Title
The Power Decay Law in High Intensity Active Grid Generated Turbulence

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The Power Decay Law in High Intensity Active Grid Generated Turbulence

THESIS

submitted in partial satisfaction of the requirements
for the degree of

MASTER OF SCIENCE

in Mechanical Engineering

by

Timothy William Koster

Thesis Committee:
Professor John C. LaRue, Chair
Professor Dimitri Papamoschou
Professor Said E. Elghobashi

2015
DEDICATION

To

Family and Friends
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Abstract of the Thesis

The Power Decay Law in High Intensity Active Grid Generated Turbulence

By

Timothy William Koster

Master of Science in Mechanical Engineering

University of California, Irvine, 2015

Professor John LaRue, Chair

The work presented herein provides an assessment of the applicability of the power law decay of turbulent kinetic energy in the isotropic and homogeneous region in a high intensity flow generated by an active grid. Two sets of indicators and data collected at mean velocities of 4, 6 and 8 m/s are used to determine the isotropic region. Results of this study are inconsistent as some of the results are consistent with a power law behavior but some results are not. For example, the ratio of dissipation computed from the time derivative of the velocity, $\epsilon$, to the corresponding value computed using the power law decay for turbulent kinetic energy, $\epsilon^*$, should be one. This is indeed the value found for mean velocities of 8 and 6 m s$^{-1}$ but only when using a range of downstream positions corresponding to only one of the pairs of isotropy indicators. For mean velocity of 4 m s$^{-1}$, $\epsilon/\epsilon^* \neq 1$, nor does it equal 1 for any of the mean velocities for the isotropic range indicated by the second isotropy indicator. An alternative approach for the power decay law is proposed and is based on the determination of the range using the constraint that $\epsilon/\epsilon^* = 1$. 
1. Introduction

Turbulence is ubiquitous. It is present in the natural world and in many devices of technological interest. For example, turbulence is found in tidal flows and in the atmospheric boundary layer, as well as in combustors and premixers in gas turbine engines. However, although turbulent flows are important, their range of scales and the inherent nonlinearity in the describing equations have made it impossible to determine a general analytical solution. For this reason, much of our understanding of turbulence is based on experimental results concerning homogeneous–isotropic flows, where the describing equations are relatively simple. These experimental results, when combined with considerations, such as self-preservation, similarity, and order of magnitude analysis, have provided some of our basic understanding of turbulent flows.

Arguably, the best experimental representation of decaying homogeneous–isotropic turbulence is in a wind tunnel behind a biplane grid consisting of rods of round or square cross-sections. However, turbulence created by this type of generator produces low turbulence intensities of approximately 3% or less, with a correspondingly low Taylor Reynolds number of up to 100. In contrast, an active biplane grid that consists of rotating rods and vanes produces nearly homogenous–isotropic turbulent flows with turbulent intensities and Taylor Reynolds numbers as high as 20% and 1000, respectively (Kang et al. 2003).

In homogeneous–isotropic flow, dimensional analysis applied to the dominant terms in the turbulent kinetic energy equation (cf. Tennekes & Lumley 1972, pp. 71–73) leads to the conclusion that the downstream decay of the turbulent kinetic energy and Taylor microscale should be described by a power law. That conclusion is well supported by a large number of experimental studies in the nearly homogeneous–isotropic region downstream of passive grids (cf. Batchelor 1947; Comte-Bellot & Corrsin 1971; Mohamed & LaRue 1990; Antonia et al. 2003; Antonia & Orlandi 2004; Lavoie et al. 2005).
Whereas it seems that a power law may also reasonably describe the downstream decay of velocity variance close to the grid where the flow is not isotropic, consistent results for the decay exponent can only be determined if the decay law is applied to positions that are sufficiently downstream of the grid so the flow is homogenous and nearly isotropic (cf. Mohamed & LaRue 1990). Thus, to determine the region where a power decay law applies, criteria must be identified and applied to determine the isotropic region of the flow. In the study reported herein of the decay of the downstream component of velocity variance, careful consideration of several measures of isotropy are used to determine corresponding regions where the flow can be considered to be isotropic and where the power decay law should be observed.

Mohamed & LaRue (1990) also demonstrated that a consistent approach must be used to determine the value of the virtual origin. In that study, the virtual origin was found to be zero, i.e. at the passive grid. However, the value of the virtual origin cannot be predetermined and, as pointed out by George (1992), will likely depend on initial conditions, e.g. the geometry of the turbulent generator, the control parameters for the active grid, and, perhaps, the mean velocity. In this study, a systematic approach to determine the virtual origin that is independent of the determination of the decay exponent is developed and applied.

The description of the decay of downstream velocity variance by a power law is shown in §3 to imply power law behavior for various other parameters such as the dissipation, the integral length scale, the Taylor microscale, the Kolmogorov microscale, and the Taylor Reynolds number. This section also shows that the exponents in the power law for each of those parameters can be related to the power law exponent for the decay of the variance of the downstream velocity.

Of note, the power law exponent for the downstream variation of the Taylor length scale is a constant and is independent of the power law exponent of the decay of velocity variance. Thus, as suggested by Antonia et al. (2003), the downstream variation of the Taylor microscale provides a means
of determining the virtual origin in the decay power law that is independent of the determination of the exponent in the decay power law for the velocity variance. It is thus approach that will be used in the study described herein.

In §2, a brief review of previous studies concerning the downstream decay of the variance of the downstream velocity is presented. In this section, various measures of isotropy are discussed. §3 presents implications of the power law relevant to the downstream variation of other quantities of interest such as Taylor Reynolds number, the integral length scale and other quantities of interest. The experimental facility, sensor calibration, signal processing system, and data analysis approach are described in §4. Results and the discussion of results are presented in §5, and conclusions are presented §6.

