Title
Mass, quark-number, and sqrt sNN dependence of the second and fourth flow harmonics in ultra-relativistic nucleus-nucleus collisions

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Mass, quark-number, and $\sqrt{s_{NN}}$ dependence of the second and fourth flow harmonics in ultra-relativistic nucleus-nucleus collisions

We present STAR measurements of the azimuthal anisotropy parameter $v_2$ for pions, kaons, protons, $\Lambda$, $\bar{\Lambda}$, $\Xi$, $\Xi^-$, and $\Omega$, along with $v_4$ for pions, kaons, protons, and $\Lambda$ at mid-rapidity for Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ and 200 GeV. The $v_2(p_T)$ values for all hadron species at 62.4 GeV are similar to those observed in 130 and 200 GeV collisions. For observed kinematic ranges, $v_2$ values at 62.4, 130, and 200 GeV are as little as 10%–15% larger than those in Pb+Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV. At intermediate transverse momentum ($p_T$ from 1.5–5 GeV/c), the 62.4 GeV $v_2(p_T)$ and $v_4(p_T)$ values are consistent with the quark-number scaling first observed at 200 GeV. A four-particle cumulant analysis is used to assess the non-flow contributions to pions and protons.
and some indications are found for a smaller non-flow contribution to protons than pions. Baryon $v_2$ is larger than anti-baryon $v_2$ at 62.4 and 200 GeV perhaps indicating either that the initial spatial net-baryon distribution is anisotropic, that the mechanism leading to transport of baryon number from beam- to mid-rapidity enhances $v_2$, or that anti-baryon and baryon annihilation is larger in the in-plane direction.

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I. INTRODUCTION

In non-central heavy-ion collisions, the overlapping area has a long axis and a short axis. Re-scattering amongst the system’s constituents converts the initial coordinate-space anisotropy to a momentum-space anisotropy $1, 2, 3$. The spatial anisotropy decreases as the evolution progresses so that the momentum anisotropy is most sensitive to the early phase of the evolution — before the spatial asymmetry is washed-out $3$.

Ultra-relativistic Au+Au collisions at Brookhaven National Laboratory’s Relativistic Heavy Ion Collider (RHIC) $3$ are studied in part to deduce whether quarks and gluons become deconfined during the early, high energy-density phase of these collisions. Since the azimuthal momentum-space anisotropy of particle production is sensitive to the early phase of the collision’s evolution, observables measuring this anisotropy are especially interesting. The azimuth angle ($\phi$) dependence of particle momentum distributions can be expressed in the form of a Fourier series: $dN/d\phi \propto 1 + \sum_n 2v_n \cos n (\phi - \Psi_{RP})$, where $\Psi_{RP}$ is the reaction-plane angle $4, 5$. The Fourier coefficients $v_n$ can be measured and used to characterize the azimuthal anisotropy of particle production.

Measurements at two higher RHIC energies ($\sqrt{s_{NN}} = 130$ and 200 GeV) established that charged hadron $v_2$ rises with $p_T$ for $p_T < 2$ GeV/c and then saturates $6, 7$. As predicted by the hydrodynamic calculations $10, 11$ — where local thermal equilibrium is assumed — $v_2$ at low $p_T$ ($p_T < 1$ GeV/c) shows a characteristic dependence on particle mass $12, 13$. The $v_2$ values at $\sqrt{s_{NN}} = 130$ and 200 GeV are as large as those predicted by hydrodynamic calculations. The $v_2$ values measured at $\sqrt{s_{NN}} = 17.3$ GeV $14$, the top energy of the Super Proton Synchrotron (SPS) at CERN, however, are below the hydrodynamic models predictions. In this paper, we compare $v_2$ at $\sqrt{s_{NN}} = 17.3$ and 200 GeV to new measurements at $\sqrt{s_{NN}} = 62.4$ GeV that provide a link between the top RHIC energy and the top SPS energy.

In 200 GeV collisions, kaon, proton, $\Lambda + \bar{\Lambda}$ and $\Xi + \bar{\Xi}$ $v_2(p_T)$ at intermediate $p_T$ depends on the number $n_q$ of constituent quarks in the corresponding hadron $15$. A scaling law — motivated by constituent-quark coalescence or recombination models — can account for the observed splitting between baryons and mesons for $v_2$ in this intermediate region $15, 16$. Within these models, hadron $v_2$ ($v_2^q$) is related to the $v_2$ of quarks ($v_2^q$) in a quark-gluon phase by the relationship: $v_2^q(p_T^q) \approx n_q v_2^q(n_q p_T^q)$ $17$. Intermediate $p_T$ baryon yields also increase with collision centrality more rapidly than meson yields $15, 18$: a behavior also expected from coalescence or recombination models $17$. These models suggest that the large $v_2$ values at intermediate $p_T$ are developed during a pre-hadronic phase — a conclusion supported by the recent discovery that multi-strange baryons, thought to have smaller hadronic cross-sections $19, 20$, attain $v_2$ values apparently as large as protons or hyperons $21$. Measurements of $v_2$ for identified particles may, therefore, help reveal whether $v_2$ is developed in a deconfined quark-gluon phase and can test whether these possible conclusions may still be valid at lower $\sqrt{s_{NN}}$.

