Lawrence Berkeley National Laboratory
Recent Work

Title
Photon-Electron Interaction and Condense Beams

Permalink
https://escholarship.org/uc/item/0j8453mf

Author
Chattopadhyay, S.

Publication Date
1998-11-01
Photon-Electron Interaction and Condense Beams

S. Chattopadhyay

Accelerator and Fusion Research Division

November 1998

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
PHOTON-ELECTRON INTERACTION
AND CONDENSE BEAMS*

S. Chattopadhyay

Center for Beam Physics
Accelerator and Fusion Research Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California, Berkeley, CA 94720  USA

To be published in the proceedings of the workshop,
Quantum Aspects in Beam Physics (QABP '98),
held January 3-7, 1998, Monterey, California

* This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC 03-76SF00098.
PHOTON-ELECTRON INTERACTION
AND CONDENSE BEAMS

Swapan Chattopadhyay
Center for Beam Physics, Accelerator and Fusion Research Division,
Ernest Orlando Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720 USA
E-mail: chapon@lbl.gov

We discuss beams of charged particles and radiation from multiple perspectives. These include fundamental acceleration and radiation mechanisms, underlying electron-photon interaction, various classical and quantum phase-space concepts and fluctuational interpretations.

1 Introduction

Many intriguing aspects of beam physics revolve around the intricacies of single particle and collective dynamics of charged particles interacting with radiation fields. Understanding this behavior in great detail to the point where one can simulate and predict their mutual influence is an ongoing quest. Fascinating still are the peculiar properties exhibited by a system of charged particles and radiation under extreme conditions — super-low/ultra-high temperatures, extreme densities, intense radiation fields, etc. These have given rise to an entire new field of High Energy Density Physics, with applications in high energy particle collisions; ultracold and crystalline beams; plasma astrophysics and cosmology; high field interactions and short pulse x-rays; laser-solid-plasma interactions; ultrafast phenomena; etc.

The physics of "condensate beams" is fascinating in its own right as well. We refer the reader to the article by H. J. Leggett in these proceedings, titled "Coherent Atomic Beams from Bose Condensates". The provocative question begs itself: Can (fermionic) particle beams ever form a Bose-like condensate (analogous to He\(^3\) superfluid)? Or, put in another way, can high energy electron and proton beams ever develop "long-range" interactions and correlate into "Cooper"-like pairs?

The articulation of a "condensate" fermionic beam is vague at its best; though one cannot help feel a suggestion somewhere. For lack of this precise articulation, much of the focus was on "condense" beams instead at the workshop. Hence the title of this article. A "condense" beam can be envisioned

---

1to be published in the proceedings of the workshop, Quantum Aspects in Beam Physics (QABP '98), held January 3 - 7, 1998, Monterey, California.
in many ways depending on the context; e.g., (i) a charged particle beam under conditions of extreme electromagnetic fields such that \( \gamma \frac{|\vec{E}|}{E_c} \geq 1 \)
where \( \gamma \) is the Lorentz gamma factor of the beam, \( E_c \sim B_c = 4.4 \times 10^{13} \) Gauss is the Vacuum Breakdown Field when \( e^+ e^- \) pairs are created spontaneously from vacuum under high fields and \( |\vec{E}| \) is the actual electromagnetic field experienced by a particle. This is a measure of the fraction of energy radiated away in intense fields by particles with finite mass and is known as the \( \Upsilon \) (Upsilon) parameter \(^1\) in collider physics terminology; (ii) a super-cooled beam looked upon as a one-component plasma with a coupling parameter \(^2\) \( r = \frac{(E_{\text{Coulomb}})}{jkT} \) where \( T \) is the average temperature in any one dimension and \( (E_{\text{Coulomb}}) \) is the average strength of Coulomb interaction field between particles in the beam. For \( r < 1 \), the system is in ‘gaseous’ state; for \( r \sim 1 \), it is in ‘liquid’-like state; for \( r >> 1 \), the system behaves like a ‘crystal’.

Can crystalline beam\(^3\) ever be a reality?

