Lawrence Berkeley National Laboratory
Recent Work

Title
TRANSFER REACTIONS AT HIGH ENERGIES ON HEAVY TARGET NUCLEI

Permalink
https://escholarship.org/uc/item/0pr8r6n9

Author
Kovar, D.G.

Publication Date
1973-03-01
TRANSFER REACTIONS AT HIGH ENERGIES ON HEAVY TARGET NUCLEI

D. G. Kovar

March 1973

Prepared for the U. S. Atomic Energy Commission under Contract W-7405-ENG-48
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
TRANSFER REACTIONS AT HIGH ENERGIES ON HEAVY TARGET NUCLEI

D. G. Kovar
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

I. INTRODUCTION

In the last few years a large number of heavy ion induced transfer reactions have been studied at bombarding energies which are well above the Coulomb barriers of the target nuclei and which might be considered high energy reactions. In the following I will discuss only a small subgroup of these transfer reactions; specifically, those transfer reactions studied on heavy nuclei (A > 90) with ions at bombarding energies which are generally not obtainable using tandems. These transfer reactions have not been studied extensively thus far primarily because of the high energies (5-10 MeV/nucleon) required, but also because of the experimental difficulties associated with the higher level densities and smaller cross sections encountered in reactions on heavier targets. The emphasis will be on the reaction dynamics inherent to heavy ion transfers at these energies and in specific to their importance in studies of the spectroscopy of states at lower excitation. The talk will be roughly divided into two parts. The first part will consist of a discussion of what characterizes transfers reactions on heavy nuclei at high energies. The bulk of the experimental data which will be presented comes from studies in the Pb region where a variety of projectiles at a number of bombarding energies have been used. The second part consists of a discussion of the status of present theoretical analysis of the reaction data particularly in regard to obtaining reliable spectroscopic information. Interpretation of the experimental results will be presented from a DWBA point of view and the discussion of the results of DWBA analysis will be confined to the data of single nucleon...
transfers where one can most easily test the reliability of the reaction
theory.

2. REACTION SYSTEMATICS

To illustrate what one might expect in a high energy heavy ion
induced reaction on a heavy target nucleus I would like to begin by showing
some Russian data reported a number of years ago.\(^1\),\(^2\) In Fig. 1 are shown
the excitation spectra of some of the light reaction products observed in
the \(^{15}\text{N} + ^{232}\text{Th}\) reaction at \(E_L = 145\text{ MeV}\).\(^1\) While the thick targets used
prohibit any discussion of the details of the excitation spectra, they do
show that the region of strong excitation depends on the specific reaction and
in particular on the number of charged particles transferred; i.e., the
larger the number of protons transferred the higher in excitation the region
of large cross section. The angular distributions of these groups were found
to be bell shaped and peaked at a forward angle which was approximately the
same for all groups.\(^2\) The cross sections measured for the different reaction
products observed in this experiment are shown in Fig. 2 plotted against, \(Q_{gg}\),
the Q value for the ground state to ground state transition. The cross sections
for the reaction particles of the same \(Z\) but different isotopic number are noted
to drop dramatically as a function of Q value. It can also be seen that the
cross sections also drop, although less dramatically, with the increase in the
number of protons transferred. What is not shown here are the results in
these studies for the proton pickup reactions whose cross sections were mea-
sured to be one to three orders of magnitude smaller than that found in the
reactions with the same number of nucleons transferred in proton stripping.\(^1\)
The measurements shown in Fig. 2 were taken at 40°, near the maximum in the
angular distributions, and as noted above correspond to cross sections strengths
at different excitation regions.
While the interpretation of these results are not without ambiguities, they do very nicely illustrate some of the basic characteristics of heavy ion induced transfer reactions which are well known and which I would like to reemphasize here. First, these reactions are apparently direct reactions whose main features, notably, the bell shaped angular distributions peaked at a forward angle and the existence of a Q-window in cross section centered about an optimum Q-value, can be understood in the framework of the semi-classical model as resulting from the heavy ion reaction dynamics.\textsuperscript{3,4,5}

Second, the systematics apparent from these results clearly indicate the limitations with which one is faced when one attempts to study the spectroscopy of states in the residual nucleus. While the heavy ion energies are sufficient to produce a multitude of multinucleon transfers, the existence of a Q-window of relatively narrow width, whose location in many cases corresponds to a high excitation in the residual nucleus, severely limits what regions of excitation can be practically studied. In addition, as one looks to the multinucleon transfers, which involve more exotic reaction products and larger Q-values, the cross sections for the reactions decrease by orders of magnitude. The very small cross sections for the proton pickup reactions, for example, argues against their extensive use in spectroscopic studies. The results from better resolution studies support these conclusions and show specifically the limitations imposed on spectroscopic studies.