This article is organized as follows: in Sec. II we briefly describe the STAR detector. The analysis procedures are presented in Sec. III. In Sec. IV we present our results. This section includes subsections discussing systematic uncertainties, baryon versus anti-baryon $v_2$, quark-number scaling, the fourth harmonic $v_4$, and the collision energy dependence of $v_2$. Our conclusions are then presented in Sec. V.

II. EXPERIMENT

Our data were collected from Au + Au collisions at $\sqrt{s_{NN}} = 62.4$ and 200 GeV with the STAR detector $22$. STAR’s time projection chamber (TPC) $23$ was used for particle tracking and identification with supplementary particle identification provided by time-of-flight detectors (TOF) $24$. We analyzed events from a centrality interval corresponding to 0%–80% of the hadronic interaction cross-section. As in previous STAR analyses $13$, we define the centrality of an event from the number of charged tracks in the TPC having pseudo-rapidity $|\eta| < 0.5$, $p_T > 0.2$ GeV/c, a distance of closest approach to the primary vertex (DCA) less than 2 cm, and more than 10 measured space points $25$. Only events with primary vertices within 30 cm of the TPC center in the beam direction were analyzed.

STAR’s main TPC covers the approximate pseudo-rapidity region $|\eta| < 1.2$ (for collisions at its center) and $2\pi$ in azimuth angle. A 0.5 Tesla magnetic field allows charged particle $p_T$ to be measured above 0.1 GeV/c. At the time of data taking the TOF detectors covered $-1 < \eta < 0$ and $\pi/15$ in azimuth angle. Their timing resolutions are $\sim 110$ ps so that pions and kaons can be distinguished for $p_T < 1.8$ GeV/c and protons can be identified up to $p_T = 3.0$ GeV/c.
We identify particles using three different methods: measurement of specific ionization-energy-loss per unit length in the TPC gas \(dE/dx\), time-of-flight measurements, and weak-decay vertex finding. \(dE/dx\) measurements for a particle with a given momentum are used for identification at low \(p_T\) and in the relativistic-rise region \((p_T > 2.0 \text{ GeV}/c)\) where \(dE/dx\) increases logarithmically with \(\beta^2\) (see Ref. \[26\] and Fig. 26 in Ref. \[27\]). The pion sample in the relativistic-rise region is selected based on the deviation between the measured \(dE/dx\) of each track and the expected \(dE/dx\) for a pion in units of Gaussian standard deviations \((n\sigma_p)\). For \(p_T > 2.0 \text{ GeV}/c\), pions are selected with \(n\sigma_p > 0\) (the top half of the distribution). In this case the purity is estimated to be 98%.

The \(v_2\) of protons is measured in this region by fitting the \(dE/dx\) distribution with peaks centered at the predicted \(dE/dx\) values. From these fits we can derive the relative fractions of pions \((f_\pi)\), kaons \((f_K)\) and protons \((f_p)\) as a function of \(dE/dx\). We then measure \(v_2\) for all tracks and plot it versus the \(dE/dx\) of the track. Once the relative fractions of each particle are known for each value of \(dE/dx\), and \(v_2\) is known as a function of \(dE/dx\), \(v_2(dE/dx)\) can be fitted with function:

\[
  v_2(dE/dx) = f_\pi v_{2,\pi} + f_K v_{2,K} + f_p v_{2,p},
\]

where the \(v_2\) values for each species \((v_{2,\pi}, v_{2,K}, \text{ and } v_{2,p})\) are parameters in the fit and \(f_\pi, f_K, \text{ and } f_p\) which are extracted from the \(dE/dx\) distribution, are part of the fit function. In the relativistic rise region, kaons do not dominate the \(dE/dx\) distribution for any value of \(dE/dx\), so their \(v_2\) values are poorly constrained and are not presented here. We estimated the systematic error on the proton \(v_2\) by varying the relative fractions of the different particles within reasonable limits. The relative change in the proton \(v_2\) \((\delta v_2/v_2)\) was less than 3%. The shape and width of the peaks are determined from samples of particles identified by other means, e.g. TOF measurements and \(K_S^0\) or \(\Lambda\) decay daughters.

The reaction-plane direction is estimated for each event from the azimuthal distribution of charged tracks. We select tracks using criteria similar to those in Ref. \[2\]. To avoid self-correlations, we subtract the contribution of a given particle from the total reaction-plane vector. For particles identified through their decays, we subtract the contributions of all the decay products. The reaction-plane resolution is estimated using the sub-event method \[28\] and we correct the observed \(v_2\) to account for the dilution caused by imperfect resolution. The resolution depends on the number of tracks used in the calculation and the magnitude of \(v_2\), and therefore depends on centrality. The resolution for \(\sqrt{s_{NN}} = 62.4 \text{ GeV}\) collisions is reduced relative to \(\sqrt{s_{NN}} = 200 \text{ GeV}\) collisions by \(\approx 30\%\). For \(62.4 \text{ GeV} \text{ Au}+\text{Au}\) collisions it reaches a maximum value of approximately 0.73 in the 10%–40% centrality interval.

