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Background model and a measurement of the 2nbb decay rate in $^{130}\text{Te}$ with CUORE-0

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Background model and a measurement of the $2\nu\beta\beta$ decay rate in $^{130}\text{Te}$ with CUORE-0

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy in Physics

by

Xiaoyu Zhu

2016
Abstract of the Dissertation

Background model and a measurement of the $2\nu\beta\beta$ decay rate in $^{130}\text{Te}$ with CUORE-0

by

Xiaoyu Zhu

Doctor of Philosophy in Physics

University of California, Los Angeles, 2016

Professor Huan Z. Huang, Chair

CUORE is a ton-scale cryogenic bolometer experiment designed to search for neutrinoless double beta decay ($0\nu\beta\beta$) in $^{130}\text{Te}$. CUORE-0 was a single array experiment consisting of 11 kg of $^{130}\text{Te}$ and was designed to test the detector construction and analysis techniques for CUORE. CUORE-0 operated from 2013 to 2015 in the Laboratori Nazionali del Gran Sasso in Italy and collected a data sample of 9.8 kg·yr of $^{130}\text{Te}$.

This dissertation will focus on a method for reconstructing the background spectrum and extracting the two neutrino double beta decay ($2\nu\beta\beta$) signal of $^{130}\text{Te}$ in CUORE-0. Identifying and reducing the most significant background contributions are keys to improving the sensitivity of future $0\nu\beta\beta$ experiments. The background spectrum of CUORE-0 was fit using the spectra of radioactive contaminants generated by Monte Carlo simulations and the $2\nu\beta\beta$ half-life was measured to be $T_{1/2}^{2\nu} = (8.27^{+1.38}_{-0.99}(\text{stat+syst})) \times 10^{20}$ yr.
The dissertation of Xiaoyu Zhu is approved.

Yifang Zhu

Lindley A. Winslow

George J. Igo

Huan Z. Huang, Committee Chair

University of California, Los Angeles
2016
To Howard and Andrew

Sharp like an edge of a samurai sword
The mental blade cut through flesh and bone
Though my mind’s at peace, the world out of order
Missing the inner heat, life gets colder
Oh yes, I have to find my path
No less, walk on earth, water, and fire
The elements compose a magnum opus
My modus operandi is amalgam steel packed tight in micro
Chip on my armor, a sign of all-pro
The ultimate reward is honor not awards
At odds with the times in wars with no Lords
A Freelancer
A battle cry of a hawk make a dove fly and a tear dry
Wonder why a lone wolf don’t run with a clan
Only trust your instincts and be one with the Plan

Shingo Annen – Battle cry
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Vita

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Publications and Presentations

“Simulation of Bottom Measurement with the STAR HFT Detector” Poster.

“Validation of Parylene coating to suppress alpha contamination on the copper surface in CUORE bolometers” Poster. TAUP 2013, Asilomar, CA. September 2013.


“Latest results from CUORE-0 and the status of CUORE” Talk. LLWI 2016, Lake Louise, Canada. February 2016.


CHAPTER 1

Introduction

Physical phenomena can often be explained elegantly through underlying symmetries. The conservation of energy, for example, is the consequence of time translation symmetry. Symmetries are a powerful tool in unifying different physics, the Standard Model (SM) of particle physics combines three of the four known forces under one symmetry group and has accurately predicted almost all experimentally tested phenomena to date. However, the SM is still considered an incomplete model particularly in regard to the smallest and largest scales of the Universe.

One of the major questions on the cosmological scale is the baryon asymmetry problem in the Universe. According to the Big Bang theory, matter and antimatter should have been produced in equal amounts in the early Universe. However, the observable Universe is dominated by matter; some physical phenomena, perhaps a greater overarching symmetry, must act differently on matter over antimatter.

One of the most promising models for disentangling this asymmetry lies in the neutrino sector of particle physics. Neutrinos are the lightest elementary particles currently known and the SM does not have a prediction for many of their properties. In fact, before the 1990s neutrinos were thought to be completely massless; the values of their masses are still an unanswered question. Neutrino mass potentially serves as a unique probe to both the largest and smallest scales in physics.

This thesis focuses on one experimental method to resolve the properties of neutrino mass. Before delving into the intricacies of neutrino physics and how it
relates to the baryon asymmetry problem, we will first review how neutrinos fit into the SM.

1.1 The Standard Model

The SM is built on the symmetry group $SU(3) \otimes SU(2) \otimes U(1)$ where $SU(3)$ corresponds to the strong interactions of quantum chromodynamics and $SU(2) \otimes U(1)$ corresponds to the Glashow-Weinberg-Salam model of electroweak theory. Maximal parity violation in weak interactions is characterized by the maximal asymmetry between left and right chirality; only left-handed fermions interact via the weak force. Thus, it is convenient to work in terms of states with definite chirality. The chiral projections of fields $\psi$ can be defined as

$$
\psi_L = \left( \frac{1 - \gamma_5}{2} \psi \right)
$$

$$
\psi_R = \left( \frac{1 + \gamma_5}{2} \psi \right)
$$

(1.1)

where the $\gamma_i$ represent the $4 \times 4$ Dirac matrices.

Using the chiral projection format, the fermion content within the SM can be summarized as

$$
q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \quad u_R, d_R
$$

(1.2)

$$
L_l \equiv \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \quad l_R; \quad l \in e, \mu, \tau
$$

(1.3)

where the lepton sector $L_l$ differs from the quark sector $q$ through the assumption that right-handed neutrinos ($\nu_{lR}$) do not exist.

1.1.1 Higgs mechanism

The requirement of gauge invariance prohibits fermions from having mass terms directly in the SM Lagrangian. Instead, all elementary fermions acquire their mass
through spontaneous symmetry breaking via the Higgs mechanism. The SM Higgs field is a doublet \( \Phi \) and a contraction between the Higgs doublets and left-handed fermion doublets produces SU(2) invariant objects. Further contraction of the contracted Higgs and left-handed fermion doublets with a right-handed fermion singlet produces the familiar SU(2)\( \otimes \)U(1) invariant objects matching the SM Lagrangian. These contractions are called \textit{Yukawa couplings}, using the electron as an example

\[
\mathcal{L}_{\text{Yuk}} = y_e \bar{e}_R \Phi_0^\dagger \begin{pmatrix} \nu_eL \\ e_L \end{pmatrix} + h.c. \tag{1.4}
\]

When the electroweak symmetry is spontaneously broken during the electroweak phase transition, the Higgs field acquires a non-zero vacuum expectation value \( \langle \Phi_0 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \) and the SU(2)\( \otimes \)U(1) objects transform into the observed U(1) symmetry of electromagnetism.

\[
\mathcal{L}_{\text{Yuk}} = y_e \left[ \bar{e}_R \Phi_0^\dagger \begin{pmatrix} \nu_eL \\ e_L \end{pmatrix} + (\bar{\nu}_e, \bar{e}_L) \Phi_0 e_R \right] \\
= y_e (\bar{e}_R v e_L + \bar{e}_L v e_R) \\
= y_e v \bar{e} e
\tag{1.5}
\]

This term corresponds to a mass term for the electron with a mass of \( m_e = y_e v \) and is called the \textit{Dirac mass}. Without a right-handed singlet neutrino, neutrinos are unable to acquire mass through Yukawa couplings in the SM.

### 1.2 Neutrino oscillation

The first hint at physics beyond the SM in neutrino physics came from solar neutrino experiments. Experiments pioneered by the Davis experiment \cite{LCD92} observed a deficit in the electron neutrino flux coming from the sun compared to
Figure 1.1: Results from Solar neutrino experiments show the ratios of solar neutrino fluxes from various experiments to the SSM without neutrino oscillations. Filled circles are experimental data while open circles are theoretical expectations based on SSM (Figure from [MV04]).

predictions from the Standard Solar Model (SSM) creating what is now known as the Solar Neutrino Problem (Figure 1.1).

One of the proposed solutions was to give neutrinos a non-zero rest mass, allowing their flavor eigenstates to be a superposition of their propagating mass eigenstates [Bon05]. The probability of measuring a particular flavor of neutrino varies or “oscillates” as the neutrino propagates through space in its mass eigenstate. Oscillations between mass and flavor eigenstates are a well known phenomena in the quark sector; the two bases are connected by a unitary mixing matrix called the Cabbibo-Kobayashi-Makai (CKM) matrix.

The basics of neutrino oscillation can be illustrated using a simplified two neutrino flavor case. The neutrino flavor states $\nu_e$ and $\nu_\mu$ can be written as a linear combination of neutrino mass eigenstates $\nu_1$ and $\nu_2$ with masses $m_1$ and
As neutrinos are created in definite flavor eigenstates rather than mass eigenstates, their wave-functions are initially a superposition of mass eigenstates

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

(1.7)

The neutrino moves with $p \gg m$ so its velocity is essentially $c$ and the approximation $t = \frac{L}{c}$ can be made. Applying the time evolution operator $U(L)$ on $|\nu_e\rangle$

$$U(L)|\nu_e\rangle = \cos \theta e^{-iE_1t}|\nu_1\rangle + \sin \theta e^{-iE_2t}|\nu_2\rangle$$

$$= [\cos^2 \theta e^{-iE_1t} + \sin^2 \theta e^{-iE_2t}]|\nu_e\rangle$$

$$+ [\cos \theta \sin \theta (e^{-iE_1t} - e^{-iE_2t})]|\nu_e\rangle$$

(1.8)

As the neutrinos have definite momentum, we can approximate their energy as $E_i = \sqrt{p^2 + m^2} \simeq p(1 + \frac{m^2}{2p^2})$. The probability that a $\nu_e$ stays as a $\nu_e$, the survival probability, can be calculated

$$P(\nu_e \rightarrow \nu_e) = |\langle \nu_e |U(L)|\nu_e\rangle|^2$$

$$= 1 - 2(\cos \theta \sin \theta)^2 \left[1 - \cos^2 \left( \frac{E_2 - E_1}{2} t \right) \right]$$

(1.9)

$$= 1 - \sin^2 (2\theta) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

where $\Delta m^2 = m_2^2 - m_1^2$.

The neutrino oscillation hypothesis was confirmed by the combination of the SNO and KamLAND experiments. The SNO experiment confirmed, through neutral current measurements, that while the electron neutrino flux was much lower
than predicted by the SSM, the total flux of neutrinos was accurately predicted by the SSM \cite{Ahm02} (Figure 1.1). The KamLAND experiment was able to confirm the oscillation shape of the survival probability of neutrinos coming from nuclear reactors \cite{Abe08} (Figure 1.2).

Generalizing to the three neutrino flavor case, the mixing matrix can be parameterized by 4 parameters: 3 mixing angles $\theta_{ij}$ and 1 CP violating phase $\delta_{CP}$. The most common form of the matrix is Pontecorvo-Maki-Nakagawa-Sakata (PMNS)
parameterization

\[ U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13}
\end{pmatrix} \times \begin{pmatrix}
  e^{i\alpha_1/2} & 0 & 0 \\
  0 & e^{i\alpha_2/2} & 0 \\
  0 & 0 & 1
\end{pmatrix} \]

(1.10)

where \( c_{ij} = \cos\theta_{ij} \) and \( s_{ij} = \sin\theta_{ij} \). Two additional phases \( \alpha_i \), called the Majorana phases, are necessary and have observable consequences only if neutrinos are Majorana particles. The consequences of Majorana neutrinos will be discussed in the next section.

Neutrino oscillation experiments have currently nailed down all 3 of the mixing angles as well as the mass-squared difference parameters \( \Delta m_{23}^2 \) and \( |\Delta m_{13}|^2 \simeq |\Delta m_{12}|^2 \) (Table 1.1). Most of the remaining questions in neutrino physics are related to the mass of the neutrino. Because the sign of mass splitting \( |\Delta m_{13}|^2 \) is still unknown, it is possible for the ordering of the neutrino mass eigenstates to be either normal \((m_1 < m_2 < m_3)\) or inverted \((m_3 < m_1 < m_2)\) creating the neutrino mass hierarchy ambiguity (Figure 1.3). Although oscillation experiments can measure the mass-squared differences in neutrino mass, the current generation of experiments are not sensitive to the individual neutrino mass eigenvalues. The absolute mass scale of the neutrino can be accessed through either direct neutrino mass measurements or through cosmological limits.

### 1.3 Incorporating neutrino mass

With the confirmation of a non-vanishing neutrino rest mass, a major theoretical question becomes how to extend the SM to incorporate neutrino mass as well as
Table 1.1: Best-fit and 3σ intervals for neutrino mixing parameters from a global three-flavor fit to oscillation data. The values (values in brackets) correspond to $m_1 < m_2 < m_3$ $(m_3 < m_1 < m_2)$. The definition of $\Delta m^2$ used is: $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$ (Table from [CFL14]).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Fit ($\pm 1\sigma$)</th>
<th>3σ Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{12}^2$ [10$^{-5}$ eV$^2$]</td>
<td>$7.54^{+0.26}_{-0.22}$</td>
<td>6.99 – 8.18</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2</td>
<td>$ [10$^{-3}$ eV$^2$]</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.308 \pm 0.017$</td>
<td>$0.259 – 0.359$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$, $\Delta m^2 &gt; 0$</td>
<td>$0.437^{+0.033}_{-0.023}$</td>
<td>$0.374 – 0.628$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$, $\Delta m^2 &lt; 0$</td>
<td>$0.455^{+0.039}_{-0.031}$</td>
<td>$0.380 – 0.641$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$, $\Delta m^2 &gt; 0$</td>
<td>$0.0234^{+0.0020}_{-0.0019}$</td>
<td>$0.0176 – 0.0295$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$, $\Delta m^2 &lt; 0$</td>
<td>$0.0240^{+0.0019}_{-0.0022}$</td>
<td>$0.0178 – 0.0298$</td>
</tr>
<tr>
<td>$\delta/\pi$ (2σ range quoted)</td>
<td>$1.39^{+0.38}<em>{-0.27}$ ($1.31^{+0.29}</em>{-0.33}$)</td>
<td>($0.00 - 0.16$) $\oplus$ ($0.86 – 2.00$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($0.00 – 0.02$) $\oplus$ ($0.70 – 2.00$)</td>
</tr>
</tbody>
</table>
explain why the neutrino mass is so elusively small. The natural extension of the SM to incorporate neutrino mass is to add new particles, however, the LEP experiment has excluded new particles coupling to the Z boson with \( m < M_Z/2 \) [Acc98]. New particles added to the SM either have be right-handed such that they cannot couple to the Z or have to be extremely heavy. In this section, we will review different methods to incorporate neutrino mass into the SM.

### 1.3.1 Majorana vs Dirac

The Dirac equation is the relativistic wave equation describing fermions in free space

\[
(i\gamma_\mu \frac{\delta}{\delta x_\mu} - m)\psi = 0 \quad (1.11)
\]

where \( \psi \) denotes a four-component spinor. The Dirac equation follows from the Dirac Lagrangian in free space

\[
\mathcal{L} = i\bar{\psi}\gamma^\lambda \delta_\lambda \psi - m\bar{\psi}\psi \quad (1.12)
\]
which contains the mass term $m\bar{\psi}\psi$ of the fermion corresponding to the Dirac mass. Theoretically any term that satisfies Lorentz invariance can be added to the Lagrangian. Another Lorentz scalar, $m\psi^T C^{-1} \psi$, where $C$ is the charge conjugation matrix, represents another mass term called the Majorana mass.

The difference between the two mass terms arises from their behavior under a U(1) symmetry transformation, $\psi \rightarrow e^{i\alpha} \psi$. The Dirac mass term is invariant under a U(1) transformation meaning there exists a conserved charge. The Majorana mass term, however, is not invariant under U(1) symmetry; fermions with Majorana mass cannot have a conserved charge. In other words, if the fermion has any conserved charge value, it cannot have a Majorana mass. Without a conserved charge ($\psi = \psi^c$), the mass term $\bar{\psi}\psi$ can be reduced to the Majorana mass term $\psi^T C^{-1} \psi$; the fermion becomes its own anti-particle. Due to the existence of electric charge, Majorana mass terms are impossible for all elementary fermions with the exception of the neutrino.

While the neutrino has no electric charge, it has experimentally been observed to conserve lepton number. However, lepton number conservation is an accidental symmetry in the SM and has not been associated with a fundamental symmetry. There are no theoretical reasons why lepton number should be a conserved quantum number as the conservation of B+L has already been broken in the SM through the triangle anomaly from the chiral nature of electroweak interactions. As a consequence, the search for a lepton number violating process is crucial to determining the nature of neutrino mass.

As both Dirac and Majorana mass terms are theoretically allowed for the neutrino, the most general mass term for neutrinos is a combination of both Dirac
and Majorana mass terms and can be represented by a matrix

\[
\mathcal{L} = \frac{1}{2} m_D (\bar{\psi}_L \psi_R + \bar{\psi}_L^c \psi_R^c) + m_L \bar{\psi}_L \psi_R + m_R \bar{\psi}_L^c \psi_R^c + h.c.
\]

\[
= \frac{1}{2} (\bar{\psi}_L^c, \bar{\psi}_L) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_R^c \end{pmatrix}
\] (1.13)

1.3.2 Seesaw mechanism

The simplest method to introduce neutrino mass in the SM is to extend the fermionic content of the model and add a right-handed singlet neutrino $\nu_R$. This results in a Yukawa coupling similar to other fermions

\[
\Delta \mathcal{L} = y_\nu \bar{\ell}_L \sigma_2 \Phi^* \nu_R
\] (1.14)

resulting in a Dirac mass term $m_D = -y_\nu v$. This method is somewhat unsatisfactory as current limits of the neutrino mass are at the sub-eV level which would require the Yukawa coupling to be incredibly small, $y_\nu \sim 10^{-11}$, creating an unnatural explanation for the smallness of neutrino mass.

A more natural explanation can be made if neutrinos exhibit Majorana mass in addition to Dirac mass. Common extensions of the SM can be classified into three tree-level methods called the seesaw mechanisms. While the physical picture of neutrino mass could likely be described by a mixture of these methods, they serve as a basis on which other methods of introducing neutrino mass can be developed [Sen11].

1.3.2.1 Type I

The first and most commonly used form of the seesaw mechanism introduces a heavy singlet right-handed Majorana neutrino $N_R$ per generation of neutrino, allowing for both a Dirac mass term as well as a Majorana mass term [Min77]. Written for one generation of neutrino only, the addition to the Lagrangian be-
comes
\[
\Delta \mathcal{L} = y_D \bar{l}_L \sigma_2 \Phi^* N_R + \frac{M_R}{2} \nu_R^T C N_R + h.c.
\] (1.15)

Introducing the two neutrino states \( \nu_L \equiv \nu_L + C \bar{\nu}_L^T \) and \( N \equiv \nu_R + C \bar{\nu}_R^T \) the mass matrix becomes

\[
M = \begin{pmatrix}
0 & m_D \\
m_D^T & M_R
\end{pmatrix}
\] (1.16)

Three scenarios occur depending on the relationship between \( M_R \) and \( m_D \)

1. \( M_R \ll m_D \): the neutrino predominantly has a Dirac mass.

2. \( M_R \simeq m_D \): the neutrino is a mixture of both Majorana and Dirac masses.

3. \( M_R \gg m_D \): the neutrino is predominantly Majorana.

The last case can lead to interesting possibilities as the approximate mass eigenstates are \( N \) with mass \( M_N \equiv M_R \) and \( \nu \) with mass

\[
M_\nu = -m_D^T \frac{1}{M_N} m_D
\] (1.17)

This is called the Type I seesaw mechanism, the seesaw effect of \( M_\nu \) decreasing as the heavy Majorana mass \( M_N \) increases serves as an explanation for the smallness of neutrino masses \[SV06\]. For a \( \sim 1 \) eV neutrino mass \( M_\nu \) the heavy right-handed Majorana partner will have a mass \( \sim 10^{14} \) GeV.

### 1.3.2.2 Type II and III

If no new fermionic content is introduced, neutrino mass can be generated through the addition of a scalar triplet “Higgs”. This is commonly known as the Type II seesaw mechanism \[MW80\].
Instead of extending the fermionic content by a singlet, a SU(2) triplet fermion can produce the same Yukawa coupling as (Equation 1.15); this is known as the Type III seesaw mechanism [FLH].

1.4 Baryogenesis

The baryon asymmetry in the Universe can be observed via precise temperature measurements of the Cosmic Microwave Background (CMB). The total baryon content of the Universe is constrained through the acoustic fluctuations of the CMB and the amount of asymmetry measured between baryons and antibaryons is conventionally expressed in terms of the average photon number density $n_\gamma$ [GD11]

$$5.7 \times 10^{-10} \leq \eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \leq 6.7 \times 10^{-10}.$$

An explanation to the baryon asymmetry in the Universe was first studied by Sakharov [Sak67]. He concluded that the small baryon asymmetry maybe have been produced in the early Universe if three necessary “Sakharov” conditions are satisfied

1. Baryon number violation
2. Violation of C and CP
3. Departure from thermal equilibrium

All three conditions are actually already met within the SM: baryon number is violated by non-perturbative processes (more precisely, B+L is violated while B-L is conserved), CP is violated by a phase in the CKM matrix, and the space-time expansion gives a departure from equilibrium. The problem of baryon asymmetry could have been generated without the need for any processes beyond the SM.
Unfortunately, experiments have confirmed that the CP-violation within the quark sector does not supply a sufficient production of the baryon asymmetry we observe today. The SM alone does not provide a mechanism to drive baryogenesis.

1.4.1 Baryogenesis from Leptogenesis

Turning to extensions of the SM for answers, heavy Majorana neutrinos are a promising tool for generating the baryon asymmetry. An excess of lepton number can be used produce baryon number violation, creating what is known as baryogenesis via leptogenesis [FY86].

The heavy Majorana neutrinos $N_i$ may decay in the early Universe; two particular decay branches that are interesting as they violate lepton number are

\[ N_i \rightarrow H + l_L \]
\[ N_i \rightarrow H^* + \bar{l}_L. \]

In the minimal case of leptogenesis, only the lightest of the heavy Majorana neutrinos, $N_1$ decays. Through CP violation of $N$ decays from tree-loop interference diagrams [Figure 1.4], an asymmetry in the rates of the two processes can
be generated to produce an excess lepton number

\[
\epsilon_1 = \frac{\Gamma(N_i \rightarrow Hll_L) - \Gamma(N_i \rightarrow H^*\bar{l}_L)}{\Gamma(N_i \rightarrow Hll_L) + \Gamma(N_i \rightarrow H^*\bar{l}_L)} \approx \frac{3}{16\pi} \sum_{i>1} \frac{\text{Im}((y^i_\nu y^i_\nu^\dagger)_{1i})}{(y^i_\nu y^{i\dagger}_\nu)_{11}}. \tag{1.18}
\]

The excess lepton number generated can then be converted to a baryon number through sphaleron transitions that are active in the early Universe. The baryon asymmetry caused by heavy Majorana neutrino decays can be estimated as

\[
\eta_B \simeq -C\epsilon_1\kappa \tag{1.19}
\]

where \( C \) represents the fraction of conversion from L to B through sphaleron transitions and \( \kappa \) represents an efficiency factor that requires the departure from thermal equilibrium \cite{Buc14b}. 


