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Financial Super-Markets: Size Matters for Asset Trade*

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Abstract: This paper presents a new theoretical framework to analyse financial markets in an international context. We build a two-country macroeconomic model in which agents are risk averse, assets are imperfect substitutes, the number of financial assets is endogenous, and cross-border asset trade entails transaction costs. We show that demand effects have important implications for the link between market size, asset prices and financial market development. These effects are consistent with the existing empirical evidence. Due to co-ordination failures, the extent of financial market incompleteness is inefficiently high. We also analyse the impact of domestic transaction costs and issuing costs on financial markets and returns.  
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1) Introduction

This paper presents a new theoretical framework to analyse financial markets in an international context. We study a two-country macro-economic world with four key characteristics: i) agents are risk averse, ii) assets are imperfect substitutes, iii) the number of financial assets is endogenous, iv) cross-border asset trade entails some transaction costs.

This general framework makes sense of several empirical findings. First, a series of papers have found significant demand effects on asset prices\(^1\). This evidence goes against traditional CAPM models but arises naturally in our model because of imperfect substitution between assets. Second, some recent empirical papers have pointed towards significant effects of market size and financial market integration on the cost of capital\(^2\). These papers typically find that companies which are listed on a non-US stock market and then become listed on the New York Stock Exchange experience an increase in their share price. In our model, the interaction between imperfect substitutability and international transaction costs on asset trade implies that larger markets indeed benefit from higher asset prices. Finally, the US is sometimes described as a “super-market” for financial assets. American markets offer a wide range of financial assets and are both very broad and liquid. In our model, large economies indeed support a disproportionately high number of different assets.

In our model, the extent of financial market incompleteness is endogenous. The decision by one agent to develop a new risky investment and to put a new security on the market enhances risk-sharing opportunities for all agents in the world. Because of co-ordination failures, the extent of market incompleteness is inefficiently high and depends negatively on the size of the economy. The model also puts forward the importance of size of

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\(^1\) Shleifer (1986) and Bagwell (1992) find evidence of downward sloping demand and upward sloping supply curves for stocks. Hence, in both studies a demand shift for a stock induces a significant change in price.

economies and transaction costs for gross trade flows in assets. This is consistent with recent empirical evidence on bilateral gross cross-border equity flows (Portes and Rey, 1999). We show that our framework can be extended to discuss intertemporal smoothing considerations and the impact of domestic transaction and issuing costs on asset prices and financial market development.

Our approach is related to the financial and macro-economic literature on incomplete asset markets and risk-sharing as well as to the literature on trade under uncertainty. Allen and Gale (1994) provide an account of the literature on financial innovation and risk sharing. But in their work, the number of risky projects is exogenously given (unlike in our model). They introduce issuing costs but they have no transaction costs, nor do they analyse international asset trade. More closely related to this paper is Pagano (1993). He looks at the decision of flotation of companies on the stock market and introduces trading externalities. But his model is a pure exchange closed economy; we endogenise the investment decisions of entrepreneurs and analyse international capital flows.

Our modelling approach is linked to Acemoglu and Zilibotti (1997), which builds on a market structure (endogenous number of Arrow-Debreu projects) similar to ours. A major difference, besides the fact that their world has no transaction costs, is that we adopt a monopolistically competitive environment - coming naturally from the imperfect substitutability of financial assets - as opposed to their competitive framework. They focus exclusively on capital accumulation and growth. We study the interactions between size, incompleteness of markets, and price of financial assets in open economies.

The literature on trade and uncertainty has been pioneered by Helpman and Razin (1978). They introduced a stock market economy à-la-Diamond into a framework that fits the

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3 See also Magill and Quinzii (1996) who survey the general equilibrium theory of incomplete markets and Obstfeld and Rogoff (1996).
standard Ricardian and Hecksher-Ohlin models of international trade. More recent contributions have extended this line of work, including Svensson (1988), Persson-Svensson (1989). Some of the issues raised by these authors are similar to ours, but the approach adopted is very different: in their papers the number of securities traded is exogenous and the analysis of asset trade is based on autarky prices. One important aspect of our analysis is to go beyond this Ricardian interpretation of trade in assets and apply some of the insights of the “new trade” theory (Krugman, 1979 and Dixit and Norman 1980) to financial flows. We are however interested in questions, which are absent from this literature, such as financial market incompleteness, risk sharing, stock market volatility and intertemporal trade.

The general framework is presented in Section II. Section III derives the equilibrium demands and supplies on asset markets. Sections IV and V analyse the impact of country size on asset prices, financial market development, risk diversification and the current account. The case of perfect competition is briefly discussed in section VI. Welfare implications of the model are derived in section VII. The effect of intertemporal smoothing on asset prices is analysed in section VIII. The impact of domestic transaction costs and issuing costs is presented in section IX. Section X concludes.

II. The general framework

We consider a two-period model with two countries or financial areas, A and B, large and small. They are respectively populated with \( n_A \) and \( n_B \) risk-averse immobile identical agents with \( n_A > n_B \). In the first period all agents in the world are endowed with \( y \) units of a freely traded good (the numéraire), which they can choose to consume or invest in fixed size risky projects. In the second period, there are \( N \) exogenously determined and equally likely states of nature and \( M \) different contingent projects whose pay-offs are the following:
project \(i\) pays \(\begin{cases} d \text{ if state } i \in \{1, \ldots, N\} \text{ occurs} \\ 0 \text{ otherwise} \end{cases}\)

Shares of these projects (claims on the risky dividends) are traded on the stock markets of the two countries. This implies that investing in a specific project (either directly or through the stock market) is equivalent to buying an Arrow-Debreu asset that pays only in one state of nature. This formalisation is close to the one of Acemoglu and Zilibotti (1997). It captures the first main feature of our model: different projects and assets are imperfectly correlated so that assets are imperfect substitutes and variety improves safety.