At the workshop, the two discussion topics — (i) photon-electron interaction and (ii) production, control and use of ‘condense’ beams — merged into a single Working Group for a whole week. The goal was to sharpen the issues further, raise critiques, develop various analyses and syntheses and chart out future direction of further research. What emerged could best be characterized as an impressionistic painting on a large canvas rather than a fully work out miniature art in detail. I will attempt at portraying this development in my own personal way, with intentional broad strokes of brush. I hope this will leave plenty of room for innovation by the imaginative reader.

While I accept the sole responsibility for the authorship of this personalized record of these developments in the working group (which I had the privilege to chair), I must acknowledge the very many participants of this working group who made it happen. Their works are scattered throughout these proceedings and need no further exposition. Two individuals however, who have influenced me significantly during the Workshop, should not remain anonymous. I owe a lot to Max Zolotorev and Kwang-Je Kim for many illuminating discussions. Many of the ideas in this summary reflect their thinking in various topics as well as mine. I give these collective thoughts expression in written words, with explicit credit at appropriate places.

The topics discussed at the workshop naturally grouped themselves under five categories as follows: (1) Acceleration and Radiation; (2) Phase Space Control and Cooling; (3) Fluctuations and “Condense” Coherent Beams; (4) Classical and Quantum Emittance and Entropy; and (5) Particle and Radiation Sources.

We discuss these in the following.
2 Acceleration and Radiation

There is an intimate connection, often overlooked but of fundamental importance, between acceleration of charged particles in externally imposed electromagnetic fields and the radiation, both spontaneous and stimulated, of the same charged particles in the electromagnetic configuration so imposed. Much of the exposé that I will present on this subject is based on a lucid tutorial offered by Max Zolotorev during the Workshop. These discussions were provoked by talks by Kirk McDonald on temporary acceleration of electrons in intense electromagnetic fields and by James Spencer on the Stanford laser acceleration experiment.

We envision a set of N electrons travelling in a group (a 'bunched' beam) in presence of boundaries 'B' (conducting or otherwise), as depicted in Fig. 1. The electrons move in a region 'R' where we have imposed an external field \( \vec{E}_{\text{ex}}(\omega, \vec{r}) \). Here \( \omega \) is the frequency component of the external field at a position with coordinate \( \vec{r} \). A single electron will radiate spontaneously in the presence of this external field and the apertures bounded by the boundaries.

![Figure 1: A set of charged particles moving in free space with boundaries](image)
We denote this spontaneous radiation field from a single electron by \( e_s (\omega, \vec{r}) \).

The fields due to \( N \) electrons will add linearly by the principle of superposition. The total field in the ‘far field’ region can then be written as:

\[
\vec{E} = \vec{E}_{ex} (\omega, \vec{r}) + \vec{e}_s (\omega, \vec{r}) N .
\]  

(1)

The field energy density in region \( R \) can be written as:

\[
\mathcal{E} \sim \int \left| \vec{E} \right|^2 d^3 \vec{r} d\omega
= \int \left| \vec{E}_{ex}(\omega, \vec{r}) \right|^2 d^3 \vec{r} d\omega + N^2 \int |\vec{e}_s (\omega, \vec{r})|^2 d^3 \vec{r} d\omega
+ 2 \text{Re} \left[ N \int \vec{E}_{ex} (\omega, \vec{r}) \cdot \vec{e}_s (\omega, \vec{r}) d^3 \vec{r} d\omega \right] .
\]  

(2)

The first term on the right hand side represents the externally ‘stored’ energy in region \( R \), the second term represents the energy loss from the set of charged particles due to coherent spontaneous radiation and the third or last term represents the energy exchange with particles leading to energy gain from (‘decelerated’) or loss to (‘accelerated’) particles. This last term is analogous to ‘stimulated’ absorption or radiation of energy in terms of atomic transitions in lasers.

