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Equalizing Opportunity for Racial and Socioeconomic Groups in the United States Through Educational Finance Reform

by

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Abstract

We analyze the reallocations of educational expenditures required to equalize opportunities (taken to be wage income), according to the theory of Roemer (1998). Using the NLSYM data set, we find that implementing an equal-opportunity policy across men of different races, by using educational finance as the instrument, and ensuring that no race received less than the average observed nationally, would require spending nine times as much on black students, per capita, as on white students. Even the lower bound of bootstrapped confidence intervals for the policy estimates suggests large reallocations between races. The equal-opportunity policy across men from different socio-economic backgrounds that ignores race does almost nothing to equalize wages across races. For inter-racial allocations, we find evidence of a tradeoff between equity and total product, with reallocation lowering the wage bill by about 5%. In contrast, for reallocations based on parental education, equalization increases the wage bill by about 2% because the impact of school spending appears to be slightly higher for those with less highly educated parents.
1. **Introduction**

Education is perhaps the main tool that democracies use to attempt to equalize economic opportunities among citizens. It is commonly thought that opportunity equalization, in that dimension, is implemented by the provision of equal educational resources to all students. We will argue here that that is not so, and we will attempt to compute the distribution of educational spending in public schools in the United States that would equalize opportunities for a measure of economic welfare, namely, earning capacity.

Notably, in the United States lawsuits over the last 35 years have challenged the constitutionality of public education finance systems in most states. Subsequent court orders have typically acted to reduce gaps in spending per pupil between have- and have-not districts, while increasing the power of state governments to control spending. \(^1\) Further, these court cases have tended to shift in focus over time from the simpler view of equal opportunity described above, namely equalizing resources, towards an alternative that instead espouses equalizing outcomes such as test scores and graduation rates. This approach is much closer, but still not identical, to the definition of equal opportunity presented in this paper. This shift away from equal resources to equal outcomes has been embraced by the “school adequacy” movement, which through court cases has argued that all schools should be held to a set of minimum outcome standards. In many cases adequacy proponents have successfully argued that holding all schools to equal absolute standards means that society must spend more on schools that serve less affluent

\(^1\) See Evans, Murray and Schwab (1997) for a review of court-ordered spending equalization in the United States.
students. Hoff (2004) writes that “Plaintiffs’ success in adequacy-based school finance suits began with the 1989 Kentucky Supreme Court decision that declared the state’s school system unconstitutional and ordered the legislature to appropriate enough money ‘to provide each child in Kentucky an adequate education.’ The decision shifted the legal debate away from ‘equitable’ funding, or money spread fairly among districts to ‘adequate’ funding, or whether the state spends enough.”

In one well known adequacy case, the Campaign for Fiscal Equity v. State of New York, the plaintiff sued on the grounds that the status quo did not offer New York City students the “sound, basic education” promised by the state constitution. In late 2004 a court referee panel recommended an increase in spending for New York City schools by $5.6 billion, or 45 percent. (Hoff, 2004)

Over the last thirty years, and throughout the last century, public school systems have also radically increased real spending per pupil. (See e.g. Hanushek and Rivkin, 1997 or Betts, 1996.) Significant bodies of empirical work examine the impact of school spending on adults’ earnings. This literature has yielded mixed results, but most papers indicate that increased school spending is associated with, at best, rather small gains in adult earnings. Relatively little work has used this literature to estimate the magnitude of educational reform required to equalize opportunities across workers from different backgrounds. An analysis requires estimates of the impact of finance reform on earnings for each type of worker, and an analysis of the required reallocation, or increase, in education dollars needed to level the playing field. This paper seeks to provide estimates of the extent to which increasing spending per pupil contributes to creating equality of opportunity.
We intend our work as a positive analysis of what is possible, rather than as a normative analysis of what should be done. Indeed, proponents and opponents of equal opportunity alike should share a desire for a better understanding of what might be achieved through re-targeting educational dollars, and a clearer knowledge of the cost of such programs.

The next section outlines the theory of equal opportunity, and discusses what equality of opportunity has come to mean in the United States over the last thirty years. Section 3 describes the data and presents regression estimates of the impact of school spending. Section 4 summarizes the algorithm used to compute the equal-opportunity policy and the optimal spending per pupil that we derive using this algorithm. That section also examines the implications of a “race-blind” equal-opportunity policy for the black-white wage gap. Section 5 compares the costs and benefits of reallocating educational expenditures. Section 6 concludes with a summary of the most important policy implications that emerge.

2. The theory of equality of opportunity

Our goal is to calculate the reallocation of educational spending needed to equalize opportunities among students for future earning capacity. To do so first requires a short review of a theory of equal opportunity that one of us has recently elaborated (Roemer [1998]), a theory that attempts to formalize the 'level the playing field' metaphor. The troughs of the playing field, in that metaphor, are the disadvantages that individuals suffer, with regard to attaining some goal (here, the capacity to earn income), due to circumstances for which society believes they should not be held accountable --
such as their race, or the socio-economic status of their parents. In contrast to circumstances, an equal-opportunity ethic maintains that differences in the degree to which individuals achieve the goal in question that arise from their differential expenditure of effort are, morally speaking, perfectly all right. It is crucial to understand that by effort we mean not only the extent to which a person exerts himself or herself, but all the other background traits of the individual that might affect his or her success, but which we exclude from the list of circumstances. The partition of causes into circumstances and effort is the central move that distinguishes an equal-opportunity ethic from an equal-outcome ethic. While an equal-outcome ethic implicitly holds the individual responsible for nothing, an equal-opportunity ethic emphasizes that an individual has a claim against society for a low outcome only if he expended sufficiently high effort.  

Five words constitute the vocabulary of the equal-opportunity theory: circumstances, type, effort, objective, and instrument. A type is the set of individuals with the same circumstances. The objective is the condition for which opportunities are to be equalized (the 'opportunity equalisandum'), and the instrument is the policy intervention -- in our case, educational finance—used to effect that equalization. We may state, verbally and somewhat imprecisely, that the equal-opportunity (EOP) policy is the value (or specification) of the instrument which makes it the case that an agent's expected value of the objective is a function only of his effort and not of his circumstances. Thus, educational finance, if it is to equalize opportunities for future earning capacity, should

\footnote{It is possible that the disadvantage that children from less educated parents face is not only social and cultural, but genetic. In either case, the disadvantage has a source beyond...}
make it the case that a young person's expected wage be a function only of his effort and not of his circumstances; that is, it should compensate the individual for disadvantageous circumstances.

We can formulate this in a precise manner as follows. We suppose that a list of circumstances has been specified, as has a unidimensional measure of effort. First, we partition the relevant population into $T$ types. We suppose that the expected value of the objective for individuals in type $t$ is a function $u'(x,e)$, where $x$ is the 'resource' that the individual is allocated by the policy instrument and $e$ is the effort she expends. Suppose, for the moment that all those in type $t$ are allocated an amount $x'$ of the resource -- in our case, educational finance. Then there will ensue a distribution of effort in that type, to be denoted by a probability distribution $F'(\cdot, x')$. ($x'$ is a parameter of the distribution; its support is an interval of effort levels.) These distributions will differ across types, even should different types be assigned the same amount of the resource. Note that the distribution functional $F^t$ is a characteristic of the type, not of any individual. This apparently trivial remark is important.

Equality of opportunity holds that individuals should not be held responsible for their circumstances, that is, for their type. In constructing an inter-type-comparable measure of effort, we must therefore take account of the fact that some individuals come from types that have 'good' distributions of effort, and some from types with 'poor' distributions -- for coming from a type with a poor distribution of effort should not count against a person. We therefore take the inter-type comparable measure of effort to be the quantile of the effort distribution in his type at which an individual sits. We say that all the control of the individual, and hence should be rectified at the bar of equal
individuals at the $\pi^{th}$ quantile of their effort distributions, across types, have tried equally hard\(^3\).

To restate this important point, it would be wrong to pass judgments on the quality of effort expended by individuals in different types by looking at their pure expenditure of effort, for those raw effort levels are polluted, as far as our theory is concerned, by being drawn from distributions for which we do not wish to hold the individuals responsible. The distribution of effort of a type is a characteristic of the type, not of any individual, and as such, it is a circumstance as far as the individual is concerned. To the extent that an individual’s effort is low in absolute terms because he belongs to a type with a low mean effort, the individual should not be held responsible. We therefore say that the best measure of an individual’s effort is his effort relative to effort of others in his type, which is captured by his rank or quantile on the effort distribution of his type. Using the quantile measure of effort sterilizes out the 'good' or 'bad' nature of the distribution of effort in the type. We thus treat two individuals in different types, who sit at the same quantile of the effort distributions of their types, as having tried equally hard.

Our task is therefore: to find that value of the policy which makes it the case that, at each quantile, the expected value of the objective across types, is 'equal.' Since equality will virtually never be possible, we really mean 'maximin' where we just wrote 'equal.' Unfortunately, even this instruction is incoherent, for it amounts to maximizing

\[^3\text{We admit this is arbitrary, yet it would be worse to attempt to make no correction for the fact that absolute levels of effort are not the right measures to compare, across types, in deciding how hard individuals have tried. Discovering the ‘right’ way to compare}\]
many objectives simultaneously, and so some second-best approach must be taken. In our analysis, we make the compromise as follows.

