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Heterogeneity and State Dependence in Household Car Ownership: A Panel Analysis Using Ordered-Response Probit Models with Error Components

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Heterogeneity and State Dependence in Household Car Ownership: A Panel Analysis Using Ordered-Response Probit Models with Error Components

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ABSTRACT

Dynamic, disaggregate choice models which use longitudinal data are known to have clear advantages over cross-sectional models, but they also have their own unique estimation problems. The correlation among unobserved error components (“heterogeneity”) that is likely to exist in such data sets can be the source of apparent state dependence, but true state dependence is also possible. A review of car ownership models reveals that the issue of heterogeneity versus true state dependence has not been adequately addressed in the transportation literature. This paper develops computationally convenient ordered-response probit models for panel data, estimates models of car ownership, and performs tests of heterogeneity versus true state dependence. Conclusions in (the more general) one-factor models are found to differ from those obtained from (the more restricted) components of variance models, and the issue of initial conditions is also found to affect the conclusions.

INTRODUCTION

Many disaggregate models of household car ownership, car-type choice, and utilization have been proposed during the past decade and a half. These developments have been driven by the recognition that the aggregate time-series models that had previously dominated automobile demand forecasting were deficient in some respects. Specifically, they did not capture the causal relationships underlying household behavior, thus limiting their accuracy, versatility, and policy sensitivity (see, e.g., Manski, et al., 1978, for a review of aggregate automobile demand models). In contrast, disaggregate models formulated at the household level possess the structure necessary for depicting the causal mechanisms that govern household behavior.

However, recently it has been recognized that disaggregate models based on cross-sectional data are subject to their own sets of limitations. They may be flawed because elasticities evaluated using a cross-sectional model may not be identical to longitudinal elasticities associated with behavioral changes of each behavioral unit, thus they may not offer accurate forecasts. Furthermore, the presence of unobserved contributing factors that are correlated with observed variables will lead to biased coefficient estimates, which in turn will produce false elasticities and forecasts. This motivates the use of dynamic models that are based on longitudinal observation of individual behavioral units.

There are many additional behavioral, as well as statistical, reasons to favor such dynamic models (for discussions, see Heckman, 1981, Davies and Pickles, 1985, Goodwin, et al., 1987, Goodwin, et al. 1989, Kitamura, 1989). For example, Goodwin and Mogridge (1981) note the “resistance to change” as one of the dynamic aspects of car ownership behavior that previous cross-sectional models have failed to account for. Factors that motivate the use of dynamic models include:

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• asymmetry in the magnitude of response (i.e., elasticity may be different depending on the direction of change, say, between income increase and decrease),

• asymmetry in the speed of response (i.e., the time lag between the time when a change takes place in the travel environment and the time a response takes place may be different depending on the direction of change),

• influence on behavior of past experiences or future expectations (e.g., brand loyalty), and,

• effects of temporal changes and trends (e.g., increasing license ownership).

Possibly underlying apparent behavioral asymmetry are the effects of incomplete information, searching, experimentation, learning, inertia or habit, risk aversion behavior under uncertainty, and various constraints that prohibit immediate response. As a result, behavior may be dependent on its past history, exhibiting asymmetry and hysteresis.¹

Many modeling approaches can be taken to capture such behavioral characteristics. For example, explanatory variables from previous time points (lagged independent variables) may be introduced to represent response lags. The increase and decrease over time in an contributing factor may be represented by separate variables to capture asymmetric responses. Dependence on past experience may also be incorporated by introducing variables that represent past behavior (lagged dependent or endogenous variables).²

The apparent dependence of current choices on past choices ("state dependence"), as shown through simple lagged dependent variable models, may actually be due to heterogeneity. Heterogeneity here refers to variations in unobserved contributing factors across (otherwise observationally equivalent) behavioral units. If behavioral differences are largely due to unobserved factors, and if unobserved factors are invariant over time but correlated with the measured explanatory variables, then estimates of model coefficients will be biased if this heterogeneity is not taken into account. In particular, this may offer a false indication that behavior has been fundamentally altered by previous choices ("true state dependence"). Distinguishing between true state dependence and apparent state dependence resulting from heterogeneity (or, "spurious state dependence") is critical for proper characterization of behavioral variation. Having made distinctions among various sources of state dependence, in the remainder of this paper the phrase "state dependence" will refer to true state dependence.

The problem of distinguishing heterogeneity from state dependence is well-recognized as an important issue in econometrics. Addressing this problem with discrete choice models, however, requires elaborate models and estimation procedures. The beta-logistic model (Heckman and Willis, 1977) is a discrete-choice version of an error component model that allows for heterogeneity in longitudinal behavior; however, it can only make use of explanatory variables from the first time period, and thus cannot incorporate endogenous variables for modeling state dependence. Davies (1984) generalizes the beta-logistic model to accommodate time-varying variables and so-called "feedback effects," i.e., information on behavior from prior periods, and estimates a renewal model of residential mobility. A few examples of beta-logistic model applications exist in the transportation field (Uncles, 1987, Smith, et al., 1989). To our knowledge, however, no single study has performed a rigorous test of state dependence versus heterogeneity in travel behavior studies using discrete choice. In fact, as the review of household car ownership models of the next section reveals, most "dynamic" discrete choice models with lagged endogenous variables are estimated with the restrictive assumption that heterogeneity does not exist.
This study aims at a rigorous treatment of heterogeneity and state dependence in household car ownership behavior observed over time. Dynamic ordered-response probit models with error components are formulated and estimated using the algorithm described in Bunch and Kitamura (1989). The error-component ordered-response probit models allow more flexible formulation of the error terms, and thus offer a better accounting of heterogeneity than do beta-logistic models.

The objective of the study is to determine the characteristics of household car ownership behavior through thorough examination of hypotheses on state dependence and heterogeneity by using alternative specifications of the error terms. It is anticipated that coefficient estimates may vary greatly depending on the formulation of the dynamics of household car ownership, potentially leading to drastically different forecasts. The intent of the study is to present how dynamic characteristics of car ownership can be statistically examined, and how resulting predictions are influenced by the hypotheses adopted.

The paper begins with a review of dynamic models of car ownership which have appeared in the transportation literature. Section 3 presents detailed definitions of heterogeneity and state dependence which are needed for the development of model formulations in Section 4. Sections 5 and 6 present empirical results and conclusions, respectively.

**Dynamic Models of Household Car Ownership: A Review**

Dynamic, disaggregate choice models of household car ownership (i.e., those which attempt to incorporate lagged dependent variables) are relatively few, and those that properly account for the endogenous nature of the lagged dependent variables are even fewer. Some models are formulated using behavioral information related to past choices, e.g., a dummy variable in a car-type choice model for the same make as the previously owned car. Yet in virtually every model that incorporates them, such endogenous variables are treated as if they are exogenous, which gives rise to a variety of statistical difficulties.