In Fig.\[1\] the minimum-bias, mid-rapidity \(v_2\) values are shown for inclusive charged hadrons, pions, kaons, protons, \(\Lambda + \bar{\Lambda}, \Xi + \bar{\Xi}, \text{ and } \Omega + \bar{\Omega}\). The gross features of \(v_2\) at \(\sqrt{s_{NN}} = 62.4 \text{ GeV}\) are similar to those observed at \(\sqrt{s_{NN}} = 200 \text{ GeV}\) \[12, 13\]. For \(p_T < 1.5 \text{ GeV}/c\), a mass hierarchy is observed with \(v_2\) smaller for heavier particles. The \(p_T\) and mass dependencies are qualitatively (not necessarily quantitatively) consistent with expectations from hydrodynamic calculations that assume the mean-free-path between interactions is zero \[10\]. For \(p_T > 2 \text{ GeV}/c\), \(v_2\) reaches a maximum, the mass ordering is broken, and \(v_2\) for protons and hyperons tend to be larger than for either pions or kaons. The \(v_2\) values for protons and \(\Lambda + \bar{\Lambda}\) above \(p_T = 2 \text{ GeV}/c\) are similar. In this region, the multi-strange baryons also exhibit \(v_2\) values similar to protons. While hadrons containing strange quarks are expected to be less sensitive to the hadronic stage, we do not see a statistically significant reduction in the \(v_2\) values of strange baryons compared to protons. Statistical uncertainties, however, still do not exclude the possibility of some strangeness content dependence for \(v_2\). If \(v_2\) or its hadron species dependence is developed through hadronic interactions, \(v_2\) should depend on the cross-sections of the interacting hadrons (with hadrons with smaller cross-sections developing less anisotropy). The large \(v_2\) values for \(\Xi + \bar{\Xi}\) and \(\Omega + \bar{\Omega}\) are consistent with \(v_2\) having been developed before hadronization.

The centrality dependence of identified hadron \(v_2(p_T)\) for \(\sqrt{s_{NN}} = 62.4 \text{ GeV}\) is shown in Fig.\[2\]. Similar \(p_T\) and mass dependencies are observed for each of the centrality intervals: 0%–10%, 10%–40%, and 40%–80%. The data from the 0%–10% interval are most affected by non-flow effects \[5\] while the 10%–40% interval is least affected by these uncertainties. The particle-type dependence of non-flow will be discussed in the following section.

### A. Systematic Uncertainties

Systematic uncertainties are shown in Fig.\[4\] as bands around \(v_2 = 0\). The errors are asymmetric. The portions of the band above zero represent the negative errors so that the difference between the measurement and zero is more visually evident. These uncertainties take into account effects from weak-decay feed-down, tracking artifacts, detector artifacts, and non-flow effects. Non-flow effects are dominant. In Fig.\[2\] the tracking and non-flow systematic uncertainties are shown as bands around \(v_2 = 0\) and the weak-decay feed-down uncertainties are included in the error bars on the pion data points.

The number of tracks coming from weak-decays that are included in the \(v_2\) analysis depends on the experimental setup and track selection criteria. Pions produced in \(K_S^0\), \(\Lambda\), or \(\bar{\Lambda}\) decays tend to be distributed at low \(p_T\) with \(v_2\) values larger than the pions from other sources. We have calculated their effect on the observed pion \(v_2\).
We assume exponential $m_T$ spectra for $K_S^0$ and $\Lambda$ with inverse slope parameters of 285 and 300 MeV respectively. For relative abundances, we take $K_S^0/(\pi^+ + \pi^-)$ and $(\Lambda + \bar{\Lambda})/(\pi^+ + \pi^-)$ ratios of 0.06 and 0.054 respectively. The $v_2$ of $K_S^0$ and $\Lambda$ are taken from data. We then use a full detector simulation to estimate what fraction of the weak-decay products will fall within our detector acceptance and pass our track selection criteria. We find that for our analysis, feed-down will increase $v_2$ by approximately 13% (as a fraction of the original $v_2$) at $p_T = 0.15$ GeV/c. The increase falls to approximately 3% relative at $p_T = 0.25$ GeV/c and is negligible for $p_T > 0.4$ GeV/c. Modifications to the observed proton $v_2$ from $\Lambda$ and $\bar{\Lambda}$ decays are negligible due to the similarity of proton and hyperon $v_2$.

$v_2$ measurements can also be distorted by anti-correlations that arise from tracking errors (e.g. track-merging and hit-sharing). These anti-correlations can be eliminated by correlating tracks with $\eta > 0$ ($\eta-$subevents). This method also has a different sensitivity to the spurious correlations arising from jets and resonance decays (non-flow effects discussed in the next paragraph). In this paper, $\eta$-subevents are used to analyze pion, $K_S^0$, proton and $\Lambda + \bar{\Lambda}$ $v_2$. The remaining systematic uncertainties from detector artifacts are estimated by comparing data taken with different field settings: 0.5 Tesla (full-field) and 0.25 Tesla (half-field). The STAR experiment did not collect half-field data during the 62.4 GeV data taking period so we use the 200 GeV data to estimate the uncertainties in the 62.4 GeV measurements. From these studies, we assign an uncertainty to $v_2$ for all particles of $\pm 0.0035$ (absolute).