Let \( v'(\pi, x') \) be the (average) value of the objective for individuals in type \( t \), at quantile \( \pi \) of the effort distribution in type \( t \), if the type is allocated \( x' \) in resource by the policy instrument. (In the application we will study, \( v'(\pi, x') \) is the logarithm of the wage at the \( \pi \)th quantile of the wage distribution of individuals of type \( i \) if \( x' \) was invested in their education. We define the typology later.) If we fix a particular value of \( \pi \) in the interval \( [0,1] \), there will be a policy \( x(\pi) = (x_1', x_2', ..., x_T') \) that solves the following program:

\[
\begin{align*}
& \text{Maximize} & \quad & \min_{i} v'(\pi, x_i') \\
& \text{subject to} & & (x_1', x_2', ..., x_T') \in X
\end{align*}
\]

where \( X \) is the feasible set of policies. \( x(\pi) \) is the policy that maximizes the minimum value of the objective for all agents of all types at effort quantile \( \pi \). If \( x(\pi) \) were the same policy for all \( \pi \), that would be, unambiguously, the equal-opportunity policy. But that will almost never be the case in actual applications, and so our compromise will be to average these policies: that is, we declare the equal-opportunity policy to be:

\[
x^{EOp} = \frac{1}{0} \int_{X} \text{ArgMax} \min_{i} v'(\pi, x_i') d\pi. \tag{2.1}
\]

If \( X \) is a convex set, then \( x^{EOp} \) is feasible.

For example, suppose we look at ten deciles of wages in each type. We would compute, for each decile, the investment policy that maximized the minimum wage in effort across types is a problem intrinsically as complex as comparing the subjective welfares of very different individuals.
that decile, across the various types. This would, in general, give us ten different investment policies. We declare the EOp policy to be the average of these ten policies.

Thus, given a specification of the circumstances, the effort measure, the objective, and the instrument, and given the data necessary to calculate the functions \( v' \), we can solve for the equal-opportunity policy. Note that the equalization of opportunities according to this formulation is always relative to a given resource constraint, specified by the feasible set \( X \).

In what follows, we apply this theory -- which the reader can find elaborated at more length, and philosophically justified, in Roemer (1998) -- to educational policy in the United States.

*Equality of Opportunity in Practice*

To what extent does the theory of equal opportunity outlined above correspond to the intent behind current legislation and practices related to affirmative action in the United States? As argued in Roemer (1998), one conception of what equal opportunity requires is the principle of non-discrimination. In labor markets, this approach says that employers should judge job applicants solely on their productivity, rather than upon characteristics such as race or nationality. The non-discrimination requirement lies at the heart of the Civil Rights Act of 1964.

But a second definition of equal opportunity, and the one that we use in this paper, argues that non-discrimination is insufficient for equalizing opportunities. One must compensate for historical inequities to the extent that they adversely affect the
circumstances of living individuals. This view has come to dominate the practical
application of affirmative action in the United States in recent years.

Donohue (1994) argues persuasively that in recent years employment law has
evolved from a ‘non-discrimination’ view, one that advocates ‘intrinsic equality’, toward
an approach resembling our conception of equal opportunity, one he refers to as
‘constructed equality’. For the two decades following the second world war, Donohue
says that law -- particularly the Civil Rights Act of 1964 -- sought to establish intrinsic
equality in the workplace, where wages of different workers are judged to be intrinsically
equal if they are those that would be forthcoming in a perfectly competitive labor market.
Thus, to the extent that low wages paid to black workers are the consequence of market
imperfections, which allow discriminatory employer attitudes to survive, then law
requiring that wages be ‘intrinsically equal’ will provide a remedy. Of course, a set of
perfectly competitive markets is meritocratic -- employees, in particular, will be paid
their marginal value products, and those productivities reflect circumstances as well as
effort. Thus, intrinsic equality does not implement equality of opportunity in our sense.

But Donohue writes that in recent years, law has sought to attain a higher goal
that he calls ‘constructed equality.’ Under constructed equality, ‘the dictates of law are
defined no longer through some abstract market paradigm but rather through considering
what steps would be necessary to define a fair society (Donohue [1994, p.2611]).’ The
Americans with Disabilities Act (ADA), passed in 1991, is his primary example. The
ADA does not require employers to pay disabled workers their (competitive) market
values, but rather to provide them with ‘reasonable accommodations’ that enable these
workers to become more productive. As Donohue summarizes, ‘Thus, the
transformation that has occurred in the realm of civil rights is that the ideal nondiscriminatory market solution, which previously was both the benchmark of intrinsic equality and what the law demanded, is now regarded as the obstacle to social justice (p. 2609).’ In our language, the ADA requires employers to supply extra resources to disabled workers on account of their disadvantageous circumstances, so that their productivity is more truly reflective of their effort. Donohue conjectures that economically disadvantaged classes may proceed, on the example of the ADA, to seek remedies from employers to compensate for their objectively lower productivity, due to economic and social circumstances⁴. If this indeed occurs, it will mark a transformation of employment law to an opportunity-equalizing device.

To summarize, the Americans with Disabilities Act specifically adopts the view that society must compensate for circumstances beyond a person’s control, as in the theory of equal opportunity outlined in this paper and in Roemer (1998).

Other examples of equal opportunity in the real world are provided by current educational practice. In 1975 the Education for All Handicapped Children Act put into place requirements for schools to provide additional services to handicapped children. This provides a clear example of equal-opportunity legislation, since it attempts to level the playing field by spending more than the average on students with learning or physical disabilities.⁵

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⁴ ‘The ADA has paved the way for the possibility that economically disadvantaged minorities such as blacks…will employ the ADA’s rationale to argue that the effects of the factors that have undermined their productivity -- including very poor schooling and broken families -- are now to be corrected by employers ( Donohue p.2612).’

⁵ For a description of this legislation, and its impact on overall educational spending between 1980 and 1990, see section IV of Hanushek and Rivkin (1997).
The way in which American universities admit applicants provides a final example of how equal opportunity, rather than non-discrimination, has come into common use in the United States. Under a non-discriminatory admissions policy, a university would select students based on grades or test scores. But instead of using purely meritocratic procedures, admissions committees supplement students’ grades and test scores with information on personal and family background. Typically, universities have set lower admissions standards for minorities in the belief that this could help to correct the racial imbalance still observed in many skilled occupations. This practice provides a clear example of how society in recent decades has pursued equal-opportunity policies with a view to compensating for disadvantageous circumstances.

Of course, in the last few years court decisions and voter initiatives have led public universities in Texas and California to end their policy of using race as a marker of disadvantage when making admission decisions. In both states, universities are now actively considering alternative forms of affirmative action in admissions, that, for instance, take into account whether either parent of a student has attended university. As will be shown below, a switch from a race-based equal-opportunity program to one that conditions on socioeconomic traits such as parental education leads to radically different recommendations. We consider this to be one of the important findings of the ensuing analysis.
3. **Data and regression results for spending per pupil**

*Data*

We choose as objective the logarithm of an individual’s weekly wages as a young adult. We model log weekly earnings from the National Longitudinal Survey of Young Men (NLSYM), computed as the log of the product of hours per week and hourly wages, and adjusted to 1990 prices using the Consumer Price Index. Spending per pupil in the student’s district, gathered from a 1968 survey of high schools, is also included in the analysis as the policy instrument. Betts (1996) finds that existing estimates of the impact of spending per pupil on wages based on the NLSYM fall roughly in the middle of published empirical estimates. 6 Furthermore, the confidence intervals of the black-white estimates provided below encompass most of the results in the published literature. The regression sample for each race consists of all wage observations between 1966 and 1981 for workers who were 18 or older and who were not enrolled in school or college in the given year. We drop a wage observation if weekly earnings are below $50 or above $5000 in 1990 prices. 7

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7 One issue in the past literature has been whether there is measurement error in district reports of spending per pupil. This data-set does not contain repeat measures of spending per pupil, but other papers point to, at best, a modest effect of measurement error. Betts (1995), in a model of log wages as a function of school-level resources using the National Longitudinal Survey of Youth 1979, instruments school-level measures of resources with state-level averages and does not find an increase in the level of significance, even though the state-level measures by themselves are significant if placed in the log wage equation. One interpretation is that the state-level aggregates are measuring something orthogonal to resources at the high school level. Grogger (1996) performs a similar analysis with High School and Beyond, modeling log wages as a function of spending per pupil. Uniquely, his paper also has two measures of spending per pupil at the district
Outline of the Empirical Estimates on Spending per Pupil

We will examine the reallocation of spending per pupil that would be necessary to equalize opportunities for (weekly) earnings. Such reallocations have been at the heart of court-mandated school reform over the last quarter century. We at first focus on reallocations of spending per pupil across types of student, given a fixed educational budget. However, since such reallocations are virtually guaranteed to reduce spending per pupil for certain types, we also calculate EOp solutions where the constraint is not a fixed budget but a requirement that no type receive less than a pre-specified amount per pupil. Since no students become worse off in an absolute sense, this second approach is perhaps more politically realistic, but is potentially quite costly.

