Title
Numerical simulation model of run of river hydropower plants: Concepts, Numerical modeling, Turbine system and selection, and design optimization

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Numerical simulation model of run of river hydropower plants: Concepts, Numerical modeling, Turbine system and selection, and design optimization

THESIS

submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in Civil Engineering

by

Veysel Yildiz

Thesis Committee:
Asst. Prof. Jasper A. Vrugt, Chair
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Asst. Prof. Kristen A. Davis

2015
DEDICATION

This thesis is dedicated to my eldest brother, Esref Yildiz, who has been a source of motivation over the years for my education and intellectual development.
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Hydropower is a relatively cheap, reliable, sustainable, and renewable source of energy that does not consume natural resources nor produces emissions and toxic waste. In fact, compared to all other energy sources, hydropower is the least expensive and most efficient method for generating electricity, with a price competitive to traditional energy sources such as fossil fuels, gas, and biomass. Most hydroelectric power that is being generated in the world today comes from (large) hydroelectric dams that generate electricity by converting the potential energy of falling or running water from human-made reservoirs. These reservoir-fed plants distort significantly the local environment and ecosystem, and hence much opposition exists towards their use and construction. Run of the river (RoR) hydroelectric stations are a viable alternative to large-scale plants as they require no reservoir capacity, so that the water coming from upstream must be used for generation at that moment, or must be allowed to bypass the station. This is a key reason why such RoR plants are often referred to as environmentally friendly, or green power. Here, we introduce a numerical model, called HYdroPowER or HYPER, which simulates the daily power production of a RoR plant in response to a historical record of daily discharge values, and design and operation variables. HYPER constitutes the first numerical model that takes into explicit consideration the design flow, penstock diameter, penstock thickness, specific speed, rotational speed, cavitation,
and suction head in evaluating the technical performance, production, cost, and profit of a RR plant. The model simulates both single and parallel turbine systems involving Kaplan, Francis, Pelton and crossflow turbines and combinations thereof. HYPER is coded in MATLAB and includes a built-in evolutionary algorithm that optimizes automatically the design of the hydropower system of the RoR plant for a given record of river flows and objective function (maximization of net profit or power production). This algorithm can be called from the main model script and maximizes (among others) the type and number of turbines, their design flow, and the penstock diameter. Finally, we introduce a graphical user interface (GUI) of HYPER which simplifies numerical simulation and interpretation of the results. Three different case studies are used to illustrate the power of HYPER. The model and its different components is available upon request from the authors.
Chapter 1

Introduction

Hydropower is a relatively cheap, reliable, sustainable, and renewable source of energy. Kinetic energy from falling or running water is transformed into mechanical energy, which can be used to generate electricity. Hydropower is a 100% clean, green energy, source that does not consume natural resources, nor produces emissions and toxic waste. In fact, compared to all other energy sources, hydropower is the least expensive, most efficient method for generating electricity, with a price competitive to traditional energy sources such as fossil fuels, gas, and biomass. Indeed, hydropower is the only regenerative energy source to supply electricity on an industrial scale at competitive prices. However, less than one third of the available hydroelectric potential is currently exploited (Basso and Botter, 2012).

In 2010, the world commission of dams has estimated the potential production of hydropower energy to be more than four times the current annual worldwide generation (IJHD, 2010). Currently, hydropower plants produce about 20% of the total energy demand and this percentage is only expected to increase significantly in the coming decades as we are seeking better and cleaner environmental alternatives to burning fossil fuels. Currently, more than 60 countries in the world derive at least half of their entire electricity production (demand)
from hydropower plants (GVR, 2014; Lafitte, 2014).

Small scale hydropower plants are an under-used but viable, clean and cost-effective alternative to their reservoir-fed counterparts. They provide the option of decentralized power production and rural electrification in for instance less-developed countries. Such plants also hold great promise for continents such as Europe which has exhausted the possibilities for large-scale hydropower production, and is seeking better and less invasive ways of energy production. Indeed, reservoir-fed plants have large environmental impacts, and legislation in many countries therefore prohibits further construction of such plants (Paish, 2002a,b). Compared to their large-scale counterparts, small hydropower plants require a rather large construction investment, yet have a relatively long lifespan, low operation and maintenance cost and relatively small socioeconomic impacts (Kumar et al., 2011). These latter three characteristics are especially desirable and have propelled the use of smaller hydro-electric plants to the center stage of the energy debate (Okot, 2013). Note that it is particularly difficult to delineate exactly between a large and a small-scale plant. No formally accepted guidelines exist that help determine what is considered large or not, in part because of the sheer differences in size and population of countries, and differing developmental policies (Paish, 2002a; Egré et al., 2002; Kumar et al., 2011). Nevertheless, a hydropower plant with a maximum production capacity of 15,000,000 watts (15 MW) per year is generally considered to be small.

The large majority of hydropower plants in the world are classified as small and so called run of the river (RoR) hydroelectric plants. These plants divert the river’s streamflow (up to 95% of mean annual discharge) through a pipe and/or tunnel leading to the hydropower system (turbines), and then return the water back to the river downstream (Douglas, 2007; KPS, 2008). The power production of RoR plants is not constant but varies dynamically between days, weeks and seasons due to natural variations (fluctuations) in river discharge. Indeed, RoR plants can only generate electricity if a sufficient amount of water is available
in the river network. If the river discharge falls below the minimum technical inflow of
the turbines, power production ceases. Note, RoR projects with pondage, as opposed to
those without pondage, can store water for peak load demand or continuously for base load,
especially during wet seasons (Dwivedi et al., 2006)

RoR power plants generate electricity in a way similar to large dam-based power plants yet
their production is not as constant/effective and design and appearance rather different.
Nevertheless, RoR power plants have several desirable advantages over their reservoir fed
counterparts that make them suitable for energy production. For instance, RoR power plants
have a much lower impact on the local ecosystem (construction), their design is less complex
and they require an overall lower capital investment. RoR plants are also much better
amenable to smaller water heads, and their construction time is much shorter than that of
reservoir fed counterparts. What is more, RoR plants offer the possibility of decentralized
electrification at a relatively low operational and maintenance cost. Thus, RoR plants are
generally considered to be environmental friendly, flexible to operate, and ideally suited for
localized energy production.

Despite the many environmental advantages of RoR power plants, their design and oper-
ation is a challenging task. The design of the RoR plant should be robust and flexible
enough so that it can steadily produce power in the presence of large dynamic fluctuations
in river discharge and thus turbine inflows. Generally, the amount of water diverted into the
turbines is time variant and varies considerably particularly between seasons depending on
the prevailing climatic conditions of the river network (watershed). The optimal RoR plant
design (e.g. turbine type, number, and size, penstock diameter) should maximize energy
production under fluctuating boundary conditions (turbine inflows). An investigation of the
performance of hydropower plants in Malaysia has shown that a 1 percent enhancement of
turbine efficiencies could lead to a 1.25% improvement in earnings (Al-Zubaidy et al., 1997).
Of course some power loss is unavoidable during energy conversion, yet this loss can be
minimized with a proper design.

During the past three decades much research has been devoted to the design, operation, analysis and optimization of RoR hydropower plants. That research has focused primarily on five different issues: (1) the determination of the optimum RoR plant capacity, (2) the development of specialized metrics (indexes) that convey properly the economic performance (profitability) and energy production of RoR power plants, (3) the development of efficient optimization approaches that can reliably solve the RoR design optimization problem, (4) the design, operation, analysis and performance of turbines, and (5) the importance of streamflow processes and surface hydrology on the overall performance of the RoR plant.

Research into the optimum size (capacity) of RoR plants (1) has focused primarily on maximization of investment profitability and/or economic return (Sharma et al., 1980; Gingold, 1981; Fahlbuch, 1983, 1986; Da Deppo et al., 1984; Najmaii and Movaghar, 1992; Sharma et al., 2002; Voros et al., 2000; Montari, 2003; Hosseini et al., 2005; Anagnostopoulos and Papantonis, 2007; Haddad et al., 2011; Santolin et al., 2011; Basso and Botter, 2012). General consensus is that it is particularly difficult to define an optimum size of the turbines of a RoR plant, in large part because of the rather unrealistic assumption of constant turbine inflows (efficiencies). Mishra et al. (2011) presents the review on the research work in the area of optimum installation of small hydropower plants. Moreover, the optimum size of the turbines depends strongly on the characteristics of the installation site, actual turbines used, and the main performance metric of the RoR plant (Fahlbuch, 1983; Da Deppo et al., 1984; Papantonis and Andriotis, 1993; Voros et al., 2000; Kaldellis et al., 2005; Hosseini et al., 2005). Detailed numerical simulation is required to estimate the maximum capacity of a RoR plant for given site characteristics and record of discharge observations (Lopes de Almeida et al., 2006; Anagnostopoulos and Papantonis, 2007; Papantonis and Andriotis, 1993; Haddad et al., 2011).

Research into performance measures of RoR power plants (2) has led to the development
of several specialized metrics. These metrics can be grouped into three main categories including variables that measure the economic performance (Voros et al., 2000; Hosseini et al., 2005; Motwani et al., 2006; Nouni et al., 2006), operational efficiency (Liu et al., 2003), and power production (Karlis and Papadopoulos, 2000; Arslan et al., 2008; Niadas and Mentzelopoulos, 2008). Economic indices include criteria such as the net present value (Da Deppo et al., 1984; Brealey and Myers, 2002; Karlis and Papadopoulos, 2000; Hosseini et al., 2005; Kaldellis et al., 2005; Nouni et al., 2006; Anagnostopoulos and Papantonis, 2007; Santolin et al., 2011; Basso and Botter, 2012), efficiency maintenance, operational maintenance (Liu et al., 2003), internal rate of return (Karlis and Papadopoulos, 2000; Kaldellis et al., 2005; Santolin et al., 2011; Basso and Botter, 2012; Kaldellis et al., 2005), pay-back time (Karlis and Papadopoulos, 2000) and benefit-cost ratio (Hosseini et al., 2005; Karlis and Papadopoulos, 2000; Nouni et al., 2006; Anagnostopoulos and Papantonis, 2007). The operational efficiency of the RoR plant can be defined using metrics such as the overall efficiency, ideal efficiency, and reachable efficiency (Liu et al., 2003). The power production is simply equivalent to the total mechanical energy produced by the RoR plant. Of all these metrics, the net profit value and annual power production are used most commonly in the hydropower literature to evaluate the performance of RoR hydroelectric plants.

Research into optimization algorithms (3) has led to the development and use of linear, nonlinear and quadratic programming, hybrid mixed-integer variants, interior point methods, (quasi)-Newton method, and more flexible stochastic and evolutionary optimization algorithms (Da Deppo et al., 1984; Najmaii and Movaghar, 1992; Montari, 2003; Hosseini et al., 2005; Fleten and Kristoffersen, 2005; Finardi et al., 2005; Lopes de Almeida et al., 2006; Anagnostopoulos and Papantonis, 2007; Yoo, 2009; Haddad et al., 2011; Baños et al., 2011; Basso and Botter, 2012). The application of optimization techniques to power system planning and operation has been an area of active research since the early 1960s. Many different approaches have been developed for optimal power flow tracing in an electric network (Momoh et al., 1999a,b), and some of these approaches have found application and use in
the optimization of RoR plants. Recently, Baños et al. (2011) presented a comprehensive review of optimization algorithms for design, planning and control of hydropower plants. Altogether, published findings demonstrate that evolutionary algorithms are superior to more simplistic analytic approaches and/or linear/nonlinear/quadratic programming methods, in large part because of their easy of implementation and use, and ability to handle efficiently many different decision variables.

