be eliminated by

$$eE = -\frac{1}{n_e} \frac{\partial p_e}{\partial x} = -e \frac{\partial \varphi}{\partial x} . \qquad (6)$$

Equations (3), (4), and (6) replace the continuity and the equation of motion of the electrons as well as the Poisson equation. Equations (1)–(3) and (6) can be combined to give

$$\frac{\partial}{\partial t} \sum \rho_i V_i + \frac{\partial}{\partial x} \left(p_e + \sum \rho_i V_i^2 \right) = 0 , \qquad (7)$$

where ρ_i is the ion density. This should be supplemented by the equation of energy conservation for the electrons:

$$\frac{\partial}{\partial t} \left(\rho_e \epsilon_e + \sum \frac{1}{2} \rho_i V_i^2 \right) + \frac{\partial}{\partial x} \left((p_e + \rho_e \epsilon_e) V_e + \frac{1}{2} \sum \rho_i V_i^3 \right) = 0 . \qquad (8)$$

$\epsilon_e = \epsilon_e(p_e, \rho_e)$ is given by the equation of state of the electrons. The initial equations can be thus reduced to five equations for the unknown functions N_1, V_1, N_2, V_2, and p_e. These equations can not be solved analytically except for special cases. We shall show by simple examples that ion separation is possible if the mass ratio is large enough.

Small perturbations propagate with the ion sound speed. To see this let us linearize the flow equations. The result is

$$\frac{\partial}{\partial t} \delta N_i + N_i \frac{\partial}{\partial x} \delta V_i = 0 , \qquad (9)$$

$$M_i \frac{\partial}{\partial t} \delta V_i = -\frac{m_e}{n_e} C_s^2 \frac{\partial}{\partial x} (\delta N_1 + \delta N_2) , \quad i = 1,2 . \qquad (10)$$

δN_i and δV_i are the perturbed number density and particle speed. N_i and n_e refer to the initial quantities (assumed constant in space). $C_s^2 = (\partial p_e / \partial \rho_e)_s$ is the adiabatic electron sound speed. The energy conservation equation for the electrons reduces to entropy conservation. This is because entropy increase is only of second order. Thus C_s was used. Fourier analyzing Eqs. (9) and (10) give the dispersion relation:

$$\hat{c} = \omega / k = \pm \left(\frac{N_1}{n_e} \frac{m_e}{M_1} + \frac{N_2}{n_e} \frac{m_e}{M_2} \right)^{1/2} C_s . \qquad (11)$$

Initial perturbations will propagate with this speed. For a one species plasma it is the usual ion sound speed. Each of the perturbed field functions can be expressed as a sum of right and left propagating waves:

$$\delta f = \delta f^-(x - \hat{c}t) + \delta f^+(x + \hat{c}t) .$$

They all can be expressed in terms of one perturbed function, for example, the perturbed electron density. As $(\partial/\partial t)(\delta V^{\mp}) = \mp (1/\hat{c}) . (\partial/\partial x)(\delta V^{\mp})$ and $\delta V = 0$ for $\delta n_e = 0$ we get

$$\delta V_i^{\mp} = \pm \frac{m_e}{M_i} \frac{C_s^2}{\hat{c}} \frac{\delta^{\mp} n_e}{n_e} ; \quad \delta V_e^{\mp} = \pm \hat{c} \frac{\delta n_e^{\mp}}{n_e} . \qquad (12)$$

Thus the particle speed behind the wave is inversely proportional to their mass. These speeds are, of course, much smaller than the sound speed. If $M_2 \gg M_1$ and $N_1 \approx N_2$, $\delta V_1^{\mp} \simeq 2 \delta V_e^{\mp}$ while $\delta V_2^{\mp} = (2M_1/M_2) \delta V_e^{\mp}$.

Another simple case occurs when one of the ion species has a small concentration. It does not contribute to the electric field, but is, however, dragged along by it. The flow equations for a continuous flow are

$$\frac{d\rho}{dt} + \rho \frac{\partial V}{\partial x} = 0 , \qquad \rho \frac{dV}{dt} = -\frac{\partial p_e}{\partial x} , \qquad \frac{dS}{dt} = 0 , \qquad (13)$$

where ρ, V, and S are the plasma density, speed, and specific entropy. Note that ions and electrons move together. Ions carry the directional kinetic energy and electrons carry the thermal energy. They exchange energy via the electric field which is given by Eq. (6). For an ideal gas with constant γ

$$eE = -\frac{\gamma}{\gamma - 1} \frac{\partial}{\partial x} \left(\frac{p_e}{n_e} \right) . \qquad (14)$$

Thus the motion of the minority ions is determined by the equation

$$M \frac{dV}{dt} = -\frac{\gamma}{\gamma - 1} \frac{\partial}{\partial x} \left(\frac{p_e}{n_e} \right) . \qquad (15)$$

The minority ions move relative to the majority ions. In the case of a cylindrical pinch they are accelerated faster toward the center if they are lighter and vice versa. In a hollow pinch after the light ions cross the inner boundary they implode freely. Their initial kinetic energy can be estimated in the following way. Assume an isentropic flow. Then $p_e = p_e(0) \delta^\gamma$, where $\delta = n_e / n_e(0)$. For such a flow (for simplicity we omit the subscript e)

$$-e \frac{\partial \varphi}{\partial x} = -\frac{\gamma}{\gamma - 1} \frac{p(0)}{n(0)} \frac{\partial}{\partial x} \left[\frac{p}{p(0)} \right]^{(\gamma - 1)/\gamma}$$

or (16)

$$e\varphi(x,t) = \frac{\gamma}{\gamma - 1} \frac{p(0)}{n(0)} \left[\left(\frac{p(x,t)}{p(0)} \right)^{(\gamma - 1)/\gamma} - \left(\frac{p_s(t)}{p(0)} \right)^{(\gamma - 1)/\gamma} \right] ,$$

where $p_s(t)$ is the boundary pressure. The majority ions form a potential well for the minority light ions. Thus if a light particle starts at a point x_0 at $t = 0$ and leaves the inner boundary at time t (at the inner boundary $p = 0$) its kinetic energy gain is

$$\delta \left(\frac{1}{2} M v^2 \right) = -\delta(e\varphi) \approx \frac{\gamma}{\gamma - 1} \frac{p(0)}{n(0)} \left[\frac{p_s(t)}{p(0)} \right]^{(\gamma - 1)/\gamma} = \frac{\gamma}{\gamma - 1} kT(0) \left[\frac{p_s(t)}{p(0)} \right]^{(\gamma - 1)/\gamma} \qquad (17)$$