Thus the cross-term, \( 2 \text{Re} \left[ N \int \vec{E}_{ex} (\omega, \vec{r}) \cdot \vec{e}_s (\omega, \vec{r}) d^3 \vec{r} d\omega \right] \), representing interference of ‘external’ and ‘radiated’ fields, signifies acceleration or deceleration of particles. For this term to have a finite, non-zero value, the spectra and mode-pattern of spontaneous radiation \( \vec{e}_s (\omega, \vec{r}) \) must have finite non-zero ‘overlap’ with the spectra and spatial mode-pattern of the externally imposed field \( \vec{E}_{ex} (\omega, \vec{r}) \).

Hence there cannot be any net ‘acceleration’ or ‘deceleration’ if the electromagnetic configuration forbids ‘radiation’ into the accelerating or decelerating field modes. And so arises what we call the FUNDAMENTAL THEOREM OF ACCELERATION and RADIATION: “For optimal acceleration/deceleration, linear or otherwise in \( \vec{E}_{ex} \), there must be optimal overlap of ‘acceleration’ mode \( \vec{E}_{ex} (\omega, \vec{k}) \) with ‘radiation’ mode \( \vec{e}_s (\omega, \vec{k}) \).” The particles are accelerated if the coherent radiation loss (second term in eq. (2)) is less than the energy gain (third term in eq. (2)), leading to the restriction, \( N \leq \left( \frac{\vec{E}_{ex}}{e} \right)^2 \lambda^2 \), on the number of particles in a bunch that can be effectively accelerated before losses start to dominate, leading to deceleration (here \( \lambda \) is the reduced wavelength).\(^4\)
The above considerations immediately lead to a quick, unambiguous and transparent way of determining the possibility or impossibility of charged particle acceleration under various conditions of 'open' and 'closed' systems, non-linear interaction with lasers, etc. We illustrate three typical cases that have been subjects of numerous debates in the past.

2.1 Case 1: OPEN SYSTEM

Consider two focussed laser beams, intersecting at a finite angle at their waists, as shown in Fig. 2. The electric fields of the fundamental TM Gaussian modes of the two beams do add up to produce a net longitudinal electric field in the z-direction. Is this field capable of producing a net acceleration of a charged particle travelling through the focus from infinite past to distant future? This is an "open" system with no bounding apertures. For not too strong laser fields, electron trajectories are simply straight lines. There cannot be any spontaneous radiation in free space in linear motion without boundaries. The last term in eq. (2) vanishes. (The coupling to radiation from temporary local linear acceleration is rather weak indeed). Hence we cannot expect any particle acceleration that is linear in $|E_{\text{laser}}|$. A detailed calculation will support this conclusion. This is often referred to as the Lawson-Woodward Theorem.\textsuperscript{5}
2.2 Case 2: CLOSED SYSTEM

One can make the configuration in Fig. 2 "closed" by imposing boundaries as shown in Fig. 3. These boundaries form an effective cavity that allows the particle to go through the apertures in the walls and yet bounce the laser beams, making them cross at the cavity center. This configuration allows spontaneous radiation through the apertures via Cerenkov or transition radiation effect. One can verify that the allowed radiation has a component that is fundamental Gaussian laser-beam like in its spectrum and mode-pattern — a 'cavity mode'. Thus acceleration or deceleration is possible depending on the restriction on the number of particles per bunch as mentioned before, optimizing the energy gain by accelerating field against energy loss to radiated fields. This is the basis of the experiments planned at Stanford.

![Crossed laser beams within cavity boundaries: a "closed" system](image)

2.3 Case 3: NONLINEAR INTERACTION

When an intense laser is focussed onto a very small spot, the local electromagnetic field can be high enough to distort a charged particle trajectory, which bends in the strong fields (See Fig. 4). The intense laser acts as an electromagnetic wiggler and the resulting quivering motion of the electron via interaction with the strong laser field leads to transverse acceleration \( \ddot{z} \) which is propor-
tional to the laser field intensity $E_{\text{laser}}$. The electron radiates in this strong field, with radiation field intensity given by:

$$e_s \sim \hat{x} \propto E_{\text{laser}}.$$  \hspace{1cm} (3)

The interference term in eq. (2) leads to an energy loss/gain which is now not linear in the external laser field $E_{\text{laser}}$, but rather quadratic:

$$\Delta E|_{\text{gain}} \sim (E_{\text{laser}} \cdot e_s) \propto E_{\text{laser}}^2 = A_L \left( \frac{r_e}{Z_R} \right)$$  \hspace{1cm} (4)

where $A_L$ is the 'flash energy' of the laser, $Z_R$ the laser Raleigh length and $r_e$ the classical electron radius.