2. Power law decay: a brief review of analytical prediction, results from previous experimental studies, and measures of isotropy

In a homogeneous–isotropic flow, where smaller order terms are neglected, the equation for the turbulent kinetic energy can be reduced to the following:

$$\epsilon^* = -\frac{3}{2} \frac{d \overline{u^2}}{dx}$$

where \( \overline{U} \) is the mean velocity, \( \overline{u^2} \) is the variance of the velocity defined as \( \overline{u^2} \equiv (u - \overline{U})^2 \), \( \epsilon^* \) is the dissipation rate of turbulent kinetic energy and \( x \) is the distance in the downstream direction with origin at the grid. As described in Tennekes & Lumley (1972, pp. 71–13), the application of dimensional analysis, along with estimates of the time scale of the energy transfer from large to small scales and the decay time of the large eddies, suggests that the velocity variance, \( \overline{u^2} \), should follow a power law decay proportional to \( x^{-n} \), and the Taylor microscale and other length scales should increase in proportion to \( x^{n/2} \). That analysis suggests that \( n = 1 \). However, Tennekes & Lumley (1972) note that the assumptions used in the
analysis are “crude”; and therefore, it is expected that the values for $n$ found experimentally will not equal 1.

Kolmogorov (1941) and Saffman (1967) determined values for the decay exponent based on the assumption that the Loitsianskii (1939) integral needs to be constant, i.e.

$$I_n = \int_0^\infty r^n \langle u(x)u(x + r) \rangle dr = \text{Constant},$$  \hspace{1cm} (2)

where $\bar{n}$ denotes the order of the Loitsianskii integral, $r$ is the separation distance, $u$ is the downstream velocity, and $\langle \dots \rangle$ indicates the time average. Kolmogorov (1941) assumes that the fourth-order ($\bar{n} = 4$) Loitsianskii integral is a constant, whereas Saffman (1967) assumes that the second-order integral is a constant. Both researchers assume that the velocity correlation is self-similar, i.e. that $\langle u(x)u(x + r) \rangle = \langle u^2 \rangle f(r/L)$, and that the dissipation is related to the integral scale, i.e. $C_e = \langle \varepsilon \rangle L/\langle u^2 \rangle^{3/2}$.$\sim$Constant. Kolmogorov (1941) predicts a value of the decay exponent of $10/7$, whereas Saffman (1967) predicts a value of the decay exponent of $6/5$.

However, as noted in the preceding, whereas numerous experimental studies of the isotropic flow downstream of a passive grid support the conclusion, with the addition of a virtual origin, that the downstream variation of the velocity variance follows a power law of the form

$$\frac{\overline{u^2}}{\overline{v^2}} = A \left( \frac{x}{M_u} - \frac{x_0}{M_u} \right)^{-n},$$ \hspace{1cm} (3)

where $x$ is the position downstream of the grid, $x_0$ is the virtual origin, $M_u$ is the mesh spacing of the grid, and $A$ is a coefficient that may depend on initial conditions (cf. George 1992), $n$ is found to range from 1.2 to 1.309, as shown in Table 1.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>$R_\lambda$ ($x/M$)</th>
<th>$n$</th>
<th>Virtual origin $x_0/M_u$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comte-Bellott &amp; Corrsin</td>
<td>N/A(30)</td>
<td>1.300</td>
<td>2.0</td>
<td>$\overline{u^2}/\overline{v^2}$ longest linear</td>
</tr>
<tr>
<td>(1966)</td>
<td>N/A (30)</td>
<td>1.270</td>
<td>2.0</td>
<td>range</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>-------</td>
<td>-----</td>
<td>-------</td>
</tr>
<tr>
<td>Mohamed &amp; LaRue (1990)</td>
<td>28.37 (40)</td>
<td>1.309</td>
<td>0</td>
<td>Least-square fit to $\bar{u}^2/\bar{U}^2$</td>
</tr>
<tr>
<td></td>
<td>43.85 (40)</td>
<td>1.299</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Antonia et al. (2003)</td>
<td>N/A (40)</td>
<td>1.32</td>
<td>-0.177</td>
<td></td>
</tr>
<tr>
<td>Lavoie et al. (2005)</td>
<td>42 (40)</td>
<td>1.20</td>
<td>6.0</td>
<td>Constant decay exponent</td>
</tr>
<tr>
<td></td>
<td>40 (60)</td>
<td>1.29</td>
<td>3.0</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 1. Decay exponent for the decay of velocity variance downstream of passive grids

None of the values is close to 1, and none is as high as the value of 1.43 predicted by Kolmogorov (1941). However, many of the values found experimentally are similar to the value of 1.2 predicted by Saffman’s (1967). The differences in the decay exponent, $n$, could be caused by different initial conditions, different indicators for the isotropic range, different virtual origins, and different methods used to calculate the virtual origin.

The Taylor microscale squared, $\lambda^2$, in homogenous isotropic decay turbulence is directly proportional to the downstream position. This relationship is first pointed out by Von Karman & Howarth (1938) and later reiterated in George's (1992) similarity analysis of the decay of homogenous–isotropic turbulence. George (1992) also pointed out that the decay exponent, $n$, in (3) and the virtual origin, $x_0/M_U$, can be extracted from a linear fit to $\lambda^2 = m(x/M_U) + B$. This extraction is shown for the two passive grids as presented in Table 2.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Turbulence generation</th>
<th>$m$</th>
<th>$B$</th>
<th>Virtual origin $x_0/M_u$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>George (1992)</td>
<td>Passive grid</td>
<td>0.00625</td>
<td>-0.0284</td>
<td>4.54</td>
<td>-1.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00314</td>
<td>-0.0281</td>
<td>8.95</td>
<td>-1.20</td>
</tr>
<tr>
<td>Antonia et al. (2003)</td>
<td>Passive grid</td>
<td>$3.961 \times 10^{-4}$</td>
<td>$7.012 \times 10^{-5}$</td>
<td>-0.177</td>
<td>-1.32</td>
</tr>
</tbody>
</table>