3. HIGH RESOLUTION EXPERIMENTAL RESULTS

A. General Features

In studies under more favorable experimental conditions one is able to associate better energy resolution excitation spectra with these earlier cross section measurements and to see in much better detail what makes up the cross section. In Fig. 3, for example, are shown the excitation spectra obtained
from the measurement of the light reaction products $^{15,16,17}_N$ resulting from the $^{16}_O + ^{208}_Pb$ reaction studied at Berkeley at an $^{16}_O$ energy of $E_L = 140$ MeV. The cross section observed can roughly be divided into two parts corresponding to the region of clearly resolved low lying bound states and to the region of unresolved unbound "states" at higher excitation. The large cross section at higher excitation must in most cases correspond to the major part of the cross section reported in the poorer resolution studies. In the Pb studies the centroid of this cross section strength was found to lie fairly close to the $Q_{opt}$ predicted by simple semi-classical theory. From a comparison of the three spectra in Fig. 3 one can see that 1) the cross section decreases rapidly, and 2) the Q-window appears to move slightly to higher excitation as more nucleons are transferred, consistent with the results shown previously. Similar results are observed for the other light reaction products measured where in particular the cross section strength moves to higher excitation as more protons are transferred. In the spectra obtained for the two proton transfers resulting in carbon isotope reaction products the cross section maxima are located at such high excitation energy in the residual nuclei ($E_x \sim 15$ MeV) that the low lying states are all but outside the Q-window (e.g. see Fig. 4) and are very weakly excited ($d\sigma/d\Omega \sim 10^{-20}$ b/sr). These limitations imposed by the reaction dynamics make it important to determine whether different projectiles and bombarding energy have any effect in displacing or changing the region of excitation in a manner favorable to studying states of interest.

The change in the $^{208}_Pb(16,15N)^{209}Bi$ excitation spectra as a function of the bombarding energy can be seen in Fig. 5 where a comparison is made of the spectra obtained at $^{16}_O$ energies of $104$ MeV and $140$ MeV. The relative intensities of the single particle states populated are observed to change
and this will be discussed in more detail later. However, more striking is the significant increase in the cross section at higher excitation due to higher bombarding energy. A second effect of going to a higher bombarding energy can be seen in the spectra for the $^{208}\text{Pb}(^{16}\text{O},^{18}\text{O})^{206}\text{Pb}$ reaction at the two energies shown in Fig. 6. The cross section strength moves by 2-3 MeV to higher excitation while the width of the Q-window remains about the same as can be seen from the reduced strengths of the ground state and the low lying $2^+$ state in $^{206}\text{Pb}$. These results would indicate a shift in $Q_{\text{opt}}$ as a result of higher bombarding energies, but no spreading of the Q-window which might have been expected from simple semi-classical arguments$^3$ and the prediction of DWBA.$^6$

The projectile dependence can be seen from a comparison of the spectra obtained for the $^{208}\text{Pb}(^{11}\text{B},^{10}\text{Be})^{209}\text{Bi}$ reaction at $E_L = 113.5$ MeV$^8$ (Fig. 7) to the spectra for the $(^{16}\text{O},^{15}\text{N})$ reaction at a bombarding energy of $E_L = 140$ MeV (Fig. 5) which correspond to reactions at approximately the same energy above the Coulomb barrier. The most notable difference in the spectra is the very much larger cross section at higher excitation in the $^{11}\text{B}$ induced reaction. This can be understood at least qualitatively as the result of the location of $Q_{\text{opt}}$ at higher excitation ($E_X \approx 13.0$ MeV) in the $^{11}\text{B}$ reaction as compared to the $Q_{\text{opt}}$ ($E_X \sim 7.0$ MeV) in the $^{16}\text{O}$ reaction. The larger cross sections observed for the single particle states at higher excitation as well as the excited states of $^{10}\text{Be}$ as compared to $^{15}\text{N}$ appear to also be consistent with these Q-value arguments.

