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The Hyperon Composition of Neutron Stars

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Abstract

Neutron stars are studied in the framework of relativistic interacting field theory of nucleons, hyperons, and mesons. A large component of strange baryons is found, and in the interior the neutron population is minor.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
A description of neutron star matter is developed here in the framework of a relativistic field theory of interacting nucleons, hyperons, and mesons. The theory is solved in the mean field approximation. Relativistic covariance of the theory is retained throughout.

The important bulk properties of nuclear matter are imposed as constraints. These are the saturation density, binding energy, compressibility, and isospin symmetry energy. Subject to these constraints the equation of state is calculated for cold dense charge-neutral matter that is in chemical equilibrium.

The range of validity of the theory is from nuclear matter densities up to the density where a description of nuclear forces between hadrons that is mediated by meson exchange becomes increasingly unreliable. Above this density the quarks and gluons would have to be considered as the basic fields. This critical density is of great interest but is unknown. Based on the radius of quark bag models these occur anywhere from nuclear to fifteen times nuclear density. The lower limit is presumably ruled out by the degree to which the nuclear shell model provides a successful first orientation in nuclear physics. At even higher densities a phase transition to an asymptotically free gas of quarks and gluons may occur. This transition point, in cold matter, is estimated to be in excess of ten times nuclear density.

Our starting point is a Lagrangian describing interacting fields of nucleons and the scalar and vector mesons $\sigma$ and $\omega^\mu$. The saturation density and binding of symmetric nuclear matter can be accounted for in this way. Boguta and Bodmer introduced self-interactions of the scalar meson, and these are essential for softening the extremely stiff equation of state of the Walecka model, which has a compression modulus $\sim 550$ MeV. A
large number of single-particle properties of finite nuclei can be accounted for quite well in addition to the saturation density, binding, and compressibility of nuclear matter. These are the charge density of nuclei over the entire mass range, the single-particle states including the spin-orbit splitting,\(^6\) the energy dependence of the optical potential\(^9\) and the spectra of \(\Lambda\) hypernuclei.\(^10\) The symmetry energy coefficient of the Walecka model is only about half as large as it should be. Therefore, the isovector rho meson is an essential ingredient for asymmetric matter.\(^7\)

As the density of matter rises, the Fermi energy of the neutrons will eventually exceed the threshold for decay, first to protons and then to the hyperons, most especially the \(\Lambda, \Sigma, \Xi\), and \(\Delta\), with the emission of electrons, muons, or pions as required by charge conservation, and chemical equilibrium. The strange particle decays are, of course, not forbidden on a time scale that is large compared to \(10^{-10}\) sec. Therefore, the theory is extended, as in our earlier work, to include the hyperons.\(^11\)

At some sufficiently high density, the nature of the ground state may undergo a phase transition to the pion condensed state. We do not allow for a structural change in the ground state here, but we do allow a condensate of free pions to develop when the chemical equilibrium would require it. Likewise, of course, the electrons and muons are in chemical equilibrium with the hadrons but may be treated as free particles because the interaction is electro-weak. Our complete Lagrangian is thus (\(\hbar = c = 1\)),

\[
\mathcal{L} = \sum_{B} \overline{\psi}_B \left( i\gamma^\mu \left( \gamma^\nu \right) - m_B + g_{\omega B} \sigma - g_{\omega B} \omega_\mu \right) \psi_B \\
- g_\rho \rho_{\mu 3} \partial^\mu + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\pi - U(\sigma) \\
+ \sum_{\lambda=e,\mu} \overline{\psi}_\lambda \left( i\gamma^\mu - m_\lambda \right) \psi_\lambda
\]  \hspace{1cm} (1)
The sum $\beta$ is over the baryons $p, n, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Lambda^+$, and $\Delta^{++}, \Delta^{++}, \Delta^{++}$ and $\beta$ stands for their quantum numbers

$$\beta = J^{\beta}, I^{\beta}, I_{3\beta}, Y^{\beta}$$

being respectively the spin, isospin, 3rd component of isospin, and hypercharge. These are Yukawa coupled to the scalar and vector mesons $\sigma$ and $\omega_{\mu}$. The rho meson and nucleons are coupled through the isospin four-current of the baryons,

$$J^{\mu}_{3} = \frac{1}{2} \sum_{\beta} \bar{\psi}_{\beta} \gamma^{\mu} \tau_{3} \psi_{\beta}$$