Research into turbine selection, design, analysis, operation and performance (4) has led to approaches for direct measurement, monitoring, numerical simulation, and optimization of the turbine efficiency (Gibson, 1923; Troskolanski, 1960; IEC 41, 1991; Khosrowpanah et al., 1988; Desai and Aziz, 1994; Williams, 1994; Ye et al., 1995; Parker, 1996; Zheng, 1997; Olgun, 1998; Ye et al., 2000; Liu, 2000; Adamkowski et al., 2006; Ye-xiang et al., 2007; Wallace and Whittington, 2008; Derakhshan and Nourbakhs, 2008; Singh and Nestmann, 2009; Alexander et al., 2009; Yassi and Hashemloo, 2010; Akinori et al., 2010; Anagnostopoulos and Dimitris, 2012; Shimokawa et al., 2012; Ramos et al., 2013; Bozorgi et al., 2013; Khurana et al., 2013; Williamson et al., 2013; Laghari et al., 2013; Pimnapat et al., 2013; Cobb and Sharp, 2013; Williamson et al., 2014; Yaakob et al., 2014; Elbatran et al., 2015) The pressure-time method (Gibson, 1923; Troskolanski, 1960; IEC 41, 1991; Adamkowski et al., 2006) is one of the few methods available to measure accurately the absolute water flow rate, a key variable that determines the turbine efficiency. This requires installation of several pressure sensors at selected sections of the turbine penstock. This task is expensive when the penstock is not exposed. The excellent review of Elbatran et al. (2015) provides an in-depth summary of the performance, operation cost of low head, hydropower turbines. Other turbines have been discussed at length in the cited publications. In general, the cited literature above has demonstrated that numerical modeling, experimental investigations, and multi-criteria decision analysis are key to determining an appropriate turbine for given site characteristics and flow values. Moreover, the use of two or more turbines of different size (and or type) improves considerably the ability of a RoR plant to respond effectively to seasonal variations.
in the discharge (Anagnostopoulos and Papantonis, 2007).

Finally, research into the influence of streamflow hydrology on operation of RoR plants (5) has led to spreadsheet software and flexible parametric expressions or probabilistic/stochastic approximations of the flow duration curve (FDC) that can be used to evaluate design and economic performance (Heitz, 1982; USACE, 1985; Vogel and Fennessey, 1995; Hobbs et al., 1996; Borges and Pinto, 2008; Singh and Nestmann, 2009; Niadas and Mentzelopoulos, 2008; Peña et al., 2009; Heitz and Khosrowpanah, 2012). Probabilistic approaches are preferred as they take account explicitly for uncertainties in the turbine inflows when designing RoR plants. The mathematical expressions of the FDC introduced by Sadegh et al. (2015) are particularly powerful for RoR plant evaluation when coupled with uncertainty quantification using Bayesian inference with DREAM (Vrugt et al., 2008, 2009).

The goal of this thesis is threefold. We first introduce the different building blocks (equations) of a generic numerical model of a RoR power plant. This model, called HYdroPowER, or HYPER, is coded in MATLAB, and simulates the daily power production of a RoR plant in response to a historical record of daily river discharge observations and turbine inflows using an suite of design and variables. Unlike other numerical models approaches developed in hydropower literature (Da Deppo et al., 1984; Najmii and Movaghar, 1992; Voros et al., 2000; Montari, 2003; Hosseini et al., 2005; Anagnostopoulos and Papantonis, 2007; Haddad et al., 2011; Santolin et al., 2011; Basso and Botter, 2012), HYPER constitutes the first simulator that takes into explicit consideration the design flow, penstock diameter, penstock thickness, specific speed, rotational speed, cavitation, and suction head in evaluating the technical performance, production, cost, and profit of a RoR plant. What is more, HYPER accommodates a wide variety of flow regimes, and implements four commonly used hydroelectric turbines including Kaplan, Francis, Pelton and crossflow to test the effectiveness in producing energy. Secondly, we discuss the elements of a global optimization algorithm that can be used to optimize the design parameters of the RoR plant for given river flow
conditions, including (among others) the type and number of turbines, their design flow, and the penstock diameter. This algorithm can be called from the main model script and is easily adapted to include other design and operation (decision) variables as well. Finally, we introduce a graphical user interface (GUI) and post-processor to simplify simulation and interpretation of the results. The model with its different components (GUI, optimization algorithm) is available upon request from the first or second author.

The remainder of this thesis is organized as follows. Section 2.1 introduces the main building blocks of our numerical model, HYPER. In this section we are especially concerned with the technical characteristics of turbines, turbine selection and operation, and cavitation. In section 2.2 we discuss the differential evolution (DE) global optimization algorithm that is used to optimize the main design variables of the RoR plant. This is followed in section 2.3 with three illustrative case studies that demonstrate the power and numerical results of HYPER. The penultimate section of this paper (section 2.4) introduces the GUI of HYPER. Finally, section 2.5 concludes this paper with a summary of our main findings.
Chapter 2

Optimization of Run-of-River Hydropower Plants


One of the main goals of the present study is the development of a numerical model that simulates accurately for given site characteristics the power production of a RoR hydro-electric plant in response to a time series of daily discharge (inflow) values. This simulator is the outcome of three formal stages to model building, including development of

1 Conceptual model: Summarizes our abstract state of knowledge about the structure and workings of the RoR plant.

2 Mathematical model: Defines the computational states, fluxes, and parameters of the RoR plant and the choices regarding how system processes will be handled mathematically.
3 Computational model: Provides numerical solutions for specific site characteristics (head), design and operation parameters (decision variables), turbine and material properties (penstock), and boundary conditions (inflows)

We limit ourselves here to a description of the computation model of the RoR plant, and discuss in detail how the power production and economical costs of investment and maintenance depend on the main design variables (e.g. size of penstock), turbine selection, and site characteristics (head, inflow). The model, hereafter referred to as HYdroPowER, or HYPER, is coded in MATLAB, and differs from existing models in the hydropower literature (Da Deppo et al., 1984; Najmai and Movaghar, 1992; Voros et al., 2000; Montari, 2003; Hosseini et al., 2005; Anagnostopoulos and Papantonis, 2007; Haddad et al., 2011; Santolin et al., 2011; Basso and Botter, 2012) in that (a) the specific speed is calculated at all times and used as guiding principle to determine the most appropriate turbine(s) for given FDC and site characteristics [to the best of our knowledge, the only other study Santolin et al. (2011) used the convenient but flawed assumption of non-overlapping specific speeds of different turbines], (b) the crossflow turbine is explicitly simulated [this turbine is often ignored although used frequently in operational RoR plants], (c) parallel hydropower systems with two different turbine types are considered (note: Anagnostopoulos and Papantonis (2007) considers such systems but does not take into account the specific speed in turbine selection, and (among others) ignore problems with cavitation as well). In summary, HYPER constitutes the first numerical model that takes into explicit consideration the design flow, penstock diameter, penstock thickness, specific speed, rotational speed, cavitation, and suction head in evaluating the technical performance, production, costs, and profit of a RoR plant. Moreover, the built-in optimization algorithm allows optimization of the design parameters of four different turbines (Francis, Kaplan, Pelton and crossflow) and their possible parallel combinations. We next discuss the main equations of our model.
2.1.1 Energy Production

The amount of energy, $E$ (kWh) produced by a hydropower plant over a time period, $\Delta T$, can be calculated as follows

$$E = \int_0^{\Delta T} \rho \ g \ \eta_g \ H_{\text{net}} \{t, D\} \sum_{j=1}^{n} [q_j(t) \ \eta_j \{q_j(t), O_{dj}\}] \ dt,$$

where $t$ (s) denotes time, $\rho$ (kg/m$^3$) is the density of water, $g$ (m/s$^2$) signifies the gravitational constant, $\eta_g$ (-) represents the overall generator efficiency, $H_{\text{net}}$ (m) characterizes the net pressure head of water across the turbine, $D$ (m) denotes the diameter of the penstock, $n$ (-) signifies the number of turbines, $q_j$ (m$^3$/s) is the volume flux of water passing through the $j$th turbine and $\eta_j$ (-) is the efficiency of the $j$th turbine. Braces are used to denote a dependency of $H_{\text{net}}$ and $\eta_j$ on time and the diameter of the penstock, and the inflow and design flow respectively.

As can be seen in Equation (2.1) the power production depends on the local characteristics of the installation site, $H_{\text{net}}$, the turbine inflows, and turbine and generation efficiencies. Turbine efficiency is determined by the ratio of the volume flux of water it receives per time unit, $q_j$ (inflow) and its design flow, $O_{d}$, hence $\eta_j \sim q_j(t)/O_{d_j}$. In the next section we will discuss in detail the characteristics of different commonly used turbines, and depict their efficiency curves. Note that the turbine inflow and net head have a linear effect on production.

Equation (2.1) cannot be solved analytically for dynamically varying boundary conditions and we therefore resort to numerical integration. A $N$-vector of mean daily turbine inflows, $Q = \{q(1), \ldots, q(N)\}$ is used to solve Equation (2.1) with a fixed daily integration time step. This vector of inflows is derived from observations of the hydrograph of the river.
network, \( \mathbf{Q}_r = \{g_r(1), \ldots, g_r(N)\} \). If such discharge measurements are not readily available then streamflow records from nearby catchments can be used, or alternatively a synthetic hydrograph can be defined. In the illustrative case studies presented herein we specify the \( N \) values of the discharge record by sampling from the FDC of the river network. This function depicts graphically the relationship between the exceedance probability of streamflow and its magnitude. Not all the streamflow that is transported by a river network can be diverted to the RoR plant. A fixed quantity of discharge, also called the residual flow, should be transported by the river network at all times to minimize adverse affects on the biotic environment within the stream. We therefore calculate the turbine inflow as follows

\[
q(t) = \max(q_r(t) - q_{md}, 0),
\]

where \( q_r(t) \) (\( \text{m}^3/\text{s} \)) signifies the river discharge at time \( t \), and \( q_{md} \) (\( \text{m}^3/\text{s} \)) represents the minimum river discharge required to maintain perfectly the ecosystem.

The net head, \( H_{\text{net}} \) (\( \text{m} \)), varies as function of time and can be calculated as follows

\[
H_{\text{net}}(t) = H_g - H_f(t) - H_o(t),
\]

where \( H_g \) denotes the gross head, \( H_f \) (\( \text{m} \)) represents the friction loss in the penstock, and \( H_o \) (\( \text{m} \)) is an aggregate term of all hydraulic losses in the conveyance system. The gross head, \( H_g \) (\( \text{m} \)), depends on the difference in water elevation between the upstream water level and the water level at the tail race or nozzle jet for reaction and impulse turbines, respectively.
and hence can be calculated as follows

$$H_g = \begin{cases} 
Z_{up} - Z_{d} & \text{for reaction turbines} \\
Z_{up} - Z_{d} - \delta_{jet} & \text{for impulse turbines,}
\end{cases} \quad (2.4)$$

in which, \(Z_u\) (m) signifies the upstream water elevation, \(Z_d\) (m) denotes the downstream water elevation, also referred to as the elevation at the tail race, and \(\delta_{jet}\) is the nozzle jet height from the tail race. The variables \(H_f\) and \(H_o\) are time dependent due to dynamic variations in penstock flow (inflow to the turbines). This causes the hydraulic loses in the hydropower system to vary temporally. The friction loss, \(H_f\), is a function of the penstock diameter, \(D\) (m), the length, \(L\) (m), and the friction factor, \(f\) (-), and can be calculated using the Darcy-Weisbach equation

$$H_f(t) = f \frac{L V(t)^2}{D 2g}, \quad (2.5)$$

where \(V\) (m/s) denotes the mean velocity of the turbine inflow. The flow velocity is simply calculated by dividing the inflow, \(q(t)\) by the penstock area. The composite term of all other singularities losses, \(H_o\), depends mainly on the geometry of the hydraulic conveyance system (bends, fittings, valves, etc.) and can be computed from

$$H_o(t) = k_{sum} \frac{V(t)^2}{2g}, \quad (2.6)$$

where \(k_{sum}\) (-) is an aggregate resistance term that represents the composite effect (summation) of screen, entrance, bend, and valve losses.

The penstock transports water from a reservoir or river to the turbine system, and constitutes
one of the most important components of a RoR hydropower plant. It is usually made of steel and is equipped with a gate system to control the water inlet. The penstock diameter, \( D \) (m), should be large enough to transport a sufficient amount of water, but cannot be too large otherwise the penstock will be too expensive and too difficult to install (among others), hence

\[
D_{\text{min}} \leq D \leq D_{\text{max}},
\]  

(2.7)

where \( D_{\text{min}} \) (\( D_{\text{max}} \)) is the minimum (maximum) diameter of the penstock. The smaller the diameter of the penstock, the lower the investment cost, yet the smaller the net head due to substantial friction losses. On the contrary, a large penstock can maintain a larger net head but is more expensive. The diameter of the penstock is an important design parameter whose value requires a detailed optimization analysis of energy produced versus investment and maintenance cost. We will elaborate on this in a later section. The minimum penstock thickness, \( k \) (m), satisfies the following equation (Ramos et al., 2000)

\[
k = 0.0084D + 0.001,
\]  

(2.8)

so that the penstock can withstand the maximum imposed pressure.