For example, it is quite common for dynamic studies to treat initial conditions as if they are fixed and exogenous, even though the initial lagged dependent variables are stochastic. As Heckman (1981b) has shown, this may led to serious estimation errors. In addition, problems arise when estimating discrete choice models with lagged endogenous variables in the presence of serially correlated errors, as noted above. Although practical methods exist for linear models with lagged dependent variables, estimating nonlinear models is computationally much more demanding under the assumption of serially correlated errors. In many of the studies of household car ownership reviewed below, the fact that lagged dependent variables are endogenous is not acknowledged at all. Among those studies which do, most facilitate model estimation by assuming that all error terms are serially independent.

Consequently, most studies have been unable to directly address such questions as state dependence versus heterogeneity. The presence of heterogeneity in car ownership is shown in Uncles (1987) and Smith, et al. (1989) using beta-logistic models. These studies, however, do not address the issue of state dependence. Although Kitamura (1988) incorporates the possibilities of both state dependence and heterogeneity, no rigorous statistical test is performed in that study. In the discussion below, disaggregate household car ownership models in the literature are reviewed, focusing on their treatment of lagged variables.

The use of lagged variables became prevalent when discrete choice models of car ownership level were extended to include vehicle-type choice. In one of the earliest studies of this kind, Manski, Sherman, and Ginn (1978) estimate a multinomial logit car-type choice model, where car-types are defined by combinations of make, model, and vintage. The search-
and-transaction cost is represented by a dummy variable "which takes the value zero for vehicles currently owned by the household and one for all vehicles obtainable on the market" (op. cit., p. 26). Manski, et al. recognize that the search/transaction cost dummy variables form "a set of lagged endogenous variables" (op. cit., p. 29), but they estimate the model assuming the absence of serial correlation. Their justification is that the omission of transaction dummy variables would have led to more serious specification errors than ignoring serial correlation.3

In their development of dynamic household vehicle transaction models, Hocherman, et al. (1983) use similar dummy variables to represent the effect of transaction costs (search costs and information costs), brand loyalty, and income effects. These authors also acknowledge that problems might arise from ignoring temporally correlated errors, but estimate their nested-logit models assuming that serial correlation is not present.

This is also the case for the system of models developed by Train and Lohrer (1982) for predicting the number of cars, car-types, and utilization. They use transaction dummy variables in their "class/vintage" submodels to represent "the psychic, search, and other transaction costs associated with buying a new vehicle" (op. cit., p. 41). The endogenous nature of these variables is not discussed and possible estimation problems are not recognized in this study. Similarly, the models of Berkovec and Rust (1985) include transaction dummy variables without discussion of their endogeneity.

Mannering and Winston (1985) present a "dynamic" model system similar to Train and Lohrer (1982). It is comprised of nested-logit models of car ownership level (number of cars) and vehicle-type choice (make, model and vintage), combined with linear utilization models (vehicle-miles traveled). The paper emphasizes dynamic aspects of car ownership and utilization behavior, e.g., stationarity, state dependence, "brand preference" and "brand loyalty" (op. cit., p. 216).

Most striking in Mannering and Winston (1985) is the complete neglect of the possible intertemporal correlation in the error terms, despite the prominence of lagged dependent variables in their model system. The "vehicle-type choice models" contain up to two-period lagged utilization variables for the "same vehicle" and vehicles of the "same make" that the household may have owned. The same lagged variables are also used in the linear vehicle utilization models (for which consistent and practical estimation methods have existed). These lagged endogenous variables are treated as if they are fixed, exogenous variables, constituting an assumption of nonstochastic initial conditions. In a subsequent paper, Mannering discusses the same model system, but in this account gives explicit recognition to the "endogenous variable problem" (Mannering, 1986, p. 3). A decision is taken to assume that disturbances are serially independent because of "the difficulty in accounting for serial correlation in the presence of lagged endogenous variables in discrete choice models" (op. cit.).

The issue of state dependence versus heterogeneity is not addressed in any of the studies discussed so far. It is also evident that these attempts to include dynamic elements may have resulted in model specifications which were too complex for the estimation methods that were readily available. The inclusion of lagged endogenous variables, and the introduction (either explicitly or implicitly) of the assumption of serially independent errors require a careful re-examination of the various behavioral characteristics which these models are hoping to capture.

An extensive discussion of dynamic elements--"heterogeneity, nonstationarity, and intertemporal dependence"--is given by Hensher and Wrigley (1986). In particular, they note "confoundment" due to the correlation between temporally invariant observed and unobserved variables, and discuss the issue of true versus spurious state dependence. They also suggest
the use of beta-logistic models (Heckman and Willis, 1977) and their generalizations (see discussion below).

Another model system of car ownership, car-type choice, and utilization is proposed by Hensher, et al. (1987). A unique feature of this model system is the inclusion of "expectation" and "experience" terms as components of the "conditional intertemporal indirect utility" (op. cit., p. 8). Both expectation and experience terms are formulated for each alternative as a geometric series of its attributes observed over time, and a multiplier (the absolute difference between the attribute of an alternative and the corresponding attribute of the chosen alternative is used to represent the experience term of the former). Another unique feature is the inclusion of an "initial condition term" (op. cit., p. 8) which is again expressed as a geometric series of observed attributes.

Hensher, et al. (1987) avoid the problem of serial correlation by "substituting the exogenous choice-determining variables in previous periods for the previous period endogenous-choice variables" (op. cit., p. 9). The treatment of the initial conditions in this study is based on the same idea: exogenous variables are used as instruments, and are used in place of the lagged endogenous elements. Presumably because of this, the issue of heterogeneity versus state dependence is not examined in Hensher, et al. (1987). Indeed there is no point in doing so if obtaining model coefficients is the concern because, once lagged endogenous variables have been replaced by exogenous variables, consistent estimates can be obtained in the presence of serial correlation.

Despite these conscious efforts, the estimations in Hensher, et al. may not be entirely flawless. The "exogenous" variables comprising the "experience" effects in the indirect utility function appear to be, in fact, endogenous because they include the attributes of the chosen alternative. Dubin and McFadden (1984) propose three alternative approaches to account for this type of endogeneity in a vehicle choice-utilization context. No such treatment is apparent in the effort by Hensher, et al.

As previously noted, an application of the beta-logistic model to travel behavior can be found in Uncles (1987). The study, however, is subject to the limitations inherent in the beta-logistic model as it was originally proposed (Heckman and Willis, 1977). Specifically, the model assumes that the choices made over time by each behavioral unit follow a Bernoulli process comprised of repeated independent draws from a set of time-invariant binary choice probabilities. The model allows for heterogeneity—the probabilities are assumed to vary across behavioral units due to unmeasured as well as measured effects—but assumes that choices are state independent, and that exogenous variables are invariant over time.

