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The Inequality Factor: Skewness and Kurtosis as a Measure of Set-class Cohesion

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Online Supplement

Part 1: Mathematical Options

1.1. Calculating the individual deviations

The algorithms for calculating the individual deviations, central as they are to many similarity measures, have been well-discussed in the literature.[1] Although these different algorithms have a fairly minimal effect on the inequality factor, overall, the nuances may be of interest to some readers. The first two approaches—Morris' (which we term the "city-block" metric), and Isaacson's (which we term the "vector-length" metric)—both begin by calculating a six-dimensional "difference vector," representing the absolute value of the differences between the two vectors for each dimension. The city-block metric simply sums the values in each place of the difference vector. The vector-length metric determines the length of the six-dimensional difference vector, by calculating the square root of the sum of the squares of each place in the difference vector.

Alternatively, we could use a radial-angle metric. While for many areas of scientific study this approach produces the most accurate results, for the music-theoretic purposes of this paper the results may be counterintuitive or even misleading. The radial-angle metric does not involve a difference vector, instead calculating the angle between the two original vectors (the average interval-class vector and each individual interval-class vector). The formula [arc cosine (the-dot-product-of-the-two-vectors / the-product-of-the-length-of-them)] yields the number of radians between the two vectors, ranging from 0 to 1.57.

1.2. Deviations from the deviational norm

The individual deviations represent a distribution of values away from the average interval-class vector "baseline" (see Figure 1 in the article). This distribution may be described with an average deviation (as above) or instead with a ***standard deviation*** (SD). The average deviation is the sum of the individual deviations—that is, the total deviation—divided by the cardinality; the standard deviation is the square root of the average of the squares of the individual deviations. In general, the differences between the average deviation and the standard deviation are not significant for the present purposes.[2]

Having calculated the average deviation or the standard deviation, we next determine the individual deviations' deviation from that norm. Those second-order deviational norms can be calculated in the same two ways: as an average or a standard deviation from the first-order deviational norm. While it would be possible to calculate the average deviation from the standard deviation or the standard deviation from the average deviation, in the discussion below we will restrict ourselves to the ***average deviation from the average deviation*** (the ADAD, as described in the paper) or the ***standard deviation from the standard deviation*** (SDSD).

1.3. Calculating the inequality factor

The straightforward method for calculating the inequality factor, as described in the article text, is to multiply the first-order deviational norm by the second-order deviational norm (average deviation*average deviation from the average deviation). Because the calculations for the average deviation and average deviation from the average deviation differ somewhat from those for the standard deviation and standard deviation from the standard deviation, we differentiate between average deviation*average deviation from the average deviation (termed the "inequality factor—average," or IFA) and standard deviation*standard deviation from the standard deviation (termed the "inequality factor—standard," or IFS). In addition to the fairly simple inequality factor—average or inequality factor—standard approaches, a third alternative is to consider the first- and second-order deviation values as a pair in a two-dimensional vector. The angle between the [1st-Order, 2nd-Order] vector and a [1st-Order, 0] vector can be determined using the radial angle formula given above; this number is then multiplied by the magnitude of the [1st-Order, 2nd-Order] vector as measured by the square of the vector's length[3], producing a slightly different inequality factor (termed the "inequality factor—radial," or IFR).[4]

1.4. Discussion

Given the number of choices at each step—three methods for determining individual deviations, two each for the first- and second-order deviational norms, and two for determining the final inequality factor, it is clear that there is an overabundance of complete calculation combinations—twenty-four, to be exact.

Even restricting ourselves to 1) the two difference vector techniques used for calculating the individual deviations (see 1.1, above); and 2) the same type of deviation norm measurement for both the first- and second-order deviational norms (see 1.2, above), the total number of possible combinations of calculations still stands at eight. For simplicity's sake, the paper itself employs only the inequality factor—average (referred to as the inequality factor); Part 3 of this online supplement, however, lists city-block metric/inequality factor–average (the CB-IFA), vector-length metric/inequality factor—standard (the VL-IFS), and the vector-length metric/inequality factor—radial (the VL-IFR). It should be clear that whichever method of calculation is used, the absolute numbers are only relevant when compared to other values computed with the same metric.

Part 2: Network Topologies and the Inequality Factor

2.1. Segmentation and the inequality factor: contextual vs. abstract contexts

Beyond the insights into set classes discussed in the article, we have found that the results of the inequality factor measure model our musical intuitions in substantive ways. Most importantly, the inequality factor measure complements the findings of the authors' segmentational theory [1], but in an abstract context. Following the work of other theorists [33, 34], the segmentational theory hypothesizes that the most salient segments demonstrate a balance between cohesion (internal to the segmental group) and discontinuity (along segmental boundaries). Similarly, the inequality factor is a measure of regularity (consistency) and discontinuity (skewness and kurtosis)—in abstract set classes, as opposed to within the context of a musical passage. Both the segmentation theory and the inequality factor measure implicitly posit a complex network of relationships between sets or groups of notes that are generally considered only intervallically related.

The two basic principles influencing the formulation of our general theory of segmentation [1] can be generalised as follows, keeping in mind that our theory privileges the listener:

> Similarity among stimuli will tend to result in the perception of those stimuli as a cohesive whole, distinct from other groups of stimuli.
>
> Dissimilarity among stimuli will tend to result in the perception of those stimuli as distinct from one another.

External dissimilarity (dissimilarity between members and non-members of a segment) results in the beginning- and ending-tones of a segment—the "edge events"—being highly distinct, while the non-edge events are less so. Both the segmentation theory and the inequality factor values suggest, however, that it is *not* the case that a *preponderance* of edge events will distinguish a group of notes from another that is less "edgy": on the contrary, we found that the music that had the most coherent segmentational groups had a *balance* of edge events and non-edge events.

2.2. Network topologies as a model for set class coherence and regularity

In this sense, an abstract set of pitch classes that meets the criteria of having a low degree of skewness may be thought of as exhibiting a continuity of intervallic value bounded by discontinuity of value vis-à-vis non-member tones. Of course, the actual realization of a set in a musical passage will affect the prominence of different relationships between the pitches (as indicated in Section 3 of the article, and discussed more in Section 4). Equally obviously, the relationships between pitches in musical space are affected by the abstract relationships between a set's pitch classes. As such, member tones that generate the lowest individual deviations would be more neutral in character than other tones, those generating moderate individual deviations might emerge as being the most "central," while those generating the highest individual deviations would likely be considered outliers.[5]

As these member tones work together within a set, a musical "network topology"—a structure with logical connections between the nodes (pitch classes)—is created. In a given topology, the tones exhibiting low (but not the lowest) individual deviation values tend to advance and those at the extremes of individual deviation values tend to recede, or possibly create edge events. The interconnections between these notes are the similarity patterns crucial to the cohesiveness of all the subsets of a set class. When inequalities do not exist—because either all of the tones are equivalent to one another, or the inequalities are so vast that the networks of intervallic similarity are too diffuse to knit the different levels into one unified topology—the sets in a set class will exhibit less cohesiveness.

Given the relationship between our segmentation research and the abstract properties of set classes, it seems logical to conclude that a set class which is most completely cohesive, both in terms of distinguishing member tones from non-member tones and distinguishing member tones from one another, would feature a *low non-zero inequality factor value*. Such a value would ensure the maximal degree of consistency while still allowing sufficient kurtosis to produce the type of network topologies discussed above.

2.3. Visual analogy

To clarify the relationship between skewness, kurtosis, and the resulting topologies, we will employ a visual analogy.[6] Consider the three six-dot figures presented in Figure 3.

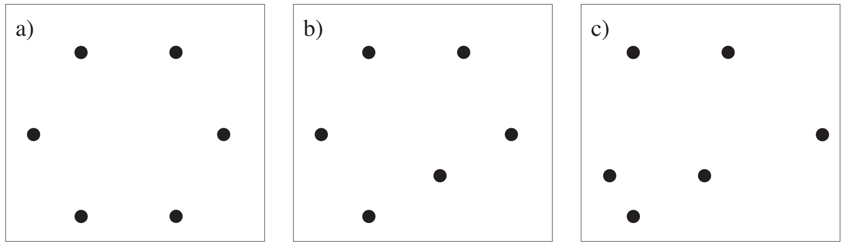
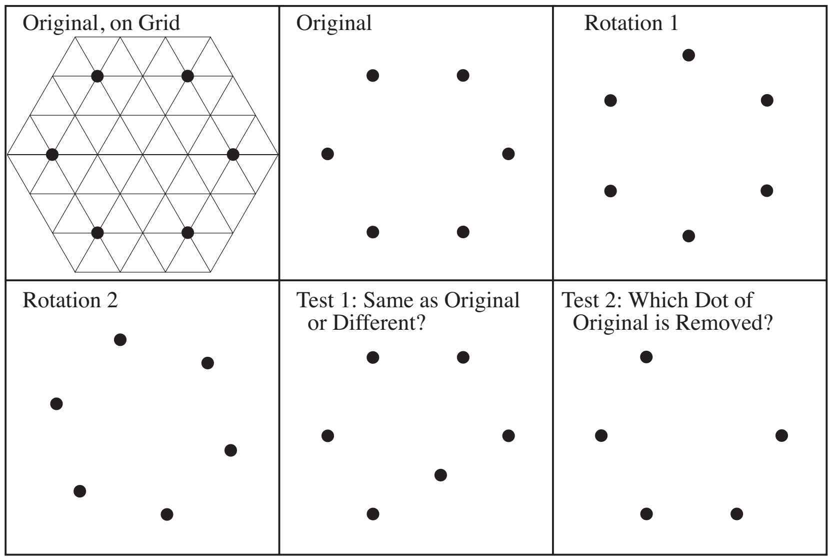