The fixed size investment projects are costly to develop. An agent \(h_A \in \{1, \ldots, n_A\}\) chooses to develop \(z_{hA}\) different projects (\(z_{hB}\) for an agent \(h_B \in \{1, \ldots, n_B\}\) in the small country). \(M\), the total number of projects (and assets) in the world is \(\sum_{h_A=1}^{n_A} z_{hA} + \sum_{h_B=1}^{n_B} z_{hB}\) since, in equilibrium, agents will have no interest in duplicating a project that has already been developed and all agents of the same country will develop the same number of projects. The set of projects that have been developed in country A and B are \(M_A\) and \(M_B\) respectively. The equilibrium total number of assets in the world \(M = M_A + M_B\) is endogenous. We restrict parameters so that \(M < N\): markets in general will not be complete, meaning that it will not be possible to eliminate all risk by holding a portfolio of all traded assets. In some states of the world, there will be no production. Hence, in general, the matrix of the pay-offs will have the following form:

Matrix of payoffs:

\[
\begin{bmatrix}
d & 0 & 0 & \ldots & 0 & 0 \\
0 & d & 0 & \ldots & 0 & 0 \\
0 & 0 & d & \ldots & 0 & 0 \\
0 & 0 & 0 & d & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 \\
\end{bmatrix}
\]
The cost of each new project is increasing with the number of projects an agent is performing: we assume that the monitoring of each project becomes more complex and costly as the number of projects increases. The total cost in units of the numeraire of the investment in risky projects of an agent \( h_A \) is \( f(z h_A) \), with \( f'(z) > 0 \) and \( f''(z) > 0 \). The investment cost function in country B is similar. There is no restriction on the development of new projects. This will determine the equilibrium number of projects and therefore the equilibrium number of assets. One way to interpret the model is that the risky projects that agents develop are combined to create firms so that each agent creates a firm with possibly a different number of projects.

**Transaction costs**

In the first period, agents raise capital by selling shares of their projects and they buy shares of other projects. The second essential feature of the model is the presence of international transaction costs on asset markets. When agents trade assets, they incur a transaction cost \( \tau \), which is paid in units of the share itself. The same transaction cost also applies on the stochastic dividend and is paid in units of the dividend. For our set up to make sense, we need to assume that these transaction costs cannot be evaded by going through the goods market on which, for convenience, we assume no transaction costs.

The transaction cost is modelled as an iceberg cost: part of the share and part of the dividend “melt” during the transit. The iceberg form greatly simplifies the results because it eliminates the need to introduce financial intermediaries as an additional sector. It also implies that the elasticity of demand for an asset with respect to its price is the same whether the transaction cost is paid or not, that is whether the asset is a domestic or a foreign one.

The presence of international transaction costs on assets captures different types of costs: 1) banking commissions and variable fees; 2) exchange rate transaction costs; 3) some
information costs. Gordon and Bovenberg (1996), also use the same type of proportional transaction costs on capital flows and focus on the asymmetry of information between foreign and domestic investors to justify them.

There are two ways to introduce these transaction costs on the international trade in assets. The first is to make buyers of the assets bear the transaction cost. In this case, the amount paid by an agent \( h_B \) located in country B to buy an asset sold on the stock market in country A by an agent \( h_A \) is: \( p_{h_A} s_{h_B}^{h_A}(1 + \tau) \) where \( p_{h_A} \) is the price of a share of a project developed by agent \( h_A \) and \( s_{h_B}^{h_A} \) is agent \( h_B \) demand for an asset sold by agent \( h_A \). In the rest of the paper, superscripts will identify the seller and subscripts the buyer. If an asset pays a dividend \( d \) in period 2, then a shareholder in country B will receive only \((1-\tau)d\) per share. Profits generated by projects in country A are denominated in currency of country A so that agents in country B have to incur the transaction cost at that stage too\(^4\).

The second possible way to introduce transaction costs is to have project owners bear the transaction cost. These two ways of introducing transaction costs produce the same results as long as we assume that international transaction costs paid by agents buying shares and by project owners selling shares are identical.

\textit{Budget constraint}

We choose to present the model so that buyers pay the transaction costs as they anyway bear the cost. In this configuration, the budget constraint for an agent \( h_A \) in country A is:

\[
c_{lh_A} + \sum_{i \in z_A} M_A p_i s_{h_A}^i + \sum_{j} M_B (1 + \tau) p_j s_{h_A}^j = y + \sum_{k} p_{h_A}^k \alpha_{h_A}^k \tag{1A}
\]

\(^4\)The transaction cost could be eliminated either on the purchase of assets or on the dividends (but not both of course) without changing any of our qualitative results.
where $c_{1hA}$ is consumption of agent $hA$ in period 1. The second term on the left-hand side is the cost of investment in risky projects. The two last terms on the left-hand side represent the demands for domestic and foreign assets. There are $(M_A - zh_A)$ different domestic assets that agent $hA$ will demand as he will only buy assets of projects he has not developed himself. There are $M_B$ different foreign assets on which he will have to incur the transaction cost $\tau$. On the revenue side, in addition to endowment $y$, agent $hA$ will sell a portion $\alpha^k_{hA}$ of each project $k \in zh_A$ that he has developed. The budget constraint of an agent $hB$ in country $B$ is symmetric:

$$c_{1hB} + f(z_{hB}) + \sum_{i \in zhB} p^i s^i_{hB} + \sum_{j} (1 + \tau) p^j s^j_{hB} = y + \sum_{k} p^k_{hB} \alpha^k_{hA}$$  \hspace{1cm} (1B)