Figure 4: A laser pulse focussed tightly to a small spot, giving rise to intense fields which bends particle trajectories and produces non-zero acceleration in free space quadratic in the electric field.

Such gain in electron energy in presence of strong laser fields — quadratic in laser fields and linear in its flash energy — have been observed in many experiments to date. The basic interaction being nonlinear and second order in $E_{\text{laser}}$ with a small coefficient, both the spontaneous radiation and the resulting acceleration are relatively weak compared to direct acceleration linear in $E_{\text{laser}}$ in suitable geometries.
Much progress has been made recently in controlling and cooling the phase space of systems of charged particles such as ions in ion traps and high intensity particle beams in storage rings. During the workshop, Jeff Hangst presented new results on the formation of large ion crystals in an ion trap and the formation of a super-cooled space-charge dominated ion beam achieved by laser cooling of ions in a storage ring — both performed at the University of Aarhus in Denmark. A new idea — called the laser-electron storage ring — was presented by Z. Huang, where radiative cooling in the strong field of a laser is used in a compact storage ring to damp an electron beam at relatively low energy (a few MeV to a few hundred MeV) and to generate very intense x-rays. These works are reported elsewhere in these proceedings.

Such “cold ion beams” or ion crystals represent ideal examples of strongly coupled one component plasmas and provide fertile ground for novel plasma physics studies. Being highly nonlinear, they are also prone to exhibit phenomena such as phase transitions, diffusion, chaos, etc. of typical statistical physical nature. Applications to atomic/molecular physics are obvious — such cold well-localized systems with long interaction times lend themselves to novel spectroscopic studies. Manipulation of single ions allowed by these systems provide excellent opportunities in quantum optics — studies of quantum jumps, quantum computing, cavity quantum electrodynamics, etc. Finally, frequency/time standards can be revisited as a major application in applied physics.

Phase space cooling of particle beams in a storage ring has many outstanding applications as well — super-damped electron and positron bunches for linear colliders, luminosity enhancement of hadron colliders by controlling phase-space diffusion (and damping the colliding beam phase space, thus increasing the probability of collisions), high-brightness source of short wavelength x-rays, etc.

Laser cooling of ionic beams in a storage ring is restricted however, to certain ions only, as the laser wavelength has to match the atomic transitions in the ions. This is not the case with conventional stochastic beam cooling scheme where one utilizes a microwave (several GHz bandwidth) feedback loop to resolve and affect microscopic samples of a particle beam. This technique has been successful in cooling exotic systems such as a beam of antiprotons. A typical microwave stochastic beam cooling scheme is shown in Fig. 5. Non-destructive near-field microwave pick-ups detect incoherent Schottky signals from an antiproton beam, say, at the level of $10^{19}$ watts/particle (for a beam of $10^8$ particles, this signal power is at the level of 10 picowatts). The finely
structured Schottky signal, containing phase space information of the beam, together with added electronic/thermal noise, get suitably processed by an electronic system with a typical bandwidth and gain of $1 \text{ GHz}$ and $10^{13}$ respectively, and the modified amplified signal is applied back to the beam via electromagnetic kickers with wide bandwidths at power levels of 100 watts. The typical signals are at nanoVolts level while the incoherent noise is at the microVolts level. Cooling times as fast as fractions of a second can be achieved.

