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Radiation from laser accelerated electron bunches: 
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Abstract—Electron beam based radiation sources provide electromagnetic radiation for countless applications. The properties of the radiation are primarily determined by the properties of the electron beam. Compact laser driven accelerators are being developed that can provide ultra-short electron bunches (femtosecond duration) with relativistic energies reaching towards a GeV. The electron bunches are produced when an intense laser interacts with a dense plasma and excites a large amplitude plasma density modulation (wakefield) that can trap background electrons and accelerate them to high energies. The short pulse nature of the accelerated bunches and high particle energy offer the possibility of generating radiation from one compact source that ranges from coherent terahertz to gamma rays. The intrinsic synchronization to a laser pulse and unique character of the radiation offers a wide range of possibilities for scientific applications. Two particular radiation source regimes are discussed: coherent terahertz emission and x-ray emission based on betatron oscillations and Thomson scattering.

Index Terms—Lasers, Plasmas, Electron accelerators, Electromagnetic radiation, X-rays

I. INTRODUCTION

LASER-driven, plasma-based accelerators [1] hold the potential of becoming compact alternatives to conventional radio-frequency-based linear accelerators (linacs). Accelerating gradients in the 10’s to 100’s of GV/m have been measured in laser wakefield accelerator (LWFA) experiments [2]–[8] which are three orders of magnitude higher than in conventional linacs. In the self-modulated LWFA regime, acceleration of electrons has also been demonstrated, [2]–[8] with bunches containing up to several nC of charge and energy spectra characterized by a Boltzmann-like distribution with an equivalent temperature ranging from the few MeV to tens of MeV. In this regime the laser pulse is modified during its propagation in the plasma leading to large amplitude plasma waves capable of trapping and accelerating background plasma electrons. Work is also under way to produce narrow energy spread beams using laser triggered injection techniques [9]–[14] and via control of the plasma properties (transverse and longitudinal) and laser parameters. Recent experiments at Lawrence Berkeley National Laboratory (LBNL) have demonstrated the production of narrow energy spread (few percent), high energy (∼100 MeV), high charge (∼0.5 nC) electron bunches from a LWFA that used a preformed plasma density channel [15]. Narrow energy electron bunches were also observed in the UK and France [16], [17].

Although the use of laser-plasma accelerators for high energy physics applications will require performance well beyond today’s achievements, several applications with less stringent beam property requirements are becoming possible. One of these is radio-isotope production through (γ, n) reactions with laser accelerated bunches [18], [19], which only requires a sufficient number of electrons with energy in excess of 10’s of MeV, a condition that is relatively straightforward to obtain using LWFAs. Two other applications are being explored which exploit the short pulse nature and high charge of the accelerated bunches. The first is the production of coherent terahertz (10^{12} Hz) radiation when the femtosecond electron bunches cross the plasma-vacuum boundary and emit transition radiation [20]–[22]. The second aims at the generation of femtosecond x-ray pulses, which are produced either by the betatron radiation emitted as the electron beam propagates through the plasma, or via Thomson scattering of an intense laser off the electron beam. These latter applications are the focus of the present paper.

The first part of this paper provides an overview of recent experiments [20], [22] and theory [21] on generation of coherent terahertz radiation. This electron beam-based radiation source relies on two novel components: (1) extremely dense, sub-ps electron bunches produced with a compact laser-plasma accelerator and (2) the production of coherent transition radiation by these bunches at the boundary between a plasma and vacuum. This source has the potential for generating up to tens of μJ per pulse, several orders of magnitude beyond conventional THz radiation sources. The theory of transition radiation generated by a relativistic electron bunch crossing a plasma-vacuum boundary is summarized in Sec. II-A. Sections II-B – II-F discuss the experimental set-up and present evidence for the coherent nature of the radiation at a frequency of 94 GHz.
and in the 0.3 to 19 THz range, polarization measurements, and preliminary frequency spectrum measurements. Using these preliminary spectral measurements, bunch durations on the order of 30–50 fs are inferred. Also compared are the measured and calculated total radiated energy. Methods for increasing the radiated energy beyond what has been achieved in the present experiments are discussed in the conclusion.

The second part of the paper discusses two methods for generating femtosecond x-ray pulses. The first method relies on the emission of betatron radiation, i.e., synchrotron radiation emitted as the accelerated electrons undergo betatron oscillations within a plasma channel [23], [24]. These betatron oscillations are due to the radial electric field component of the large amplitude plasma wave (wake) excited within a plasma channel, the axial component of which is responsible for electron acceleration. This process can occur in both laser-driven and electron beam-driven plasma based accelerators. Betatron radiation from a plasma driven by an intense electron or laser beam has recently been observed [25], [26]. Section III-A presents the theory describing the betatron radiation mechanism, various scaling laws, and examples for the design of such an x-ray source.

Alternatively, x-ray radiation may be produced via Thomson scattering (TS) using an external laser pulse [27]–[33]. The first demonstration of the generation of sub-picosecond duration x-ray pulses using TS was implemented at the Beam Test Facility of the Advanced Light Source at LBNL [28]–[30]. The scattering geometry was configured for 90° [34] so that the generated x-ray pulse duration of 300 fs (FWHM) was determined by the convolution between the laser pulse duration (100 fs) and the crossing time of the laser across the tightly focused electron beam (200–250 fs). The number of x-rays and peak brightness of the 90° TS source experiments at LBNL were in part limited by the fact that the laser beam only interacted with about a 100 fs long electron beam slice (or 0.3% of all the available electrons), as well as by the relatively high transverse emittance of the electron beam and low peak laser power used in the experiment. To increase the photon yield and source brightness, high quality femtosecond electron bunches are needed to enable femtosecond x-ray production through 180° laser backscattering [35]. As indicated above, laser driven accelerators are now capable of producing such intense femtosecond electron bunches. Section III-B presents the theory describing the Thomson scattered radiation spectrum, and presents examples of TS x-ray sources for various electron energy distributions produced by laser-plasma accelerators.

II. COHERENT TERAHertz RADIATION FROM LASER ACCELERATED BUNCHES

Laser-triggered solid-state based sources of THz radiation have been developed that rely on switched photoconducting antennas (e.g., Ref. [36] and references therein) or optical rectification of femtosecond pulse trains [37]. Large aperture biased GaAs structures, operated at 1 kHz repetition rate, have produced on the order of 0.5 μJ/pulse [38]. Most other sources using either laser switched structures or optical rectification have operated at high frequency (10’s of MHz) with μW–mW level average power. To date, the experiments carried out at the L’OASIS facility of LBNL [20] have collected and focused coherent radiation with energies near the 100 nJ/pulse level, in which the measured energy was limited primarily by the small collection angle (≈100 mrad) and the small transverse size (≈100 μm) of the plasma.

A. Transition Radiation Theory

Modeling of the radiation process can be done by assuming that the plasma (with dielectric constant \( \epsilon = 1 - \omega_p^2/\omega^2 \), where \( \omega_p \) and \( \omega \) are the plasma and radiation frequencies, respectively) is equivalent to a conductor with a sharp conductor-vacuum boundary. The transition radiation will be generated by electron beam induced polarization currents at plasma densities below the critical density (where \( \omega_p = \omega \)) for the radiation wavelength. The plasma density profile used in the experiments discussed in Sec. II-B had a sufficiently large gradient such that the dielectric constant satisfied \( |\epsilon| \gg 1 \) within a distance on the order of a skin depth, and therefore the plasma can be well-modeled as a conductor for frequencies \( \omega < \omega_p \). In addition, provided the plasma scale length is short compared to the radiation formation length, then the dielectric interface radiates as if it were a sharp dielectric-vacuum boundary.