Recall that we partition each person’s traits into two sets, those against which we wish to indemnify the person (circumstances), and those for which we hold the person accountable (effort). The former traits are used to partition people into types; the latter traits are treated as the person’s choice variables. If we define many types, for instance by distinguishing people not only by race but also by marital status, geographic location and so forth, our EOp policy in general will call for a more differentiated allocation of expenditures.

level, for two different years. When he instruments one measure with the other measure, coefficients do rise, suggesting some measurement error in the data. However, his preferred estimates suggest an elasticity of wages with respect to spending per pupil that is quite close to our own estimates. For instance in our black-white typology we find an elasticity of 0.116 and 0.119 for blacks and whites respectively. Grogger, using OLS, obtains an elasticity of 0.068, but when he instruments one measure of district spending per pupil with the other district measure, his average elasticity rises to 0.097, which is still slightly below our estimate.
With this in mind, we begin with a relatively conservative approach, in which we define only two types -- black and white -- thus holding each person in our sample accountable for all other traits, such as family background, and geographic location (both region of the country and rural/urban/suburban residence). The use of two types also allows for an intuitive discussion of the optimal policy. We then consider outcomes using parental education as an additional or alternative factor in determining type.

The theory outlined earlier emphasizes that the impact of school spending on earnings for a given type of worker may vary with the person’s ranking in the earnings distribution, conditional upon school spending. Quantile regression provides a technique that almost perfectly fits with this theory. We estimate models of log weekly wages that condition on spending per pupil in the district in which the worker attended school.

We estimate a series of quantile regressions for a given type of worker:

$$\log w_i^t = \alpha_{tq} + \beta_{tq} x_i^t + Z_i^t \theta_{tq} + \epsilon_i^t, \quad q=0.1,0.2,\ldots,0.9$$

(3.1)

where t indexes the worker’s type, i indexes the observation, q is the discrete quantile that corresponds with the continuous variable $\pi$ in the theory developed earlier, $w_i^t$ is weekly wages, $x_i^t$ is spending per pupil for observation i and worker type t, $Z_i^t$ is a row vector of other regressors, $\epsilon_i^t$ is an error term and the remaining Greek symbols indicate coefficients. Here

$$Quan_q(\log w_i^t \mid x_i^t, Z_i^t) = \alpha_{tq} + \beta_{tq} x_i^t + Z_i^t \theta_{tq}$$

(3.2)

is the conditional quantile for the given quantile q. We estimate this model nine times for each type of worker for quantiles q=0.1, 0.2,…, 0.9. What quantile regression allows us
to do is to estimate the impact of spending per pupil on workers at different points in
the conditional wage distribution. By conditional wage distribution we mean the ranking
of workers in terms of the outcome variable, after conditioning, or taking account of, the
individual worker’s values for spending per pupil and the other regressors in $Z_i^t$.

The coefficient estimates are calculated by minimizing the following objective
function for the $q$-th quantile for type $t$:

$$\sum_i \log w_i^t - \alpha_{i}^q - \beta_{i}^q x_i^t - Z_i^t \theta_{i}^q \lambda_i$$

where $\lambda_i$ are weights defined by

$$\lambda_i = \begin{cases} 
2q, & \text{if } \log w_i^t - \alpha_{i}^q - \beta_{i}^q x_i^t - Z_i^t \theta_{i}^q > 0 \\
2(1-q), & \text{otherwise} 
\end{cases}$$

These weights are central to understanding how quantile regression works. A key
feature of quantile regression is that by construction a proportion $1-q$ of the observations
will have positive residuals with the remaining observations having negative residuals.
Because of this, the weights will give proportionately more weight to workers whose
values of log earnings, conditional upon the regressors, are “close” to the quantile in
question. Two examples illustrate. For $q=0.5$, $\lambda_i=1$ for all observations. That is, all
workers are treated equally in this regression. In this case quantile regression reduces to
the well known Least Absolute Deviations (LAD) estimator. It is very similar to
Ordinary Least Squares (OLS). The difference is that OLS yields the values of the
coefficients for the mean observation in the population whereas the LAD estimator yields
the value of the coefficients for the median observation.

Now consider $q=0.2$. In this case the weights will be 0.4 for the 80% of workers
whose residual log earnings from the model will be positive. For the 20% of workers
whose residual log earnings will be negative, the weight is 1.6. In essence, this places greater weight on workers whose earnings are in the bottom 20% of the conditional earnings distribution. In this way, quantile regression yields regression coefficients specific to different points in the conditional wage distribution.\footnote{For more details on quantile regression estimation see for instance Koenker and Bassett (1982).}

We condition not only on spending $x_1'$ but also on a vector of other regressors $Z_i'$. These other variables, while exogenous to the individual worker, might influence his earnings. Without taking account of family background, for instance, our estimates of the impact of school spending on earnings could suffer from omitted variable bias. Accordingly, we include in our vector $Z_i'$ the worker’s age and its square, dummies for whether the person’s mother and father were present in the home when the person was 14, and the number of siblings. In addition, in the black/white typology we also condition on the level of education of the more highly educated parent. We do not condition on the worker’s own level of education because this is a choice variable, and the impact of spending per pupil may work partly through its influence on students’ subsequent years of education completed.

This method has two distinct advantages. It is entirely consistent with the theory outlined earlier in that $\pi$ is defined conditional upon $x_1'$. Second, the pattern of coefficients obtained from the nine quantile regressions performed for each type of worker $t$ allows for non-linearities in the relation between wages and spending per pupil $x_1'$ and between wages and the quantile.
These quantiles conform closely to the quantiles of “effort”, that is, the person’s percentile ranking by log wages, conditional upon type and spending per pupil. Thus, roughly speaking, the coefficient estimates for \( q=0.9 \) describe the determinants of wages for people ranked at the 90th percentile of log wages after conditioning upon the regressors, or, in terms of the theory, for people ranked at the 90th percentile of effort. Recall that “effort” is just short-hand for what we more accurately called the aspect of autonomous volition in a person’s behavior. In reality, effort is a multi-dimensional variable, which includes not only years of schooling but marital status, region, and other personal choices. Not only will various personal choices be captured in effort, so measured, but so will be luck. An individual who earns a high wage simply by virtue of inheriting his father’s good job will be classified as one who expended high effort. It is important to bear in mind the conservative nature\(^9\) of this assumption when considering the estimates presented below of the extent to which school resources would be reallocated to maximize the EOp objective.

\[ \text{Regression Results} \]

We next present estimates based on three different partitions of the sample of workers into types. First, we partition workers into blacks and whites. Second, we examine a race-blind typology that assumes that workers should be compensated not for their race but rather the level of education of their parents. By not taking account of race,

\(^9\) Conservative in that sense that Robert Nozick (1974) says that a person is morally entitled to benefit by virtue of luck -- the luck, for instance, of being born into a wealthy family.
such a typology runs against the nature of recent affirmative action programs. But ballot and court decisions in California and Texas have led to prohibitions on the use of race as an identifying variable in affirmative action programs such as those related to college admissions. It therefore is salient to study the implications of a race-blind equal-opportunity policy. Finally, we examine a hybrid typology that divides black and white workers separately into two approximately equally sized groups, based on the years of schooling of the more highly educated parent. This typology yields four types in total -- it is an appropriate partition if society takes into account that more than race influences a young person’s chances in life. For young black men, we partitioned the sample approximately in half by including men whose more highly educated parent had fewer than ten years of schooling in one type and those whose parental education was 10 or more years in the other type. The closest we could come to partitioning the white sample in half was to use “fewer than 12 years of education for the more highly educated parent” as the criterion for the less advantaged type.

Tables 1 to 3 show the full regression results for the median regression $q=0.5$ for each of the typologies. The standard errors are based on results from 1500 bootstrap samples. The empirical results generally conform to past results using this and similar datasets. Family socioeconomic status, especially number of siblings and parental education are strongly related to log wages of workers later in life. Earnings rise with age but at a decreasing rate. Spending per pupil appears to be positively and significantly related to earnings, as past research with the NLS-YM has suggested. (See Betts, 1996, for a review.) In the final typology, that divides workers based on both race and parental
education, the estimated effect of school spending is estimated less precisely than for the other typologies.

While the estimated effect of school spending varies among types at q=0.5, there is no definitive relationship between the coefficient on school spending and the degree of a person’s advantage. Although the type with the highest degree of advantage in each typology appears to benefit the least from additional spending per pupil, this relationship is not particularly strong in a statistical sense.

Space constraints prevent us from displaying all nine models for q=0.1, 0.2,...,0.9 for each type, but the results are available from the authors on request.