### 2.1.2 Turbine Selection: Technical Characteristics, Operation and Cavitation

The turbine system is at the heart of the power plant and converts flowing water into mechanical energy. The selection which turbine(s) to use depends in large part on the
characteristics of the installation site, for example the available net head and anticipated
dynamics of the river discharge, and is therefore arguably one of the most difficult decisions
in the design of a hydropower plant. Many different turbines have been developed in the
past decades, to handle effectively the large range of heads and inflows observed in real-world
river networks. Each of these turbines has been designed for a particular range of inflows
- outside this range the ability of the turbine to generate electrical power is significantly
compromised.

In the present study, we investigate some of the most commonly used turbines including
the Kaplan, Francis, Pelton and crossflow turbines. These turbines can be classified based
on how they operate and produce energy. The Pelton and crossflow turbines are impulse
turbines. In these two turbines, a nozzle directs a stream of high velocity water tangential to
the turbine disc to which are affixed radial blades. The blades move in the direction of the
water jet, with at least one blade always intercepting the stream. Energy is thus produced
by water impinging on the blades of the runner.

The Francis and Kaplan turbines are reaction turbines, and use the force exerted by the
water to rotate the runner inside the turbine, in a way similar to how the engines of an
airplane create thrust. Reaction turbines exhibit a rather poor efficiency at low flows (see
Figure 2.1), despite their relatively high specific speed. Their operation and use is also more
difficult than that of impulse turbines, as they use profiled blades with special casings and
guide vanes. Such blades are rather costly, and hence the Kaplan and Francis turbines are
commonly considered to be rather expensive for operational use in RoR plants.

What sets impulse turbines apart from other turbines is that they exhibit relatively high
efficiencies at low flows (see Figure 2.1). In addition, they have relatively simple designs,
which makes them easy to fabricate and maintain, and their performance is not much affected
by sand and other dissolved particles. Indeed, Pelton and crossflow turbines are generally
considered to be cost effective in small RoR plants.
The efficiency curve is of crucial importance in the selection of an adequate turbine for a given installation site and river system. Each turbine has an efficiency curve (also called characteristic curve) that can be obtained from the turbine manufacturer. The curve depicts graphically the relationship between the flow and design flow, $q/O_d$ (-) and the efficiency $\eta$ (-). This ratio is also referred to in the literature as design flow proportion, or percentage of full load (aka load). Efficiency curves are used to analyze how each turbine performs under specified conditions for simulation study. Typical efficiency curves are shown in the Figure 2.1. The following main conclusions can be drawn from this turbine chart.

Impulse turbines (crossflow and Pelton) achieve high efficiencies (up to 90%) at rather low loads. The Pelton turbine seems to be particularly appealing as it achieves relatively high efficiencies for a large range (0.2 - 1.25) of loads. Indeed, they maintain a high efficiency even when operating much below the design flow. Reaction turbines (Francis and Kaplan)
accommodate a smaller range of loads (upward from 20 - 30% of design flow) but can achieve a higher efficiency at full load and beyond (1-1.25). The crossflow turbine exhibits the overall smallest efficiency, perhaps a reason why this turbine is often discarded as a viable candidate for a RoR plant in the hydropower literature. However, crossflow turbines (and Pelton) are relatively cheap to construct, build and repair, and maintain a rather constant efficiency for a large range of loads.

The net head and the design flow are used to determine the turbines applicable to the installation site of the RoR hydropower plant. The selection chart in Figure 2.2 depicts schematically the operable range of net heads and design flows (inflows) for each of the turbines simulated by HYPER including Pelton (green), crossflow (black), Kaplan (blue) and Francis (red). In general, impulse turbines (crossflow and Pelton) can handle a large range of heads (3 - 1,000 m) with low-medium/high design flows of 0 - 9 (m$^3$/s). Reaction turbines, on the contrary cover a smaller range of heads from low (Kaplan: 3 m) to medium/high (Francis: 350 m) but their design flow can vary from 0 - 50 m$^3$/s. Note that the different turbines overlap. This makes it difficult to decide a-priori which turbine to select for a given head and anticipated (mean) river discharge (e.g. inflow). For design flows of 0.75 - 1 m$^3$/s and net heads that vary between 50 - 170 m, one can select up to three different turbines for the RoR plant, namely crossflow, Francis and Pelton. Yet, this does not mean that each of them will produce an equivalent monthly or annual power. This all depends on their characteristic curve (Figure 2.1). What is more, each turbine will have a different operation and maintenance cost. Indeed, this warrants an in-depth study of the merits of each turbine using HYPER. Their annual power production and construction and maintenance costs have to be computed and compared (amongst others), of course taking into account as well the anticipated flow variability at the installation site. Some turbines can handle a larger range of inflows than others and still operate at a high efficiency.

The final choice which turbine(s) to select involves an optimization analysis of costs versus
benefits. Figure 2.2 alone is thus insufficient to determine an appropriate turbine for a given installation site. Instead, the graph only dictates which turbines should be considered for a given head and inflow rate in the optimization analysis. Many different publications can be found in the literature that have studied the efficiency, cost, and performance of impulse and reaction turbines (Elbatran et al., 2015 and many references therein). This includes advanced numerical modeling with the finite element method. The difference with our work presented herein is (among others) that we do not study the behavior of turbines in isolation, but rather consider their operation and performance in a RoR plant under dynamically varying inflow conditions, and when configured in parallel with other turbines. The technical characteristics of the turbines are discussed next.
2.1.2.1 Technical Characteristics of the Turbines

After selecting the suitable turbines for a given installation site, the technical characteristics of each turbine must be examined before a final decision can be made which turbines to use. This includes the specific speed and rotational speed.

The specific speed, $\omega_s$, is a dimensionless number that is defined as the ratio of the rotational speeds of two different turbines that are geometrically similar to each other but with differing size of their turbine runners. It can be derived from the laws of similarity and calculated from

$$
\omega_s = \frac{1}{60} \frac{\omega}{\sqrt[3]{(gH_{\text{net}})^4}},
$$

(2.9)

where $\omega$ (rpm) denotes the rotational speed of the turbine, and the other variables have been defined previously. The multiplication factor of $1/60$ is used for unit conservation. The rotational speed in Equation (2.9) is determined by the frequency, $f_e$ (Hz) of the electric system, and the number of poles, $p$ (Hz) of the turbine generator or

$$
\omega = \frac{60f_e}{p} \quad (p = 2, 4, 6, ..., 28).
$$

(2.10)

However, since a turbine can be coupled with a speed increaser to reach the desired generator speed, the range of turbine speed is upgraded from a discrete to a continuous function restricted by the upper limit of Equation (2.10)

$$
\omega \leq \omega_{\text{max}},
$$

(2.11)
Table 2.1: Ranges of the specific speed, $\omega_s$ (unit) for the four different turbines simulated by HYPER (ESHA, 2004; EUMB, 2009)

<table>
<thead>
<tr>
<th>Turbine</th>
<th>Specific speed range.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelton</td>
<td>$0.005 \times \mu^{0.5} \leq \omega_s \leq 0.025 \times \mu^{0.5}$ $^a$</td>
</tr>
<tr>
<td>Crossflow</td>
<td>$0.04 \leq \omega_s \leq 0.21$</td>
</tr>
<tr>
<td>Francis</td>
<td>$0.05 \leq \omega_s \leq 0.33$</td>
</tr>
<tr>
<td>Kaplan</td>
<td>$0.19 \leq \omega_s \leq 1.55$</td>
</tr>
</tbody>
</table>

$^a$\(\mu\) denotes the number of nozzles

where $\omega_{\text{max}}$ is the maximum allowable speed of the turbine, derived at $p = 2$ poles.

Table 2.1 summarizes the ranges of the specific speeds that are used in our simulation experiments presented herein. The listed values illustrate that for given heads and inflow values reaction turbines rotate faster than impulse turbines. We present purposely dimensionless values so as to be able to compare directly the specific speeds of the Kaplan, Pelton, Francis and crossflow turbines.

### 2.1.2.2 Cavitation

Cavitation is the formation of vapor cavities in a liquid, also called bubbles or voids, that are the consequence of forces acting upon the liquid. It usually occurs when a liquid is subjected to rapid changes of pressure that cause the formation of cavities where the pressure is relatively low. Turbines are particularly prone to cavitation around the turbine runner, and in the draft tube. This deformation and/or disintegration of the blades and runner severely compromises the performance of the turbine.

To determine whether the turbine is susceptible to cavitation we calculate the turbine’s runner position, also referred to as admissible suction head. A negative value of the suction head, $H_s$ essentially means that the hydrodynamic pressure in the turbine can fall below the
water vapor pressure, a requirement for cavitation to occur. The admissible suction head, $H_s$ (m) can be computed from

$$H_s = \frac{P_{atm} - P_v}{\rho g} + \frac{V_{out}^2}{2g} - \sigma H_d,$$

(2.12)

where $P_v$ (Pa) denotes the water vapor pressure, $V_{out}$ (m/s) is the average water outlet velocity of the turbine, $H_d$ (m) represents the design head which is the average of the net heads for the proposed design parameters and $\sigma$ signifies Thoma’s coefficient (ESHA, 2004). The atmospheric pressure $P_{atm}$ (Pa) above sea level is approximated using

$$P_{atm} = P_o \exp\left(-\frac{z_{el}}{7000}\right),$$

(2.13)

where $P_o$ (Pa) signifies the sea level atmospheric pressure, and $z_{el}$ (m) represents the elevation of the power house above sea level. Thoma’s coefficient is a function of the specific speed, $\omega_s$, and can be calculated as follows (ESHA, 2004)

$$\sigma_F = 1.2715\omega_s^{1.41} + \frac{V_{out}^2}{2gH_d}$$

(2.14)

$$\sigma_K = 1.5241\omega_s^{1.46} + \frac{V_{out}^2}{2gH_d},$$

(2.15)

where the subscript F and K are used to denote the Francis and Kaplan turbines respectively.

Turbines will not be susceptible to cavitation if their admissible suction head remains larger than zero during the operation of the RoR plant. This requires an in-depth analysis of the
interplay between the anticipated average turbine outflow velocity, atmospheric pressures and net heads at the installation site, that determine jointly with the water vapor pressure and other properties (gravitational head and density of water) the admissible suction head. If cavitation is expected based on preliminary evaluation of the design of the RoR plant, excavation can be used to bury the turbines at a depth \( H_s \) below the current position (surface). Of course, this is not without burden as it increases the financial costs of the RoR plant. HYPER calculates such economic variables as well because of their trade-off with power production - of which more later.

### 2.1.2.3 Turbine Operation

The design flow and the net head are two key variables that determine the optimal properties and configuration of the hydropower system. The design flow determines the portion of turbine inflow, also called workable flow, that can pass through the turbine, and as such is the maximum flow rate a RoR plant can accommodate. Once the range of potential design flows is obtained from the FDC of the river network, the final decision which turbine to use necessitates nonlinear optimization in which power production is maximized for a given total investment.

Turbines have technical flow constraints and can only operate effectively within certain flow ranges. If only one turbine is installed and used in a hydropower system, then three different operation levels can be distinguished

\[
q(t) = \begin{cases} 
0, & q \leq q_{\text{min}} \\
q, & q_{\text{min}} < q < q_{\text{max}} \\
q_{\text{max}}, & q_{\text{max}} \leq q
\end{cases}
\]  

(2.16)
Table 2.2: Values of $\gamma^{-}$ and $\gamma^{+}$ for the different turbines simulated by HYPER. The Francis and Kaplan are classified as impulse turbines, whereas crossflow and Pelton constitute reaction turbines.

<table>
<thead>
<tr>
<th>Turbine</th>
<th>$\gamma^{-}$</th>
<th>$\gamma^{+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelton</td>
<td>0.1</td>
<td>1.25</td>
</tr>
<tr>
<td>Crossflow</td>
<td>0.1</td>
<td>1.25</td>
</tr>
<tr>
<td>Francis</td>
<td>0.3</td>
<td>1.25</td>
</tr>
<tr>
<td>Kaplan</td>
<td>0.2</td>
<td>1.25</td>
</tr>
</tbody>
</table>

where $q_{\text{min}}$ is the minimum flow level, also called cutoff flow, below which power generation ceases, and $q_{\text{max}}$ denotes the maximum flow a turbine can process. The values of $q_{\text{min}}$ and $q_{\text{max}}$ depend on the design flow and turbine characteristics which may vary from one manufacturer to another. In practice, the values of $q_{\text{min}}$ and $q_{\text{max}}$ are set as fixed multiple of the design flow, or $q_{\text{min}} = \gamma^{-}O_{\text{d}}$, and $q_{\text{max}} = \gamma^{+}O_{\text{d}}$. Table 2.2 lists the values of $\gamma^{-}$ and $\gamma^{+}$ for the impulse and reaction turbines considered in HYPER.