The energy radiated, per unit frequency \( d\omega \) and per unit solid angle \( d\Omega \), from a single electron traversing the dielectric boundary is given by the well-known result [39]–[41]

\[
\frac{d^2W_e}{d\omega d\Omega} = \frac{r_e m_e c}{\pi^2} \frac{u^2 (1 + u^2 \sin^2 \theta)}{(1 + u^2 \sin^2 \theta)^2},
\]

where \( \theta \) is the observation angle with respect to the electron trajectory (assumed to be normal to the plasma surface and along the z-axis), \( u = \gamma v/c \) is the electron momentum normalized to \( m_e c \), \( c \) is the speed of light, \( m_e \) is the electron rest mass, and \( r_e = e^2/m_e c^2 \) is the classical electron radius. The radiation pattern is zero along the axis \( (\theta = 0) \) and peaks at a radiation cone angle of \( \theta \approx 1/\gamma \) (assuming \( \gamma \gg 1 \)). The differential energy radiated by a single electron \( d^2W_e/d\omega d\Omega \) is independent of frequency. In practice, however, the maximum wavelength radiated will be limited, for example, by the physical dimensions of the system, as will be discussed below.

Integrating over solid angle \( d\Omega = 2\pi \sin \theta d\theta d\theta \) yields

\[
\frac{dW_e}{d\omega} = \frac{2r_em_ec}{\pi} \left[ \frac{(1 + 2u^2)}{u\sqrt{1 + u^2}} \tanh^{-1} \left( \frac{u}{\sqrt{1 + u^2}} \right) - 1 \right],
\]

which, in the highly-relativistic limit \( \gamma \gg 1 \), reduces to

\[
\frac{dW_e}{d\omega} \approx (2/\pi)r_em_ec \ln(\gamma).\]

The radiation from the beam of electrons sums incoherently for wavelengths short compared to the bunch length, i.e., \( W_{CTR} \approx N_b W_e \), where \( N_b \) is the number of electrons in the bunch, and a monoenergetic divergenceless beam was assumed. For wavelengths long compared to the bunch length, the radiation sums coherently, i.e., \( W_{CTR} \approx N_b^2 W_e \). In particular, the total coherent radiated energy over all angles and frequencies is given by

\[
W_{tot} \approx (4r_em_ec^2) N_b^2 \ln(\gamma)/\lambda_{min},\]

\[
\lambda_{min} \approx \lambda_{min} \text{ (the minimum wavelength for which the bunch radiates coherently and is determined by the electron bunch dimensions,}
\]

[1]
as will be discussed below. The total coherent energy can be written in practical units as

\[ W_{\text{tot}}[J] \approx 3.6 \times 10^{-2}(Q[nC])^2 \ln(\gamma)/\lambda_{\text{min}}[\mu\text{m}], \]

where \( Q \) is the bunch charge. For example, \( Q = 5 \) nC \((N_b = 3.1 \times 10^{10}), \gamma = 10, \) and \( \lambda_{\text{min}} = 200 \) \( \mu \text{m} \) give \( W_{\text{tot}} \approx 10 \) mJ, which is several orders of magnitude beyond that of conventional sources.

The above analysis can be generalized for the case of an arbitrary electron beam momentum distribution including the effects of coherence and the transverse size of the dielectric structure (diffraction from the plasma edge). Assuming a normalized energy distribution of the electron beam \( g(u) \), the differential radiated energy by a bunch travelling normal to the plasma-vacuum boundary is [21]

\[
\frac{d^2W}{d\omega d\Omega} = \frac{r_e m_e c}{\pi^2} N_b (N_b - 1) \sin^2 \theta \times \left( \int \! d\omega g(u) F(\omega, \rho, \theta, u) \right)^2, \tag{4}
\]

where

\[
F = \int d\zeta d\zeta_\perp \! e^{-ik_\perp \cdot \zeta_\perp} e^{-i\omega \zeta/v} f(\zeta_\perp, z) \tag{5}
\]

is the Fourier transform of the electron beam spatial distribution \( f(\zeta_\perp, z) \), or spatial form factor; \( \vec{k} \) is the radiation wave vector, and

\[
D = 1 - J_0(bu \sin \theta) \left[ bK_1(b) + b^2 K_0(b)/2 \right] - b^2 K_0(b)J_2(bu \sin \theta)/2 \tag{6}
\]

is the correction due to diffraction from the transverse edge of the plasma. Note that (5) assumes that there are no correlations between position and momentum. The parameter \( b = k\rho/u \) describes the relative influence of the transverse boundary (i.e., the ratio of the transverse size of the plasma \( \rho \) to the transverse extent of the self-fields of the relativistic electrons \( \sim \gamma \lambda \)), and \( K_0, K_1, J_0, \) and \( J_2 \) are Bessel functions. For large transverse size (i.e., \( b \gg 1 \)), \( D \approx 1 \), and the coherent radiation is well-described by transition radiation. For \( b \sim 1 \), (i.e., transverse plasma size small compared to the effective source size \( \gamma \lambda \) diffraction effects will strongly modify the radiated energy spectrum.

The degree of coherence in the bunch will be determined by the form factor (5). For a bunch with an uncorrelated Gaussian spatial distribution, the spatial form factor is \( F = F_\perp F_\parallel \) where \( F_\perp = \exp[-(k\sigma_\perp \sin \theta)^2/2] \) and \( F_\parallel = \exp[-(\omega\sigma_\parallel/\nu)^2/2] \), with \( \sigma_\perp \) the rms transverse bunch radius and \( \sigma_\parallel \) the rms longitudinal bunch length. Fully-coherent emission in both the transverse and longitudinal directions requires \( F \approx 1 \). This condition is satisfied when \( \sigma_\perp \sin \theta \) and \( \sigma_\parallel \) are much less than the radiation wavelength. Measurement of the energy spectra of the transition radiation provides a method for obtaining information about the spatial and temporal distributions of the electron beam.

As (4) indicates, the finite transverse extent of the plasma produces a wavelength dependence in the differential energy for fully-coherent radiation. The spectral content of the radiation is no longer constant, as is the case for transition radiation from a fully-coherent beam traversing a boundary of infinite transverse extent, and the spectra is strongly modified by the diffraction effects for parameters \( b \sim 1 \) (i.e., \( \lambda \sim \rho/\gamma \)). Distortion of the spectra increases with larger energy, and the spectra are suppressed for decreasing transverse size. For large transverse size (\( b \gg 1 \)), the fully-coherent spectra becomes constant (i.e., the limit of transition radiation from an infinite transverse boundary). In general, the spectral region of coherent radiation is approximately \( 2\pi\sigma_z < \lambda < 2\pi\rho/\gamma \), where the lower bound is due to longitudinal coherence \([F \in (4)] \), and the upper bound is due to diffraction effects \([D \in (4)] \).