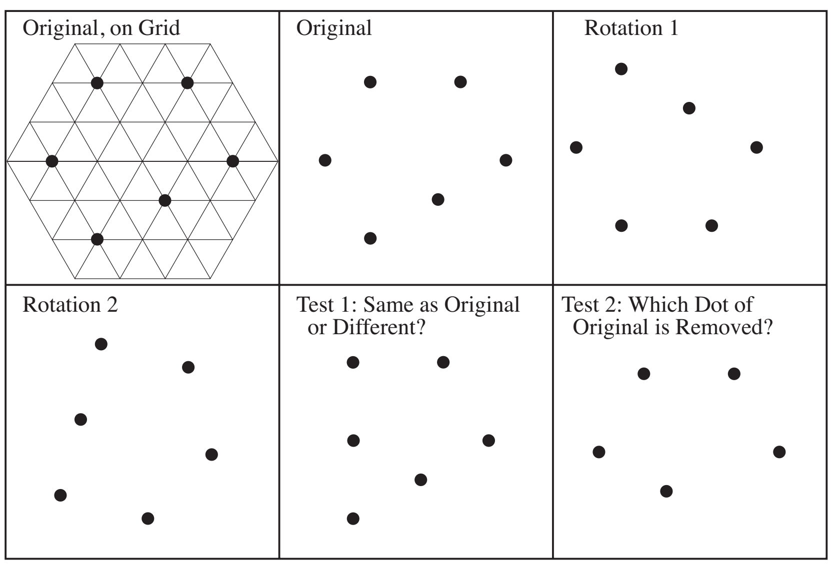
Figure 3. Visual stimuli, with a) perfect, b) near-perfect, and c) overly-skewed consistency

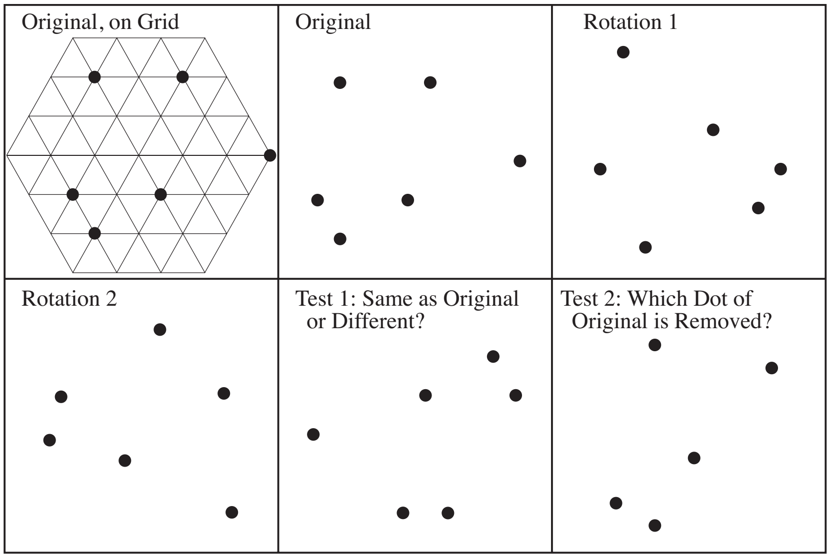
For each set of dots, consider the following operations: rotation, and rotation paired with the removal of one dot. The hypothesis is that because of the *perfect* consistency—zero skewness—in distance-relationships between the six dots in Figure 3a, the distance-relationships will easily cohere, yet the individual dots will not be differentiable from one another on the basis of those distance-relationships themselves. If this stimuli is rotated, it would not be difficult to determine that the resulting image is another rotation of the original image or a different network of six dots with altered distance-relationships. In a rotated five-dot subset of the original image, however, it would be impossible to determine which of the original dots has been removed.

Because of the *near-perfect* consistency—low skewness together with leptokurtosis—in distance-relationships between the six dots in Figure 3b, by contrast, the rotation of the original image would again be easily distinguished from a different group of six dots; additionally, there is sufficient kurtosis to quickly identify which of the original six dots has been removed in a five-dot subset of the original image. Figure 3c, however, has such an overabundance of skewness and platykurtosis that neither determination (distinguishing a rotation from an alteration, or identifying the missing dot) will be easy: the complexity of distance-relationships makes it difficult to quickly distinguish the role of one dot from another within the network.

The analogy between interval- and distance-based skewness and kurtosis can be made more apt by adding only one constraint to the visual stimuli: that, as with musical intervals, the distances be *discrete and quantised* (as opposed to smooth and continuous). If the dots are placed on the intersection points of a rectilinear (Cartesian) or hexagonal (Chinese checkers) grid, and distances are reckoned by the number of grid segments between dots, then the interval-class vector/average interval-class vector/individual interval-class vector mathematics used would be nearly identical (the only difference being that the resulting "distance-classes" do not have complements, since the space is not cyclic). The mathematics and sample stimuli for the same three figures are shown in Figures 4a-c and Tables 1a-c.







Figures 4a-c. Visual stimuli for shape with perfect, near-perfect, and overly skewed consistency

Distance Vector:	[0 6 0 9]	Avg. Distance Vector:[0 2 0 3]		
Ind.DV_1:	[0 2 0 3]	ID = 0		
Ind.DV_2:	[0 2 0 3]	ID = 0		
Ind.DV_3:	[0 2 0 3]	ID = 0	TD = 0.00	AD = 0.00
Ind.DV_4:	[0 2 0 3]	ID = 0	ADAD = 0	IF = 0
Ind.DV_5:	[0 2 0 3]	ID = 0		
Ind.DV_6:	[0 2 0 3]	ID = 0		

Distance Vector:	[0 6 3 6]	Avg. Distance Vector:[0 2 1 2]		
Ind.DV_1:	[0 2 0 3]	ID = 2		
Ind.DV_2:	[0 2 1 2]	ID = 0		
Ind.DV_3:	[0 2 1 2]	ID = 0	TD = 8	AD = 1.33
Ind.DV_4:	[0 2 1 2]	ID = 0	ADAD = 1.33	IF = 1.78
Ind.DV_5:	[0 2 0 3]	ID = 2		
Ind.DV_6:	[0 2 3 0]	ID = 4		

Distance Vector:	[1 3 5 3 3]	Avg. Distance Vector:[1/3 1 5/3 1 1]		
Ind.DV_1:	[1 1 1 1 1]	ID = 3.33		
Ind.DV_2:	[1 1 1 1 1]	ID = 1.33		
Ind.DV_3:	[0 1 2 1 1]	ID = 0.67	TD = 17.33	AD = 2.89
Ind.DV_4:	[0 1 2 2 0]	ID = 2.67	ADAD = 1.33	IF = 3.85
Ind.DV_5:	[0 0 2 0 3]	ID = 4.67		
Ind.DV_6:	[0 2 3 0 0]	ID = 4.67		

Tables 1a-c. IF math for shape with perfect, near-perfect, and overly skewed consistency

As is clear from Figure 4 and Table 1, a shape—just as with a set class—with perfect consistency returns an inequality factor of zero; a shape with near-perfect consistency will return a low but noticeable inequality factor; and a shape with an overabundance of skewness and high kurtosis returns a much higher inequality factor. Thus, the first shape's inequality factor of zero reflects the absence of a hierarchy of individual deviations, the last shape's high inequality factor reflects the dispersed network of individual deviations, while the second shape's moderate inequality factor reflects the likelihood that, given the individual deviation distribution, the network of dots could both cohere and establish specific roles for individual dots, establishing some kind of network hierarchy.

A hierarchy of individual deviations will exist only when the second-order deviational value (the average deviation from the average deviation —i.e., the kurtosis—and therefore the inequality factor) is greater than zero. If it is much greater than zero, however, then likely either the first-order deviations are too large for internal cohesiveness to exist, or that there exists one or two "outliers"—a member which produces values far out of the ordinary—which overly "skew" the data and could not be integrated into the network topology.

2.4. Set class examples

Some simple examples will correlate the above discussion of visual stimuli to musical stimuli. Whole-tone set class 6-35 [02468A] is the most consistent hexachord possible—with zero skewness and zero kurtosis, analogous to Figure 3a. Without distinct cues on the musical surface, no tone will stand out as distinct. By contrast, set class 6-48 [012579] is one of the less consistent hexachords, with very high average deviation values. As with Figure 3c, the high first-order deviational norm makes it unlikely that a network hierarchy of tones could arise.