TABLE 2. Decay of $\lambda^2$ to obtain the virtual origin

The use of this approach to find $x_0$ and $n$ as noted by Antonia et al. (2003) leads to approximately the same range of exponent and virtual origin values found using the power decay law for the velocity variance.
The power decay law exponent and virtual origin have also been determined for the flow downstream of an active grid. These results are summarized in Table 3. Values of the exponent are found to vary from 1.21 to 1.43. The variation in exponent value does not seem to vary in a consistent manner with the Taylor Reynolds number. This is not consistent with the prediction of George (1992). However, the lack of consistent behavior and differences in exponent values may depend more on how the virtual origin is determined and whether the flow region used in the study is sufficiently isotropic.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Turbulence generation</th>
<th>$R_\lambda (x/M)$</th>
<th>$n$</th>
<th>Virtual origin $x_0/M_u$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Makita &amp; Sassa (1991)</td>
<td>Active grid</td>
<td>387 (50)</td>
<td>1.43</td>
<td>-12.0</td>
<td>$x_0$ determined by least square fit</td>
</tr>
<tr>
<td>Mydlarski &amp; Warhaft (1996)</td>
<td>Active grid</td>
<td>319 (68)</td>
<td>1.21</td>
<td>0</td>
<td>$x_0 = 0$ assumed</td>
</tr>
<tr>
<td>Kang et al. (2003)</td>
<td>Active grid</td>
<td>676 (30)</td>
<td>1.25</td>
<td>0</td>
<td>$x_0 = 0$ assumed</td>
</tr>
<tr>
<td>Mordant (2008)</td>
<td>Active grid</td>
<td>240 (16)</td>
<td>1.24</td>
<td>0</td>
<td>$x_0 = 0$ assumed</td>
</tr>
</tbody>
</table>

TABLE 3. Decay exponent for the decay of velocity variance downstream of active grids

2.1. Measures of isotropy

Typical measures of isotropy are outlined by Mohamed & LaRue (1990). The first measure is that the skewness of velocity fluctuations, $S(u) = \overline{u^3}/\overline{u^2}^{3/2}$, should be zero. The second measure, or indicator, is based on the analysis of Kolmogorov where $M_m = \frac{\langle \frac{\partial u}{\partial x} \rangle^m}{\langle \left( \frac{\partial u}{\partial x} \right)^{2\frac{m}{2}} \rangle} = \text{Constant}$. In this study the second indicator of isotropy indicator is that skewness of the velocity derivative, $S(\partial u/\partial x) = \overline{(\partial u/\partial x)^3}/(\overline{\partial u/\partial x})^{23/2}$, is found to be a constant. A third measure of isotropy is that $S(\partial u/\partial x)R_\lambda = \text{Constant}$. This measure is based on George’s (1992) analysis where he assumes the velocity power spectrum and transfer power spectrum can be described as functions of a similarity function and $S(\partial u/\partial x)$ can described as $S(\partial u/\partial x) = -\frac{3\sqrt{30}}{14}\left(\int_0^\infty k^2 T(k)dk\right)/\left[\int_0^\infty k^2 E(k)dk\right]^\frac{3}{2}$, where $T(k)$ is the transfer power spectrum, $k$ is the wavenumber, and $E(k)$ is the velocity power spectrum. These measures of
isotropy are evaluated as a function of downstream distance and aim to determine the isotropic region in the flow. It should be noted that this indicator is not consistent with the previously mentioned indicator. George's (1992) analysis is in contradiction with Van Atta & Antonia (1980) paper that shows that over a verity of turbulent flows that \( S(\partial u/\partial x) \) is proportional to \( R_\lambda \).

3. Extension of the power law

The fact that the downstream decay of the velocity variance or, equivalently, the turbulent kinetic energy can be represented by a power law implies that other quantities of interest, such as the downstream decay of the dissipation and the downstream growth of the length scales, must also follow a power-law behavior. In this section, the forms of those power laws are presented, as are other implications. A slight modification to in eqn. (1) leads to the following equation where from which relates the dissipation rate to the time rate of decay of the turbulent kinetic energy

\[
\epsilon = \frac{1}{2} \frac{d q^2}{dt} = \frac{\bar{U}}{2} \frac{d q^2}{dx}
\] (4)

Substituting the power law form for the downstream variation of the velocity variance, seen in eqn. (3) into the left side of eqn. (1) leads to, after some simple algebra, the following equation, which describes the downstream decay of the dissipation rate:

\[
\epsilon^* = \frac{3}{2} n AU^3 \left( \frac{x}{M U} - \frac{x_0}{M U} \right)^{-n-1}
\] (5)

Combining the power-law expressions for the downstream variance of the velocity variance and the dissipation, as appropriate, with the defining equations for the Taylor microscale, the Kolmogorov length scale, the integral length scale, and the Taylor Reynolds number, power law expressions for the downstream variations of those quantities can be determined.

The defining equations for those quantities are as follows:
Explicit forms for the power-law behavior can be found by substituting eqns (3) and (5) into eqns (6) to (9). The power-law expressions after some manipulation are as follows:

\[ \lambda \left( \frac{15 \nu \cdot u^2}{\epsilon} \right)^{0.5} = \lambda \left( \frac{\nu^3}{\epsilon} \right)^{0.25} \]  
\[ \eta = \left( \frac{\nu^3}{\epsilon} \right)^{0.25} \]  
\[ L_U = \frac{\epsilon}{u^{2.15}} \]  
\[ R_\lambda = \frac{\nu^{0.5} \lambda}{v^2} \]

In these equations, \( R_{MU} = UMU/\nu \) is the Reynolds number based on the grid mesh size. The exponents \( n_\lambda, n_\eta, n_L, n_{R_\lambda} \), and \( n \) should be equal. However, in this study, the exponents found experimentally may differ in value, and the subscript is used to denote the expression corresponding to a particular exponent and power-law fit.