Using (4), assuming \( b \gg 1 \), the total coherent radiation energy into a small collection angle \( \theta \leq \theta_{\text{coll}} < 1/(u) \), and for a bandwidth \( \Delta \omega/\omega \) is

\[
W_{\text{CTR}} \approx m_e c^2(r_e/\lambda)N_b^2(\Delta \omega/\omega)\theta_{\text{coll}}^2(u)^2, \tag{7}
\]

assuming \((u)^2 > 1 \). Laser-plasma generated electron beams in the self-modulated regime can be characterized by a Boltzmann momentum distribution \( g(u) = u_{\text{T}}^{-1}\exp[-u/u_{\text{T}}] \), where \( u_{\text{T}} \) is the temperature of the distribution. The amount of radiated energy in the collection cone half-angle \( \theta_{\text{coll}} < 1/\omega \) and frequency bandwidth \( \Delta \omega/\omega \) is given by \( \Delta W_{\text{CTR}} \approx 4m_e c^2(r_e/\lambda)N_b^2(u_{\text{T}}/\theta_{\text{coll}})^4(\Delta \omega/\omega) \). For example, \( \Delta W_{\text{CTR}} = 30 \) \( \mu \text{J} \) within \( \theta_{\text{coll}} = 50 \) mrad for a 1.5 nC bunch with temperature \( u_{\text{T}} = 5 \) in a bandwidth 0.3 to 3 THz. Due to the strong dependence on electron energy and angle, \( W_{\text{CTR}} \propto (\gamma \theta_{\text{coll}})^2 \), the measured energy can easily be increased by increasing either the electron energy \( \gamma \) or the cone angle \( \theta_{\text{coll}} \) of the collection optics (for \( \theta_{\text{coll}} < 1/(u) \)).

B. Experiment

The experiments described in this paper used the short pulse, high peak power and high repetition rate Ti:Al_2O_3 laser system [18] of the L’OASIS facility at LBNL. Low energy laser pulses (of wavelength \( \lambda \approx 0.8 \mu\text{m} \)) from a Ti:Al_2O_3 laser oscillator were first temporally stretched, amplified to 1 J/pulse level, and then compressed using a grating based optical compressor. Following compression, the laser beam was focused to a spot size \( w = 6 \mu\text{m} \) with a 30 cm focal length (F/4) off-axis parabola onto a pulsed gas jet. The peak power \( P \) of the laser was varied using both the pulse duration and laser energy. At optimum compression [55 fs full-width-half-maximum (FWHM) duration], \( P \approx 8.3 \) TW, resulting in a calculated peak intensity \( I = 2P/\pi r_0^2 \approx 1.5 \times 10^{19} \) W/cm^2 and a normalized laser strength \( a_0 = 8.6 \times 10^{-10} \lambda^{1/2}[\mu\text{m}]T^{1/2}[\text{W/cm}^2] \approx 2.6 \). The gas jet was backed with up to 70 bar helium gas. The profile of the jet had a 1.8 mm flat top at a density of about \( 3 \times 10^{19} \) cm\(^{-3}\) with a ramp of 1 mm length to zero density on either side (a total length of just under 4 mm). The plasma density profile, measured with a folded-wave interferometer using 400 nm wavelength, 50 fs duration laser pulses, had a typical transverse size of 100 \( \mu\text{m} \pm 15 \mu\text{m} \). The lay-out of the experiment is shown in Fig. 1.
The charge per bunch and electron beam spatial profile were measured using an integrating current transformer (ICT) and a phosphor screen, located 50 cm and 75 cm away from the exit of the gas jet, respectively. The response of the phosphor screen (number of counts on the CCD vs. deposited charge) was calibrated against the ICT. The energy distribution was obtained using an imaging magnetic spectrometer [42] and a phosphor screen with CCD camera. At a given excitation current of the magnet, the momentum acceptance of the magnetic spectrometer was $\delta p/p = \pm 2\%$. A spectrum ranging from 0–50 MeV was obtained by recording the total light yield on the CCD image (corrected for background counts) for excitation currents ranging from 0 to 45 A.

**C. Electron Beam Properties**

The present experiments were operated using a single laser pulse to ionize the gas jet and excite a plasma wake that subsequently traps and accelerates background electrons. For the particular gas jet density profile (absolute gas density and length of the jet) and laser parameters used in the present experiments, a typical energy spectrum is shown in Fig. 2 and is reasonably well approximated by a Boltzmann distribution with a temperature of 4.6 MeV.

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The electron beam profile was found to be dependent on laser power. Two typical images of the electron beam on the phosphor screen in the accelerator operating regime for which the data were taken are shown in Fig. 3. The beam divergence vs. laser pulse duration is shown in Fig. 4. The laser pulse duration was changed by changing the distance between the gratings in the pulse compressor. The asymmetry in the data curve for points on the left and on the right hand side of the minimum pulse duration is consistent with previous observations of a dependence of electron yield on the shape of the laser pulse. [7], [43]

For power levels near the maximum power, electron beams were observed with a divergence less than 10 mrad (FWHM) [see Fig. 3(b)]. The amount of charge contained in the small beams was typically about 5\% of the main beam and ranged between 30–50 pC. The energy of this part of the beam was reasonably well approximated by a Boltzmann distribution with an effective temperature of 4.6 MeV.
rms geometric "emittance" for the high power case can be
estimated to be 0.01 mm mrad, by simply multiplying the
assumed source size and measured divergence angle. Simi-
larly low unnormalized “emittance” values between 0.01 and
0.1 π mm mrad have been reported previously [44], [45].
However, prior to interpreting this quantity into an electron
beam emittance value, as commonly used for conventional
accelerators, a thorough understanding of all phase space
properties of the beam is required, including energy (mean
and spread) and bunch length, as a function of propagation
distance. Upon exiting the plasma, the high density electron
bunch is subjected to significant space charge forces that can
result in transverse beam blow-up as well as modifications to
the longitudinal bunch shape and energy distribution. In gen-
eral, longitudinal space charge forces are expected to lengthen
the bunch, thereby reducing the density and, consequently,
the effect of transverse space charge forces while the beam
is propagating through vacuum [46], [47]. Also, the large
energy spread can result in ballistic spreading of the electron
bunches since fast electrons would out run the slower ones.
More complicated modifications of the energy distribution and
correlations in momentum and physical space can still occur
however, complicating the interpretation of the data. As an
example, if the bunch initially has slow electrons in the front
and back, and fast electrons in the middle, bunch compression
can occur during the propagation resulting in a temporary
increase in space charge forces. Detailed measurements of
the energy (mean and spread), bunch length, and transverse
characteristics of these low divergence beams are needed to
fully understand their behavior during propagation in vacuum
and to properly assign an emittance value. Pepper pot based
measurements of electrons that were energy filtered around
55 MeV have recently been carried out in a similar laser
wakefield setup to ours and indicate a normalized emittance
of 3 π mm mrad at the spectrometer location [48].

D. Radiation Measurements

The radiation measurements were done from 0.3–19 THz
using a liquid helium cooled bolometer. To verify the depen-
dence of radiated energy on collection angle, two different
set-ups were used. In the first set-up, with a collection angle
of 30 mrad, radiation was reflected out of the vacuum chamber
through a window using a 5 μm thick metal coated nitrocellu-
lose foil 30 cm from the gas jet. (Although transition radiation
will be generated when the beam propagates through this
foil, the emission is incoherent due to bunch lengthening [47]
and transverse beam size increase at that location, measured
with a phosphor screen.) The radiation then propagated in air
and through laser beam attenuators, and was focused with an
F/30 metal coated spherical mirror onto a cooled (4.2 K) Si-
based bolometer, equipped with an internal 3 THz low-pass
filter, to measure the radiated energy. The spectral transmission
properties of all solid materials used in the terahertz path were
measured using a ZnTe based optical rectification system [49].