Utility

The utility of an agent $hA$ in country $A$ has the following form:

$$U_{hA} = c_{1hA} + \beta E \left( \frac{c^{1-1/\sigma}_{zhA}}{1-1/\sigma} \right)$$  \hspace{1cm} (2)

where $\beta$ is the rate of discount of the future. The utility of agents in country $B$ is similar. $\sigma$ is the inverse of the degree of risk aversion and is also the elasticity of substitution between assets. The linearity of the utility function in the first period enables us to derive simple closed form analytical solutions, while keeping all the insights of the model. We analyse the CES case and the induced wealth effects in detail in section VIII. The state of the world is revealed at the beginning of the second period. Hence, given the description of the payoffs of the different projects, the expected utility of agent $hA$ is:

$$EU_{hA} = c_{1hA} + \beta \frac{1}{N} d^{1-1/\sigma} \left( \sum_{i \in zhA} p^i s^i_{hA} \right) + \beta \frac{1}{N} d(1-\tau) \left( \sum_{j} p^j s^j_{hA} \right) + \beta \frac{1}{N} d^{1-1/\sigma} \left( \sum_{k} (1-\alpha^k_{hA}) \right)$$  \hspace{1cm} (3)
The second element in equation (3) is the expected consumption in states i that are backed by assets of risky projects developed by agents in country A other than those developed by agent \( h_A \) himself. The third element is the expected consumption in states j backed by assets of risky projects developed by agents in country B. The last element is the expected consumption in states which are backed by assets of risky projects developed by the agent \( h_A \) himself. The measure of the extent to which he has decided not to diversify his own risk is therefore:

\[ 1 - \alpha_{h_A}^k \] for each project/asset \( k \in \{1, \ldots, z_{h_A}\} \). The expected utility of an agent in country B is symmetric.

### III) Equilibrium demand and supply on asset markets

Agents maximise utility under their budget constraint. Agent \( z_{h_A} \) in country A chooses consumption in period 1, \( c_{1h_A} \), the number of projects \( z_{h_A} \) he will develop, the demands for the different assets (domestic and foreign) and the portion of each of his projects that he will retain in the second period: \( 1 - \alpha_{h_A}^k \) for each project/asset \( k \in \{1, \ldots, z_{h_A}\} \). When buying shares on the stock market, agents are price takers. The fixed cost that is required to develop a new project insures that no agent will ever find it optimal to replicate an already existing project. The reason is that if he were to do so, the supply of the corresponding asset would necessarily increase so that its equilibrium price would decrease. Because we assume that the choice of projects by all agents is public knowledge, it is more profitable to develop a project that has not been opened yet.

**Market structure**

Each agent has a potential monopoly power on the projects that he has developed and therefore on the sale of the assets that correspond to these projects. This is a departure from
the Arrow-Debreu world where asset markets are assumed to be perfectly competitive. It is easy to check that the perceived elasticity of demand for any asset \(k\) with respect to its price is:

\[
\frac{\partial \alpha_k}{\partial p^k}/(\alpha_k/p^k) = -\sigma, \quad k \in M.
\]

The owner of the asset will exploit this imperfectly competitive structure and will sell only a portion of his project. The fact that firms extensively buy and sell their own stocks to affect the price of their shares suggests that this structure is quite realistic.

We however show in section VI that the monopolistic competition structure on asset market is not essential for most of our results. This structure of the market also implies that \(\sigma\), the price elasticity, is necessarily more than one. Otherwise the model would be degenerate, as asset suppliers would always be better off selling less of the asset at a higher price.

Because all agents in the same country are identical and projects are symmetric, the demands for assets of a given country by agents of the same nationality will be symmetric. Even though agents, in equilibrium, will not be identical because they will hold different amounts of the different assets, they will be symmetric in the sense that their diversification choice will be identical. Also, the price of all projects/assets developed by agents of the same country will be identical for the same reason. Hence, from now on we will in general omit notations that refer to the identity of the agents and of the assets. Agents (and their projects/assets) will only be identified by their nationality A or B. As for the demands for assets, the superscript denotes the origin of the asset and the subscript denotes the nationality of the buyer. Hence, for example, \(s_A^B\) is the demand for an asset of country B by an agent of country A.

The first order conditions are such that we find the different demands for shares as a function of \(\alpha_A\) and \(\alpha_B\) which represent the extent of diversification in each of the two countries.
\[
\begin{align*}
    s_A^b &= (1 - \alpha_A) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma; \quad s_A^b = (1 - \alpha_A) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma \left( \frac{(1 - \tau)^{\sigma - 1}}{(1 + \tau)^\sigma} \left( \frac{p_A}{p_B} \right) \right)^\sigma \\
    s_B^b &= (1 - \alpha_B) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma; \quad s_B^b = (1 - \alpha_B) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma \left( \frac{(1 - \tau)^{\sigma - 1}}{(1 + \tau)^\sigma} \left( \frac{p_B}{p_A} \right) \right)^\sigma \\
    s_A^b &= p_A^{-\sigma} \left( \frac{\beta}{N} d^{1-1/\sigma} \right)^\sigma; \quad s_B^b = p_B^{-\sigma} \left( \frac{\beta}{N} d^{1-1/\sigma} \right)^\sigma 
\end{align*}
\]

Result 1: Agents in both countries do not fully diversify their domestic portfolio: \(1 - \alpha_A > s_A^b\) and \(1 - \alpha_B > s_B^b\).