CHAPTER 2

Double Beta Decay

2.1 Double beta decay

The quest to answer the remaining questions in neutrino physics has led to the experimental search for a lepton number violating process. One of the most promising methods to search for lepton number violation is a class of experiments involving double beta decay. Double beta decay is a class of extremely rare second-order weak decays in which the mother nucleus (Z,A) transitions to its isobar (Z+2,A) with the emission of two electrons. Two neutrino double beta decay ($2\nu\beta\beta$) is a process allowed in the SM as it conserves lepton number and has been experimentally observed in 11 isotopes (Table 2.1). If neutrinos have Majorana mass, an additional mode of decay violating lepton number conservation called neutrinoless double beta decay ($0\nu\beta\beta$) becomes possible. The search for $0\nu\beta\beta$ has become an important research direction in neutrino and nuclear physics.

2.1.1 Two neutrino double beta decay

$2\nu\beta\beta$ can be described as two beta decays occurring simultaneously

$$(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\nu_e$$

and its decay rate was first calculated by Maria Goeppert-Mayer [Goe35] (Figure 2.1). The calculation is complicated as it is impossible to distinguish between the combinations in which the electron-neutrino systems appear in intermediate nuclear states. As a result, all of the configurations must be summed over. High-
Table 2.1: Experimentally measured half-lives of $2\nu\beta\beta$ for various isotopes. The values listed are the weighted average of various experiments [Bar10].

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$2\nu\beta\beta T_{1/2}$ (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>$4.4^{+0.6}_{-0.5} \times 10^{19}$</td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>$(1.5 \pm 0.1) \times 10^{21}$</td>
</tr>
<tr>
<td>$^{82}\text{Se}$</td>
<td>$(9.2 \pm 0.7) \times 10^{19}$</td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td>$(2.3 \pm 0.2) \times 10^{19}$</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>$(7.1 \pm 0.4) \times 10^{18}$</td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>$(2.8 \pm 0.2) \times 10^{19}$</td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>$(1.9 \pm 0.4) \times 10^{25}$</td>
</tr>
<tr>
<td>$^{130}\text{Te}$</td>
<td>$6.8^{+1.2}_{-1.1} \times 10^{20}$</td>
</tr>
<tr>
<td>$^{136}\text{Xe}$</td>
<td>$(2.165 \pm 0.016 \pm 0.059) \times 10^{21}$</td>
</tr>
<tr>
<td>$^{150}\text{Nd}$</td>
<td>$(8.2 \pm 0.9) \times 10^{18}$</td>
</tr>
<tr>
<td>$^{238}\text{U}$</td>
<td>$(2.0 \pm 0.6) \times 10^{21}$</td>
</tr>
</tbody>
</table>
Figure 2.1: Feynman diagram for $2\nu\beta\beta$ (left) and $0\nu\beta\beta$ through the exchange of a light Majorana neutrino (right).

lighting the important aspects of the derivation from [Zub11], the decay rate of $2\nu\beta\beta$ from $0^+ \rightarrow 0^+$ can be described by second-order time-dependent perturbation theory

$$
\lambda_{2\nu} = \frac{G_F^4 \cos^4 \theta_C}{8\pi^7} \int_{m_e}^{Q+m_e} F(Z, E_{e1}) p_{e1} E_{e1} dE_{e1} \\
\times \int_{m_e}^{Q+2m_e-E_{e1}} F(Z, E_{e2}) p_{e2} E_{e2} dE_{e2} \\
\times \int_0^{Q+2m_e-E_{e1}-E_{e2}} E_{\nu1}^2 E_{\nu2}^2 dE_{\nu1} \sum_{m,m'} A_{mm'} 
$$

(2.1)

where $Q$ is nuclear transition energy (Q-value), $E_{ei}$ and $E_{\nu i}$ are the electron and neutrino energies, and $G_F \cos \theta_C$ is the weak coupling constant. $F(Z, E)$ is called the Fermi function, a correction term for the interaction between the electrons with the Coulomb field of the daughter nucleus, and the quantity $A_{mm'}$ contains the Gamow-Teller nuclear matrix elements and typical energy denominators from the perturbation calculations summed over the intermediate $1^+$ nuclear states $m$ and $m'$. The matrix elements $A_{mm'}$ are defined as

$$
A_{mm'} = \langle 0_f^+ | t^- \sigma | 1_f^+ \rangle \langle 1_f^+ | t^- \sigma | 0_i^+ \rangle \langle 0_f^+ | t^- \sigma | 1_i^+ \rangle \langle 1_f^+ | t^- \sigma | 0_i^+ \rangle \\
\times \frac{1}{3} (K_m K_{m'} + L_m L_{m'} + \frac{1}{2} K_m L_{m'} + \frac{1}{2} L_m K_{m'})
$$

(2.2)
where $t_{\pm}$ are the isospin ladder operators, $\sigma$ is the spin operator, and

$$K_m = \frac{1}{E_{Nm} + E_{e1} + E_{\nu1} - E_i} + \frac{1}{E_{Nm} + E_{e2} + E_{\nu2} - E_i}$$

$$L_m = \frac{1}{E_{Nm} + E_{e1} + E_{\nu2} - E_i} + \frac{1}{E_{Nm} + E_{e2} + E_{\nu1} - E_i}.$$  \hfill (2.3)

$$L_m = \frac{1}{E_{Nm} + E_{e1} + E_{\nu2} - E_i} + \frac{1}{E_{Nm} + E_{e2} + E_{\nu1} - E_i}.$$  \hfill (2.4)

As the Gamow-Teller matrix element depends on the energies of both the leptons and the intermediate nuclear states, the integration over lepton energies should be carried out individually for each intermediate nuclear state. Two approximations can be made at this point to simplify the calculation of the decay rate. The first approximation separates the nuclear physics and the kinematic parts of the equation by replacing all lepton energies with their corresponding average value, $E_e + E_\nu \sim Q/2 + m_e$ which simplifies $K_m$ and $L_m$ to

$$K_m \approx L_m \approx \frac{1}{E_{Nm} - E_i + Q/2 + m_e}.$$  \hfill (2.5)

The second approximation simplifies the Fermi function by using the Primakoff-Rosen approximation, a non-relativistic approach [PR59]. Under these two approximations, the decay rate of $2\nu\beta\beta$ can be written as

$$\lambda_{2\nu}/\ln 2 = (T^{2\nu}_{1/2})^{-1} = G^{2\nu}(Q, Z)|M_{GT}^{2\nu}|^2 + \frac{g_V^2}{g_A^2}|M_F^{2\nu}|^2$$

$$= G^{2\nu}(Q, Z)|M^{2\nu}|^2.$$  \hfill (2.6)

with $G^{2\nu}(Q, Z)$ as the four-particle phase space factor and the matrix elements are given by

$$M_{GT}^{2\nu} = \sum_j \frac{\langle 0^+_j|t_-\sigma|1^+_j\rangle\langle 1^+_j|t_-\sigma|0^+_i\rangle}{E_j - Q/2 + m_e - E_i}.$$  \hfill (2.7)
\[ M_{F}^{2\nu} = \sum_{j} \frac{\langle 0_+^j || t_- || 1_+^j \rangle \langle 1_+^j || t_- || 0_+^i \rangle}{E_j - Q/2 + m_e - E_i}. \]  \hspace{1cm} (2.8)

Technically both the Fermi and Gamow-Teller operators can contribute to the transition, however, only the Gamow-Teller component is implemented in practice since the Fermi component is heavily suppressed by isospin selection rules.

### 2.2 Neutrinoless double beta decay

The simplest form of $0\nu\beta\beta$ occurs through the exchange of a light Majorana neutrino and, when first considered by Racah \[\text{Rac37}\], can be described as a neutron decaying with the emission of a right-handed $\bar{\nu}_e$ and then absorbed at a second neutron within the same nucleus as a left-handed $\nu_e$

\[ (Z, A) \rightarrow (Z + 1, A) + e^- + \bar{\nu}_e \]

\[ (Z + 1, A) + \nu_e \rightarrow (Z + 2, A) + e^- \]

For this process to occur, the neutrino must be identical to the anti-neutrino (Majorana particle) and the neutrino must have mass to allow for helicity matching (Figure 2.1). Though not all processes that produce $0\nu\beta\beta$ utilize a Majorana neutrino mass as the dominant mechanism, Schechter and Valle have shown in their “black box” theorem that all realizations of $0\nu\beta\beta$ require a Majorana neutrino mass to some degree \[\text{SV82}\] (Figure 2.2). The discovery of $0\nu\beta\beta$ therefore necessitates a Majorana mass for the neutrino.

The decay rate for $0\nu\beta\beta$ takes a similar form to that of $2\nu\beta\beta$ \[\text{BKI13}\]

\[ \lambda_{0\nu}/\ln2 = (T_{1/2}^{0\nu})^{-1} = G^{0\nu}(Q, Z)|M^{0\nu}|^2|f(m_i, U_{ei})|^2. \]  \hspace{1cm} (2.9)

where $G^{0\nu}$ and $M^{0\nu}$ represent the phase space factor and nuclear matrix element for $0\nu\beta\beta$, respectively. The noticeable difference is the quantity $f(m_i, U_{ei})$, containing the physics information beyond the SM including the neutrino masses
Figure 2.2: The black box theorem for the generation of a Majorana neutrino mass regardless of the driving mechanism behind $0\nu\beta\beta$ (Figure from [SV82]).

$m_i$ as well as the PMNS mixing matrix elements $U_{ei}$. $f(m_i, U_{ei})$ is a model dependent quantity so the physics information extracted from a $0\nu\beta\beta$ search depends on the model being used.

In the simplest case, through the exchange of a light Majorana neutrino, $f(m_i, U_{ei}) = \langle m_{\beta\beta} \rangle_{m_e}$ where $m_{\beta\beta}$ is called the effective Majorana neutrino mass and is defined as a coherent sum over all the mass eigenstates

$$m_{\beta\beta} = \left| \sum_{i=0}^{3} U_{ei}^2 m_i \right|.$$  \hspace{1cm} (2.10)

$0\nu\beta\beta$ experiments are unique in their ability to probe the additional Majorana phases $\alpha_i$ in the PMNS mixing matrix (Equation 1.10). If CP is conserved, $\alpha_i = k\pi$ with $k = 0, 1, 2, \ldots$ and the effective Majorana neutrino mass reduces to

$$m_{\beta\beta} = |m_1 U_{e1}^2 \pm m_2 U_{e2}^2 \pm m_3 U_{e3}^2|.$$  \hspace{1cm} (2.11)


2.2.1 Nuclear matrix elements

In practice, the calculations of the nuclear matrix element (NME) are complicated, many-body nuclear physics problems. The NME essentially describes the overlap of the nuclear wave functions of both the initial and final states. A model for calculating NME typically has a set of single-particle states with a number of possible wave function configurations and will aim to diagonalize a Hamiltonian in a mean background field. The wave functions of the states are very sensitive to small modifications of the Hamiltonian, thus the NME calculations come with a large amount of uncertainty [Rod11].

While the matrix elements for $2\nu\beta\beta$ and $0\nu\beta\beta$ look similar in form, there are key differences in their calculation. For an exact calculation of the NME, all potential intermediate states between the initial and final states of the $0\nu\beta\beta$ must be considered. To avoid these computational complications, the calculations of the $0\nu\beta\beta$ NME are performed using the closure approximation where the energies of the intermediate states are replaced with a constant value called the closure energy. With this approximation, only the initial and final state wave-functions are required and the complex calculations of the intermediate states can be avoided. This approximation is not typically applied to $2\nu\beta\beta$ as the closure approximation does not take into account the interference between the individual intermediate states.

The five primary approaches to calculate NMEs are the Quasi-particle Random Phase Approximation (QRPA), the Interacting Shell Model (ISM), the Interacting Bosom Model (IBM), the Generating Coordinate Method (GCM), and the Projected Hartree-Fock-Bogoliubov model (PHFB). The details of these methods can be found in [BKI13] [BI09] [SFR08] [CS09] [MPC09] [RCC13] [RM10]. Recent NME calculations from different groups using the different methods are shown in Figure 2.3. NME calculations represent the largest theoretical uncertainty for $0\nu\beta\beta$.
Figure 2.3: Matrix element calculations from various calculation approaches for all of the potential decay isotopes of $0\nu\beta\beta$.

### 2.3 Experimental searches for $0\nu\beta\beta$

In principle, any nucleus $(Z,A)$ can undergo $2\nu\beta\beta$ as long as its isobar $(Z+2,A)$ is lighter; any nucleus that can undergo $2\nu\beta\beta$ can also potentially undergo $0\nu\beta\beta$. However, in practice, the $2\nu\beta\beta$ signal can only be experimentally observed if the single beta decay channel $(Z+1,A)$ is forbidden. As a consequence, only even-even nuclei are chosen as candidate isotopes for double beta decay experiments (Figure 2.4).

The basic approach for measuring double beta decay is to measure and sum together the energies of the two decay electrons. The signature of $2\nu\beta\beta$ decay
Figure 2.4: Even-even nuclei are the isotopes of choice for double beta decay experiments as the single beta decay channel is energetically forbidden.
Figure 2.5: The signature of $0\nu\beta\beta$ in the summed electron spectrum is a monoenergetic peak at the Q-value of the decay.

appears in the summed electron energy spectrum as a continuous spectrum from 0 up to the Q-value of the decay. As there are no neutrinos in the final state of $0\nu\beta\beta$, all of the decay energy is shared by the two electrons and the signature becomes a mono-energetic peak at the Q-value of the decay in the summed electron energy spectrum (Figure 2.5).

2.3.1 Detector design

There are two design philosophies for $0\nu\beta\beta$ experiments; one focuses on maximizing the collection efficiency of the decay electrons by using the decay source as the detector while the other focuses on tracking the decay electrons, keeping the decay source and the detector separate (Figure 2.6). Using the detector as the decay source maximizes the ability to measure the decay electrons, increasing the experimental sensitivity which scales directly with efficiency. Using the detector as the decay source also allows experiments to grow large in size without sacrificing the detection efficiency. Alternative, constructing a detector separate from
the decay source allows for the tracking of decay electrons as well as background rejection. Particle tracking can also be used to distinguish among the various underlying processes that drive the decay once the decay is discovered. These two design philosophies are not mutually exclusive, an experiment can be built using both techniques.

The experimental sensitivity figure-of-merit can be maximized by focusing on three primary design aspects: source mass, background level, and energy resolution. As current limits of $0\nu\beta\beta$ half-lives near $10^{25}$ yr, a large number of source nuclei is required to expedite the observation process, requiring experiments to have a large source mass. Cleanliness of the source and surrounding materials is critical to reducing backgrounds that can spoil a potential signal. A high resolution detector means a smaller energy window can be used to evaluate the $0\nu\beta\beta$ signal, reducing the potential background contribution within the window.

When it all boils down, experiments searching for $0\nu\beta\beta$ are looking for a peak on top of a background spectrum. An analytic figure-of-merit for experimental sensitivity can be derived based on the number of background events ($B$) written as a function of the background density ($b$) with units of $c/\text{keV/kg/yr}$

$$B = bMt\delta E. \quad (2.12)$$

With the present technology available, we can expect the background counts of
an experiment to increase with the source mass $M$ as well as the experimental running time $t$. A key assumption made is that the background level scales linearly with the source mass which may not be accurate if surface contaminants dominate the background spectrum at the Q-value of $0\nu\beta\beta$. The expression also assumes the background under the $0\nu\beta\beta$ peak is approximately constant over the energy interval $\delta E$ around the $0\nu\beta\beta$ signal. This assumption holds as long as the detector has a high enough energy resolution such that the $2\nu\beta\beta$ signal does not strongly leak into $\delta E$ [11Z05].

The number of $0\nu\beta\beta$ events observed $C_{\beta\beta}$ can be expressed in terms of the $0\nu\beta\beta$ decay rate

$$C_{\beta\beta} = \lambda_{\beta\beta} A_0 \frac{a \epsilon}{W} M t$$

(2.13)

where $A_0$ is Avagadro’s number, $W$ is the molecular weight of the source, $a$ is the isotopic abundance of the source, $M$ is the mass of the detector, $t$ is the detector live time, and $\epsilon$ is the detection efficiency of $0\nu\beta\beta$ events. Assuming a Poisson distribution, a general criteria for discovery potential can be set as

$$C_{\beta\beta} = n \sigma \sqrt{B + C_{\beta\beta}}$$

(2.14)

where $n \sigma$ is the confidence level. Combining Equation 2.12, 2.13, and 2.14 we obtain

$$\lambda_{\beta\beta} A_0 \frac{a \epsilon}{W} M t = n \sigma \sqrt{b M t \delta E + C_{\beta\beta}}.$$  

(2.15)

If we require a signal to background ratio of $C_{\beta\beta} \approx B$, the expression can be simplified to

$$\lambda_{\beta\beta} = n \sigma \sqrt{\frac{A_0}{a \epsilon}} \frac{W}{M t} \sqrt{2bR E \delta E}. $$

(2.16)

Converting the decay rate to half-life and absorbing the constants into a constant $\alpha$, we arrive at the sensitivity figure-of-merit used to judge $0\nu\beta\beta$ experiments

$$T_{1/2}^{0\nu} = \alpha \left( \frac{a \epsilon}{W} \right) \sqrt{\frac{M t}{b R E \delta E}}.$$  

(2.17)
If the phase space factor and nuclear matrix element from Equation 2.9 are treated as calculable constants, the sensitivity to $T_{1/2}^{0\nu}$ can be converted to a sensitivity for the observable $m_{\beta\beta}$ representing the physics information beyond the SM. The phase space of $m_{\beta\beta}$ is conventionally represented as a function of the lightest neutrino mass (Figure 2.7). Two distinct bands of allowed values appear due to the two potential neutrino mass hierarchies; the two bands converge in the quasi-degenerate region where $m_{\beta\beta} \gg \Delta m^2$. The thickness of the bands corresponds to the variances in the oscillation parameters.

$0\nu\beta\beta$ experiments set upper limits on $m_{\beta\beta}$ and therefore exclude horizontal regions from the parameter space in the Figure 2.7. The past generation of $0\nu\beta\beta$ experiments have mostly explored the quasi-degenerate region and the current generation of experiments are projected to begin exploring the inverted hierarchy parameter space ($m_{\beta\beta} < 50$ meV). Future generations will aim to completely cover the inverted hierarchy.
Figure 2.7: \( m_{\beta\beta} \) plotted against the lightest neutrino mass \( m_{\text{lightest}} \) with limits from the current generation experiments. The exclusion bands for each isotope are truncated for visualization purposes only. The \(^{76}\text{Ge} \) limit comes from GERDA + Hidelberg Moscow + IGEX data \cite{Ago13}. The \(^{136}\text{Xe} \) data comes from EXO-200 + KamLAND-Zen \cite{Alb14,Gan13}. The \(^{130}\text{Te} \) limit comes from Cuoricino+CUORE-0 \cite{And11,Alf15}. 
CHAPTER 3

Detector Description

3.1 CUORE

Cryogenic Underground Observatory for Rare Events (CUORE) is a bolometric experiment searching primarily for $0\nu\beta\beta$ in $^{130}$Te at Gran Sasso National Laboratory (LNGS) \cite{Ard05, Art15}. Bolometers are low temperature calorimeters that extract the energy of an incident particle by measuring temperature change \cite{EM08}. Bolometers consist of 3 main components: an energy absorber, a temperature sensor, and a thermal link to a heat sink \cite{Figure 3.2}.

CUORE builds on the success of its predecessor experiment Cuoricino, a tower of 62 TeO$_2$ bolometers containing 11 kg of $^{130}$Te operated from 2003 to 2008 at LNGS. For its time, Cuoricino produced the most stringent limit on the $0\nu\beta\beta$ half-life of $^{130}$Te at $T_{1/2}^{0\nu\beta\beta} > 2.8 \times 10^{24}$ yr (90\% C.L.) \cite{Ale11}. CUORE aims to improve upon Cuoricino in every possible way without changing the source isotope. Referring back to the experimental sensitivity figure-of-merit (Equation 2.17), CUORE plans to scale up mass by nearly 20 times, reduce background by 15 times, improve energy resolution to $\leq 5$ keV FWHM, and improve the duty cycle of the cryostat by using a custom cryogen-free dilution refrigerator. Ultimately, CUORE should improve upon the $0\nu\beta\beta$ half-life sensitivity by roughly two orders of magnitude.

Full sized CUORE will consist of 19 towers arranged in a cylindrical matrix. Each tower consists of 52 TeO$_2$ bolometers (988 bolometers total for a total mass
Figure 3.1: The layout of the underground laboratory at LNGS. There are three main experimental halls, CUORE is located in Hall A but also has R&D facilities in Hall C.

![Diagram of the underground laboratory at LNGS]

Figure 3.2: Example schematic of a bolometer.

of 206 kg of $^{130}$Te) arranged in 13 floors with 4 bolometers on each floor. To validate and expedite the construction of CUORE, a single tower module, CUORE-0 [CUOa], was built to debug the detector construction process as well as test the improved detector performance.

3.1.1 TeO$_2$ crystals

The energy absorber converts energy deposited by incident particles to phonons. Following the Debye model at low temperatures [Kit04], the heat capacity $C$ is given by

$$C = \frac{m}{M} \frac{12}{5} \pi^4 N_A k_B \left( \frac{T}{\Theta_D} \right)^3$$

(3.1)
where $m$ is the mass of the crystal, $M$ is the molar mass of the absorber, $T$ is the operating temperature, and $\Theta_D$ is the Debye temperature which is material dependent. The heat capacitance of a CUORE crystal is low enough such that single particle interactions produce measurable changes in temperature. CUORE bolometers are composed of $5 \times 5 \times 5 \text{cm}^3$ TeO$_2$ crystals with individual masses of 750 g. TeO$_2$ doubles as both the energy absorber as well as the source of double beta decay (via $^{130}$Te). The greatest advantage of this technique is the high efficiency of detecting $0\nu\beta\beta$ events (98.5% via simulations); $\beta\beta$ decays occurring within a single crystal are fully contained within the crystal. The Debye temperature of TeO$_2$ is high ($232 \pm 7 \text{ K}$), resulting in a heat capacity of 1 MeV/0.1 mK at operating temperatures of 10 mK.