To improve collection efficiency, the second set-up used an
off-axis 90° parabola (effective focal length of 127 mm) to
collimate THz light onto a second off-axis parabola (effective
focal length of 177.8 mm). After exiting the vacuum chamber
through a silicon window, the radiation was incident on
the bolometer which was positioned in the calculated focal
position. Note that a hole was drilled on axis through the first
parabola to allow the 800 nm laser beam to propagate through
it, resulting in a collection angle (half-cone) covering emission
angles between 80 < θ < 300 mrad. The complete bolometer
system sensitivity (volts per incident energy) including a gain
factor of the internal amplifer of 1000 and the bolometer
internal collection efficiency was calibrated and found to be
1 V of signal per 4.0 pJ of incident THz energy. The total
transmission of all materials in the THz beam path was
measured by a far-infrared interferometer in combination with
a hot Hg lamp and found to be 2.5 × 10⁻⁵.

For θ_{coll} = 30 mrad, and electron beams containing up

To the calculated radiated energy per pulse between

For the experimental parameters (ρ = 100 μm ± 15 μm,

Fig. 4. Electron beam divergence vs. laser pulse duration. The laser pulse
duration was varied by changing the distance between the gratings in the pulse
compressor and the duration was measured using a single-shot second order
autocorrelator. Each data point represents 10 shots.
E. Polarization Measurement

Since transition radiation is in principle radially polarized for an axisymmetric e-beam, a further test of the validity of our model was carried out by measuring the polarization characteristics of the source. A far-infrared wire-grid linear polarizer was placed at the entrance plane of the bolometer, where the THz radiation is brought to a focus. The polarizer angle was rotated from $0^\circ$ to $180^\circ$ in steps of $30^\circ$ and at each position about 30 shots were acquired. Within the error bars, the transmitted signal was found to be independent of the polarizer angle, consistent with radially polarized radiation.

F. Radiation Spectrum

As a first attempt to obtain spectral information of the THz source, the total radiated energy was collected in the 0.3–19 THz and in the 0.3–2.9 THz ranges (and hence 2.9–19 THz range). This was done with the use of two separate filters that were inserted between the bolometer window and the bolometer detector element. The first long pass filter consisted of a thin polyethylene filter with a diamond scattering layer, and had a cut-on wavelength of 16 $\mu$m (19 THz). The second long pass filter, made of crystal quartz with a broadband anti-reflection coating, had a cut-on wavelength of 103 $\mu$m (2.9 THz). Radiation at wavelengths longer than 1 mm (0.3 THz) was not detectable with the bolometer through its intrinsic design.

Analysis of the raw experimental data (averaging of several tens of shots) shows that 22% of the observed radiation was emitted in the frequency range 0.3–2.9 THz, while 78% of the radiation was emitted within the frequency range 2.9–19 THz range. To model this data, the radiated spectrum was calculated including diffraction (see Sec. II-A) from a 100 $\mu$m radius plasma column and again using a 4.6 MeV Boltzmann distribution for the electron bunch energy distribution. The spectrum of the radiation can be calculated from (4) assuming a longitudinal charge distribution and transverse size of the dielectric (plasma). Figure 6 shows the calculated spectrum of the radiation for a Gaussian longitudinal charge distribution with rms length $\sigma_z = 13.5 \mu$m and $\nu_T = 9$, transitioning out of a plasma with a transverse half-width of 100 $\mu$m. The short wavelength cut-off is due to the longitudinal coherence effects ($\lambda_{\text{min}} \sim \sigma_z$), and the long wavelength cut-off is due to the influence of diffraction radiation from the transverse edge of the plasma ($\lambda_{\text{max}} \sim \rho/\nu_T$). For these values of plasma size and electron beam properties, the spectrum shown in Fig. 6 contains 22% of the radiation emitted in the frequency range 0.3 to 2.9 THz, and 78% of the radiation emitted within the frequency range 2.9 to 19 THz range. An rms bunch length on the order of 10 $\mu$m is consistent with self-modulated LWFA simulations [50] which typically predict electron beam bunch lengths to be on the order of the laser pulse duration.

III. FEMTOSECOND X-RAY GENERATION

The ultra-short pulse nature of the accelerated electron bunches produced by a laser-plasma accelerator offers the possibility of generating femtosecond x-ray pulses. Two different methods will be considered. The first relies on the emission of betatron radiation as the electrons propagate through a plasma channel in the presence of the radial wakefield excited by the laser pulse. The second method relies on the use of an external laser to Thomson scatter off the accelerated electron bunch.

A. Betatron Radiation from Plasma Focusing Channels

Betatron radiation will be analyzed starting with the single particle orbits to calculate the radiation spectrum. The on-axis radiation spectrum is calculated, as well as the asymptotic behavior of the spectrum for large values of the betatron strength parameter. Examples of the radiated spectrum are given for a single electron as well as for an electron beam with a Guassian radial distribution of electrons.
1) Single Particle Orbits: The electron motion in a plasma focusing channel is governed by the relativistic Lorentz equation, which may be written in the form $du/dt = \mathbf{\nabla}\Phi$, where $\Phi = e\Phi/m_e c^2$ is the normalized electrostatic potential of the focusing channel, $u = p/m_e c = \gamma\beta$ is the normalized electron momentum, and $\gamma = (1 + u^2)^{1/2} = (1 - \beta^2)^{-1/2}$ is the relativistic factor. Here only the transverse focusing force of the plasma is considered. Near the axis, $r^2 \ll r_0^2$, the space charge potential is assumed to have the form $\Phi = \Phi_0(1 - r^2/r_0^2)$, such that the normalized radial electric field is $E_r = -\partial\Phi/\partial r = 2\Phi_0 r/r_0^2$, where $\Phi_0$ and $r_0$ are constants. The electrostatic potential is related to the electron plasma density by $\nabla^2\Phi = k_p^2(n_e/n_0 - 1)$, where a uniform background of plasma ions of density $n_0$ is assumed. The maximum focusing field occurs when the plasma electrons are completely expelled (blown out) from the channel, $n_e = 0$. Notice that in the blowout regime, $E_r = k_p^2 r/2$, hence, $\Phi_0/r_0^2 \leq k_p^2/4$. Assuming the electron orbit lies in the $(x, z)$ plane, the betatron orbit is given by

$$\beta_x \simeq k_\beta r_\beta \cos(k_\beta ct),$$

$$\tilde{x} \simeq r_0 \sin(k_\beta ct),$$

$$\beta_z \simeq \beta_m (1 - k_\beta^2 r_\beta^2/4) - \beta_m (k_\beta^2 r_\beta^2/4) \cos(2k_\beta ct),$$

$$\tilde{z} \simeq z_0 + \beta_m (1 - k_\beta^2 r_\beta^2/4) ct - \beta_m (k_\beta^2 r_\beta^2/8) \sin(2k_\beta ct),$$

where $\beta_m = (2\Phi_0/\gamma_{z0} r_0^2)^{1/2}$ is the betatron wavenumber, $r_\beta$ is the amplitude (assumed constant) of the betatron orbit, $u_z = u_{z0}$ is the initial axial momentum of the electron (a constant of the motion), $\gamma_{z0} = (1 + u_{z0}^2)^{1/2}$, $\beta_{x0} = u_x/\gamma_{z0} \gamma_{\beta0}$ and $z_0$ is a constant. The above expressions are the leading order contributions to the orbits, assuming $k_\beta^2 r_\beta^2/2 \ll 1$. Notice that in the blowout regime $\Phi_0 = k_p^2 r_\beta^2/4$, which gives $k_\beta = k_p/(2\gamma_{z0})^{1/2}$.