The two other spectra in Fig. 7 serve to illustrate two other interesting features which can be seen in other data. First, it is noted that pickup reactions, such as the $^{208}\text{Pb}(^{11}\text{B},^{12}\text{B})^{207}\text{Pb}$ reaction whose spectrum is shown in Fig. 7, as a rule show much smaller cross section strength at higher
excitation as compared to what is observed in the stripping reactions, and perhaps provides a clue to the nature of this cross section. Second, the Q-value for the single neutron stripping reaction $^{208}\text{Pb}(^{11}\text{B},^{10}\text{B})^{209}\text{Pb}$ ($Q = -7.5 \text{ MeV}$) is large compared to the semi-classical prediction of $Q_{\text{opt}} \approx 0 \text{ MeV}$ which would imply that the cross section for these transfers should be small. Yet as observed in this reaction, as well as in several other reactions, notably the $(^{16}\text{O},^{15}\text{O})$ reactions on $^{90}\text{Zr}$ and $^{208}\text{Pb}$ with $Q = -8.5 \text{ MeV}$ and $Q = -11.7 \text{ MeV}$, respectively, the cross sections are found to be respectable ($d\sigma/d\Omega \approx 1-3 \text{ mb/sr}$). These results would indicate that at sufficiently high bombarding energies the Q-window for the neutron stripping must either be significantly shifted or broadened to overcome mismatches in $Q$ of as much as $\approx 10 \text{ MeV}$.

In the discussion above the general features of the excitation spectra as a function of projectile and bombarding energy have been considered, where I have tried to focus attention on the behavior which is relevant to investigations of the structure of the states at lower excitation. However, as was observed, the most striking feature was the large cross section at higher excitations. It is not the point of this talk to go into the details of this interesting and at the present time puzzling cross section which appears to have little or no pronounced structure, is energy dependent, and is reported to have rather dramatically different angular distributions as a function of excitation energy. In the low excitation region it is observed that the individual levels are populated strongly if their $Q$-values are near $Q_{\text{opt}}$ and with an increase of bombarding energy the Q-window moves to higher excitation. An examination of the spectra provides conflicting evidence as to whether the Q-window spreads with increase in bombarding energy. If one is at a more negative $Q$ than $Q_{\text{opt}}$ then an increase in bombarding energy appears to help, however, if one is less negative than $Q_{\text{opt}}$ an increase in bombarding
energy appears to not help and may even decrease the cross sections. In this discussion I have taken a very simple minded approach in order to see whether the general features showed any simple systematics and to give a feel for the experimental data. There is ample evidence that the reaction dynamics and in particular the Q dependence is more complicated and has an L-dependence.\textsuperscript{9,11} To consider this in detail let us now look at some of the systematics which emerge from the data of single nucleon transfers.

B. Single Nucleon Transfers

Typical spectra obtained in the single proton stripping reactions induced by \textsuperscript{12}C ions at 78 MeV and \textsuperscript{16}O ions at 104 MeV on \textsuperscript{208}Pb are shown in Fig. 8. The single particle states dominate these spectra, no evidence for the excitation of the particle-core coupled states in \textsuperscript{209}Bi is observed, and in these reactions no evidence for the excitation of the outgoing projectile is observed. What is of interest here is not so much what states are excited, but the relative intensities at which the states are excited. They are different in the two reactions and it is observed that in both reactions the $J = l+1/2 \ (\equiv J_\uparrow)$ final states are more strongly excited than are the $J = l-1/2 \ (\equiv J_\downarrow)$ final states (e.g., $\sigma_{f_5/2} < \sigma_{f_7/2}$). Similar results have been found in the study of these reactions of different nuclei at the same energies\textsuperscript{9} as well as in studies with other projectiles.\textsuperscript{13} A plot of the ratios of the cross sections measured for the $J_\uparrow$ states to the $J_\downarrow$ states in several nuclei where both states were observed in both reactions is shown in Fig. 9.\textsuperscript{9} The ratios are different for the two reactions and in both cases are greater than unity. It is instructive to attempt to understand these results using DWBA.