$^{130}$Te is an appealing source isotope for CUORE due to its high natural isotopic abundance (34.08%) as well as its relatively high Q-value for $0\nu\beta\beta$ (2527.515 keV) \cite{Sci09}\cite{RMM09}. A high natural isotopic abundance reduces the necessity for expensive isotopic enrichment; CUORE is able to achieve a large source mass without expensive isotopic enrichment. The high energy Q-value avoids most of the naturally occurring gamma lines at lower energy, excluding the 2615 keV gamma line from $^{208}$Tl, and also increases the decay rate of $0\nu\beta\beta$ by increasing the phase space (calculated by the Primakov-Rosen approximation: $\lambda_{0\nu\beta\beta} \propto (\frac{Q^2}{30} - \frac{2Q^2}{3} + Q - \frac{2}{3})$).

As bolometers measure phonon signals, their theoretical energy resolution is only limited by the thermodynamic fluctuations of phonons, $\Delta E = \sqrt{k_B C T^2}$. Under the typical operating conditions of CUORE bolometers, the thermodynamic limit is $\Delta E \sim 10 \text{ eV}$. However, in practice the resolution observed is several orders of magnitude above the thermodynamic limit and is spoiled by other sources of noise.

The crystals are grown at the Shanghai Institute of Ceramics, Chinese Academy of Sciences (SICCAS) following a strict cleaning and machining procedure to limit
bulk and surface contamination [Arn10]. TeO$_2$ was chosen over pure metallic tellurium as metallic tellurium could not withstand the mechanical stresses from thermal cycling. The crystals are grown to a size larger than specifications as the crystallization process pushes impurities to edges of the crystal. The crystal is then trimmed and polished to the specified size limit removing the contaminants. To limit cosmogenic activation of the crystals, the crystals endure a shipping process by sea from Shanghai to LNGS [Wan15a]. Once on site, four crystals from each shipment are randomly chosen to undergo a cryogenic test to validate their radioactive properties. A total of 10 of these CUORE Crystal Validation Runs (CCVR) were performed and all of the crystals met their radioactive specifications [Ale12]. The rest of the shipment of crystals are stored in three layers vacuum wrap under constant nitrogen flux to prevent recontamination.

Figure 3.3: Left: A TeO$_2$ CUORE crystal. Right: Six TeO$_2$ crystals packed for transportation and storage.

3.1.2 NTD Ge thermistor

The temperature sensor converts small temperature changes from deposited energy into a measurable voltage signal. CUORE uses Neutron-transmutation-doped (NTD) Ge thermistors as temperature sensors which are thermally coupled to the
TeO$_2$ crystals through glue [Hal84] (Figure 3.4).

At high temperatures, semiconductors like germanium naturally conduct via the thermal excitation of electrons from the valence band to the conduction band. As the temperature approaches zero Kelvin, thermal excitations become insufficient at producing the same type of excitation so the conductivity of the semiconductor naturally approaches zero. Doping can be used to increase conductance of semiconductors at low temperatures; the thermal activation of the dopant impurities are able to generate the conduction, effectively reducing the energy gap between the conduction and valence bands. A critical doping density, the metal-insulator transition, exists such that below the transition the conductivity goes to zero at zero Kelvin and above the transition the conductivity is always finite, regardless of the temperature.

The process of neutron transmutation doping exposes a germanium wafer to a neutron flux, converting the germanium isotopes into unstable isotopes via neutron capture. Natural germanium contains four chemically identical isotopes that are uniformly and randomly distributed within a bulk of germanium, neutron transmutation doping provides a reproducible method of uniformly doping germanium [Ens05].

CUORE thermistors are doped to a level slightly below the transition to a metal. In this regime, called the variable hopping regime, electrons are able to tunnel from one impurity site to another with the assistance of phonons. The resistivity $\rho$ of the thermistor in this regime depends heavily on its temperature

$$\rho(T) = \rho_0 e^{(T_0/T)^{1/2}}$$

where $T_0$ depends on the doping level and $\rho_0$ depends on both the doping level and the geometry of the thermistor [ES75]. Each crystal is equipped with one NTD thermistor; a temperature change within the crystal due to energy deposition will
generate a change in resistance of the thermistor resulting in a measurable voltage signal.

3.1.3 Tower support structure

The TeO$_2$ crystals are structurally held in place by polytetrafluoroethylene (PTFE) holders supported by a copper frame structure. The PTFE holders also double as the thermal link to the copper frame heat sink, restoring the detector to thermal equilibrium after every particle interaction [Ped03]. Since the thermal links between the crystals, PTFE, and copper frames are weak, each bolometric signal lasts on the order of seconds. The slowness of its signals is one of the primary weaknesses of CUORE bolometers.

Electronic Tough Pitch (known as NOSV) copper was the material of choice for the supporting frames of the CUORE towers [Heu95]. NOSV copper contains low amounts of hydrogen and exhibits low radioactivity. Hydrogen at room temperature consists of 75% orthohydrogen (triplet state) and 25% parahydrogen (singlet state). As hydrogen is cooled, orthohydrogen is thermodynamically unstable and will release energy while transitioning to the parahydrogen state, essentially creating a virtual heat leak. Low hydrogen content of materials is an
important property in accelerating the cooling process in cryogenics.

The copper support structure represents the largest mass directly exposed to the crystals, special care is necessary to minimize its radioactive contamination. As CUORE bolometers have no active rejection of background events, all support structure used in detector construction must follow a very stringent cleaning and treatment procedure to limit both bulk and surface contamination. Several cryogenic test runs were designed to characterize cleaning methods developed to minimize contaminants on the surface of the copper structure. The copper cleaning techniques will be discussed further in chapter 4.

3.1.4 Si heater

Cryogenic detectors are susceptible to slight drifts in temperature which can spoil the energy resolution of the detector. To counteract drifts in temperature and stabilize the gain of the bolometers, CUORE bolometers are equipped with a silicon semiconductor chip as a joule heater [And12] (Figure 3.5). During the typical operation of CUORE bolometers, a heat pulse containing a fixed amount of energy is injected into each crystal every 300 s. The heat pulses mimic particle interactions and are used in an offline analysis to correct any deviations in detector response arising from temperature drifts. The heat pulses can also be used to map out the efficiency of the software triggering algorithm at low energy.

3.1.5 Electronic readout

Each of the NTD thermistors and heaters are attached to and read out through a set of copper tracks etched onto insulator tapes made with and Polyethylene 2.6 Naphthalate (CuPEN) substrate. The CuPEN cables are attached to two opposite sides of each CUORE tower and are designed for low crosstalk, microphonism, and radioactivity. A total of 4 cables are needed for each side of the tower as each
Figure 3.5: Left: Photograph of a CUORE silicon heater. Right: Drawing of the heater geometry.

CuPEN cable can only instrument the thermistors of 3-4 floors of each tower side and one separate cable is necessary for the heaters. The CuPEN cables extend above the CUORE tower and are plugged into ribbon cables which carry the signal to the rest of the electronic readout chain sitting at room temperature [Figure 3.6].

To read out the temperature signal, a circuit consisting of a bias voltage source and two load resistors in parallel with the thermistor is used [Figure 3.7]. The total resistance of the load resistors is chosen to be much greater than the resistance of the thermistor at operating temperatures such that the current across the thermistor is approximately constant \( I = \frac{V_{\text{bias}}}{R_L} \). The temperature signal is then read out by recording the voltage across the thermistor at all times.

### 3.2 CUORE detector construction

Consistency and reproducibility are important in the construction of a large, granular detector like CUORE. The detectors need to be built in a consistent manner to provide a uniform response. The detector construction process also needs to limit human interaction with detector materials positioned near the bolometers to limit potential recontamination. To meet the detector quality goals set out for CUORE, a specialized assembly line was designed for the construction of all
the towers. The detector construction was split into two primary phases: crystal gluing and tower assembly. The tower construction process took place in a class 1000 clean room in the CUORE hut in Hall A.

3.2.1 Crystal gluing

TeO$_2$ crystals were instrumented with both the NTD thermistor and the Si heater during the gluing phase. The chips were attached to the crystals using Araldite Rapid, a two-component epoxy with a rapid curing time ($\sim$1 hr), low radioactivity, and high thermal conduction. Because the glue, crystal, and semiconductor chips have different thermal contraction coefficients, a solid film of glue between the crystal and chips would incur an excessive amount of temperature induced mechanical stress. Mechanical stresses can affect the resistance of the thermistor and, in the worst case scenario, potentially destroy the semiconductor chip. R&D studies have shown a $3 \times 3$ matrix of glue spots on each thermistor and a 5 dot matrix on each heater provided the optimal thermal coupling while also minimiz-
Figure 3.7: Bias circuit of NTD, typical values for CUORE bolometers are $R_L \sim 60 \text{ G}\Omega$, $R_{\text{th}} \sim 100 \text{ M}\Omega$, and $V_{\text{bias}} \sim 1 \text{ V}$.

Ining thermal stress \cite{Ped04}. Controlling the reproducibility of the glue distribution and the glue dot uniformity on the chips was also critical as the quality of thermal coupling between the chips and the crystal largely determines the quality of the bolometric response.

To minimize recontamination via human interaction with the crystals, the gluing cycle was semi-automated and performed by using two robots within a glovebox. The first robot, a robotic arm, was designed to handle the crystals before and after gluing. The second robot was designed to dispense glue spots in a quick and uniform manner. In the first stage of gluing, the semiconductor chips were first placed manually onto a precise positioning platform to specify the chip locations on the crystal. Then glue spots were dispensed onto the chips and visually inspected for uniformity. \textbf{Figure 3.8} shows an example of semiconductor chips with glue spots that passed inspection. Once the glue matrix passed inspection, the robotic arm then placed the crystal onto teflon cradles which were then lowered onto the platform onto the chips. Crystals were glued one at a time and each gluing cycle lasted approximately half an hour. As a safeguard no activity can be performed inside the glovebox for the first half-hour after a crystal was lowered onto the platform. Once the crystals were glued, they were placed in groups of 4 in vacuum.
sealed boxes and stored in cabinets that were continuously flushed with nitrogen.

### 3.2.2 Tower assembly

The tower construction phase was composed of three steps each requiring a different glovebox set-up [Buc14a]. Rather than transporting the in-construction tower into separate gloveboxes and risking recontamination from exposure to air, the tower was stored under constant nitrogen flux in a garage under a Universal Working Plane (UWP) on which all of the gloveboxes can be attached. The tower can then be raised or lowered into the garage until the gloveboxes were ready for use.

The first step of assembly involves the physical construction of the tower. Each tower was built by hand by a team of technicians in a specialized glovebox one floor at a time. The tower was gradually lowered into the storage garage as it grows in height. Once assembled, the technicians then instrument the tower with two CuPEN strips each attached to a wire tray; the CuPEN strips are first glued to wire trays using Araldite before they are attached to the sides of the tower. Finally, a team of two wire bond the semiconductor chips to the copper pads etched onto the CuPEN strips. After an inspection of the bonded wires, a
protective copper cover was installed over the wire trays. The tower was then stored in an acrylic storage container in the clean room under constant nitrogen flux.

### 3.3 CUORE-0

CUORE-0 was the first tower built using the assembly line and was used both as a tool to debug the assembly and analysis procedures for CUORE as well as a stand alone physics experiment. CUORE-0 is a single CUORE-like tower containing 52 TeO$_2$ bolometers adding up to a total mass of 39 kg TeO$_2$ (10.9 kg $^{130}$Te) \[\text{CUOa}\]. As CUORE-0 is similar in size to Cuoricino, it can conveniently be housed in the same cryostat as Cuoricino in Hall A at LNGS \[\text{Figure 3.9}\].
Figure 3.10: Plot of the accumulated exposure of CUORE-0 over time. The left vertical axis corresponds to TeO$_2$ exposure while the right vertical axis corresponds to $^{130}$Te exposure.

3.3.1 CUORE-0 operation

CUORE-0 acquired physics data in two campaigns, the first began in March 2013 and lasted until September 2013 and the second campaign started in November 2013 and lasted until February 2015. In between the two campaigns, the experiment was shut down to maintain the Cuoricino cryostat. The second data campaign saw a reduction in noise as well as an improvement in the detector duty cycle due to the improved cryogenic conditions. Data accumulation was segmented into roughly two-day long runs; normal detector operation was only halted by the refilling of the main bath with LHe$_2$, producing a down time of five hours per run. The runs were then grouped into roughly month-long datasets with a 2-3 day detector calibration separating each dataset. The progress of CUORE-0 data collection can be seen in Figure 3.10.
3.3.1.1 Optimizing working conditions

The conditions of the detector must be optimized before data collection. First the thermistors were characterized by mapping gain as a function of bias to produce load curves (Figure 3.11). The electronic bias, gain, and offset of each thermistor were set manually to maximize the signal amplitude for low energy events of each detector while still maintaining a large enough dynamic range so that high energy events can be contained within the dynamic range of the ADC. A measurement of the thermistor resistances, called the “working point” of the detector, gives a good estimate on the cryogenic conditions of the tower. The working point of the detector is measured at the end of every run to monitor the cryogenic status of the detector; a new optimized electronic configuration must be set each time the cryostat suffers a major disturbance.

Figure 3.11: I-V curve (blue) and pulse amplitude curve (red) used to optimize the electronic configuration of a bolometer.
3.3.2 CUORE-0 events

The CUORE-0 data acquisition (DAQ) system digitizes and samples the bolometer signals at a frequency of 125 Hz. The waveforms of each detector were sampled continuously and stored in two forms, a triggered format used in all of the standard analysis and a continuous format used primarily in low energy analysis. Triggered events were flagged in three categories: signal, pulser, and noise.

**Signal** - Signal triggers were the basis of physical events and occur when the slope of the bolometer waveform exceeds a given threshold for a set amount of time [*Figure 3.12*]. The threshold of each bolometer depends on the signal to noise ratio of each bolometer and was set manually to maximize low amplitude pulses while minimizing triggering on noise spikes. Each signal triggered event lasts for 5.008 s (626 samples with the sampling frequency of 125 Hz) and includes 1 s of data before the trigger for evaluation of the bolometer conditions as well as 1 s of dead window where no other signal triggers can occur.

**Pulser** - Pulser triggers occur every 300 s when the pulser board injects a predetermined amount of energy in each bolometer. The pulser boards fire in cycles of ten pulses with eight of the pulses corresponding to energies in the 3-4 MeV region, one in the lower energy region between 1.5-2.3 MeV, and one in the higher energy region between 6.5-8.5 MeV. The pulses within the 3-4 MeV region were used to stabilize the gain of each bolometer as they were closest in energy to the $0\nu\beta\beta$ Q-value while the other two were used to check the gain corrections at different energies. To avoid overlap with the derivative trigger, a 100 ms dead window was placed after the pulser event to flag the signal trigger fired from the heater pulse as a secondary trigger. These secondary triggers can be used in offline analysis to determine the signal trigger efficiency at different energies.
Noise - The noise trigger was programmed to fire every 200 s on all bolometers, acquiring a snapshot of the baseline of each bolometer regardless of the presence of signal events. Noise triggers were intended to capture the random noise behavior of each bolometer and events generated by this trigger were used to build the noise power spectra of the bolometer.

3.3.3 Calibration

CUORE-0 was calibrated using two thoriated tungsten wires utilizing the radioactive isotopes from the $^{232}$Th chain. The two wires were strung opposite of each other between the outer vacuum chamber of the cryostat and the external lead shield. Each calibration source string was produced with an activity of roughly 50 Bq with respect to the Th parent, totaling 500 Bq for the entire chain as the $^{232}$Th chain contains ten nuclei and was approximately in secular equilibrium. The electronic settings of the tower were configured with the calibration sources inserted to provide higher statistics for real-time feedback. The detector threshold of each
Figure 3.13: Calibration Spectrum of CUORE-0. Highlighted gamma lines are from the $^{232}$Th decay chain: (1) 511 keV ($e^+e^-$ annihilation), (2) 583 keV ($^{208}$Tl), (3) 911 keV ($^{228}$Ac), (4) 965 and 969 keV ($^{228}$Ac), (5) 1588 keV ($^{228}$Ac), (6) 2104 keV (single escape of $^{208}$Tl), and (7) 2615 keV ($^{208}$Tl).

bolometer was set such that the counting rate of each bolometer was between 40 and 70 mHz with the calibration source. The calibration spectrum of CUORE-0 can be seen in Figure 3.13.

### 3.3.4 Data production

CUORE-0 data processing, the procedure to reconstruct the energy of events in the detectors from raw data pulses, is briefly described in this section. The procedure is described in great detail in [Alf15], [Ald16], and [Oue15].

Amplitude evaluation of thermal pulses is critical to data analysis as the pulse amplitude is directly correlated to the energy of an incident event. We utilized two parallel pulse-filtering techniques, optimal filter (OF) and decorrelated optimal filter (DOF), to evaluate the pulse amplitudes of the bolometers by maximizing the signal-to-noise ratio. The OF technique was a standard used in Cuoricino while the DOF was developed to reduce correlated noise between adjacent bolometers.
Two thermal gain stabilization (TGS) techniques, heater-TGS and calibration-TGS were used to correct for drifts in the energy-to-amplitude response of the bolometer over time. Heater-TGS, used in Cuoricino, utilizes monoenergetic heater pulses to correct for temperature drifts while calibration-TGS uses the 2615 keV $^{208}$Tl calibration line for the same correction. The stabilized spectra is converted to energy by correlating fitted peak means in the calibration data to known gamma lines between 511 keV and 2615 keV using a second-order polynomial function. The second-order polynomial function was found to be more stable when extrapolating to energies beyond 2615 keV and provided the same performance at lower energies.

Combinations of the two different amplitude evaluation and gain stabilization techniques produced four distinct energy estimators. We adopted a pick-and-choose method where the best performing energy estimator of each bolometer was used for the $0\nu\beta\beta$ analysis.

While double beta decay events are mostly confined within one bolometer, background sources may potentially deposit energy in multiple bolometers within a small time window. We group events occurring within a ±5 ms time window in different bolometers into multiplets. The anti-coincidence spectrum (M1) and the double-hit spectrum (M2) were used in our study to facilitate the characterization of background sources.

Periods of cryostat instability or malfunction were discarded to maintain the quality of the data. Pile-up on a bolometer occurs when two or more events occur in the same crystal within a time window. As CUORE bolometer signals are slow to develop as well as stabilize, we require that the time since the previous event and the time until the subsequent event, on the same bolometer, be greater than 3.1 s and 4.0 s respectively to reject pile-up events.

Each bolometer waveform also must pass six pulse-shape parameter conditions, tuned on prominent gamma peaks, that require the waveform to be consistent
with a waveform template of each bolometer. The efficiency of the pulse-shape parameters as a function of energy can be modeled as an exponential function evaluated as the fraction of events that survive the cuts on a bin-by-bin basis of the M2 spectrum (detailed in subsection 5.7.5). After data selection was performed, we obtained a detector exposure of 35.2 kg·yr used for the $0\nu\beta\beta$ analysis.

### 3.3.5 CUORE-0 performance

CUORE-0 saw many performance improvements over its predecessor Cuoricino. The improved construction techniques led to detectors that were more robust to thermal cycling: 51 out of 52 NTD channels were kept alive with the only lost channel being a NTD that was never wire bonded. The improved reproducibility of the detector construction process has also led to improved stability CUORE-0 bolometers; in a period of one month, the detector resistance values varied by less than 3% (Figure 3.14).

In Cuoricino, the energy resolution was evaluated using a single Gaussian func-
tion on the 2615 keV gamma line of $^{208}$Tl. Several empirical detector response models were developed for CUORE-0 as the single Gaussian response function proved unreliable in properly fitting the 2615 keV photopeak due to a secondary peak substructure with energy slightly below the primary photopeak. The secondary peak structure was also observed in Cuoricino but only in the bolometers with the best energy resolution and is currently an ongoing topic of study. The most successful method to fit the photopeak used a double Gaussian function

$$f_{\text{Det}}(\mu, \sigma; E) = (1 - \alpha_{\text{Sub}}(C))Gauss(E; \mu, \sigma) + \alpha_{\text{Sub}}(C)Gauss(E; r_{\text{Sub}}(C)\mu, \sigma)$$  

(3.3)

with $\alpha_{\text{Sub}}$ characterizing the amplitude of the sub-peak and $r_{\text{Sub}}$ as the position of the sub-peak. Both Gaussians were assumed to have the same resolution, sharing the same $\sigma$. The energy resolution of each bolometer was evaluated individually on each calibration dataset using the FWHM of the double Gaussian lineshape [Figure 3.15]. The distribution of energy resolutions of CUORE-0 bolometers in comparison to Cuoricino is shown in [Figure 3.16]. An effective FWHM for the entire tower can be calculated through a harmonic mean by weighting the resolution of each bolometer by the exposure of the bolometer.

$$\sigma_{\text{Eff}} \equiv \frac{\sum_o T_{\text{Bkg},i}}{\sum_i \frac{T_{\text{Bkg},i}}{\sigma_i}}$$  

(3.4)

The effective FWHM is a proxy to the sensitivity contribution of each bolometer. The effective FWHM of CUORE-0 was evaluated as 4.9 keV, reaching the 5 keV FWHM goal set out for CUORE.
Figure 3.15: Fit to the $^{208}\text{Tl}$ 2615 keV line with normalized residuals (top).