In between those two extremes we might place set class 6-32 [024579]. Set class 6-32, the diatonic hexachord, contains within it three pairs of member tones whose interval-class relationships are identical, and overall there is a high degree of emphasis on interval class 2 and interval class 5. At the same time, the differences between the tones' interval-class relationships —the low non-zero average deviation from the average deviation —distinguishes one member tone from another; the sufficient degree of kurtosis produces the type of network topology so crucial to the quality of coherence referred to in the discussion of Figure 3b.

2.5. Relationship of set classes to their sub- and super-sets

2.5.1 Unique additive or subtractive pitch classes

One potential confounding factor of set class topologies is the close relationship that some sets have with strongly distinguishable subsets or supersets. To address that problem, two additional approaches to subset/superset relationships are proposed. ***Unique subtractive pitch class*** (USPC) and ***unique additive pitch class*** (UAPC) are defined as follows:

> For any given set class of cardinality *x*, does there exist a unique pitch class which, if 1) *Subtracted*, would produce a subset with a significant first-order deviational norm value lower than those of all of the other subsets (of cardinality *x*-1); or 2) *Added*, would produce a superset with a significant first-order deviational norm value lower than those of all of the other supersets (of cardinality *x*+1).[7]

It should be noted that the value in question is not the inequality factor: since the issue is whether or not the subset or superset acts so as to *clarify the similarity relationships* between the pitches, the relevant value is the average deviation, and not the inequality factor.

2.5.2 Determination of significance

The determination of a subset (or superset) with a significant average deviation value lower than all of the other subsets (or supersets) is a somewhat ad hoc process, involving both a *significance* and a *margin* from other set classes. We have devised two methods for determining "significance." The first is to choose some arbitrary value below which a high degree of consistency is exhibited. With this method, values chosen for each cardinality from four through eight are as follows.[8] (SD is standard deviation.)

Cardinality	SD Value	# of Set Classes With SD Value Equal or Smaller
4	1.000	13
5	2.240	15
6	2.222	14
7	2.612	9
8	2.938	11

Table 2. Suggested values for determining "significance"

A less arbitrary method is to choose as a "significant value" anything below the average average deviation.

In addition to meeting the criteria for significance, the average deviations of all the subsets (or supersets) must be compared to find the one value that is lower than all others. But by what margin? One method would be to not specify any margin; another method might be to choose as a margin at least one standard deviation away from the average of all average deviations. The standard deviations of the average deviations from the average average deviation for each cardinality are as follows. (AD is average deviation.)

Cardinality	Average AD	SD from Average ADs
4	1.672	0.716
5	2.598	0.576
6	2.640	0.752
7	2.907	0.431
8	2.817	0.652

Table 3. Average ADs and standard deviation from the average ADs

Finally, we must match a level of significance with a margin value. Part 3 of this online supplement includes two methods: 1) to use the suggested values in Table 2 with no margin, and 2) to use the average average deviation values with the average deviation from that average as the margin.[9, 10]

2.5.3 Interpretation

The unique subtractive pitch class identifies whether or not a single outlier skews the inequality factor value. Because that pitch class diverges from the similarity patterns of the others, the set class's high inequality factor value essentially results from that particular tone. If it were subtracted, the inequality factor values would drop significantly. We can therefore conclude that the intervallic relationships of the member tones might cohere more than the high inequality factor value indicates. In contrast, the presence of more than one such pitch class with such characteristics indicates that the high inequality factor value cannot be ascribed to a *single* pitch and, therefore, the intervallic relationships are dispersed at a rate commensurate with the inequality factor value.

In contrast, the unique additive pitch class identifies whether or not there is a single pitch that can be *added* to "flatten" the individual deviation values so that all of the pitch classes relate to one another more equally. If there is a pattern of similarity with two or more "edge" tones, the presence of a unique additional tone would reduce or remove altogether the intervallic differences associated with those edge tones. In this situation we can conclude that the intervallic relationships of the member tones might exhibit somewhat more coherence than the inequality factor value would indicate. For example, the addition of pitch class A (10) transforms 5-33 [02468] (inequality factor 0.164) into the whole-tone 6-35 (inequality factor 0.0).

Alternatively, if there is more than one pitch class that would flatten the individual deviation values, then we can conclude that the set class's inequality factor value reflects a more balanced spread of individual deviations and that the inequality factor sufficiently reflects the balance of intervallic relationships. An important exception to this principle, however, is if there is a *pair* of pitch classes that can be added to produce the *same* superset class. This would be the case, for example, with 6-1, to which two different tones can be added to produce 7-1, a set class with slightly reduced "edginess" as compared to 6-1. In Part 3, such set classes are distinguished from those that have a "truly unique" additive pitch class only by having a pair of pitch classes listed in the appropriate column.

Part 3: IF values for set classes with cardinalities 3-10

Notes on the Table:

Within the USPC or UAPC columns, the numbers before the parentheses indicate the relevant (subtractive or additive) pitch class. Those within the parentheses indicate the resultant set class. A bullet before the entry indicates that the set class satisfies the criteria of the second method described within Part 2 (that is, having a margin from other relevant sub- or super-sets).

Low Values (1) indicate the lowest value for each column, for all sets within the cardinality.

Low Values (2) indicate the lowest value for each column, for sets within the cardinality *other than* those that return zero IF values.

Averages indicate the average of all for values for each column.

High Values indicate the highest value for each column.

Ordinals indicate ranking by relevant IF value. Set classes that return zero IF values have sub-ranks indicating the first-order deviational norms. Within equivalent ordinal rankings, set classes are listed in ascending order by catalog number.

The Table begins on the next page.

TRICHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(3-1)	[012]	[210000]	2.667	0.889	0.296	**0.263**	1.886	0.667	0.225	**0.150**	0.230		•3B (4-1)
(3-2)	[013]	[111000]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		
(3-3)	[014]	[101100]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		
(3-4)	[015]	[100110]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		
(3-5)	[016]	[100011]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		•7 (4-9)
(3-6)	[024]	[020100]	2.667	0.889	0.296	**0.263**	1.886	0.667	0.225	**0.150**	0.230		•6A (4-21)
(3-7)	[025]	[011010]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		
(3-8)	[026]	[010101]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		•8 (4-25)
(3-9)	[027]	[010020]	2.667	0.889	0.296	**0.263**	1.886	0.667	0.225	**0.150**	0.230		•59 (4-23)
(3-10)	[036]	[002001]	2.667	0.889	0.296	**0.263**	1.886	0.667	0.225	**0.150**	0.230		•9 (4-28)
(3-11)	[037]	[001110]	4.000	1.333	0.000	**0.000**	2.449	0.816	0.000	**0.000**	0.000		
(3-12)	[048]	[000300]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
Trichordal:	Low Values (1):		0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
	Low Values (2):		2.667	0.889	0.296	**0.263**	1.886	0.667	0.225	**0.150**	0.230		
	Averages:		3.222	1.074	0.099	**0.088**	2.057	0.699	0.075	**0.050**	0.077		
	High Values:		4.000	1.333	0.296	**0.263**	2.449	0.816	0.225	**0.150**	0.230		

Ordinals:

By C-B **IFA**				By V-L **IFS**				(By V-L IFR)			
0_1.	3-12	0_2.	3-8	0_1.	3-12	0_2.	3-8	0_1.	3-12	0_2.	3-8
0_2.	3-2	0_2.	3-11	0_2.	3-2	0_2.	3-11	0_2.	3-2	02.	3-11
0_2.	3-3	1.	3-1	0_2.	3-3	1.	3-1	0_2.	3-3	1.	3-1
0_2.	3-4	1.	3-6	0_2.	3-4	1.	3-6	0_2.	3-4	1.	3-6
0_2.	3-5	1.	3-9	0_2.	3-5	1.	3-9	0_2.	3-5	1.	3-9
0_2.	3-7	1.	3-10	0_2.	3-7	1.	3-10	0_2.	3-7	1.	3-10

TETRACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(4-1)	[0123]	[321000]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000		•4B (5-1)
(4-2)	[0124]	[221100]	8.000	2.000	0.500	**1.000**	4.738	1.225	0.314	**0.385**	0.318		•3 (5-1)
(4-3)	[0134]	[212100]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000		2 (5-1)
(4-4)	[0125]	[211110]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209		6 (5-6)
(4-5)	[0126]	[210111]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209		7 (5-7)
(4-6)	[0127]	[210021]	8.000	2.000	1.000	**2.000**	4.576	1.225	0.444	**0.544**	0.453		•68 (5-7)
(4-7)	[0145]	[201210]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000		6B (5-6)
(4-8)	[0156]	[200121]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000		7B (5-7)
(4-9)	[0167]	[200022]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
(4-10)	[0235]	[122010]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000		