In Fig. 10 are plotted the predicted no-recoil DWBA\textsuperscript{10} cross sections for different L-transfers as a function of reaction Q-value for the \textsuperscript{208}Pb(\textsuperscript{16}O, \textsuperscript{15}N)\textsuperscript{209}Pb reaction at 104 MeV. To separate the kinematics of the reaction from the nuclear
structure, the same form factor (a simple exponential tail) was used for all Q-values and L-transfers. It is seen that all the L-transfers have a Q-window which is centered about the same $Q_{opt}$ corresponding to the point at which the elastic partial wave amplitudes in the incoming and outgoing channel have the greatest overlap. Off $Q_{opt}$ a larger L-transfer will have a larger cross section and a wider Q-window than a smaller L for any Q-value. When the nuclear structure is put into the calculation by using the form factor appropriate for each final state (here the form factor was calculated using the binding energy of the state at its observed excitation energy and used for all other Q-values), the Q-dependence of the L-transfer remains about the same, as one would expect, but the relative magnitudes of the cross sections change dramatically as shown in Fig. 11. The magnitude of the form factor at the distance of interaction, which depends to a large extent on the number of radial nodes in the wave function, is largely responsible for the magnitude of the cross section. Experimental evidence for this Q-dependence of the cross section for different L-transfers can clearly be seen in the Argonne studies in the calcium region. By using the L-dependence of the cross section favoring large L-transfers, the dependence of the cross section on the magnitude of the form factor at the interaction distance, and the selection rules given in the no-recoil DWBA formalism, one is able, for example, to qualitatively predict the magnitude of the cross sections observed in the $(^{16}_0, ^{15}_N)$ reaction on $^{208}$Pb and in particular the preferential population of the $j>$ states. Specifically, the preferential population of the $2f_{7/2}$ ($L = 4$) single particle state in $^{209}$Bi as compared to the $2f_{5/2}$ ($L = 2$) state in the $(^{16}_0, ^{15}_N)$ reaction can be understood to be simply a result of the larger L-transfer allowed by no-recoil selection rules. Quantitatively, however, no-recoil DWBA overpredicts the differences in the cross section for the $j<$ and $j>$ states by a
factor of 3 to 4.\textsuperscript{23} This, as will be discussed later, is an indication of the failure of no-recoil DWBA. Despite this failing, the basic features of the discussion above remain valid. Interpretation of the relative magnitudes of the states populated can also be qualitatively obtained using semi-classical treatments in which the reaction dynamics as manifested in matching conditions in angular and linear momenta lead to preferred L-transfers.\textsuperscript{12,13,14}

When the bombarding energy is changed, DWBA predicts the Q-windows to change their width, becoming wider for higher bombarding energies, and to shift. From Fig. 11 it is easy to envision that such changes in the Q-dependence of the cross section could make large changes in the relative cross sections of states at fixed excitation energies. In Fig. 12 are plotted the cross sections observed in the proton stripping and neutron pickup reactions in the \textsuperscript{12}C and \textsuperscript{16}O induced reactions on \textsuperscript{208}Pb as a function of the energy above the Coulomb barrier at which the reactions were studied.\textsuperscript{6,18,15} These large changes in the cross section with bombarding energy and the difference observed with different projectiles show that heavy ion transfers could be useful for assigning J-values, but also illustrate why the reaction theory must reproduce this Q-dependence quite precisely if one hopes to extract spectroscopic information. Since similar cross section behavior should also be observed in multinucleon transfers, it is clear that attempts to extract structure information from cross sections without an adequate reaction theory is a dangerous business.

Examples of the type of angular distributions obtained are shown in Fig. 13 from a study of the \textsuperscript{12}C,\textsuperscript{11}B and \textsuperscript{12}C,\textsuperscript{13}C reactions on \textsuperscript{208}Pb at \textsuperscript{12}C bombarding energies of 77, 98 and 116 MeV at Oak Ridge.\textsuperscript{15} The angular distributions move to forward angle with increasing bombarding energies with their maximum at approximately the grazing angle. Careful examination shows, however, that while the maximum in the angular distributions for the neutron
pickup changes slowly with excitation energy, as might be expected, the maximum in the proton stripping angular distributions remains fixed as a function of excitation. This is shown in Fig. 14. The lines in the figure represent phenomenological fits to the data in a semi-classical model which will not be discussed here. As will be discussed in the next section this Q-dependence of the angular distributions cannot be reproduced by DWBA\textsuperscript{23} and is not understood at the present time. It is also appropriate to comment here that while it is generally true that the angular distributions look very much the same, there are small differences which depend not only on the excitation energy, but also on the L-transfer. This will be pointed out for some of the angular distributions which will be shown together with DWBA fits in the next section.

4. DWBA ANALYSIS OF SINGLE NUCLEON TRANSFERS

Only a few analyses of transfer reactions using DWBA have been reported for the studies considered here, primarily because of the large number of partial waves needed in the calculations. Without going into the details of the formalism which have been discussed extensively in the literature,\textsuperscript{10,19,20,25} I will present here the results of analyses of single nucleon transfer reactions which show how well one can hope to extract quantitative spectroscopic factors from heavy ion induced reactions at the present time.