Substituting eqn (10) into eqns (11) and (12) yields the following constructions:

\[ \frac{\lambda^2 U}{\nu M_U} = 10 \frac{x - x_0}{M_U} \]  
\[ \frac{\eta^4 U^3}{v^5 M_U} = \frac{2}{3 A_\eta n_\eta} \left( \frac{x - x_0}{M_U} \right)^{n_\eta + 1} \]  
\[ \frac{L_U}{M_U} = \frac{2 A^{0.5}_L}{3 n_L} \left( \frac{x - x_0}{M_U} \right)^{1 - 0.5 n_L} \]  
\[ \frac{R_\lambda}{R_{MU}} = \frac{10 A_{R_\lambda}}{n_{R_\lambda}} \left( \frac{x - x_0}{M_U} \right)^{1 - n_{R_\lambda}} \]

Equation (15) shows that, unless \( n = 1 \), \( L_U \) is not directly proportional to \( \lambda \).
3.1. Infinite Reynolds number

Von Karman & Howarth (1938) propose that the decay exponent \( n \) equals 1 when \( R_\lambda \to \infty \).

Setting \( n = 1 \) in eqns (10) to (13) yields the following:

\[
\frac{\epsilon M_U}{U^3} = \frac{3A_e}{2} \left( \frac{x}{M_U} - \frac{x_0}{M_U} \right)^{-2}
\]

(16)

\[
\frac{\lambda^2 U}{\nu M_U} = 10 \left( \frac{x}{M_U} - \frac{x_0}{M_U} \right)
\]

(17)

\[
\frac{\eta^4 U^3}{\nu^3 M_U} = \frac{2}{3A_\eta} \left( \frac{x}{M_U} - \frac{x_0}{M_U} \right)^2
\]

(18)

\[
\frac{L_U^2}{M_U^2} = \frac{4A_L}{9} \left( \frac{x}{M_U} - \frac{x_0}{M_U} \right)
\]

(19)

\[
\frac{R_\lambda^2}{R_{M_U}} = 10 A_{R_\lambda}
\]

(20)

Equation (20) is equivalent to eqn (69) in George (1992). Of note, the dependence on the virtual origin in eqn (20) disappears where \( n = 1 \). Equation (18) is equivalent to the equation noted in Batchelor (1947, p. 136, footnote).

Finally, for \( n = 1 \), eqns (17), (18), and (19) show that \( \lambda, L_U, \) and \( \eta \) are directly proportional to each other. The fact that \( \lambda \) is directly proportional to \( L_U \) is in agreement with George (1992) but, again, this only occurs when \( n = 1 \). Setting \( n = 1 \) for eqns (14) to (15), yields the following:

\[
\eta^2 \left( \frac{U^3}{\nu^3 M_U} \right)^{0.5} = \left( \frac{1}{150A} \right)^{0.5} \left( \frac{\lambda^2 U}{\nu M_U} \right)^{0.5}
\]

(21)

\[
\frac{L_U^2}{M_U^2} = \frac{2A}{45} \left( \frac{\lambda^2 U}{\nu M_U} \right)
\]

(22)

Which shows that only at \( n = 1 \).

\[ \eta \sim \lambda \sim L_U \]

i.e. that the length scales are proportional to each other.
4. Experimental setup

The experiment was conducted in a close-return wind tunnel using a hot-wire anemometer to obtain a time resolve velocity. The turbulence was created by an active grid. The mean velocity was varied between 4, 6 and 8 m s\(^{-1}\) to vary the initial turbulence intensity from 16\%, 18\% and 19\% respectively. Points where taken every 50 mm downstream starting 1.87 meters downstream of the active grid.

4.1. Sensors

Sensors are used in this study to measure the mean speed, mean temperature, and time-resolved velocity in the downstream direction. The mean velocity is obtained using a Pitot tube connected to a Baratron MKS Model 698A11TRE differential pressure transducer (MSK Instruments, Andover, MA). The Pitot tube not only measures the mean velocity of the tunnel, but is also used to calibrate the hot-wire described below. Mean temperature is measured using a platinum resistance thermometer (PRT). The PRT is a three-wire probe made by Omega Engineering Inc. (Bridgeport, NJ) and is connected to a custom-built Wheatstone bridge through a twisted three-wire cable. The Wheatstone bridge is based on an Analog Devices 5B34-01 isolated, linearized RDT Input Module. The PRT measures the mean temperature in the wind tunnel and is used to correct the hot wire for mean temperature drift that occurs inside the tunnel throughout the course of the data-collection period. The PRT and Pitot tube are mounted at the same vertical location, as is the hot wire, but are displaced, horizontally, 12 mm from the hot wire.

The hot wire used in the study measures the time-resolved velocity in the downstream direction and uses an AN-1005 constant temperature anemometer (CTA) manufactured by AA Labs Systems (Westminster, CA). The hot wire is fabricated in house with a 5.08-µm diameter (D) platinum wire 1 mm in length (L), yielding an L/D ratio of approximately 200. The hot wire is operated with an overheat ratio of 1.75. Based on a square-wave test, the hot wire is estimated to have a frequency response of 40 kHz at a velocity of 8 m s\(^{-1}\).
The analog signals coming from the CTA pass through a series of analog signal conditioners before being digitized by a Measurement-Computing 16-bit USB-1608HS (Norton, MA) analog-to-digital converter (A/D). The A/D has an input range of ±10 volts. After the CTA, as shown in fig. 1, the signal passes through a four-pole Butterworth low-pass filter to remove the high-frequency electronic noise before the signal is split. As shown in fig. 1, both signals pass through an amplification/attenuation stage to ensure the fluctuating signal levels exceed at least half of the dynamic range of the A/D but do not exceed the dynamic range of the A/D. The output of the upper-processing path, as shown in fig. 1 (hereafter, referred to as the straight signal), then passes to the input of the A/D converter. An analog differentiator is added to the second set of processing electronics (the lower path on the right side of fig. 1), and the output of that path is used to determine the time-resolved velocity derivative. The amplifiers and differentiator have a frequency response of 70 kHz and 18 kHz, respectively.