The dominant systematic uncertainties in $v_2$ measurements arise from correlations unrelated to the reaction plane (thought to be primarily from correlations between particles coming from jets or resonance decays or other correlations intrinsic to p+p collisions). When $v_2$ is mea-
FIG. 2: (color online). The unidentified charged hadron, charged pion, $K^0_S$, charged kaon, proton and $\Lambda + \bar{\Lambda}$ $v_2$ as a function of $p_T$ for 10%-40%, 0%-10% and 40%-80% of the Au+Au interaction cross section at $\sqrt{s_{NN}} = 62.4$ GeV. Weak-decay feed-down errors are included in the error bars on the data points while non-flow and tracking error uncertainties are plotted as bands around $v_2 = 0$, which apply to all identified particles. The errors are asymmetric and the portion of the error band above (below) zero represents the negative (positive) error.

Data from the 10%-40% centrality interval are used. For $n\sigma_{\pi} > 0$, approximately 98% of the charged tracks are pions. For $-5 < n\sigma_{\pi} < -2.5$, the sample contained approximately 75% protons, 19% kaons and 6% pions. The ratio of the event-plane $v_2$ ($v_2\{EP\}$) to the cumulant $v_2\{4\}$ for the pion sample and the proton sample are listed in Table I. In the $p_T$ region below 1 GeV/c, proton $v_2$ does not appear to manifest any non-flow correlations for either energy. For pions in this region, however, non-flow correlations seem to account for 10% of the $v_2$ measured with the event-plane analysis.

At intermediate $p_T$, $v_2\{EP\}/v_2\{4\}$ is greater than unity for protons and pions. This shows that non-flow correlations increase the observed $v_2\{EP\}$ for both protons and pions. At 62.4 GeV, the increase is the same (within errors) for both particles. With the larger 200 GeV data set however, we observe a larger non-flow fraction for pions than protons: the pion $v_2\{EP\}/v_2\{4\} = 1.22 \pm 0.02$ and $v_2\{EP\}/v_2\{4\}$ for the proton sample $= 1.16 \pm 0.02$. Pion $v_2\{EP\}$, therefore, appears to be measured using an event-plane analysis ($v_2\{EP\}$) [28], these correlations can bias the experimental estimation of the reaction plane (the event-plane angle) and change the apparent $v_2$ values (non-flow). A four-particle cumulant analysis of $v_2 \{v_2\{4\}\}$ is less sensitive to non-flow effects than a standard analysis but yields larger statistical uncertainties [8]. Although $v_2\{4\}$ has been shown to significantly reduce non-flow uncertainties, some sources of uncertainty may remain: e.g. if $v_2$ fluctuates from event-to-event $v_2\{4\}$ may yield values smaller than the mean $v_2$ [29].

For $p_T < 1$ GeV/c, a four-particle cumulant analysis is carried out for pions and protons identified with greater than 98% purity. To study the hadron-type dependence of non-flow effects at intermediate $p_T$, we analyze two samples of charged hadrons at 2.4 < $p_T$ < 3.6 GeV/c: one with $n\sigma_{\pi} > 0$, the other with $-5 < n\sigma_{\pi} < -2.5$. For $p_T < 1$ GeV/c, a four-particle cumulant analysis is carried out for pions and protons identified with greater than 98% purity. To study the hadron-type dependence of non-flow effects at intermediate $p_T$, we analyze two samples of charged hadrons at 2.4 < $p_T$ < 3.6 GeV/c: one with $n\sigma_{\pi} > 0$, the other with $-5 < n\sigma_{\pi} < -2.5$.
more susceptible to non-flow correlations than \( v_2 \) for particles in the proton sample.

TABLE I: The ratio \( v_2 \{\text{EP}\}/v_2 \{4\} \) (\( v_2 \) from a standard event-plane analysis over \( v_2 \) from a four-particle cumulant analysis) for the centrality interval 10%–40% in three \( p_T \) ranges (units for \( p_T \) are GeV/c). The sample from 2.4 < \( p_T \) < 3.6 GeV labeled as protons contains contamination from pions (6%) and kaons (19%).

<table>
<thead>
<tr>
<th>( p_T )</th>
<th>62.4 GeV</th>
<th>200 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>protons</td>
<td>pions</td>
<td>protons</td>
</tr>
<tr>
<td>0.3 – 0.5</td>
<td>1.09 ± 0.01</td>
<td>1.01 ± 0.10</td>
</tr>
<tr>
<td>0.5 – 0.7</td>
<td>1.10 ± 0.01</td>
<td>0.98 ± 0.08</td>
</tr>
<tr>
<td>2.4 – 3.6</td>
<td>1.08 ± 0.04</td>
<td>1.11 ± 0.05</td>
</tr>
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</table>

B. Baryon vs. Anti-baryon \( v_2 \)

To our knowledge, no prediction for a difference between baryon and anti-baryon \( v_2 \) exists in the literature. Previous measurements at RHIC of identified baryon \( v_2 \) reported no differences between \( \Lambda \) and \( \bar{\Lambda} \) \( v_2 \) or between proton and anti-proton \( v_2 \). Typically the particle and anti-particle samples were combined. These measurements were made with smaller data samples and at higher energies where the anti-baryon to baryon yield ratios are much closer to unity. Several scenarios can lead to a difference between anti-baryon and baryon \( v_2 \) that is larger when the anti-baryon to baryon yield ratio is smaller: (1) baryons may develop larger momentum-space anisotropies through multiple rescattering as they are transported to mid-rapidity, (2) if the initial spatial net-baryon density is anisotropic, flow developing in a later stage could convert that spatial anisotropy to an observable momentum-space anisotropy, and (3) annihilation of anti-baryons in the medium can reduce the anti-baryon yield, with the reduction larger in the more dense, in-plane direction than the out-of-plane direction.