A. No-Recoil DWBA

The results of the single proton stripping reactions induced by \textsuperscript{12}C and \textsuperscript{16}O on \textsuperscript{208}Pb from studies at Berkeley\textsuperscript{9,23} have been analyzed using no-recoil DWBA.\textsuperscript{10} The calculations were done using finite range form factors\textsuperscript{16} in the distorted wave code DWUCK. Optical model parameters were obtained by fitting the elastic scattering in the incident channel and the bound state parameters were taken from the literature (see Table I). The fits to the transitions to the six single particle states in \textsuperscript{209}Bi in the \((\textsuperscript{16}O,\textsuperscript{15}N)\) reactions at 10\textsuperscript{4} MeV...
and 140 MeV and the \( ^{12}\text{C},^{11}\text{B} \) reaction at 78 MeV are shown in Figs. 15 and 16. DWBA predicts angular distributions which shift to larger angles for states at higher excitation, contrary to experiment. This shift is largest for the \(^{12}\text{C} \) reaction at 78 MeV, noticeable in the \(^{16}\text{O} \) reaction at 104 MeV, and not detectable in the \(^{16}\text{O} \) reaction at 140 MeV. Attempts to remove this shift were made by varying the optical model parameters and the bound state parameters without success. This predicted shift is observed to be even larger in two proton transfers\(^7\) and also appears to be present in inelastic scattering\(^{17}\) where the interference dip in the angular distributions appears to lie in many cases at angles forward of the predicted dip for states at higher excitation. This would indicate that the problem perhaps lies in the choice of optical parameters, however, in this study these parameters could not be found. The shapes of the angular distributions in the \( ^{12}\text{C},^{11}\text{B} \) reaction were found to have an L-dependence, where the larger L-transfers drop more rapidly at larger angles. DWBA reproduced this behavior for all but the large L-transfer (L = 7) to the \(^{11}\text{Li}_{13/2} \) state.

To extract spectroscopic factors the integrated cross sections were fit (shown by the dashed curves). The relative spectroscopic factors obtained are listed in Table II together with the L-transfers allowed in no-recoil formalism. It is observed that the spectroscopic factors in the \( ^{16}\text{O},^{15}\text{N} \) reaction for the \( j_\text{<} \) states uniformly exceed those deduced for the \( j_\text{>} \) states (by about a factor of \( \sim 4 \) at 104 MeV and \( \sim 8 \) at 140 MeV). In contrast, the spectroscopic factor for the \( j_\text{<} \) states obtained in the \( ^{12}\text{C},^{11}\text{B} \) reaction are found to be about half those deduced for the \( j_\text{>} \) states. The results of no-recoil DWBA analysis are: 1) it is not possible to obtain consistent spectroscopic factors simultaneously for both \( j_\text{<} \) and \( j_\text{>} \) states in either reaction. 2) The spectroscopic factors obtained in the \(^{12}\text{C} \) and \(^{16}\text{O} \) induced reactions
show opposite deviations. 3) The discrepancies in the spectroscopic factors for the \( J_\text{<} \) and \( J_\text{>} \) states found in the \(^{16}\text{O} \) reactions increase with increasing bombarding energies. Similar results shown in Fig. 17 are found in analysis of data over a large target mass region and at different bombarding energies. 27

B. DWBA Including Recoil

One of the more questionable approximations made in the DWBA theory used is the no-recoil approximation. 18,19 As has been pointed out in the literature, the neglected recoil terms have the effect of introducing additional angular momentum which may significantly change the predicted cross section. 12,20 Calculations were performed in a DWBA formalism which included in first order the effects of recoil terms. 20 The cross section expression in this treatment can be written as;

\[
\frac{\, d\sigma}{\, d\Omega} = \sum_{L} |T_L + K T'_L|^2 = \sum_{L} |T_L + K \left( \sum_{L_R} T_{L_R} \right)|^2 , \tag{1}
\]

where \( T_L \) is the normal no-recoil transition amplitude, \( T'_L \) is the recoil correction term which in a general perturbative treatment would include several no-recoil transition amplitudes calculated with several different \( L \)-transfers. In the first order treatment the number of transition amplitudes used to evaluate the recoil term is greatly reduced and only a single transition amplitude (for \( L_R = l_2 \) ) is needed. The coefficient \( K \) contains a mass factor \( (m_p/M_c_1) \), the wave number, \( k_i \), at the distance of interaction, and a nuclear overlap integral. 20 Under the approximation of a weakly bound final state (made in this calculation) the coefficient can be written as;

\[
K = (2l_1 + 1) \left( \frac{m_p}{M_c_1} \right) \left( \frac{k_i}{\kappa_2} \right), \tag{2}
\]

where \( \kappa_2 \) is the bound state decay constant for the final state. DWBA calcu-
tions were made using this formalism and the predicted angular distributions fit to the experimental data. The shapes of the angular distributions were essentially the same as those calculated in no-recoil DWBA. The spectroscopic factors obtained from fitting the integrated cross sections are listed in Table II together with the L-transfers allowed and the $L_R$ used to evaluate the recoil correction terms. The effect of the recoil corrections is to significantly reduce the differences in the spectroscopic factors for the $j_-$ and $j_>$ states in both the $^{12}$C and $^{16}$O induced reactions. While there are still problems, such as the predicted energy dependence of the $^{16}$O,$^{15}$N transfers to the $j_>$ states, the agreement with the light ion results is much improved. Better agreement is probably hindered by the approximations made in the present treatment. Most serious is the approximation for the overlap integral which is only valid for weakly bound states. These results do indicate that the effects of recoil can be large and must be taken into account if one hopes to obtain reliable spectroscopic information from heavy ion transfer reactions. The treatment above implies recoil effects will be more important at high bombarding energies and for weakly bound states. Similar results have been reported from studies on lighter mass targets.