![Signal Flow Chart](chart.png)

**Figure 1. Signal flow chart.**

The first step in the calibration of the hot wire is to determine the temperature of the wire, \( T_w \), and the exponent \( n \) in King’s law,

\[
\frac{E_{HW}^2}{T_w - T_g} = (A_{HW} + B_{HW} \cdot U^n),
\]

where \( T_g \) is the temperature of the gas, \( E_{HW} \) is the voltage from the hot wire, and \( A_{HW} \) and \( B_{HW} \) are the calibration constants to be found. The first calibration is used to find \( T_w \) and \( n \). The calibration is achieved
by taking at least 35 different values of the velocity and temperature from a range of velocities from 4 to 20 m s\(^{-1}\) and temperatures between 23 and 60°C. With this information and the use of a nonlinear solver in Microsoft Excel add-ins called Solver, \(n\) and \(T_w\) are obtained by finding the best set of constants from (23) that collapse the data to a single line on a log–log plot. From this calibration, \(T_w\) is found to be 249.9°C, and \(n\) is found to be 0.399. A detailed process of how \(n\) is calculated can be found in Appendix A: Calibration procedure determination of \(T_w\) and \(n\). The second step in the calibration is to determine the calibration coefficients \((A_{HW}\) and \(B_{HW}\) in eqn.(23)). These coefficients are determined by measuring the output of the CTA corresponding to at least 10 different velocities. See Table 4. Velocity calibration ranges for the ranges of calibration velocities for the experiments presented herein. In the second calibration step, the mean temperature of the flow is held constant. Calibrations are performed before and after the data collection period, and data are only analyzed if application of the pre and post calibration leads to no more than a 2% variation in the calculated velocity and velocity derivative statistics.

<table>
<thead>
<tr>
<th>Mean speed (m s(^{-1}))</th>
<th>Low velocity (m s(^{-1}))</th>
<th>High Velocity (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

**TABLE 4. Velocity calibration ranges**

4.2. Wind tunnel

The experimental study presented in this thesis is performed in a closed-return wind tunnel at the University of California, Irvine, which has a test section that is 0.61 m wide, 0.91 m high, and 6 m long. The top and bottom walls of the test section diverge to account for the boundary layer growth to maintain a nearly constant mean velocity with downstream distance. Without any flow apparatus in place, the wind tunnel mean speed is stable to within ±0.05 m s\(^{-1}\) and is homogeneous to within 1% across the test section. The background turbulence intensity is 0.17% at the entrance and 0.22% at the exit. The contraction section and test section have a 9.36:1 area ratio that goes from 5.15 to 0.55 m\(^2\). A schematic of the wind
tunnel is shown in Figure 2 (Selzer 2001). The tunnel is equipped with a traverse that can move the sensors upstream and downstream, as well as vertically, in the wind tunnel test section. The vertical position is measured by a LM10 Renishaw magnetic encoder (Hoffman Estates, IL) with a USB interface to a resolution of 5 µm. Throughout the experiment measuring the downstream decay of turbulence, the traverse height is maintained to within ±1 mm from the center of the tunnel. The downstream position is measured to a resolution of 1.5 mm.

![Figure 2. Schematic of the wind tunnel.](image)

4.3. Turbulence generator

The turbulence is generated by an active grid based on Makita’s 1991 design (Makita & Sassa 1991), as implemented by Mydlarski & Warhaft (1996). The grid consists of 12 vertical and 18 horizontal rods with a diameter of 9.5 mm and spaced 50.4 mm apart. (That spacing will be, hereafter, referred to as the mesh size, $M_U$, of the grid.) There are 187 square agitator flaps that measure 34.3 mm on each side.
and are 1.55 mm thick. As shown in Figure 3, the flaps are center-mounted, which is different from the other active grid studies.

**FIGURE 3.** Close-up photo of active grid.

The rods are controlled by thirty Anaheim Automation 17MD102S-00 stepper motors (Anaheim, CA) that have a resolution of 200 steps per revolution. The motors are controlled by two Propeller Proto USB Boards using a P8X32A-Q44 Propeller chip made by Parallax Inc. (Rocklin, CA). Each microcontroller controls 15 of the motors. The microcontrollers take four inputs from the user: The first two control the mean rotational speed and percent variance of that speed, and the second two inputs control the mean time for change in rotation direction and the variance of that time change. At a certain
time point, each pair of motors is given a random rotation direction and speed that is within the variance chosen by the user. Then, all of the motors for a particular microcontroller are given a new time interval, speed, and direction. The parameters for this experiment are designated as a set of four numbers e.g., 2, 25, 250, and 50, where 2 is the mean rotation rate in revolutions per second, 25 is 25% variance on the mean rotation rate so the rods can rotate at any speed between 1.5 and 2.5 revolutions per second, 250 is the rotation period in ms, and 50 corresponds to 50% variance of the mean rotation rate so the period can be anywhere between 125 and 375 ms before a new rotation rate and direction are chosen. In contrast, the condition designator, 2 0 250 50, expresses that the rotation rate is fixed at 2 revolutions per second with 0 variance and with a rotation period of 250 ms with a 50% variance. The power spectra for the downstream velocity for these control conditions, 35 mesh lengths downstream of the active grid at a mean speed of 8 m s\(^{-1}\), are shown in fig.4. A bump or spike in the power spectrum at twice the rotation rate, or 4 Hz can be observed. This same phenomenon was observed by Mydlarski & Warhaft (1996) at a fixed rotation rate. They suggest that a variable rotation rate will minimize the relative amplitude of the spike. Velocity power spectra for the downstream for conditions at 2 25 250 50 and 2 0 250 50 for a mean velocity of 8 m s\(^{-1}\) at 35 mesh lengths downstream of the grid Figure 4. Downstream velocity power spectrum for conditions at 2 25 250 50 and 2 0 250 50 for a mean velocity of 8 m s\(^{-1}\) at 35 mesh lengths downstream of the grid.
Figure 4. Downstream velocity power spectrum for conditions at 2 25 250 50 and 2 0 250 50 for a mean velocity of 8 m s$^{-1}$ at 35 mesh lengths downstream of the grid.