Figure 3.16: Energy resolution distribution comparison between CUORE-0 and Cuoricino.
CHAPTER 4

Surface alpha contamination suppression

4.1 Background results from Cuoricino

CUORE bolometers currently have no particle identification ability so any combination of energy depositions that sum to the Q-value of $0\nu\beta\beta$ can potentially mimic a $0\nu\beta\beta$ signal. Three primary contributors to the background in the region of interest (ROI) of $0\nu\beta\beta$ were identified during background studies of Cuoricino (see Figure 4.1) [Sil06]. The first major contribution comes from multiple Compton scattered gammas from the 2615 keV gamma line of $^{208}$Tl. In this case, the gamma may originate from the crystals, nearby copper structure, or the cryostat shields and can Compton scatter, depositing only part of its energy within a crystal. The second major contributor is degraded alphas from nearby copper structure directly facing the crystals. Alpha radiation with an energy of 4 to 8 MeV may lose part of its energy within the supporting copper structure before depositing the rest of its energy in a crystal. The final major contributor to the background in the ROI is from surface alpha contamination originating from the crystal itself. For a crystal surface facing the external copper shield, a partially contained alpha particle may lose part of its energy within the crystal and deposit the rest of its energy in an inert material. The signature of degraded alphas originating from both copper or crystal surface can be approximated by the flat continuum from 2.7 MeV to 3.9 MeV in the energy spectrum. The degraded alpha continuum is assumed to extend into the $0\nu\beta\beta$ ROI. From the background model of Cuoricino, the Compton scattered gamma contributes to $40\% \pm 10\%$ of the background in
Figure 4.1: Three types of background sources of CUORE bolometer modules are illustrated: 1) γ background from bolometer material and cryostat, 2) α background from the copper supporting structure, and 3) α background from TeO₂ crystal surfaces.

the ROI while the degraded alpha from the copper and the degraded alpha the crystals make up 50%±20% and 10%±10% of the ROI background respectively [Sil06].

Many efforts were taken to reduce the background levels from the primary three sources when upgrading from Cuoricino to CUORE. Cuoricino and CUORE-0 were housed in a 25-year-old dilution refrigerator built with no special precaution taken to reduce to the natural uranium and thorium isotopes in its components. The gamma background can be significantly reduced by careful radio-purity material screening and CUORE will be operated in a cryostat custom-built with ultra-low radioactive impurity levels. The extrapolated gamma background in the ROI is about 20 times smaller than that of Cuoricino [Ard05].

Alpha contaminations from the crystal surfaces can be greatly reduced with improved cleaning and handling procedures as well as coincidence analysis among different towers. Since CUORE has 19 closely packed towers, an alpha particle
emitted from a crystal surface will likely hit a nearby crystal instead of the inert copper shielding. Coincidence analysis utilizing the summed energy spectrum will identify the partially contained alpha particles from crystal surfaces and reject the background contamination due to crystal surface contamination in the ROI.

Unfortunately, the coincidence analysis cannot reject alpha contamination originating from the surface of nearby copper structures. Therefore, to reduce the impact of alpha background from copper surfaces, the collaboration re-designed the copper support structures to reduce the copper surface area directly facing each bolometer. The impact of copper thermal shielding is also reduced when scaling from Cuoricino to CUORE as the multi-tower arrangement allows the outer towers to shield the inner towers from directly seeing the thermal shields.

In this chapter we will discuss two bolometric test runs, the Three Tower Test (TTT)\cite{Ale13} and the Parylene Test\cite{Zhu15}, designed to investigate various copper surface treatment techniques for the suppression of surface alpha contamination.
4.1.1 Copper surface treatment with the Three Tower Test (TTT)

In the TTT, three 12-crystal bolometer arrays were constructed to validate and compare three different techniques used to mitigate the surface alpha background. All the crystals were inherited from the Cuoricino experiment but were cleaned with the surface treatment procedure developed for CUORE. The copper structure used in the TTT was fabricated from the same batch of NOSV copper used in CUORE and cleaned using the same basic procedure. The major difference in the towers comes from the treatment of the surface of copper support structure (Figure 4.2).

Polyethylene Tower (T1) - The copper structure was wrapped with a 70 micron thick layer of polyethylene (PE) film after cleaning with Micro-90 soap, water, and citric acid. The goal of the PE film is to range out problematic alpha particles between 5 to 7 MeV.

LNGS Tower (T2) - The copper was cleaned using a purely chemical process with ultra-pure reagents developed at LNGS. The process includes cleaning the copper with soap and water before subjecting the copper to chemical electroerosion, etching, and passivation.

Legnaro Tower (T3) - The copper was cleaned at Legnaro National Lab (LNL) using a technique including tumbling, electropolishing, chemical etching, and magneto plasma etching (TECM).

T2 and T3 attempt to reduce surface alpha contamination through cleaning techniques while T1 attempts to suppress the surface alpha contamination using PE film. For both T2 and T3, a surface layer of copper was etched off to eliminate surface contamination introduced in the fabrication process. Special care was taken in packaging and handling to avoid re-contamination after the surface treatment. The TTT was constructed in summer 2009 and acquired data from September
2009 to January 2010 in the Hall A Cuoricino cryostat at LNGS. The results of the TTT will be summarized in section 4.4, ultimately, the TECM technique was chosen for the construction of CUORE-0 and CUORE.

4.2 Validation of Parylene coating

4.2.1 Parylene coating

The Parylene Test run utilizes a conformal coating of Parylene film over the CUORE copper frames to provide a barrier between the copper and the TeO$_2$ crystals. Similar to the PE coated tower of the TTT, alphas particles emitted from the copper surface may stop in the Parylene film given an adequate thickness of the film.

Parylene is a trade name for the Poly-para-Xylylene coating film formed by chemical vapor deposition (for more info, see, e.g. [Wik13]). Parylene film can be coated conformally at room temperature with a uniform and controllable thickness. It is widely used in printed circuit boards and medical devices as a moisture and dielectric barrier. There are many different kinds of Parylene but Parylene-C is the most widely used polymer. Parylene-C has one chlorine group per p-Xylylene repeat unit.

The deposition process starts with [2.2]paracyclophane, otherwise known as dimer. When heated up under vacuum, e.g. 90°C for Parylene-C dimer, the dimer vaporizes and enters a higher temperature furnace at about 690°C, where it breaks into its monomeric form. The monomer finally reaches the deposition chamber at room temperature and deposits on all surfaces conformally as a thin polymer film. For parylene-C, the deposition speed is on the order of 1 nm per second.

We used SRIM [Zie13] to simulate the stopping range of 1 to 10 MeV alphas in
Figure 4.3: Alpha stopping range in Parylene-C. A 50 µm-thick layer of Parylene-C film was applied to stop the 5.3 MeV alpha from $^{210}$Po. The curve is simulated using SRIM.

Parylene-C film, as shown in Figure 4.3. For CUORE, the most problematic 5.3 MeV alpha from $^{210}$Po stops in about 40 µm of Parylene-C film. In our validation run, we coated, on average, 50 µm-thick Parylene-C film on the copper frames to range out potential surface alpha contamination.

4.2.2 Dimer selection

Before applying the Parylene coating, we first studied the radioactive properties of potential dimers to limit the introduction of additional radioactive background. Three different types of Parylene dimers were assayed using Inductively Coupled Plasma Mass Spectrometry (ICP-MS) at the LNGS underground counting facil-
Table 4.1: Radioactivity assay of three different kinds of Parylene dimers using ICP-MS technique at LNGS underground counting facility. The second sample was selected to be used in the validation run.

<table>
<thead>
<tr>
<th>Dimer Type</th>
<th>Pb (ppb)</th>
<th>K (ppb)</th>
<th>U (ppt)</th>
<th>Th (ppt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUORE-I</td>
<td>300</td>
<td>15</td>
<td>20</td>
<td>8200</td>
</tr>
<tr>
<td>CUORE-II</td>
<td>&lt; 200</td>
<td>&lt; 2</td>
<td>&lt; 20</td>
<td>&lt; 40</td>
</tr>
<tr>
<td>CUORE-III</td>
<td>200</td>
<td>130</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

ity. The results are listed in Table 4.1. The second sample, called Galxyl C (Parylene-C type) dimer from a northern Italian vendor, provided the best overall radioactivity limits among the samples and was chosen to be the dimer of choice for the bolometric validation run.

4.2.3 Bolometer array preparation and assembly

A 12-crystal bolometer array was assembled for cryogenic bolometric testing after all the copper components of the array were coated with $\sim 50 \mu$m-thick Parylene-C film.

To establish a direct comparison between the PE tower of the TTT and the Parylene tower, all of the copper parts were cleaned following the standard procedure established in the PE tower cleaning of TTT [Ale13]. The copper frame parts were first assembled together with Teflon spacers in place without the crystals (See Figure 4.4 left). The assembled structure was then coated in Labcoter 2010 from Parylene Deposition System with Galxyl C dimers. The coating process lasted about 12 hours and deposited a 50 $\mu$m-thick layer Parylene-C film. The lateral copper shield closest to the copper frames was also coated with Parylene-C film of a similar thickness.
All the crystals used in the Parylene tower were re-used from the CUORE Crystal Validation Runs (CCVR) \cite{Ale12}. Unlike CUORE-0 and CUORE, two NTD thermistors and one silicon resistive heater were glued on each crystal. The Parylene tower used thermistors and heaters with pre-bonded wires so two thermistors were used to provide better redundancy and flexibility in terms of performance for data analysis.

The assembled bolometer array (See Figure 4.4 right) was then attached to the CUORE R&D cryostat in Hall C of LNGS for the bolometric test. Data taking started in October 2012 and lasted for two months.

### 4.3 Data production

Parylene tower data processing is based off of the same procedure as Cuoricino and is slightly different from the data processing procedure of CUORE-0 described in subsection 3.3.4. The key differences are that the amplitude of raw pulses are
evaluated using only the OF technique and the gain instabilities are corrected by using the heater-TGS technique or by using the 5407 keV alpha line from $^{210}$Po for crystals without heaters. Energy is calibrated using $^{232}$Th gamma sources inserted into the external shields of the cryostat and the calibration is performed using a third-order polynomial function fit using gamma peaks from the $^{232}$Th spectrum as well as the $^{210}$Po alpha peak. A third-order polynomial function can be used without fear of calibration instabilities experienced in CUORE-0 at high energy due the high statistics in the 5407 keV peak from $^{210}$Po.

### 4.3.1 Baseline jumps

ADC saturation from detector baseline jumps due to thermal disturbances or electronic noise is common in CUORE bolometers. Detectors in the Parylene tower experienced an abnormal amount of baseline jumps in comparison to other bolometric runs (Figure 4.5). One preliminary hypothesis for the phenomenon was that the baseline jumps were caused by an effect of the thermal properties of Parylene that would settle out with time. However, we observed a consistent number of baseline jumps throughout the entire data sample. The excessive number of baseline jumps were also seen in the data of other bolometer modules running concurrently with the Parylene Tower. Unfortunately, the issue was never fully understood and the intervals were removed during data production (Figure 4.6).

### 4.3.2 Pulser flagging

During data collection in CUORE-0, the heater pulses are flagged in the DAQ upon being sent so that they can easily be filtered out in the data selection process. Unfortunately, the heater flag was not active during the Parylene tower run. In order to flag the heater pulses of each channel, an algorithm was developed to flag pulser events based off of the timing interval of the pulser events for each channel.
Figure 4.5: Baseline (mV) vs. Time (ns) plot for a noisy dataset with a large number of bad intervals (Dataset 6079, Channel 6).

4.3.3 Data selection

Data used in the analysis must pass two levels of data selection cuts (listed in Table 4.2). The first level cuts are global which are decided a priori to maintain detector performance. Global cuts are designed to reject time intervals with excessive baseline noise or ADC saturation as well as re-triggered events, events with physical pulses and noise spikes within the same acquisition window. Because of the subtraction of time intervals, these cuts induce a dead time within detectors that can be accounted for by reducing the live time of the detector.

The second level of cuts are event-based which comprise of pulse shape requirements, pile-up rejection, and coincidence selection. Pulse shape cuts are used to reject non-physical shaped pulses and need to be tuned based off of the quality of the dataset. Pulse shape parameters include the OF filtered rise/decay time as
well as variables that measure deviation of the OF filtered pulse from the average detector pulse. The pulse shape cut values were tuned by channel using data from dataset 6073. Pile-up events, where there are more than one physical pulse within one acquisition window, inhibit a proper evaluation of the pulse amplitude and are rejected by imposing a paralyzable dead window of 7.1 s to each triggered event (3.1 s before each trigger and 4.0 s after each trigger).

4.3.4 Energy spectra and resolution

The energy resolution of each channel was evaluated by performing Gaussian fits on the 2615 keV peak of each channel and taking the FWHM (Table 4.3). As some crystals had multiple operational thermistors attached, the thermistor with the best energy resolution, or the primary channel, on each crystal was chosen for
Table 4.2: Global and pulse shape cuts

<table>
<thead>
<tr>
<th>Cuts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>IsSignal</td>
</tr>
<tr>
<td>Filter_ReTrigger</td>
</tr>
<tr>
<td>Filter_RejectBadIntervals</td>
</tr>
<tr>
<td>NumberOfPulses==1</td>
</tr>
<tr>
<td>TimeSinceSignalEvent_SameChannel &gt; 3.1</td>
</tr>
<tr>
<td>TimeUntilSignalEvent_SameChannel &gt; 4.0</td>
</tr>
<tr>
<td><a href="mailto:PulserFlagByRegularTiming@IsPulser.fValue">PulserFlagByRegularTiming@IsPulser.fValue</a>==0</td>
</tr>
<tr>
<td>OF_SecondAmplitude==0</td>
</tr>
<tr>
<td>abs(BaselineSlope)&lt;0.1</td>
</tr>
<tr>
<td>Multiplicity==1 or Multiplicity==2</td>
</tr>
</tbody>
</table>

the final analysis. The total single hit spectrum, shown in Figure 4.7, is a sum of primary channels 2, 4, 6, 7, 8, 10, 12, 13, 15, and 16.

### 4.3.5 Efficiency calculation

The pile-up rejection cut creates a loss in efficiency for the collection of true events as events may lie within the dead time window of the detector. For a paralyzable dead window, the measured event rate $m$ depends on the true event rate $r$ and the dead time $T$

$$m = r e^{-rT}. \quad (4.1)$$

The pile-up efficiency is defined to be the ratio of the two rates where the approximation of $m = r$ in the exponential can be used as the counting rate in the bolometers is low.

The pulse shape cut efficiency was evaluated by integrating over a ±50 keV
Table 4.3: Energy resolution (in units of keV) of each channel evaluated as the FWHM of a Gaussian fit to the 2615 keV peak.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Dataset 6073</th>
<th>Dataset 6079</th>
<th>Dataset 6082</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.79 ± 0.14</td>
<td>3.85 ± 0.13</td>
<td>3.50 ± 0.28</td>
<td>3.71 ± 0.18</td>
</tr>
<tr>
<td>3</td>
<td>10.29 ± 0.35</td>
<td>11.36 ± 0.5</td>
<td>8.99 ± 0.74</td>
<td>10.22 ± 0.53</td>
</tr>
<tr>
<td>4</td>
<td>4.18 ± 0.11</td>
<td>4.27 ± 0.11</td>
<td>3.90 ± 0.16</td>
<td>4.12 ± 0.13</td>
</tr>
<tr>
<td>5</td>
<td>4.81 ± 0.21</td>
<td>4.71 ± 0.21</td>
<td>4.17 ± 0.32</td>
<td>4.56 ± 0.25</td>
</tr>
<tr>
<td>6</td>
<td>4.65 ± 0.16</td>
<td>4.34 ± 0.17</td>
<td>4.69 ± 0.36</td>
<td>4.56 ± 0.23</td>
</tr>
<tr>
<td>7</td>
<td>5.02 ± 0.18</td>
<td>5.06 ± 0.35</td>
<td>5.22 ± 0.25</td>
<td>5.10 ± 0.26</td>
</tr>
<tr>
<td>8</td>
<td>3.34 ± 0.45</td>
<td>5.17 ± 0.69</td>
<td>4.54 ± 1.4</td>
<td>4.35 ± 0.85</td>
</tr>
<tr>
<td>9</td>
<td>3.81 ± 0.09</td>
<td>3.90 ± 0.08</td>
<td>3.91 ± 0.14</td>
<td>3.87 ± 0.10</td>
</tr>
<tr>
<td>10</td>
<td>3.05 ± 0.09</td>
<td>3.10 ± 0.08</td>
<td>3.18 ± 0.17</td>
<td>3.11 ± 0.11</td>
</tr>
<tr>
<td>11</td>
<td>6.12 ± 0.21</td>
<td>5.99 ± 0.17</td>
<td>7.78 ± 0.37</td>
<td>6.63 ± 0.25</td>
</tr>
<tr>
<td>12</td>
<td>4.80 ± 0.14</td>
<td>4.99 ± 1.7</td>
<td>4.34 ± 0.21</td>
<td>4.71 ± 0.68</td>
</tr>
<tr>
<td>13</td>
<td>2.22 ± 0.39</td>
<td>4.41 ± 1.61</td>
<td>5.13 ± 2.03</td>
<td>3.92 ± 1.34</td>
</tr>
<tr>
<td>14</td>
<td>4.80 ± 0.61</td>
<td>5.75 ± 0.7</td>
<td>4.54 ± 2.0</td>
<td>5.03 ± 1.11</td>
</tr>
<tr>
<td>15</td>
<td>7.25 ± 1.15</td>
<td>6.19 ± 1.48</td>
<td>8.52 ± 3.38</td>
<td>7.32 ± 2.00</td>
</tr>
<tr>
<td>16</td>
<td>2.94 ± 0.06</td>
<td>3.52 ± 0.07</td>
<td>3.73 ± 0.1</td>
<td>3.40 ± 0.08</td>
</tr>
</tbody>
</table>

window around the 2615 keV peak. The efficiency was defined to be the ratio of the pulses that passed the pulse shape cuts and the total number of pulses. The 2615 keV peak was chosen because the pulse shape cuts are energy dependent as it was the closest peak to the Q-value of the $0\nu\beta\beta$ decay. The anti-coincidence cut efficiency was evaluated similarly to the pulse shape cut efficiency except on the 1460 keV peak of $^{40}$K. The $^{40}$K peak was chosen as it is not emitted in coincidence with any other gamma source. The combined efficiencies are shown in Table 4.4.
4.3.6 Polonium subtraction

While the crystal growth process is effective at eliminating contaminants from within the crystal bulk, one contaminant it cannot eliminate is polonium. Polonium is difficult to remove during the growth process as it is chemically similar to tellurium; as a result, there is an inherent bulk contaminant of $^{210}$Po in all of the crystals. $^{210}$Po decays via alpha emission (5407 keV Q-value) with a half-life of 174 days and due to its relatively short half-life, is expected to decay to equilibrium with the rest of the $^{238}$U chain in CUORE. However, the Parylene tower uses relatively young crystals from CCVR where the $^{210}$Po contamination has not decayed away.

We applied an analysis technique to subtract the excess counting rate due to
Table 4.4: Efficiencies of various cuts for each channel. The anti-coincidence efficiency was only evaluated for the primary channel on each bolometer.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Pulse Shape</th>
<th>Anti-coincidence</th>
<th>Pile-up</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>98.61%</td>
<td>94.12%</td>
<td>93.25%</td>
<td>86.55% ± 1.67%</td>
</tr>
<tr>
<td>3</td>
<td>90.91%</td>
<td>N/A</td>
<td>94.56%</td>
<td>85.96% ± 3.70%</td>
</tr>
<tr>
<td>4</td>
<td>96.43%</td>
<td>92.07%</td>
<td>92.72%</td>
<td>82.32% ± 0.82%</td>
</tr>
<tr>
<td>5</td>
<td>97.33%</td>
<td>N/A</td>
<td>84.64%</td>
<td>82.37% ± 1.03%</td>
</tr>
<tr>
<td>6</td>
<td>95.83%</td>
<td>95.63%</td>
<td>82.31%</td>
<td>75.43% ± 2.19%</td>
</tr>
<tr>
<td>7</td>
<td>98.82%</td>
<td>87.61%</td>
<td>93.42%</td>
<td>80.88% ± 0.86%</td>
</tr>
<tr>
<td>8</td>
<td>97.50%</td>
<td>94.92%</td>
<td>94.61%</td>
<td>87.55% ± 1.53%</td>
</tr>
<tr>
<td>9</td>
<td>97.60%</td>
<td>N/A</td>
<td>94.52%</td>
<td>92.26% ± 1.15%</td>
</tr>
<tr>
<td>10</td>
<td>97.13%</td>
<td>95.24%</td>
<td>92.37%</td>
<td>85.44% ± 0.83%</td>
</tr>
<tr>
<td>11</td>
<td>97.77%</td>
<td>N/A</td>
<td>88.61%</td>
<td>86.63% ± 0.89%</td>
</tr>
<tr>
<td>12</td>
<td>98.01%</td>
<td>92.51%</td>
<td>93.02%</td>
<td>84.34% ± 0.81%</td>
</tr>
<tr>
<td>13</td>
<td>97.08%</td>
<td>96.22%</td>
<td>87.92%</td>
<td>82.13% ± 1.17%</td>
</tr>
<tr>
<td>14</td>
<td>93.67%</td>
<td>N/A</td>
<td>95.83%</td>
<td>89.76% ± 2.81%</td>
</tr>
<tr>
<td>15</td>
<td>96.13%</td>
<td>96.38%</td>
<td>94.43%</td>
<td>87.48% ± 1.04%</td>
</tr>
<tr>
<td>16</td>
<td>97.71%</td>
<td>94.62%</td>
<td>94.90%</td>
<td>87.74% ± 1.06%</td>
</tr>
</tbody>
</table>

the $^{210}$Po on the surface of the younger crystals [Ale13]. If the $^{210}$Po contamination in the crystal is close enough to the surface for the alpha to escape, it can release part of its energy into either another detector (creating an M2 event) or inert material nearby (creating an M1 event). We subtract off M2 events with a total energy of $5407 \pm 50$ keV to correct for the excess due to surface contamination of $^{210}$Po. For the correction to the M1 rate, the number of M2 events that pass the total energy cut from the polonium peak was multiplied by the ratio of the crystal faces facing inert material and the total number of crystal faces (32 of the 60 faces). This correction assumes the polonium contamination is uniformly
Table 4.5: Counting rates in various sections of the alpha region for both anti-coincidence (M1) and double-hit (M2) spectra. Units are in c/keV/kg/yr.

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Continuum (2700-3200 keV)</th>
<th>$^{190}$Pt (3200-3400 keV)</th>
<th>Continuum (3400-3900 keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.0680 ± 0.013</td>
<td>0.392 ± 0.051</td>
<td>0.0444 ± 0.011</td>
</tr>
<tr>
<td>M2</td>
<td>0.0131 ± 0.006</td>
<td>0.0065 ± 0.006</td>
<td>0.0078 ± 0.005</td>
</tr>
<tr>
<td>U and Th (4000-5000 keV)</td>
<td></td>
<td>$^{210}$Po (5000-6000 keV)</td>
<td>U and Th (6000-8000 keV)</td>
</tr>
<tr>
<td>M1</td>
<td>0.102 ± 0.012</td>
<td>20.16 ± 0.162</td>
<td>0.0209 ± 0.004</td>
</tr>
<tr>
<td>M2</td>
<td>0.021 ± 0.005</td>
<td>0.587 ± 0.028</td>
<td>0.0039 ± 0.002</td>
</tr>
</tbody>
</table>

distributed on all crystal faces.