We consider scenario (1) and (2) to be distinct. In scenario (1), extra \( v_2 \) is built up while the baryons are being transported to mid-rapidity, while in scenario (2) the \( v_2 \) is established through rescattering after the baryons are transported to mid-rapidity.

In Fig. 3 we show the ratio of \( \bar{\Lambda} \) \( v_2 \) to \( \Lambda \) \( v_2 \). The data are from minimum bias Au+Au collisions at 62.4 and 200 GeV. The bands represent the mean values of the ratios within the indicated \( p_T \) ranges. For \( \Lambda \) \( v_2 \) at 62.4 GeV the net \( \Lambda \) \( v_2 \) (the asymmetry of the quantity \( \Lambda - \bar{\Lambda} \)) is 12% ± 3% larger than the \( v_2 \) of all other \( \Lambda \)s or \( \bar{\Lambda} \). At 200 GeV it is 13% ± 4% larger. The larger \( \Lambda \) \( v_2 \) is not anticipated from the RQMD hadronic transport model which at mid-rapidity, the ratio of anti-proton \( v_2 \) to proton \( v_2 \) is 1.148 ± 0.084 and the ratio of \( \bar{\Lambda} \) \( v_2 \) to \( \Lambda \) \( v_2 \) is 1.142 ± 0.123. We note however that this model does not reproduce the overall magnitude of \( v_2 \) at this energy either.

In Fig. 4 we display the centrality dependence of the \( p_T \) integrated \( \bar{\Lambda} \) \( v_2 \) to \( \Lambda \) \( v_2 \) ratio. A Monte-Carlo Glauber model is used to convert the centrality intervals defined by multiplicity into mean impact parameter values. Given the errors we are unable to make a definitive statement about a possible dependence of the ratio on

![Figure 3](image)

![Figure 4](image)
centrality.

C. Quark-number Scaling

Models of hadron formation by coalescence or recombination of quarks successfully reproduce many features of hadron production in the intermediate $p_T$ region ($1.5 < p_T < 5$ GeV/c) \cite{15, 16, 17}. These models find that at intermediate $p_T$, $v_2$ may follow a quark-number ($n_q$) scaling with $v_2(p_T/n_q)/n_q$ for most hadrons falling approximately on one curve. In these models, this universal curve represents the momentum-space anisotropy developed by quarks prior to hadron formation. This scaling behavior was observed in Au+Au collisions at 200 GeV \cite{15}. Approximate quark number scaling of $v_2$ also exists in RQMD models where the scaling is related to the additive quark hypothesis for hadronic cross-sections \cite{31, 32}. The RQMD model, however, underpredicts the value of $v_2$ by approximately a factor of two. Pre-hadronic interactions are therefore necessary to generate a $v_2$ as large as that observed at RHIC. If $v_2$ is predominantly established in this pre-hadronic phase, the hadronic cross-sections might not play a dominant role in establishing the particle-type dependence of $v_2$.

Fig. 5 shows $v_2$ scaled by the number of valence quarks in the hadron ($n_q$) as a function of $p_T/n_q$ (a) and $(m_T - m_0)/n_q$ (b) for identified hadrons at $\sqrt{s_{NN}} = 62.4$ GeV. A polynomial function has been fit to the scaled values of $v_2$ for all particles except pions, which, for reasons discussed below, may violate the scaling. To investigate the quality of agreement between hadron species, the data from the top panel are scaled by the fitted polynomial function and plotted in the bottom panels (c) and (d) of Fig. 5. In panel (c), for $p_T/n_q > 0.6$ GeV/c, the scaled $v_2$ of $K^0_S$, $K^\pm$, $p + \bar{p}$, and $\Lambda + \bar{\Lambda}$ lie on a single curve, within errors. The 62.4 GeV data for these species are therefore consistent with the scaling observed in 200 GeV collisions. The $\Xi^+ + \Sigma$ may lie below the curve but the current errors do not permit a strong conclusion regarding deviations between $\Xi + \Sigma$ and $p + \bar{p}$ or $\Lambda + \bar{\Lambda} v_2$. At $p_T/n_q < 0.6$ GeV/c, the scaling breaks down.