The free Lagrangians for the mesons are denoted by $L_{\sigma}, \ldots L_{n}$, the lepton Lagrangians by $L_{\lambda}$, and finally the $\sigma$-field self-interactions by $U(\sigma)$,

$$U(\sigma) = [b_{m} n + c \left(g_{\sigma}\right) \left(g_{\sigma}\right)] \left(g_{\sigma}\right)^{3}$$

The mean field approximation consists of replacing all nucleon currents in the Euler-Lagrange equations derived from (1), by a ground state expectation value, with the Fermion ground state consisting of a degenerate Hartree state constructed from the solutions of the field equation for the baryons in which meson fields are replaced by their mean values. The resulting equations are solved self-consistently for the mean fields. The Dirac equations for the baryons are

$$\left[p - g_{\omega_{\beta}} \omega_{\beta} - \frac{1}{2} g_{\rho_{\beta}} \tau_{3} \rho_{3} - (m_{\beta} - g_{\sigma_{\beta}}\sigma)\right] \psi_{\beta} = 0$$

which have the spin degenerate eigenvalue spectrum for particles given by

$$E_{\beta}(p) = g_{\omega_{\beta}} \omega_{\beta} + g_{\rho_{\beta}} \rho_{3} I_{3\beta} + [(p - g_{\omega_{\beta}} \omega_{\beta} - g_{\rho_{\beta}} \rho_{3})^{2} + (m_{\beta} - g_{\sigma_{\beta}}\sigma)^{2}]^{1/2}$$

(The notation of Bjorken and Drell is employed.\(^{12}\) In particular $\sigma \equiv \gamma_{\mu} p^{\mu}$ and the metric is $g_{\mu \nu} = (1, -1, -1, -1)$.\(^{12}\) From the Dirac equation (5) the
nucleon source currents in the meson field equations can be evaluated. We find that the mean meson fields satisfy

\[ m_{o}^2 = - \frac{dU}{d\sigma} + \sum_{B} g_{\omega B} \langle \bar{\psi}_B \psi_B \rangle \]

\[ = - \frac{dU}{d\sigma} + \sum_{B} g_{\omega B} \frac{(2J_B + 1)}{2\pi^2} \int_{0}^{k_B} \frac{m_{B} - g_{\omega B}}{\sqrt{p^2 + (m_{B} - g_{\omega B})^2}} p^2 dp. \tag{7} \]

\[ m_{\omega}^2 = \sum_{B} g_{\omega B} \langle \bar{\psi}_B \gamma_0 \psi_B \rangle \]

\[ = \sum_{B} g_{\omega B} (2J_B + 1) \frac{k_B^3}{6\pi^2} \sum_{B} g_{\omega B} \eta_B, \quad \eta_B = \sum_{B} \eta_B \tag{8} \]

\[ \omega = 0 = \rho_3 \tag{9} \]

\[ m_{p}^2 \rho_{03} = \sum_{B} g_{\rho B} \frac{1}{2} \langle \bar{\psi}_B \gamma_0 \tau_3 \psi_B \rangle \]

\[ = \sum_{B} g_{\rho B} (2J_B + 1) I_{3B} \frac{k_B^3}{6\pi^2}. \tag{10} \]

Charge neutrality and chemical equilibrium impose relations among the Fermi momenta \( k_B \). The total charge is

\[ Q = \sum_{B} q_B (2J_B + 1) \frac{k_B^3}{6\pi^2} - \sum_{\lambda} \frac{k_\lambda^3}{3\pi^2} - \rho_{\pi} \theta (\mu_{\pi} - m_{\pi}) \tag{11} \]

where the charge on species \( B \) is

\[ q_B = I_{3B} + \gamma_B / 2 \]

The three terms are the electric charge densities of baryons, leptons, and negative pions. For a neutron star, the electric charge, \( Q \), must vanish; otherwise the Coulomb repulsion would counterbalance the gravitational attraction.*