4.4 Background index

The primary goal of the Parylene Test run was to evaluate the effectiveness of Parylene coating in mitigating surface alpha contaminants. As CUORE bolometers cannot distinguish between alpha, beta, and gamma particles, it is difficult to extract the contribution of the alpha background index directly from the $0\nu\beta\beta$ ROI. Instead, we can utilize the flat continuum region above the 2615 keV energy gamma peak as a proxy for the alpha background index in the ROI. We chose the continuum region from 2700 to 3900 keV, excluding the region from 3200 to 3400 keV where there was a contribution from the alpha line from $^{190}$Pt, to evaluate the alpha background index. The counts of each channel were corrected by their respective cut efficiencies. The M1 and M2 background rates in various regions are shown in Table 4.5 and plotted in Figure 4.8.

4.4.1 CUORE-0 background index

One of the primary goals of CUORE-0 was to test the cleaning and assembly procedures for CUORE. The copper support structure of both CUORE-0 and CUORE are cleaned using the TECM technique developed for the TTT. The background
Figure 4.8: Alpha region of the background spectrum highlighting various sections depending on the sources present. The event rates of each region are listed in Table 4.5.

A similar study of the background index in the alpha region was performed on the CUORE-0 data. CUORE-0 performed beyond expectations and saw an alpha background index of almost 7 times lower than that of Cuoricino. The full comparison of the M1 background rates of the Parylene Test, TTT, Cuoricino, and CUORE-0 is shown in Table 4.6.
Figure 4.9: Comparison between the anti-coincidence background spectra of CUORE-0 (red) and Cuoricino (blue).

Table 4.6: Background rate comparison of different towers without multiplicity cuts.

<table>
<thead>
<tr>
<th>Tower</th>
<th>Rate (2.7-3.9 MeV) (c/keV/kg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUORE-0</td>
<td>0.016 ± 0.001</td>
</tr>
<tr>
<td>CUORICINO</td>
<td>0.116 ± 0.0002</td>
</tr>
<tr>
<td>T1</td>
<td>0.068 ± 0.006</td>
</tr>
<tr>
<td>T2</td>
<td>0.120 ± 0.012</td>
</tr>
<tr>
<td>T3</td>
<td>0.070 ± 0.008</td>
</tr>
<tr>
<td>Parylene</td>
<td>0.0812 ± 0.010</td>
</tr>
<tr>
<td>Parylene (Po-sub)</td>
<td>0.0732 ± 0.010</td>
</tr>
</tbody>
</table>
CHAPTER 5

Background model of CUORE-0

5.1 Background studies

The experimental sensitivity of the current generation of $0\nu\beta\beta$ experiments is often limited by the level of background contamination. Reducing potential background sources and mitigating backgrounds already present are important to improving the sensitivity of future experiments. The key to a quantitative understanding of radioactive background sources comes from an accurate model of the background spectrum. A model of the background that can provide accurate background source contamination levels and source locations is an invaluable tool for improving future experiments.

5.2 Background sources in CUORE-0

There are a large number of potential background sources for CUORE-0 coming from a multitude of locations. To narrow down the background candidates, we take advantage of background models and experience gained from previous experiments. As CUORE-0 was operated in the same cryostat as Cuoricino, many of the gamma emitting backgrounds originating from the cryostat observed in Cuoricino were expected in CUORE-0. Most of the material close to the detectors have also been screened using various techniques; Table 5.1 shows the 90% C.L. upper limits of bulk contamination for materials screened prior to the experiment and provide a basis for the expected background activities. Materials were screened at
the LNGS [Lau04], Milano-Bicocca [Sal13] and Baradello [BCF04] laboratories.

The event rates of each of the major gamma lines were evaluated in comparison to the rates of the same gamma lines from Cuoricino taking into account possible decay due to the time difference between the two experiments. For this study, plausible radioactive sources with clear spectrum identifications based off of either material screening or previous experience with Cuoricino were considered. Table 5.2 lists the gamma peaks observed in the background spectrum of CUORE-0 as well as their most probable source isotopes.

Table 5.1: 90% C.L. upper limits on bulk contamination of various detector components screened through various techniques. Techniques include bolometric, Neutron Activation Analysis (NAA), Inductively Coupled Plasma Mass Spectrometry (ICP-MS), and High Purity Ge diode gamma spectrometry (HPGe).

<table>
<thead>
<tr>
<th>Detector Component</th>
<th>$^{232}$Th (Bq/kg)</th>
<th>$^{238}$U (Bq/kg)</th>
<th>$^{40}$K (Bq/kg)</th>
<th>Screening Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>TeO$_2$</td>
<td>$\leq 8.4 \times 10^{-7}$</td>
<td>$\leq 6.7 \times 10^{-7}$</td>
<td>$\leq 1 \times 10^{-3}$</td>
<td>bolometric + HPGe</td>
</tr>
<tr>
<td>Glue</td>
<td>$\leq 8.9 \times 10^{-4}$</td>
<td>$\leq 1.0 \times 10^{-2}$</td>
<td>$\leq 47 \times 10^{-3}$</td>
<td>NAA</td>
</tr>
<tr>
<td>Gold Wires</td>
<td>$\leq 4.1 \times 10^{-2}$</td>
<td>$\leq 1.2 \times 10^{-2}$</td>
<td></td>
<td>ICP-MS</td>
</tr>
<tr>
<td>Si Heaters</td>
<td>$\leq 3.3 \times 10^{-4}$</td>
<td>$\leq 2.1 \times 10^{-3}$</td>
<td></td>
<td>NAA</td>
</tr>
<tr>
<td>NTD Ge Thermistors</td>
<td>$\leq 4.1 \times 10^{-3}$</td>
<td>$\leq 1.2 \times 10^{-2}$</td>
<td></td>
<td>Producer spec.</td>
</tr>
<tr>
<td>CuPEN Cables</td>
<td>$\leq 1.0 \times 10^{-3}$</td>
<td>$\leq 1.3 \times 10^{-3}$</td>
<td>$\leq 1.3 \times 10^{-2}$</td>
<td>NAA + HPGe</td>
</tr>
<tr>
<td>PTFE Supports</td>
<td>$\leq 6.1 \times 10^{-6}$</td>
<td>$\leq 2.2 \times 10^{-5}$</td>
<td></td>
<td>HPGe</td>
</tr>
<tr>
<td>NOSV Copper</td>
<td>$\leq 2.0 \times 10^{-6}$</td>
<td>$\leq 6.5 \times 10^{-5}$</td>
<td>$(7 \pm 2) \times 10^{-4}$</td>
<td>NAA + HPGe</td>
</tr>
<tr>
<td>Otokumpo Copper</td>
<td>$\leq 4.4 \times 10^{-4}$</td>
<td>$\leq 6.7 \times 10^{-4}$</td>
<td>$(3 \pm 1) \times 10^{-3}$</td>
<td>NAA + HPGe</td>
</tr>
<tr>
<td>Roman Lead</td>
<td>$\leq 3.3 \times 10^{-5}$</td>
<td>$\leq 4.5 \times 10^{-5}$</td>
<td>$\leq 2.3 \times 10^{-5}$</td>
<td>NAA + HPGe</td>
</tr>
<tr>
<td>External Lead</td>
<td>$\leq 2.6 \times 10^{-4}$</td>
<td>$\leq 4.1 \times 10^{-4}$</td>
<td>$\leq 9 \times 10^{-3}$</td>
<td>HPGe</td>
</tr>
</tbody>
</table>

5.2.1 Natural radioactive sources

The natural radioactivity of materials make up the majority of the background spectrum of CUORE-0. Two long-lived radioactive decay chains found naturally in almost all materials are the $^{232}$Th and $^{238}$U chains; gamma lines originating from both decay chains are noticeably present in the CUORE-0 background spectrum.
The $^{232}$Th decay chain has relatively short-lived daughters so there are few points where the secular equilibrium of the chain can be broken. $^{208}$Tl (3.053 m half-life), a daughter of the $^{232}$Th decay chain, is significant to the evaluation of $0\nu\beta\beta$ as its 2615 keV gamma line is the only naturally occurring gamma line with a higher energy than the Q-value of $^{130}$Te.

Unlike $^{232}$Th, the $^{238}$U chain has multiple long-lived daughters where secular equilibrium can be broken. The primary gamma lines from the $^{238}$U chain originate from $^{214}$Bi (19.9 m half-life). One particular break-off point is $^{222}$Rn (3.823 d half-life), a noble gas produced through $^{238}$U decays in the environment nearby the detector. $^{222}$Rn is carried in air and attaches to the surface of materials that are exposed to the environment; $^{222}$Rn is also a parent to $^{214}$Bi. CUORE-0 limits the potential $^{222}$Rn contamination by keeping the cryostat under a constant nitrogen flux.

$^{40}$K ($1.27 \times 10^9$ yr half-life) is another long-lived naturally occurring radioiso-
otope present in almost all materials. Unlike $^{232}$Th or $^{238}$U, the signature of $^{40}$K is a single gamma line at 1460 keV.

Natural radioactivity in materials doesn’t always manifest itself in gamma peaks; high energy beta radiation incident on lead may generate bremsstrahlung radiation creating a continuous gamma spectrum without peaks. In CUORE-0, we observed a continuous bremsstrahlung spectrum due to $^{210}$Bi (5.01 d half-life) contained in the external lead shielding of the cryostat up to 1000 keV.

5.2.2 Cosmogenically activated sources

Radioisotopes may be produced by cosmic ray spallation on materials as the materials are created or delivered while on the surface of the Earth. To limit potential cosmogenic activation, most materials used for the detector, such as the TeO$_2$ crystals, are shipped by sea and by land to LNGS rather than delivered by plane where the cosmic ray flux is greater [Wan15a].

In CUORE-0, both the copper and the tellurium may be cosmogenically activated to produce $^{60}$Co (5.27 yr half-life). The signature of $^{60}$Co is two gamma lines produced simultaneously at 1173 keV and 1332 keV. The gamma cascade from $^{60}$Co can also produce a summed gamma peak of 2505 keV near the Q-value of $0\nu\beta\beta$. Copper can also be activated to produce $^{54}$Mn (312.5 d half-life), emitting a gamma at 834.8 keV.

Tellurium can be further activated to produce $^{125}$Sb (2.8 yr half-life). Traces of $^{125}$Sb can be found in the gamma lines at 428 keV and 600 keV as well as a gamma line at 145 keV from its decay product $^{125m}$Te (58 d half-life).

5.2.3 Fall-out products

Man-made nuclear fall-out products may pollute materials such as lead and copper. We identified the signatures of both $^{207}$Bi (31.55 yr half-life) and $^{137}$Cs (30.07
yr half-life) in the background spectrum of CUORE-0. The 661 keV gamma line from $^{137}\text{Cs}$ and the 1063 keV gamma line from $^{207}\text{Bi}$ were observed in Cuoricino and attributed to contaminations in the internal copper cryostat shields and the Roman lead shield respectively.

### 5.2.4 External environmental sources

Cosmic ray interactions may produce secondary particles like muons. Even though the cosmic ray muon flux is greatly mitigated by a factor of $10^6$ by going underground to LNGS, stray muons are still able to penetrate the rock and interact directly in the detector or in materials surrounding the detector. A muon passing directly through the detector is likely to trigger on multiple bolometers creating a clear coincidence signal that can be rejected. However, a muon interacting near the detector may produce secondary particles that can generate a continuous spectrum in the detector. The muon flux at LNGS has been accurately measured by the LVD and MACRO experiments to be $3 \times 10^{-8}$ $\mu/(s\cdot cm^2)$ and is used as a constant throughout the background studies \cite{Amb95}.

Neutrons produced either by interactions of muons in the rock or by $(\alpha, n)$ fission reactions in the rock can also generate a continuous spectrum within the detector. The thermal neutron spectrum was estimated through calculation and its contributions to the CUORE-0 background spectrum are negligible.

### 5.2.5 Two neutrino double beta decay

When all of the other background sources have been eliminated, the last standing background to $0\nu\beta\beta$ is the $2\nu\beta\beta$ decay spectrum. While its impact on the $0\nu\beta\beta$ ROI can be mitigated with good energy resolution, $2\nu\beta\beta$ is an irreducible background source so a precise evaluation of its decay rate is not only a sanity check of the background model but also critical to evaluating the $0\nu\beta\beta$ signal.
Table 5.2: Gamma lines observed in the CUORE-0 background spectrum.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Source Isotope</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>$^{154}$Eu</td>
</tr>
<tr>
<td>145</td>
<td>$^{125m}$Te</td>
</tr>
<tr>
<td>511</td>
<td>$e^+e^-$ (Annihilation)</td>
</tr>
<tr>
<td>583</td>
<td>$^{208}$Tl</td>
</tr>
<tr>
<td>599</td>
<td>$^{207}$Bi</td>
</tr>
<tr>
<td>609</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>661</td>
<td>$^{137}$Cs</td>
</tr>
<tr>
<td>723</td>
<td>$^{154}$Eu</td>
</tr>
<tr>
<td>768</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>803</td>
<td>$^{210}$Po</td>
</tr>
<tr>
<td>836</td>
<td>$^{54}$Mn</td>
</tr>
<tr>
<td>911</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>934</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>965</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>969</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>1063</td>
<td>$^{207}$Bi</td>
</tr>
<tr>
<td>1120</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1173</td>
<td>$^{60}$Co</td>
</tr>
<tr>
<td>1238</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1332</td>
<td>$^{60}$Co</td>
</tr>
<tr>
<td>1378</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1408</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1461</td>
<td>$^{40}$K</td>
</tr>
<tr>
<td>1496</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>1509</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1588</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1593</td>
<td>$^{208}$Tl (Double Escape)</td>
</tr>
<tr>
<td>1621</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1631</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>1730</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1764</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1847</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2104</td>
<td>$^{208}$Tl (Single Escape)</td>
</tr>
<tr>
<td>2118</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2204</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2448</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2615</td>
<td>$^{208}$Tl</td>
</tr>
</tbody>
</table>
5.3 Background simulations

CUORE-0 simulations are based off of the Geant4 toolkit [Ago03]. The CUORE-0 geometry implementation, MCuoreZ, models the full CUORE-0 tower geometry as well as its surrounding cryostat and shields (Figure 5.2). The implemented detector elements are listed in Table 5.3 along with the volume, surface area, and mass of each element. Each detector element represents both a source where backgrounds can be generated as well as an absorber where particles can undergo interactions.

Potential backgrounds can be distributed both uniformly within the detector element (Bulk) or near the surface of the detector element following either an exponential density profile or a uniform density profile (Surface). The exponential density profile was used as the conventional standard under the hypothesis that contaminations on the surface of a material gradually diffuse into the bulk of the material. The background sources can also be generated in arbitrary geometric distributions such as points, lines, disks, cylindrical, and spherical distributions.

5.3.1 Source generation

A variety of source particles can be generated in Geant4 and have been implemented in MCuoreZ. Single particles such as electrons, positrons, muons, photons, alphas, and neutrons can be generated individually or in any combination with variable kinetic energies. Radioactive isotopes can also be generated using the built-in radioactive decay database in Geant4. Radioactive decays can also be assigned initial energies to account for the possibility of decaying in flight as well as decaying from an excited metastable state.

While Geant4 has built-in functions for single radioactive isotope decays, there are no implementations for generating entire radioactive decay chains. The major implementation issue for radioactive decay chains is the temporal correlations of
Figure 5.2: Geant4 implementation of the CUORE-0 geometry. The full cryostat with the tower is shown on the left and the tower is shown individually on the right.
Table 5.3: Cryostat elements implemented in the CUORE-0 simulations along with their volume, surface area, and mass values.

<table>
<thead>
<tr>
<th>Element</th>
<th>Volume (cm$^3$)</th>
<th>Surface (cm$^2$)</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TeO$_2$ Crystals</td>
<td>6500</td>
<td>7800</td>
<td>39000</td>
</tr>
<tr>
<td>Copper Frames</td>
<td>291.3</td>
<td>2314.02</td>
<td>2610.04</td>
</tr>
<tr>
<td>Copper Box</td>
<td>773.41</td>
<td>9467.18</td>
<td>6929.71</td>
</tr>
<tr>
<td>PTFE Spacers</td>
<td>101.61</td>
<td>1498.69</td>
<td>223.54</td>
</tr>
<tr>
<td>PEN Tapes</td>
<td>21.87</td>
<td>583.63</td>
<td>20.45</td>
</tr>
<tr>
<td>Cu Pads</td>
<td>0.31</td>
<td>63.39</td>
<td>2.76</td>
</tr>
<tr>
<td>Wire Trays</td>
<td>67.51</td>
<td>1457.62</td>
<td>604.89</td>
</tr>
<tr>
<td>NTD Thermistors</td>
<td>0.47</td>
<td>15.6</td>
<td>2.49</td>
</tr>
<tr>
<td>50mK Shield</td>
<td>1894.79</td>
<td>15948.3</td>
<td>16977.32</td>
</tr>
<tr>
<td>Roman Lead Shield</td>
<td>17823.3</td>
<td>20898.8</td>
<td>202294.46</td>
</tr>
<tr>
<td>600mK Shield</td>
<td>2196.7</td>
<td>18310.8</td>
<td>19682.43</td>
</tr>
<tr>
<td>IVC Shield</td>
<td>5089.32</td>
<td>28109</td>
<td>45600.31</td>
</tr>
<tr>
<td>OVC Shield</td>
<td>20167.9</td>
<td>87370.2</td>
<td>180704.38</td>
</tr>
<tr>
<td>Superinsulation</td>
<td>1830</td>
<td>36432.95</td>
<td>163200</td>
</tr>
<tr>
<td>Mixing Chamber</td>
<td>480.1</td>
<td>648.09</td>
<td>4301.71</td>
</tr>
<tr>
<td>Main Bath</td>
<td>16296.7</td>
<td>69095.6</td>
<td>146018.43</td>
</tr>
<tr>
<td>External Lead Shield</td>
<td>2177600</td>
<td>233888</td>
<td>24693984</td>
</tr>
</tbody>
</table>

subsequent decays in a chain. It is impractical to generate an entire decay chain at one decay site as many radioactive isotopes have half-lives longer than the live time of the detector.

To simulate decay chains in MCuoreZ, a list of radioactive isotopes containing the decay chain ordering and branching ratio was used as an input. Radioactive isotopes in the chain are generated in sequence following the ordering in the chain while taking into account any potential branching. A simulation time threshold of one year is set to account for decays with extremely long half-lives. Whenever the time difference of a decay with respect to the previous decay in the chain is greater than the time threshold, the location of the subsequent decay is changed.
Using this method, the decay chain can be generated in secular equilibrium and the temporal correlations of decays with short half-lives are kept.

The decays of $2\nu\beta\beta$ and $0\nu\beta\beta$ can be simulated from both the 0+ or 2+ states. The two betas are distributed according to the theoretical shape calculated either via the Primakov-Rosen approximation or according to Kotila and Iachello calculation [PR59] [KI12].

### 5.3.2 Simulation output

Simulations performed with MCuoreZ idealize the bolometers, the only signal recorded in each bolometer is the energy deposited through particle interactions. In a physical bolometer, particle depositions convert into heat, ionization, and light signals which are then converted into electric signals creating the signal pulses seen in Figure 3.12. As MCuoreZ does not simulate signal pulses, external factors that can affect the detector response like temperature and noise fluctuations are not included in the simulations. In order to add aspects of detector response to the simulations, a secondary program based on C++ called g2root was used.

Minimal time information is kept from the output of MCuoreZ, a timing algorithm is applied to mold the raw Geant4 output into a data format similar to that of the physical detector:

1. **Timing Resolution** - The timing resolution representing the slow signal response of each bolometer is first set by integrating energy depositions occurring in the same bolometer within a time window of 10 ms.

2. **Absolute Timing** - The absolute timing of events is then set by first correlating physical coincidences and then randomly distributing events with an event rate of 1 mHz while maintaining the physical coincidences. The event rate is an input measured from the background data of CUORE-0.
3. **Coincidence Selection** - A time window of 10 ms, identical to the background data processing, is used to correlate coincident multiplets occurring in multiple bolometers.

4. **Pile-up Rejection** - A paralyzable dead time window of 7.1 s (3.1 s before and 4.0 s afterwards) is imposed on each bolometer to remove pile-up events. The pile-up rejection window reflects the same restriction placed on the CUORE-0 background data.

The detector response properties of bolometers can also be added with g2root by smearing the energy of the simulated data. Two response functions can be used to smear the input data, the standard detector response function is a single Gaussian over all bolometers with a FWHM that varies linearly with energy. The second detector response function is the calibration detector response (Equation 3.3), a double Gaussian function with a shifted secondary Gaussian energy empirically obtained from the calibration data of CUORE-0. The variation of the FWHM of the Gaussians across all energy scales was evaluated by fitting gamma peaks from 145 to 2615 keV with Gaussians. The distribution of FWHM from the gamma peaks was fitted with a linear function and used as an input to both of the detector response functions (Figure 5.3).

The threshold efficiency of each bolometer was measured through specialized low energy pulser “N-pulses” runs which were performed after periods of cryostat or electronic maintenance. The threshold efficiency for each bolometer is expected to follow a Gaussian cumulative distribution function and can be parameterized in g2root by fitting the trigger efficiency with an error function [Pip15]

\[ \epsilon(E) = \frac{1}{2} \left( \frac{Erf\left(\frac{E - \mu}{\sigma\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2} \right) \]  

(5.1)

where \( \mu \) represents the minimum energy of the threshold trigger and \( \sigma \) represents the baseline resolution (resolution at 0 keV) of the detector. The input parameters
for the threshold of each bolometer is shown in Figure 5.4. The thresholds of bolometers 1 and 10 were set to a flat cutoff of 50 keV as they did not have active heaters for the N-pulses measurements.

G2root also has the ability to correct for any energy-scale non-linearities. Alpha particles experience an energy quenching effect in CUORE bolometers resulting in a higher observed energy compared to gamma particles. Since CUORE-0 bolometers are calibrated using gamma lines, the alpha energies in the simulations are shifted higher by roughly 0.8% of their theoretical energy to reflect the energy quenching effect. The energy quenching effect in the alpha region is a phenomena that is still not well understood for CUORE bolometers and will be a topic for future R&D.