(4-11)	[0135]	[121110]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	
(4-12)	[0236]	[112101]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	
(4-13)	[0136]	[112011]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	
(4-14)	[0237]	[111120]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	8 (5-20)
(4-15)	[0146]	[111111]	12.000	3.000	0.000	**0.000**	4.899	1.225	0.000	**0.000**	0.000	
(4-16)	[0157]	[110121]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	6 (5-7)
(4-17)	[0347]	[102210]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000	•8B(5-21)
(4-18)	[0147]	[102111]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	
(4-19)	[0148]	[101310]	9.000	2.250	0.375	**0.844**	4.732	1.225	0.320	**0.392**	0.323	•59 (5-21)
(4-20)	[0158]	[101220]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000	•49 (5-21)
(4-21)	[0246]	[030201]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000	•8A (5-33)
(4-22)	[0247]	[021120]	8.000	2.000	0.500	**1.000**	4.738	1.225	0.314	**0.385**	0.318	•9 (5-35)
(4-23)	[0257]	[021030]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000	•9A (5-35)
(4-24)	[0248]	[020301]	7.000	1.750	0.750	**1.312**	4.510	1.225	0.488	**0.598**	0.500	•6A (5-33)
(4-25)	[0268]	[020202]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000	•4A (5-33)
(4-26)	[0358]	[012120]	4.000	1.000	0.000	**0.000**	2.828	0.707	0.000	**0.000**	0.000	A (5-35)
(4-27)	[0258]	[012111]	10.000	2.500	0.500	**1.250**	4.828	1.225	0.208	**0.255**	0.209	
(4-28)	[0369]	[004002]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000	
(4-29)	[0137]	[111111]	12.000	3.000	0.000	**0.000**	4.899	1.225	0.000	**0.000**	0.000	
Tetrachordal:	Low Values (1):		0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000	
	Low Values (2):		7.000	1.750	0.375	**0.844**	4.510	1.225	0.208	**0.255**	0.209	
	Averages:		6.690	1.672	0.263	**0.600**	3.615	0.920	0.129	**0.158**	0.131	
	High Values:		12.000	3.000	0.750	**2.000**	4.899	1.225	0.488	**0.598**	0.500	

Ordinals:

C-B **IFA**

0_1. 4-9
0_1. 4-25
0_1. 4-28
0_2. 4-1
0_2. 4-3
0_2. 4-8
0_2. 4-7
0_2. 4-10
0_2. 4-17
0_2. 4-20
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0_2. 4-23
0_2. 4-26
0_2. 4-15
0_2. 4-29
1. 4-19
2. 4-2
2. 4-22
3. 4-4
3. 4-5
3. 4-11
3. 4-12
3. 4-13
3. 4-14
3. 4-16
3. 4-18
3. 4-27
4. 4-24
5. 4-6

VL-**IFS**

0_1. 4-9
0_1. 4-25
0_1. 4-28
0_2. 4-1
0_2. 4-3
0_2. 4-8
0_2. 4-7
0_2. 4-10
0_2. 4-17
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0_2. 4-21
0_2. 4-23
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0_2. 4-29
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1. 4-16
1. 4-18
1. 4-27
1. 4-2
2. 4-2
2. 4-22
3. 4-19
4. 4-6
5. 4-24

VL-IFR

0_1. 4-9
0_1. 4-25
0_1. 4-28
0_2. 4-1
0_2. 4-3
0_2. 4-8
0_2. 4-7
0_2. 4-10
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0_2. 4-20
0_2. 4-21
0_2. 4-23
0_2. 4-26
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1. 4-18
1. 4-27
1. 4-2
2. 4-2
2. 4-22
3. 4-19
4. 4-6
5. 4-24

PENTACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(5-1)	[01234]	[432100]	8.000	1.600	0.320	**0.512**	4.319	0.894	0.234	**0.210**	0.237		•5B (6-1)
(5-2)	[01235]	[332110]	11.200	2.240	0.448	**1.004**	5.693	1.166	0.254	**0.296**	0.256		4 (6-1)
(5-3)	[01245]	[322210]	10.400	2.080	0.416	**0.865**	5.438	1.131	0.314	**0.356**	0.318		3 (6-1)
(5-4)	[01236]	[322111]	15.200	3.040	0.512	**1.556**	6.926	1.414	0.287	**0.405**	0.288	•6 (4-1)	5 (6-3)
(5-5)	[01237]	[321121]	15.200	3.040	0.768	**2.335**	6.767	1.414	0.415	**0.587**	0.421	•7 (4-1)	8 (6-38)
(5-6)	[01256]	[311221]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		7 (6-6)
(5-7)	[01267]	[310132]	9.600	1.920	0.256	**0.492**	4.840	0.980	0.153	**0.149**	0.153	•2 (4-9)	8 (6-7)
(5-8)	[02346]	[232201]	13.600	2.720	0.832	**2.263**	6.845	1.442	0.459	**0.663**	0.467	•3 (4-21)	
(5-9)	[01246]	[231211]	15.200	3.040	0.768	**2.335**	6.767	1.414	0.415	**0.587**	0.421	•1 (4-21)	5 (6-6)
(5-10)	[01346]	[223111]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		
(5-11)	[02347]	[222220]	12.800	2.560	0.768	**1.966**	6.809	1.414	0.385	**0.545**	0.390	•2 (4-17)	
(5-12)	[01356]	[222121]	14.400	2.880	0.768	**2.212**	6.642	1.386	0.398	**0.552**	0.403	•3 (4-8)	
(5-13)	[01248]	[221311]	18.400	3.680	0.704	**2.591**	8.178	1.673	0.355	**0.595**	0.358		
(5-14)	[01257]	[221131]	15.200	3.040	0.768	**2.335**	6.767	1.414	0.415	**0.587**	0.421	•1 (4-23)	6 (6-6)
(5-15)	[01268]	[220222]	11.200	2.240	1.024	**2.294**	5.769	1.265	0.530	**0.671**	0.545	•1 (4-25)	7 (6-7)
(5-16)	[01347]	[213211]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		
(5-17)	[01348]	[212320]	13.600	2.720	0.512	**1.393**	7.113	1.442	0.238	**0.343**	0.239	•8 (4-3)	79 (6-19)
(5-18)	[01457]	[212221]	14.400	2.880	0.384	**1.106**	6.855	1.386	0.202	**0.280**	0.203	•7 (4-7)	
(5-19)	[01367]	[212122]	12.800	2.560	0.896	**2.294**	5.668	1.233	0.495	**0.610**	0.507	•3 (4-9)	
(5-20)	[01568]	[211231]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		7 (6-38)
(5-21)	[01458]	[202420]	7.200	1.440	0.192	**0.276**	4.200	0.849	0.120	**0.102**	0.121		•9 (6-20)
(5-22)	[01478]	[202321]	13.600	2.720	0.512	**1.393**	7.113	1.442	0.238	**0.343**	0.239	•4 (4-8)	
(5-23)	[02357]	[132130]	11.200	2.240	0.448	**1.004**	5.693	1.166	0.254	**0.296**	0.256		A (6-32)
(5-24)	[01357]	[131221]	15.200	3.040	0.768	**2.335**	6.767	1.414	0.415	**0.587**	0.421	•0 (4-21)	8 (6-26)
(5-25)	[02358]	[123121]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		
(5-26)	[02458]	[122311]	18.400	3.680	0.576	**2.120**	8.238	1.673	0.294	**0.492**	0.295		
(5-27)	[01358]	[122230]	10.400	2.080	0.416	**0.865**	5.438	1.131	0.314	**0.356**	0.318		A (6-32)
(5-28)	[02368]	[122212]	12.800	2.560	0.896	**2.294**	5.668	1.233	0.495	**0.610**	0.507	•3 (4-25)	
(5-29)	[01368]	[122131]	15.200	3.040	0.512	**1.556**	6.926	1.414	0.287	**0.405**	0.288	•0 (4-23)	5 (6-25)
(5-30)	[01468]	[121321]	18.400	3.680	0.704	**2.591**	8.178	1.673	0.355	**0.595**	0.358		
(5-31)	[01369]	[114112]	14.400	2.880	0.768	**2.212**	6.233	1.327	0.461	**0.611**	0.469	•1 (4-28)	4 (6-27)
(5-32)	[01469]	[113221]	12.000	2.400	0.320	**0.768**	5.414	1.095	0.166	**0.182**	0.167		
(5-33)	[02468]	[040402]	6.400	1.280	0.128	**0.164**	3.973	0.800	0.093	**0.074**	0.093	•4 (4-25)	•A (6-35)
(5-34)	[02469]	[032221]	13.600	2.720	0.832	**2.263**	6.845	1.442	0.459	**0.663**	0.467	•9 (4-21)	
(5-35)	[02479]	[032140]	8.000	1.600	0.320	**0.512**	4.319	0.894	0.234	**0.210**	0.237		•5 (6-32)
(5-36)	[01247]	[222121]	17.600	3.520	0.576	**2.028**	8.139	1.649	0.265	**0.438**	0.267		
(5-37)	[03458]	[212320]	13.600	2.720	0.512	**1.393**	7.113	1.442	0.238	**0.343**	0.239	•4 (4-26)	9B (6-44)
(5-38)	[01258]	[212221]	14.400	2.880	0.384	**1.106**	6.855	1.386	0.202	**0.280**	0.203	•2 (4-20)	

Pentachordal: Low Values (1):	6.400	1.280	0.128	**0.164**	3.973	0.800	0.093	**0.074**	0.093
Low Values (2):	6.400	1.280	0.128	**0.164**	3.973	0.800	0.093	**0.074**	0.093
Averages:	12.989	2.598	0.542	**1.481**	6.199	1.278	0.297	**0.393**	0.300
High Values:	18.400	3.680	1.024	**2.591**	8.238	1.673	0.530	**0.671**	0.545