2) Single Particle Radiation Spectrum: The energy spectrum of the radiation emitted by a single electron on an arbitrary orbit $\tilde{r}(t)$ and $\beta(t)$ can be calculated from the Lienard-Wiechert potentials [51],

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c^2} \int_{-T/2}^{T/2} dt \left[ \mathbf{n} \times \left( \mathbf{n} \times \frac{\partial}{\partial t} \right) \right] e^{i\omega(t - \mathbf{n} \cdot \tilde{r}/c)}^2,$$

where $d^2 I/d\omega d\Omega$ is the energy radiated per frequency, $\omega$, per solid angle, $\Omega$, during the interaction time, $T$, and $\mathbf{n}$ is a unit vector pointing in the direction of observation. Using the betatron orbits given above, the radiation spectrum can be calculated with conventional techniques [27], [52]. In the limits $\gamma_{z0} \gg 1$, $\theta^2 \ll 1$, and $(1 + a_\beta^2/2)/\gamma_{z0}^2 \ll 1$, the radiation spectrum can be written as

$$\frac{d^2 I}{d\omega d\Omega} = \sum_{n=1}^\infty \frac{\alpha n^2}{1 + a_\beta^2 + \gamma_{z0}^2 \theta^2} R_n(\omega, \omega_n) \times \left[ \frac{a_\beta^2 c^2}{2} + 4\gamma_{z0}^2 \theta^2 \tilde{C}_z^2 - 4 a_\beta \gamma_{z0} \theta C_z \tilde{C}_z \cos \phi \right],$$

where

$$\tilde{C}_z = \sum_{m=-\infty}^{\infty} J_m(\alpha_z) \left[ J_{n+2m-1}(\alpha_z) + J_{n+2m+1}(\alpha_z) \right],$$

$$\alpha_z = \frac{n(\omega/\omega_n) a_\beta^2}{1 + a_\beta^2 + \gamma_{z0}^2 \theta^2},$$

$$R_n = \frac{\sin^2 \left[ \pi n N_{\beta}(\omega/\omega_n - 1) \right]}{\pi N_{\beta}(\omega/\omega_n - 1)^2},$$

where $n$ is the harmonic number,

$$a_\beta = \gamma_{z0} k_\beta r_\beta$$

is the betatron strength parameter, $N_{\beta} = L/\lambda_\beta$ is the number of betatron periods that the electron undergoes, $L = cT$ is the interaction length, $J_m$ are Bessel functions, and $\alpha_f = e^2/\hbar c \simeq 1/137$ the fine structure constant. Here $\theta$ is the observation angle with respect to the electron propagation axis and $\phi$ is the azimuthal observation angle.

The resonance function $R_n(\omega, \omega_n)$ determines many properties of the radiation spectrum. Provided the number of betatron periods is large, $N_{\beta} \gg 1$, radiation is emitted in a series of harmonics and is confined in a narrow bandwidth about the resonant frequency ($\omega = \omega_n$) of each harmonic. The intrinsic frequency width $\Delta\omega_n$ of the spectrum $R_n$ about $\omega_n$ is given by $\Delta\omega_n/\omega_n = 1/n N_{\beta}$. Furthermore, $R_n(\omega, \omega_n) \rightarrow \Delta\omega_n \delta(\omega - \omega_n)$ as $N_{\beta} \rightarrow \infty$. For a single harmonic $n$, the angular width $\Delta\theta_f$ about the axis of a cone containing
radiation with frequencies in a small bandwidth $\Delta \omega$ about $\omega_n$ is given by

$$\Delta \theta_l^2 \approx \left(1 + \frac{a_{\beta}^2}{2}\right) \times \left\{ \frac{\Delta \omega_n / \omega_n}{\Delta \omega \leq \Delta \omega_n}, \quad \frac{\Delta \omega / \omega_n}{\Delta \omega \geq \Delta \omega_n} \right\}, \quad (21)$$

3) On-Axis Radiation: Of particular interest is the radiation emitted along the axis, $\theta = 0$, where only the odd harmonics are finite, i.e., the even harmonics vanish. Setting $\theta = 0$ in the above expressions gives, for the $n^{th}$ odd harmonic, $\alpha_x = 0$, $\alpha_z = a_n$, and

$$\frac{d^2 I_n(0)}{d \omega d \Omega} = \alpha_f \left(\frac{\omega}{\omega_n}\right)^2 \frac{\gamma_{20}^2 N_\beta^2 a_{\beta}^2}{(1 + a_{\beta}^2/2)^2} R_n(\omega, \omega_n) \times \left[J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n)\right]^2, \quad (22)$$

where $\alpha_n = n(\omega / \omega_c)(a_{\beta}^2/4)/(1 + a_{\beta}^2/2)$ and $\omega_n = 2\gamma_{20}^2 c k \beta / (1 + a_{\beta}^2/2)$ is the on-axis resonant frequency.

An expression for the number of photons ($N_n$) radiated along the axis per solid angle, $dN_n / d\Omega$, per electron for photons in a narrow bandwidth $\Delta \omega$ about the resonant frequency $\omega_n$ is obtained by integrating the above expression over $\Delta \omega$ and by dividing by the energy per photon ($h \omega_n$).

$$\frac{dN_n}{d\Omega} \approx \alpha_f \frac{\Delta \omega / \omega_n}{(1 + a_{\beta}^2/2)^2} \times \left[J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n)\right]^2, \quad (23)$$

where $\Delta \omega_l = \Delta \omega$ for $\Delta \omega \leq \Delta \omega_n$ and $\Delta \omega_l = \Delta \omega_n$ for $\Delta \omega \geq \Delta \omega_n$, with $\Delta \omega_n = \omega_n / nN_\beta$ the intrinsic bandwidth and $\alpha_f$ the fine structure constant. The total number of photons radiated per electron in the bandwidth $\Delta \omega$ about $\omega_n$ is given by multiplying $dN_n / d\Omega$ by the solid angle $2\pi(\Delta \theta_l^2/2)^{1/2}$, where $\Delta \theta_l$ is given above. This yields

$$N_n \approx \pi \alpha_f \frac{\Delta \omega N_\beta a_{\beta}^2}{\omega_n (1 + a_{\beta}^2/2)} \times \left[J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n)\right]^2, \quad (24)$$

for all values of $\Delta \omega^2 \ll \omega_n^2$.

In the limit $a_{\beta}^2 < 1$, this reduces to $N_n \approx \pi \alpha_f (\Delta \omega / \omega_n) N_\beta a_{\beta}^2$. By integrating the full expression for $d^2 I / d\omega d\Omega$ over all frequencies and angles, one finds that the total number of photons radiated is given by

$$N_T = (\pi/3)\alpha_f N_\beta a_{\beta}^2, \quad (25)$$

assuming $a_{\beta}^2 < 1$.

4) Asymptotic Behavior: For values of $a_{\beta}^2 < 1$, the emitted radiation will be narrowly peaked about the fundamental resonant frequency, $\omega_1$ ($n = 1$). As $a_{\beta}$ approaches unity, emitted radiation will appear at harmonics of the resonant frequency as well, $\omega_n = n\omega_1$. When $a_{\beta} \gg 1$, high harmonic ($n \gg 1$) radiation is generated and the resulting synchrotron radiation spectrum consists of many closely spaced harmonics. Finite variations in the parameter $a_{\beta} = \gamma_{20} k \beta r_{\beta}$ within an electron beam can broaden the linewidth and cause the spectrum to overlap. Hence, in the asymptotic limit, i.e., $a_{\beta} \gg 1$, the gross spectrum appears broadband, and a continuum of radiation is generated which extends out to a critical frequency, $\omega_c$, beyond which the radiation intensity diminishes.