This original beta-logistic model has been extended to incorporate multinomial choice (Dunn and Wrigley, 1985), and to allow for time-varying exogenous variables, "feedback effects," and initial conditions (Davies, 1984). Smith, Hensher and Wrigley (1989) adopt Davies' beta-logistic model in their analysis of household vehicle transactions. The analysis in Smith, et al. is based on 373 households which maintained exactly one vehicle during a four-wave survey period. Transactions behavior is described as the binary choice between replacing versus keeping the car. Vehicle age, an endogenous variable, is included in the model formulation, roughly representing the state dependence of vehicle transactions behavior (holding duration would be a more precise indicator). However, state dependence based on previous choices is not considered, and thus no hypothesis tests are performed to examine the specific issue of current interest.

Presumably the study that most closely addresses the issue is the panel analysis of car ownership by Kitamura (1988), which explicitly incorporates both state dependence and
heterogeneity (as expressed by serially correlated errors). Heckman-type correction terms are used to obtain consistent coefficient estimates. The single-equation estimation method used, however, is not efficient and, as noted earlier, performing a rigorous test of state dependence versus heterogeneity was outside the scope of that work.\(^4\)

As this review indicates, the development of dynamic models of car ownership, car-type choice, or transactions, is unfortunately somewhat limited. This is mainly due to the difficulty in estimating discrete choice models with lagged endogenous variables under the presence of serially correlated errors.\(^5\) Consequently no study has rigorously examined alternative hypotheses involving heterogeneity versus state dependence. One purpose of this study is to overcome these limitations.

**OPERATIONAL DEFINITIONS OF HETEROGENEITY AND STATE DEPENDENCE**

Mathematical expressions of heterogeneity and state dependence will naturally vary depending on how these concepts are specifically defined. For example, heterogeneity may imply the variation in *observed* exogenous variables, or may refer to the variation in *unobserved* variables. Similarly, there are various possible types of state dependence, some of which appear in the studies reviewed in the previous section. For example, Hensher and Wrigley (1986) list: Markovian state dependence on the current state; dependence on the number of visits to the respective states; duration of the occupancy at the current state; and durations of the occupancy at previous states. Even when a particular type of state dependence is specified, there are many possible model specifications that can represent the dependence.

As noted earlier, heterogeneity in this study refers to the variation in unobserved contributing factors across behavioral units. If these variations are purely random across behavioral units and over time, then "heterogeneity" is inconsequential, presenting no difficulties for model estimation, and providing no interesting behavioral implications. This study is concerned with the case where the unobserved errors are cross-sectionally independent but temporally correlated for each behavioral unit. One possible approach is to model the errors by an autoregressive process

\[
\varepsilon(i, t) = \rho \varepsilon(i, t - 1) + U(i, t), \quad t = 1, 2, ..., T, \text{ and } i = 1, 2, ..., I, \tag{1}
\]

where \(\varepsilon(i, t)\) is the error term for individual \(i\) at period \(t\), \(\rho\) is the coefficient of serial correlation, \(U(i, t)\) is an independent random error term, \(T\) is the number of time periods, and \(I\) is the number of behavioral units in the data. We further assume that, for \(i = 1, 2, ..., I\) and \(t = 1, 2, ..., T\),

\[
\begin{align*}
E[\varepsilon(i, t)] &= E[U(i, t)] = 0, \\
E[\varepsilon(i, 0)\varepsilon(i', 0)] &= 0, \\
E[\varepsilon(i, 0)U(i, t)] &= E[\varepsilon(i, t)U(i', t')] = E[U(i, t)U(i', t')] = 0 \text{ for } i \neq i', \\
E[U(i, t)U(i, t')] &= 0 \text{ for } t \neq t', \text{ and,} \\
E[U(i, t)^2] &= \sigma_U(t, t).
\end{align*}
\]

Note that under this formulation the error term \(\varepsilon(i, t+1)\) is conditionally independent of \(\varepsilon(i, t')\), for \(t' = 0, 1, ..., t - 1\), given \(\varepsilon(i, t)\).

A more simple alternative formulation for heterogeneity may be defined by introducing an individual-specific, time-invariant error component, \(q(i)\),

\[
\varepsilon(i, t) = q(i) + U(i, t), \tag{2}
\]
where \( E[q(i)] = 0, E[q(i)U(i, t)] = E[q(i)U(i', t)] = 0, E[q(i)^2] = \sigma_i^2 \) and \( U(i, t) \) is as defined above. In this approach, unobserved attributes are assumed to be distributed across behavioral units that are otherwise observationally identical, and this endowment of attributes does not vary over time.

Equation (2) defines the components of variance model ("CVAR"). It is a special case of the one-factor model ("OFAC") discussed by Heckman (1981a), given by

\[
\epsilon(i, t) = \alpha^*(t)q(i) + U(i, t),
\]

where \( \alpha^*(t) \) is a constant factor-loading for period \( t \). These two schemes provide certain computational advantages, and are used in combination with ordered-response probit models to represent heterogeneity in the empirical analysis presented below. Model formulations are discussed in more detail in the next section.

Let \( Y(i, t) \) be a measure of behavior at time \( t \). Models are easily formulated which include direct dependencies of current behavior, \( Y(i, t) \), on behavior observed at previous time periods, i.e., \( Y(i, t'), t' = 1, 2, ..., t - 1 \). In particular, consider the expression

\[
Y(i, t) = X(i, t)\beta + \delta Y(i, t - 1) + \epsilon(i, t),
\]

where \( X \) is a vector of exogenous variables, \( \beta \) is a vector of coefficients, and \( \delta \) is a scalar coefficient. Here the behavioral dependence is captured by the simple additive term \( \delta Y(i, t - 1) \). A more general formulation would allow the coefficient vector \( \beta \) to vary as a function of \( Y(i, t-1) \); however, we assume that the past does not affect the marginal contribution of each exogenous variable. This is in part due to the convenience it offers in model estimation; the more general formulation should be tested in future efforts.

Note that this model formulation may have multiple behavioral interpretations. For example, equation (4) may be interpreted either as (i) a model including experience effects, (ii) a model of partial adjustment, (iii) a model including response lags due to habit persistence, cost of change, etc., or (iv) an adaptive expectations model depicting planning behavior. In the context of this study, \( Y(i, t) \) is a discrete choice variable, and hence we interpret the model as a state dependence model analogous to a first-order Markov process. Examination of a more extensive range of model formulations and interpretations, including the use of lagged exogenous variables, also belongs to future efforts.

MODEL FORMULATION

The models considered here fall into the category of discrete choice models, and generalize the work of Heckman (1981a) and Heckman and Willis (1977). The focus of the attention here is on ordered choice models, i.e., choice models whose discrete dependent variable has a natural interpretation as an increasing integer. Specifically, the study is concerned with the number of cars owned by a household (0, 1, and 2 or more), observed at equi-spaced discrete time points.