C. Exact Finite Range DWBA

The best way to treat the recoil problem is to do the full finite range treatment in which the six dimensional transition amplitude integral is evaluated explicitly. DeVries has written such a code which has been used to calculate heavy transfer cross sections for lighter mass targets where the number of partial waves and the integration radius are not too large. He has extended this calculation to include enough partial waves to calculate the transfer cross sections for the $^{16}$O,$^{15}$N reaction on $^{208}$Pb at 104 MeV. The results of his preliminary calculation for the $3p_{3/2}$ and $3p_{1/2}$ states in $^{209}$Bi
are in good agreement with the results of light ion studies. These results are particularly encouraging since they promise the opportunity of obtaining reliable quantitation spectroscopic information from transfer reactions at high energies on heavy targets.

5. SUMMARY

The kinematics in heavy ion induced reactions are known to be important and to a large extent determine what regions of excitation can be practically studied. In many cases, particularly for multinucleon transfers, this corresponds to high excitation in the residual nucleus. With use of different projectiles and different bombarding energies the regions of strong excitation can be shifted and modified so as to better study the states of interest and use the selectivity of the reactions for spectroscopy. In particular, studies at high bombarding energies indicate reactions with highly unfavored Q-values can have sizeable cross section. Results of DWBA analyses indicate 1) that recoil effects are important in high energy transfer reactions on heavy targets and much be included to obtain reliable spectroscopic information, and 2) there exist a Q-dependence of the measured angular distributions which cannot at present be reproduced. While the major part of the discussion has largely been to point out the limitations of heavy ion induced transfer reactions and the present shortcoming of DWBA treatments, it was the main point of the discussion that we are rapidly reaching the time where our understanding of and the techniques for the treatment of heavy ion transfer reactions will allow for the extraction of spectroscopic information about nuclei and their structure which have up till now been unattainable.
REFERENCES

Work performed under the auspices of the U. S. Atomic Energy Commission.


6. D. G. Kovar, et al., to be published.

7. F. D. Becchetti, et al., to be published.


13. F. Pougheon and P. Roussel, to be published.


17. F. D. Becchetti; Symposium on Heavy-Ion Transfer Reaction, ANL Report (1973).

18. K. R. Greider; Nuclear Reactions Induced by Heavy Ions, ed. by R. Bock and W. Hering (North-Holland, Amsterdam, 1970).


22. M. S. Zisman, et al., to be published.


Table I.

Optical-Model and Bound State Parameters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}_O^*$</td>
<td>104</td>
<td>40</td>
<td>15</td>
<td>1.31</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$^{16}_O^*$</td>
<td>140</td>
<td>30</td>
<td>15</td>
<td>1.31</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$^{12}_C^+$</td>
<td>78</td>
<td>40</td>
<td>15</td>
<td>1.31</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>B.W. $^{16}_O$</td>
<td></td>
<td></td>
<td></td>
<td>1.20</td>
<td>0.65</td>
<td>0</td>
</tr>
<tr>
<td>B.W. $^{12}_C$</td>
<td></td>
<td></td>
<td></td>
<td>1.20</td>
<td>0.65</td>
<td>0</td>
</tr>
<tr>
<td>B.W. $^{208}_Pb$</td>
<td></td>
<td></td>
<td></td>
<td>1.28</td>
<td>0.76</td>
<td>$\sim 18$</td>
</tr>
</tbody>
</table>

*Same parameters used for $^{15}_N$.
† Same parameters used for $^{11}_B$. 
Table II.