Ordinals:

C-B **IFA**		6.	5-27	14.	5-12	V-L **IFS**		6.	5-38	14.	5-12	V-L IFR		6.	5-35	13.	5-30
1.	5-33	7.	5-2	14.	5-31	1.	5-33	7.	5-2	15.	5-5	1.	5-33	7.	5-17	14.	5-11
2.	5-21	7.	5-23	15.	5-8	2.	5-21	7.	5-23	15.	5-9	2.	5-21	7.	5-22	15.	5-12
3.	5-7	8.	5-18	16.	5-15	3.	5-7	8.	5-17	15.	5-14	3.	5-7	7.	5-37	16.	5-5
4.	5-1	8.	5-38	16.	5-19	4.	5-6	8.	5-22	15.	5-24	4.	5-6	8.	5-2	16.	5-9
4.	5-35	9.	5-17	16.	5-28	4.	5-10	8.	5-37	16.	5-13	4.	5-10	8.	5-23	16.	5-14
5.	5-6	9.	5-22	17.	5-5	4.	5-16	9.	5-3	16.	5-30	4.	5-16	9.	5-36	16.	5-24
5.	5-10	9.	5-37	17.	5-9	4.	5-20	9.	5-27	17.	5-19	4.	5-20	10.	5-4	17.	5-34
5.	5-16	10.	5-4	17.	5-14	4.	5-25	10.	5-4	17.	5-28	4.	5-25	10.	5-29	17.	5-8
5.	5-20	10.	5-29	17.	5-24	4.	5-32	10.	5-29	18.	5-31	4.	5-32	11.	5-26	18.	5-31
5.	5-25	11.	5-11	18.	5-13	5.	5-1	11.	5-36	19.	5-8	5.	5-18	12.	5-3	19.	5-19
5.	5-32	12.	5-36	18.	5-30	5.	5-35	12.	5-26	19.	5-34	5.	5-38	12.	5-27	19.	5-28
6.	5-3	13.	5-26	19.	5-34	6.	5-18	13.	5-11	20.	5-15	6.	5-1	13.	5-13	20.	5-15

HEXACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(6-1)	[012345]	[543210]	10.667	1.778	0.296	**0.527**	5.550	0.943	0.184	**0.173**	0.185		•6B (7-1)
(6-2)	[012346]	[443211]	16.667	2.778	0.556	**1.543**	7.814	1.333	0.288	**0.383**	0.290	6 (5-1)	•5 (7-1)
(6-3)	[012356]	[433221]	14.667	2.444	0.667	**1.630**	7.205	1.247	0.340	**0.424**	0.344		
(6-4)	[012456]	[432321]	14.667	2.444	0.296	**0.724**	7.442	1.247	0.132	**0.164**	0.132		•3 (7-1)
(6-5)	[012367]	[422232]	16.667	2.778	0.593	**1.646**	7.756	1.333	0.329	**0.439**	0.332	3 (5-7)	8 (7-7)
(6-6)	[012567]	[421242]	16.000	2.667	0.000	**0.000**	6.928	1.155	0.000	**0.000**	0.000		8B (7-7)
(6-7)	[012678]	[420243]	10.667	1.778	0.593	**1.053**	5.333	0.943	0.319	**0.301**	0.325		
(6-8)	[023457]	[343230]	13.333	2.222	0.593	**1.317**	7.848	1.333	0.260	**0.346**	0.261		
(6-9)	[012357]	[342231]	18.667	3.111	0.593	**1.844**	8.861	1.491	0.203	**0.303**	0.204		
(6-10)	[013457]	[333321]	14.667	2.444	0.667	**1.630**	8.513	1.453	0.315	**0.457**	0.317	5 (5-16)	8 (7-37)
(6-11)	[012457]	[333231]	15.333	2.556	0.815	**2.082**	8.285	1.453	0.458	**0.665**	0.465	0 (5-10)	B (7-36)
(6-12)	[012467]	[332232]	17.333	2.889	1.111	**3.210**	8.623	1.528	0.526	**0.803**	0.535	•4 (5-7)	5 (7-7)
(6-13)	[013467]	[324222]	13.333	2.222	0.296	**0.658**	6.266	1.054	0.144	**0.152**	0.144		9A (7-31)
(6-14)	[013458]	[323430]	12.000	2.000	1.111	**2.222**	7.171	1.333	0.607	**0.810**	0.626	•3 (5-21)	9 (7-21)
(6-15)	[012458]	[323421]	17.333	2.889	0.519	**1.498**	8.726	1.491	0.330	**0.491**	0.332	•2 (5-21)	•9 (7-21)

(6-16)	[014568]	[322431]	17.333	2.889	0.370	**1.070**	8.775	1.491	0.290	**0.433**	0.292	•6 (5-21)	•9 (7-21)
(6-17)	[012478]	[322332]	20.667	3.444	0.630	**2.169**	10.230	1.732	0.307	**0.531**	0.308	•4 (5-7)	
(6-18)	[012578]	[322242]	16.667	2.778	0.593	**1.646**	7.756	1.333	0.329	**0.439**	0.332	5 (5-7)	•6 (7-7)
(6-19)	[013478]	[313431]	13.333	2.222	0.889	**1.975**	7.348	1.291	0.414	**0.534**	0.420	•1 (5-21)	9 (7-22)
(6-20)	[014589]	[303630]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
(6-21)	[023468]	[242412]	21.333	3.556	1.037	**3.687**	9.121	1.633	0.607	**0.991**	0.620	•3 (5-33)	
(6-22)	[012468]	[241422]	21.333	3.556	1.185	**4.214**	9.080	1.633	0.625	**1.021**	0.639	•1 (5-33)	
(6-23)	[023568]	[234222]	13.333	2.222	0.296	**0.658**	6.266	1.054	0.144	**0.152**	0.144		•9B (7-31)
(6-24)	[013468]	[233331]	19.333	3.222	0.556	**1.790**	9.943	1.667	0.178	**0.296**	0.178	8 (5-10)	
(6-25)	[013568]	[233241]	14.667	2.444	0.667	**1.630**	7.205	1.247	0.340	**0.424**	0.344		•A (7-35)
(6-26)	[013578]	[232341]	14.667	2.444	0.296	**0.724**	7.442	1.247	0.132	**0.164**	0.132		•A (7-35)
(6-27)	[013469]	[225222]	16.000	2.667	0.000	**0.000**	6.928	1.155	0.000	**0.000**	0.000		•7 (7-31)
(6-28)	[013569]	[224322]	23.333	3.889	0.370	**1.440**	10.507	1.764	0.211	**0.373**	0.212		
(6-29)	[023679]	[224232]	20.000	3.333	0.444	**1.481**	9.247	1.563	0.264	**0.413**	0.265		
(6-30)	[013679]	[224223]	16.000	2.667	0.000	**0.000**	7.970	1.333	0.115	**0.153**	0.115		•4A (7-31)
(6-31)	[014579]	[223431]	17.333	2.889	0.519	**1.498**	8.726	1.491	0.330	**0.491**	0.332	•7 (5-31)	•8 (7-21)
(6-32)	[024579]	[143250]	10.667	1.778	0.296	**0.527**	5.550	0.943	0.184	**0.173**	0.185		•AB (7-35)
(6-33)	[023579]	[143241]	16.667	2.778	0.556	**1.543**	7.814	1.333	0.288	**0.383**	0.290	•3 (5-35)	•A (7-35)
(6-34)	[013579]	[142422]	21.333	3.556	1.037	**3.687**	9.121	1.633	0.607	**0.991**	0.620	•0 (5-33)	
(6-35)	[02468A]	[060603]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
(6-36)	[012347]	[433221]	17.333	2.889	0.963	**2.782**	8.412	1.491	0.514	**0.767**	0.524	7 (5-1)	
(6-37)	[012348]	[432321]	21.333	3.556	1.185	**4.214**	9.816	1.700	0.465	**0.791**	0.471	•8 (5-1)	
(6-38)	[012378]	[421242]	16.000	2.667	0.000	**0.000**	6.928	1.155	0.000	**0.000**	0.000		69 (7-7)
(6-39)	[023458]	[333321]	19.333	3.222	0.556	**1.790**	9.943	1.667	0.178	**0.296**	0.178	4 (5-25)	
(6-40)	[012358]	[333231]	15.333	2.556	0.815	**2.082**	8.285	1.453	0.458	**0.665**	0.465	1 (5-25)	6 (7-36)
(6-41)	[012368]	[332232]	17.333	2.889	0.667	**1.926**	9.034	1.528	0.258	**0.395**	0.260		7 (7-7)
(6-42)	[012369]	[324222]	20.000	3.333	0.444	**1.481**	9.247	1.563	0.264	**0.413**	0.265		
(6-43)	[012568]	[322332]	13.333	2.222	0.741	**1.646**	7.401	1.291	0.385	**0.497**	0.391		
(6-44)	[012569]	[313431]	13.333	2.222	0.889	**1.975**	7.348	1.291	0.414	**0.534**	0.420	•0 (5-21)	8 (7-22)
(6-45)	[023469]	[234222]	22.667	3.778	0.741	**2.798**	10.375	1.764	0.349	**0.616**	0.352		
(6-46)	[012469]	[233331]	14.667	2.444	0.667	**1.630**	8.513	1.453	0.315	**0.457**	0.317	2 (5-32)	5 (7-17)
(6-47)	[012479]	[233241]	17.333	2.889	0.963	**2.782**	8.412	1.491	0.514	**0.767**	0.524	•1 (5-35)	
(6-48)	[012579]	[232341]	21.333	3.556	1.185	**4.214**	9.816	1.700	0.465	**0.791**	0.471	•1 (5-35)	
(6-49)	[013479]	[224322]	13.333	2.222	0.296	**0.658**	6.266	1.054	0.144	**0.152**	0.144		•6A (7-31)
(6-50)	[014679]	[224232]	13.333	2.222	0.296	**0.658**	6.266	1.054	0.144	**0.152**	0.144		•3A (7-31)
Hexachordal:	Low Values (1):		0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
	Low Values (2):		10.667	1.778	0.296	**0.527**	5.333	0.943	0.115	**0.152**	0.115		
	Averages:		15.840	2.640	0.577	**1.639**	7.748	1.331	0.293	**0.423**	0.297		
	High Values:		23.333	3.889	1.185	**4.214**	10.507	1.764	0.625	**1.021**	0.639		