As can be seen in Equation (2.16), a turbine can only operate within a certain envelope of inflows. For this reason it may be advantageous to install several smaller turbines instead. The use of two or more turbines configured in parallel could, at least theoretically, enhance the workable range of inflows, and thus overall efficiency of the RoR plant when confronted with variations in river discharge. What is more, the sharing of water between two or more turbines allows for a higher rotational speed, so the turbine torque will be lower leading to a more stable and reliable operation (Penche, 1998). Our numerical model, HYPER, implements the option of two parallel turbines of different size and type. Of course, the choice whether to use one large turbine or two parallel turbines depends on energy production, construction and operational costs. Whatever hydropower system is used, their design and operation should maximize power production for a given financial investment.

Figure 2.3 provides a schematic overview of how two turbines working in parallel distribute the available inflow. The first of the two turbines (labeled with subscript '1') is assumed to
have the larger design flow. In general, we can discern the following three operational modes in a parallel system consisting of two turbines.

(a) The hydropower power system is down due to insufficient flow
(b/c) Only one of the two turbines is working
(d/e) Both turbines are working - possibly at maximum capacity

![Diagram of operational rules](image)

Figure 2.3: Schematic overview of the operational rules of a parallel hydro turbine system consisting of two different turbines.

With respect to (a) no power is produced if the current flow rate is below the minimum flow of the second (smaller) turbine, labeled "2". With respect to (b/c), If the inflow is larger than the minimum flow of second turbine one, and smaller than that of its counterpart two, the smaller turbine one will be activated and generate power. For inflows between $q_{1\text{ min}}$ and $q_{1\text{ max}}$, the decision which of the two turbines to operate depends on their anticipated production (calculated from Equation 2.1). Finally, with respect to (d/e), both turbines are in operation if the current inflow is larger than the maximum flow of turbine one. Both turbines operate at full capacity (load) if the current flow exceeds the sum of their maximum flows (Anagnostopoulos and Papantonis, 2007).

This two turbine parallel hydropower system is easily coded in MATLAB using the five operational rules defined in Figure 3. We are currently also investigating ways how we can
generalize the operational rules to hydropower systems consisting of more than two turbines. This could involve a combination of parallel and serially configured turbines.

### 2.1.3 Numerical solution of HYPER

The different equations of HYPER are solved numerically using a daily integration time step. This Eulerian integration scheme is unconditionally stable for the present application with daily discharge and turbine inflow time series. The output of HYPER consists of daily values of the produced energy, \( E \) (solution of Equation (2.1)) and net profit (of which more later).

Now we have discussed the different building blocks (equations) of HYPER, we will now summarize in words how the model operation. Figure 2.4 provides a schematic overview of the model. Model inputs are outputs are color coded in red and blue, respectively. A \( N \)-vector with daily observations of the river discharge, \( Q_r \) is used as main input to the model, and used to derive the corresponding vector of turbine inflow values, \( Q \), by subtracting the value of the residual flow, \( q_{md} \) (see Equation 2.2). At each time \( t \) (days), the net head, \( H_{\text{net}} \) is calculated by subtracting from the gross head (known constant), the friction and other singular losses in the conveyance system (see Equation 2.3). Based on the values of the design parameters, \( D \) (penstock diameter), \( O_{d1} \) and \( O_{d2} \) (design flow first and second turbine) (single/parallel operation), the rotational and specific speed of the turbine(s) are calculated (see Equation 2.9 and 2.10). If a reaction turbine is used (e.g. Francis, Kaplan), the suction head, \( H_s \) is also computed (see Equation 2.12). The efficiency of the turbines(s) is subsequently computed from the ratio between the turbine inflow(s) and design flow using the efficiency curves depicted in Figure 2.1. Now all main variables of the RoR plant have been calculated, Equation (2.1) can be solved numerically, and the energy production between time \( t \) and \( t+1 \) is calculated. At the end of simulation, the net profit value of the RoR plant
is computed by subtracting from the price of the total energy production the investment and maintenance costs. Details of how the different costs of the RoR plant are calculated are provided in the next section. The net profit of the RoR plant depends in large part on its

![Flow diagram of HYPER](image)

Figure 2.4: Flow diagram of HYPER. Model input (discharge time series) and output ($E$ and net profit, NPV) color coded in red and blue, respectively. The 'DE' box constitutes the global optimization algorithm used to estimate the design/operation parameters (purple). The main prognostic (state) variable of the model is the net head, $H_{net}$ and turbine efficiency.

design and the turbine(s) that is/are being used. HYPER includes a built-in optimization algorithm, differential evolution, that can be used to optimize the plant design (and turbine selection) for a given monetary investment. This component of the model will be discussed in the next section.
2.2 Nonlinear Optimization and Objective Functions

2.2.1 The Optimization Algorithm

In the present study, the design of the RoR hydropower plant is optimized to maximize the energy production for a given financial costs. We consider in our optimization analysis the two most common decision variables including the penstock diameter, $D$ and the turbine design flow, $O_d$, see Figure 2.5. If a setup with two parallel turbines is used then the optimization involves the design flow of the first and second turbine, also referred to as $O_{d1}$ and $O_{d2}$, respectively, see Figure 2.6. The thickness of the penstock, $k$, the suction head, $H_s$, and rotational speeds, $\omega_1$ and $\omega_2$ of the first and second turbine are derived from the design parameters, along with of course the annual power production, $E$ (kW) and NPV ($M). We will consider in our optimization analysis all individual turbines separately, and their possible combinations. The Differential Evolution (DE) algorithm is employed to estimate the optimal model parameters. DE is a derivative-free population-based global optimization algorithm that has become increasingly popular in the past decade to solve complicated search problems involving non-ideal response surfaces (objective function mapped over the parameter space). DE has been demonstrated to be able to cope with strong correlation among decision variables, and exhibits rotationally invariant behavior (Storn and Price, 1997). The DE method starts by creating an initial population, $X$ of $N$ parents, $X_0 = \{x_0^1, \ldots, x_0^N\}$ by sampling from the prior parameter ranges, $x_0^i \in X_0 \in \mathbb{R}^d$ where $d$ denotes the dimensionality of the search space. The objective function, NPV, is subsequently calculated for each individual parent, $i = \{1, \ldots, N\}$ and stored in a $N$-vector $F = \{NPV(x_0^1), \ldots, NPV(x_0^N)\}$.

We now create offspring (children) from the parent population, $X$ as follows. If $A$ is a subset of $m$-dimensions of the original parameter space, $\mathbb{R}^m \subseteq \mathbb{R}^d$, then the offspring for the $i$th parent, $z^i$, $i = \{1, \ldots, N\}$ at iteration $t = \{1, \ldots, T\}$ is calculated using (Storn and Price,
Figure 2.5: Schematic overview of a single turbine RoR power plant. The variables \(L\) and \(D\) signify the length and diameter of the penstock, \(H_g\) signifies the gross head, and \(T_1\) represents the turbine.

\[z_{i,A} = x_{t-1}^{w_i,A} + \lambda_{DE}(x_{t-1}^{v_i,A} - x_{t-1}^{y_i,A})\]
\[z_{i,\neq A} = x_{t-1}^{i,\neq A},\]  

(2.17)

where \(w, v,\) and \(y\) are selected without replacement from \(\{1, \ldots, i-1, i+1, \ldots, N\}\), and \(\lambda_{DE} \in (0, 2]\) is a control parameter that determines offspring diversity. Once the offspring population, \(Z = \{z^1, \ldots, z^N\}\) has been created, their objective function values are calculated, \(G = \{NPV(z^1), \ldots, NPV(z^N)\}\). Finally, we pairwise compare each parent with its respective child. If \(G^i \geq F^i\), then the child replaces the parent, \(x^i = z^i\) and \(F^i = G^i\), otherwise the \(i\)th parent is kept in the population, \(i = \{1, \ldots, N\}\). If the maximum number of generations, \(T\), has not been reached, go back to Equation 2.17, otherwise stop, and return the parent of \(X_T\) with the highest (or lowest, if appropriate) objective function value as solution to the optimization problem. A detailed description of the DE can be found in Storn and Price 1997; Price et al., 2005)
The number of dimensions stored in $A$ ranges between 1 and $d$ and depend on the actual crossover value used. In our calculations we use a crossover value, $CR = 0.8$, and determine the dimensions of $A$ as follows. Each time a child is created, we sample a vector $\tau = \{\tau_1, \ldots, \tau_d\}$ with $d$ standard uniform random labels is drawn from a standard multivariate uniform distribution, $\tau \sim U_d(0,1)$. All those dimensions $j$ for which $\tau_j \leq CR$ are stored in $A$ and span the subspace of the child that will be sampled. In the case that $A$ is empty, one dimension of $x_{t-1}$ will be sampled at random. This simple randomized strategy, activated when $CR < 1$, constantly introduces new directions outside the subspace spanned by the current parent population and enables single-parameter updating, multi-parameter sampling (a group of parameters), and full-dimensional search (all dimensions).

In practice, it is possible that the DE algorithm samples combinations of the $d$ design pa-
rameters of the RoR plant that lead to turbine specific speeds that are outside their feasible ranges. These solutions are penalized immediately by assigning them a zero value of the NPV objective function. This avoids the DE algorithm to converge to unfeasible and unreliable solutions for the proposed hydropower site.

2.2.2 Objective Function

Once the technical feasibility of a site has been established, the remain key consideration question is to determine whether the hydropower project will be profitable or not. The proposed objective function includes all the expenses and the earnings in order to decide whether the investment is economically feasible or not. The total development cost of a RoR plant consists of the three main components, including the cost for the civil works, $C_{cw}$, the electric, hydro-mechanical equipment, $C_{em}$, and the penstock, $C_{p}$. The yearly revenue of the plant, $R$, is simply the difference between the amount of money made per year for selling the electricity, and the yearly operation and maintenance cost, $C_{om}$. Part of these revenues is used to pay-off the initial investments, possibly with interest. It is well known that hydropower plants have relatively higher installation costs and lower operating and maintenance costs which means that a large proportion of the project’s budget will be spent during construction stage and hence the cost and the energy production should be optimized carefully.

The objective function that we maximize in the present paper is the net present value, NPV, which is an economical index used to determine whether a prospective investment will be profitable or not. The NPV is equal to the cumulative sum of all discounted cash inflow generated by the power plant minus the sum of all its cost during the lifetime of the project. To be adequately profitable, an investment should have a NPV value greater than zero.

Quite a few number of empirical equations have been proposed in the hydropower literature
Table 2.3: The values of $a$, $b$ and $c$ in the cost functions four different turbines simulated by HYPER. Tabulated values of $b$ and $c$ are rounded two three significant digits.

<table>
<thead>
<tr>
<th>Turbine Type</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crossflow</td>
<td>8,846</td>
<td>−0.364</td>
<td>−0.281</td>
</tr>
<tr>
<td>Francis</td>
<td>25,698</td>
<td>−0.560</td>
<td>−0.127</td>
</tr>
<tr>
<td>Kaplan</td>
<td>33,236</td>
<td>−0.583</td>
<td>−0.113</td>
</tr>
<tr>
<td>Pelton</td>
<td>17,692</td>
<td>−0.364</td>
<td>−0.281</td>
</tr>
</tbody>
</table>

To quantify the investment and maintenance costs of a RoR plant (Gordon and Penman, 1979; Voros et al., 2000; Kaldellis et al., 2005; Aggidis et al., 2010; Singal et al., 2010). Most of these equations cannot be readily applied to a newly to be developed hydropower plant, because these equations are either site specific, only applicable to certain regions in the world, or simply no longer representative for current prices. Ogayar and Vidal (2009) introduced several empirical cost equations for $C_{em}$ ($\), the electro-mechanical equipment

$$C_{em} = \xi a P^{(b+1)} (H_d)^c,$$

(2.18)

where $\xi$ is the EUR/USD exchange rate, $P$ (kW) denotes the power produced by the RoR hydropower plant, and $a$, $b$, and $c$ are constants whose values are listed in Table 2.3 and differ for each of the four turbines considered in HYPER and listed in Table 2.1. Note that Ogayar and Vidal (2009) do not specify an equation for $C_{em}$ for crossflow turbines. The values listed for $a$, $b$, and $c$ of this turbine are derived based on the observation that the electro-mechanical costs of the crossflow turbine are about half of those of the Pelton turbine for the same design head and power (RE, 2004).