Asymptotic properties of the radiation spectrum for $a_{\beta}^2 \gg 1$ and for large harmonic numbers, $n \gg 1$, can be obtained with standard methods. In particular, the asymptotic spectrum along the axis $\theta = 0$, is given by

$$\frac{d^2 I(0)}{d \omega d \Omega} \approx \left(6/\pi^2\right) \alpha_f N_\beta \gamma_{20}^2 c^2 K_{2/3}(\xi), \quad (26)$$

where $\xi = \omega / \omega_c$ and $\omega_c = 3a_{\beta}^2 \gamma_{20}^2 c k \beta$ is the critical frequency (corresponding to a critical harmonic number of $n_c = 3a_{\beta}^2/4$). The function $Y(\xi) = \xi^2 K_{2/3}(\xi)$ is maximum at $\xi = 1/2$ and decreases rapidly for $\xi > 1$. Half the total power is radiated at frequencies $\omega < \omega_c/2$ and half at $\omega > \omega_c/2$.

5) Radiation from a Beam: For a single electron undergoing betatron motion in a plasma focusing channel, the resonant frequency of the radiation emitted along the axis is $\omega = 2\gamma_{20}^2 n a_{\beta}^2 c k \beta / (1 + a_{\beta}^2/2)$. Here, $a_{\beta} = \gamma_{20} k \beta r_{\beta}$ is a function of both the electron energy $\gamma_{20}$ and the radial position of the electrons via the betatron amplitude $r_\beta$. If a monoenergetic beam of finite radius is injected into a focusing channel (without any special tapering), electrons at different radii will have different betatron amplitudes $r_\beta$, different values of $a_{\beta}$, and hence different resonant frequencies. In general, the spectral energy density of the radiation emitted by a finite radius beam will be significantly different from that of a single electron, especially in the limit $a_{\beta} \gg 1$.

Consider the case of a monoenergetic, axisymmetric (round) beam in cylindrical geometry in the limit of zero emittance. In this case, $a_{\beta} = \gamma_{20} k \beta r_{\beta}$ represents the normalized radial position of the electron, since the initial radial position of the electron at the channel entrance is assumed to be equal to $r_\beta$. The radiation spectrum from a beam, $d^2 I / d\omega d\Omega$, can be approximately calculated from $d^2 I / d\omega d\Omega$ by multiplying by the electron distribution function, $f_c$, and integrating over both radius ($a_{\beta}$) and over $\phi$ (from 0 to $2\pi$ for an axisymmetric beam). For an axisymmetric beam,

$$\frac{d^2 I_B(\omega, \theta)}{d \omega d \Omega} = \int_0^{2\pi} d\phi \int_0^{\infty} da_{\beta} a_{\beta} f_c(a_{\beta}) \frac{d^2 I(\omega, a_{\beta}, \theta)}{d \omega d \Omega}. \quad (27)$$

For simplicity, a Gaussian radial beam distribution $f_c(a_{\beta}) = (2/a_{rms}^2) \exp(-a_{\beta}^2/a_{rms}^2)$ is assumed, where $a_{rms} = \gamma_{20} k \beta r_{rms}$ and $r_{rms}$ is the normalized RMS beam radius.

For a large number of betatron periods, the radial integration can be approximated analytically. Let

$$S_R = \int_0^{\infty} da_{\beta} a_{\beta} f_c \frac{d^2 I}{d \omega d \Omega} \equiv \int_0^{\infty} da_{\beta} a_{\beta} f_c S_n(\omega, a_{\beta}, \theta), \quad (28)$$

where $R_n$ is the resonance function. At resonance $\omega = \omega_n(a_{\beta})$ or, alternatively, $a_{\beta} = a_\omega(\omega)$, where $a_\omega^2 = 2\left[(n)^2 \gamma_{20}^2 c k \beta / \omega - (1 + \gamma_{20}^2 \theta_\omega^2)\right]$. Furthermore, in the limit $N_{\beta} \rightarrow \infty$, $R_n \rightarrow \Delta a_{\beta} a_\omega(\theta_\omega - a_\omega)$, where $\Delta a_{\beta} = 2\gamma_{20}^2 c k \beta / (N_{\beta} a_{\beta} \omega)$. Hence, for $N_{\beta} \rightarrow \infty,$

$$S_R \approx \frac{2\gamma_{20}^2 c k \beta}{N_{\beta} \omega} f_c(a_{\beta} = a_\omega) S_n(a_{\beta} = a_\omega). \quad (29)$$

In this limit, the spectrum of the radiation emitted along the
axis (θ = 0) from a Gaussian beam profile is

\[
\frac{d^2I_B(0)}{d\omega d\Omega} = \sum_n \alpha_f \frac{n\gamma_0^2 N\beta a_\beta^2}{(1 + a_\beta^2/2)}
\times \left[ J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n) \right]^2,
\]

where the sum is over odd harmonics n and the argument of the Bessel functions is \( \alpha_n = (n - \omega)/2 \), where \( \omega = \omega_0/2\gamma_0 c k_\beta \), i.e., evaluated at \( a_\beta^2 = 2(n/\omega - 1) \).

Using the above expression, other quantities of practical interest, such as the photon flux and brightness, can be calculated. Assuming that the collection angle, \( \theta_d \), is small \( \theta_d < (\Delta \omega/\omega)^{1/2} < (1/N)^{1/2} \) so the intensity distribution is flat over the solid angle \( \Delta \Omega_d = \pi \theta_d^2 \), the number of photons intercepted in a small bandwidth, \( \Delta \omega \), about \( \omega \) and solid angle, \( \Delta \Omega_d \), for an electron bunch with \( N_B \) electrons is

\[
N_B = \frac{N_B d^2I_B(0)}{d\omega d\Omega} \frac{\Delta \omega}{\omega} \pi \theta_d^2.
\]

The average flux in photons per second, \( F_{ave} \), in the collection angle \( \theta_d \) and with bandwidth \( \Delta \omega \), is \( N_B \) multiplied by the repetition rate, \( f_{rep} \), of the laser/electron beam, i.e., \( F_{ave} = N_B f_{rep} \). The average source brightness is given by

\[
B_{ave} = \frac{N_B f_{rep}}{(2\pi)^2 r_0^2 \theta_d^2} = \frac{N_B}{4\pi^2} \frac{d^2I_B(0)}{d\omega d\Omega} \frac{\Delta \omega}{\omega},
\]

where \( r_0 \) is the electron bunch radius. The peak flux and brightness are, respectively, \( F_{pk} = F_{ave}/(\tau_x f_{rep}) \) and \( B_{pk} = B_{ave}/(r_x f_{rep}) \), where \( \tau_x \) is the x-ray pulse duration, which is assumed to be approximately equal to the electron bunch duration.

The radiation spectra for a single electron with \( a_\beta = 2 \) and from an axisymmetric beam with a Gaussian radial distribution with \( a_{rms} = 2 \) are shown in Figs. 7-8. The result for a single electron with \( N_B = 4 \) and \( a_\beta = 2 \), is shown in Fig. 7, which shows the spectral density \( d^2I/d\omega d\Omega \) (normalized to \( \alpha_f \gamma_0^2 \)) versus normalized frequency \( \tilde{\omega} = \omega/2\gamma_0^2 c k_\beta \) and angle \( \theta = \gamma_0 \theta \), for (a) \( \phi = 0 \) and (b) \( \phi = \pi/2 \). The results of averaging over only \( \phi \) for \( a_\beta = 2 \) are shown in Fig. 8(a), whereas the results of averaging over both \( \phi \) and \( a_\beta \) are shown in Fig. 8(b). The effects of averaging over \( a_\beta \) leads to a dramatic smoothing of the radiation spectrum.