While the simulation framework is successful at basic energy reconstruction, there are still aspects that can be improved in future updates to provide simulation data more compatible with physical data. Simulation of signal pulses is the
Figure 5.4: Threshold parameters, the $\mu$ of each channel is plotted at the markers while the $\sigma$ of each channel is represented by the error bars. Channels 1 and 10 are missing as they have a flat cutoff of 50 keV; Channel 49 is a dead channel.

first major step. The energy of CUORE bolometers is evaluated from the amplitude of signal pulses in the data processing procedure, simulated signal pulses can provide feedback on potential systematics of the data processing procedure. The detector response function of CUORE-0 is distinctly non-Gaussian and still not fully understood. Simulating signal pulses can also potentially debug any systematics associated with the detector response function as well as probe the origins of the response shape.

Energy scale non-linearities originating from the alpha peaks are also a major point of future study. CUORE bolometers have no particle identification but alpha peaks are evaluated at a higher energy than gamma peaks after calibration. This effect could be a product of detector response or simply due to the extrapolation of the calibration function. A model of the phonon interactions representing the heat deposited by each type of particle interaction could potentially help disentangle the energy quenching effects seen in alpha particles.
5.3.3 Monte Carlo comparison to calibration data

Before we can be confident in utilizing the Monte Carlo simulations to analyze the background data, the simulations were vetted with high statistics data acquired from calibration runs. We simulated two $^{232}$Th wire strings placed outside of the OVC within the external lead shield at the same site as the physical locations of the calibration source strings. A trigger rate of 2.6 Hz, mirroring the calibration data, was applied to generate the calibration simulation.

The calibration simulation was then normalized to the calibration data between 2000 keV and 3000 keV so a comparison between the two spectra could be made. The simulation spectrum matches well with the calibration data except below 500 keV where both the M1 and M2 simulation spectra show slight deviations from the data (Figure 5.5). Different normalization ranges were also used and produced compatible results.

To test the accuracy of our simulation cuts as well as our ability to project simulated source activities, we extrapolated the source activity and compared it to HPGe measurements of the calibration strings. The activity of the simulated calibration source strings can be reconstructed by first counting each of the major gamma peaks from the $^{232}$Th chain after normalization to the calibration data, then dividing by the simulation efficiency of each peak, and finally dividing by the run time of the calibration data. The Monte Carlo simulations produced a projected activity of $\sim 96$ Bq, consistent with the $\sim 90$ Bq we obtained through HPGe measurements of the calibration strings.
Figure 5.5: Comparison of calibration simulation (red) with calibration data (blue).
5.4 Background model

This work models the background of CUORE-0 by fitting the background spectrum using a frequentist approach. This analysis was performed in parallel with a Bayesian analysis performed using JAGS (Just Another Gibbs Sampler [Plu03]). The two methods are used to cross check each other and their outputs converge.

5.4.1 Log-Likelihood derived $\chi^2$

The principle of maximum likelihood may be used as a starting point to derive a goodness-of-fit test statistic (with weights $w_i = 1/\sigma_i^2$) when the parent distributions are Gaussian with variances $\sigma_i^2$ [BC84]. Starting with the likelihood function of the Poisson distribution

$$L_p(y; n) = \prod_i^n \frac{e^{-y_i}y_i^n}{n_i!}$$

(5.2)

where $y_i$ is the number of events predicted by the model in the $i^{th}$ bin and $n_i$ is the number of observed events in the $i^{th}$ bin, we can first construct the likelihood ratio

$$\lambda = \frac{L(y; n)}{L(m; n)}$$

(5.3)

where $m$ are the true (unknown) values of $n$. Using the likelihood ratio test theorem, we can create a test statistic that asymptotically obeys a $\chi^2$ distribution according to Wilks’ theorem.

$$\chi^2_\lambda = -2 \ln \lambda = -2 \ln (L(y; n)) + 2 \ln (L(m; n))$$

(5.4)

For Poisson distributed histograms, the unknown $m$ can be replaced by the
measured values $n$ which leads to the Poisson likelihood $\chi^2_\lambda$

$$\chi^2_\lambda = 2 \sum_i y_i - n_i + n_i \ln (n_i/y_i). \quad (5.5)$$

The advantage of using $\chi^2_\lambda$ over Neyman or Pearson $\chi^2$ as a test statistic is twofold. Firstly, the maximum likelihood method preserves the area of Poisson distributed data while the other methods either overestimate or underestimate the area. Secondly, the likelihood ratio test allows for confidence interval setting using $\chi^2_\lambda$ as a general $\chi^2$ statistic.

5.4.2 Adaptive binning algorithm

$\chi^2$ fits to binned data are sensitive to the choice of binning: the optimal bin width depends heavily on the distribution being fit. If the binning is too fine, the bin content may not be normally distributed and the $\chi^2$ approximation will fail. Minimum bin content becomes a problem in the “alpha region” of the energy spectrum above 3000 keV where events from gamma sources become scarce. Likewise, if the binning is too coarse, the spectral shape information may be lost and background sources with similar spectral shapes may become indistinguishable. This property is important in the “gamma region” of the energy spectrum below 3000 keV. To maintain an optimal binning of the spectrum, an adaptive binning algorithm was adopted to minimize the constraints of binning (Figure 5.6):

1. **Base binning of 1 keV:** A base binning of 1 keV was chosen so that the spectral shape in the gamma region (below 3000 keV) is maintained.

2. **Minimum bin content of 50 counts:** To make sure the bin content in each bin is normally distributed, a minimum bin content of 50 counts is required for all bins above 50 keV. Starting from 50 keV and working up to 7000 keV, if any bin has less than 50 counts, it is combined with the next
bin until the number of counts is $\geq 50$. Below 50 keV the base binning of 1 keV is kept to maintain the spectral shape due to detector thresholds.

3. **Scaling bin content by bin width:** To maintain spectral shape for the combined bins, the number of events in each bin ($n_i$) is multiplied by the base bin width and divided by the bin width ($w_i$).

4. **Error bars:** The error bars are similarly scaled for each bin, for $n_i$ events in the $i^{th}$ bin, the error bar on the bin content is $\sigma_i = (1/w_i)\sqrt{n_i}$. The error bars are used for visualization purposes only and not used in the calculation of $\chi^2$.

5.4.3 **Binning in the alpha region**

Along with the adaptive binning algorithm, special care was taken when binning the alpha region. Since CUORE-0 was calibrated on the gamma lines of $^{232}$Th, the final calibration point with high statistics stops at 2615 keV. The calibration function needs to be extrapolated to the higher energy alpha lines. When summing data over multiple channels as well as multiple datasets, minor calibration non-linearities combine and produce alpha peaks that exhibit non-Gaussian features. In addition, we observed that the energy of alpha peaks may be slightly quenched relative to gamma peaks; alpha peaks calibrated using the extrapolated calibration function are observed at a slightly higher energy than their theoretical values. This becomes a problem when fitting the spectrum as many sources originating from the surface of materials also contain distorted peak features. We are unable to distinguish between a peak distortion due to physical means or an irregularity due to a summing of calibration non-linearities in the alpha region beyond the calibration energy range.

To compensate for these issues, the alpha region uses the adaptively binned spectra and adds to it by binning such that each alpha peak is fully contained
within one bin. This solution simplifies the spectrum for fitting but introduces a problem with degenerate spectra. Multiple combinations of surface sources can now combine to provide similar solutions to fit the spectrum. We studied the potential systematic errors of this binning and the details are in section 5.7.

5.4.4 Platinum peak

$^{190}$Pt is a naturally occurring isotope of platinum decaying via alpha emission with a Q-value of 3249 keV ($2.0 \times 10^{15}$ yr half-life). Platinum contamination can be introduced into the bulk of CUORE crystals when the crystals are grown in platinum crucibles. We observed a peak at 3265 keV in the M1 spectrum of CUORE-0 that we attribute to $^{190}$Pt as there are no other long-lived alpha emitters within an energy of ±100 keV of the observed peak. As the contamination appears only in the M1 spectrum, it is likely due to a contamination within the bulk of the crystals. Because bulk alpha contamination does not contribute to the rest of the background spectrum and because the shift in energy of the Q-value peak is inconsistent with the 0.8% quenching factor observed in the other alpha peaks, the 3265 keV peak is excluded from the background model.
Figure 5.6: Histogram of the background spectrum of M1 (top) and M2 (bottom) using variable binning.

Figure 5.7: Alpha region binning
5.4.5 Fitting function

The fitting function for the background model \( F \) was constructed as a linear combination of each individual source probability density function (PDF) constructed from Monte Carlo simulations. A fitter based on the Minuit minimization package [JR75] and the ROOT data analysis toolkit [BR97] was used in this work.

\[
F = \sum_i \sum_j \sum_k \left( \frac{c_{ij}}{N_{M1}} f_{ijk} \right)
\]

(5.6)

where \( c_{ij} \) are floating parameters for each PDF \( f_{ijk} \). The floating parameters are normalized to unity by the integral of the M1 spectrum \( N_{M1} \) to avoid large derivative changes which can create computation problems for the fitter. Essentially each \( c_{ij} \) represents the percentage of the anti-coincidence (M1) spectrum that each PDF \( f_{ijk} \) accounts for. The double-hit (M2) spectrum was normalized such that the \( \frac{M1}{M2} \) ratio of each PDF before normalization is fixed.

The maximum likelihood \( \chi^2_{\lambda} \) minimized by the fitter is defined as

\[
\chi^2_{\lambda} = 2 \times \sum_i^{\text{bins}} \left( F_i - n_i + n_i \ln \frac{n_i}{F_i} \right)
\]

(5.7)

where \( F_i \) is the model at the \( i^{\text{th}} \) bin and \( n_i \) is the number of counts from the data in the \( i^{\text{th}} \) bin. As the maximum likelihood \( \chi^2_{\lambda} \) is defined from the Poisson distribution, the statistical error of each bin is properly accounted for by using \( n_i \) instead of the bin content value \( \frac{n_i}{w_i} \).

5.5 Toy Monte Carlo fitting validation

A set of Toy Monte Carlo data was used to test the validity and robustness of the fitter against known background sources. Four toy datasets were produced, each with the projected statistics of CUORE-0 and a different combination of Monte Carlo generated output. The first two toy datasets (combi1 and combi2)
are simple with no potential degeneracy in background sources while the last two toy datasets (scombi1 and scombi2) contain a large list of background sources.

To test the fitter, each toy dataset was passed through the adaptive binning algorithm and then fit with its known constituents. The results of the fits are shown in Figure 5.8, 5.9, 5.10, and 5.11.

For toy spectra with a small number of input Monte Carlo spectra, the fitter was able to correctly reproduce the chosen Monte Carlo combination. However, toy spectra 3 and 4 have a large number of input Monte Carlo spectra and even though the fit quality was maintained, the fitter was not able to uniquely distinguish between all of the input spectra. This suggests the degeneracy of individual Monte Carlo spectra due to similarity in their spectral shape as well as lack of statistics in the combined data.

### 5.5.1 Grouping degenerate spectra locations

With a significant number of degenerate spectra, it is possible that there may be representative Monte Carlo simulations that are able to describe different but similar source locations. To study the degenerate Monte Carlo spectra further, we replaced a PDF originally present in the toy dataset with one similar in spectral shape (for example, replacing Copper Frame $^{232}$Th with the Tower Shield $^{232}$Th) and performed the fit to see if the fit quality was still maintained. Toy dataset 3 and dataset 4 were primarily used to perform these tests as they contained the most degenerate spectra. From the tests, we split the cryostat elements into several groups starting with volumes closest to the bolometers. The grouped locations are listed in Table 5.4.

Cryostat elements with the degenerate spectra were grouped approximately according to their location from the crystals. For degenerate cryostat elements directly facing the crystals (copper holder), the spectra were combined and nor-
Figure 5.8: Toy dataset 1 (combi1) spectrum fit for M1 (top) and M2 (bottom).

Toy dataset 1 is composed of 4 sources placed close to the detector.
Figure 5.9: Toy dataset 2 (combi2) spectrum fit for M1 (top) and M2 (bottom).
Toy dataset 2 is composed of 7 background sources and is a combination of gamma and alpha sources.
Figure 5.10: Toy dataset 3 (scombi1) spectrum fit for M1 (top) and M2 (bottom).
Toy dataset 3 is composed of 6 background sources consisting only of gamma sources.
Figure 5.11: Toy dataset 4 (scombi2) spectrum fit for M1 (top) and M2 (bottom).
Toy dataset 4 is composed of 17 gamma sources placed throughout all cryostat locations.
malized under the assumption that the source elements have the same contami-
nation per unit surface area as the cryostat elements were cleaned using the same 
surface treatment technique. For degenerate cryostat elements farther away from 
the crystals (internal and external shields), the spectra were combined and nor-
malized under the assumption that they have the same contamination per unit 
volume as the cryostat elements are built from the same material.

Table 5.4: Cryostat element grouping.

<table>
<thead>
<tr>
<th>Source Group</th>
<th>Constituent Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>TeO₂ Crystals</td>
<td>TeO₂ Crystals</td>
</tr>
<tr>
<td>Copper Holder</td>
<td>Copper Frames, Tower Shield, PTFE Spacers</td>
</tr>
<tr>
<td>Internal Shields</td>
<td>50mK Shield, 600mK Shield, Inner Vacuum Chamber</td>
</tr>
<tr>
<td>Roman Lead Shield</td>
<td>Roman Lead Shield</td>
</tr>
<tr>
<td>External Shields</td>
<td>Outer Vacuum Chamber, Super Insulation, Main Bath</td>
</tr>
<tr>
<td>External Lead Shield</td>
<td>External Lead Shield</td>
</tr>
</tbody>
</table>

5.6 Background fit

A simultaneous fit was performed on the M1 and M2 spectra from 500 keV to 7000 
keV. The region below 500 keV was excluded from the fitting range due to the 
deviations in the comparison between MC and calibration data (Figure 5.5). The 
region between 3150 keV and 3400 keV was combined into one bin and skipped 
in the M1 spectrum due to the shifted ¹⁹⁰Pt peak.

5.6.1 Simulation and data selection

Only a select number of background sources are used to avoid over-parameterizing 
the model. All possible radioactive sources were generated in the bulk of each 
source group unless previous experimental evidence suggested otherwise. The 
selection of the list of background sources that contribute to the alpha region will
Table 5.5: Cuts used on the CUORE-0 background data.

<table>
<thead>
<tr>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>NumberOfPulses == 1</td>
</tr>
<tr>
<td>TimeUntilSignalEvent_SameChannel &gt; 4.0</td>
</tr>
<tr>
<td>TimeUntilSignalEvent_SameChannel &lt; 0</td>
</tr>
<tr>
<td>TimeSinceSignalEvent_SameChannel &gt; 3.1</td>
</tr>
<tr>
<td>TimeSinceSignalEvent_SameChannel &lt; 0</td>
</tr>
<tr>
<td>NormBaselineSlope &lt; 4.8 &amp;&amp; NormBaselineSlope &gt; -4</td>
</tr>
<tr>
<td>NormRiseTime &lt; 4.8 &amp;&amp; NormRiseTime &gt; -4</td>
</tr>
<tr>
<td>NormDecayTime &lt; 4.8 &amp;&amp; NormDecayTime &gt; -4</td>
</tr>
<tr>
<td>NormDelay &lt; 4.8 &amp;&amp; NormDelay &gt; -4</td>
</tr>
<tr>
<td>NormTVL &lt; 5.3 &amp;&amp; NormTVL &gt; -6</td>
</tr>
<tr>
<td>NormTVR &lt; 5.3 &amp;&amp; NormTVR &gt; -6</td>
</tr>
<tr>
<td>Multiplicity.Sync == 1 or 2</td>
</tr>
</tbody>
</table>

be detailed in section 5.7 A total of 50 plausible sources was used in the total fit; the PDFs of each of the sources are shown in Appendix A.

All of the unblinded data, except for dataset 2049, from the $0\nu\beta\beta$ analysis was used for the background fit. Dataset 2049 was excluded as it was collected under the first data campaign where the cryostat suffered unstable conditions with high detector noise. All data passed through the standard cuts used in the $0\nu\beta\beta$ analysis (listed in Table 5.5), producing a total live time of 0.874 yr corresponding to a detector exposure of 33.42 kg-yr.

5.6.2 Fit results

The best fit spectra, which will be referred to as the reference fit, for both M1 and M2 returned by the fitter is shown in Figure 5.12. The normalized residuals (defined as $\sigma_i = \frac{n_i - x_i}{\sqrt{n_i}}$ where $n_i$ and $x_i$ are the data and model of each $i^{th}$ bin) of the reference fit with the data and the distribution of the residuals are shown in Figures 5.13 and 5.14. The residual distribution is fit with a Gaussian function and represents the overall quality of the fit. The fit quality statistics are listed in Table 5.6 with the “Probability” statistic defined to be the probability for an observed $\chi^2$ exceeding the $\chi^2$ value by chance.
We studied the progression of the goodness-of-fit $\chi^2_{\lambda}$ statistic with energy (Figure 5.16) and found that the major increases in $\chi^2_{\lambda}$ occur at the 1460 keV line from $^{40}$K and the 2615 keV line from $^{208}$Tl in the M1 spectrum. As a fine base binning of 1 keV was used, the fit quality seems to be heavily predicated on the fit quality of the model matching the high statistics gamma lines. Because the peaks were not fit perfectly by the model, we double checked the reconstruction of the gamma peaks by calculating the areas under each major gamma peak through fitting the peaks with Gaussian functions; then compared the model with the data (Figure 5.17).

Figure 5.15 shows the covariance matrix of the fit parameters. Elements 35, 39, 44, and 47, corresponding to $^{238}$U within the Roman lead, OVC, and external lead as well as the external lead $^{210}$Bi bremsstrahlung spectra have the most correlation with the $2\nu\beta\beta$ spectrum. The covariance value for each element of the matrix is converted to a dimensionless correlation value lying between -1 and 1 by weighting the covariance with the errors of the parameters

$$\rho_{ij} = \frac{\text{cov}[x_i, x_j]}{\sigma_i \sigma_j}. \quad (5.8)$$

The background contributions in the $0\nu\beta\beta$ ROI extracted from the reference fit are listed in Table 5.7. The contributions are split into several groups separated by decay isotope ($^{232}$Th, $^{238}$U, and $^{60}$Co), as well as contamination region (surface and bulk). The background in the $0\nu\beta\beta$ is dominated by Compton scattered gamma events from the 2615 keV line. A comparison to the background model of Cuoricino in the $0\nu\beta\beta$ ROI is also made. A detailed breakdown of all source contributions to the $0\nu\beta\beta$ ROI can be found in Appendix A.

The activity of each source $A_i$ in units of Bq/kg can be reconstructed by

$$A_i = \frac{c_i N_{M1}}{\epsilon_i M_i T}. \quad (5.9)$$

where the product $c_i N_{M1}$ is the number of counts for the source in the M1 spec-
trum, $\epsilon_i$ is the MC efficiency of the source, $M_i$ is the mass of the source element, and $T$ is the live time of the experiment. The extracted activities of each source element are listed in Table 5.8 and Table 5.9.

Table 5.6: Fit quality statistics.

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>NDF</th>
<th>$\chi^2$/NDF</th>
<th>Probability</th>
<th>Residual RMS (M1)</th>
<th>Residual RMS (M2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3048.48</td>
<td>1929</td>
<td>1.58</td>
<td>$8.84 \times 10^{-54}$</td>
<td>1.26</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 5.7: Contributions to $0\nu\beta\beta$ ROI (2470 to 2570 keV) split into various groups. A comparison with the Cuoricino background model is made in the last column.

<table>
<thead>
<tr>
<th>Source</th>
<th>Event Rate (counts/keV/yr)</th>
<th>Percentage (%)</th>
<th>Cuoricino (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{238}$U Chain</td>
<td>$0.48 \pm 0.07$</td>
<td>$19 \pm 2$</td>
<td></td>
</tr>
<tr>
<td>$^{232}$Th Chain</td>
<td>$2.02 \pm 0.13$</td>
<td>$65 \pm 4$</td>
<td></td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>$0.59 \pm 0.07$</td>
<td>$15 \pm 2$</td>
<td></td>
</tr>
<tr>
<td>Surface (TeO$_2$)</td>
<td>$0.064 \pm 0.02$</td>
<td>$2 \pm 1$</td>
<td>$10 \pm 10$</td>
</tr>
<tr>
<td>Surface (Copper)</td>
<td>$0.547 \pm 0.07$</td>
<td>$18 \pm 2$</td>
<td>$50 \pm 20$</td>
</tr>
<tr>
<td>Bulk (Gamma)</td>
<td>$2.46 \pm 0.15$</td>
<td>$80 \pm 5$</td>
<td>$40 \pm 10$</td>
</tr>
</tbody>
</table>
Figure 5.12: Fit results for M1 (top) and M2 (bottom) between 500 keV and 7000 keV. The lower panel on each plot shows the ratio of data/model.
Figure 5.13: Normalized residuals for M1 (top) and M2 (bottom).
Figure 5.14: Distribution of the normalized residuals for M1 (left) and M2 (right). The distributions are fit with a Gaussian.

Figure 5.15: Covariance Matrix generated by Minuit. Refer to Table 5.8 and Table 5.9 for the sources associated with each index number.
Figure 5.16: Progress of the $\chi^2_\lambda$ with respect to energy.

Figure 5.17: The ratio (data/model) of the area of prominent gamma peaks. The area of the gamma peaks in both the model and data are evaluated by a fit using a Gaussian function.
Table 5.8: Background sources as well as their normalizations within the fit and their source activities converted to the units of Bq/kg. Parameters that are “Limited” reach 0 in the fitter.