Because all assets are symmetric, full domestic diversification implies that agents keep no more ownership of their own projects than they buy of projects developed by other agents in the same country. If agents indeed fully diversified in country A, they would then set: \(1 - \alpha_A = s_A^b\). From equation (4), it is easy to check that this not the case (\(\sigma > 1\)) and that agents in both countries keep more shares of their own project than they buy of those developed by agents in the same country. By doing so, each agent exploits the non-competitive structure of the asset market. For the monopolistic competition structure to make sense the parameters must be such that the number of projects each agent develops is small relative to the total number of projects. Otherwise, agents would take into account the effect of their pricing policy on the aggregate outcome. If we interpret firms in our framework to be combinations of various projects, then firms have, in equilibrium, a “nationality”. There is an agent with a specific nationality who chooses optimally to keep a higher share of the project he has himself developed.

The last line of equation (4) just says that the demand for assets are decreasing in the price and increasing in the dividend \(d\). We use the equilibrium conditions for each stock market and for each asset, that implies that the amount of shares offered for a specific asset equals the aggregate domestic demand plus the aggregate foreign demand inclusive of transaction costs:
In this case, we get that the portions of each project sold on the stock markets, are respectively:

\[
\alpha_A = \frac{n_A - 1 + n_B \phi}{n_A - 1 + n_B \phi + \left(\frac{\sigma}{\sigma - 1}\right)^\sigma}; \quad \alpha_B = \frac{n_B - 1 + n_A \phi}{n_B - 1 + n_A \phi + \left(\frac{\sigma}{\sigma - 1}\right)^\sigma}
\]  

(6)

where \( \phi = \left(1 - \frac{\tau}{1 + \tau}\right)^{\sigma-1} \) is a useful transformation of transaction costs and is less than 1. When \( \phi \) increases, transaction costs are lower.

**Result 2:** There is more diversification in the large country than in the small one: \( \alpha_A > \alpha_B \).

Project owners in the large country choose to retain fewer shares of their projects and to sell more on the stock market. In this sense the financial markets are more developed in the large country so there is a market size effect on financial markets.

**IV. Prices, financial market development and risk diversification**

Using the first order conditions and the market equilibrium, we obtain the prices for shares on the different markets:

\[
p_A = \frac{\beta}{N} d^{1-\frac{1}{\sigma}} \left[ n_A - 1 + n_B \phi + \left(\frac{\sigma}{\sigma - 1}\right)^\sigma \right]^{1/\sigma}
\]

\[
p_B = \frac{\beta}{N} d^{1-\frac{1}{\sigma}} \left[ n_B - 1 + n_A \phi + \left(\frac{\sigma}{\sigma - 1}\right)^\sigma \right]^{1/\sigma}
\]

(7)

**Result 3:** The price for shares of projects developed by agents located in the large country is higher than those developed in the small country: \( p_A > p_B \).

Also, the price ratio of the large country assets to the small country assets is increasing in the level of the international transaction costs. If international transaction costs were zero (\( \phi = 1 \)),
then asset prices would be equal in the two countries. Note also that if agents were risk neutral which in our set up means that assets are perfect substitutes ($\sigma \to \infty$), then again the price difference between the two countries vanishes. Result 3 has an immediate implication on the expected returns of an asset which is just: $d/(Np_i)$, $i = A, B$. Hence, the expected return on assets is smaller in the large country than in the small one.

Next, we determine the optimal choice for $z_A$ and $z_B$ the number of projects developed by each agent in country A and B. Because the number of projects must be a natural number, we have to assume that $N$ is large enough so that the equilibrium can be considered as an approximation:

$$f'(z_A) = p_A ; f'(z_B) = p_B$$

(8)

Due to perfect competition on the market for developing projects, the choice for the number of projects, $z_A$ and $z_B$, is such that the price of the asset ($p_A$ and $p_B$ respectively) is equal to the marginal cost of the last project. As long as the cost function $f$ is convex, the large country, which has also the high asset price, will have more projects per agent.

**Result 4:** The number of projects per agent developed in the large country is higher than in the small country: $z_A > z_B$.

We can interpret this result as saying that firms in the large country are made of more projects. It is also possible to endogenize $d$, which can be interpreted as the size of each project in the model. If we assume that, subject to a convex cost function, agents can choose larger projects (projects with larger dividends), then it is easy to show that not only each agent of the large country will develop more projects, but also projects of larger size.

Depending on the exact form of the cost function for developing new projects, the number of assets may be lower than or equal to the number of states of the world, so that markets may be complete or incomplete. We only consider the more realistic and interesting case where financial markets are incomplete: not all states of the world will be covered by an
Arrow-Debreu asset. In some states of the world consumption will be zero in the second period.

To gain intuition on these results, we come back to the first order conditions of the agents in the large country. When choosing how much to sell of their own projects on the domestic stock market, agents set the marginal cost of doing this equal to the marginal gain (the Lagrangian is equal to 1 because of linearity of utility in first period) so that:

$$\frac{B}{N} d^{1-\sigma/\alpha} (1-\alpha)^{-\sigma/\alpha} = p_A \left( \frac{\sigma - 1}{\sigma} \right); \quad \frac{B}{N} d^{1-\sigma/\beta} (1-\beta)^{-\sigma/\alpha} = p_B \left( \frac{\sigma - 1}{\alpha} \right)$$

(9)