The velocity power spectrum at the condition of 2 0 250 50 exhibits a significant spike with a relative height of over one decade at twice the rotation speed. The area under that spike corresponds to approximately 0.5% of the value of $\overline{u^2}$. In contrast, for the same mean speed and downstream location, the condition of 2 25 250 50 leads to a broader spike or bump in the power spectrum but with a significantly lower relative amplitude of approximately 0.5 decade. The area under that bump corresponds to less than a 0.1% increase of variance in velocity and velocity derivative, i.e. in $\overline{u^2}$ and $(\overline{du/dt})^2$. That contribution to the measured value of $\overline{u^2}$ and $(\overline{du/dt})^2$ is considered low enough that no correction is made to any of the measured values of $\overline{u^2}$ and $(\overline{du/dt})^2$.

4.4. Coordinate system

The origin of the coordinate system for this experiment is located at the center plane of the active grid. Measurements of the time-resolved velocity and its derivative are made in the range of $x/Mu = 35$ to $x/Mu = 142$. The vertical and horizontal coordinates are represented as $y$ and $z$, respectively. The wind
tunnel is 18 mesh lengths tall and 12 mesh lengths wide. See fig. 5 for a schematic of the coordinate system.

![Diagram of test section and active grid]

**FIGURE 5.** Coordinate system

### 4.5. Sample length and Stationarity

The approach involves the collection of long-time data sets and the determination of the time required for the statistical parameter obtained from subsets of the data to exhibit a variability of <3.5%. These long-time data sets are obtained at \( x/Mu = 35 \) for mean speeds of 4 and 8 m s\(^{-1}\). The corresponding number of data blocks (continuously collected data), duration for each block, and sample rate for the two mean velocities are shown in Table 5. Long-time data sets used to determine sampling time.

<table>
<thead>
<tr>
<th>Mean velocity</th>
<th>Number of data blocks</th>
<th>Duration of each data block (seconds)</th>
<th>Samples rate (sample/second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 m s(^{-1})</td>
<td>30</td>
<td>240</td>
<td>20,000</td>
</tr>
<tr>
<td>8 m s(^{-1})</td>
<td>30</td>
<td>240</td>
<td>48,000</td>
</tr>
</tbody>
</table>

**TABLE 5.** Long-time data sets used to determine sampling time
Table 6. Uncertainty in statistical quantity at sampling times of 120 seconds and 90 seconds for \( U = 4 \) and 8 m s\(^{-1}\) at \( x/M_U = 35 \), respectively. shows the variability of various statistical quantities due to the lack of stationarity at sampling times of 120 seconds and 90 seconds for \( U = 4 \) and 8 m s\(^{-1}\) at \( x/M_U = 35 \), respectively. At a mean velocity of 4 m s\(^{-1}\) for a sample duration of 120 seconds, the variation in the statistical quantity of interest is about 2.9%, and at 8 m s\(^{-1}\), this variation is about 2.18%. Note that since \( S(u) \) approaches a value of 0, it is not normalized but is reported as a standard deviation. No direct study of the effect of sample length in seconds is performed for the mean speed of 6 m s\(^{-1}\), and the conservative sample length of 120 seconds is chosen for that mean velocity.

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Sampling time</th>
<th>( U )</th>
<th>( \bar{u}^{0.5} )</th>
<th>( S(u) )</th>
<th>( \frac{\partial u}{\partial t}^{0.5} )</th>
<th>( S \left( \frac{\partial u}{\partial t} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 m s(^{-1})</td>
<td>120 s</td>
<td>0.75%</td>
<td>1.23%</td>
<td>0.1922</td>
<td>1.24%</td>
<td>2.99%</td>
</tr>
<tr>
<td>8 m s(^{-1})</td>
<td>90 s</td>
<td>0.72%</td>
<td>1.34%</td>
<td>0.2595</td>
<td>1.19%</td>
<td>2.18%</td>
</tr>
</tbody>
</table>

Table 6. Uncertainty in statistical quantity at sampling times of 120 seconds and 90 seconds for \( U = 4 \) and 8 m s\(^{-1}\) at \( x/M_U = 35 \), respectively.

The sampling rate at different downstream position for mean velocities of 4, 6 and 8 m s\(^{-1}\) are shown in Table 7. Filter and sampling rates for downstream location. Filter and sampling rates for downstream location are so that electronic noise has a contribution of no more that 3% to value of \( (\frac{\partial u}{\partial t})^{2.5} \). The effect of electronic noise on the directly computed value of \( (\frac{\partial u}{\partial t})^{2.5} \) is determined by integrating the power spectrum for the \( (\frac{\partial u}{\partial t}) \) up to the frequency where electronic noise is noted and comparing that value to the value obtained directly from the time series. The sampling rate is chosen to be at least two times the corner frequency to minimize aliasing effects.