Heckman (1981a) gives a very general framework for panel analysis of binary choices, which we extend in this study to trinary, ordered choices. The Heckman framework encompasses many complex stochastic processes, but for our purposes the discussion can be simplified by considering the following latent random variable model:

\[
V(i, t) = Z(i, t)\beta + D_1(i, t - 1)\gamma_1 + D_2(i, t - 1)\gamma_2 + \epsilon(i, t),
\]
and

\[
Y(i, t) = \begin{cases} 
0 & \text{if } -\infty < V(i, t) \leq \tau_1 \\
1 & \text{if } \tau_1 < V(i, t) \leq \tau_2 \\
2 & \text{if } \tau_2 < V(i, t) < +\infty 
\end{cases}
\]  

(5b)

where

\( V(i, t) \) is the latent ordinal preference index for individual \( i \) in time period \( t \).

\( Y(i, t) \) is the observed ordered-response choice (in this case the number of cars owned),

\( Z(i, t) \) is a \( 1 \times k \) vector of exogenous explanatory variables,

\( D_1(i, t) = 1 \) if \( Y(i, t) = 1 \), and 0 otherwise,

\( D_2(i, t) = 1 \) if \( Y(i, t) = 2 \), and 0 otherwise.

\( \beta \) is a \( k \)-vector of parameters,

\( \gamma_1 \) and \( \gamma_2 \) are weights for the state in the previous period.

\( \tau_1 \) and \( \tau_2 \) are constant thresholds, and

\( \epsilon(i, t) \) is a random variable with \( E[\epsilon(i, t)] = 0 \).

As noted, the model is formulated for trinomial ordered-response choice. \( Y(i, t) \) is a latent one-dimensional random variable of increasing household preference for cars, and the \( D_i \)'s are dummy variables representing the observed (discrete) choices.

The implication of the lagged dummy variables is that two households that are otherwise observationally equivalent in time period \( t \) may have different choice probabilities due to latent preference shifts associated with having experienced the choice made during the last period. This is an example of true state dependence, where we have specifically chosen a first-order Markov model in this instance. Such preference shifts could be associated with having experienced the convenience and flexibility associated with owning a car (or cars).

Heckman (1981b) uses a binary version of this model in his study of estimation problems associated with initial conditions. In another study, Heckman (1981c) applies a similar (binary) model to an empirical study of labor force participation, in which the shift is a function of the sum of the dummy variables for all previous periods. The binary character of these examples is an important distinction, since they neatly fit into the framework of random utility maximization developed by McFadden (1981) and others. In contrast, our ordinal latent variable model, while natural for the problem of car ownership level, does not fall into this framework, and requires a different behavioral interpretation.

From the point-of-view of practical estimation for panel analysis, there are two important issues. First, equation (5) must be combined with an assumed error process for the \( \epsilon(i, t) \)'s so that the resulting panel model is computationally tractable. This will be a direct function of the intertemporal covariance structure of the \( \epsilon(i, t) \)'s. Second, allowance must be made for the problem of initial conditions described by Heckman (1981c). Each of these is considered next.

**Covariance Structures for \( \epsilon(i, t) \)**

The error component models of equations (2) and (3) can be readily combined with equation (5) to form ordered-response panel data models, as we now describe. Normal distributions are used for all error components, leading to models with normal mixing distributions (a mixing distribution refers to the distribution of error components). With the distributional assumptions already introduced in the previous section, the models lead to likelihood functions which require only two-dimensional integration for a sequence of choices made in \( T \) periods. This can be shown for the components of variance model as follows. For
normalization purposes, let $E[U(i, t)^2] = 1$ for $t = 1, 2, ..., T$. For notational convenience define

$$W(i, t) = Z(i, t) + \alpha_{1}(t) + \alpha_{2}(t),$$

for $t = 1, 2, ..., T$, and denote a series of $T$ choices by $(c(1), c(2), ..., c(T))$, where $c(t) = 0, 1, 2$ for $t = 1, 2, ..., T$. Since the $U(i, t)$'s are temporally uncorrelated, the probability of observing $(c(1), c(2), ..., c(T))$ is given by

$$Pr\left[Y(i, 1) = c(1), Y(i, 2) = c(2), ..., Y(i, T) = c(T)\right] = \frac{\Phi(\tau_{c(1)} - W(i, 1) - q(i)) - \Phi(\tau_{c(1)} + 1 - W(i, 1) - q(i))}{f_{q}(q(i))dq(i)},$$

where $\Phi$ is the standard normal cumulative distribution function, $f_{q}$ is the density function of $q(i)$, $\tau_{0} = -\infty$, and $\tau_{3} = \infty$. The integration involved is two-dimensional, one to evaluate $\Phi$ and the other taken with respect to $q(i)$. (Equation 6 is used here for illustrative purposes. Normalization restrictions are required for the estimations discussed below.)

**Components of Variance (CVAR) Model:** This model uses the error structure defined in equation (2). We add the distributional assumptions that both components are iid normal, i.e., $E[U(i, t)^2] = \sigma_{U}(t, t) = \sigma_{U}^2$ for all $i$ and $t$, with $U(i, t) \sim N(0, \sigma_{U}^2)$, and that $q(i) \sim N(0, \sigma_{q}^2)$ for all $i$. Define the disturbance vector for individual $i$ by $\varepsilon(i) = (\varepsilon(i, 1), \varepsilon(i, 2), ..., \varepsilon(i, T))^T$. The covariance matrix for $\varepsilon(i)$ has identical diagonal elements $\sigma_{q}^2$ and identical off-diagonal elements $\sigma_{q}^2$. To obtain estimates, it is necessary to choose a normalization so that the model is identified. The scale of the model must be fixed: one choice is to estimate the intertemporal correlation $\rho = \sigma_{q}^2/(\sigma_{U}^2 + \sigma_{q}^2)$. (Other possibilities are to assign either $\sigma_{U}^2$ or $\sigma_{q}^2$ to be an arbitrary constant.) Note that $\rho$ is non-negative in this formulation; heterogeneity in a CVAR model is expressed by the temporally invariant individual specific term, $q(i)$, which always produces non-negative intertemporal correlation. The other requirement concerns the $W(i, t)$'s and the $\tau$'s. Because choice is a function of differences in these terms, only relative values can be identified. In this study the $Z(i, t)$'s contain a constant term, and the normalization $\tau_{1} = 0$ is used to fix location.