These are respectively the optimality condition for the representative agent in A and in B. The expected marginal cost of selling one more share of the project developed by the agent is the expected welfare loss due to consumption thus foregone (left hand side of the equation). Note that because of the concavity of expected utility in consumption, this marginal cost is naturally rising with the portion of the project sold. The marginal gain is less than the price of the asset as an increase in the supply of the asset implies a decrease in its price. At the optimum, the price of a share is equal to its marginal cost multiplied by the mark up $\sigma/(\sigma-1)$. The market size effect comes on the demand side. Note that this market size effect is reminiscent of the home market effect in the new trade and new geography literatures. Using the equilibrium on asset markets, and the demands given in (4), we get:

$$\alpha_A = \left( \frac{\beta d^{1-\sigma/\alpha}}{N} \right)^{\sigma} p_A^{\sigma} (n_A - 1 + n_B \phi); \quad \alpha_B = \left( \frac{\beta d^{1-\sigma/\beta}}{N} \right)^{\sigma} p_B^{\sigma} (n_B - 1 + n_A \phi)$$

(10)

Because there are more agents in country A, and because of the existence of transaction costs ($\phi < 1$), the total demand for an asset of the large country will be larger than the demand for an asset in the small country for a given price. As can also be seen from the equation above, demands in both countries are decreasing in the price. On graph 1, we illustrate the
determination of the prices of assets, \( p_A \) and \( p_B \), and of the extent of diversification, \( \alpha_A \), and \( \alpha_B \) which are also measures of the supply of assets:

**Graph 1: determination of asset price and diversification**

![Graph 1: determination of asset price and diversification](image)

**V. Characteristics of the equilibrium**

**V.1. Capitalisation**

The market size effect also shows up in the market capitalisation of the two countries. In our model, this is the market value of shares traded on the stock market. Calling \( C_A \) and \( C_B \), the market capitalisations per capita as a share of income in the two countries, then:

\[
C_A = \frac{z_A p_A \alpha_A}{y} ; C_B = \frac{z_B p_B \alpha_B}{y} \tag{11}
\]

**Result 5:** Market capitalisation per capita in percentage of income is larger in the large country than in the small one: \( C_A > C_B \).
From that point of view, the financial markets of the large country are more developed than those of the small country. There are more assets traded on these markets both in absolute terms and relative to income ($z_A > z_B$). Financial markets are also more developed because project owners choose to sell more of their shares on the stock market ($\alpha_A > \alpha_B$). Finally the value of these shares is higher ($p_A > p_B$).

V.2. Variance of stock indices

We can also derive the variance of returns on each market. Suppose that one dollar is invested equally in each asset of a country. This is the closest measure of stock market indices in our model. The variance of the return of this stock market index is then:

$$\text{var}(I_d) = \frac{d^2}{N^3 p_d^2 n_d z_d^2} (N - n_d z_d)^2$$

$$\text{var}(I_d) = \frac{d^2}{N^3 p_d^2 n_d z_d^2} (N - n_d z_d)^2$$

(12)

**Result 6:** The variance of the return of the stock index of the large country is smaller.

The intuition is simply that the stock market of the large country offers more diversification and therefore less risk because the number of assets on the large market is higher.

V.3. Home bias

There are several ways to define a “home bias” in the context of our model. Here, we derive the share of domestic assets in the portfolio and compare it to the share of the economy in the world. The value of the non-traded portion of wealth (the part of each project kept by the project owner) is given by the indirect utility function which at the optimum is valued at the market price. Computing this ratio, we get that a home bias exists in country A and in country B if:
\[
\frac{n_A - 1 + \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma}}{n_A} > \frac{n_A}{n_A + n_B}
\]

\[
\frac{n_B - 1 + n_B \phi + \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma}}{n_B} > \frac{n_B}{n_A + n_B}
\]  

(13)

It is easy to check that indeed this is the case in both countries as long as international transaction costs exist \( (\phi < 1) \) or that the asset market is imperfectly competitive \( (\sigma \) is finite).  

V.4. Current account

The current account of the large country is simply aggregate output minus consumption and investment:

\[
CA_A = n_A \left[ y - c_{1A} - f(z_A) \right] =
\]

\[
\frac{\beta}{N} a^{1-\sigma} n_A n_B \phi \left[ z B \left[ n_B - 1 + n_B \phi + \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \right]^{1/\sigma - 1} - z A \left[ n_A - 1 + n_B \phi + \left(\frac{\sigma}{\sigma - 1}\right)^{\sigma} \right]^{1/\sigma - 1} \right]  
\]

(14)

The current account in country B is just the opposite. The sign of the current account of the large country is ambiguous. It is easy to show that it is negative if:

\[
p_A^{1-\sigma} z_A > p_B^{1-\sigma} z_B
\]

which will be the case if parameters are such that:

\[
f'(z) > (\sigma - 1)zf''(z) \quad \text{or} \quad \frac{\partial z}{\partial p} p > \sigma - 1
\]

(16)

Hence, the large country runs a current account deficit if the price elasticity of investment projects is large relative to the elasticity of substitution between assets. The intuition is that in this case, the large country offers many more assets than the small country as the high price of assets in the large country induces agents to invest more in risky projects. This will be the case if the cost function is not too convex. Moreover, when the elasticity of substitution
between assets is low (the relative risk aversion is high) agents of the small country will accept to bear the high price of the large country assets so as to diversify risk. In this case, the total value of assets bought by the small country from the large country will be high and the large country will run a current account deficit, selling more assets than it buys.

The model generates bilateral gross trade flows in assets, which are positively correlated with the size of the economies and negatively correlated with transaction costs. Recent empirical evidence described in Portes and Rey 1999 supports these results strongly: a “gravity” specification explains very well the bilateral distribution of gross cross-border equity trade. In that paper, transaction costs are interpreted as information costs, which are increasing with distance.

