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Publication Date
1979-11-21
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November 21, 1979
A NOTE ON CAPITAL BUDGETING AND THE THREE R's

In a standard capital budgeting analysis, three measures of return are involved. These are the internal rate of return or expected return (R), the disequilibrium required rate of return measured on investment cost (R), and the equilibrium required cost of return measured on market value (R*). On attractive investment projects, the three returns will be unequal with $\bar{R} > R > R^*$. The three are defined as:

$$R = R_f + \lambda \text{Cov}(\tilde{X}, \tilde{R}_m)/C_o$$

$$R^* = R_f + \lambda \text{Cov}(\tilde{X}, \tilde{R}_m)/V_o$$

$E(\tilde{V}_1)$ is the expected end-of-period value of the investment, $C_o$ is the cost of investment, $R_f$ is the risk-free rate, $\lambda$ is the market risk factor, $\tilde{R}_m$ is the random return on the market and $\tilde{X}$ is the random net cash flow of the period.

Rendleman (2) presents a clear distinction between the disequilibrium and equilibrium required rates of return. He also notes that

"potential errors can arise when ranking projects on the basis of their expected excess returns if market-determined measures of systematic risk rather than measures reflecting project cost are employed to determine the required rates of return. Thus, another challenge is presented to the practical usefulness of the model." (p. 41)
But this criticism of the practical use of the equilibrium required rate of return is based on an inappropriate application. Consistent formulations derived from the NPV rule establish that either the disequilibrium or equilibrium required rates of return can be used for ranking capital projects if applied appropriately.

First, we illustrate the use of the equilibrium required rate of return, using \[ NPV_j = V_o - C_o \] where \[ V_o = E(V_1)/(1+R^*_j) \]. For the one-period case,

\[ NPV_j = \frac{C_o(\bar{R}_j-R^*_j)}{(1+R^*_j)} \]  \hspace{1cm} (1)

Here \( NPV \) is the excess of the internal rate of return over the equilibrium rate of return, weighted by the initial investment cost and capitalized by one plus the equilibrium rate of return. Note that \( R^*_j \) is determined without the knowledge of \( V_o \), the gross present value of the project. \[ \text{[Fama (1), equation (3)]} \]

We next derive the appropriate use of \( R^*_j \), the disequilibrium required rate of return. Making use of Fama (1), equations (6) and (9), we have

\[ NPV = [C_o(1+\bar{R}_j) - \lambda \text{Cov}(X_j, \bar{V}_m) - C_o(1+R_f)]/(1+R_f) \]  \hspace{1cm} (2)

This simplifies to

\[ NPV = \frac{C_o(\bar{R}_j-R^*_j)}{(1+R_f)} \]  \hspace{1cm} (2a)

The numerator in Equation (2a) is the same as the Rubinstein (3, p. 174) formulation for choosing between mutually exclusive investments. It represents the excess of the expected rate of return over the required return, calculated on investment cost, with the difference weighted by the cost of the investment. When this amount is discounted by one plus the risk-free rate of return in a one-period model, we obtain the net present value of the
project.

For projects of the same scale, ranking on the basis of either \((\bar{R}_j - R_j)\) or \((\bar{R}_j - R_j)/(1+R_j)\) will lead to the same result. If projects are of different scale, the excess rates of return must be weighted by the initial investment costs. But this is equivalent to computing NPV, so the NPV rule applies in either case. As demonstrated, the NPV criterion can incorporate either the disequilibrium or equilibrium return measures.

However, the equilibrium rate of return is more generally applicable. For example, in a multiperiod model, Fama (1) has established that the equilibrium return measure must be employed. The reason is that in the multiperiod setting, no simple analog to (2a) that uses the disequilibrium discount rate \(R_j\) and \(\bar{R}_j\) to arrive at the equilibrium net present value is available. Therefore, the general multiperiod model requires the use of the equilibrium measure \(\bar{R}_j\) exclusively.

References