The total costs of the civil works, $C_{cw}$ ($\) is proportional to the costs of the electro-mechanical
equipment (Gordon and Penman, 1979; Voros et al., 2000; Kaldellis et al., 2005)

\[ C_{cw} = \alpha C_{em}, \quad (2.19) \]

where \( \alpha \) (-) is also known as the site factor and takes on values between 0.8 and 2 (Kaldellis et al., 2005). Note, the cost of the penstock is excluded from Equation 2.19, in order to take into explicit consideration in our optimization the dependency of the civil costs, \( C_{cw} \) on the diameter of the penstock. A value of \( \alpha = 0.5 \) is therefore deemed appropriate.

The yearly maintenance and operation cost of the hydropower plant is proportional to the electromechanical equipment cost, \( C_{em} \), (Gordon and Penman, 1979; Voros et al., 2000; Kaldellis et al., 2005; Hosseini et al., 2005; IRENA, 2012) and can therefore be calculated using

\[ C_{om} = \beta C_{em}, \quad (2.20) \]

where \( \beta \) (-) is a coefficient whose value ranges between 0.01 and 0.04 (IRENA, 2012).

The penstock cost, \( C_p \) ($), constitutes a major expense in the total budget, and depends on the current market price

\[ C_p = \pi(D + 2k) k d_s L P_{ton}, \quad (2.21) \]

where \( k \) (m) signifies the thickness of the penstock, \( d_s \) represents the steel density (ton/m\(^3\)) and \( P_{ton} \) ($/ton) denotes the cost of the penstock per ton weight.
From an economic point of view, it is also important to note the effect of the life span of the electro-mechanical equipment to the total cost. In essence, turbines, as well as most other electro-mechanical equipments, have life spans of about 25 years. Thus, the total value of the equipment at time of purchase is approximately equal to the costs of renovation and reconstruction of the equipment 25 years later (Hosseini et al., 2005). Therefore the total cost of the investment, $C_t$, for a RoR plant life time of 50 years can be calculated as follows

$$C_t = C_{cw} + 2C_{em} + C_p \cdot \quad (2.22)$$

In this study, we conveniently assume the RoR plant to be connected to a central-grid which stores the entire energy production. This avoids having to specify the load. By further assuming a fixed energy price, and an infinite energy demand, the NPV can be computed from

$$\text{NPV} = \left[ \frac{R_1 - C_{om}}{1 + r} + \frac{R_2 - C_{om}}{(1 + r)^2} + \ldots + \frac{R_{Ls} - C_{om}}{(1 + r)^{Ls}} - C_t \right] C_{tf} \quad (2.23)$$

where

$$R_l = e_p E_l \quad (l = 1, 2, \ldots, L_s) \quad (2.24)$$
and

\[ C_{rf} = \frac{r(1 + r)^{L_s}}{(1 + r)^{L_s} - 1}, \quad (2.25) \]

where \( R \) represents the yearly revenue in dollars, \( r \) (\%) denotes the interest rate, \( C_{rf} \) (-) signifies the capital recovery factor and \( L \) is the life time of the project.

The annual NPV of a RoR hydropower plant is typically calculated for a constant power production with a constant inflow of cash Equation (2.23) becomes

\[ \text{NPV} = \left( (R - C_{om}) \frac{1 - (1 + r)^{-L_s}}{r} - C_t \right) C_{rf}, \quad (2.26) \]

which after a few mathematical manipulations can be simplified to

\[ \text{NPV} = R - C_{om} - C_t C_{rf}. \quad (2.27) \]

### 2.3 Case Studies and Results

We now illustrate the main results of HYPER using three illustrative case studies. These three examples cover a relatively wide range of heads and inflow conditions (and their variability) for which different turbines (and their combinations in parallel) will be deemed most effective in power production (Figure 2.2). For example, the Kaplan and crossflow turbines are most effective at heads smaller than 20 m, whereas Francis and Pelton are designed to
Table 2.4: Glossary of main variables used in our model, their value and units.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_g$</td>
<td>0.9</td>
<td>-</td>
<td>ESHA (2004)</td>
</tr>
<tr>
<td>$\delta_{jet}$</td>
<td>1</td>
<td>m</td>
<td>Ramos et al. (2000)</td>
</tr>
<tr>
<td>$e_p$</td>
<td>10</td>
<td>$\hat{c}$/kWh</td>
<td>RECIs (2015)</td>
</tr>
<tr>
<td>$p_{ton}$</td>
<td>800</td>
<td>$$/ton</td>
<td>alibaba.com</td>
</tr>
<tr>
<td>$z_{el}$</td>
<td>900</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>-</td>
<td>Kaldellis et al. (2005)</td>
</tr>
<tr>
<td>$r$</td>
<td>5</td>
<td>%</td>
<td>Santolin et al. (2011)</td>
</tr>
<tr>
<td>$N$</td>
<td>50</td>
<td>year</td>
<td>Hosseini et al. (2005)</td>
</tr>
<tr>
<td>$k_{sum}$</td>
<td>1.08</td>
<td>m</td>
<td>Arslan et al. (2008)</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>2</td>
<td>m/s</td>
<td>ESHA (2004)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.5</td>
<td>%</td>
<td>IRENA (2012)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.3</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$q_{md,1}$</td>
<td>0</td>
<td>m³/s</td>
<td>Najmaii and Movaghar (1992)</td>
</tr>
<tr>
<td>$q_{md,2}$</td>
<td>0.1</td>
<td>m³/s</td>
<td>Santolin et al. (2011)</td>
</tr>
<tr>
<td>$q_{md,3}$</td>
<td>0</td>
<td>m³/s</td>
<td>Karlis and Papadopoulos (2000)</td>
</tr>
</tbody>
</table>

accommodate (much) larger heads. In each study, we summarize the power production and NPV of the RoR plant using a $N = 365$ day period of daily discharge values sampled from their respective FDCs (see Figure 2.7). The exceedance probability is simply divided into 365 equidistant intervals (between near zero and one) and the corresponding flow values then define a $N$-vector of ascending discharge values that is used as main input to HYPER. In each study, we will report the results for the optimized design parameters with DE assuming a single turbine (Figure 2.5) and two turbines configured in parallel (Figure 2.6). The variables used in case studies are listed in Table 2.4
2.3.1 Case Study I

The first case study uses the FDC depicted graphically in red in Figure 2.7, and adopted from Najmaii and Movaghar (1992). The discharge fluctuates considerably between 0.2 and 24 m$^3$/s, with mean flow value of about 4 m$^3$/s. In our calculations we assume that value of the generator efficiency, $\eta_g = 0.9$ and set the value of gross head $H_g = 55$ (m) and the penstock length, $L = 5,000$ (m) equivalent to Najmaii and Movaghar (1992). Under these conditions the maximum achievable annual power should equate to about 1,942 using either the Francis, Kaplan or crossflow turbines (Figure 2.2) or combinations thereof. We optimize the design parameters with DE using a population size of $N = 25$ individuals in combination with $T = 100$ generations. Figure 2.8 displays trace plots of the optimized design parameters for the NPV objective function assuming RoR plant operation with a single Kaplan turbine (top row: A,B,C) or two Kaplan turbines configured in parallel (bottom row: D,E,F,G). The last panel at the right-hand-side plots the evolution of the optimized NPV values of the parent population. The most important results are as follows. First, about 50 generations with DE appear sufficient for the algorithm to converge adequately to a single optimum solution. Repeated independent trials with DE provide the same optimum design parameter estimates, irrespective of the size of the initial population size. Moreover, our results are confirmed separately by the Nelder-Mead simplex algorithm, a derivative-free local optimizer. This inspires confidence in the ability of the DE algorithm to solve for the design parameters of a RoR plant. Second, the use of two parallel configured Kaplan turbines enhances the net profit (NPV) from 0.781 (single turbine) to 0.846 ($M), an increase of about 8%. Apparently, the use of two Kaplan turbines increases the range of workable flows and enables more effective power production when confronted with highly variable streamflows. Thirdly, the optimized value of the diameter of the penstock of around 2 meters is rather similar for both cases, whereas the design flows of the turbines differ substantially. Indeed, when using a single turbine for power production, the optimized design flow equates to about
Figure 2.7: Flow duration curves (FDCs) of the three case studies considered herein. Color coding is used to denote the different cases. The FDCs of the three studies differ substantially, both in magnitude and variability of the streamflows.

4.87 m$^3$/s. For the parallel case, the optimum design flows are equivalent to 7.15 (first, bigger turbine) and 2.17 (second, smaller turbine). Thus, the use of two Kaplan turbines allows for a larger design flow of the first turbine and hence accommodates more easily a range of inflows. Table 2.5 lists the optimum values of the design parameters for all single and parallel turbine configurations of the RoR plant. The first column lists the name(s) of the turbine(s) (single/parallel), whereas the second and third column present the optimized value of NPV and the corresponding annual power production, respectively. Columns four to five/six summarize the optimized design parameters, and the last four columns tabulate values of the specific speed of the turbine(s), mean suction head, and thickness of the penstock, calculated from the various equations provided in this paper. The use of two turbines not
Table 2.5: Case Study I: Optimized values of the design parameters, \(O_{d1}, O_{d2}\) and \(D\) derived from DE using a total of 2,500 HYPER model evaluations. These listed values maximize the energy production (second column). The first three rows tabulate the results of single turbines operating individually (hence only two design parameters), whereas the remaining rows list the results for all different combinations of two turbines configured in parallel. The values in the last four columns are derived from the optimized design parameters. Blue and red values correspond to the optimum results for the NPV and power production, respectively.

<table>
<thead>
<tr>
<th>Turbines</th>
<th>NPV</th>
<th>Power</th>
<th>(O_{d1})</th>
<th>(O_{d2})</th>
<th>(D)</th>
<th>(\omega_1)</th>
<th>(\omega_2)</th>
<th>(H_s)</th>
<th>(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Francis</td>
<td>0.737</td>
<td>1,177</td>
<td>4.12</td>
<td>-</td>
<td>1.87</td>
<td>750</td>
<td>-</td>
<td>0.14</td>
<td>1.7</td>
</tr>
<tr>
<td>Kaplan</td>
<td>0.781</td>
<td>1,267</td>
<td>4.87</td>
<td>-</td>
<td>1.94</td>
<td>600</td>
<td>-</td>
<td>0.92</td>
<td>1.7</td>
</tr>
<tr>
<td>Crossflow</td>
<td>0.778</td>
<td>1,283</td>
<td>6.74</td>
<td>-</td>
<td>2.06</td>
<td>500</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
</tr>
<tr>
<td>Francis - Francis</td>
<td>0.853</td>
<td>1,445</td>
<td>4.64</td>
<td>2.07</td>
<td>2.08</td>
<td>600</td>
<td>1000</td>
<td>0.80</td>
<td>1.9</td>
</tr>
<tr>
<td>Kaplan - Kaplan</td>
<td>0.846</td>
<td>1,534</td>
<td>7.15</td>
<td>2.17</td>
<td>2.23</td>
<td>500</td>
<td>1000</td>
<td>-0.37</td>
<td>2.0</td>
</tr>
<tr>
<td>Crossflow - Crossflow</td>
<td>0.785</td>
<td>1,373</td>
<td>6.74</td>
<td>1.84</td>
<td>2.17</td>
<td>500</td>
<td>750</td>
<td>-</td>
<td>2.0</td>
</tr>
<tr>
<td>Francis - Kaplan</td>
<td>0.849</td>
<td>1,462</td>
<td>4.51</td>
<td>2.45</td>
<td>2.10</td>
<td>600</td>
<td>1000</td>
<td>-1.25</td>
<td>1.9</td>
</tr>
<tr>
<td>Francis - Crossflow</td>
<td>0.845</td>
<td>1,439</td>
<td>4.53</td>
<td>2.96</td>
<td>2.14</td>
<td>600</td>
<td>750</td>
<td>2.05</td>
<td>1.9</td>
</tr>
<tr>
<td>Kaplan - Crossflow</td>
<td>0.848</td>
<td>1,490</td>
<td>7.13</td>
<td>1.85</td>
<td>2.21</td>
<td>500</td>
<td>1000</td>
<td>0.85</td>
<td>2.0</td>
</tr>
<tr>
<td>Kaplan - Francis</td>
<td>0.854</td>
<td>1,521</td>
<td>7.15</td>
<td>1.84</td>
<td>2.21</td>
<td>500</td>
<td>1000</td>
<td>0.83</td>
<td>2.0</td>
</tr>
<tr>
<td>Crossflow - Francis</td>
<td>0.839</td>
<td>1,468</td>
<td>4.50</td>
<td>4.29</td>
<td>2.19</td>
<td>600</td>
<td>750</td>
<td>2.21</td>
<td>2.0</td>
</tr>
<tr>
<td>Crossflow - Kaplan</td>
<td>0.831</td>
<td>1,481</td>
<td>4.50</td>
<td>4.47</td>
<td>2.21</td>
<td>600</td>
<td>600</td>
<td>1.41</td>
<td>2.0</td>
</tr>
</tbody>
</table>
only significantly enhances the net profit (NPV) but also dramatically enhances the annual power production from about 1,250 kW to approximately 1,500 kW. This amounts to an production increase of about 20%. Altogether, the Kaplan (large) - Francis (small) turbine system exhibits the highest value of the NPV for given head and flow conditions. A close contender would be the Kaplan - Kaplan system, as this configuration maximizes the annual power production. Yet, excavation is required to offset its negative (mean) suction head \( H_s = -0.37 \) m. The financial cost of excavation is not included in the calculation of the NPV, in large part because of its dependency on the properties of the terrain and subsurface (e.g. soil type, depth to bedrock). For the present FDC and installation site, the Kaplan - Francis turbine system is therefore preferred.