**One-Factor (OFAC) Model:** This model uses the error structure defined in equation (3). The distributional assumptions for $q(i)$ are the same as for CVAR, but those for $U(i, t)$ are relaxed to allow the variances to change over time, i.e., $U(i, t) \sim N(0, \sigma_{U}(t, t))$, for $t = 1, ..., T$. This yields a covariance matrix for $\varepsilon(i)$ which has diagonal elements given by

$$E[\varepsilon(i, t)^2] = \sigma_{U}(t, t) = \alpha^{*}(t)^2\sigma_{q}^2 + \sigma_{U}(t, t),$$

and off-diagonal elements given by
\[
E[\varepsilon(i, t)\varepsilon(i, t')] = \alpha^*(t)\alpha^*(t')\sigma_q^2, \text{ for } t' \neq t. \tag{8}
\]

As before, a normalization is required to render the model estimable. One normalization used in Heckman (1981c) can be written in terms of the correlation matrix for \(\varepsilon(i)\), and the \(\sigma_q^2(t, t)\)'s. The special one-factor structure gives that the correlation between \(\varepsilon(i, t)\) and \(\varepsilon(i, t')\) is
\[
\rho_{tt'} = \alpha(t)\alpha(t'), \text{ where the } \alpha(t)'s \text{ are parameters to be estimated. The normalization } \sigma_q(1,1) = 1 \text{ is also required, along with } \tau_1 = 0 \text{ as before. (Note that } \alpha(t) \neq \alpha^*(t), \text{ and that } \sigma_q^2 \text{ and } \sigma_U(t, t) \text{ are not identified in this normalization.) As previously noted, this special structure results in probability calculations which require only two-dimensional integrals. See Heckman (1981a) for expressions involving binary choices; for our ordered models, the univariate normal CDFs are replaced by the appropriate univariate standard normal integrals.}

The OFAC model is an attractive alternative for panel analysis, exhibiting much more flexibility than the CVAR model, while at the same time retaining the computational advantages. However, the OFAC structure does imply some special restrictions, as described in Heckman (1981a). For \(T > 3\) the OFAC model requires that the error process defined in equation (3) be nonstationary to be interesting. In other words, restricting \(\sigma_{q}(t, t) = \sigma_{q}^2\) for all \(t\) essentially forces the OFAC model to be equivalent to the CVAR model. Unfortunately, error processes which modelers find appealing are often stationary, and the bottom line is that many interesting error structures simply cannot be one-factor analyzed.

For the special case \(T = 3\) the above restrictions do not apply, and there are some interesting stationary processes which can be captured by OFAC models. In particular, the first order stationary Markov process characterized by
\[
\varepsilon(i, t) = \rho \varepsilon(i, t - 1) + U(i, t) \tag{9}
\]
may be accommodated by restricting \(\alpha(1) = \alpha(3)\) and \(\alpha(2) = 1\) (along with \(\sigma_q(1, 1) = \sigma_q(2, 2) = \sigma_q(3, 3) = 1\))—see Heckman (1981c). Finally, we note that models more general than CVAR and OFAC may be formulated by assuming a general error structure for the \(\varepsilon(i, t)\)'s. However, such an approach requires multivariate integration of dimension \(T\), and produces estimation problems of the same level of difficulty as the multinomial probit model.

**Initial Conditions**

One technical problem that arises for dynamic models with lagged dummy variables when applied to "short" discrete panel data sets (i.e., data sets with small \(T\)) is the treatment of initial conditions. The choice behavior being studied here is viewed as a discrete-time stochastic process in which the behavior at time \(t\) may depend on that at time \(t-1\); other models (not considered here) could include even more lagged terms, as well as lagged exogenous variables. Such processes must in principle have starting points. Formally, at the true beginning of the stochastic process any "lagged" information will be available and be represented as nonstochastic initial conditions.

Unfortunately, it is essentially always the case that observed panel data sets do not include the beginning of the process. Let the full process be represented by observations taken at \(t = 1, 2, ..., T\), plus any nonstochastic initial conditions at periods \(0, -1\), etc. Typical panel data sets are observed only for \(t = J, J+1, ..., T\), where \(J >> 1\). Even if some pre-sample information is available, this information is usually also stochastic, coinciding with previous points in the process rather than with the nonstochastic initial conditions. Ideally, maximum
likelihood estimation should be performed using data from the entire process, including the initial conditions. Or at the very least, a long panel could be estimated, minimizing the effects of the missing initial conditions.

Heckman (1981b) addresses this problem directly, noting that most social scientists simply ignore the issue. He illustrates the deleterious effects of ignoring the problem in short panels via Monte Carlo simulations, and recommends two possible solutions. Unfortunately, one of the procedures is relatively difficult to implement, and the other is rather ad hoc and sketchy. The second of these is used in an empirical study of female labor force participation (Heckman, 1981c), and Heckman acknowledges that the first procedure has not been attempted in an empirical study.

The Heckman ad hoc procedure is an approximate solution to the initial conditions problem, and he claims that it is acceptable for testing the null hypothesis of no state dependence, e.g., $\gamma_1 = \gamma_2 = 0$ in equation (5). We have implemented a version of this procedure, which we now describe.

First, we note that there were four periods of data available for our study. Due to the considerations described above for the OFAC model, estimating models with $T=3$ rather than with $T=4$ was attractive due to the potential added flexibility of the model specifications. In addition, if models were estimated using periods (2, 3, 4), then lagged variables would be available from period 1. However, simply using lagged variables from period 1 as though they were nonstochastic could cause estimation problems.

A cross-sectional probit model was estimated for period 1, and then predicted probabilities for each household were generated. The probabilities corresponding to the dummy variables in equation (5) were used as explanatory variables for period 2 in place of the observed lagged dummies. A separate set of coefficients was estimated for these explanatory variables, which represented "predicted pre-sample information" in our first-order model. Before proceeding to empirical results, we discuss some issues pertaining to the choice of model formulations in our study.

**Error-Component Ordered-Response Probit versus Beta-Logistic Models**

The fundamental concepts underlying the error-component ordered-response model proposed here and the beta-logistic model are the same: a series of choices made by an individual (or household) can be described by a series of cross-sectional discrete choice models which have been written as conditional on the individual-specific error components. Since the individual-specific terms are unobserved, the probability of observing the actual sequence of choice outcomes can be evaluated by assuming an error distribution and integrating out the error components. Normal distributions are used to derive the panel data ordered-response probit model. In motivating the beta-logistic model, on the other hand, it is simpler to assume the final functional form for the mixing distribution of the choice probabilities and work backwards. Specifically, the distribution of choice probabilities is conditioned on a single set of exogenous variables and is assumed to have a beta distribution. This is convenient, since the mean choice probabilities then reduce to the standard logit model--see Heckman and Willis (1977).

There are advantages to using the error-component ordered-response probit models defined above. First of all, representation of multinomial ordered choice does not increase the model's complexity or computational requirements versus binary models. Secondly, its computational requirements in model estimation are modest because its likelihood function can be evaluated through one-dimensional numerical quadrature. (As is shown in equation (6), the
integrand includes one-dimensional normal integrals; hence, the full integration is two-
dimensional.)