Ordinals:

C-B **IFA**

0_1.	6-20	6.	6-8	14.	6-9
0_1.	6-35	7.	6-28	15.	6-41
0_2.	6-6	8.	6-29	16.	6-19
0_2.	6-27	8.	6-42	16.	6-44
0_2.	6-30	9.	6-15	17.	6-11
0_2.	6-38	9.	6-31	17.	6-40
1.	6-1	10.	6-2	18.	6-17
1.	6-32	10.	6-33	19.	6-14
2.	6-13	11.	6-3	20.	6-36
2.	6-23	11.	6-10	20.	6-47
2.	6-49	11.	6-46	21.	6-45
2.	6-50	11.	6-25	22.	6-12
3.	6-4	12.	6-5	23.	6-21
3.	6-26	12.	6-18	23.	6-34
4.	6-7	12.	6-43	24.	6-22
5.	6-16	13.	6-24	24.	6-37
		13.	6-39	24.	6-48

V-L **IFS**

0_1.	6-20	6.	6-7	17.	6-31
0_1.	6-35	7.	6-9	18.	6-43
0_2.	6-6	8.	6-8	19.	6-17
0_2.	6-27	9.	6-28	20.	6-19
0_2.	6-38	10.	6-2	20.	6-44
1.	6-13	10.	6-33	21.	6-45
1.	6-23	11.	6-41	22.	6-11
1.	6-49	12.	6-29	22.	6-40
1.	6-50	12.	6-42	23.	6-36
2.	6-30	13.	6-3	23.	6-47
3.	6-4	13.	6-25	24.	6-37
3.	6-26	14.	6-16	24.	6-48
4.	6-1	15.	6-5	25.	6-12
4.	6-32	15.	6-18	26.	6-14
5.	6-24	16.	6-10	27.	6-21
5.	6-39	16.	6-46	27.	6-34
		17.	6-15	28.	6-22

V-L IFR

0_1.	6-20	6.	6-9	17.	6-3
0_1.	6-35	7.	6-28	17.	6-25
0_2.	6-6	8.	6-41	18.	6-45
0_2.	6-27	9.	6-8	19.	6-43
0_2.	6-38	10.	6-29	20.	6-19
1.	6-30	10.	6-42	20.	6-44
2.	6-4	11.	6-2	22.	6-11
2.	6-26	11.	6-33	22.	6-40
3.	6-13	12.	6-16	23.	6-37
3.	6-23	13.	6-17	23.	6-48
3.	6-49	14.	6-10	24.	6-36
3.	6-50	14.	6-46	24.	6-47
4.	6-24	15.	6-7	25.	6-12
4.	6-39	16.	6-5	25.	6-21
5.	6-1	16.	6-18	25.	6-34
5.	6-32	16.	6-15	26.	6-14
		16.	6-31	27.	6-22

HEPTACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(7-1)	[0123456]	[654321]	16.000	2.286	0.490	**1.120**	7.266	1.069	0.258	**0.276**	0.260		7B (8-1)
(7-2)	[0123457]	[554331]	19.429	2.776	0.478	**1.327**	9.409	1.385	0.336	**0.466**	0.340	•7 (6-1)	6 (8-1)
(7-3)	[0123458]	[544431]	21.143	3.020	1.085	**3.276**	10.062	1.539	0.558	**0.859**	0.570	•8 (6-1)	7 (8-4)
(7-4)	[0123467]	[544332]	18.857	2.694	0.840	**2.262**	8.999	1.355	0.435	**0.589**	0.442	2 (6-13)	5 (8-1)
(7-5)	[0123567]	[543342]	18.857	2.694	0.548	**1.477**	9.223	1.355	0.320	**0.433**	0.323	3 (6-6)	
(7-6)	[0123478]	[533442]	22.286	3.184	0.945	**3.007**	10.497	1.552	0.403	**0.625**	0.407	4 (6-38)	9 (8-8)
(7-7)	[0123678]	[532353]	16.571	2.367	0.583	**1.380**	7.894	1.178	0.344	**0.406**	0.349	3 (6-7)	5 (8-6)
(7-8)	[0234568]	[454422]	24.000	3.429	0.816	**2.799**	10.791	1.591	0.396	**0.630**	0.400	4 (6-23)	
(7-9)	[0123468]	[453432]	25.714	3.673	0.700	**2.570**	11.913	1.726	0.290	**0.501**	0.291		
(7-10)	[0123469]	[445332]	21.714	3.102	0.851	**2.641**	10.531	1.552	0.384	**0.595**	0.387	2 (6-27)	7 (8-13)
(7-11)	[0134568]	[444441]	20.000	2.857	0.653	**1.866**	10.235	1.498	0.329	**0.493**	0.332		
(7-12)	[0123479]	[444342]	20.571	2.939	0.630	**1.851**	10.324	1.512	0.335	**0.506**	0.337	2 (6-49)	6A (8-13)
(7-13)	[0124568]	[443532]	22.286	3.184	0.630	**2.005**	10.663	1.552	0.298	**0.462**	0.300		
(7-14)	[0123578]	[443352]	18.857	2.694	0.548	**1.477**	9.223	1.355	0.320	**0.433**	0.323	5 (6-38)	
(7-15)	[0124678]	[442443]	23.429	3.347	1.539	**5.152**	11.035	1.690	0.620	**1.049**	0.633	•4 (6-7)	
(7-16)	[0123569]	[435432]	21.714	3.102	0.851	**2.641**	10.555	1.552	0.369	**0.573**	0.373	1 (6-27)	8 (8-18)
(7-17)	[0124569]	[434541]	18.286	2.612	0.700	**1.828**	9.066	1.340	0.347	**0.465**	0.351		8A (8-19)