*The ratio of net charges to mass number of a star must not exceed \( 10^{-36} \).
Chemical equilibrium with respect to the various transmutations of the particles $p + e^- \leftrightarrow n$, $p + \pi^- \leftrightarrow n$, $p + \mu^- \leftrightarrow n$, $\Lambda^0 \leftrightarrow n + \gamma$, etc. can be expressed in terms of two independent chemical potentials $\mu_N$ and $\mu_e$ (corresponding to baryon and electric charge conservation) by

$$\mu_B = \mu_n - q_B \mu_e, \quad \mu_\pi = \mu_\mu = \mu_e$$  \hspace{1cm} (13)

where $\mu_B$ is the chemical potential for baryon type $B$. The Fermi momenta, $k_B$, of the baryons therefore satisfy the equations

$$\mu_B = g_{\omega B} \omega_0 + g_{\rho B} \rho_{03} I_3 + \sqrt{k_B^2 + (m_B - g_{\rho B} \sigma)^2}$$ \hspace{1cm} (14)

when these equations have real solutions; otherwise $k_B$ vanishes. Similarly

$$\mu_e = \sqrt{k_e^2 + m_e^2} = \sqrt{k_\mu^2 + m_\mu^2} = \sqrt{k_\pi^2 + m_\pi^2}$$ \hspace{1cm} (15)

Since the pions are bosons, they can condense in the zero momentum state when $\mu_e \geq m_\pi$.

Equations (7-15) comprise a set of coupled transcendental equations for the $(8 + N)$ quantities

$$\sigma, \omega_0, \rho_{03}, \mu_n, \mu_e, k_e, k_\mu, k_\pi, k_p, k_n \ldots$$

which determine the properties of the system at the density (8) and charge (11), when $\mu_e \leq m_\pi$. The lowest energy state has $\mu_e \leq m_\pi$. When the equality holds, then $\rho_\pi$ replaces $\mu_e$ as an unknown, and its value is prescribed by (11).

The energy density $\rho$ and pressure $P$ can be found from the canonical expression for the stress-energy tensor

$$T_{\mu\nu} = -g_{\mu\nu} \mathcal{L} + \sum \frac{\delta \mathcal{L}}{\delta (a_{\mu}^\phi)} \partial_{\nu} \phi$$ \hspace{1cm} (16)
being the time-like and the (equal) space-like diagonal components. We find
\[
\rho = U(\sigma) + \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} m_{\omega}^2 \omega_0^2 + \frac{1}{2} m_{\rho}^2 \rho_{03}^2 + \rho_{\pi}^2 m_{\pi}^2
\]
\[
+ \sum_{\beta} \frac{2J_{\beta} + 1}{2\pi^2} \int_0^{k_B} \frac{\sqrt{p^2 + (m_{\beta} - g_{\sigma\beta})^2 \rho_{03}^2}}{p^2 dp} \tag{17}
\]
\[
P = -U(\sigma) - \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} m_{\omega}^2 \omega_0^2 + \frac{1}{2} m_{\rho}^2 \rho_{03}^2
\]
\[
+ \frac{1}{3} \sum_{\beta} \frac{2J_{\beta} + 1}{2\pi^2} \int_0^{k_B} \frac{p^4 dp}{\sqrt{p^2 + (m_{\beta} - g_{\sigma\beta})^2 \rho_{03}^2}} \tag{18}
\]
These are required to find the structure of a neutron star. The matter then
arranges itself according to the solution of the Oppenheimer-Volkoff
equations, which follow from the general theory of relativity for a static
spherically symmetric geometry. 13

Although the four bulk nuclear properties mentioned in the introduction
are the most important constraints on the theory that we can think of that
would influence the structure of the star, they are not sufficient to
determine all the parameters. The single-particle properties of finite nuclei
serve to further narrow the choice. 8, 9 Following Moszkowski's 14
application of SU(3) symmetry and based on quark counting arguments the
nucleons and Δs are coupled with the same constants to the meson fields, but
the couplings of strange baryons relative to nucleons are reduced by a factor
\[(g_S/g_N)^2 = 2/3.\] With the nucleon-meson coupling constants
\[
(g_{\sigma}/m_{\sigma})^2 = 9.957, \quad (g_{\omega}/m_{\omega})^2 = 5.354
\]
\[
(g_{\rho}/m_{\rho})^2 = 6.2, \quad b = 0.00414, \quad c = 0.00716
\]
we obtain the correct saturation density and binding of 0.145 fm$^{-3}$ and 15.95 MeV per nucleon respectively, a compressibility coefficient $K = 285$ MeV and a symmetry coefficient of 36.8 MeV, in accord with the droplet model determination from the atomic masses.$^{15}$