VI. Perfect competition on asset markets

The monopolistic structure of the asset markets is a natural consequence of the imperfect substitutability of assets in our model. It is not key to most of our results, however. To see this, suppose that project owners, when selling shares of their projects on the stock market, do not exploit their monopolistic power. In this case, it is easy to show that in all expressions derived above, the term \( (\sigma / \sigma - 1)^{\varphi} \) is replaced by 1. This term measures the mark-up over marginal cost that project owners are able to impose when they exploit their monopolistic power. It therefore goes to one in the perfect competition case. Except for result 1, all the following results are qualitatively unchanged. Result 1 no longer holds because in this case, agents will hold as much of their own project as of the projects of the other domestic agents. Hence, there will be full home diversification although the home bias will remain.

Graph 1 makes clear what happens when project owners are not able to impose any mark-up over their marginal cost. In the two countries, the price of assets will be lower than in the non-competitive setting but it will still be higher in the large country than in the small
country as \( p_A = \frac{\beta}{N} d^{1-1/\sigma} [n_A + n_B \phi]^{\sigma} \) and \( p_B = \frac{\beta}{N} d^{1-1/\sigma} [n_B + n_A \phi]^{\sigma} \) in this case. This also implies that the number of risky projects per agent will be lower in the perfect competition case. Also, the diversification will be larger in both countries but still more important in the large country as \( \alpha_A = \frac{n_A - 1 + n_B \phi}{n_A + n_B \phi} \) and \( \alpha_B = \frac{n_B - 1 + n_A \phi}{n_B + n_A \phi} \) in this case.

VII. Welfare implications

The market equilibrium is not efficient for two reasons. First, a world planner would choose a higher number of projects per person and therefore issue a higher number of assets than in the market equilibrium. This sub-optimally high extent of market incompleteness is due to the existence of a coordination failure. An agent, when developing a new project, does not internalise the benefits that other agents get from the risk-diversification provided. This is because in the decentralized equilibrium, the asset price reflects the marginal utility of an extra share of a given project but not the marginal utility of opening a new market. The other source of inefficiency in the model is the imperfectly competitive structure of the asset market, which leads agents to choose to retain too much ownership of the projects they have developed themselves so that in equilibrium there is too little diversification.

To compare the market and the planner’s equilibrium we choose the symmetric case where both countries are identical (\( n_A = n_B \)) so as to ignore any distribution problem. The planner maximises the utility of a representative agent in A under the following resource constraint: \( y = c_1 A + f(z_A) \). The planner's solution is the following:

\[
S_A^A = \frac{1}{n_A (1 + \phi)} \quad ; \quad S_B^A = \frac{1 - \tau}{1 + \tau} \frac{n_A (1 + \phi)}{n_A (1 + \phi)} \quad ; \quad f'(z_A) = \frac{\beta}{N} d^{1-1/\sigma} \frac{\sigma}{\sigma - 1} [n_A (1 + \phi)]^{\sigma} \] (17)

which we can compare to the market equilibrium in the case of identical countries:
\[ s_A^A = \frac{1}{n_A(1+\phi)-1+\left(\frac{\sigma}{\sigma-1}\right)^\sigma}; \quad s_A^B = \frac{1-\tau}{n_A(1+\phi)-1+\left(\frac{\sigma}{\sigma-1}\right)^\sigma}; \]

\[ f'(z_A) = \frac{\beta}{N} d^{1-1/\sigma} \left[ n_A(1+\phi)-1+\left(\frac{\sigma}{\sigma-1}\right)^\sigma \right]^{1/\sigma} \tag{18} \]

**Result 7:** The extent of diversification is too small in the market equilibrium: \( s_A^A \) and \( s_A^B \) in the market equilibrium are smaller than in the planner's solution. The number of projects per agent is also smaller in the market equilibrium than in the planner's solution as long as the cost function is convex.

At first glance, comparing \( z_A \) in (17) and in (18), it is not obvious that the former is larger than the latter. This is because there are two market failures that have contradictory effects on the choice of \( z_A \) in the market equilibrium. On the one hand the coordination failure already described means that there will be too few projects developed. On the other hand, in the market equilibrium, because the asset market is not perfectly competitive, the price of an asset is above its marginal cost. This induces agents to develop more projects. However, it can be shown that this second effect is always less important than the coordination failure effect so that in equilibrium too few projects are developed, and too few assets traded.

It is also easy to show that to attain the social optimum in the market equilibrium, a single subsidy on the demand for traded assets is sufficient. This subsidy must be financed by a lump sum tax in the first period. It increases demand for assets and therefore diversification and also the price level so that in equilibrium the optimal number of assets is developed. The value of this subsidy is simply \( s = 1/\sigma \), the degree of risk aversion. This makes sense as the more risk-averse agents are, the greater the monopolistic power of asset issuers and the more the welfare cost of the insufficient number of assets and market incompleteness.
VIII. Wealth effects and intertemporal substitution

Until now, we have abstracted from any wealth effect by assuming a linear utility in first period. To analyze such effects, we use the same framework as before except for the utility function which becomes:

\[ U_{h_A} = \left( \frac{c_{1A}^{1-1/\sigma}}{1-1/\sigma} \right) + \beta E \left( \frac{c_{2A}^{1-1/\sigma}}{1-1/\sigma} \right) \tag{19} \]

for an agent \( h_A \) in country A and the symmetric for an agent in country B. In this case, the equilibrium we obtain is:

\[ \alpha_A = \frac{(n_A - 1)c_{1A} + n_B \phi c_{1B}}{(n_A - 1)c_{1A} + n_B \phi c_{1B} + c_{1A} \left( \frac{\sigma}{\sigma - 1} \right)^\sigma} \quad ; \quad \alpha_B = \frac{(n_B - 1)c_{1B} + n_A \phi c_{1A}}{(n_B - 1)c_{1B} + n_A \phi c_{1A} + c_{1B} \left( \frac{\sigma}{\sigma - 1} \right)^\sigma} \tag{20} \]

and

\[
p_A = \frac{\beta}{N} d^{1-1/\sigma} \left( (n_A - 1)c_{1A} + \phi n_B c_{1B} + c_{1A} \left( \frac{\sigma}{\sigma - 1} \right)^\sigma \right)^{1/\sigma} \quad ; \\
p_B = \frac{\beta}{N} d^{1-1/\sigma} \left( (n_B - 1)c_{1B} + \phi n_A c_{1A} + c_{1B} \left( \frac{\sigma}{\sigma - 1} \right)^\sigma \right)^{1/\sigma} \tag{21} \]