(We assumed that at $t = 0$ the pressure was constant up to the boundary.) If $p_s(t)$ increases in time the particles will leave the inner boundary with increasing speed. Thus they will accumulate on a narrow imploding annulus. On the other hand, heavy minority ions will move backward relative to

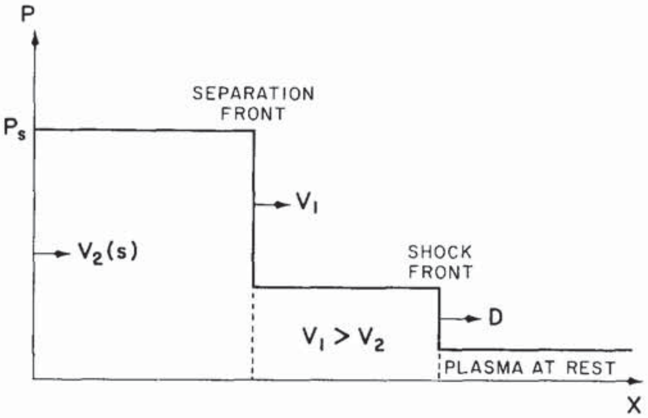

FIG. 1. Qualitative picture of the shock wave structure. The plasma behind the shock front contains the two species flowing with different speeds. Behind the separation front only the heavy ions exist. $V_2(s)$ is the piston speed and P_s is the pressure close to the piston ($M_1 < M_2$).

the majority ions and thus be accumulated on the external boundary. This phenomenon was observed in Ref. 2.

If the external magnetic pressure increases fast enough, a shock wave forms inside the plasma. In order to compute the jump conditions for such a shock wave we use Eqs. (1), (2), (7), (8) together with (3) and (4). We integrate over the jump and let its width go to zero. If the speeds are taken relative to the shock front, the following conservation relations are obtained:

$$\begin{aligned} &N_i V_i = \text{const}, \\ &e\varphi + \tfrac{1}{2} M_i V_i^2 = \text{const}, \\ &p_e + \sum \rho_i V_i^2 = \text{const}, \\ &(p_e + \rho_e \epsilon_e) V_e + \tfrac{1}{2} \sum \rho_i V_i^3 = \text{const}. \end{aligned} \tag{18}$$

For a single ion plasma there are four parameters (N, V, φ, p_e) which can be calculated by the four equations (18), leaving the shock speed undetermined. For two species plasma there are six parameters which can be calculated from Eqs. (18) leaving again the shock speed undetermined.

In the first case we get the well known Hugonio relations[4]

$$\begin{aligned} &V_1/D = 1 - N_0/N_1, \\ &p_1 - p_0 = M N_0 D V_1, \\ &\frac{N_1}{N_0} = \frac{(\gamma+1)p_1 + (\gamma-1)p_0}{(\gamma-1)p_1 + (\gamma+1)p_0}, \\ &e(\varphi_1 - \varphi_0) = \tfrac{1}{2} M D^2 [1 - (N_0/N_1)^2], \end{aligned} \tag{19}$$

where V_1, N_1, p_1, and φ_1 are the particle speed, number density, pressure, and potential behind the shock, and D is the shock speed. Subscript 0 refers to the quantities ahead of the shock. The plasma ahead of the shock was assumed to be at rest.

The shock is characterized by a large electric field within the front accompanied by a discontinuity in the potential and thus the particle kinetic energy and density. For $\gamma = 5/3$ and $p_1/p_0 \gg 1$, $N_1/N_0 \simeq 4$, $V_1/D \simeq 3/4$, and $e\delta\varphi \simeq (5/2)kT_1$.

Impurity ions will be accelerated by the electric field inside the shock front. Their kinetic energy will be at most $e\delta\varphi$. If their mass is low enough they will be trapped by the shock front and move with it. If the impurity ions are heavier than the plasma ions they will be left behind and then swept by the magnetic piston.

The jump relations in the general case are more complicated. The basic phenomena, however, remain unchanged. The shock front accelerates the different species to different speeds. These speeds are determined by the shock speed (or the pressure behind the front). If $M_2 \gg M_1$

$$V_2 \simeq (M_1/M_2)[(1 - V_1)/2D] V_1, \tag{20}$$

where V_1 and V_2 are the speeds behind the front. As $V_1 < D$, $(M_1/M_2)(V_1/2) < V_2 < (M_1/M_2)V_1$. The light ions move faster, thus forming a tail of discontinuity (a separation surface). This discontinuity moves with the speed V_1. Equations (18) can again be used to match the flow functions on both sides of the separation surface. Thus if the pressure ahead of the magnetic piston is known the full problem can be solved. A qualitative picture of such a flow is given in Fig. 1.

In the general case the full flow equations should be solved. This can, however, be done only numerically. Such a calculation is now under way and will be published in a later paper. In this letter we set the basic physical mechanism for the separation and show in simple, but quite realistic cases, that a separation may occur.

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