![Diagram](image)

**Figure 5: Microwave Stochastic Beam Cooling**

The microwave stochastic cooling gets increasingly limited with ever higher demands on faster cooling times and brighter beams in phase-space. This is so since the number of independent "samples" that can be observed and controlled in a beam is at best the number of microwave 'coherence volumes' contained in the beam. The coherence volume of microwaves in phase-space is typically in the scale of their wavelengths (mms. to cms.) while the transverse phase space volume of beams required in today's experiments is already much smaller than this. Thus, while microwaves can resolve a beam into many independent samples in the longitudinal time-like direction, it cannot resolve the beam any further in the transverse direction.

This situation can be overcome by using optical technology instead of microwaves, since the coherence volume of light (microns) is typically less than the beam emittance. In addition to providing finer and more numerous independent samples in time, a light beam can also discriminate a particle beam in its transverse microstructure. Thus a particle beam is fully resolved in space and time by a light beam.
The sampling of particle and radiation (microwave and light) beams against each other is illustrated in Fig. 6. It is obvious that in addition to faster cooling times, ‘optical’ sampling and feedback allows for controlling and shaping the “beam halo” in transverse directions. The phase space cooling of particle beams by optical sampling and feedback has been termed Optical Stochastic Cooling. It has been proposed several years ago and is under active study at present.\textsuperscript{11}

\begin{equation}
\Delta x_{o} \Delta \phi_{o} = 2 \pi \epsilon_{\perp}
\end{equation}

Particle

\begin{align*}
E & \left( \Delta x_{o}, \Delta \phi_{o} \right) \\
\beta &
\end{align*}

\begin{align*}
\Delta x_{p} \Delta \phi_{p} &= 2 \pi \epsilon_{\perp} \\
\text{Particle beam is fully resolved in space and time by light beam} \\
\text{Coherence Volume of Light} &< \text{Beam Emittance}
\end{align*}

Radiation

\begin{align*}
\hbar \omega & \left( \Delta x_{r}, \Delta \phi_{r} \right) \\
Z_{r} &
\end{align*}

\begin{align*}
\Delta \phi_{r} \Delta x_{r} &= \hbar / 2
\end{align*}

Figure 6: Transverse Sampling of Particle Beams by Radiation Beam

The Optical Stochastic Cooling scheme is illustrated in Fig. 7. A far-field radiator like an undulator is used to obtain information about the discrete graininess of the particle beam at the 1 \( \mu \)m (\( \sim \) optical wavelength) level. An optical laser amplifier with a typical ‘bandwidth’ of \( 3 \times 10^{14} \) Hz and ‘gain’ of \( 10^{7} \) is used to process, filter and amplify the light, before using it in a reciprocal arrangement to affect the beam phase space via an undulator kicker. Radiation being a manifestation of the discreteness of the radiating charge, the radiation from a charge particle beam contains information about the discrete and grainy particle distribution in phase space. The true phase space signal manifests in...
the number of discrete photons emitted per charged particle, which is typically
given by $\alpha N_W K^2$ — an electron or elementary charge emitting typically $\alpha$ photons per simple ‘bend’ for $N_W$ bends in an undulator, with a scaled strength parameter $K \approx 9 B_{[\text{tesla}]} \lambda_{u \text{ [cm]}}$ where $\alpha$ is the fine structure constant, $\lambda_u$ the undulator wavelength and $B$ the undulator field. Depending on the parameters, $n_y/n_e$ could range from 1/137 to 1 spanning the entire range from classical-to-quantum-noise dominated regime. Clearly signal-to-noise ratio and radiation coherence are important issues.