The next step involves using these regression estimates to compute the EOp policy. We need to boil down the individual predicted wages from these models to a simple summary consisting of the pair \((a^q, b^q)\) that predicts average log wages for type t conditional upon quantile q and spending per pupil \(x_t\):

\[
v_t^q(q, x_t) = a^q + b^q x_t
\]

where \(v_t^q(q, x_t)\) is the log of weekly earnings predicted for workers of type t at quantile q who received spending per pupil of \(x_t\). Our estimate of \(b^q\) is simply \(\beta^q\) from (3.1). To obtain our estimates of the part of predicted weekly log earnings that does not depend on school spending, \(a^q\), we must first identify those workers in type t who belong to a given quantile q. Therefore after each quantile regression we rank observations i in type t by the residuals and assign observation i in type t a ranking \(\rho_i^q\) such that \(\rho_i^q \in [0,1]\), and \(\rho_i^q = 1\) indicates the wage observation with the largest residual in the quantile regression for that type. We selected observations i in type t with \(\rho_i^q\) within ±0.05 of a given q, and
calculated the mean predicted log wage of those workers assuming that $x^t=0$ and that all workers are aged 30, that is

$$a^{tq} = \hat{\alpha}^{tq} + (Z^t_i | \text{age} = 30) \hat{\theta}^q$$

(3.6)

where circumflexes indicate estimated coefficients. Note that we remove variations in predicted wages related to age because it is unlikely that policymakers would aim to remove all age-related variations in earnings among types. However, we leave in our estimate of $a^{tq}$ variations in predicted log earnings resulting from other background variables such as the number of siblings. In sum, these intercept estimates are estimates of predicted earnings of workers who are close to the given quantile, after setting the workers’ age to 30, and spending per pupil to zero. They are essentially the “constant” part of predicted earnings for workers at a given quantile.

The EOp policy will not remove variations in predicted earnings within types, but the policy will attempt to compensate for variations across types at given quantiles.

4. Calculation of the spending allocations that implement equal opportunity

a) Main results

We solve a discrete version of program (2.1), where the effort quantile, $\pi$, takes on nine values, which we denote $q = 1, \ldots, 9$. For each quantile $q$ and type $t$, we have an estimated relationship, as described in section 3:
\( v'(q, x') = a^{q} + b^{q} x' \) \hspace{1cm} (4.1)

where \( v \) is logarithm of the future wage and \( x' \) is the amount invested in the education of the student. The set \( X \) is defined by the budget constraint:
\[ \sum_{t} p^{t} x^{t} = R \] \hspace{1cm} (4.2)

where \( p^{t} \) is the fraction of individuals of type \( t \), and \( R \) is spending per student. Thus for each \( q \) we solve:

\[ x(q) = \text{ArgMax}_{x} \text{Min}_{t} (a^{q} + b^{q} x^{t}) \]
\[ \text{subject to } \sum_{t} p^{t} x^{t} = R, \] \hspace{1cm} (4.3)

and we then define the equal-opportunity policy as:

\[ x^{EOp} = \frac{1}{9} \sum_{q=1}^{9} x(q). \] \hspace{1cm} (4.4)

Program (4.3) is solved by solving a series of linear programs. Typically, at the solution of (4.3), the most disadvantaged type will be the worst-off at the solution, and so the solution of (4.3) is the solution of the following linear program, where type one is the most disadvantaged type:

\[ \text{Max}_{x} (a^{q} + b^{q} x^{1}) \]
\[ \text{subject to } a^{q} + b^{q} x^{t} \geq a^{q} + b^{q} x^{1}, \quad t = 2, 3, 4 \]
\[ \text{and } \sum_{t} p^{t} x^{t} = R. \] \hspace{1cm} (4.5)

To solve (4.3), we solve four linear programs, where, in turn, each of the four types is assumed to be the worst-off type at the solution, and we then take the solution to be the one of these four which maximizes the value of (4.3).

We report on various aspects of the EOp policies. In order to generate confidence intervals for these policies, we bootstrapped the EOp policy using a bootstrap sample of
One remark is in order. For a small proportion of the bootstrap estimates, the coefficient $b_{tq} < 0$. The solution to (4.5) in these cases would entail $x_{tq} = 0$. Instead of taking this to be the solution, we set $x_{tq} = R$ for all $(t, q)$ for which $b_{tq} < 0$.

Beginning with the simple black/white typology in Table 1, we first calculated the optimal allocation of educational funding under the assumption that average spending per pupil ($R$) is $2500 in 1990 prices, which is approximately the average in the NLSYM sample.10

Egalitarian policies are criticized for being ‘inefficient’, that is, for decreasing output. It is possible, but not certain, that when one reallocates educational expenditures between different types, the overall wage bill will shrink, if the marginal product of educational resources is higher for the type from which funding is being removed. Therefore we also calculate the ratio of the wage bill that is predicted to result from the EOp policy to the wage bill under the equal resource policy, in which all students receive the same amount of the financial resource. Our calculations based on the black/white typology in Table 1 assume that 12.0% of the population is black and that 88.0% is white, which matches the population frequencies in 1966 in the NLSYM.11

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10 Taking all observations in 1966, the weighted mean spending per pupil, in 1990 prices, was $2233. Spending per pupil has grown steadily since then. Current expenditures per pupil in American public schools during the 1990-91 school year were $4847. (National Center for Education Statistics, 1991, p. 155).

11 The bottom portion of Tables 1, 2 and 3 show estimates of the distribution of the population of men in 1966 by type, and mean spending per pupil by type. Both of these were calculated using sampling weights from 1966, on all available 1966 observations. The bottom of these tables also show weekly earnings by type averaged over all wage observations in all years, using sample weights. The frequencies of worker types in the three tables do not exactly add up due to a slightly smaller sample once observations missing covariates such as parental background are removed.
We also calculate the required aggregate budget which assures that, under the EOp policy, all types would receive at least (approximately) $2,500 per capita. This exercise assumes that such a 'no-lose' option might be politically necessary in order to implement an EOp policy in reality.

For each of our 1500 bootstrap estimates under the ‘no-lose’ scenario, we calculated the value of $R$ at which the most advantaged type would receive an investment in the interval ($2450, 2550$). These results are reported in the bottom three lines of Tables 4, 5 and 6.

We report the results for three partitions of the sample into typologies: (i) a two-type typology, black (B) and white (W), (ii) a four type typology, where the circumstance is the educational level of the more highly educated parent, and (iii) a four – type typology obtained by crossing \{B, W\} with \{L, H\}, where L and H stand for low or high parental education. In other words, in typology (i), we level the playing field with respect to the race of individuals only; in typology (ii), we level the playing field with respect only to the educational background of the family the individual came from, and in typology (iii) we level the playing field with respect to both a person’s race and the educational background of his family.

(i) Type partition: Black and white

The results are reported in Table 4. At the EOp solution, in our point estimate, blacks receive approximately 18 times what whites receive when $R = 2.5$. The .025 and .975 values of this ratio from the bootstrap samples are 7.76 and 79.17. We can thus assert, conservatively, that equalizing opportunities for this typology, and at this budget,
requires an investment in black students of *at least seven or eight times* the investment in whites.

If $R$ is increased to the point where whites receive approximately $2,500 per capita, then this ratio falls, so that blacks receive approximately nine times as much per capita, and the confidence interval on this ratio from the bootstrap samples is (5.39, 21.49).

The columns labeled $w^B$ and $w^W$ show estimated average weekly earnings of black and white workers under the two scenarios in thousands of dollars, and corresponding confidence intervals. In comparison to the average earnings of blacks shown in Table 1, the predicted wages after EOp policy is implemented in Table 4 are much higher for blacks. The average wages of the two types are not equalized exactly. The lack of perfect equalization follows directly from the stipulation that all students of a given race receive the same $x^t$. (Policymakers under an EOp policy would aim to equalize outcomes on average across types while not attempting to remove variations in the outcome within types that are attributed to variations in “effort”.)

The second and third columns from the right-hand side of Table 4 report the average weekly wage at the equal-resource (ER) and EOp solutions, respectively (in thousands of dollars), and the last column is the ratio of these two numbers, our measure of ‘efficiency.’ We see there would be a substantial decrease in the average wage if we implemented the EOp policy for this typology, in comparison to implementing the equal-resource policy. Under both the fixed-budget and the ‘no-lose’ EOp policies, the total wage bill drops by roughly 5%.
The reallocation of school resources needed to equalize opportunity between black and white men is substantial. Note, though, that our wage sample is drawn from the years 1966-1981. To check whether it is possible that today smaller reallocations would be required, we examined data on usual weekly earnings of full-time male workers by race, as reported for the year 1996 in the Current Population Survey. Strikingly, the size of the wage gap between black and white men is almost identical in 1996 to the average value observed in the NLSYM data. The ratio of blacks’ earnings to those of whites in 1996 was 71.0%, compared to 72.2% in our sample of wages over the period 1966-1981. In absolute terms, the black-white wage gap in the NLSYM data was $149 per week in 1990 prices (Table 1). In 1996, the same gap was $140. Some readers may be surprised that the ratio and absolute gap in wages between black and white male workers changed so little between 1966-81 and 1996, although a number of researchers including Bound and Freeman (1992) have documented the slowing of the convergence in wages between blacks and whites during the 1980’s.

The implication for our analysis is simple. Although our wage observations are centered in the 1970’s, the black-white wage gap has changed so little over the last two decades that our results would be virtually unchanged if we used recent wage distributions.