As an example, consider a laser wakefield accelerator producing a narrow energy spread electron beam, such as in the proposed colliding pulse injection method [10], [12], [13], or as in the recent experiments on the channel-guided LWFA [15]. The high energy part of the electron spectrum is assumed to exhibit a narrow peak at 100 MeV (3% energy spread) with a total charge of 0.3 nC and a bunch radius of 3 \( \mu \)m. The plasma is assumed to be 1 mm long with a density of \( 3 \times 10^{19} \) cm\(^{-3} \), and the wakefield is assumed to be in the blowout regime. Note that for an electron with a betatron radius equal to the beam radius (3 \( \mu \)m), \( a_\beta \approx 26 \). The expected radiated spectrum for a detection angle of \( \theta_d = 3 \) mrad, normalized to a bandwidth of \( \Delta \omega/\omega = 0.1 \%), is shown in Fig. 9, as obtained from (30). At the peak of the spectrum (10 keV), the number of photons is \( N_B = 2 \times 10^4 \) photons/shot and the brightness is

\[
B_{ave} = 6.3 \times 10^6 \text{ photons/shot/0.1\%BW/mm}^2/\text{mrad}^2,
\]

assuming a 3 mrad detection angle and 0.1% bandwidth, as obtained from (31) and (32).

B. Thomson Scattering using an External Laser Pulse

The analysis of this x-ray source will again start with a discussion on the orbit of a single electron and the spectrum will be calculated in a similar way as for betatron radiation. The scattered spectrum for a single electron will be calculated as well as that from an electron beam with finite energy spread. The obtained expressions will be used to design a polychromatic and quasi-monochromatic source, obtained by scattering off an electron bunch with a broad (100%) and a narrow (few %) energy distribution, respectively.

1) Single Particle Orbits: Consider a one-dimensional (1D) laser field propagating in the negative z direction with a normalized vector potential, \( a = eA/m_e c^2 \), of the form \( a = a_0 \cos(k_0 z + c t) \), which is linearly polarized in the \( z \) direction with frequency \( \omega_0 = k_0 c \). Here, the parameter \( a_0 \) is related to the laser intensity \( I_L \) and wavelength \( \lambda_0 \) by \( a_0 \approx 8.5 \times 10^{-10} \lambda_0 [I_L \text{[W/cm}^2\text{]}]^{1/2} \). The electron motion is governed by the relativistic Lorentz equation,

\[
d\mathbf{u}/dt = \mathbf{a}/\gamma - (\mathbf{u}/\gamma) \times (\nabla \times \mathbf{a}).
\]

(33)

In the 1D limit, there exist two constants of the motion, \( \mathbf{u}_\perp = \mathbf{a}_\perp \) and \( \gamma + u_z = \gamma_{0z} + u_{0z} = \gamma_{0z}(1 + \beta_{0z}) \), where \( u_{0z} = 1 \).
The radiation spectrum for a single electron scattered by a laser with \( a_0 = 1 \) and \( N_0 = 10 \) is shown in Fig. 10. The figure shows the spectral density \( d^2I/d\omega d\Omega \) (normalized to \( \alpha f(\gamma_{r0}) \)) versus normalized frequency \( k = k/2\gamma_{r0}^2\beta z \) and angle \( \phi = \gamma z_0^2 \theta \), for (a) \( \phi = 0 \) and (b) \( \phi = \pi/2 \).

3) Spectrum for an Electron Bunch with Finite Energy Spread: To include the effect of energy spread, the spectral flux density is integrated over the electron energy distribution, \( f(\gamma) \),

\[
\frac{d^2I_{nT}}{d\omega d\Omega} \approx \int d\gamma f(\gamma) \frac{d^2I_n}{d\omega d\Omega}. \tag{37}
\]

Beam emittance is neglected since the angular width of the spectrum over the photon energies of interest is much broader than typical beam divergences.

The electron bunch produced by a LWFA will have a finite energy spread. The SM-LWFA is typically characterized by a very broad energy distribution, e.g., a Boltzmann distribution. For a standard LWFA with laser-triggered injection, the energy spread is smaller, typically a few percent. In the following, \( f(\gamma) \) is assumed to be slowly varying compared to \( R_n(\omega, \omega_n) \) for fixed \( \omega \). For fixed \( \omega \), the energy width \( \Delta \gamma_n \) of the resonance function \( R_n \) about the resonant energy \( \gamma_n \) is given by

\[
\gamma_n = \frac{4\gamma_{r0}^2n\omega_0}{(1 + a_0^2/2 + \gamma_{r0}^2\theta^2)}. \tag{36}
\]
by

$$\Delta \gamma_n = \int d\delta \mu R_n = \left( \frac{\gamma_n}{2nN_0} \right) \left( 1 - \frac{\omega \theta^2}{4n\omega_0} \right)^{-1}, \tag{38}$$

where $\delta \gamma = \gamma - \gamma_n$. Here, the resonant energy $\gamma_n$ is found by inverting the expression $\omega = \omega_n$, i.e.,

$$\gamma_n = \left( \frac{\omega \gamma_n^2}{4n\omega_0} \right)^{1/2} \left( 1 - \frac{\omega \theta^2}{4n\omega_0} \right)^{-1/2}, \tag{39}$$

where $\gamma_n^2 \gg \gamma_0^2$ and $\theta \ll 1$ have been assumed. Notice that in the backscattered direction, $\Delta \gamma_n / \gamma_n = 1/2nN_0$. Hence, the condition that $f(\gamma)$ be slowly varying compared to $R_n(\omega, \omega_n)$ for fixed $\omega$ implies that $\Delta \gamma_n / \gamma_n \ll \Delta \gamma / \gamma$ ($\Delta \gamma / \gamma$ is the energy spread of the bunch), or $\Delta \gamma / \gamma \gg 1/2nN_0$, which is easily satisfied for a large number of laser periods $N_0 \gg (2n\Delta \gamma / \gamma)^{-1}$. In the limit of a large number of laser periods ($N \to \infty$), $R_n \to \Delta \gamma_n \delta(\gamma - \gamma_n)$.

The above integration over momentum can be simplified by approximating the resonance function as a delta function, i.e., $R_n(\omega, \omega_n) \simeq \Delta \gamma_n \delta(\gamma - \gamma_n)$. Hence, (37) reduces to

$$\frac{d^2I_n}{d\omega d\Omega} \simeq \Delta \omega_n f(\gamma_n) S_n(\gamma_n), \tag{40}$$

where $d^2I_n/d\omega d\Omega = R_n(\omega, \omega_n)S_n$. The notation on the right side of the above equation indicates that in the functions $f(g)$ and $S_n$, $u$ and $\gamma$ are to be evaluated at their resonant values, $\gamma = \gamma_n$ and $\gamma;0 = \gamma_n$.

The radiation spectrum for an electron beam with a 5 MeV Boltzmann energy distribution scattering against a laser with $a_0 = 1$ and $N_0 = 10$ is shown in Fig. 11.