<table>
<thead>
<tr>
<th>( U )</th>
<th>( x/M_U )</th>
<th>( f_c )</th>
<th>Sampling rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 m s(^{-1})</td>
<td>35</td>
<td>7300 Hz</td>
<td>20 000 Hz</td>
</tr>
<tr>
<td></td>
<td>66</td>
<td>4500 Hz</td>
<td>14 000 Hz</td>
</tr>
<tr>
<td></td>
<td>96</td>
<td>3500 Hz</td>
<td>10 000 Hz</td>
</tr>
<tr>
<td>6 m s(^{-1})</td>
<td>35</td>
<td>9000 Hz</td>
<td>36 000 Hz</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>7000 Hz</td>
<td>28 000 Hz</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>6000 Hz</td>
<td>24 000 Hz</td>
</tr>
<tr>
<td>8 m s⁻¹</td>
<td>91</td>
<td>5000 Hz</td>
<td>19 000 Hz</td>
</tr>
<tr>
<td>--------</td>
<td>----</td>
<td>---------</td>
<td>-----------</td>
</tr>
<tr>
<td></td>
<td>121</td>
<td>4500 Hz</td>
<td>18 000 Hz</td>
</tr>
<tr>
<td></td>
<td>131</td>
<td>4000 Hz</td>
<td>15 600 Hz</td>
</tr>
</tbody>
</table>

| 4.6. Homogeneity |

Assessing the homogeneity at the end of the test section for the slowest speed (4 m s⁻¹) is important to ensure that the flow is homogenous throughout the isotropic range. Figure 6 shows the variation of $U$, $\overline{u^2}$, $S(u)$, $\overline{(\partial u/\partial t)^2}$, and $S(\partial u/\partial x)$ in the transverse direction at $x/M_u = 142$ at 4 m s⁻¹. Over the range of $-10 \leq y/M_u \leq 10$, the mean velocity is seen to varies by less than ±1%. For $-8 \leq y/M_u \leq 10$, $\overline{u^2}/U^2$ varies by less than ±4%; for $-10 \leq y/M_u \leq 10$, $\overline{(\partial u/\partial t)^2}$ varies by about ±5% and for $-10 \leq y/M_u \leq 10$, $S(\partial u/\partial x)$ varies by less than ±5%. In summary, based on these measurements, homogeneity is seen to occur for most quantities of interest statistics for $-10 \leq y/M_u \leq 10$. 

TABLE 7. Filter and sampling rates for downstream location
Figure 6. (A) $\overline{U}$, (B) $\overline{u^2}$, (C) $S(u)$, (D) $\left(\frac{\partial u}{\partial t}\right)^2$, and (E) $S\left(\frac{\partial u}{\partial t}\right)$ for $\frac{x}{M_U} = 142$ and $\frac{x}{M_U} = 0$ for 4 m s$^{-1}$. 
4.7. Experimental condition

Experimental data are collected at various downstream position for mean velocities of 8, 6 and 4 m s\(^{-1}\). The corresponding mesh Reynolds numbers are 12,959, 19,438 and 25,918.

5. Results

The power-law decay for the downstream variation of the turbulent kinetic energy is only predicted for the homogeneous–isotropic region of the flow. However, even when downstream positions close to the grid are included, and where the flow is not isotropic, experimentally, a power law of the form

\[
q^2 = A \left( \frac{x}{M_0} - \frac{x_0}{M_0} \right)^{-n}
\]

is found to provide a reasonably good description of the downstream variation of the turbulent kinetic energy. However, although this aspect of the power law has been exhibited, other predicted results from the power are not inconsistent. As discussed in the analysis sections, other quantities, such as the downstream variation of the dissipation rate, \(\epsilon\), and length scales (specifically, \(\eta\), \(L_U\), \(\lambda\), and \(R_\lambda\)), should have power-law behaviors, and the exponents and coefficients in those power laws should have a specific relationship, as implied in eqns (10) to (13).

Furthermore, the ratio of the measured dissipation rate, \(\epsilon\), based on the time derivative of the velocity,

\[
\epsilon = \frac{15}{U^2} \left( \frac{dU}{dt} \right)^2
\]

should be equal to the dissipation rate, \(\epsilon^*\), computed using the turbulent Reynolds average energy equation, seen in eqn. (1), where it is assumed that the flow is isotropic and \(\overline{u^2} = \overline{v^2} = \overline{w^2}\). The equality does not hold when data from the non-isotropic portion of the flow are included in the determination of the power law exponent. Specifically, if the flow is not isotropic, \(\epsilon \neq \epsilon^*\).
In the next section, the approaches used to assess the isotropy of the flow are presented. The parameters used in this assessment include the skewness of the velocity, $S(u)$; the skewness of the velocity derivative, $S(\partial u/\partial x)$, which, based on the analysis of Kolmogorov (1941), should approach a constant; and the product of the skewness of the velocity derivative and the Taylor Reynolds number, $S(\partial u/\partial x)R_\lambda$, which is also predicted to reach a constant value in isotropic flows (c.f. George 1992). George (1992) asserts that the constancy of the latter measure is a better predictor of isotropy in finite Reynolds number flows, whereas the former is a more appropriate indicator of isotropy as the Reynolds number approaches infinity.

The power-law results are presented not only for downstream positions, where $(\partial u/\partial x) \approx $ Constant, but also for the separate set of downstream positions, where $S(\partial u/\partial x)R_\lambda \approx $ Constant. For the positions where these criteria are applied, $S(u) \approx 0$. In the next section, the variation of $S(u)$, $S(\partial u/\partial x)$, and $S(\partial u/\partial x)R_\lambda$ with downstream positions are presented. Then, the process used to determine the virtual origin and values of the virtual origin for each mean velocity and the two measures of isotropy will be discussed.

5.1. Determination of downstream positions where flow is isotropic

In this section, the downstream variation of $S(u)$, $S(\partial u/\partial x)$, and $S(\partial u/\partial x)R_\lambda$ are presented. These data are used to identify the flow positions that correspond to regions where the flow is isotropic. The skewness of the velocity $S(u) = \frac{-u^3}{\langle u^2 \rangle^2}$, as a function of downstream position for the three mean velocities, are shown in fig. 7.
Figure 7. Skewness of the velocity (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ as a function of normalized downstream position.
An inspection of fig. 7 shows that for 8 m s\(^{-1}\), the position where \(S(u) \rightarrow 0\) is further downstream than it is for 6 and 4 m s\(^{-1}\). The observed scatter is approximately 0.05. For this reason, the downstream position where \(|S(u)| \leq 0.1\) is chosen as the region where this measure indicates an isotropic flow. The corresponding downstream positions for each mean velocity where the value of \(S(u)\) indicates an isotropic flow are shown in Table 8. Downstream position where \(|S(u)| \leq 0.1\)

| Mean velocity (m s\(^{-1}\)) | Downstream position where \(|S(u)| \leq 0.1\) |
|-----------------------------|---------------------------------------------|
| 8                           | 100                                         |
| 6                           | 90                                          |
| 4                           | 80                                          |