In the interpretation of the results in Table 2.5, it is important to be aware of the negative dependency between the rotational speed of a turbine and the size of the runner diameter. The larger the rotational speed of a turbine the smaller the required diameter of the runner of the turbine, and thus overall size of the turbine for the same design flow. For example, we refer to the first turbine of all parallel combinations considered by HYPER as the largest turbine. In practice, however the size of both turbines depends on their optimized rotational speeds. For instance, the rotational speed of the Kaplan turbine is 600 rpm (single turbine), but this value decreases to 500 rpm when this turbine is configured in parallel with a Francis turbine (1,000 rpm). Hence, the Kaplan-Francis system constitutes a combination of two turbines, the first of which has the largest diameter of the runner blades.

The annual power production of about 1,500 kW is considerably lower than the maximum achievable value of 1,942 derived previously for a 'perfect' turbine without any hydraulic losses in the conveyance system of the hydropower system. In practice, the RoR plant efficiency can therefore only reach up to about 75-80% of the theoretical maximum attainable productivity. Note that the crossflow turbine generates most power from all individual turbines, yet the gain of two crossflow turbines configured in parallel is rather minimal.
Indeed, the crossflow-crossflow configuration exhibits the lowest NPV and annual production from all possible turbine combinations.

To provide more insights into the performance of the Kaplan-Francis hydropower system, please consider Figure 2.9 that presents time series plots of the daily (A) discharge and (B) power production (solid red line). For completeness we also include with a solid green line the power production of the single Kaplan turbine system. This latter system achieves the highest NPV from all single turbine configurations. The bottom graph, also referred to as power duration curve (PDC) is simulated by HYPER using the observed FDC (red line Figure 2.7) and the optimum design parameters listed in Table 2.5. The area under the PDC equates to the annual energy production. Apparently, the single (green) and parallel (red) hydropower systems receive a similar performance when confronted with intermediate streamflow levels observed between days 55 and 310 of the synthetic daily discharge data record. During the first (0-40) and last (325-365) forty days of the PDC, however the parallel Kaplan-Francis combination clearly outperforms the Kaplan turbine. The daily power production of the parallel system is considerably larger than that of the single Kaplan turbine configuration. Particularly problematic is the finding that the Kaplan turbine cannot generate electricity during the last 50-days of the record, simply because the inflows are lower than the minimum operable flow. Thus, if a single turbine were to be used energy production would cease for about 2 months. In rural areas without available power storage this could lead to severe and prolonged power outages. We conclude that the use of two turbines guarantees power production over the entire range of flow values observed in the FDC (and plotted in the PDC). Indeed, a single turbine is unable to accommodate effectively a large range of streamflows. A parallel configuration with two or more turbines is hence preferred as such system enhances the range of workable flows and thus can sustain energy production in the presence of highly variable streamflows.
Figure 2.8: Case study I: Evolution of the values of the design parameters sampled by the DE global optimization algorithm. (A,B,C) single Kaplan turbine; (D,E,F,G) two Kaplan turbines configured in parallel.
Figure 2.9: Case study I: Power duration curve. A) Time series of daily discharge values sampled from the FDC by dividing the exceedance probabilities in 365 equidistant intervals, starting from near zero, B) daily energy produced by the Kaplan (green line) and Kaplan-Francis (red line) turbine combination.
2.3.2 Case Study II

The second case study uses the FDC depicted graphically in blue in Figure 2.7, and adopted from Santolin et al. (2011). The discharge of this watershed is quite constant compared to the first study, and fluctuates between values of 0 and 15 m$^3$/s, with mean flow value of about 12 m$^3$/s. We assume a generator efficiency, $\eta_g = 0.9$, set the value of the gross head, $H_g = 41.5$ (m) equivalent to Santolin et al. (2011) and define a penstock length of $L = 85$ m. This results in a maximum achievable annual power of about 4,546 kW. According to Figure 2.2, the available turbines are Francis, Kaplan and crossflow, and hence these turbines will be considered in our numerical simulations with HYPER. The design parameters are optimized with DE using a population size of $N = 25$ individuals in combination with $T = 100$ generations.

Figure 2.10 displays trace plots of the optimized design parameters assuming a hydropower system composed of a single Francis turbine (top row: A,B,C) and two parallel Francis turbines (bottom row: D,E,F,G). The last panel at the right-hand-side plots the evolution of the optimized NPV values of the parent population. The design parameters converge rapidly, within about 50 generations with DE to a single optimum solution the values of which match exactly with those separately derived from local optimization with the simplex algorithm (not shown). The optimized NPV value of both turbine configurations is about equal and amounts to 3.338 and 3.328 ($M) for the single and parallel hydropower systems respectively. This rather insignificant gain (actually a small loss) of the two turbine system is not surprising. The turbine inflow at this particular site is nearly constant, and hence a single turbine suffices to maximize energy production. A second turbine only marginally increases the energy production while adding additional costs to the construction and maintenance of the hydropower system. These results are further confirmed in Table 2.6 which lists, in a style similar to Table 2.5, the optimized annual power production, NPV, design parameters and derived variables for all single turbines and their parallel combinations. The results
pertain to maximization of the NPV. The results in Table 2.6 demonstrate a marginal benefit in the power production from using a second turbine (except again crossflow-crossflow). Again, this result is explained by the functional shape of the FDC. For the large majority of the year, the streamflow ranges between 10 and 15 (m$^3$/s), hence producing a rather steady inflow to the turbines. A single turbine is sufficient to handle uniform flows, and the anticipated benefits from a second turbine thus rather marginal. In other words, the incremental power revenue is not enough to warrant additional costs for capital investment and maintenance of the second turbine. We now proceed with investigation of the PDC of this RoR hydropower plant assuming (Figure 2.11B) a single Francis turbine (green line) and two Francis turbines configured in parallel (red line). The top panel (Figure 2.11A) plots the daily record of discharge values used for numerical simulation with HYPER. Both turbines produce a virtually similar amount of energy during the first 340 days of the hypothetical record of ascending discharge values. Around day 346 the discharge drops below 5 m$^3$/s and the single Francis turbine with an optimized design flow of 13.67 m$^3$/s ceases to operate. The Francis-Francis turbine system, however continues to operate and generate power for another eleven days, just eight days short of the entire year. The optimized design flow of the second turbine of 3.95 m$^3$/s enables power generation at much lower streamflows observed in the hypothetical daily discharge record at the end of the year. Altogether, the Francis turbine alone can deliver up to 87.5% of the maximum attainable power, whereas the Francis-Francis combination achieves an efficiency of 88.9%. Thus, careful optimization analysis of the FDC is required to determine an appropriate turbine system for a RoR plant. 19 days without electricity can cause particularly large problems in rural areas without access to power storage. We revisit again the importance of specific speed in the decision what turbine to select. The optimized rotational speed of the Francis turbine is 375 rpm, whereas this value changes to values to 428 and 750 rpm for two Francis turbines run concurrently in parallel. Hence, it may still be advantageous to use such system instead of one large turbine, even if the net benefit is rather marginal. From all single turbines, the Francis turbine is
Figure 2.10: Case study II: Sampled values of the design parameters as function of the number of generations of the DE global optimization algorithm. (A,B,C) single Francis turbine; (D,E,F,G) two Francis turbines configured in parallel.
Table 2.6: Case Study II: Optimized values of the design parameters, $O_{d1}$, $O_{d2}$ and $D$ derived from DE using a total of 2,500 HYPER model evaluations. These listed values maximize the energy production (second column). The first three rows tabulate the results of single turbines operating individually (hence only two design parameters), whereas the remaining rows list the results for all different combinations of two turbines configured in parallel. The values in the last four columns are derived from the optimized design parameters. Blue and red values correspond to the optimum results for the NPV and power production, respectively.

<table>
<thead>
<tr>
<th>Turbines</th>
<th>NPV</th>
<th>Power</th>
<th>$O_{d1}$</th>
<th>$O_{d2}$</th>
<th>$D$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$H_s$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Francis</td>
<td>3.338</td>
<td>3,980</td>
<td>13.67</td>
<td>-</td>
<td>2.50</td>
<td>375</td>
<td>-</td>
<td>0.22</td>
<td>2.2</td>
</tr>
<tr>
<td>Kaplan</td>
<td>3.285</td>
<td>3,940</td>
<td>14.22</td>
<td>-</td>
<td>2.50</td>
<td>333</td>
<td>-</td>
<td>1.35</td>
<td>2.2</td>
</tr>
<tr>
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<td>3,584</td>
<td>15.93</td>
<td>-</td>
<td>2.50</td>
<td>272</td>
<td>-</td>
<td>-</td>
<td>2.2</td>
</tr>
<tr>
<td>Francis - Francis</td>
<td>3.328</td>
<td>4,042</td>
<td>10.16</td>
<td>3.95</td>
<td>2.50</td>
<td>428</td>
<td>750</td>
<td>0.28</td>
<td>2.2</td>
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<tr>
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<td>9.63</td>
<td>4.57</td>
<td>2.50</td>
<td>428</td>
<td>600</td>
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<tr>
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<td>2.926</td>
<td>3,558</td>
<td>9.90</td>
<td>4.11</td>
<td>2.50</td>
<td>333</td>
<td>500</td>
<td>-</td>
<td>2.2</td>
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<tr>
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<td>4,018</td>
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<td>3.87</td>
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<td>428</td>
<td>600</td>
<td>0.84</td>
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<tr>
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<td>3,955</td>
<td>10.87</td>
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<td>428</td>
<td>500</td>
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<td>3.231</td>
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<td>14.35</td>
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<td>333</td>
<td>600</td>
<td>1.29</td>
<td>2.2</td>
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<tr>
<td>Kaplan - Francis</td>
<td>3.260</td>
<td>3,986</td>
<td>7.55</td>
<td>6.48</td>
<td>2.50</td>
<td>500</td>
<td>500</td>
<td>1.56</td>
<td>2.2</td>
</tr>
<tr>
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<td>3,790</td>
<td>7.50</td>
<td>6.81</td>
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<td>375</td>
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<td>3,755</td>
<td>7.50</td>
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<td>375</td>
<td>500</td>
<td>0.52</td>
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</tbody>
</table>
Figure 2.11: Case study II: Power duration curve. A) Time series of daily discharge values sampled from the FDC by dividing the exceedance probabilities in 365 equidistant intervals, starting from near zero, B) daily energy produced by the Francis (green line) and Francis-Francis (red line) turbine combination.