4) Backscattered Radiation: In the backscattered direction ($\theta = 0$) in the limit $\gamma_n \gg \gamma$, the spectrum integrated over the electron energy distribution is given by [53]

$$\frac{d^2I_n}{d\omega d\Omega} = 2\alpha f N_0 \gamma_n^3 \frac{f(\gamma_n)}{\gamma_n^2} \times \left[ J_{(n-1)/2}(\alpha_z) - J_{(n+1)/2}(\alpha_z) \right]^2 \tag{41}$$

where $\gamma \simeq (\gamma / \omega_n)^{1/2}$ and $\alpha_z \simeq na_0/(4\gamma_n^2)$. 5) Low Intensity Limit ($a_0^2 \ll 1$): In the limit $a_0^2 \ll 1$, only the fundamental ($n = 1$) radiation is significant. The fundamental radiation spectrum in the backscattered direction, averaged over the electron distribution, is given by

$$\frac{d^2I}{d\omega d\Omega} = \alpha f N_0 \gamma_n^3 \frac{f(\gamma_n)}{2}, \tag{42}$$

where $\gamma_n \simeq (\omega / 4\omega_0)^{1/2}$. The total number of photons radiated over all frequencies and angles is given by

$$N_{tot} = 4\pi\alpha f N_0 N_0 a_0^2, \tag{43}$$

where $N_0$ is the number of electrons.

Of particular interest is the photon flux and brightness of the TS radiation. Assuming that the collection angle, $\theta_d$, is small ($\theta_d < (\Delta \omega / \omega)^{1/2} / \gamma < (1/N)^{1/2} / \gamma$) so that the intensity distribution is flat over the solid angle $\Delta \Omega_d = \pi \theta_d^2$, the number of photons intercepted in a small bandwidth, $\Delta \omega$, about $\omega$ and solid angle, $\Delta \Omega_d$, for an electron bunch with $N_b$ electrons is

$$N_T = N_b \frac{d^2I}{d\omega d\Omega} \frac{\Delta \omega}{\omega} \pi \theta_d^2 \tag{44}$$

The average flux in photons per second, $F_{ave}$, in the collection angle $\theta_d$ and with bandwidth $\Delta \omega$, is $N_T$ multiplied by the repetition rate, $f_{rep}$, of the laser/electron beam, i.e., $F_{ave} = N_T f_{rep}$. The average source brightness (in
The number of scattered x-ray photons is given by

\[ B_{ave} = \frac{F_{ave}}{(2\pi)^2 r_b^2 \theta_d^2} \approx \frac{\alpha f N_0 \theta_0^2}{64\pi r_b^2} \left( \frac{\omega}{\omega_0} \right)^{3/2} f(\gamma = \gamma_0) \frac{\Delta \omega}{\omega}, \]  

where \( r_b \) is the electron bunch radius. The peak flux and brightness are, respectively, \( F_{pk} = F_{ave}/(\tau x f_{rep}) \) and \( B_{pk} = B_{ave}/(\tau x f_{rep}) \), where \( \tau x \) is the x-ray pulse duration, which is assumed to be approximately equal to the electron bunch duration. Note that the region of validity of (16)–(18) is for \((\omega/\omega_0)^{1/2} \geq \gamma_{min}\), where \( \gamma_{min} \) is the minimum \( \gamma \) of the electrons in the distribution \( f(\gamma) \).

6) Example I: Low energy x-ray source (\(<10 \text{ keV}\) ): Laser wakefield accelerators operating in the self-modulated regime typically produce energy spectra consisting of an exponentially decreasing distribution of electrons, with a total charge of 1-10 nC [20], [22]. A typical radiation spectrum from Thomson scattering off such an electron energy distribution is shown in Fig. 12, where the x-ray spectrum is normalized to a detection cone. The laser is assumed to operate at 800 nm and to produce 1 J in 2 ps.

\[ N_{tot} \approx 0.1 \times 10^9, \] from (43), assuming a 1 J, 2 ps Ti:sapphire laser focused to a spot size of \( r_0 = 12 \mu m \) with \( a_0 = 0.32 \).

The spectral width of the peak radiated by a single electron is \( \delta \omega/\omega = 1/N_0 \), whereas the spread in scattered frequencies as a result of finite electron energy spread \( \Delta \gamma/\gamma \) is \( \delta \omega/\omega \approx 2\Delta \gamma/\gamma \). For a sufficiently long laser pulse with \( N_0 \gg (2\Delta \gamma/\gamma)^{-1} \), the width of the single electron resonance function \( R_n \) will be much narrower than the width of the electron distribution function. In this limit the scattered spectrum can be estimated using (44), and the width of the peak in the x-ray spectrum is determined by the width of the energy spread of the electrons. The expected spectrum for a detection angle of \( \theta_d = 3 \text{ mrad} \) and an electron beam with a narrow peak at 50 MeV with a 3% energy spread (FWHM) is shown in Fig. 13.

Three source designs are summarized in Table I. The key difference between the three cases is the laser that is used for scattering. The first case assumes a long-pulse, Nd:YAG, Q-switched laser operating at 1064 nm and producing 1 J energy in a 20 ns pulse. The second and third cases use a more powerful Ti:Al_{2}O_{3} laser operating at 800 nm wavelength. The laser, based on chirped pulse amplification, delivers 1 J energy in a pulse that is either uncompressed (case 2) or partially compressed (case 3). In all cases the interaction length is approximately the Rayleigh length \( Z_R = \pi r_0^2/\lambda \) with \( r_0 = 12 \mu m \), such that the total number of photons is proportional to the laser power. Furthermore, non-uniform pulse profile effects are neglected [54]. These examples show that excellent performance of a femtosecond x-ray source with sufficient flux for various pump-probe experiments can potentially be achieved using a moderately powerful laser.

IV. CONCLUSION

Radiation produced by laser accelerated electron bunches has been analyzed. In the long wavelength range of the electromagnetic spectrum (THz radiation), the ultra-short pulse nature of the accelerated electron bunches provides the possibility of producing high energy coherent radiation. In proof-of-principle experiments, coherent transition radiation emis-
sion was observed when electron bunches with a few MeV mean energy propagated through the plasma-vacuum boundary. Since the radiation was produced at this boundary, bunch densities, which would be reduced by space charge effects during propagation of these low density in vacuum, remained high. Due to the high amount of charge, this simple source can in principle produce 10’s of µJ/pulse. Diffraction effects arising from the finite transverse plasma size can significantly increase the opening angle of the radiation, limiting the amount of collected energy unless collection optics with large open angle can be used. For laser wakefield accelerators that produce high energy electron beams (i.e., much less susceptible to space charge effects), metal foils can be used as in conventional CTR experiments and very high source efficiency can be expected.

In the short wavelength range (x-rays), two mechanisms for the generation of femtosecond x-rays have been discussed. The first one relies on the emission of betatron radiation when a relativistic electron beam propagates inside a plasma channel in the presence of large focusing fields. The second one uses an external laser to Thomson backscatter off the relativistic electrons. For 100 MeV class laser wakefield accelerators, betatron radiation x-rays will be in the few keV range. Thomson scattered x-rays can be in the 60 keV range using a 50 MeV beam and a laser operating at 0.8 μm. Using TW-class lasers for Thomson scattering and electron beams achievable by present day laser wakefield accelerators, photon fluxes from TS (betatron radiation) on the order of 5 × 10^9 photons/shot/0.1%BW (2 × 10^4 photons/shot/0.1%BW) are expected at 60 keV (10 keV) in truly femtosecond duration pulses (10–50 fs). Such a source will have applicability for pump-probe experiments studying melting, shock propagation, and other phenomena in materials.

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REFERENCES


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