It was shown that for 200 GeV at $m_T - m_0 < 0.8$ GeV/$c^2$, $v_2(m_T - m_0)$ is a linear function and independent of hadron mass \cite{33}. In Fig. 5 panels (b) and (d) we combine $m_T$ scaling and $n_q$ scaling so that a single curve can be used to approximately describe $v_2$ throughout the measured range. This is the same scaling as used in Ref. 34 where the figures are labelled $KE_T$ ($m_T - m_0$ is the transverse kinetic energy). This combined scaling works because in the range where $v_2$ is a linear function of $m_T - m_0$, dividing by $n_q$ does not alter the shape of the curve. Once it is observed that $v_2$ for all particles follow the same linear function for $m_T - m_0$, the scaling of $v_2(m_T - m_0)$ with $n_q$ becomes trivial. At higher $p_T$, $v_2$ is only weakly dependent on $p_T$ so that changing the axis variable from $p_T/n_q$ to $(m_T - m_0)/n_q$ does not effect the scaling significantly.

Pion $v_2$ deviates significantly from the fit function in both panels (a) and (b). The contribution of pions from resonance decays to the observed pion $v_2$ may account for much of the deviation for $p_T < 1.5$ GeV/c \cite{35}. For $p_T > 1.5$ GeV/c, non-flow correlations discussed previously may contribute to the deviation. From the results in Table I we conclude that non-flow effects tend to be larger for pions than protons. Particularly for the 200 GeV data, removing non-flow contributions will increase the difference between pion and proton $v_2$ and will improve the agreement between pion $v_2/n_q$ and $v_2/n_q$ for the other measured particles. It has also been suggested that constituent-quark-number scaling may be violated for pions because the pion mass is much smaller than the masses of its constituent-quarks. This implies a larger binding energy and a wider wave-function for the pion. As a result, the approximation that hadrons coalesce from constituent-quarks with nearly co-linear momenta is broken \cite{35}.

Fig. 6 shows $v_2/n_q$ versus $(m_T - m_0)/n_q$ for 0%–10%, 10%–40%, and 40%–80% most central Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ GeV. $v_2/n_q$ for each centrality interval is scaled by the mean eccentricity of the initial overlap region. The eccentricity is calculated from the mean $x$ and $y$ positions of the participating nucleons using a Monte-Carlo Glauber model. The coordinate system is shifted and rotated so that (0,0) is located at the center-of-mass of the participants and the eccentricity is the maximum possible. This is referred to as the participant eccentricity ($\varepsilon_{\text{part}}$). Since the true reaction plane is not known, our $v_2$ measurements are sensitive to $\varepsilon_{\text{part}}$ \cite{38}. For the 0%–10%, 10%–40%, and 40%–80% centrality intervals the $\langle \varepsilon_{\text{part}} \rangle$ values respectively are 0.080, 0.247, and 0.547.

The $m_T - m_0$ and $n_q$ scalings shown for minimum bias data in Fig. 5 are also valid within the specific centrality intervals shown in Fig. 6. Early hydrodynamic calculations predicted that $v_2$ should approximately scale with the initial spatial eccentricity of the collision system \cite{4}. $v_2/\langle \varepsilon_{\text{part}} \rangle$ contradicts these expectations and rises monotonically as the centrality changes from peripheral to central. This indicates that central collisions are more efficient at converting spatial anisotropy to momentum-space anisotropy.

D. Fourth Harmonic $v_4$

Higher order anisotropy parameters ($v_4$, $v_6$, etc.) may be sensitive probes of hydrodynamic behavior and the initial conditions of the collision system \cite{39}. The authors of Ref. 38 argue that values of the ratio $v_4/v_2^2$ larger than 0.5 indicate deviations from ideal fluid behavior. When measured for identified particles, higher harmonics can also test quark-number scaling \cite{39}. $v_4$ and $v_6$ for charged hadrons at 200 GeV are shown in Ref. 10. Identified particle $v_4$ at 200 GeV is shown in Ref. 13. In Fig. 7 (top panels) we plot pion, kaon, anti-proton and $\Lambda + \bar{\Lambda}$ $v_4$ for $\sqrt{s_{NN}} = 62.4$ GeV, where the standard event-
plane analysis method has been used. In the bottom panels of Fig. 7 we show the ratio \(v_2/n_q\) for charged pions, neutral kaons, and hyperons. The uncertainty in \(v_2/n_q\) from possible non-flow leads to asymmetric errors. The ratio \(v_2/n_q\) is well above 0.5 even when errors are taken into account.

In simple coalescence models \([39]\), the ratio \(v_4/v_2^2\) for hadrons is related to \(v_4/v_2^2\) for quarks:

\[
[v_4/v_2^2]_{\text{Meson}} \approx 1/4 + (1/2) \ [v_4/v_2^2]_{\text{Quark}} \tag{2}
\]

\[
[v_4/v_2^2]_{\text{Baryon}} \approx 1/3 + (1/3) \ [v_4/v_2^2]_{\text{Quark}} \tag{3}
\]

where here \(p_T\) is the quark \(p_T\). The \(v_4/v_2^2\) for mesons can also be related to \(v_4/v_2^2\) for baryons:

\[
[v_4/v_2^2]_{\text{Baryon}}^{\text{Baryon}} \approx 1/6 + (2/3) \ [v_4/v_2^2]_{\text{Meson}}^{\text{Meson}} \tag{4}
\]