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Normalization</th>
<th>Source Activity (Bq/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>TeO$_2$ $2\nu/3\beta$</td>
<td>0.0686 ± 0.0041</td>
<td>3.43×10$^{-5}$ ± 0.20×10$^{-5}$</td>
</tr>
<tr>
<td>1</td>
<td>TeO$_2$ $^{40}$K</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>2</td>
<td>TeO$_2$ $^{60}$Co</td>
<td>0.00326 ± 0.00055</td>
<td>3.59×10$^{-6}$ ± 0.60×10$^{-6}$</td>
</tr>
<tr>
<td>3</td>
<td>TeO$_2$ $^{125}$Sb</td>
<td>0.0156 ± 0.0020</td>
<td>8.16×10$^{-6}$ ± 1.0×10$^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>TeO$_2$ $^{232}$Th only</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>5</td>
<td>TeO$_2$ $^{228}$Ra$-^{208}$Pb</td>
<td>5.94×10$^{-5}$ ± 0.00024</td>
<td>5.70×10$^{-9}$ ± 0.23×10$^{-8}$</td>
</tr>
<tr>
<td>6</td>
<td>TeO$_2$ $^{238}$U$-^{230}$Th</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>7</td>
<td>TeO$_2$ $^{230}$Th only</td>
<td>5.49×10$^{-4}$ ± 0.88×10$^{-4}$</td>
<td>2.70×10$^{-7}$ ± 0.43×10$^{-7}$</td>
</tr>
<tr>
<td>8</td>
<td>TeO$_2$ $^{226}$Ra$-^{210}$Pb</td>
<td>0.000440 ± 0.00018</td>
<td>4.32×10$^{-8}$ ± 1.7×10$^{-8}$</td>
</tr>
<tr>
<td>9</td>
<td>TeO$_2$ Surface $^{232}$Th only 0.001 μm</td>
<td>8.317×10$^{-5}$ ± 3.9×10$^{-5}$</td>
<td>5.65×10$^{-8}$ ± 2.7×10$^{-8}$</td>
</tr>
<tr>
<td>10</td>
<td>TeO$_2$ Surface $^{228}$Ra$-^{208}$Pb 0.001 μm</td>
<td>0.00247 ± 0.00025</td>
<td>3.47×10$^{-7}$ ± 0.35×10$^{-7}$</td>
</tr>
<tr>
<td>11</td>
<td>TeO$_2$ Surface $^{238}$U$-^{230}$Th 0.001 μm</td>
<td>0.00183 ± 0.00012</td>
<td>4.07×10$^{-7}$ ± 0.28×10$^{-7}$</td>
</tr>
<tr>
<td>12</td>
<td>TeO$_2$ Surface $^{230}$Th only 0.001 μm</td>
<td>0.000312 ± 0.00013</td>
<td>2.16×10$^{-7}$ ± 0.87×10$^{-7}$</td>
</tr>
<tr>
<td>13</td>
<td>TeO$_2$ Surface $^{226}$Ra$-^{210}$Pb 0.001 μm</td>
<td>0.00311 ± 0.00022</td>
<td>5.17×10$^{-7}$ ± 0.37×10$^{-7}$</td>
</tr>
<tr>
<td>14</td>
<td>TeO$_2$ Surface $^{210}$Pb 0.01 μm</td>
<td>0.0429 ± 0.00054</td>
<td>1.51×10$^{-5}$ ± 0.02×10$^{-5}$</td>
</tr>
<tr>
<td>15</td>
<td>TeO$_2$ Surface $^{210}$Pb 1 μm</td>
<td>0.00424 ± 0.00044</td>
<td>1.34×10$^{-6}$ ± 0.14×10$^{-6}$</td>
</tr>
<tr>
<td>16</td>
<td>TeO$_2$ Surface $^{232}$Th 1</td>
<td>0.00166 ± 0.00084</td>
<td>1.77×10$^{-7}$ ± 0.90×10$^{-7}$</td>
</tr>
<tr>
<td>17</td>
<td>TeO$_2$ Surface $^{232}$Th 10 μm</td>
<td>0.00129 ± 0.00065</td>
<td>1.27×10$^{-7}$ ± 0.64×10$^{-7}$</td>
</tr>
<tr>
<td>18</td>
<td>Copper Holder Surface $^{238}$U 100 μm</td>
<td>0.0141 ± 0.0031</td>
<td>3.75×10$^{-5}$ ± 0.83×10$^{-5}$</td>
</tr>
<tr>
<td>19</td>
<td>Copper Holder Surface $^{232}$Th 100 μm</td>
<td>0.0291 ± 0.0061</td>
<td>7.78×10$^{-5}$ ± 1.6×10$^{-5}$</td>
</tr>
<tr>
<td>20</td>
<td>Copper Holder Surface $^{238}$U 10 μm</td>
<td>0.00662 ± 0.0016</td>
<td>1.07×10$^{-5}$ ± 0.26×10$^{-5}$</td>
</tr>
<tr>
<td>21</td>
<td>Copper Holder Surface $^{210}$Pb 1 μm</td>
<td>0.00242 ± 0.00051</td>
<td>1.40×10$^{-5}$ ± 0.29×10$^{-5}$</td>
</tr>
<tr>
<td>22</td>
<td>Copper Holder Surface $^{210}$Pb 0.1 μm</td>
<td>0.00432 ± 0.00071</td>
<td>2.28×10$^{-5}$ ± 0.37×10$^{-5}$</td>
</tr>
<tr>
<td>23</td>
<td>Copper Holder Surface $^{210}$Pb 0.01 μm</td>
<td>0.0198 ± 0.00062</td>
<td>1.01×10$^{-4}$ ± 0.03×10$^{-4}$</td>
</tr>
<tr>
<td>24</td>
<td>Copper Holder Surface $^{232}$Th 0.01 μm</td>
<td>5.24×10$^{-5}$ ± 0.00018</td>
<td>5.02×10$^{-8}$ ± 17.4×10$^{-8}$</td>
</tr>
<tr>
<td>25</td>
<td>Copper Holder $^{54}$Mn</td>
<td>0.00129 ± 0.00018</td>
<td>1.19×10$^{-5}$ ± 0.17×10$^{-5}$</td>
</tr>
<tr>
<td>26</td>
<td>Copper Holder $^{238}$U</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>27</td>
<td>Copper Holder $^{232}$Th</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>28</td>
<td>Copper Holder $^{40}$K</td>
<td>0.0479 ± 0.0077</td>
<td>2.38×10$^{-3}$ ± 0.38×10$^{-3}$</td>
</tr>
<tr>
<td>29</td>
<td>Copper Holder $^{60}$Co</td>
<td>0.000920 ± 0.016</td>
<td>6.07×10$^{-6}$ ± 108×10$^{-6}$</td>
</tr>
</tbody>
</table>
Table 5.9: Background sources as well as their normalizations within the fit and their source activities converted to the units of Bq/kg. Parameters that are “Limited” reach 0 in the fitter.

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Normalization</th>
<th>Source Activity (Bq/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Internal Shields $^{137}$Cs</td>
<td>0.00197 ± 0.00024</td>
<td>5.96×10$^{-6}$ ± 0.72×10$^{-6}$</td>
</tr>
<tr>
<td>31</td>
<td>Internal Shields $^{238}$U</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>32</td>
<td>Internal Shields $^{232}$Th</td>
<td>0.0305 ± 0.015</td>
<td>2.65×10$^{-5}$ ± 1.3×10$^{-5}$</td>
</tr>
<tr>
<td>33</td>
<td>Internal Shields $^{40}$K</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>34</td>
<td>Internal Shields $^{60}$Co</td>
<td>0.0110 ± 0.026</td>
<td>1.56×10$^{-5}$ ± 3.7×10$^{-5}$</td>
</tr>
<tr>
<td>35</td>
<td>Roman Lead $^{238}$U</td>
<td>0.0103 ± 0.011</td>
<td>1.69×10$^{-5}$ ± 1.8×10$^{-5}$</td>
</tr>
<tr>
<td>36</td>
<td>Roman Lead $^{232}$Th</td>
<td>0.00772 ± 0.0030</td>
<td>1.08×10$^{-5}$ ± 0.42×10$^{-5}$</td>
</tr>
<tr>
<td>37</td>
<td>Roman Lead $^{40}$K</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>38</td>
<td>Roman Lead $^{60}$Co</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>39</td>
<td>OVC $^{238}$U</td>
<td>0.0627 ± 0.0080</td>
<td>1.64×10$^{-3}$ ± 0.21×10$^{-3}$</td>
</tr>
<tr>
<td>40</td>
<td>OVC $^{232}$Th</td>
<td>0.0337 ± 0.016</td>
<td>7.20×10$^{-4}$ ± 3.3×10$^{-4}$</td>
</tr>
<tr>
<td>41</td>
<td>OVC $^{40}$K</td>
<td>0.0400 ± 0.0089</td>
<td>9.03×10$^{-3}$ ± 2.0×10$^{-3}$</td>
</tr>
<tr>
<td>42</td>
<td>OVC $^{60}$Co</td>
<td>0.0133 ± 0.020</td>
<td>1.85×10$^{-4}$ ± 2.8×10$^{-4}$</td>
</tr>
<tr>
<td>43</td>
<td>OVC $^{207}$Bi</td>
<td>0.00566 ± 0.00041</td>
<td>1.73×10$^{-4}$ ± 0.12×10$^{-4}$</td>
</tr>
<tr>
<td>44</td>
<td>External Lead $^{210}$Bi</td>
<td>0.0952 ± 0.0040</td>
<td>3.12×10$^{-4}$ ± 0.13×10$^{-4}$</td>
</tr>
<tr>
<td>45</td>
<td>External Lead $^{40}$K</td>
<td>Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>46</td>
<td>External Lead $^{232}$Th</td>
<td>0.0362 ± 0.0076</td>
<td>5.39×10$^{-5}$ ± 1.1×10$^{-5}$</td>
</tr>
<tr>
<td>47</td>
<td>External Lead $^{238}$U</td>
<td>0.0460 ± 0.0080</td>
<td>8.62×10$^{-5}$ ± 1.5×10$^{-5}$</td>
</tr>
<tr>
<td>48</td>
<td>External Lead $^{210}$Pb</td>
<td>0.00349 ± 0.00034</td>
<td>8.43×10$^{-6}$ ± 0.81×10$^{-6}$</td>
</tr>
<tr>
<td>49</td>
<td>Bottom External Lead $^{40}$K</td>
<td>0.0836 ± 0.0084</td>
<td>1.85×10$^{-4}$ ± 0.19×10$^{-4}$</td>
</tr>
</tbody>
</table>

5.7 Systematic errors

To construct a sensible but still manageable background model, several approximations were made to simplify the background model. This section summarizes how these assumptions translate to potential systematic errors for evaluation of the $2\nu\beta\beta$ signal. The results of the major systematic sources are summarized in Table 5.10.

We first constructed a reference model using the results of subsection 5.6.2 and compared all changes leading to potential systematic errors to the reference fit. The figure-of-merit used to judge the systematic error was the deviation of
the counting rate of the $2\nu\beta\beta$ spectrum from the reference fit value.

5.7.1 Stability of the fit

A potential bias of the $2\nu\beta\beta$ signal in the model was validated by performing several sanity checks. The fit bias was also evaluated using toy Monte Carlo pseudo-experiments in subsection 5.7.9.

The first check was to repeat the fit while fixing the $2\nu\beta\beta$ spectra normalization constant to 0. It is possible that the $2\nu\beta\beta$ spectra is degenerate with other background sources if the fitter is able to correctly model the spectrum through a combination of raising and lower other background sources. The result of the fit without the $2\nu\beta\beta$ source shows that the region from 1000 keV to 2000 keV in the M1 spectrum has a slight deficit and the fit quality suffers as a result ($\chi^2 = 3375.72$ without $2\nu\beta\beta$, $\chi^2 = 3048.48$ with $2\nu\beta\beta$). The M2 spectrum does not show the deficit as the $2\nu\beta\beta$ contribution in M2 is minimal (Figure 5.19).

Another sanity check was to repeat the fit while artificially increasing the $2\nu\beta\beta$ content by adding more $2\nu\beta\beta$ events into both the M1 and M2 spectrum. The fitting routine can potentially be biased towards the $2\nu\beta\beta$ spectrum which would result in an incorrect evaluation of the $2\nu\beta\beta$ amount when scaled. The results showed no bias in the fit for the $2\nu\beta\beta$ spectrum and the fitter was able to correctly identify the additional $2\nu\beta\beta$ events even in the extreme case where a large percentage of $2\nu\beta\beta$ events were added (Figure 5.18).
Figure 5.19: Fitting the background spectrum without the $2\nu\beta\beta$ spectrum. The total model (red) of the spectrum shows a clear deficit in M1 (top) but not in M2 (bottom) indicating the missing $2\nu\beta\beta$. 
Figure 5.18: Increasing the $2\nu\beta\beta$ amount by 25% of the number of counts in the total spectrum, the fitter is able to correctly reproduce the $2\nu\beta\beta$ content.

5.7.2 Binning and fit range

The choice of binning may introduce systematic biases by skewing the spectral shape. In the gamma region, the bin choice may wash out spectral shape necessary to differentiate between different contaminant locations. The systematic error of binning was evaluated using two methods. Firstly, the fit was evaluated with different minimal bin content varying from a minimum bin content of 10 counts/bin to 90 counts/bin in steps of 10 counts/bin. Secondly, the fit was evaluated varying the base bin size to 2 keV and 3 keV. The systematics involved with the large bin size in the alpha region are described in the next section.

Systematics from the potential fit range were also also studied by varying the fit starting point from 400 to 500 keV in intervals of 10 keV. The fitting range was not varied above 500 keV as the 511 keV gamma peak could be crucial in
disentangling various source locations.

5.7.3 Alpha contamination depth

Because of the loss in spectral shape from the binning in the alpha region, many combinations of surface contaminants with different characteristic depths may offer compatible solutions to the background spectrum.

We can categorize the background sources necessary to reconstruct the alpha region into groups depending on their characteristic contamination depth $l$ relative to the stopping range $s$ of a $4 - 7$ MeV alpha. A contamination with $l \ll s$ means the alpha will most likely escape its initial volume with a majority of its initial energy. A contamination with $l \simeq s$ means the alpha most likely will lose a large amount of its initial energy before it can escape its initial volume.

1. Crystal Bulk - contributes only to the Q-value peaks in the M1 spectrum as the alphas are all fully contained

2. Crystal Surface ($l \ll s$) - contributes to the alpha peaks in the M2 spectrum and the Q-value peaks in the M2 spectrum (characteristic depth from $1 \mu m$ to $100 \mu m$)

3. Crystal Surface ($l \simeq s$) - contributes to the tails of alpha peaks in the M1 and M2 spectra (characteristic depth $< 0.1 \mu m$)

4. Copper Surface ($l \ll s$) - contributes to alpha peaks in the M1 spectrum only (characteristic depth from $1 \mu m$ to $100 \mu m$)

5. Copper Surface ($l \simeq s$) - contributes to the tails of alpha peaks only in the M1 spectrum as most of the energy of the alphas will be contained within the copper (characteristic depth $< 0.1 \mu m$)

We simulated bulk and surface sources with characteristic depths from 0.001
\(\mu m\) to 100 \(\mu m\) in steps of one order of magnitude as possible sources to use in the alpha region for the reference model. We made sure to include sources corresponding to all five of the characteristic depths and allowed them all to float in a fit only in the alpha region. Sources reaching a lower limit were eliminated and the fit was repeated until we obtained a list with the minimum \(\chi^2\) with the exception of bulk sources which were allowed to reach a limit to account for a break in the secular equilibrium of a decay chain within the TeO\(_2\) crystals.

To study the potential systematics arising from the selected list of sources, we first varied the surface sources originating from within the TeO\(_2\) crystals to search for other combinations of sources that gave a similar fit quality. We then varied the surface sources originating from the copper holders searching for similar combinations. Finally, we performed the fit using these new combinations of surface sources and used the maximum deviation of the 2\(\nu\beta\beta\) decay rate from the reference model as the systematic error. A comparison between several combinations of alpha sources that provided similar fitting results in the alpha region can be seen in Figure 5.20.

5.7.4 Variation of fit with detector response function

The default detector response function for CUORE-0, which is applied to all bolometers, is a single Gaussian with a FWHM that varies linearly with energy. However, the response of each bolometer is not uniform and the peak shapes can be non-Gaussian even on an individual bolometer. An accurate response function allows for a fit using finer binning which can distinguish between degenerate source spectra. A detector response function that reflects the prominent gamma peak shapes also improves the overall fit quality of the background model.

In the 0\(\nu\beta\beta\) analysis, an empirical model of the detector response based on the calibration runs was developed using a double Gaussian function as defined in
Figure 5.20: Comparison of alpha region fitting using various source combinations. "TeO$_2$ 1" modified the contamination depth of the sources within the crystals while "Copper 1" and "Copper 2" modified the contamination depth of sources within the copper.

Equation 3.3 The calibration response function was evaluated on every bolometer and dataset combination. To better fit the gamma peaks, the calibration detector response function was applied to the Monte Carlo spectra used in the reference fit. However, as the fit was performed on the entire data sample rather than on individual datasets, an effective FWHM for each bolometer over the entire data sample was used as an input. The effective FWHM for each bolometer was calculated using the same method defined in Equation 3.4. A comparison of the two response functions for the $^{208}$Tl line at 2615 keV and the $^{40}$K line at 1460 keV can be seen in Figure 5.21.

The reference background model was produced using the calibration detector response function as it provided improved fit quality. The decay rate of $2\nu\beta\beta$
Figure 5.21: Comparison of calibration detector response (blue) and the Gaussian detector response (red). Comparison made on both $^{40}$K (left) and $^{208}$Tl (right).

obtained using the Gaussian response function is within one standard deviation of statistical errors returned by the fitter and the difference between the two models is used as the systematic error of the detector response function. The resolution values used in the calibration detector response can be seen in Appendix A. A comparison between the standard Gaussian response function and the calibration detector response function fit to the data on the $^{208}$Tl line at 2615 keV and the $^{40}$K line at 1460 keV can be seen in Figure 5.22.

5.7.5 Pulse shape selection efficiency

The efficiency of the pulse shape selection cuts for removing events can be evaluated using two methods. The standard method used in the $0
\nu\beta\beta$ analysis is to evaluate the efficiency as the fraction of events that survive the selection cuts on gamma peaks in the M1 spectrum from 146 keV to 2615 keV. Gamma peaks are used in the efficiency evaluation as they have high statistics and are composed primarily of physical events rather than spurious noise. This method was not utilized for the reference model as the efficiency function must be extrapolated below 146 keV.
Figure 5.22: Comparison of a fit on the CUORE-0 using the calibration detector response (blue) and the Gaussian detector response (red). The top frame shows the normalized residuals and the bottom frame shows the spectrum.

For the reference fit, the pulse shape selection efficiency was evaluated on a bin-by-bin basis in the M2 spectrum. Each bin in the M2 spectrum can be used for the evaluation of the pulse shape selection efficiency as the rate of accidental coincidence is low; almost all of the events in the M2 spectrum are a result of physical coincidences.

The cut efficiency is then modeled as a function of energy by an exponential

$$
\epsilon_{PSA}(E) = p_0 + p_1 e^{-p_2 E}.
$$

(5.10)

A comparison of the two efficiency curves is shown in Figure 5.23. As a systematic check, we performed the fit using both efficiency curves and the difference in the $2\nu\beta\beta$ decay rate was negligible.
5.7.6 Time varying background sources

Most background sources observed in CUORE-0 are either long-lived in comparison to the live time of the detector or have extremely short half-lives and are continuously produced. These sources exhibit a constant behavior over the live time of CUORE-0 and no special care is necessary when summing over the entire CUORE-0 data sample. Background sources that vary over the live time of CUORE-0 may generate a potential systematic error depending how on their signature varies in the background spectrum.

One time varying background source with a half-life on the order of the live time of the experiment is $^{210}$Po (138.37 d). Polonium is difficult to chemically isolate from tellurium so it is embedded within the bulk of the TeO$_2$ crystals during the growth process. $^{210}$Po is also a naturally occurring radioisotope through the $^{210}$Pb decay chain (partial chain of $^{238}$U). The isolated $^{210}$Po in the bulk of the crystals is a background source that decays over time while $^{210}$Po fed from the
Figure 5.24: Left: Event rate over time of the Q-value peak (blue) with alpha peak (red) of 210Po in the M1 spectrum. Right: Event rate over time of the Q-value peak of 210Po in the M2 spectrum.

210Pb chain is assumed to be constant over time.

As the isolated 210Po is fully contained in the bulk of the crystals, its signature is a peak in the M1 spectrum at the Q-value of 5407 keV. 210Po decaying from the 210Pb chain can be found in several locations in the spectrum and can be observed at the Q-value peak in the M1 spectrum, the alpha peak of 5304 keV in the M1 spectrum, and the Q-value peak in the M2 spectrum. To separate the two 210Po contributions, we compared the event rates of the Q-value peak in the M1 and M2 spectra as well as the alpha peak in the M1 spectrum (Figure 5.24). Only the Q-value peak in the M1 spectrum shows a clear decay over time corresponding to the isolated bulk contamination of 210Po. As the variation of bulk contamination of 210Po over time is contained within the Q-value peak and has minimal effect on the rest of the background spectrum, the average rate of 210Po over the live time of CUORE-0 was calculated and added to the model as a fixed parameter.

222Rn, parent to a prominent gamma emitter 214Bi, is a noble gas naturally produced in the environment through the 238U decay chain. The 222Rn contamination level within CUORE-0 depends on cryostat conditions, under normal operating conditions it is suppressed through flushing the cryostat with LN2 boil off. The LN2 flux can be temporarily stopped during periods of cryostat maintenance or
Figure 5.25: Comparison of event rate vs dataset for $^{214}$Bi (black), $^{40}$K (blue), and $^{208}$Tl (red). Datasets 2061, 2076, and 2088 show a significant deviation in $^{214}$Bi rate in relation to the average (dashed lines) signifying elevated radon levels during those datasets.

instability leading to elevated levels of $^{214}$Bi gamma lines.

A study of the potential variations of $^{222}$Rn levels was performed on each dataset by integrating over the most prominent gamma peaks from $^{214}$Bi, $^{40}$K, and $^{208}$Tl and normalizing by experimental exposure. The event rates of $^{40}$K and $^{208}$Tl peaks were used as a normalization standard as they are not expected to deviate over time. Three datasets, datasets 2061, 2076, and 2088, showed a significantly higher level of $^{222}$Rn compared to the rest of the data sample (Figure 5.25). The background model was produced excluding the three datasets with high $^{222}$Rn (live time 0.782 yr) and the resultant $2
\nu\beta\beta$ decay rate was found to be compatible within one standard deviation of the reference model. The difference between the two models was used as the systematic error due to radon contamination.
5.7.7 Localization of $^{40}\text{K}$

$^{40}\text{K}$ proved to be a difficult source to localize as it has a large continuous spectrum with only a single gamma peak at 1460 keV. Sources with multiple gamma lines can be differentiated by using the ratios of gamma peaks; low energy gammas are more easily shielded while high energy gammas are more penetrating. To potentially localize the $^{40}\text{K}$ contribution, a floor-by-floor event rate study was performed on the 1460 keV gamma line of $^{40}\text{K}$ on various MC source locations as well as on the CUORE-0 background data (Figure 5.26). We found that the floors at the bottom of the tower (1 – 4) in the data have an elevated event rate compared to the rest of the tower. The elevated event rate was not reproducible through any mixture of the standard Monte Carlo source locations and was only matched through an ad hoc location at the bottom of the external lead shield. The reference model was generated using the bottom external lead $^{40}\text{K}$ source, the $2\nu\beta\beta$ decay rate was evaluated both with and without the ad hoc source, and the difference was used as the systematic error of $^{40}\text{K}$ localization.