Because the marginal utility of consumption is not constant, consumption in first period affects the demand and the supply of assets and therefore their equilibrium price. Note that in addition to the size effects analysed above, an increase in home consumption now increases the price of home assets but decreases the equilibrium portion of projects sold on the stock market. We can use this framework to analyse the impact of an increase in first period endowment \( y \) on the equilibrium. Using the consumer budget constraint and differentiating it, we get that:

\[
dc_{1A} = \frac{\gamma_B}{\gamma_A \gamma_B - \delta_A \delta_B} dy_A + \frac{\delta_A}{\gamma_A \gamma_B - \delta_A \delta_B} dy_B \tag{22} \]
and a symmetric expression for consumption in B. The parameters $\gamma_A$, $\delta_A$, $\gamma_B$ and $\delta_B$ are given in the appendix. It is possible to show that $\gamma_A$ and $\gamma_B$ are positive and that $\gamma_A \gamma_B > \delta_A \delta_B$. This expression allows us to study the international transmission of endowment shocks. In particular, the parameter $\delta_A / (\gamma_A \gamma_B - \delta_A \delta_B)$ measures the financial transmission effect of a change in endowment of country B on consumption in country A. It is easy to check that in the case of financial autarky (infinite transaction costs, i.e. $\phi = 0$), this parameter goes to zero.

The effect of an increase in first period endowment in country A on asset prices in A can then be derived:

$$\frac{dp_A}{dy_A} = \frac{p_A \left[ \gamma_B \left( n_A - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma \right) + \delta_B \phi n_B \right]}{\sigma \left( (n_A - 1)c_{1A} + n_B \phi c_{1B} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A} \right) (\gamma_A \gamma_B - \delta_A \delta_B)}$$

This expression can be proved to be positive. This is also the case for:

$$\frac{dp_B}{dy_A} = \frac{p_B \left[ \gamma_B \phi n_A + \delta_B \left( n_B - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma \right) \right]}{\sigma \left( (n_B - 1)c_{1B} + n_A \phi c_{1A} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1B} \right) (\gamma_A \gamma_B - \delta_A \delta_B)}$$

Result 8: An increase in first period endowment in country A increases the price ($p_A$ and $p_B$) and therefore the number of assets ($z_A$ and $z_B$) in both countries.

When evaluated at the symmetric equilibrium ($y_A = y_B$ and $n_A = n_B$), a positive endowment shock in country A leads to an improvement of its financial terms of trade ($dp_A/dy_A > dp_B/dy_A$).

We have only considered so far the effect of a change in first-period endowment. It is not very difficult, however, to use our framework to analyse the effect of a change in second-period endowment, which we have set to zero for analytical convenience. Due to smoothing
considerations, an increase in second-period endowment (for a given level of dividends) leads consumers to decrease saving. This in turn leads to a decrease in asset prices.

IX. Domestic transaction costs and issuing costs

To simplify the exposition, we now go back to a linear utility for the first period. So far, we have not introduced domestic transaction costs on asset markets in the main analysis. Suppose however that when agents buy domestic assets, and receive the dividend on those assets, they have to bear transaction costs similar in nature to the international transaction costs we have analysed in the previous sections but which are lower than the international ones. We denote transaction costs of that sort \( \tau_A \) and \( \tau_B \) respectively on the asset markets of country A and B. In addition, when firms issue shares, they incur costs even before the transaction stage. These issuing costs could be represented, at least partly, as proportional to the amount of shares issued: we suppose that these issuing costs are \( u_A \) and \( u_B \) per share issued and again incurred in units of the share itself.

The analysis is very similar to the analysis of international transaction costs and therefore we will not repeat all the steps for finding the equilibrium. The first order conditions of agents in the two countries give the demands for assets. The different demands for the assets are given by:

\[
\begin{align*}
    s_A^A &= (1 - \alpha_A) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (1 - \tau_A)^{\sigma-1} (1 - u_A)^\sigma; \\
    s_B^B &= (1 - \alpha_B) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (1 - \tau_B)^{\sigma-1} (1 - u_A)^\sigma; \\
    s_A^B &= (1 - \alpha_A) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma \frac{(1 - \tau_B)^{\sigma-1} p_A}{p_B} (1 - u_A)^\sigma; \\
    s_B^A &= (1 - \alpha_B) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma \frac{(1 - \tau_A)^{\sigma-1} p_B}{p_A} (1 - u_A)^\sigma;
\end{align*}
\]

(25)

The no diversification result (result 1) becomes stronger when domestic transaction costs and issuing costs are taken into account.
The portions of each project sold on the stock market are now:

\[
\alpha_A = \frac{(n_A - 1)\phi_A + n_B\phi}{(n_A - 1)\phi_A + n_B\phi + (1-u_A)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^\sigma}
\]
\[
\alpha_B = \frac{(n_B - 1)\phi_B + n_A\phi}{(n_B - 1)\phi_B + n_A\phi + (1-u_B)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^\sigma}
\]