(ii) Type partition based on parental education

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12 Data for 1996 earnings by race and data for the Consumer Price Index required to deflate to 1990 prices were taken from U.S. Bureau of the Census. (1997, pages 431, 497).
Table 5 reports the results for the partition of the sample into four socio-economic types, based upon the educational attainment of the more highly educated parent.

The inequality in educational investment necessary to effect equal opportunity is strikingly less, in this typology, than in the Black-White typology. The ratio of spending for the groups with the highest and lowest spending are 4.9 and 2.9 for the fixed-budget and ‘no-lose’ scenarios, less than a third of the spending disparities required in the black/white typology. We note, as well, that the size of the average wage at the EOp policy is consistently larger than in the equal-resource policy. Thus, both equity and efficiency are improved, here, at the equal-opportunity policy. This reflects the generally larger wage responses to increased spending among the two lower parental education types relative to the two more advantaged types.

(iii) Type partition: Low-Black (LB), High-Black (HB), Low-White (LW), High-White (HW)

We make three observations from Table 6. First, in the point estimate, more is invested in the HB type than in the LB type. Indeed, the HB estimate does not quite lie within the 95% confidence interval generated by the bootstrap samples. This perhaps reflects the fact that the coefficient on $x$ is estimated less precisely for the HB type than for any other types estimated in this paper. Second, we note that three types consistently receive more than their per capita share at the EOp estimates (all but HW), whereas, at the higher values of $R$ in the last three lines of the table, both white types receive less than their per capita share. Finally, we note that the wage of the HB type, for four rows
of the table, is larger than the wage of the $HW$ type. This is an instance of the ecological fallacy, and occurs because of our averaging technique for arriving at the EOp investment. Although, for each quantile, it may be that the wage of the $HW$ type is greater than the wage of the $HB$ type, to arrive at the EOp investment, we averaged the maximin investments over quantiles, and this can explain the effect on $HB$ wage: higher effort quantiles of the $HB$ type benefit more than higher effort quantiles of the $HW$ types from this averaging of investments across quantiles.

b) Do Race-Blind EOp Policies Do Much to Reduce Black-White Inequality?

An interesting inference follows from the results reported here for the on-going debate on affirmative action. As we wrote earlier, the emerging view in the United States seems to be that affirmative action, at least with regard to university admissions, is desirable when it is used to favor students of low socio-economic status, but not when it is used to favor students of color\textsuperscript{13}. In our language, this view holds that the type partition into types based on socio-economic circumstances is ethically acceptable, but not so for the types that predicate on race. The natural question is, to what extent will opportunities be equalized in our society by recognizing differential socio-economic, but not differential racial, circumstances?

Our results suggest a pessimistic conclusion. Far less would be invested in black students under the EOp policy associated with the socio-economic typology of Table 5 than under the EOp policies which predicate upon race. It is, however, important to note

\textsuperscript{13} Indeed, Ward Connerly, who spear-headed the initiative on the University of California Board of Regents to abolish race-based affirmative action admissions holds this view. He
that the large investments in blacks, of Tables 4 and 6, contrasted with the relatively small investment in the most disadvantaged socio-economic type of Table 5, are due not only to the extra disadvantage of blacks, but also to the fact that blacks are a small share of the population (low \( p \) values), and so it is relatively cheap to subsidize them in the EOp policy. In other words, it would be wrong to infer that blacks are three times as disadvantaged as the most disadvantaged socio-economic type (\( E1 \)) because approximately three times as much is invested in the former compared to the latter at the EOp policies in their respective typologies.

To study more formally the impact on blacks if EOp policies condition on socioeconomic status rather than education, we calculate the percentage of black men in the regression samples in each of the earnings quintiles before and after the various EOp policies are put into place. We adjust each worker's earnings as follows. For a given typology, we assign each worker a level of spending \( x^t \) dependent on his type in that typology. To calculate the earnings that would result for that worker, we multiply the change in spending that he would receive by the coefficient on spending from the black/white typology, and the worker's actual quantile. We then find the quantile \( q \) that for worker \( i \) solves

\[
q^t_i = \text{ArgMin}_{q \in \{0.1, 0.2, ..., 0.9\}} |q - p^t_{iy}| \quad \text{where } t = B, W \tag{4.6}
\]

for each wage observation \( i \) in the black and white types. This is simply the quantile that most closely matches the individual wage observation. In addition, to put workers of said, “UC should use economic status and other genuine hardships when making special admissions, not race”. (Sacramento Bee, May 20, 1995, p. B1)
different ages on an equal footing, we adjusted the weekly wages of each worker to the predicted value had he been age 30.

Table 7 shows the results. The top row shows that in the raw data, blacks predominantly occupy the bottom two earnings quintiles. For the sake of comparison, we also show the result when all students receive spending of 2.5. The results are quite similar to the raw data. This finding suggests that the court struggles that have been waged over the last three decades to equalize spending across schools, even when successful, will have done little to equalize earnings between blacks and whites.

We then estimate the wages each worker would earn if various reallocations were put into effect. The fixed-budget EOp policy (B/W, r=2.5) greatly improves the earnings of blacks relative to whites, so that the median black now occupies the middle earnings quintile, and the percentage of blacks in the top earnings quintile triples. The alternative B/W EOp policy, with r=4.85, pushes blacks away from the middle three quintiles and toward the top and bottom quintiles, where they are now over-represented. Again, however, the median black belongs to the middle earnings quintile, suggesting a dramatic interracial equalization compared to the raw data or even the school spending equalization shown in the first two rows.

A quite remarkable result is shown in the next two rows: when type is defined independently of race, and only parental education is used, the EOp reallocations leave the distribution of black workers across earnings quintiles little changed from the status quo in row 1. Even though 42% of blacks in the regression sample are in the type with low parental education, and so receive spending of 5.36, this is a small reallocation relative to the more advantaged type, which receives 1.10. This limited reallocation,
combined with the fact that 19% of whites also fall into the bottom socioeconomic group, implies that the gap in mean earnings between blacks and whites is little changed after the EOp policy is implemented.

Further, we note that by choosing a four-way typology based on parental education, we have guaranteed a more radical reallocation of resources than would occur in a simpler typology. In earlier work, we found that, when we partitioned workers into two types, based on whether the more highly educated parent had or lacked a high school diploma, implementing an EOp policy created virtually no change in the distribution of blacks across the earnings distribution. One might conjecture that, in the 4-way typology that we use in this paper, being in the bottom socioeconomic category – that is, having parents with eight or fewer years of schooling – would almost characterize impoverished blacks. But this is not the case: fully 70% of workers in this disadvantaged category are white.

Our analysis focuses on primary and secondary school, and so cannot speak directly to the postsecondary issue. But it seems clear that using proxies for race, such as parental education, will at best lead to equality-of-opportunity policies that are far less compensatory than what would be needed to equalize opportunity across races. Our calculations suggest that an equality-of-opportunity policy based on two or even four levels of parental education does very little to improve the income share of black men.

Overall, our results suggest that reallocation of spending per pupil can significantly alter the distribution of earnings. However, the marginal effect of spending per pupil on adult wages is very small. Consequently, if society were to take equality of opportunity seriously, radical reallocations of educational expenditures would be
required. These reallocations go far beyond merely equalizing spending across student types. This fact is noteworthy, since court-mandated reforms in school finance over the last 30 years have typically ordered at most equalization of spending across schools.


We now compare the costs and the benefits of various EOp policies. We work with the typology \{B,W\}. We measure benefits as the value of the EOp objective function, that is, the mean of the lower envelope of the earnings:q functions for blacks and whites. (The lower envelope is the function whose value is the value of the objective, at each quantile, of the worst-off type.) To be precise, we define the weekly benefits from a policy \(\varphi\) as

\[
\left(\frac{1}{9} \sum_{q=0.1}^{0.9} \text{Min}_{t} \exp(\nu'(q, \varphi))\right),
\]

where \(\exp(\nu'(q, \varphi))\) is the average of the wage (the exponential of the dependent variable, the logarithm of the weekly wage) of individuals at the qth quantile of the effort distribution of type \(t\) when the policy is \(\varphi\).

Table 8 shows the value of the EOp objective function for various scenarios. The table presents this mean in dollar terms to aid understanding. The “base case” scenario is one in which mean x is $2500 (r=2.5).\(^{14}\) The value of the mean along the lower envelope, which in the base case consists of blacks at every quantile is $464.58 per week.

\(^{14}\) We use $2500 to provide comparability with the simulations based on the EOp solutions presented in the previous section. Since the actual mean spending per pupil was slightly below $2500 in the sample, we increased spending per pupil proportionately across workers, and calculated the predicted gain in earnings using the quantile regression results.
The second row ("equal resources") shows the gains that would result if all schools spent exactly $2500 per pupil. As shown, the average gain in earnings for workers on the lower envelope is $1.10 per week, or about 0.25%. The next two rows show the mean of wages on the lower envelope for the two EOp solutions, first where average spending is held constant at $2500 per week, and then the cost-increasing intervention in which both types receive at least $2500 per week. The gains in average earnings along the lower envelope are very large in both cases, between $46 and $66 per week, with increases in the average wage well over 10% above the base case.