![Optical Stochastic Cooling]

The typical laser amplifiers required for applications are under development at present. The undulator radiators and kickers, on the other hand, are straightforward to build in today’s technology. However, they tend to be physically large. Much effort is focused at present on alternative schemes that provide a compact Schottky monitor and kicker in the optical range. Cerenkov and Smith-Purcell radiators involving gas-jets, near-field slotted structures etc. are under active investigation.
4 FLUCTUATIONS AND "CONDENSE" COHERENT BEAMS

While an ordinary incoherent light source is characterized by a source size $\sigma$ and source divergence $\sigma'$ such that $\sigma \sigma' \gg \lambda$ (wavelength of light), a beam of transversely coherent laser light in its most ordered state is characterized by its fundamental Gaussian mode-profile, giving for its size as a function of propagating distance, $s$, the well known Raleigh relation:

$$\sigma^2 (s) = \frac{\lambda}{\pi} Z_R \left( 1 + \frac{s^2}{Z_R^2} \right)$$

and a characteristic diffraction-limited phase-space volume:

$$\sigma \sigma' \approx \frac{\lambda}{2\pi} = \lambda$$

where $Z_R$ is the Raleigh length. The description can be carried over in an analogous fashion to particle beams by replacing $Z_R$ by $\beta^* \lambda$ (with usual definition of a low-beta function of a beam) and $(\lambda/\pi)$ by the beam phase-space emittance $\epsilon$ for conventional 'incoherent' beams, but by $\lambda_c$, the reduced Compton wavelength, for a fully quantum-mechanically 'coherent' beam, described by:

$$\sigma^2 (s) = \lambda_c \beta^* \left( 1 + \frac{s^2}{\beta^* \lambda_c^2} \right)$$

$$\sigma \sigma' \approx (\lambda_c/2)$$

Could such a quantum coherent beam be ever realized in the laboratory? And if so, would it be detectable via its fluctuations?

It is expected that "condense" beams close to quantum coherence will acquire new collective macroscopic properties, very different from their equivalents in relatively dilute phase equilibria, characterized by all-pervasive fluctuations with correlations described by a fluctuation power spectra $P_0 (\omega) = \langle E_0 (\omega) E^*_0 (\omega) \rangle$. The modified and new correlations, long- or short- ranged, established by the condensed beam will distort the fluctuation and correlation power spectrum to:

$$P (\omega) = \langle E (\omega) E^* (\omega) \rangle = \frac{P_0 (\omega)}{\varepsilon (\omega)^2}$$
where \( \varepsilon(\omega) = [1 + \chi(\omega)] \) is the generalized dielectric permittivity and \( \chi(\omega) \) the generalized dielectric susceptibility of the condensed phase in the variable \( E(\omega) \). Depending on "condensate" properties and corelations, the fluctuation noise power could be collectively enhanced or suppressed.

The distortion of fluctuations from an incoherent beam to a coherent beam is pictorially illustrated in Fig. 8. The figure is for the purpose of qualitative exposition only with no quantitative relevance. It is meant to reinforce the concept that understanding of such distortions in the power spectrum of fluctuations in "condense" or "condensate" beams would be crucial towards designing proper diagnostic for such states.

\[
\sigma^2(s) = \varepsilon^* \left( 1 + \frac{s^2}{\beta^2} \right)
\]

\[
P_\omega(\omega) = \langle x_\omega x_\omega^*(\omega) \rangle
\]

\[
P(\omega) = \langle x(\omega) x^*(\omega) \rangle
\]

Figure 8: Incoherent (a) and coherent (b) beams and their fluctuation spectra

5 Classical and Quantum Emittance and Entropy

Charged particle beams, in addition to carrying 'energy', can transmit 'information'. The information carried by a beam is characterized by its entropy '\( S_b \)', which in turn is determined by its emittance, '\( \varepsilon \)'. The emittance is the classical phase space volume occupied by the beam:

\[
\varepsilon \sim \frac{1}{\pi} \int (\delta x \delta p_z) / (\beta \gamma)
\]

(10)
and the associated entropy, for a uniform phase-space 'painting' or distribution is given by: \( S_b \sim k N \log \left( \frac{\pi \xi}{\delta \Omega} \right) \) (11)

where \( k \) is the Boltzmann constant and \( N \) is the total number of particles distributed in the phase space \((x, p_x)\) with elemental cell volume \( \delta \Omega \). (See Fig. 9).