Within this simple model, the large \(v_4/v_2^2\) ratios presented here indicate a large quark \(v_2\). At intermediate \(p_T\), where quark-scaling is thought to be valid, we use the equations above to fit \(v_4/v_2^2\) simultaneously for mesons and baryons, with \(v_4/v_2^2\) for quarks as a free parameter. The fit range is \(p_T > 1.2\) GeV/c for mesons and \(p_T > 1.8\) GeV/c for baryons. A good \(\chi^2\) per degree-of-freedom (4.4/13) is found with quark \(v_4/v_2^2 = 1.93 \pm 0.29\). The grey bars in the bottom panels of Fig. 7 show the corresponding \(v_4/v_2^2\) values for mesons and baryons, \(\langle v_4/v_2^2 \rangle\) values for \(p_T/n_q > 0.6\) GeV/c from data and the fit are listed in Table II. Since pion \(v_2\) is known to devi-

FIG. 5: (color online). Identified particle \(v_2\) from minimum bias collisions at \(\sqrt{s_{NN}} = 62.4\) GeV scaled by the number of valence quarks in the hadron \(n_q\) and plotted versus \(p_T/n_q\) (a) and \((m_T - m_0)/n_q\) (b). In each case a polynomial curve is fit to all particles except pions. The ratio of \(v_2/n_q\) to the fit function is shown in the bottom panels (c) and (d).

FIG. 6: (color online). \(v_2/n_q\) scaled by the mean eccentricity of the initial overlap region versus \((m_T - m_0)/n_q\) for 0%-10%, 10%-40%, and 40%-80% most central Au+Au collisions at \(\sqrt{s_{NN}} = 62.4\) GeV.
from 17.3 and/or 200 GeV scaled

using interval. The TOF

where

Table II. The systematic errors on the

our data are consistent with quark-number scaling where

fit excluding the pion data points (fit II) which yielded

The small $\chi^2$ values for both fits indicate that

are with quark-number scaling where quark $v_4/v_2^2$ is approximately 2.

TABLE II: The ratio $v_4/v_2^2$ for $p_T/n_q > 0.6$ GeV/c from a

for fit I and excluded for fit II. The $\chi^2$ per degree-of-freedom is

E. Collision Energy Dependence

In Fig. 7 (top panel) we plot pion and proton $v_2$

at $\sqrt{s_{NN}} = 62.4$ and 17.3 GeV. The 62.4 GeV data are from

and $dE/dx$ measurements combined. Middle and bottom panel: ratios of $v_2$ for $\pi^+ + \pi^-$, $K^0_S$, $p + \bar{p}$, $\Lambda + \bar{\Lambda}$ and

at different center-of-mass energies scaled by the values at 62.4 GeV. The grey and yellow bands represent systematic uncertainties in the $v_2$ ratios arising from non-flow effects. The grey bands (above unity) are the uncertainties for the 200 GeV/62.4 GeV data and the yellow bands (below unity) are for the 17.3 GeV/62.4 GeV data.

article allow us to show the 17.3 GeV to 62.4 GeV $v_2$

decrease at higher $p_T$ than the 200 GeV data extends. In order

to approximately match the centrality interval used for the 17.3 GeV data, the 17.3 to 62.4 GeV ratios are taken using respectively 0%-43.5% and 0%-40% centrality intervals. The STAR data at 62.4 and 200 GeV are measured within the pseudo-rapidity interval $|y| < 1.0$ and the 17.3 GeV data are from the rapidity interval $0 < y < 0.7$. These intervals represent similar $y/y_{beam}$ values. The same method is used to analyze the 200 and 62.4 GeV data.

Systematic errors from weak-decay feed-down and tracking errors will mostly cancel when taking the ratio of $v_2$ at 200 and 62.4 GeV. Possible non-flow errors are larger at 200 GeV than at 62.4 GeV. In the lower panels of Fig. 8, the shaded bands around unity show the uncertainty in the energy dependence of the $v_2$ ratio arising from possible changes in the magnitude of non-flow effects at different energies. The portion of the band above unity applies to the ratio of 200 and 62.4 GeV data while the portion below unity only applies to the ratio of...
the 17.3 and 62.4 GeV data.

The $v_2$ data for pions and kaons at 62.4 GeV tends to be about 5% smaller than the 200 GeV data (although at $p_T > 1$ GeV/c the difference is within systematic uncertainties). The anti-proton data at 62.4 and 200 GeV are consistent within errors. The data exclude a proton $v_2$ variation between 62.4 and 200 GeV greater than approximately 15%. The $\Lambda + \overline{\Lambda}$ data show a potentially interesting $p_T$-dependence: for $p_T < 1.5$ GeV/c the 200 GeV $\Lambda + \overline{\Lambda}$ $v_2$ is systematically smaller than the 62.4 GeV data while for $p_T > 1.5$ GeV/c the 200 GeV $\Lambda + \overline{\Lambda}$ $v_2$ data are consistent with or larger than the 62.4 GeV data. Such a dependence can arise if the system in 200 GeV collisions develops a larger expansion velocity.