We now ask a related question: what are the relative sizes of the costs of implementing the various programs? Starting from a base of $2500 per pupil, equalizing spending at that level or implementing the EOp plan with mean spending r=2.5 have no impact on costs. Of course, even equalization of spending across schools, let alone the radical reallocation suggested by EOp with r=2.5, may not be politically feasible, since some types (whites, in the present analysis) face lower spending per pupil after the reallocation.

Consider next the cost of the EOp program with minimum spending of $2500 per person of either type. To evaluate its cost per pupil, we assume that any change in spending occurs from kindergarten through the year in which the pupil leaves school, which is appropriate since our measure of spending per pupil is measured for the school district in which the student attended school. Using the empirical distribution of years of schooling, we then calculate the cumulative change in spending per pupil from kindergarten up to the year in which the student left school (or Grade 12 in the case of those with more than 12 years of schooling). We convert all expenditures to their value
in the year in which the student would have been in Grade 12, using a discount rate of 2.67%, which is the mean real interest rate over the period 1953 to 1997.\footnote{This real interest rate was calculated as the yield on ten-year federal bonds minus the percentage change in the Consumer Price Index (for all urban consumers). The period 1953 to 1997 represents the widest time span possible with the available data. Sources are the Economic Report of the President (Council of Economic Advisers, 1998) and the Bureau of Labor Statistics respectively.}

The EOp plan increases the mean earnings along the lower envelope by $65.79 per week. But the costs of achieving EOp in this way are extremely large: in terms of present value of spending in the year in which the person turns 18, the cost is over $34500 per person. This figure is obtained by dividing total program cost by the number of people in the entire population. All of this additional spending is directed toward blacks, who on average would receive an extra $293,000 while in school. This is spread out over the entire population, bringing the cost down to roughly $34500 per person.

Note that in Table 8 it is inappropriate to compare the costs and benefits directly since the costs are the present value of accumulated spending for all students in all grade levels, while our measure of benefit focuses on workers who are on the lower envelope only, and represents the gains during a typical week, rather than over the entire working lifetime. Clearly, though, both the benefits and the costs are sizeable. The predicted earnings gain works out to about $3400 per year for each black worker assuming 52 weeks of work or paid vacation annually. If we think of this as an investment project, the upfront cost of $293,000 per black would yield an annual payback of about 1.2%.

There are two reasons why increasing the rate of return on increasing school expenditure through the EOp algorithm is relatively small. The first reason is that spending per pupil has a very modest impact on students’ subsequent earnings. The
second reason for the relatively low cost effectiveness of increasing school spending is that under the “no-lose” EOp plan average spending rises dramatically. Furthermore, the value of the EOp objective at its optimum, viewed as a function of \( r \) (the per capita resource endowment), is a concave increasing function, and the ratio of this ‘value function’ (our ‘benefit’) to \( r \) is a convex, decreasing function. Therefore, the benefit-cost ratio of an EOp program that increases dramatically the resources spent on education will be small.  

6. Concluding Comments

We conclude by briefly reiterating some of the more important policy implications of our analysis.

First, even though court battles on educational finance have typically centered on the goal of equalizing spending across schools, our analysis suggests that this alone will do little to equalize opportunity, especially across races. The reason is that the impact of school spending on students’ subsequent wages is rather modest compared to the racial gap in earnings. We estimate that full equalization of spending per pupil would increase the weekly earnings of workers along the lower envelope by only $1.10 or about 0.2%.

\[16\] In an early draft of this paper we also calculate the impact and cost of increasing the school-leaving age by one year, based on OLS regressions of log earnings on years of schooling. This is only a very rough estimate of what increasing the school-leaving age might do in practice. Nonetheless, the results are illuminating. Along the lower earnings envelope in our black/white typology, average weekly earnings are predicted to increase by $2.38, at a cost in present value terms of $142.25. Both the predicted impact and the costs are extremely small compared to the impact and cost of the ‘no-lose’ EOp scenario. When compared to the EOp program that increases total spending per pupil, increasing the school-leaving age is predicted to have a proportionately bigger impact on the objective function than on cost. However, this is to be expected given our earlier argument that the ratio of the benefit of EOp to spending increase \( r \) is a decreasing function, which reduces the effectiveness of large increases in expenditure per pupil.
Second, in order to equalize opportunity across races at a given budget, government would have to reallocate spending radically. Our results vary depending on whether overall spending is held constant, or spending is increased such that no type experiences a decrease in school funding. In the first case, equalizing opportunity between races entails spending eighteen times as much on blacks as on whites. In the second case, nine times as much must be spent on blacks. These estimates are of course subject to uncertainty. However, we have directly controlled for statistical uncertainty by bootstrapping our estimates of optimal policy. We note that even the lower bound of our 95% confidence interval yield black/white spending ratios of eight and five, which similarly suggest that simple equalization of spending across schools can accomplish little. One possibility is that our estimates are too high because, in extrapolating so far, we are missing increasing returns to school expenditures. While this may be true, we note that work by Betts and Johnson (1997) does not find strong evidence of increasing returns to school resources, and in fact finds weak evidence of the opposite. Our central point remains: mere equalization in spending achieves little.

Third, we compared the upfront costs of the EOp reforms with the annual payback as measured by the increase in weekly earnings along our objective function. Under the EOp policy that holds spending constant, the cost is by definition zero but the benefits to workers along the lower envelope are substantial -- an increase in weekly earnings of 10%. The political drawbacks of this zero-cost reallocation are obvious, as it is financed by reducing spending for whites. Thus, such a reform is likely to be much less politically feasible than a more expensive one that guarantees that no student sees a reduction in school spending. Our second EOp policy sets a floor on spending per pupil
for both races, and is predicted to achieve more, increasing weekly earnings of workers along the lower envelope by just over 14%. But the cost is large: about $293,000 per black student, or about $34,500 per student when the cost is distributed across all students.

Fourth, it matters enormously whether a program to equalize opportunity takes race into consideration. This insight is important given recent moves in California and Texas to eliminate race as a factor that is considered in university admissions. We found that a “color-blind” EOp program that equalizes opportunities between types of student differentiated only by parental education does almost nothing to change the distribution of blacks across earnings quintiles. In the language of our model, given such a race-neutral policy, any variations in earnings that are correlated with race would be attributed to variations in “effort” rather than “circumstance”. Thus, a color-blind EOp program based on socioeconomic traits other than race costs relatively little, but achieves relatively little as well. This has important implications for the affirmative action debate: affirmative action programs that do not take race into account explicitly are likely to do little to reduce variations in outcomes between races -- unless, that is, they succeed in proxying for race by use of other, correlated characteristics.

Both opponents and proponents of affirmative action should have an interest in learning about the costs of implementing equal opportunity through educational finance reform. This paper has offered a positive analysis of the benefits and costs of such policies. But it is important to discuss the practical implementation of the educational financial reforms analyzed here. Implementing such reforms, which allocate more money to disadvantaged types than to advantaged ones, is a remote possibility in a
society that has not yet fully implemented the more moderate 'equal resource' policy. It is important to separate the positive analysis from a discussion of what reforms are politically feasible, or even desirable. (One might believe, for example, that the cost in average income associated with equalizing opportunities subject to the dual type Black-White typology is too great.) Knowing what theory and the data imply, the public will be better prepared to reform educational finance subject to political reality and to their own values.

Finally, our findings suggest that money alone will not suffice to equalize educational opportunity. This realization suggests the urgent need for finding complementary means of improving educational and life outcomes for the disadvantaged.
Table 1

Estimates of Impact of Spending per Pupil on Log Weekly Earnings by Race for q=0.5

<table>
<thead>
<tr>
<th>Race:</th>
<th>Black</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spend. per Pupil (’000s)</td>
<td>0.0553</td>
<td>0.0530</td>
</tr>
<tr>
<td></td>
<td>(0.0163)**</td>
<td>(0.0054)**</td>
</tr>
<tr>
<td>Number of siblings</td>
<td>-0.0169</td>
<td>-0.0090</td>
</tr>
<tr>
<td></td>
<td>(0.0038)**</td>
<td>(0.0018)**</td>
</tr>
<tr>
<td>Father present age 14</td>
<td>0.0828</td>
<td>0.0088</td>
</tr>
<tr>
<td></td>
<td>(0.0219)**</td>
<td>(0.0097)</td>
</tr>
<tr>
<td>Mother present age 14</td>
<td>0.0212</td>
<td>0.0954</td>
</tr>
<tr>
<td></td>
<td>(0.0429)</td>
<td>(0.0302)**</td>
</tr>
<tr>
<td>Age</td>
<td>0.1049</td>
<td>0.1612</td>
</tr>
<tr>
<td></td>
<td>(0.0165)**</td>
<td>(0.0067)**</td>
</tr>
<tr>
<td>Age squared</td>
<td>-0.0015</td>
<td>-0.0022</td>
</tr>
<tr>
<td></td>
<td>(0.0003)**</td>
<td>(0.0001)**</td>
</tr>
<tr>
<td>Parental ed. &lt; 9 years</td>
<td>-0.1658</td>
<td>-0.0993</td>
</tr>
<tr>
<td></td>
<td>(0.0293)**</td>
<td>(0.0100)**</td>
</tr>
<tr>
<td>Parental ed. 9-11 years</td>
<td>-0.0524</td>
<td>-0.0378</td>
</tr>
<tr>
<td></td>
<td>(0.0307)</td>
<td>(0.0110)**</td>
</tr>
<tr>
<td>Parental ed. &gt; 12 years</td>
<td>0.0949</td>
<td>0.0206</td>
</tr>
<tr>
<td></td>
<td>(0.0515)</td>
<td>(0.0079)**</td>
</tr>
<tr>
<td>Constant</td>
<td>4.2540</td>
<td>3.4055</td>
</tr>
<tr>
<td></td>
<td>(0.2276)**</td>
<td>(0.1014)**</td>
</tr>
<tr>
<td>Observations</td>
<td>2737</td>
<td>14475</td>
</tr>
</tbody>
</table>