(26)

where \( \phi_i = \left(1 - \frac{\tau_i}{\tau_i + 1}\right)^{\sigma-1} \); \( i = A, B \) and is less than 1, and decreasing in transaction costs. The prices of assets in the two countries are:

\[
p_A = \frac{\beta}{N} d^{1-1/\sigma} (1-u_A)^{-1/\sigma} \left[ (n_A - 1)\phi_A + n_B\phi + (1-u_A)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^\sigma \right]^{1/\sigma}
\]
\[
p_B = \frac{\beta}{N} d^{1-1/\sigma} (1-u_B)^{-1/\sigma} \left[ (n_B - 1)\phi_B + n_A\phi + (1-u_B)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^\sigma \right]^{1/\sigma}
\]

(27)

The impact of the financial market equilibrium on the choice of the number of risky projects is given by the modified condition on the optimum number of projects per agent:

\[
f'(z_A) = p_A (1-u_A) \quad f'(z_B) = p_B (1-u_B)
\]

(28)

**Result 9:** Markets with high domestic transaction costs and issuing costs will be less developed (\( \alpha \) will be smaller). Asset prices will be lower on markets with high domestic transaction costs and higher on markets with high issuing costs. Both high transaction costs and issuing costs induce agents to develop less risky projects.

Proof: comparative statics on equations (26), (27) and (28).

The intuition can again be understood in reference to a graph with the demand and supply of assets (see graph 2 below). Higher domestic transaction costs reduce the domestic demand for assets which shifts the demand curve downwards. The supply curve is in this case unaffected.
In the case of issuing costs, the marginal cost of issuing a share is increased by \(1/(1-u_A)\) and \(1/(1-u_B)\) respectively which shifts the supply curve to the left. The demand curve, not inclusive of issuing costs is unaffected. On graph 2, we illustrate the impact of transaction costs and issuing costs on asset prices and on the diversification choice, i.e. the supply of assets (here not inclusive of the issuing costs paid in shares).

**Graph 2: The impact of domestic transaction costs and of issuing costs**

![Graph showing the impact of domestic transaction costs and of issuing costs](image)

**X. Conclusion**

The paper has presented a two-country macroeconomic model with an endogenous number of financial assets. This framework can be used to analyse questions. It links the size of economies to the determination of asset returns, the breadth of financial markets and the degree of risk sharing. These issues have been largely overlooked by the traditional macroeconomic and finance literature. They arise very naturally in our model because we have endogenously incomplete asset markets, imperfect substitutability of assets and transaction costs. The model is very simple and conveys clear intuitions. In particular, if world financial
markets are segmented, then large areas will have a higher market capitalisation per capita than small areas, *ceteris paribus*. The model makes sense of several empirical findings such as significant demand and market size effects on asset prices and market breadth, which have been unexplained so far in a unified model.

The theoretical framework developed here can be applied to analyse the impact of regional financial integration on welfare and on the geographical location of financial centers (see Martin and Rey, 2000). It would be also interesting to look at other types of demand effects such as changes in the demographic structure of countries on financial markets. Another possible extension of our framework would be to introduce non-traded goods. So far the theoretical literature has failed to provide a clear link between non-tradability in goods and asset holdings. This model could allow us to make progress on that front. Marrying monopolistic competition on the asset markets and the good markets could also bring interesting results. Finally, our framework is a natural vehicle to analyse in detail the international transmission of shocks through the channel of financial markets. We leave these considerations for future research.
Appendix:

\[ \gamma_A = 1 + \frac{p_A^2}{f''(z_A)} \frac{c_{1A}}{\sigma} \left[ \frac{n_A - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma}{(n_A - 1)c_{1A} + n_B \phi c_{1B} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A}} \right]^2 \]

\[ + \frac{\sigma}{\sigma - 1} p_A n_B c_{1B} z_A \phi \left[ \frac{n_A - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma}{(n_A - 1)c_{1A} + n_B \phi c_{1B} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A}} \right]^2 \]

\[ + \frac{p_B^2}{f''(z_B)} \frac{n_A n_B \phi^2 c_{1A}}{\sigma} \left[ \frac{1}{(n_B - 1)c_{1B} + n_A \phi c_{1A} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A}} \right]^2 \]

\[ + \frac{n_B - 1}{\sigma} n_A c_{1B} \phi \left[ \frac{1}{(n_B - 1)c_{1B} + n_A \phi c_{1A} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A}} \right]^2 \]

\[ \delta_A = - \frac{p_A^2}{f''(z_A)} \frac{c_{1A}}{\sigma} n_B \phi \left[ \frac{n_A - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma}{(n_A - 1)c_{1A} + n_B \phi c_{1B} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A}} \right]^2 \]

\[ + \frac{p_A n_B z_A \phi}{\sigma} \left[ (n_A - 1)c_{1A} + n_B \phi c_{1B} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1A} \right]^2 \]

\[ + \frac{\sigma}{\sigma - 1} p_A n_B c_{1A} z_B \phi \left[ \frac{n_B - 1 + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma}{(n_B - 1)c_{1B} + n_A \phi c_{1A} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1B}} \right]^2 \]

\[ - \frac{p_B^2}{f''(z_B)} \frac{n_B \phi c_{1A}}{\sigma} \left[ (n_B - 1)c_{1B} + n_A \phi c_{1A} + \left( \frac{\sigma}{\sigma - 1} \right)^\sigma c_{1B} \right]^2 \]

and the symmetric expressions for \( \gamma_B \) and \( \delta_B \).
References