In addition to the $^{40}\text{K}$ location in the cryostat shields, $^{40}\text{K}$ located within the bulk of the crystals may also be a source of systematic error. Normally, $^{40}\text{K}$ decays via electron capture to $^{40}\text{Ar}$ emitting a 1460 keV gamma and a neutrino which goes undetected. However, around 1% of the time the potassium atom also emits a 3 keV X-ray. If the potassium source is located outside of the crystal, only the 1460 keV gamma will be observed. However, if the source is contained within the crystals, the 1460 keV gamma and the 3 keV X-ray can simultaneously be absorbed within the crystal. The combination of the gamma and X-ray will produce a slight shift in the gamma peak mean so the best way to disentangle a potential $^{40}\text{K}$ source within the crystals is through fitting the 1460 keV peak well (Figure 5.27). The calibration response function fit was used in the reference fit as it fit the 1460 keV peak well. The contribution of $^{40}\text{K}$ within the crystals was negligible and used to produce a limit (subsection 5.8.2).
Figure 5.26: A floor-by-floor comparison of the $^{40}\text{K}$ event rate between the experimental data and various background locations generated in the MC. Note that the combined rate in floor 10 is lower because of the dead channel 49.

Figure 5.27: Simulation of $^{40}\text{K}$ in the crystals shows a shifted mean due to the 3 keV X-ray emission of potassium.
5.7.8 $2\nu\beta\beta$ signal shape differences

In the past, the $2\nu\beta\beta$ spectrum was obtained using the non-relativistic Primakoff-Rosen approximation for the Coulomb interaction between the nucleus and the daughter electrons. Recently, Kotila and Iachello have updated the calculation of the $2\nu\beta\beta$ spectrum shape using exact Dirac wave-functions with finite nuclear size and electron screening [KI12]. The reference model was obtained using the improved $2\nu\beta\beta$ spectrum shape and the comparison between the $2\nu\beta\beta$ summed electron spectra is shown in Figure 5.28. The $2\nu\beta\beta$ rate was also evaluated using the Primakoff-Rosen spectrum and the difference in the rate was used as the systematic for the signal shape.

5.7.9 Fit validation with Toy Monte Carlo

For models with many parameters, a maximum likelihood fit is not guaranteed to produce unbiased parameter estimates or yield correct statistical errors. Toy Monte Carlo pseudo-experiments were used to further check the fitter for potential biases as well as the correctness of errors. 1000 Monte Carlo toy datasets were
generated from the reference model to the same level of exposure as the data. Each pseudo-experiment was then fit using the fitter.

The pull statistic is defined to be

\[ \frac{\tau_{mc} - \tau_{bf}}{\sigma_{mc}} \]  

(5.11)

where \( \tau_{mc} \) is the fit value on the toy experiment, \( \tau_{bf} \) is the reference fit value to the data, and \( \sigma_{mc} \) is the statistical error returned by the fitter on the toy data. For an unconstrained maximum likelihood fit, we expect \( \tau_{mc} \) to be approximately Gaussian distributed about \( \tau_{bf} \). For a large number of fits to toy datasets, the pull distribution of an unbiased fit should be a Gaussian with mean at 0 and \( \sigma = 1 \).

The pull distribution is shown in Figure 5.29 and exhibits a width that is greater than 1; the statistical errors generated by the fitter are underestimated. The mean of the pull distribution is also greater than 0, meaning the average half-life value from the toy pseudo-experiments is greater than the reference fit value. The pull distributions of the parameters most correlated with the \( 2\nu\beta\beta \) half-life are shown in Appendix A.
Toy data was generated with statistics equivalent to 5000 times the exposure of the data and fit using the fitter. The high statistic toy fits resulted in a $2\nu\beta\beta$ half-life slightly higher than the input value, suggesting that the biases shown in the previous toy fits are potentially from the fitting routine. To account for the potential bias in the fitter, the difference between the mean of the half-life distribution in Figure 5.29 and the reference fit value was used as a systematic error.

We applied the profile likelihood method in the next section to accurately evaluate the errors of the fit [RLC05].

5.8 Interpretation of fit results

5.8.1 $2\nu\beta\beta$ Half-life

The half-life of $2\nu\beta\beta$ can be calculated by

$$T_{1/2}^{2\nu} = \ln 2 \frac{\epsilon T N}{n_{2\nu\beta\beta}}$$

(5.12)

where $\epsilon$ is the detection efficiency of $2\nu\beta\beta$, $T$ is the live time of the experiment, $N$ is the number of $^{130}$Te nuclei, and $n_{2\nu\beta\beta}$ is the number of $2\nu\beta\beta$ events extracted by the background model. The total number of $^{130}$Te nuclei can be calculated by

$$N = N_A \frac{a M}{W}$$

(5.13)

where $N_A$ is Avogadro’s number, $M$ is the detector mass, $W$ is the molar mass of TeO$_2$ (158.6 g/mol), and $a$ is the isotopic abundance of $^{130}$Te (34.167%). The $2\nu\beta\beta$ half-life evaluated from the result of the fitter is

$$(8.27 \pm 0.49) \times 10^{20} \text{ yr (stat)}.$$

The different contributions to systematic error in Table 5.10 are assumed to be uncorrelated and Gaussian in nature. The errors are summed together in
Figure 5.29: Top: Pull distribution; Bottom: Distribution of $2\nu\beta\beta$ half-lives from toy fits with the red dashed line representing the input half-life value for the toy data.
quadrature \( (\sigma_{syst}^2 = \sum_i \sigma_i^2) \) to obtain a total systematic error; the statistical and total systematic errors are then combined into a total \( \chi_{tot}^2 \) distribution

\[
\frac{1}{\chi_{tot}^2} = \frac{1}{\chi_{stat}^2} + \frac{1}{\chi_{syst}^2}.
\] (5.14)

The nuisance parameters \( \nu \) can be eliminated by evaluating the profile likelihood of the \( 2\nu\beta\beta \) half-life through the likelihood ratio test

\[
\lambda(\tau_0) = \frac{L_{BF}(\tau_0, \nu_0)}{L(\tau, \nu)}
\] (5.15)

where \( \tau_0 \) is the best fit half-life and \( \tau \) is mapped around the best fit value. As \(-2\log\lambda\) approaches a \( \chi^2 \) distribution, we can utilize the \( \chi^2_\lambda \) detailed previously to evaluate the confidence intervals around the best fit \( 2\nu\beta\beta \) half-life. Taking values where \( \Delta\chi^2 = 1 \), we obtain the \( \pm 1\sigma \) confidence intervals for the \( 2\nu\beta\beta \) half-life both with and without systematic errors.

\[
(8.27^{+0.50}_{-0.44}) \times 10^{20} \text{ yr (stat only)}
\]

\[
(8.27^{+1.38}_{-0.99} \text{ (stat+syst)}) \times 10^{20} \text{ yr}
\]

The profile negative log-likelihood can be seen in Figure 5.30. As expected from the toy MC fits, the evaluated statistical error from the profile likelihood is greater than the error returned by the fitter above the \( 2\nu\beta\beta \) half-life.

5.8.2 Limit setting for background sources

Several sources have their normalization parameters limited to zero in the background model fit. The error matrix in Minuit can only assign symmetric errors so it becomes unsafe to use the errors outputted by the fitter to calculate parameter limits. To set an upper limit on the limited sources, we calculated the values of \( \Delta\chi^2/2 \) by varying the limited parameter while fixing all of the other parameters at their best fit values. The 90% C.L. upper limit for the parameter corresponds
Figure 5.30: Profile Negative Log-Likelihood of $2\nu\beta\beta$.

to $\Delta \chi^2/2 = 1.65$. We then integrated the limited source using the 90% C.L. nor-
malization and then converted to an activity in terms of Bq/kg. An example of
the procedure is shown in Figure 5.31 performed on the $^{238}$U source in the bulk
of the internal shields. All of the obtained limits are shown in Table 5.11.

We compared the reconstructed source activity values and limits from the
background model and compared them to limits obtained by various screening
tests (Table 5.12). All parameters adhere directly to the screened limit within the
statistical errors returned by the fitter.
Figure 5.31: Activity limit evaluated for $^{238}\text{U}$ in the bulk of the internal shields.

Table 5.11: 90% C.L. upper limits for source activity in units of Bq/kg. The index corresponds to the index of Table 5.8 and Table 5.9.

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Activity Limit (Bq/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\text{TeO}_2$ $^{40}\text{K}$</td>
<td>$\leq 1.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>$\text{TeO}_2$ $^{232}\text{Th}$ only</td>
<td>$\leq 2.7 \times 10^{-8}$</td>
</tr>
<tr>
<td>6</td>
<td>$\text{TeO}_2$ $^{238}\text{U}$-$^{230}\text{Th}$</td>
<td>$\leq 2.0 \times 10^{-8}$</td>
</tr>
<tr>
<td>26</td>
<td>Copper Holder $^{238}\text{U}$</td>
<td>$\leq 6.6 \times 10^{-6}$</td>
</tr>
<tr>
<td>27</td>
<td>Copper Holder $^{232}\text{Th}$</td>
<td>$\leq 7.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>31</td>
<td>Internal Shields $^{238}\text{U}$</td>
<td>$\leq 1.8 \times 10^{-6}$</td>
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<td>Internal Shields $^{40}\text{K}$</td>
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<td>Roman Lead $^{40}\text{K}$</td>
<td>$\leq 1.9 \times 10^{-5}$</td>
</tr>
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<td>Roman Lead $^{60}\text{Co}$</td>
<td>$\leq 1.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>45</td>
<td>External Lead $^{40}\text{K}$</td>
<td>$\leq 5.4 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Table 5.12: 90% C.L. upper limit comparison between measured activity and reconstructed activity from the background model. Measured limits were from Table 5.1.

<table>
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<tr>
<th>Detector Component</th>
<th>Measured Limit (Bq/kg)</th>
<th>Reconstructed Rate (Bq/kg)</th>
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</thead>
<tbody>
<tr>
<td>TeO₂ ⁴⁰K</td>
<td>≤ 1.0 × 10⁻³</td>
<td>≤ 1.2 × 10⁻⁶</td>
</tr>
<tr>
<td>TeO₂ ²³²Th</td>
<td>≤ 8.4 × 10⁻⁷</td>
<td>≤ 2.7 × 10⁻⁸</td>
</tr>
<tr>
<td>TeO₂ ²³⁸U</td>
<td>≤ 6.7 × 10⁻⁷</td>
<td>≤ 2.0 × 10⁻⁸</td>
</tr>
<tr>
<td>TeO₂ ⁶⁰Co</td>
<td>≤ 3.0 × 10⁻⁷</td>
<td>(3.6 ± 0.6) × 10⁻⁶</td>
</tr>
<tr>
<td>Copper Holder ²³²Th</td>
<td>≤ 2.0 × 10⁻⁶</td>
<td>≤ 6.6 × 10⁻⁶</td>
</tr>
<tr>
<td>Copper Holder ²³⁸U</td>
<td>≤ 6.5 × 10⁻⁵</td>
<td>≤ 7.5 × 10⁻⁶</td>
</tr>
<tr>
<td>Internal Shields ²³²Th</td>
<td>≤ 4.4 × 10⁻⁴</td>
<td>(2.7 ± 1.3) × 10⁻⁵</td>
</tr>
<tr>
<td>Internal Shields ²³⁸U</td>
<td>≤ 6.7 × 10⁻⁴</td>
<td>≤ 1.8 × 10⁻⁶</td>
</tr>
<tr>
<td>Internal Shields ⁶⁰Co</td>
<td>≤ 1.8 × 10⁻⁴</td>
<td>(1.6 ± 3.7) × 10⁻⁵</td>
</tr>
<tr>
<td>Roman Lead ⁴⁰K</td>
<td>≤ 2.3 × 10⁻⁵</td>
<td>≤ 1.9 × 10⁻⁵</td>
</tr>
<tr>
<td>Roman Lead ²³²Th</td>
<td>≤ 3.3 × 10⁻⁵</td>
<td>(1.1 ± 0.42) × 10⁻⁵</td>
</tr>
<tr>
<td>Roman Lead ²³⁸U</td>
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<td>(1.7 ± 1.8) × 10⁻⁵</td>
</tr>
<tr>
<td>External Shields ⁶⁰Co</td>
<td>≤ 4.2 × 10⁻⁵</td>
<td>(1.9 ± 2.8) × 10⁻⁴</td>
</tr>
<tr>
<td>External Lead ²³²Th</td>
<td>≤ 2.6 × 10⁻⁴</td>
<td>(5.4 ± 1.1) × 10⁻⁵</td>
</tr>
<tr>
<td>External Lead ²³⁸U</td>
<td>≤ 4.1 × 10⁻⁴</td>
<td>(8.6 ± 1.5) × 10⁻⁵</td>
</tr>
</tbody>
</table>
CHAPTER 6

Conclusions

This dissertation has presented a measurement of the $2\nu\beta\beta$ half-life of $^{130}\text{Te}$ as well as a model of the background spectrum of CUORE-0. CUORE-0 was a success; the experiment achieved the energy resolution goal set out for CUORE as well as greatly reduced the background from degraded alpha events.

6.1 Comparison with previous measurements

The $2\nu\beta\beta$ half-life of $^{130}\text{Te}$ has been measured from previous experiments: Cuoricino [Kog11], Mi-DBD [Arn03], and NEMO-3 [Arn11]. The result of this work is compared with previous measurements in Table 6.1. To make the comparison, the errors of each experiment was combined in quadrature. For errors that are asymmetric, the error in the direction of the mismatch is used in the error combination. This work agrees with the results of the other experiments to within $\pm 1\sigma$.

6.2 CUORE and beyond

The CUORE experiment has finished tower construction and is in the final stages of cryostat commissioning. The installation of the towers as well as the data taking are expected to begin in the middle of 2016.

A major technical accomplishment for CUORE will be to reach a background
Table 6.1: Comparison with previous measurements of the $2\nu\beta\beta$ half-life of $^{130}\text{Te}$.

The errors quoted include both statistical and systematic errors.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Half-life ($10^{20}$ yr)</th>
<th>Difference ($10^{20}$ yr)</th>
<th>Discrepancy ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>8.27$^{+1.38}_{-0.99}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cuoricino</td>
<td>$9.81 \pm 1.07$</td>
<td>$-1.5 \pm 1.8$</td>
<td>0.83</td>
</tr>
<tr>
<td>NEMO-3</td>
<td>$7.0 \pm 1.42$</td>
<td>$1.3 \pm 1.7$</td>
<td>0.76</td>
</tr>
<tr>
<td>Mi-DBD</td>
<td>$6.1^{+3.22}_{-3.77}$</td>
<td>$2.2 \pm 3.4$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

index of 0.01 c/keV/kg/yr in the $0\nu\beta\beta$ ROI. The background goal is achievable with the current projections of the CUORE-0 background simulations to the CUORE geometry [CUO5], allowing CUORE to reach an effective Majorana neutrino mass range of $\langle m_{\beta\beta} \rangle < 51 – 133$ meV after five years of operation. Further background reduction is necessary in order to reach deeper into or potentially cover the inverted hierarchy.

One of the major deficiencies in CUORE is the lack of particle identification. The signature of $0\nu\beta\beta$ comes from electrons while the majority of the background in CUORE comes from degraded alpha and Compton scattered gamma events; identifying the decay electrons of $0\nu\beta\beta$ can greatly reduce the background. Energy deposited through particle interactions is typically split between heat and light. The magnitude of each interaction type depends on the particle type as well as the material in which the energy is deposited. Measuring the signal from different interaction types can potentially be used to distinguish particle interactions.

CUORE Upgrade with Particle IDentification (CUPID) [Wan15c] is a proposed ton-scale bolometric experiment searching for $0\nu\beta\beta$. CUPID will utilize the infrastructure and experience developed for CUORE in order to improve on the sensitivity of CUORE, aiming for an effective Majorana neutrino mass range of $\langle m_{\beta\beta} \rangle < 15 – 50$ meV. The R&D efforts of CUPID will be split into two paths, technologies utilizing TeO$_2$ and technologies utilizing an alternative source.
TeO$_2$ bolometers have been demonstrated to be a robust and efficient technology with high resolution. However, TeO$_2$ bolometers have limited light yield; they do not scintillate and only produce a small amount of Cherenkov light from electrons. The focus of the TeO$_2$ R&D path will first be to isotopically enrich the bolometers as well as develop high resolution light detectors capable of clearly identifying Cherenkov light from the decay electrons.

Alternative isotopes for CUPID are all based on scintillating bolometers. Scintillating bolometers using $^{82}$Se, $^{100}$Mo, and $^{116}$Cd have already been demonstrated to reach the level of background rejection required for CUPID. However, they have only been demonstrated at the single detector level. The R&D path of the alternative isotopes will be focused on crystal production in order to scale up to the ton-scale.
Appendix A

Figures

This appendix will cover extra plots that are not in the main chapters of the thesis.

A.1 Monte Carlo spectra

The y-axis of each spectra is normalized to arbitrary units.

Figure A.1: Bulk $^{232}$Th contamination (M1).
Figure A.2: Bulk $^{238}\text{U}$ contamination (M1).

Figure A.3: Bulk $^{60}\text{Co}$ contamination (M1).
Figure A.4: Bulk $^{40}$K contamination (M1).

Figure A.5: Bulk contamination from miscellaneous sources (M1).
Figure A.6: Surface $^{232}$Th contamination from TeO$_2$ (M1).

Figure A.7: Surface $^{238}$U contamination from TeO$_2$ (M1).
Figure A.8: Surface $^{210}$Pb contamination from TeO$_2$ (M1).

Figure A.9: Surface $^{232}$Th contamination from Copper Holders (M1).
Figure A.10: Surface $^{238}$U contamination from Copper Holders (M1).

Figure A.11: Surface $^{210}$Pb contamination from Copper Holders (M1).
A.2 Zoomed sections of background model

Figure A.12: Low energy extrapolation of the background fit. The low energy discrepancy could be due to missing background sources, threshold effects, or a mismatch of the MC.
Figure A.13: Zoomed in section on the $^{40}$K peak at 1460 keV.
Figure A.14: Zoomed in section on the $^{208}$Tl peak at 2615 keV.
Figure A.15: Alpha region of the background.
A.3 Stacked histograms of gamma peaks

Figure A.16: Stacked histogram for the 2615 keV peak of $^{208}$Tl (M1).

Figure A.17: Stacked histogram for the 1460 keV peak of $^{40}$K (M1).
Figure A.18: Stacked histogram for the 1173 keV peak of $^{60}$Co (M1).

Figure A.19: Stacked histogram for the 1332 keV peak of $^{60}$Co (M1).
A.4 Toy Monte Carlo pull distributions

Figure A.20: Pull distribution of Toy MC fits for Roman Lead $^{238}$U.

Figure A.21: Pull distribution of Toy MC fits for OVC $^{238}$U.
Figure A.22: Pull distribution of Toy MC fits for External Lead $^{210}\text{Bi}$.

Figure A.23: Pull distribution of Toy MC fits for External Lead $^{238}\text{U}$. 
A.5 $0\nu\beta\beta$ ROI breakdown

Table A.1: Percentage contributions to $0\nu\beta\beta$ ROI (2470 to 2570 keV) from each background source. Sources that contribute negligibly are excluded.

<table>
<thead>
<tr>
<th>Name</th>
<th>Percentage (%)</th>
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<tbody>
<tr>
<td>TeO$_2$ $^{60}$Co</td>
<td>19 ± 3</td>
</tr>
<tr>
<td>TeO$_2$ Surface $^{210}$Pb 1 µm</td>
<td>1 ± 1</td>
</tr>
<tr>
<td>TeO$_2$ Surface $^{232}$Th 10 µm</td>
<td>1 ± 1</td>
</tr>
<tr>
<td>Copper Holder Surface $^{238}$U 100 µm</td>
<td>4 ± 1</td>
</tr>
<tr>
<td>Copper Holder Surface $^{232}$Th 100 µm</td>
<td>7 ± 1</td>
</tr>
<tr>
<td>Copper Holder Surface $^{210}$Pb 10 µm</td>
<td>6 ± 2</td>
</tr>
<tr>
<td>Copper Holder Surface $^{210}$Pb 1 µm</td>
<td>1 ± 1</td>
</tr>
<tr>
<td>Copper Holder $^{60}$Co</td>
<td>1 ± 11</td>
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<tr>
<td>Internal Shields $^{232}$Th</td>
<td>8 ± 4</td>
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<tr>
<td>Internal Shields $^{60}$Co</td>
<td>1 ± 3</td>
</tr>
<tr>
<td>Roman Lead $^{232}$Th</td>
<td>2 ± 1</td>
</tr>
<tr>
<td>OVC $^{238}$U</td>
<td>1 ± 1</td>
</tr>
<tr>
<td>OVC $^{232}$Th</td>
<td>60 ± 28</td>
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<tr>
<td>External Lead $^{232}$Th</td>
<td>30 ± 6</td>
</tr>
<tr>
<td>External Lead $^{238}$U</td>
<td>1 ± 1</td>
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</table>

A.6 Monte Carlo detector resolutions

The secondary Gaussian function from Equation 3.3 is parameterized in the Monte Carlo by 3 parameters: the distance from the primary Gaussian ($d = -3.05082$), the fraction of events in the secondary Gaussian ($f = 0.0624326$), and the energy dependent slope ($s = d \times (1 + E \times s) = 0.0004529$). The resolution of each crystal is parameterized by $FWHM = E_0 + b \times E$ where $E_0$ represents the FWHM at 0 keV and $b$ represents the slope of the FWHM as a function of energy (Table 6.2).
Table 6.2: Parameters used for the energy resolution of each channel in the MC.

<table>
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<tr>
<th>Channel</th>
<th>$E_0$ (keV)</th>
<th>b</th>
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<td>1</td>
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<td>2.0844</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>4.72816</td>
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<td>6</td>
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REFERENCES


[Oue15] Jonathan Ouellet. The Search for $0\nu\beta\beta$ decay in $^{130}$Te with CUORE-0. PhD thesis, University of California, Berkeley, 5 2015.