Estimated share of population, 1966
- 12.0% 88.0%

Estimated mean earnings of workers in this type, 1966-81
- 385.34 533.96

Mean spending per pupil, (’000s)
- 2.091 2.243

Standard errors, based on 1500 bootstraps, in parentheses. The final three lines of the table are based on weighted averages of the entire NLSYM sample. Log weekly earnings and spending per pupil are adjusted to 1990 prices.
* significant at 5% level; ** significant at 1% level
### Table 2
Estimates of Impact of Spending per Pupil on Log Weekly Earnings by Parental Education for q=0.5

<table>
<thead>
<tr>
<th>Typology:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental Ed. &lt; 9</td>
<td>0.0659</td>
<td>0.1006</td>
<td>0.0473</td>
<td>0.0396</td>
</tr>
<tr>
<td>Parental Ed. 9-11</td>
<td>(0.0111)**</td>
<td>(0.0135)**</td>
<td>(0.0075)**</td>
<td>(0.0131)**</td>
</tr>
<tr>
<td>Parental Ed. = 12</td>
<td>-0.0236</td>
<td>-0.0162</td>
<td>-0.0127</td>
<td>-0.0096</td>
</tr>
<tr>
<td>Parental Ed. &gt; 12</td>
<td>(0.0030)**</td>
<td>(0.0035)**</td>
<td>(0.0028)**</td>
<td>(0.0049)**</td>
</tr>
<tr>
<td>Number of siblings</td>
<td>-0.0236</td>
<td>-0.0162</td>
<td>-0.0127</td>
<td>-0.0096</td>
</tr>
<tr>
<td>Father present age 14</td>
<td>0.0019</td>
<td>0.1176</td>
<td>0.0318</td>
<td>0.1237</td>
</tr>
<tr>
<td>Mother present age 14</td>
<td>0.0740</td>
<td>-0.0671</td>
<td>0.2158</td>
<td>-0.0790</td>
</tr>
<tr>
<td>Age</td>
<td>0.1268</td>
<td>0.1619</td>
<td>0.1551</td>
<td>0.1936</td>
</tr>
<tr>
<td>Age squared</td>
<td>-0.0017</td>
<td>-0.0023</td>
<td>-0.0021</td>
<td>-0.0026</td>
</tr>
<tr>
<td>Constant</td>
<td>3.8388</td>
<td>3.3602</td>
<td>3.3672</td>
<td>2.9844</td>
</tr>
<tr>
<td>Observations</td>
<td>3907</td>
<td>3201</td>
<td>6860</td>
<td>3244</td>
</tr>
<tr>
<td>Estimated share of population, 1966</td>
<td>21.1</td>
<td>17.0</td>
<td>39.0</td>
<td>23.0</td>
</tr>
<tr>
<td>Estimated mean earnings of workers in this type, 1966-81</td>
<td>493.54</td>
<td>523.45</td>
<td>586.40</td>
<td>631.72</td>
</tr>
<tr>
<td>Mean spending per pupil, ('000s)</td>
<td>2.138</td>
<td>2.202</td>
<td>2.179</td>
<td>2.261</td>
</tr>
</tbody>
</table>

Standard errors, based on 1500 bootstraps, in parentheses. The final three lines of the table are based on weighted averages of the entire NLSYM sample. Log weekly earnings and spending per pupil are adjusted to 1990 prices.

* significant at 5% level; ** significant at 1% level
Table 3  
Estimates of Impact of Spending per Pupil on Log Weekly Earnings by Parental Education and Race for q=0.5

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Black, Ed. &lt; 10</td>
<td>Black, Ed. &gt;= 10</td>
<td>White, Ed. &lt; 12</td>
<td>White, Ed. &gt;= 12</td>
</tr>
<tr>
<td>Spend. per Pupil (‘000s)</td>
<td>0.0521</td>
<td>0.0648</td>
<td>0.0666</td>
<td>0.0463</td>
</tr>
<tr>
<td></td>
<td>(0.0203)*</td>
<td>(0.0289)*</td>
<td>(0.0080)**</td>
<td>(0.0061)**</td>
</tr>
<tr>
<td>Number of siblings</td>
<td>-0.0273</td>
<td>-0.0108</td>
<td>-0.0104</td>
<td>-0.0064</td>
</tr>
<tr>
<td></td>
<td>(0.0056)**</td>
<td>(0.0048)*</td>
<td>(0.0024)**</td>
<td>(0.0025)*</td>
</tr>
<tr>
<td>Father present age 14</td>
<td>0.1147</td>
<td>0.0593</td>
<td>-0.0389</td>
<td>0.0472</td>
</tr>
<tr>
<td></td>
<td>(0.0335)**</td>
<td>(0.0322)</td>
<td>(0.0163)*</td>
<td>(0.0135)**</td>
</tr>
<tr>
<td>Mother present age 14</td>
<td>-0.1401</td>
<td>0.1335</td>
<td>-0.0076</td>
<td>0.1485</td>
</tr>
<tr>
<td></td>
<td>(0.0653)*</td>
<td>(0.0498)**</td>
<td>(0.0414)</td>
<td>(0.0353)**</td>
</tr>
<tr>
<td>Age</td>
<td>0.1216</td>
<td>0.1106</td>
<td>0.1511</td>
<td>0.1681</td>
</tr>
<tr>
<td></td>
<td>(0.0256)**</td>
<td>(0.0241)**</td>
<td>(0.0098)**</td>
<td>(0.0083)**</td>
</tr>
<tr>
<td>Age squared</td>
<td>-0.0018</td>
<td>-0.0015</td>
<td>-0.0021</td>
<td>-0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.0005)**</td>
<td>(0.0004)**</td>
<td>(0.0002)**</td>
<td>(0.0001)**</td>
</tr>
<tr>
<td>Constant</td>
<td>4.1104</td>
<td>3.9717</td>
<td>3.6140</td>
<td>3.2100</td>
</tr>
<tr>
<td></td>
<td>(0.3387)**</td>
<td>(0.3408)**</td>
<td>(0.1420)**</td>
<td>(0.1178)**</td>
</tr>
<tr>
<td>Observations</td>
<td>1467</td>
<td>1270</td>
<td>5078</td>
<td>9397</td>
</tr>
<tr>
<td>Estimated share of population, 1966</td>
<td>5.6%</td>
<td>5.0%</td>
<td>30.4%</td>
<td>59.0%</td>
</tr>
<tr>
<td>Estimated mean earnings of workers in this type, 1966-81</td>
<td>369.94</td>
<td>409.55</td>
<td>508.36</td>
<td>546.35</td>
</tr>
<tr>
<td>Mean spending per pupil, (‘000s)</td>
<td>2.017</td>
<td>2.148</td>
<td>2.248</td>
<td>2.242</td>
</tr>
</tbody>
</table>

Standard errors, based on 1500 bootstraps, in parentheses. The final three lines of the table are based on weighted averages of the entire NLSYM sample. Log weekly earnings and spending per pupil are adjusted to 1990 prices.