<table>
<thead>
<tr>
<th>Ex (MeV)</th>
<th>nlj</th>
<th>L 104 MeV</th>
<th>140 MeV</th>
<th>L 78 MeV</th>
<th>L 104 MeV</th>
<th>140 MeV</th>
<th>L 78 MeV</th>
<th>Ref. 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1h9/2</td>
<td>4</td>
<td>3.04</td>
<td>3.84</td>
<td>4.6</td>
<td>0.63</td>
<td>4,(5)d</td>
<td>1.32</td>
</tr>
<tr>
<td>0.90</td>
<td>2f7/2</td>
<td>4</td>
<td>0.30</td>
<td>0.48</td>
<td>2.4</td>
<td>0.99</td>
<td>(3),4c</td>
<td>1.00c</td>
</tr>
<tr>
<td>1.61</td>
<td>1i13/2</td>
<td>7</td>
<td>0.66</td>
<td>0.40</td>
<td>5.7</td>
<td>0.86</td>
<td>(6),7</td>
<td>0.80</td>
</tr>
<tr>
<td>2.84</td>
<td>2f5/2</td>
<td>2</td>
<td>3.20</td>
<td>3.20</td>
<td>2.4</td>
<td>0.56</td>
<td>2,(3)</td>
<td>1.12</td>
</tr>
<tr>
<td>3.12</td>
<td>3p3/2</td>
<td>2</td>
<td>0.92</td>
<td>0.48</td>
<td>0.2</td>
<td>1.56</td>
<td>(1),2</td>
<td>1.28</td>
</tr>
<tr>
<td>3.64</td>
<td>3p1/2</td>
<td>0</td>
<td>2.80</td>
<td>4.80</td>
<td>2.4</td>
<td>0.47</td>
<td>0,(1)</td>
<td>0.82</td>
</tr>
</tbody>
</table>

(a, b) Normalization factor \( N = 1.25(3.20) \) used in obtaining \((^{16}_0, ^{15}_N)\) and \((^{12}_C, ^{11}_B)\) spectroscopic factors, respectively.

c Normalization factors obtained by normalizing these spectroscopic factors to unity.

d The L values in parentheses are the \( L_R \) used in recoil corrections. The others are those allowed in no-recoil DWBA theory.
FIGURE CAPTIONS

Fig. 1. Energy spectra of the C, B, and Be ions produced in the multi-proton stripping reactions \((^{15}\text{N} - x \text{ protons}), (^{15}\text{N} - x \text{ proton} + 1 \text{ neutron})\), and \((^{15}\text{N} - x \text{ proton} + 2 \text{ neutrons})\) reactions occurring in the bombardment of \(^{232}\text{Th}\) by 145 MeV \(^{15}\text{N}\).

Fig. 2. The differential cross sections, \(\frac{d\sigma}{d\Omega}_{40^\circ}\), for the production of carbon, boron, beryllium, lithium, and helium isotopes in the \(^{232}\text{Th} + ^{15}\text{N}\) collision as function of Qg.g., the Q-value for formation of the reaction products in their ground states.

Fig. 3. Excitation spectra obtained for \(^{209}\text{Bi},^{208}\text{Bi},^{207}\text{Bi}\) from the measurement of the reaction products \(^{15,16,17}\text{N}\) produced in the \(^{16}\text{O} + ^{208}\text{Pb}\) reaction at \(E_L = 140\) MeV at \(\theta_L = 30\) deg.

Fig. 4. Comparison of the excitation spectra for \(^{208}\text{Pb}(^{16}\text{O},^{15}\text{N})^{209}\text{Bi}\) at bombarding energies of 104 MeV and 140 MeV. Spectra are at angles corresponding to the maximum in the angular distributions.

Fig. 5. Excitation spectra obtained for the \(^{208}\text{Pb}(^{16}\text{O},^{18}\text{O})^{206}\text{Pb}\) reaction at bombarding energies of 104 MeV and 140 MeV.

Fig. 6. Excitation spectrum observed in the \(^{208}\text{Pb}(^{16}\text{O},^{14}\text{C})^{210}\text{Po}\) reaction at \(E_L = 140\) MeV.

Fig. 7. Excitation spectra for single proton and single neutron stripping and single neutron pickup in the \(^{11}\text{B} + ^{208}\text{Pb}\) reaction at 113.5 MeV. (Ref. 8.)

Fig. 8. Excitation spectra obtained in the single proton stripping reactions \((^{12}\text{C},^{11}\text{B})\) at 78 MeV and \((^{16}\text{O},^{15}\text{N})\) at 104 MeV on \(^{208}\text{Pb}\). The single particle states in \(^{209}\text{Bi}\) are labeled according to their shell model orbitals.
Fig. 9. Comparison of the cross-section ratios of the $j = l + 1/2$ states to the $j = l - 1/2$ states observed in the ($^{16}$O, $^{15}$N) and ($^{12}$C, $^{11}$B) reactions on nuclei where both states are populated.

Fig. 10. DWBA calculations of the $Q$-dependence of the cross section for different $L$-transfers to demonstrate the kinematics of the ($^{16}$O, $^{15}$N) reaction on $^{208}$Pb prefer large $L$-transfers. A fixed exponential form factor is used in all calculations to remove any structure effects.