Appreciable differences are seen between the 17.3 GeV and 62.4 GeV data. At $p_T > 0.5$ GeV/c, for both pions and protons, the $v_2$ values measured at 62.4 GeV are approximately 10%–25% larger than those measured at 17.3 GeV [14, 11]. Although the magnitude of $v_2$ is different at the lower energy, the systematics of the particle-type dependencies are similar. In particular, pion $v_2$ and proton $v_2$ cross over each other (or attain similar values) at $p_T$ near 1.7 GeV/c for $\sqrt{s_{NN}} = 17.3, 62.4$ and 200 GeV data. Due to the limited kinematic range covered by the 17.3 GeV data, a quark-number dependence of $v_2$ at intermediate $p_T$ can neither be confirmed nor excluded.

The increase in the magnitude of $v_2$ from 17.3 GeV to 62.4 GeV and the similarity of 62.4 GeV $v_2$ to 200 GeV $v_2$ has been taken as a possible indication for the onset of a limiting behavior [12]. In a collisional picture, a saturation of $v_2$ could indicate that for $\sqrt{s_{NN}}$ at and above 62.4 GeV the number of collisions the system constituents experience in a given time scale can be considered large and that hydrodynamic equations can therefore be applied. Hydrodynamic model calculations of $v_2$ depend on the model initialization and the poorly understood freeze-out assumptions [10, 11]. As such, rather than comparing the predicted and measured values at one energy, the most convincing way to demonstrate that a hydrodynamic limit has been reached may be to observe the onset of limiting behavior with $\sqrt{s_{NN}}$. For this reason, $v_2$ measurements at a variety of center-of-mass energies are of interest. Contrary to the large differences reported in Ref. [12], we find that when the 17.3 and 62.4 GeV $v_2(p_T)$ data are compared within similar $y/y_{beam}$ ranges and when possible non-flow systematic uncertainties are accounted for (the yellow bands in the bottom panel of Fig. 8), the differences between $v_2(p_T)$ within the data sets may be as small as 10%–15%. As such, a large fraction of the deviation between the SPS data and hydrodynamic models arises due to the wide rapidity range covered by those measurements ($v_2$ approaches zero as beam rapidity is approached [12]), increased $\langle p_T \rangle$ values at RHIC and the larger $v_2$ values predicted for the lower colliding energy by hydrodynamic models.

V. CONCLUSIONS

We presented measurements of $v_2$ for pions, kaons, protons, $\Lambda, \overline{\Lambda}, \Xi + \Xi$, and $\Omega + \Omega$ from Au+Au collisions with $\sqrt{s_{NN}} = 62.4$ GeV. We compared these measurements to similar measurements at $\sqrt{s_{NN}} = 17.3$ and 200 GeV. The 62.4 GeV pion, kaon, proton, and hyperon $v_2$ data are, within a few percent, consistent with the equivalent data at 200 GeV. Within similar $y/y_{beam}$ intervals and after we account for systematic uncertainties, we find that for a given identified particle species the difference between 17.3 and 62.4 GeV $v_2$ data may be as small as 10%–15%. We find that $v_2$ is larger than $\Lambda v_2$ at 62.4 and 200 GeV and that the difference is larger at 62.4 GeV where the anti-baryon to baryon yield ratio is smaller. At both energies our measurements are consistent with net $\Lambda$ $v_2$ being approximately 10%–15% larger than $\overline{\Lambda}$ and pair-produced $\Lambda$ $v_2$.

Our $v_2$ measurements at 62.4 GeV are consistent with the quark-number scaling of $v_2$ first observed from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The 17.3 GeV data do not extend to high enough $p_T$ to test quark-number scaling. We note, however, that the $p_T$ where the $v_2$ values for mesons and baryons cross over each other (or, in the case of 17.3 GeV data, become similar) is approximately the same at all three center-of-mass energies. This indicates that identified particle $v_2$ at 17.3 GeV may also be consistent with quark-number scaling.

We also reported measurements of the higher harmonic term, $v_4$, for pions, kaons, protons, and $\Lambda + \overline{\Lambda}$. These measurements are also consistent with quark-number-scaling laws arising from coalescence or recombination models [39]. This quark-number dependence may indicate that in ultra-relativistic heavy-ion collisions collective motion is established amongst quarks and gluons before hadrons are formed. This view is supported by the large $v_2$ values measured for multi-strange baryons at $\sqrt{s_{NN}} = 62.4$ and 200 GeV [21]. Collisions involving lighter nuclei and larger, deformed nuclei (U+U) will provide another opportunity to study mass and quark number systematics for $v_2$. The possible approach to limiting values for $v_2$ (where the $p_T$ and mass dependence at $p_T < 1$ GeV/c are consistent with hydrodynamic models) along with the evidence presented here that the relevant degrees of freedom in the early system may be sub-hadronic (e.g. constituent quarks) suggests that a strongly coupled matter with sub-hadronic degrees of freedom may be created in heavy-ion collisions at RHIC.

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[6] The reaction-plane angle is defined by the angle (relative to the laboratory frame) of the vector that connects the centers of the two colliding nuclei.
[41] We note that we are comparing $p+\bar{p}$ $v_2$($p_{T}$) for data sets with very different $\bar{p}/p$ yield ratios and in this paper we have shown that the $\bar{A}$ $v_2$ is smaller than the $\Lambda$ $v_2$.