We conclude that a single turbine is sufficient for river systems that produce a nearly constant discharge during the year. The parallel operation of two or more turbines allows for a smaller diameter of the runner blade, and enable electricity production during almost the entire year. The highest NPV in the present study is obtained with a Francis turbine operating at a design flow rate, $Q_d$ of 13.67 m$^3$/s and requiring no excavation.
2.3.3 Case Study III

The third and last case study uses the FDC depicted graphically in green in Figure 2.7, and derived from Karlis and Papadopoulos (2000). The FDC of this third study appears almost linear and hence differs substantially with their more hyperbolic shapes of the first two studies. The discharge of this watershed ranges between 0.1 and 3.5 m$^3$/s, with mean flow value of about 1.7 m$^3$/s. All flow levels are represented about equally. In the present study, we assume a generator efficiency, $\eta_g = 0.9$, and fix the values of the gross head and penstock length to $H_g = 160$ and $L = 300$, respectively, based on the study of Karlis and Papadopoulos (2000). This results in a maximum achievable annual power of about 1,175 kW. The magnitude of the observed flows is much lower than in the previous two studies, yet this is offset in part by a relatively high gross head. From Figure 2.2, we conclude the available turbines to be Francis, Pelton and crossflow, and hence these turbines will be considered in our numerical simulations with HYPER. The design parameters are optimized with DE using a population size of $N = 25$ individuals in combination with $T = 100$ generations.

Figure 2.12 presents trace plots of the optimized design parameters with the DE algorithm. The top row (A,B,C) pertain to a hydropower system with a single Pelton turbine, whereas the bottom panel depicts the results of two Pelton turbines configured in parallel. The evolution of the NPV is plotted separately at the right-hand-side (Figures 2.12C,G). The design parameters converge quickly to their optimal values, within about 50 generations with the DE algorithm. The optimized values of the design parameters are in excellent agreement with those derived separately from the Simplex algorithm (not shown). Apparently, in the present study there seems to be little benefit of using two Pelton turbines concurrently in parallel. The optimized NPV value of 0.724 is almost similar to the value of 0.722 ($\text{\$}M$) derived for a single Pelton turbine. Table 2.7 tabulates the results of all individual turbines and their parallel combinations for the present FDC and site characteristics. The following variables are listed from left to right across the Table in a way similar to Tables 2.5 and 2.6:
Figure 2.12: Case study III: Evolution of the values of the design parameters sampled by the DE global optimization algorithm. (A,B,C) single Pelton turbine; (D,E,F,G) two Pelton turbines configured in parallel.
turbine name(s), annual power production, maximum value of the NPV, optimized design parameters, and derived variables. The most important conclusions are as follows. First, for single turbines, Pelton achieves the highest NPV and annual power production. This differs from the previous two case studies in which the Francis and crossflow turbines were considered most effective, and this finding is simply explained by the site characteristics (large head) and rather low discharge values. Second, all two turbine configurations outperform their single turbine systems. In fact, two turbines increase the annual average power production with about 6%, and the net financial gain with 5%. The latter equates to about 40,000 dollars per year. The highest annual production of power of 1,028 kW is achieved with the Francis-Pelton combination, whereas the Francis-crossflow system exhibits the largest NPV of 0.760 $M. This latter system has a design flow rate, $Q_{d1}$ of 1.36 m$^3$/s (Francis) and 0.55 m$^3$/s (crossflow) and a positive installation height ($H_s = 3.21$ m). Figure 2.13 introduces the PDC of the Pelton turbine (green line) and contrasts these results against the daily power production of a Francis-Crossflow hydropower system (red line). The production lines of both turbine systems are in excellent agreement with each other for most of the discharge observations, yet with the exception of the left and right tail of the FDC, at the highest (days 0-25) and lowest streamflow values (days 340-365). The single Pelton turbine system cannot extract fully the hydraulic energy from the flowing water if the streamflow is larger than about 2 m$^3$/s. The Pelton turbine is simply not equipped to handle such relatively large flow values. What is more, production even ceases if the streamflow drops below 0.2 m$^3$/s. This happens for a 26-day period at the end of the hypothetical discharge record. In practice, such long period without power production causes severe power outages in rural areas without access to other source of energy. The Pelton turbine can extract up to 82% of the maximum attainable power, whereas parallel configurations achieve efficiencies up to as much as 85%. Finally, if the size of the design is of great concern, then the combination of the crossflow and Francis turbine might be preferred, as this parallel system exhibits the highest rotational speeds of 3,000 and 1,500 rpm, respectively, leading to the smallest
Table 2.7: Case Study III: Optimized values of the design parameters, $O_{d1}$, $O_{d2}$ and $D$ derived from DE using a total of 2,500 HYPER model evaluations. These listed values maximize the energy production (first column). The first three rows tabulate the results of single turbines operating individually (hence only two design parameters), whereas the remaining rows list the results for all different combinations of two turbines configured in parallel. The values in the last four columns are derived from the optimized design parameters. Blue and red values correspond to the optimum results for the NPV and power production, respectively.

<table>
<thead>
<tr>
<th>Turbines</th>
<th>NPV</th>
<th>Power</th>
<th>$O_{d1}$</th>
<th>$O_{d2}$</th>
<th>$D$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$H_s$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[$M$]</td>
<td>[kW]</td>
<td>[m$^3$/s]</td>
<td>[m$^3$/s]</td>
<td>[m]</td>
<td>[rpm]</td>
<td>[rpm]</td>
<td>[m]</td>
<td>[cm]</td>
</tr>
<tr>
<td>Francis</td>
<td>0.624</td>
<td>804</td>
<td>1.25</td>
<td>–</td>
<td>1.11</td>
<td>1000</td>
<td>–</td>
<td>3.55</td>
<td>1.1</td>
</tr>
<tr>
<td>Pelton</td>
<td>0.722</td>
<td>970</td>
<td>1.25</td>
<td>–</td>
<td>1.18</td>
<td>500</td>
<td>–</td>
<td>–</td>
<td>1.1</td>
</tr>
<tr>
<td>Crossflow</td>
<td>0.720</td>
<td>898</td>
<td>1.69</td>
<td>–</td>
<td>1.18</td>
<td>1500</td>
<td>–</td>
<td>–</td>
<td>1.1</td>
</tr>
<tr>
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<td>1000</td>
<td>1500</td>
<td>3.54</td>
<td>1.1</td>
</tr>
<tr>
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<td>1,006</td>
<td>1.47</td>
<td>0.26</td>
<td>1.19</td>
<td>600</td>
<td>1000</td>
<td>–</td>
<td>1.1</td>
</tr>
<tr>
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<td>945</td>
<td>1.6</td>
<td>0.35</td>
<td>1.21</td>
<td>1500</td>
<td>3000</td>
<td>–</td>
<td>1.1</td>
</tr>
<tr>
<td>Francis - Pelton</td>
<td>0.752</td>
<td>1,028</td>
<td>1.40</td>
<td>0.59</td>
<td>1.20</td>
<td>1000</td>
<td>750</td>
<td>2.53</td>
<td>1.1</td>
</tr>
<tr>
<td>Francis - Crossflow</td>
<td>0.760</td>
<td>998</td>
<td>1.36</td>
<td>0.55</td>
<td>1.20</td>
<td>1000</td>
<td>3000</td>
<td>3.21</td>
<td>1.1</td>
</tr>
<tr>
<td>Pelton - Crossflow</td>
<td>0.736</td>
<td>997</td>
<td>1.45</td>
<td>0.30</td>
<td>1.20</td>
<td>600</td>
<td>3000</td>
<td>–</td>
<td>1.1</td>
</tr>
<tr>
<td>Pelton - Francis</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>600</td>
<td>1500</td>
<td>0.88</td>
<td>1.1</td>
</tr>
<tr>
<td>Crossflow - Francis</td>
<td>0.747</td>
<td>988</td>
<td>1.00</td>
<td>1.00</td>
<td>1.20</td>
<td>3000</td>
<td>1500</td>
<td>0.85</td>
<td>1.1</td>
</tr>
<tr>
<td>Crossflow - Pelton</td>
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<td>986</td>
<td>1.11</td>
<td>0.88</td>
<td>1.22</td>
<td>1500</td>
<td>750</td>
<td>–</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Figure 2.13: Case study III: Power duration curve. A) Time series of daily discharge values sampled from the FDC by dividing the exceedance probabilities in 365 equidistant intervals, starting from near zero, B) daily energy produced by the Pelton (green line) and Francis-crossflow (red line) turbine combination.

runner diameter. We conclude that the Francis-crossflow combination is most effective in the present case with both turbines operating at a high rotational speed, and a positive installation height. However, if the electricity price goes up a little bit the Francis-Pelton combination will be most profitable as this combination extracts about 87% of the maximum attainable power.
2.4 Graphical User Interface (GUI)

For those that are not too familiar with hydro-electric modeling and simulation, we have developed a graphical user interface (GUI) of HYPER. Figure 2.14 presents a screen shot of the GUI, which, as you will notice, includes the settings of the DE optimization algorithm as well. The version of HYPER with built-in optimization algorithm is coined HYPER optimizatION, abbreviated HYPERION, the name of which was also given in the early 1980s to the first portable IBM computer. The left panel (in cyan) provides a list with project variables:

- \( Z_{up} = 65 \) m
- \( Z_d = 1 \) m
- \( L = 200 \) m
- \( q_{md} = 0.1 \) m\(^3\)/s
- \( H_0 = 1.00 \) m
- \( Z_{el} = 900 \) m
- \( U = 50 \) years
- \( r = 5 \) %
- \( e_p = 10 \) 
- \( \alpha = 0.5 \)
- \( P_{ton} = 800 \) $/ton
- \( \beta = 0.025 \)
- \( P, \delta_{jet} = 1 \) m
- \( C, \delta_{jet} = 1 \) m

The right panel (in magenta) shows the objective function with options for NPV and IRR. The middle panel (in yellow) includes turbine operation settings with options for Single and Parallel.

Figure 2.14: Screen shot of the graphical user interface of HYPER. A detailed explanation appears in the main text.
variables. These values can be changed by the user depending on the properties of the local installation site, and anticipated design and economic variables of the RoR plant. After all variables have been given a value, the user can select whether to operate the RoR plant with a single turbine or two turbines configured in parallel. In the current version of the GUI, the user does not have the option to preselect a certain turbine or combination of turbines - all four turbines (and their combinations) are separately considered in the optimization analysis with DE, the settings of which are defined in the purple box in the bottom right corner. This box allows the user to specify values for the population size and number of generations in the DE algorithm, and the feasible ranges of the most important design parameters considered in the literature in optimization analysis of RoR plants. The magenta box labeled "OF" allows the user to select which objective function to maximize, and is currently limited to the power production, net profit, internal rate of return (IRR) and pay-back (PB) of the plant. These latter two objectives have not been considered in the present study but will be studied in future publications (Yildiz and Vrugt, 2015). Finally, the user has the option to specify their own efficiency curves for the Kaplan, Francis, crossflow, and Pelton turbines. These curves are loaded from individual text files supplied by the user (name of turbine with extension .txt) and consist of two columns that list the ratio of $q$ and $O_d$ and the corresponding turbine efficiency. If such curves are not available, then the user of HYPER can resort to the default curves used in the preset study. In any case, a sufficient range of inflow values is required to be able to simulate adequately the production of power under different inflow values.

The GUI is designed especially to simplify numerical simulation with HYPER. We anticipate changes to the GUI in the coming years to satisfy users, and further enhance the number of options and flexibility of use. For instance, we can extend the optimization analysis (and thus DE box) to not only include the design parameters but also decision variables that determine RoR plant operation. Moreover, we will include the option of three parallel turbines, and simulation of additional turbines beyond the four used currently. What is more, we will also include the option of multi-criteria optimization involving the use of two or more objective
functions simultaneously. This will give rise to a Pareto front, and can help decision makers to determine the best design and/or operation for given site conditions. A detailed manual of the GUI and code will also be made available in due course.

The IRR and PB objective functions are not considered in the present study, but will be considered in future work involving multiple objective optimization with AMALGAM (Vrugt and Robinson, 2007). The IRR objective function quantifies risk, whereas the PB metric defines the number of years before the construction and design costs of the RoR plant are earned back and the plant is profitable. The output of the GUI consists of a Table which lists the optimal design parameters, energy produced (kWh) and total cost ($M) of the RoR plant for a single turbine (Figure 2.15) or two turbines configured in parallel (Figure 2.16). The results for all turbines (and their combinations) are listed. The symbols used match those used for different variables in the present paper. It is evident from Figure 2.15 and Figure 2.16 that the use of two turbines in parallel enhance the power production with about 10-20% depending on the exact turbine combination. The investment costs, however also increase. This warrants a detailed analysis of their trade-offs, which again is the subject of our future work with AMALGAM which will be published in due course.
Figure 2.15: HYPER: Optimization result of single operation.