Figure 9: Classical Phase Space

The entropy \( S_b \) can be interpreted as a measure of disorder in the system \( \text{à la} \) canonical statistical mechanical terms, e.g. incoherence, temperature, etc. The entropic interpretation of incoherence or coherence is probably less ambiguous than the notion of incoherence introduced and characterized by phase space volume \( \xi \) or emittance. This is so, as was pointed out explicitly by K. J. Kim, because in the classical case \( S_b \) involves and depends on \( \delta \Omega \), the elemental phase space volume, but the total phase space volume \( \xi \) does not:

\[
\text{classical : } S_b \equiv S_b (\delta \Omega) \\
\xi|_{\delta \Omega \to 0} = \text{Constant, independent of } \delta \Omega 
\] (12)
while the quantum mechanical generalizations imply an entropy independent of elemental cell volume in phase space, but a total phase space volume $\varepsilon$ that does depend on $\delta\Omega$ which limits to $\hbar$: \(^{19}\)

\[
\text{quantum: } S_{\text{b}|\delta\Omega} \to \hbar = \text{Constant, independent of } \delta\Omega \\
\varepsilon \equiv \varepsilon (\delta\Omega) \text{ as } \delta\Omega \to \hbar.
\] (13)

Thus if 'entropy' or the ability to carry 'information' via ordered states is the primary concern, then a class of distributions with unambiguously and uniquely quantified $\varepsilon$ can be obtained leading to the same entropy content $S_b$, independent of the geometric emittance.

The emittance and entropy concepts can be generalized to optical wavepackets generated by a charged particle beam, such as synchrotron radiation. This can be done based on a Wigner Function formalism of wave-optics for photon beams in the Gaussian approximation. \(^{15}\) A Wigner Function formalism also lends itself to the generalization of the phase-space concept for a quantum mechanical particle. The equivalence of the transformation properties of the quantum mechanical (or optical) phase space and the classical dynamical phase space based on the Wigner distribution, gets modified in presence of nonlinear transformations of phase space involving aberrations. This was discussed by A. Dragt during the workshop. The relative ambiguity in assigning magnitudes of classical beam emittance and single-particle quantum emittance can be removed by using the 'entropic emittance' concept, as introduced before, for quantum beams.

The ideas of 'entropy' and 'entropic emittance' introduced above may not be sufficient for adequate phase-space description and control for some applications. One may have to introduce concepts of higher order phase-space correlations and entropic emittances of increasing order for certain cases. We refrain from such a digression at the moment, but simply mention that there were discussions at the workshop by R. Fedele on quantum-like phase description of beams and by D. Barber on permissible equilibrium in stored proton beams.

A complete quantum beam of particles and radiation, characterized by a multiparticle collection and radiative multimodes can best be described by an overlapping collection of quantum wavepackets, localized partially along "classical" particle trajectories and rays as illustrated in Fig. 10.

The total phase space $\Phi_T$ of the quantum beam-radiation system (solid ellipse in Fig. 11) is a convolution of the single-particle quantum phase space $\Phi^Q_t$ (small concentric ellipses in Fig. 11), the classical beam phase space $\Phi^C_t$
Figure 10: Trajectories, rays and wave-packets of a multi-particle, multimode beam-radiation system.

(dotted ellipse in Fig. 11) and the radiation phase-space in the $M^{th}$ mode $\Phi^M_R$, summed over all particles $i$ and wave-modes $M$:

$$\Phi_T = \sum_{i, M} \Phi^i_Q \otimes \Phi^i_C \otimes \Phi^M_R \quad (14)$$

The total phase-space volume is minimal, leading to a maximally "bright" beam-radiation system, when the respective phase ellipses of the classical particle, quantum particle and radiation waves are "matched" in some sense with respect to their shapes, orientation, etc., leading to a macroscopically "smooth" phase space $\langle \Phi_T \rangle$ desirable for bright "coherent" radiation or particle beams.

Residual graininess or "granularity" resulting from phase-space mismatch $\delta \Phi = \Phi_T - \langle \Phi_T \rangle$ contain fluctuational information that can be used as "seed" for coherent amplification of radiation as in Free Electron Lasers; as critical diagnostics in scattering experiments, or can be a limitation on processes depending on coherent states, e.g., quantum computing.