*significant at 5% level; ** significant at 1% level
Table 4  
Point and bootstrap estimates, educational spending and wages, BW typology, R=$2,500 and xmin=$2500 with no limit on R

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>$x^B$</th>
<th>$x^W$</th>
<th>$x^B/x^W$</th>
<th>$w^B$</th>
<th>$w^W$</th>
<th>$w^{ER}$</th>
<th>$w^{EOp}$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>point est.</td>
<td>2.50</td>
<td>14.76</td>
<td>0.828</td>
<td>17.82</td>
<td>0.584</td>
<td>0.604</td>
<td>0.631</td>
<td>0.602</td>
<td>.953</td>
</tr>
<tr>
<td>.025 est</td>
<td>2.50</td>
<td>10.71</td>
<td>0.241</td>
<td>7.76</td>
<td>0.462</td>
<td>0.586</td>
<td>0.625</td>
<td>0.571</td>
<td>.905</td>
</tr>
<tr>
<td>.975 est</td>
<td>2.50</td>
<td>19.07</td>
<td>1.381</td>
<td>79.17</td>
<td>0.688</td>
<td>0.622</td>
<td>0.636</td>
<td>0.628</td>
<td>.944</td>
</tr>
<tr>
<td>xmin=2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>point est.</td>
<td>4.85</td>
<td>22.18</td>
<td>2.49</td>
<td>8.92</td>
<td>0.709</td>
<td>0.653</td>
<td>0.701</td>
<td>0.660</td>
<td>.942</td>
</tr>
<tr>
<td>.025 est</td>
<td>3.85</td>
<td>13.58</td>
<td>2.45</td>
<td>5.39</td>
<td>0.642</td>
<td>0.647</td>
<td>0.668</td>
<td>0.651</td>
<td>.807</td>
</tr>
<tr>
<td>.975 est</td>
<td>8.55</td>
<td>53.12</td>
<td>2.55</td>
<td>21.49</td>
<td>1.039</td>
<td>0.659</td>
<td>0.832</td>
<td>0.699</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note: All dollar amounts are expressed in thousands of 1990 dollars.
Table 5
Point estimates and bootstrap estimates, educational spending and wages, four-type parental education typology, \( R = 2,500 \) and \( x_{\text{min}} = 2,500 \) with no limit on \( R \).

<table>
<thead>
<tr>
<th>( R = ) 2.5</th>
<th>( x_{E1} )</th>
<th>( x_{E2} )</th>
<th>( x_{E3} )</th>
<th>( x_{E4} )</th>
<th>( w_{E1} )</th>
<th>( w_{E2} )</th>
<th>( w_{E3} )</th>
<th>( w_{E4} )</th>
<th>( \nu^E )</th>
<th>( \nu^{EOp} )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>point est.</td>
<td>2.50</td>
<td>5.36</td>
<td>3.62</td>
<td>1.88</td>
<td>1.10</td>
<td>0.656</td>
<td>0.653</td>
<td>0.638</td>
<td>0.659</td>
<td>0.633</td>
<td>0.649</td>
</tr>
<tr>
<td>.025 est</td>
<td>2.50</td>
<td>4.47</td>
<td>2.87</td>
<td>1.34</td>
<td>0.22</td>
<td>0.605</td>
<td>0.616</td>
<td>0.620</td>
<td>0.641</td>
<td>0.627</td>
<td>0.635</td>
</tr>
<tr>
<td>.975 est</td>
<td>2.50</td>
<td>6.28</td>
<td>4.20</td>
<td>2.21</td>
<td>1.14</td>
<td>0.670</td>
<td>0.674</td>
<td>0.647</td>
<td>0.692</td>
<td>0.638</td>
<td>0.655</td>
</tr>
</tbody>
</table>

\( x_{\text{min}} = 2.5 \)

<table>
<thead>
<tr>
<th>( R = ) 2.5</th>
<th>( x_{E1} )</th>
<th>( x_{E2} )</th>
<th>( x_{E3} )</th>
<th>( x_{E4} )</th>
<th>( w_{E1} )</th>
<th>( w_{E2} )</th>
<th>( w_{E3} )</th>
<th>( w_{E4} )</th>
<th>( \nu^E )</th>
<th>( \nu^{EOp} )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>point est.</td>
<td>4.33</td>
<td>7.31</td>
<td>4.75</td>
<td>3.61</td>
<td>2.51</td>
<td>0.749</td>
<td>0.714</td>
<td>0.698</td>
<td>0.694</td>
<td>0.695</td>
<td>0.710</td>
</tr>
<tr>
<td>.025 est</td>
<td>3.58</td>
<td>5.44</td>
<td>3.69</td>
<td>2.60</td>
<td>2.45</td>
<td>0.657</td>
<td>0.663</td>
<td>0.662</td>
<td>0.682</td>
<td>0.665</td>
<td>0.675</td>
</tr>
<tr>
<td>.975 est</td>
<td>4.93</td>
<td>9.69</td>
<td>6.34</td>
<td>4.53</td>
<td>2.55</td>
<td>0.821</td>
<td>0.790</td>
<td>0.716</td>
<td>0.706</td>
<td>0.714</td>
<td>0.730</td>
</tr>
</tbody>
</table>

Note: All dollar amounts are expressed in thousands of 1990 dollars.

\( E_1 = \) parental education less than or equal to eight years
\( E_2 = 8 < \) parental education < 12
\( E_3 = \) parental education = 12
\( E_4 = \) parental education > 12
Table 6  
Point estimates and bootstrap estimates, educational spending and wages, BWHL typology, $R=2,500$ and $\text{x}_{\text{min}}=2500$ with no limit on $R$.

<table>
<thead>
<tr>
<th>R</th>
<th>$x_{LB}$</th>
<th>$x_{HB}$</th>
<th>$x_{LW}$</th>
<th>$x_{HW}$</th>
<th>$w_{LB}$</th>
<th>$w_{HB}$</th>
<th>$w_{LW}$</th>
<th>$w_{HW}$</th>
<th>$w_{ER}$</th>
<th>$w_{Op}$</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>point est.</td>
<td>2.50</td>
<td>8.84</td>
<td>16.26</td>
<td>2.61</td>
<td>0.679</td>
<td>0.469</td>
<td>0.749</td>
<td>0.615</td>
<td>0.632</td>
<td>0.637</td>
<td>0.624</td>
</tr>
<tr>
<td>.025 est.</td>
<td>2.50</td>
<td>6.89</td>
<td>4.75</td>
<td>2.14</td>
<td>0.475</td>
<td>0.380</td>
<td>0.470</td>
<td>0.596</td>
<td>0.623</td>
<td>0.631</td>
<td>0.605</td>
</tr>
<tr>
<td>.975 est.</td>
<td>2.50</td>
<td>18.88</td>
<td>16.17</td>
<td>3.49</td>
<td>1.302</td>
<td>0.618</td>
<td>0.847</td>
<td>0.647</td>
<td>0.650</td>
<td>0.641</td>
<td>0.641</td>
</tr>
</tbody>
</table>

| $\text{x}_{\text{min}}=2.5$ |          |          |          |          |          |          |          |          |          |          |    |
| point est. | 4.48     | 11.10    | 23.86    | 3.92     | 2.50     | 0.489    | 0.992    | 0.673    | 0.681    | 0.699    | 0.683 | 0.977 |
| .025 est.  | 3.58     | 8.16     | 6.12     | 3.39     | 2.45     | 0.371    | 0.461    | 0.650    | 0.674    | 0.667    | 0.653 | 0.932 |
| .975 est.  | 5.08     | 29.93    | 26.06    | 4.83     | 2.55     | 0.759    | 1.216    | 0.687    | 0.688    | 0.719    | 0.699 | 1.008 |

Note: All dollar amounts are expressed in thousands of 1990 dollars.  
LB: Black and parental education 10th grade or less  
HB: Black and parental education more than 10th grade  
LW: White and parental education less than high school diploma  
HW: White and parental education greater than twelve years.
Table 7

The Percentage of Black Workers in Each Earnings Quintile in Raw Data and After Various Types of Reallocation of Educational Expenditure

<table>
<thead>
<tr>
<th>Description of Allocation</th>
<th>Earnings Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Raw Data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>46.73</td>
</tr>
<tr>
<td>r=2.5 for All Workers</td>
<td></td>
</tr>
<tr>
<td></td>
<td>46.44</td>
</tr>
<tr>
<td>EOp B/W r=2.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.43</td>
</tr>
<tr>
<td>EOp B/W xmin=2.5, r=4.85</td>
<td></td>
</tr>
<tr>
<td></td>
<td>34.27</td>
</tr>
<tr>
<td>EOp (4-type parental education) r=2.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38.29</td>
</tr>
<tr>
<td>EOp (4-type as above) xmin=2.5, r=4.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>37.96</td>
</tr>
</tbody>
</table>

Note: Earnings data are adjusted for variations in earnings by age using regression coefficients from the B/W typology. Quintile 5 refers to the fifth of the population with the lowest earnings. Calculations are based on spending under various equalization and EOp policies and regression coefficients from the B/W typology.
Table 8
Estimated Gains in the Objective Function and Costs per Student of Various Interventions Using the Black-White Typology

Note: Estimated cost per person is calculated as total program cost divided by the number of persons in the sample, where costs are calculated as a present value in the year in which the person reaches age 18. The “value of objective function” is derived from the average value of the lower envelope in log weekly wage:q space, re-expressed in average earnings per week for workers on the envelope. N/A: “not applicable”.

<table>
<thead>
<tr>
<th>Policy Description</th>
<th>Value of objective function ($$)</th>
<th>Change Relative to Base Case</th>
<th>P.D.V of Estimated Cost per Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case x-mean=2.5</td>
<td>$464.58</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Equal Resource x-mean=2.5</td>
<td>$465.68</td>
<td>$1.10</td>
<td>0</td>
</tr>
<tr>
<td>EOp x-mean=2.5</td>
<td>$510.91</td>
<td>$46.33</td>
<td>0</td>
</tr>
<tr>
<td>EOp x-min=2.5</td>
<td>$530.37</td>
<td>$65.79</td>
<td>$34,597.83</td>
</tr>
</tbody>
</table>
References