<table>
<thead>
<tr>
<th></th>
<th>NPV ($M$)</th>
<th>Power (kW)</th>
<th>D (m)</th>
<th>Od1 (m^3/s)</th>
<th>Od2 (m^3/s)</th>
<th>Hs (m)</th>
<th>w (rpm)</th>
<th>k (m)</th>
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</thead>
<tbody>
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<td>F</td>
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<td>1000</td>
<td>0.010</td>
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<tr>
<td>P</td>
<td>0.722</td>
<td>970.84</td>
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<td>1.66</td>
<td>8.54</td>
<td>500</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.720</td>
<td>898.46</td>
<td>1.17</td>
<td>1.69</td>
<td>7.61</td>
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<td>0.011</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.16: HYPER: Optimization result of parallel operation.

<table>
<thead>
<tr>
<th></th>
<th>NPV ($M$)</th>
<th>Power (kW)</th>
<th>D (m)</th>
<th>Od1 (m^3/s)</th>
<th>Od2 (m^3/s)</th>
<th>Hs,1 (m)</th>
<th>Hs,2 (m)</th>
<th>w,1 (rpm)</th>
<th>w,2 (rpm)</th>
<th>k (m)</th>
</tr>
</thead>
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<tr>
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<td>1001.67</td>
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<td>1.25</td>
<td>0.44</td>
<td>3.54</td>
<td>4.21</td>
<td>1000</td>
<td>1500</td>
<td>0.011</td>
</tr>
<tr>
<td>P-P</td>
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<td>1006.00</td>
<td>1.19</td>
<td>1.47</td>
<td>0.26</td>
<td>8.22</td>
<td>8.45</td>
<td>600</td>
<td>1000</td>
<td>0.011</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.59</td>
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<td>600</td>
<td>3000</td>
<td>0.011</td>
</tr>
<tr>
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<td>1.60</td>
<td>1.00</td>
<td>8.35</td>
<td>0.25</td>
<td>600</td>
<td>1500</td>
<td>0.011</td>
</tr>
<tr>
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<td>1.20</td>
<td>1.60</td>
<td>0.99</td>
<td>5.79</td>
<td>0.50</td>
<td>3000</td>
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<td>0.011</td>
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<tr>
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<td>0.86</td>
<td>7.32</td>
<td>8.25</td>
<td>1500</td>
<td>750</td>
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2.5 Summary and Conclusions

Small scale hydropower plants are an under-utilized but viable, clean and cost-effective alternative to large hydroelectric reservoir-fed dams. Run-of-river (RoR) plants constitute the largest majority of small plants, and use the kinetic energy of water flowing through a river stream to generate electricity. Most RoR projects, do not require a large impoundment of water, which is a key reason why such projects are often referred to as environmentally friendly, or green power. In general, RoR plants divert the river’s streamflow (up to 95% of mean annual discharge) through a pipe and/or tunnel leading to the hydropower system (turbines), and then return the water back to the river downstream.

In this paper, we have introduced the basic building blocks (equations) or a general-purpose numerical model of a RoR plant. This model is coded in MATLAB and is used to simulate the technical performance, production, cost, and profit of a RoR plant in response to a record of discharge (inflow) values, and suite of design variables (among others) the design flow, penstock diameter, penstock thickness, specific speed, rotational speed, cavitation, and suction head. HYPER differs from existing models in the hydropower literature in that (a) the specific speed is calculated at all times and used as guiding principle to determine the most appropriate turbine(s) for given FDC and site characteristics, (b) the crossflow turbine is simulated, and (c) parallel hydropower systems consisting of (among others) two different turbine types are considered. In summary, HYPER constitutes the first numerical model that takes into explicit consideration the design flow, penstock diameter, penstock thickness, specific speed, rotational speed, cavitation, and suction head in evaluating the technical performance, production, cost, and profit of a RoR plant. Moreover, the built-in optimization algorithm allows optimization of the design parameters of four different turbines (Francis, Kaplan, Pelton and crossflow) and their possible parallel combinations. The model includes a graphical user interface that the user can take advantage of to insert all project and design variables necessary to initiate numerical simulation. The GUI also includes the
option for (nonlinear) optimization of the main design parameters of the RoR plant. This includes the design flow of the turbine(s) and the penstock diameter. Their optimum values are determined using the differential evolution global optimization algorithms, and returned to the user in a single Table (see Figures 2.15 and 2.16) for all relevant turbines and their possible combinations. The turbine chart (Figure 2) is used to determine automatically the relevant turbines for the installation site (depends on net head and the design flow). The user can selected among four different objective function to be minimized (or maximized, if appropriate) including the net profit value (NPV), the total power production (POWER), internal rate of return (IRR) and pay-back time (PB) of the plant. What is more, turbine efficiency curves can be loaded separately by the GUI, if their default functions are deemed inappropriate for given usage.

Three case studies with differing site characteristics (head) and flow duration curves (FDCs) were used to illustrate the implementation and numerical results of HYPER. These results demonstrate that

1. The optimum size (design flow) and design of a RoR plant (type of turbine(s) and diameter of the penstock) balances energy production with construction and maintenance costs. This requires nonlinear optimization of the design and, possibly, operation parameters for given streamflow conditions, followed by a cost-benefit analysis of investment cost versus net profit, and power production.

2. A single turbine is unable to extract all the available power of the running water when confronted with highly variable streamflow conditions.

3. The use of two turbines in parallel (of different size and type) enhances significantly the power production, yet at the expense of an increase in the investment and maintenance costs.

4. Two turbines configured in parallel increase the range of workable flows, and hence
flexibility and effectiveness of operation of the RoR plant in the presence of time-variable inflows.

5 If the variations in discharge conditions are large, the incremental revenue of the additional power produced by two turbines out weights the costs of installation and maintenance of the second turbine.

6 For given head and flow conditions, two turbines configured in parallel allow for the use of smaller turbines and thus a larger specific speed.

7 The functional shape of the FDC determines in large part the optimum capacity and design of a RoR plant.

The size and design of a RoR plant are hence key variables that determine in large part the power production. More guidance is needed to delineate more exactly between small and large streamflows. This depends of course on the site characteristics as well.

Our future work will include multi-criteria analysis of the four (conflicting) objective functions simulated by HYPER. A multi-criteria algorithm, called AMALGAM will be coupled with the main model and used to derive the non-dominated solution set of design parameters. These Pareto solutions will provide insights into the trade-offs between the different objectives and provide decision makers with a suite of viable alternatives for the optimum design and operation of the RoR plant under given site conditions and flow values (and their variability). We will also provide an option for the user to specify the simulation time step of the model. For instance, if weekly or monthly discharge values are the only available information, then the simulated time step of HYPER will be adjusted automatically to operate at these larger time scales. Finally, we will include an additional option that allows the use to upload directly to the GUI the FDC of the river stream (daily, weekly, monthly, annual), which then serves as basis (as in present study) for numerical simulation. Alternatively, the user can instead execute the model with a record (vector) of discharge values.
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>kW</td>
<td>Electrical Power</td>
</tr>
<tr>
<td>( g )</td>
<td>m/s²</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>( \eta_g )</td>
<td>-</td>
<td>Generator efficiency</td>
</tr>
<tr>
<td>( H_{net} )</td>
<td>m</td>
<td>Net head</td>
</tr>
<tr>
<td>( H_d )</td>
<td>m</td>
<td>Design head</td>
</tr>
<tr>
<td>( f )</td>
<td>-</td>
<td>Friction factor</td>
</tr>
<tr>
<td>( D )</td>
<td>m</td>
<td>Penstock diameter</td>
</tr>
<tr>
<td>( H_s )</td>
<td>m</td>
<td>Gross head</td>
</tr>
<tr>
<td>( P_{atm} )</td>
<td>Pa</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>( H_{fr} )</td>
<td>m</td>
<td>Friction losses</td>
</tr>
<tr>
<td>( H_o )</td>
<td>m</td>
<td>Singular losses</td>
</tr>
<tr>
<td>( k )</td>
<td>m</td>
<td>Penstock thickness</td>
</tr>
<tr>
<td>( z_{el} )</td>
<td>m</td>
<td>Altitude above sea level</td>
</tr>
<tr>
<td>( q_{min} )</td>
<td>m³/s</td>
<td>Minimum turbine flow</td>
</tr>
<tr>
<td>( q_{max} )</td>
<td>m³/s</td>
<td>Maximum turbine flow</td>
</tr>
<tr>
<td>( Z_d )</td>
<td>m</td>
<td>Downstream water elevation</td>
</tr>
<tr>
<td>( f_e )</td>
<td>Hz</td>
<td>Electric system frequency</td>
</tr>
<tr>
<td>( \omega )</td>
<td>rpm</td>
<td>Turbine rotational speed</td>
</tr>
<tr>
<td>( E_l )</td>
<td>kWh</td>
<td>Annual produced energy</td>
</tr>
<tr>
<td>( C_p )</td>
<td>$</td>
<td>Penstock cost</td>
</tr>
<tr>
<td>( V_{out} )</td>
<td>m/s</td>
<td>Outlet average velocity</td>
</tr>
<tr>
<td>( N )</td>
<td>-</td>
<td>Population size</td>
</tr>
<tr>
<td>( d )</td>
<td>-</td>
<td>Dimensionalality of search space</td>
</tr>
<tr>
<td>( C_{em} )</td>
<td>$</td>
<td>Cost of electromechanical part</td>
</tr>
<tr>
<td>( a, b, c )</td>
<td>-</td>
<td>Cost function parameters</td>
</tr>
<tr>
<td>( C_{om} )</td>
<td>$</td>
<td>Annual O&amp;M cost</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-</td>
<td>O&amp;M cost coefficient</td>
</tr>
<tr>
<td>( C_{cw} )</td>
<td>$</td>
<td>Cost of civil works</td>
</tr>
<tr>
<td>( \delta_{pre} )</td>
<td>m</td>
<td>Runner position from the tailrace</td>
</tr>
<tr>
<td>( \rho )</td>
<td>kg/m³</td>
<td>Density of water</td>
</tr>
<tr>
<td>( \eta )</td>
<td>-</td>
<td>Turbine efficiency</td>
</tr>
<tr>
<td>( q )</td>
<td>m³/s</td>
<td>Processed flow rate</td>
</tr>
<tr>
<td>( O_d )</td>
<td>m³/s</td>
<td>Turbine design flow</td>
</tr>
<tr>
<td>( CR )</td>
<td>-</td>
<td>Crossover value</td>
</tr>
<tr>
<td>( L )</td>
<td>m</td>
<td>Penstock length</td>
</tr>
<tr>
<td>( V )</td>
<td>m/s</td>
<td>Mean flow velocity</td>
</tr>
<tr>
<td>( \omega_s )</td>
<td>-</td>
<td>Specific speed</td>
</tr>
<tr>
<td>( Z_{up} )</td>
<td>m</td>
<td>Upstream water elevation</td>
</tr>
<tr>
<td>( P_v )</td>
<td>Pa</td>
<td>Water vapor pressure</td>
</tr>
<tr>
<td>( d_s )</td>
<td>ton/m³</td>
<td>Density of steel</td>
</tr>
<tr>
<td>( q_{md} )</td>
<td>m³/s</td>
<td>Residual flow</td>
</tr>
<tr>
<td>( P_o )</td>
<td>Pa</td>
<td>Sea level atmospheric pressure</td>
</tr>
<tr>
<td>( r )</td>
<td>%</td>
<td>Interest rate</td>
</tr>
<tr>
<td>( R_l )</td>
<td>$</td>
<td>Annual revenue</td>
</tr>
<tr>
<td>( e_p )</td>
<td>$/kWh</td>
<td>Energy price</td>
</tr>
<tr>
<td>( N )</td>
<td>-</td>
<td>Number of generations</td>
</tr>
<tr>
<td>( \lambda_{DE} )</td>
<td>-</td>
<td>Control parameter</td>
</tr>
<tr>
<td>( C_{om} )</td>
<td>$</td>
<td>Capital recovery factor</td>
</tr>
<tr>
<td>( L_s )</td>
<td>year</td>
<td>Lifetime of the investment</td>
</tr>
<tr>
<td>( p_{ton} )</td>
<td>ton/$</td>
<td>Penstock cost per ton</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-</td>
<td>Total cost coefficient</td>
</tr>
<tr>
<td>( C_t )</td>
<td>$</td>
<td>Total investment cost</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-</td>
<td>EUR/USD exchange rate</td>
</tr>
</tbody>
</table>
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