Fluctuation as a simple diagnostic tool is best illustrated in a system with time 't' and frequency 'ω' as the two conjugate degrees of freedom. A signal in
the \((t, \omega)\) plane with a duration \(\tau\) and processed with a frequency bandwidth \(W\), is well known by Nyquist criterion to be describable by \(M\) degrees of freedom where\(^{20}\)

\[
M = 2W \cdot \tau
\]  \hspace{1cm} (15)

These \(M\) independent numbers describing the signal can be obtained as a time series of \(M\) independent fluctuation signals spanning the full signal duration \(\tau\), or \(M\) independent frequency components \((M\) amplitudes or \(M/2\) amplitudes and \(M/2\) phases\) of the fluctuation spectra as illustrated in Fig. 12.

By observing and determining the number, \(M\), of independent degrees of freedom required to characterize the fluctuation signal, frequency-analyzed via a known bandwidth \(W\), one can determine the duration of the fluctuation signal \(\tau\) — a powerful diagnostic technique based on fluctuation spectroscopy and interferometry\(^{21}\). For example, the fluctuations in the radiation spectra from a charged particle beam pulse has been successfully used to determine the pulse length fairly accurately.\(^{22}\)
PARTICLE and RADIATION SOURCES

Various particle and radiation sources are made possible via electron-photon interactions. During the workshop J. Clendenin described a high quantum yield, low emittance electron source; T. Takahashi described positron production via channelling 1.2 GeV electrons in single crystals; J. Spencer described a high brightness source of polarized neutrons; and D. Palmer described prospects for an emittance-compensated spin-polarized radio-frequency electron gun.

J. Clendenin explored novel uses of semiconductors such as GaAs in a fast-pulsed photocathode closely coupled to a rf accelerating system. This holds promise for reaching beam emittances down to 0.1 mm-mrad or below.

Discussions on emittance-compensated spin-polarized rf guns pointed out a possible incompatibility between the special magnetic optics required for space-charge emittance compensation and the "snake-optics" demanded by spin polarization preservation. This led to the concept of "spin" - emittance and polarization distribution.

On positron production by channelling 1.2 GeV electrons in Silicon and Tungsten crystals, there was a clear determination that coherent bremsstrahlung contributed to an enhancement by a factor of 2.5 to the positron production.
J. Spencer demonstrated that a low energy, stable storage ring with an undulator-based oscillator can Compton up-convert to provide quasi-monochromatic photon beams in the MeV range for neutron production. Such polarized neutron sources offer many potential research directions and applications, e.g. neutron condensates, neutron oscillation experiments, neutron electric dipole moment, etc.

Finally, E. Bessonov described a gravitational radiation source, which he named "Grasers" based on particle accelerators and A. Bogacz discussed stimulated emission of coherent radiation from a relativistic two-level system. The above topics are reported throughout these proceedings.

7 OUTLOOK

The aim of this group has been to develop a deeper understanding of: (i) the classical and quantum statistical phase space description of particle and photon beams; (ii) the energy and phase-space mapping in electron-photon interaction in beams; (iii) the production, control and use of "ultracold", "condense" beams and finally (iv) various practical applications to particle and radiation sources, "condense" beams, etc. Clearly much further work is needed. However much was achieved in stimulating new ideas and raising further issues of fundamental limits to particle and radiation phase space, coherent "condense" particle beams, physics of acceleration and radiation and electron-photon interaction in general.

Acknowledgment

I am grateful to Pisin Chen for inviting me to chair the Working Group at the QABP '98 and for his encouragement and patience towards completion of this written summary. I also thank Olivia Wong for preparing the manuscript and Illona Pittman for preparing the figures.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, of the U. S. Department of Energy, under Contract No. DE-AC03-76SF00098.

References

8. J. Hangst, these proceedings.
16. K.-J. Kim, these proceedings.
22. P. Catravas, private communication (Ph.D. dissertation work at MIT, under publication).