Although the beta-logistic model is computationally attractive in its original closed-form
representation, its advantages quickly diminish when it is extended to incorporate time-varying
exogenous variables. To achieve this flexibility, Davies (1984) proposes the use of a power
series expansion to express the choice probability in each period as a function of (i) the
probability in the "reference period" and (ii) the differences between the exogenous variables in
the period of interest and those in the reference period.

Most critically, our calculations using numerical examples indicate that the performance
of this series expansion is poor. This is especially true with nontrivial changes in the
exogenous variables, and when the choice probability is close to unity. For example, in some
cases the approximation requires at least four terms in order to get one significant digit of
accuracy.  

The proposed ordered-response probit model, on the other hand, does not require such
approximations. Perhaps the most important advantage of the probit approach is the flexibility
it allows in the specification of the error terms, and the statistical examination of alternative
hypotheses concerning heterogeneity. The CVAR and OFAC models are examined in this
study, and future studies may be extended to include more general formulations.

Finally, a comparison of error-component ordered-response probit models and the
multinomial probit model applied to discrete panel data (Daganzo, 1979, Daganzo and Sheffi,
1982, Johnson and Hensher, 1982) deserves some mention. Clearly, in the case of the CVAR
and OFAC models, the ordered models have the computational edge. They only require one-
dimensional numerical quadrature, regardless of the number of alternatives and the number of
periods. The most general ordered models require integrals of dimension T. Applications of
the multinomial probit model, on the other hand, require integrations of dimension (J-1)T,
where J is the number of choice alternatives. Of course, the latter is capable of accommodating
unordered alternatives, which may be the only appropriate formulation for many problems.

EMPIRICAL RESULTS

Alternative specifications of error-component ordered-response probit models are
estimated using a sample from the Dutch National Mobility Panel data set. This subsample
contains 605 households that participated in the panel survey from March 1984 through March
1987. Data from four survey waves conducted 12 months apart are used in the model
estimation. For the background of the Dutch Panel survey, see Golob, Schreurs and Smit
(1986) and van Wissen and Meurs (1989).

The specification of the latent variable in this study is an extension of those developed in
Kitamura (1987, 1988), and is constructed on the premise that demographic and socio-
economic attributes of a household are the major determinants of its car ownership. This
viewpoint, supported by empirical results in the literature, can also be found in Manski and
Sherman (1980).

The explanatory variables of the model are: square-root of annual household income
("SQRTInc"), number of workers in household ("Workers"), number of adults in household
("Adults"), number of children of 17 years old and over ("Child17") number of persons in
household ("HHSize"), number of drivers in household ("Drivers"), 0-1 dummy for access to
good mass transit ("Gmtgrp"), and 0-1 dummy for households with individuals with higher
education ("HHeduc"). These variables are highly correlated and, partly because of this, not
all variables are significant in every model estimated here. However, the same set of variables
is included in all the models to facilitate comparisons across models with different assumptions about heterogeneity and state dependence.

The following models are estimated:

1. Independent choice model, which assumes no state dependence or heterogeneity (equivalent to pooled cross-sectional ordered probit with no lagged dependent variables),
2. Pure state dependence model, with state dependence but with no heterogeneity,
3. Components of variance model with no state dependence assumed,
4. Components of variance model with state dependence assumed,
5. One-factor model with no state dependence assumed,
6. One-factor model with state dependence assumed,
7. One-factor model with state dependence assumed, and initial conditions assumed to be non-stochastic, and
8. One-factor model with state dependence and stationarity (temporally invariant error variances) assumed.

Maximum likelihood estimates were obtained using the algorithm described in Bunch and Kitamura (1989), and asymptotic t-scores were computed using variance estimates obtained from the inverse of the estimated Hessian matrix at the solution. The error component for each household was integrated out using four-point Gauss-Hermite quadrature—see, e.g., Butler and Moffit (1982).

Components of Variance Models

Estimated model coefficients of the components of variance models with and without true state dependence are presented in Table 1 along with those of the independent choice model and pure state dependence model. A comparison of goodness-of-fit statistics immediately indicates that the independent choice model is an inadequate specification. Accounting for intertemporal dependence is essential in dynamic analysis of household car ownership.

The components of variance model with no state dependence (Model 3) shows strong heterogeneity: the positive intertemporal correlation coefficient, $\rho$, is 0.71 with a highly significant t-statistic. Similarly, the coefficients of the lagged endogenous variables in the pure state dependence model (Model 2) are highly significant. Assuming either heterogeneity or state dependence substantially improves the fit.

The significance of heterogeneity and state dependence can be examined further by performing statistical tests using the (nested) models in Table 1. Examination of the asymptotic t-statistic for $\rho$ in Model 4 indicates that heterogeneity is not significant. Furthermore, the likelihood-ratio chi-square statistic of 1.26 with 1 degree of freedom (df), obtained from Models 2 and 4, also indicates that heterogeneity as expressed by the components of variance scheme is not statistically significant. On the other hand, the asymptotic t-statistics for the four lagged dummy terms are quite large, and the likelihood ratio test statistic for Model 3 versus Model 4 (141.9 with df = 4) is highly significant. This analysis based on components of variance models thus leads to the conclusion that intertemporal dependence in household car ownership is due to true state dependence, and that heterogeneity is not a factor.

One-Factor Models

Two one-factor models are estimated for further investigation of heterogeneity and state dependence in car ownership (Table 2). In the one-factor model without state dependence (Model 5), the factor loadings ($\alpha$'s) are almost identical for all three periods, and the $\sigma^2$'s are close to unity (recall that $\sigma^2(1, 1)$ is set to 1.0 for normalization). This one-factor model
(Model 5) is thus essentially equivalent to the corresponding components of variance model (Model 3). These models are in fact nested, and the likelihood ratio test statistic is 1.0 with 4 dfs, under the restriction $\alpha_1 = \alpha_2 = \alpha_3$ and $\sigma_e(2, 2) = \sigma_e(3, 3) = 1$.