(7-18)	[0123589]	[434442]	20.571	2.939	0.501	**1.474**	10.435	1.512	0.252	**0.382**	0.254	5 (6-50)	A (8-18)
(7-19)	[0123679]	[434343]	18.857	2.694	0.373	**1.005**	10.203	1.471	0.198	**0.291**	0.198		8 (8-9)
(7-20)	[0125679]	[433452]	22.286	3.184	0.945	**3.007**	10.497	1.552	0.403	**0.625**	0.407	9 (6-6)	8 (8-8)
(7-21)	[0124589]	[424641]	16.000	2.286	0.571	**1.306**	7.998	1.262	0.548	**0.692**	0.564	•2 (6-20)	6A (8-19)
(7-22)	[0125689]	[424542]	17.143	2.449	0.840	**2.056**	7.717	1.178	0.422	**0.497**	0.430	1 (6-49)	
(7-23)	[0234579]	[354351]	19.429	2.776	0.478	**1.327**	9.409	1.385	0.336	**0.466**	0.340	•3 (6-32)	A (8-23)
(7-24)	[0123579]	[353442]	25.714	3.673	0.700	**2.570**	11.913	1.726	0.290	**0.501**	0.291		
(7-25)	[0234679]	[345342]	21.714	3.102	0.851	**2.641**	10.531	1.552	0.384	**0.595**	0.387	2 (6-27)	1 (8-13)
(7-26)	[0134579]	[344532]	22.286	3.184	0.700	**2.228**	10.618	1.552	0.329	**0.511**	0.332	•5 (6-49)	8 (8-19)
(7-27)	[0124579]	[344451]	21.143	3.020	1.085	**3.276**	10.062	1.539	0.558	**0.859**	0.570	•1 (6-27)	6 (8-14)
(7-28)	[0135679]	[344433]	25.714	3.673	0.630	**2.313**	12.584	1.818	0.274	**0.498**	0.275		
(7-29)	[0124679]	[344352]	18.857	2.694	0.840	**2.262**	8.999	1.355	0.435	**0.589**	0.442	2 (6-50)	B (8-23)
(7-30)	[0124689]	[343542]	22.286	3.184	0.630	**2.005**	10.663	1.552	0.298	**0.462**	0.300		5 (8-19)
(7-31)	[0134679]	[336333]	11.429	1.633	0.117	**0.190**	6.303	0.904	0.075	**0.067**	0.075		•A (8-28)
(7-32)	[0134689]	[335442]	21.714	3.102	0.851	**2.641**	10.555	1.552	0.369	**0.573**	0.373	8 (6-27)	7 (8-18)
(7-33)	[012468A]	[262623]	22.286	3.184	1.539	**4.901**	9.629	1.641	0.934	**1.533**	0.977	•1 (6-35)	
(7-34)	[013468A]	[254442]	24.000	3.429	0.816	**2.799**	10.791	1.591	0.396	**0.630**	0.400	8 (6-23)	
(7-35)	[013568A]	[254361]	16.000	2.286	0.490	**1.120**	7.266	1.069	0.258	**0.276**	0.260	0 (6-32)	B7 (8-23)
(7-36)	[0123568]	[444342]	17.143	2.449	0.875	**2.142**	8.538	1.309	0.484	**0.634**	0.495	1 (6-23)	7 (8-6)
(7-37)	[0134578]	[434541]	18.286	2.612	0.700	**1.828**	9.066	1.340	0.347	**0.465**	0.351		9B (8-19)
(7-38)	[0124578]	[434442]	20.571	2.939	0.501	**1.474**	10.435	1.512	0.252	**0.382**	0.254	0 (6-13)	B (8-18)
Heptachordal:	Low Values (1):		11.429	1.633	0.117	**0.190**	6.303	0.904	0.075	**0.067**	0.075		
	Low Values (2):		11.429	1.633	0.117	**0.190**	6.303	0.904	0.075	**0.067**	0.075		
	Averages:		20.346	2.907	0.735	**2.191**	9.787	1.452	0.373	**0.550**	0.379		
	High Values:		25.714	3.673	1.539	**5.152**	12.584	1.818	0.934	**1.533**	0.977		

Ordinals:

C-B **IFA**

1. 7-31
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V-L **IFS**

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V-L IFR

1. 7-31
2. 7-19
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14. 7-16
14. 7-32
15. 7-10
15. 7-25
16. 7-8
16. 7-34
17. 7-6
17. 7-20
18. 7-22
19. 7-4
19. 7-29
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21. 7-21
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22. 7-27
23. 7-15
24. 7-33

OCTACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(8-1)	[01234567]	[765442]	18.000	2.250	0.750	**1.688**	8.993	1.173	0.337	**0.395**	0.342		
(8-2)	[01234568]	[665542]	24.500	3.062	0.578	**1.771**	11.348	1.458	0.338	**0.493**	0.341	•8 (7-1)	
(8-3)	[01234569]	[656542]	24.000	3.000	1.000	**3.000**	10.954	1.458	0.508	**0.741**	0.518	•9 (7-1)	
(8-4)	[01234578]	[655552]	24.500	3.062	0.578	**1.771**	11.012	1.414	0.327	**0.462**	0.330		
(8-5)	[01234678]	[654553]	24.500	3.062	0.594	**1.818**	11.693	1.500	0.339	**0.509**	0.342	4 (7-7)	
(8-6)	[01235678]	[654463]	18.000	2.250	0.750	**1.688**	8.952	1.173	0.354	**0.416**	0.360		
(8-7)	[01234589]	[645652]	24.000	3.000	0.750	**2.250**	11.235	1.458	0.394	**0.575**	0.399		
(8-8)	[01234789]	[644563]	21.500	2.688	1.188	**3.191**	10.201	1.369	0.508	**0.695**	0.519		
(8-9)	[01236789]	[644464]	16.000	2.000	0.000	**0.000**	9.798	1.225	0.000	**0.000**	0.000		
(8-10)	[02345679]	[566452]	24.000	3.000	0.500	**1.500**	11.473	1.458	0.263	**0.383**	0.264		
(8-11)	[01234579]	[565552]	27.500	3.438	0.453	**1.558**	12.489	1.581	0.252	**0.398**	0.253		
(8-12)	[01345679]	[556543]	23.500	2.938	0.578	**1.698**	11.655	1.500	0.360	**0.540**	0.363	•5 (7-31)	
(8-13)	[01234679]	[556453]	20.000	2.500	0.750	**1.875**	9.923	1.323	0.467	**0.618**	0.476	•2 (7-31)	
(8-14)	[01245679]	[555562]	24.500	3.062	0.578	**1.771**	11.012	1.414	0.327	**0.462**	0.330	7 (7-17)	
(8-15)	[01234689]	[555553]	27.000	3.375	0.188	**0.633**	12.912	1.620	0.141	**0.229**	0.142		
(8-16)	[01235789]	[554563]	24.500	3.062	0.594	**1.818**	11.693	1.500	0.339	**0.509**	0.342	5 (7-7)	
(8-17)	[01345689]	[546652]	24.000	3.000	0.500	**1.500**	11.366	1.458	0.328	**0.479**	0.331		
(8-18)	[01235689]	[546553]	20.500	2.562	0.828	**2.122**	9.907	1.323	0.473	**0.626**	0.482	•1 (7-31)	
(8-19)	[01245689]	[545752]	22.000	2.750	0.688	**1.891**	10.211	1.323	0.351	**0.464**	0.355	4 (7-22)	
(8-20)	[01245789]	[545662]	24.000	3.000	0.750	**2.250**	11.235	1.458	0.394	**0.575**	0.399		
(8-21)	[0123468A]	[474643]	28.000	3.500	1.500	**5.250**	13.414	1.768	0.567	**1.003**	0.576		
(8-22)	[0123568A]	[465562]	24.500	3.062	0.578	**1.771**	11.348	1.458	0.338	**0.493**	0.341	•2 (7-35)	
(8-23)	[0123578A]	[465472]	18.000	2.250	0.750	**1.688**	8.993	1.173	0.337	**0.395**	0.342		
(8-24)	[0124568A]	[464743]	24.000	3.000	1.500	**4.500**	12.085	1.620	0.596	**0.965**	0.608		
(8-25)	[0124678A]	[464644]	28.000	3.500	1.500	**5.250**	14.118	1.871	0.630	**1.179**	0.641		
(8-26)	[0134578A]	[456562]	24.000	3.000	1.000	**3.000**	10.954	1.458	0.508	**0.741**	0.518	•4 (7-35)	
(8-27)	[0124578A]	[456553]	23.500	2.938	0.578	**1.698**	11.655	1.500	0.360	**0.540**	0.363	•0 (7-31)	
(8-28)	[0134679A]	[448444]	0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
(8-29)	[01235679]	[555553]	27.000	3.375	0.188	**0.633**	12.912	1.620	0.141	**0.229**	0.142		
Octachordal:	Low Values (1):		0.000	0.000	0.000	**0.000**	0.000	0.000	0.000	**0.000**	0.000		
	Low Values (2):		18.000	2.250	0.188	**0.633**	8.952	1.173	0.141	**0.229**	0.142		
	Averages:		22.534	2.817	0.696	**2.054**	10.812	1.402	0.354	**0.521**	0.359		
	High Values:		28.000	3.500	1.500	**5.250**	14.118	1.871	0.630	**1.179**	0.641		

Ordinals:

C-B **IFA**
0_1. 8-28
0_2. 8-9
1. 8-15
1. 8-29
2. 8-10
2. 8-17
3. 8-11
4. 8-1
4. 8-6
4. 8-23
5. 8-12
5. 8-27
6. 8-2
6. 8-4
6. 8-14
6. 8-22
7. 8-5
7. 8-16
8. 8-13
9. 8-19
10. 8-18
11. 8-7
12. 8-20
13. 8-3
13. 8-26
14. 8-8
15. 8-24
16. 8-21
16. 8-25

V-L **IFS**
0_1. 8-28
0_2. 8-9
1. 8-15
1. 8-29
2. 8-10
3. 8-11
4. 8-1
4. 8-23
5. 8-6
6. 8-4
6. 8-14
7. 8-19
8. 8-17
9. 8-2
9. 8-22
10. 8-5
10. 8-16
11. 8-12
11. 8-27
12. 8-7
12. 8-20
13. 8-13
14. 8-18
15. 8-8
16. 8-3
16. 8-26
17. 8-24
18. 8-21
19. 8-25

V-L IFR
0_1. 8-28
0_2. 8-9
1. 8-15
1. 8-29
2. 8-10
3. 8-11
4. 8-4
4. 8-14
5. 8-17
6. 8-2
6. 8-22
7_1. 8-1
7_1. 8-23
7_2. 8-5
7_2. 8-16
8. 8-19
9. 8-6
10. 8-12
10. 8-27
11. 8-7
11. 8-20
12. 8-13
13. 8-18
14. 8-3
14. 8-26
15. 8-8
16. 8-21
17. 8-24
18. 8-25

NONACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(9-1)	[012345678]	[876663]	24.889	2.765	0.527	**1.457**	10.803	1.237	0.302	**0.374**	0.305		
(9-2)	[012345679]	[777663]	27.556	3.062	0.340	**1.042**	11.994	1.361	0.277	**0.377**	0.279	9 (8-1)	
(9-3)	[012345689]	[767763]	27.556	3.062	0.340	**1.042**	12.011	1.361	0.267	**0.364**	0.269		
(9-4)	[012345789]	[766773]	27.556	3.062	0.351	**1.075**	12.023	1.361	0.260	**0.354**	0.262	2 (8-19)	
(9-5)	[012346789]	[766674]	25.333	2.815	0.527	**1.483**	11.418	1.305	0.309	**0.403**	0.312	0 (8-6)	
(9-6)	[01234568A]	[686763]	29.778	3.309	0.582	**1.924**	13.567	1.556	0.387	**0.602**	0.391		
(9-7)	[01234578A]	[677673]	27.556	3.062	0.340	**1.042**	11.994	1.361	0.277	**0.377**	0.279	4 (8-23)	
(9-8)	[01234678A]	[676764]	29.778	3.309	0.582	**1.924**	14.036	1.610	0.404	**0.650**	0.408		
(9-9)	[01235678A]	[676683]	24.889	2.765	0.527	**1.457**	10.803	1.237	0.302	**0.374**	0.305		
(9-10)	[01234679A]	[668664]	21.333	2.370	0.658	**1.561**	9.723	1.176	0.474	**0.557**	0.486	•2 (8-28)	
(9-11)	[01235679A]	[667773]	27.556	3.062	0.340	**1.042**	12.011	1.361	0.267	**0.364**	0.269		
(9-12)	[01245689A]	[666963]	24.000	2.667	0.889	**2.370**	11.314	1.333	0.451	**0.601**	0.459		
Nonachordal:	Low Values (1):		21.333	2.370	0.340	**1.042**	9.723	1.176	0.260	**0.354**	0.262		
	Low Values (2):		21.333	2.370	0.340	**1.042**	9.723	1.176	0.260	**0.354**	0.262		
	Averages:		26.481	2.942	0.500	**1.451**	11.808	1.355	0.331	**0.450**	0.335		
	High Values:		29.778	3.309	0.889	**2.370**	14.036	1.610	0.451	**0.602**	0.486		

Ordinals:

C-B **IFA**
1. 9-2
1. 9-3
1. 9-7
1. 9-11
2. 9-4
3. 9-1
3. 9-9
4. 9-5
5. 9-10
6. 9-6
6. 9-8
7. 9-12

V-L **IFS**
1. 9-4
2. 9-3
2. 9-11
3. 9-1
3. 9-9
4. 9-2
4. 9-7
5. 9-5
6. 9-10
7. 9-12
8. 9-6
9. 9-8

V-L IFR
1. 9-4
2. 9-3
2. 9-11
3. 9-2
3. 9-7
4. 9-1
4. 9-9
5. 9-5
6. 9-6
7. 9-8
8. 9-12
9. 9-10

DECACHORDS			City-Block Metric				Vector-Length Metric						
Name	Normal	ICV	TD	AD	ADAD	**IFA**	TD	SD	SDSD	**IFS**	IFR	USPC	UAPC
(10-1)	[0123456789]	[988884]	25.600	2.560	0.192	**0.492**	11.281	1.131	0.086	**0.097**	0.086		
(10-2)	[012345678A]	[898884]	27.200	2.720	0.256	**0.696**	12.697	1.296	0.262	**0.339**	0.263		
(10-3)	[012345679A]	[889884]	25.600	2.560	0.256	**0.655**	11.238	1.131	0.131	**0.148**	0.131		
(10-4)	[012345689A]	[888984]	26.800	2.680	0.336	**0.900**	12.562	1.296	0.322	**0.417**	0.325		
(10-5)	[012345789A]	[888894]	25.600	2.560	0.192	**0.492**	11.281	1.131	0.086	**0.097**	0.086		
(10-6)	[012346789A]	[888885]	25.600	2.560	0.256	**0.655**	12.341	1.265	0.279	**0.353**	0.281		
Decachordal:	Low Values (1):		25.600	2.560	0.192	**0.492**	11.238	1.131	0.086	**0.097**	0.086		
	Low Values (2):		25.600	2.560	0.192	**0.492**	11.238	1.131	0.086	**0.097**	0.086		
	Averages:		26.067	2.607	0.248	**0.648**	11.900	1.209	0.194	**0.242**	0.195		
	High Values:		27.200	2.720	0.336	**0.900**	12.697	1.296	0.322	**0.417**	0.325		

Ordinals:

C-B **IFA**		V-L **IFS**		V-L IFR	
1. 10-1	2. 10-6	1. 10-1	3. 10-2	1. 10-1	3. 10-2
1. 10-5	3. 10-2	1. 10-5	4. 10-6	1. 10-5	4. 10-6
2. 10-3	4. 10-4	2. 10-3	5. 10-4	2. 10-3	5. 10-4

[1] See Isaacson [35] for an overview of the different approaches and problems associated with each, and also Buchler [3], Quinn [36], and Samplaski [37].

[2] Since the cardinality of the sets analyzed in this study range only from 3 to 10, the largest "population" of deviations is at most 10. The standard deviation, as a statistical measurement of the dispersion in a distribution, is most valuable when the population is large enough to minimise the importance of individual outliers. In this study, the standard deviation tends for the most part slightly to exaggerate differences between sets (although that is not always the case) and to exaggerate the importance of statistical outliers. While there are differences in the final values produced by the average deviations as compared to the standard deviations, it is unclear what significance, if any, can be attached to those differences. We invite readers to compare these two different sets of numbers for their comparative "accuracy" or for the value of the insights they provide.

[3] The reason why the radial angle of the vector is multiplied by the *square* of the vector's length is as follows. If there are two vectors with identical standard deviation from the standard deviation values but very different standard deviation values, the radial angle multiplied by the vector's length can counterintuitively give the longer vector a smaller value. For example, the two vectors [3,1] and [5,1] produce values of 1.018 and 1.007, respectively, if the radial angle is simply multiplied by the vector length. Squaring the vector lengths produces values of 3.218 and 5.132, respectively, which resonates with intuitive expectations.

[4] Comparing inequality factor—average vs. inequality factor—radial hexachordal ordinal rankings, (6-1) and (6-32) move from the lowest non-zero inequality factor—average to the fifth-lowest inequality factor—radial, while (6-24) moves from 13th lowest inequality factor—average to fourth lowest inequality factor—radial. Additionally, (6-30) changes from a 0.00 inequality factor—average to a low inequality factor—radial. There are occasional other such discrepancies in the ordinal rankings, but on the whole the measurements do a fair job of approximating one another.

[5] A tonal collection, for example, does not necessarily have a root or tonic note that has a higher individual deviation value than other tones in the collection. In the C major diatonic collection, C and E have individual deviations of 12/7, G and A, 14/7; F and B, 18/7; and D, 24/7. While D, with the highest individual deviation value, might assert itself in a Dorian modal context, in the context of common-practice tonality, which relies on the stacking of thirds in a harmonic context and the functionality of chords—not tones—the process is more complex, and not dependent on individual deviations of one pitch alone, but of combinations of pitches. Note, however, that the dominant seventh chord includes only tones at the higher end of the spectrum (G(14/7), B(18/7), D(24/7 and F(18/7)), while the tonic chord includes only tones at the lower end (C and E(12/7) and G(14/7)).

[6] There is no question that visual perception is different from aural perception, and that the intellectual properties involved in the two are not the same. Probably the most significant difference is that visual perception tends to be "all-at-once," and that the mind tends to grasp even complex patterns as single gestalts, while aural perception takes place through time, stimuli-by-stimuli. A psychological experiment into visual perception, however, could be constructed to take this difference into account: rather than presenting entire dot-patterns all-at-once, each constituent dot can be presented on a computer screen in random order, occupying the proper position on the screen. The subject would thus be more likely to perceive the

relationships between the dots, rather than all-at-once patterns. As a gauge of skewness and kurtosis in distance relationships, the inequality factor measure may thus be applicable. Regardless, the analogy is meant primarily in just that way: as an *analogy* to the pitch class relationships discussed above. (With thanks to Dr. Toben H. Mintz of University of Southern California for feedback from the standpoint of cognitive science and perceptual psychology.)

[7] If the answer to either question is yes, then the unique pitch class (with respect to the prime form) is listed in the the table in Part 3, followed by the Forte number of the relevant subset or superset.

[8] Note that, for trichords, nonachords, and decachords, because of the narrow range of SD values, there are no "significant" values; therefore there are no entries in the USPC column for trichords, tetrachords, and decachords, and there are no entries in the unique additive pitch class column for octachords, nonachords, and decachords.

[9] Clearly the cut-off point for significance could be changed.

[10] Note that even though the threshold for significance is higher for the second method than for the first, because of the margin involved with the second method in practice all sets that satisfy the criteria for the second method also satisfy those of the first. In the table in Part 3, therefore, relevant unique subtractive pitch class or unique additive pitch class numbers are preceded by a bullet ("•") if the second criteria are met as well as the first.