Fig. 11. DWBA calculations of the $Q$-dependence of the cross section for different $L$-transfers using the form factors appropriate for the final states to demonstrate the importance of the nuclear form factor in determining the relative cross sections. The form factor was calculated with the binding energy of the state at its observed excitation and used in the calculation at all other $Q$ values.

Fig. 12. Plot of the energy dependence of the cross sections (at the maximum in the angular distribution) for the $^{12}$C (Ref. 15) and $^{16}$O (Ref. 23) induced single nucleon transfers.

Fig. 13. Angular distributions measured for the ($^{12}$C, $^{13}$C) and ($^{12}$C, $^{11}$B) reactions on $^{208}$Pb at bombarding energies of 77, 98, and 116 MeV (Ref. 15).

Fig. 14. Center-of-mass angles at which $d\sigma/d\theta$ is maximum plotted as a function of excitation energy in the residual nuclei, $^{207}$Pb and $^{209}$Bi. Lines are the grazing angles calculated by assuming $r_0$ values as shown. Solid lines consider the initial system only, which dashed lines refer to trajectories of the system after the reaction (Ref. 15).

Fig. 15. The angular distributions, measured in the ($^{16}$O, $^{15}$N) reaction on $^{208}$Pb at bombarding energies of 104 MeV and 140 MeV. The solid curves are the DWBA fits to the data. The dashed curves are the same DWBA predictions shifted so as to fit the integrated cross section.
Fig. 16. The angular distributions, measured in the $^{12}\text{C},^{11}\text{B}$ reactions on $^{208}\text{Pb}$ at a bombarding energy of 78 MeV. The solid curves are DWBA fits to the data. The dashed curves are the same DWBA predictions but shifted so as to fit the integrated cross section.

Fig. 17. Comparison of the spectroscopic factors extracted from $^{3}\text{He},d$ and $^{16}\text{O},^{15}\text{N}$ reaction on a large variety of targets. The $^{16}\text{O},^{15}\text{N}$ reaction analysis was done using no-recoil DWBA (Ref. 27).
Fig. 1
Fig. 2

$\frac{d\sigma}{d\Omega} (\text{mb/sr})$

$Q_{gg}$ (MeV)

$^{15}N + ^{232}Th$

$E_{Lab} = 145$ MeV

XBL 732-216
Fig. 3
\[ ^{208}\text{Pb} \ (^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi} \]

- \( E_L = 104 \text{ MeV} \)
- \( \theta_L = 62.5^\circ \)

- \( E_L = 140 \text{ MeV} \)
- \( \theta_L = 45^\circ \)
Fig. 5

\[ 208\text{Pb} \left( ^{16}\text{O}, ^{18}\text{O} \right) 206\text{Pb} \]

\[ E_L = 104 \text{ MeV} \]
\[ \theta_L = 62.5^\circ \]

\[ E_L = 140 \text{ MeV} \]
\[ \theta_L = 45^\circ \]
\( {^{208}_{\text{Pb}}(^{16}_{\text{O}},^{14}_{\text{C}})^{210}_{\text{Po}}} \)

\[ E_L = 140 \text{ MeV} \]

\[ \theta_L = 35^\circ \]

Fig. 6
Fig. 7
$^{208}\text{Pb}(^{12}\text{C}, ^{11}\text{B})^{209}\text{Bi}$

$E_{c,x} = 78$ MeV

$\theta_L = 60^\circ$

$^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi}$

$E_{0,16} = 104$ MeV

$\theta_L = 65^\circ$
Fig. 9
\[ ^{208}\text{Pb}(^{16}\text{O},^{15}\text{N})^{209}\text{Bi} \]

\[ E_L = 104 \text{ MeV} \]

**FIXED FORM FACTOR**

\[ \phi \propto e^{-Kr} \]

\[ K = 1.0 \text{ fm}^{-1} \]

\[ O_{\text{PNBA}} \] (arbitrary units)

\[ Q \text{ VALUE (MeV)} \]

**Fig. 10**
$^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi}$  $E_L = 104$ MeV

**Fig. 11**

Fixed form factor using F.F. calculated with separation energy prescription:

- $r_0 = 1.28$
- $a_0 = 0.76$
- $V_{60} = 6$ MeV

Single particle levels

$Q$-value (MeV) vs. $Q_{DWBA}$ (Fermi$^2$)

$L=0$, $L=2$, $L=4$, $L=6$, $L=7$
Fig. 12
Fig. 14
Fig. 15
Fig. 16
\[ g = 1.0 \text{ FOR CLOSED SHELL} \]

\[ j = \ell + \frac{1}{2}, j = \ell - \frac{1}{2} \]

**Fig. 17**
LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.