| TABLE 1: PARAMETER ESTIMATES AND ASYMPTOTIC T-SCORES FOR COMPONENTS OF VARIANCE MODELS |
|---------------------------------|-----------------|----------------|-----------------|-----------------|
|                                | 1. No Heter.    | 2. No Heter.   | 3. CVAR          | 4. CVAR          |
|                                | No TSP         | TSP            | No TSP          | TSP             |
|                                | $\beta$       | $t$            | $\beta$        | $t$             |
| Const                          | -1.69          | -9.6           | -2.27           | -10.1           | -2.28           | -5.2           | -2.34           | -9.0           |
| SQRTInc                        | 0.27           | 8.8            | 0.14            | 3.7             | 0.41            | 6.5            | 0.15            | 3.5            |
| Workers                        | 0.02           | 5.1            | 0.02            | 3.0             | 0.25            | 2.2            | 0.03            | -1.5           |
| Adults                         | -0.13          | -1.2           | -0.07           | -0.5            | -0.29           | -1.2           | -0.08           | -0.6           |
| Child17                        | 0.16           | 3.1            | 0.05            | 0.8             | 0.03            | 0.2            | 0.06            | 0.3            |
| HHSIZE                         | -0.05          | -2.0           | -0.01           | -0.2            | -0.06           | -0.9           | -0.01           | -0.2           |
| Drivers                        | 1.38           | 26.8           | 0.77            | 9.7             | 1.99            | 16.2           | 0.88            | 7.1            |
| Gmgrp                          | -0.42          | -5.9           | -0.18           | -1.7            | -0.72           | -3.9           | -0.21           | -1.8           |
| HHeduc                         | -0.36          | -6.0           | -0.16           | -1.9            | -0.46           | -3.1           | -0.19           | -1.9           |
| $D_1$                          | 2.29           | 20.3           | 2.29            | 20.3            | 2.23            | 14.1           |
| $D_2$                          | 4.23           | 27.2           | 4.04            | 15.0            |
| Pred. $D_1$ for $t=1$          | 2.51           | 16.7           | 2.43            | 12.7            |
| Pred. $D_2$ for $t=1$          | 3.84           | 10.5           | 3.31            | 9.1             |
| $\rho$                         |                |                | 0.71            | 20.1            | 0.11            | 1.2            |
| $\tau^2$                      | 3.24           | 45.9           | 5.64            | 27.1            | 3.92            | 19.0           |
| $L(\beta)$                    | -903.8         | -652.5         | -722.8          | -651.8          |
| $-2[L(0)-L(\beta)]$           | -2180.4        | -2683.1        | -2542.4         | -2684.3         |
| $\rho^2$                       | 0.547          | 0.673          | 0.538           | 0.673           |
| adj-$\rho^2$                  | 0.542          | 0.666          | 0.632           | 0.666           |

No. of households = 605
Likelihood Ratio Chi-square Statistics for:
Significance of state dependence and heterogeneity = 503.9 (df = 5)
Significance of state dependence = 141.9 (df = 4)
Significance of heterogeneity = 1.26 (df = 1)
* "TSP" = "true state dependence"

The coefficients of the lagged endogenous variables in the one-factor model with state dependence (Model 6) are significant, but to a much lesser extent than those in the pure state dependence model (Model 2) or in the components of variance model with state dependence (Model 4). The coefficient values themselves, however, are relatively stable across these models.

The most dramatic changes are in the $\alpha$ coefficients. These coefficients are constrained to be between -1 and 1 in the estimation to allow for negative intertemporal correlation. Unlike Model 5 which has uniform $\alpha$ values, Model 6 has an $\alpha_2$ that is virtually 0, while $\alpha_1$ and $\alpha_3$ are positive and significant. The model thus implies that unobserved effects are positively correlated between period 1 and period 3, while period 2 is independent of the others. These results are parallel to those of Smith, et al. (1989) in which predicted probabilities for the second period exhibit distinct values. We have been unable to clearly identify the reason for the singularity of the second period at this point. However, note that the value for $\alpha_1$ is relatively large in comparison to $\alpha_2$ and $\alpha_3$; this is undoubtedly due to the larger unexplained random variation in period 1 caused by the correction for initial conditions—see the discussion below.
TABLE 2: PARAMETER ESTIMATES AND ASYMPTOTIC T-SCORES FOR ONE-FACTOR MODELS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 2 (No Hetero. TSP)</th>
<th>Model 5 (OFAC No TSP)</th>
<th>Model 6 (OFAC TSP)</th>
</tr>
</thead>
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<tr>
<td>Const</td>
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<td>-1.26 -6.0</td>
<td>-2.08 -6.4</td>
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<td>SQRT Inc</td>
<td>.14 3.7</td>
<td>.23 8.0</td>
<td>.13 3.0</td>
</tr>
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<td>.14 2.2</td>
<td>.03 .4</td>
</tr>
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<td>-.17 -1.5</td>
<td>-.07 -.5</td>
</tr>
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<td>Child17</td>
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<td>.01 .2</td>
<td>.08 1.1</td>
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<td>-.03 -.9</td>
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<td>2.07 5.9</td>
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<td>3.84 5.7</td>
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<td></td>
<td>2.03 8.9</td>
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<td>Pred. D2 for t=1</td>
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<td></td>
<td>3.00 6.3</td>
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<td>.77 3.1</td>
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No. of households = 605
Likelihood Ratio Chi-square statistics for:
Significance of state dependence = 161.79 (df = 4)
Significance of heterogeneity = 22.18 (df = 5)

In contrast to the components of variance models, the results obtained from the one-factor models indicate that heterogeneity and state dependence are both significant, although state dependence accounts for more variation in behavior than does heterogeneity. The likelihood ratio statistics are 161.8 (df = 4) for state dependence and 22.2 (df = 5) for heterogeneity.

This empirical analysis thus makes it clear that the significance of heterogeneity and state dependence depends on how they are specified. While the components of variance specification has led to the conclusion that heterogeneity is not significant, the second, more general, one-factor specification has shown that both heterogeneity and state dependence are significant. It is entirely possible that different conclusions could be obtained using models with an even more general specification of heterogeneity.

Although the error covariance parameters vary substantially between Models 5 and 6, the coefficients of the explanatory variables are relatively similar, despite the significance of both state dependence and heterogeneity. Particularly notable is the uniformity of the coefficients among the models with state dependence (Models 2, 4, and 6), irrespective of the assumptions on heterogeneity.

The effect of the treatment of initial conditions on coefficient estimates is examined by estimating another one-factor model. In this model, the dummy variables representing actual car ownership in the "pre-sample" period (t = 0) are used assuming they represent
nonstochastic initial conditions. The estimation results (Model 7) are compared in Table 3 with those of Model 6, whose estimation is based on instrumental variables. The table again shows that the coefficients of the explanatory variables are very similar, including the coefficients of the lagged dependent variables.

**Table 3: Parameter Estimates and Asymptotic T-Scores for One-Factor Models: Non-Stochastic Initial Conditions and Stationarity**

<table>
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<tr>
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<th>Instrument Var.</th>
<th>Non-Stochastic</th>
<th>Stationary</th>
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<td>.5</td>
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<tr>
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<td>.985</td>
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<td>σ2</td>
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<td>-551.5</td>
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<td>-1994.0</td>
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<td>-2957.5</td>
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<tr>
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<td>.728</td>
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</tr>
<tr>
<td>adj. ρ²</td>
<td>.669</td>
<td>.720</td>
<td>.669</td>
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</tbody>
</table>

| No. of households: | 605 | 616 | 605 |

* Variances are constrained to be 1 in the stationarity model.