At sub-nuclear densities, matter is not a uniform hadronic liquid but clusters into neutron rich nuclei that are arranged on a lattice and immersed in a neutron and a relativistic electron gas.$^{16}$ Therefore, below $\rho < 7 \times 10^{13}$ g/cm$^3$ we join our equation of state to that of Negele and Vautherin.$^{16}$ At even lower densities, $10^6 < \rho/(g/cm^3) < 10^{11}$, matter forms a solid Coulomb lattice of nuclei and a relativistic electron gas. In the region, $10^3 < \rho/(g/cm^3) < 10^{11}$, we use the Harrison-Wheeler equation of state.$^{17}$ These regimes of density occur only in the crust and have negligible effect on the mass of the heavier neutron stars and a small effect on the lighter ones.

Figure 1 shows the populations of dense matter as a function of the baryon number density. While neutrons are the dominant population of stable charge neutral matter at densities in the vicinity of normal nuclear density, the populations of protons, pions, and hyperons rise steeply as their thresholds are reached so that at a few times nuclear density the neutron becomes a fraction of the total baryon population. This contrasts with scenarios based only on neutrons, protons and leptons, in which case the matter remains relatively pure in neutrons.$^{18}$ Note also that the uncharged $\Lambda$ has the lowest hyperon threshold, in contradiction to argumentation based on free gases. In the latter case, the $\Sigma^-$ threshold would be lower because the $e^- + n \rightarrow \Sigma^-$ reaction absorbs an electron and neutron, both of high momentum, to produce a low momentum $\Sigma^-$. The essentially different results found here follow from several factors.
Even in a free Fermi gas model, at some density it will become more energetically favorable to populate higher mass baryons than high momentum neutron states. In this interacting theory, the threshold is additionally influenced by the charge and isospin of the baryons as can be seen in (14). In particular, the $\rho$ meson, which is essential in accounting for the known symmetry energy of nuclear matter drives the system toward isospin symmetry. This tendency is partially checked by the constraint of charge neutrality. Subject to such competing influences the populations evolve as a function of density. Through the dependence on the density, the particle populations will vary as a function of radius in a star. For a star of 1.82 solar masses (the maximum star mass for this theory) these populations are displayed in Fig. 2. The total baryon number density is represented by the top curve, and it is broken down into its components by the other curves above the axis. Below the axis (for convenience) the pion population is shown. One sees that the strange baryons represent about 2/3 of the baryon population in the deep interior and that charge neutrality is maintained not by a paucity of charged particles (as in models that consider only neutrons, protons and leptons) but by an abundance of charged particles of both signs. The electron and meson populations are both extremely small, not exceeding 0.012 and 0.003 fm$^{-3}$ anywhere. The pion condensate is very influential in quenching the lepton populations (see Fig. 1) at intermediate density.

For stable configurations of collapsed matter, this theory predicts a maximum mass of 1.82 solar masses and maximum moment of inertia of $2.1 \times 10^{45}$ g-cm$^2$, which exceed the observational lower limits of 1.6 and $1.5 \times 10^{44}$ g-cm$^2$, respectively. The fractional gravitational redshift of a spectral line emitted at the surface of this star and observed at infinity is $\Delta \lambda / \lambda = 0.42$, which for a stable star is large.
It is probably safe to say that the composition and strangeness of a neutron star will never be directly measured. However, it is not excluded that observable properties may be influenced by the hadronic composition. Several things come to mind in addition to the much studied problem of neutron star cooling. There can be vibrational modes associated with the isospin and strangeness composition. Moreover, since the charge is carried mainly by massive baryons, rather than half of it on electrons and muons, as is usually assumed, the electrical conductivity will surely be influenced. The active lifetime of pulsars depends sensitively on the electrical conductivity because it plays a decisive role in the decay rate of the magnetic field, which is thought to be responsible for their beamed radiation.

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Fig. 1. The particle populations in dense neutral matter plotted as a ratio to the total baryon number density are shown as a function of baryon density.
Fig. 2. The proper number densities (as measured in a locally inertial reference frame) of the various particles are plotted as a function of the star radius. On this scale the electron and meson populations are not much larger than the thickness of the lines. The vertical distance between lines represents the population densities.
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