Substantial differences exist, however, in the covariance parameters. Model 7, estimated assuming non-stochastic initial conditions, yields coefficients that are insignificant and variance terms that are all close to unity. It is also notable that the estimated t-statistics for the lagged dummy variables are much larger in Model 7. Despite the similarity in the model coefficients, Model 7 offers an entirely different indication of the heterogeneity of car ownership. Clearly, behavioral implications drawn by estimating a dynamic model depend critically on how the initial conditions are handled.

A final model specification, the one-factor model with stationarity (Model 8 of Table 3), was estimated. In this model the variance of the ε(i, t)'s are assumed to be invariant across all time periods. The results are essentially the same as the ones without the stationarity assumption (Model 6). A likelihood ratio test indicates that car ownership choice is stationary in the error disturbance terms.

The relative performances of Models 1 through 6 are also evaluated in terms of goodness-of-fit by comparing the predicted frequencies of car ownership over the three periods versus the observed frequencies—see Table 4. The results generally confirm our previous observations. Quite notable is the extremely large discrepancy for the independent choice
model (Model 1). The heterogeneity models without state dependence (Models 3 and 5) have chi-square statistics that are much smaller than that of Model 1, but indicate that the predictions are still significantly different from the observed frequencies. In particular, these models grossly underpredict the frequency of no change (0-0-0, 1-1-1, and 2-2-2). This offers evidence that stability in car ownership behavior cannot be represented by heterogeneity alone. Incorporation of state dependence improves the fit dramatically. In particular, the one-factor model (Model 6) exhibits a chi-squared value of 12.7 (df = 13), which implies a near perfect replication of observed frequencies.

Table 4: Comparison of Observed versus Predicted Choice Frequencies.

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<th>4.</th>
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<td>110.1</td>
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<td>12.7</td>
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An optimistic but perhaps dangerous generalization of the results obtained in this study is that the dominant source of intertemporal dependence in car ownership is state dependence and that heterogeneity may be ignored, especially when obtaining the coefficients of explanatory variables is the major concern. In our empirical results the coefficient estimates are robust irrespective of assumptions about heterogeneity or treatment of initial conditions. However, recall that conclusions about the statistical significance of heterogeneity depended on how it was specified, and upon the treatment of initial conditions. Thus, the apparent insignificance...
of heterogeneity may be due to the particular specifications used, and in future studies more
general specifications should be examined. Testing for state dependence versus heterogeneity
continues to be tricky and difficult, and will probably remain so.

SUMMARY AND CONCLUSIONS

Although correlation among unobserved errors, endogeneity of lagged dependent
variables, and problems with initial conditions have been recognized as having important
consequences in the estimation of dynamic, disaggregate models, a review of the literature
reveals that these have often been ignored in the dynamic modeling of car ownership. By
taking advantage of the ordinal nature of the problem, computationally convenient models
which take all of these issues into account can be formulated, and estimated using state-of-the-
art numerical techniques. To perform an analogous analysis using a beta-logistic approach, it
would be necessary to somehow combine the approximation procedure in Davies (1984) with
Dunn and Wrigley (1985) to produce a three-alternative model which allows time-varying
exogenous variables and "feedback effects." This would be unnecessarily restrictive,
cumbersome, and possibly inaccurate. Alternatively, one could attempt to use a multinomia/
probit approach, but for three alternatives and three time periods this would most likely be
computationally intractable using currently available methods.

The numerical results in Section 5 first demonstrate the gross inadequacy of pooled
cross-sectional analysis. Next, models assuming a components of variance error structure lead
to conclusions that (i) state dependence is strongly significant, and (ii) heterogeneity is not
significant. Presumably, these results would be similar to those obtained in a carefully
performed analysis using the beta-logistic-like approach identified above.

On the other hand, results using a more general one-factor model approach reject the
hypothesis of no heterogeneity, indicating that a more general formulation could lead to
different conclusions. This could not have been identified by a beta-logistic-type approach.

The empirical study in the literature most similar to ours is perhaps Heckman (1981c),
which estimates CVAR and OFAC models of female labor force participation (e.g., binary
choice) using three periods of data. As in Heckman's study, we accept the hypothesis of
stationarity. Unfortunately, unlike Heckman's results--which neatly lead to the conclusion that
the error process is first-order Markov--our estimated $\alpha$ parameters are very difficult to
interpret. This matter will be the subject of further study. It may be that, although
heterogeneity is present, it is not adequately modeled by the one-factor approach in this data
set, especially in the context of ordered latent variable models. Finally, our results also reveal
that acceptance or rejection of the hypothesis of no heterogeneity can be seriously affected by
the manner in which initial conditions are handled.

To conclude, we acknowledge that our results seem to indicate that the coefficient
estimates themselves may be robust to many of these concerns. However, we caution against
jumping to the conclusion that simply including lagged dummies is an adequate solution to the
dynamic modeling problem. This may lead to erroneous conclusions about true state
dependence and heterogeneity, which in turn may result in inaccurate forecasts using the
dynamic models.

ACKNOWLEDGEMENTS

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General of Transport of the Netherlands Ministry of Transport and Public Works.
FOOTNOTES


2. A model with a lagged dependent variable may be interpreted to represent a "partial adjustment" to a change, as well as dependence on past behavior. See Griliches (1967) for possible interpretations.

3. According to Train and Lohrer (1982, p. 41), Sherman, Manski and Ginn (1980) in a later report chose not to estimate the coefficient of the transaction dummy variable within the model, but its value was selected such that the observed aggregate turnover rate could be reproduced.

4. The problem of heteroskedasticity caused by truncated errors is accounted for in a later study (Kitamura and Goulias, 1989) using theoretically derived weights in the single-equation maximum likelihood estimation. Initial conditions are treated in Kitamura and Goulias (1989) via a procedure similar to that described in Heckman (1981b, pp. 188-9).

5. Notable here is the application of structural equations models to the estimation of discrete choice models with lagged endogenous variables and serially correlated errors. Golob and his associates (Golob 1989; Golob and Meurs, 1987) have applied structural equations models to various aspects of travel behavior. These models, however, adopt entirely different estimation principles; further analysis is needed to determine the relationship between the structural equation approach and the conventional econometric estimation approach.

6. Smith, et al. (1989) used up to two terms in the series expansion. Judging from the magnitude of error that takes place with this k, the results of Smith, et al., must be cautiously interpreted.

7. No adjustment is made in this study to account for possible biases in the data due to attrition because the examination of the heterogeneity versus state dependence issue, not inference of population characteristics, is the main concern of the study. For a discussion of attrition behavior in the Dutch Panel data, see Kitamura and Bovy (1987).

REFERENCES