Table 8. Downstream position where \(|S(u)| \leq 0.1\)

The downstream variation of the skewness of the velocity derivative \(S(\partial u/\partial x) = (\partial u/\partial x)^3 / (\partial u/\partial x)^{3/2}\) is shown in fig. 8.
Figure 8. Skewness of the velocity derivatives for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ as a function of normalized downstream position.
Evidence of a plateau is seen for all three mean velocities. The positions where \( S(\partial u/\partial t) \) approaches a constant value are shown in Table 9. Downstream position where \( S(\partial u/\partial x) \approx \text{Constant} \).

<table>
<thead>
<tr>
<th>Mean velocity (m s(^{-1}))</th>
<th>Starting downstream position where ( S(\partial u/\partial x) \approx \text{Constant} )</th>
<th>( S(\partial u/\partial x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>110</td>
<td>-0.58</td>
</tr>
<tr>
<td>6</td>
<td>115</td>
<td>-0.56</td>
</tr>
<tr>
<td>4</td>
<td>107</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

**Table 9. Downstream position where \( S(\partial u/\partial x) \approx \text{Constant} \)**

The value of \( S(\partial u/\partial x) \) in the plateau area decrease with decreasing mean velocity from -0.58 at \( \bar{U} = 8 \text{ m/s} \) to -0.56 at \( \bar{U} = 6 \text{ m/s} \) and -0.51 at \( \bar{U} = 4 \text{ m/s} \). This variation in value is consistent with George’s (1992) hypothesis that the value of \( S(\partial u/\partial x) \) can depend on initial conditions. It is also interesting to note that \( S(\partial u/\partial x) \approx -0.5 \) for passive grids (cf. Mohamed & LaRue 1990).

George (1992) concludes that \( S(\partial u/\partial x)R_\lambda \) should approach a constant in isotropic flows. However in the decaying turbulent flow of the current study for \( n \neq 1 \), as suggested by Eqn. (13), \( R_\lambda \) will continue to decrease with increase in downstream distance. Thus, if indeed \( S(\partial u/\partial x) \) has approached a constant value or is decreasing, it will not be possible to identify a range of downstream positions where \( S(\partial u/\partial x)R_\lambda = \text{Constant} \). Thus, for this study a range of downstream positions where \( S(\partial u/\partial x)R_\lambda \) or equivalent \( S(\partial u/\partial t)R_\lambda \) varies by at most \( \pm 3.3\% \) for 4 and 6 m s\(^{-1}\) and 3.6\% for 8 m s\(^{-1}\) will be identified. Fig. 9 shows \( S(\partial u/\partial x)R_\lambda \) as a function of downstream distance.
Figure 9. Variation of $S(\partial u/\partial t)R_\lambda$ with downstream position for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$. Red symbols show the region where $S(\partial u/\partial t)R_\lambda$ varies by less than ±4%. Black symbols show the full data range.
At 6 and 4 m s\textsuperscript{-1} regions where $S(\partial u/\partial x)R_\lambda$ approaches a nearly constant value and varies less than ±3.3%. A plateau is less obvious at 8 m s\textsuperscript{-1} but a range of downstream positions where $S(\partial u/\partial x)R_\lambda$ varies by less than ±3.6% is also identified. The approach used to determine the region where $S(\partial u/\partial x)R_\lambda$ approaches a constant value is described in detail in Appendix C. Briefly, the approach is based on the identification of a region in the flow where $S(\partial u/\partial x)R_\lambda$ is found to vary by less than twice the estimated uncertainty in $S(\partial u/\partial x)R_\lambda$ over a range of downstream positions of at least 15 $M_\nu$. The values of $S(\partial u/\partial x)R_\lambda$ that satisfy these criteria are indicated in red in fig. 9. The start and end points for the position where $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ satisfies this criteria are shown in Table 10. Downstream position where $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ However, only at 8 m s\textsuperscript{-1} does there appear to be a plateau as, $S(\partial u/\partial x)R_\lambda$ show decreasing value with increasing downstream distance.

<table>
<thead>
<tr>
<th>Mean velocity (m s\textsuperscript{-1})</th>
<th>Start of $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$</th>
<th>End of $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$</th>
<th>Range of $x/M_\nu = \text{Constant}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>111</td>
<td>127</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>123</td>
<td>141</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>131</td>
<td>21</td>
</tr>
</tbody>
</table>

**TABLE 10. Downstream position where $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$**

Comparison of the start locations where $S(\partial u/\partial x)$ and $S(\partial u/\partial x)R_\lambda$ become constant shows that for all three mean speeds, $S(\partial u/\partial x)R_\lambda$ becomes constant at positions downstream of the corresponding positions where $S(\partial u/\partial x)$ becomes a constant. In addition, in contrast to the behavior of $S(\partial u/\partial x)$, $S(\partial u/\partial x)R_\lambda$ is no longer a constant further downstream where $S(\partial u/\partial x)$ still remains a constant.

### 5.2. Determination of the virtual origin

Mohamed & LaRue (1990) used a statistical measure to determine the goodness of fit of the power law to describe the downstream variation of the turbulent kinetic energy. The researchers chose values for $n_u$ and $x_0$ that minimize the standard deviation of the difference between the measured value of $\overline{u'^2}$ and that predicted from the power law at the same downstream location.
In contrast, an alternative and more straightforward approach to determine the virtual origin, \( x_0/M_U \), and decay exponent, \( n \), based on the variation of \( \lambda^2 \) with downstream distance has been used by Antonia et al. (2003). This approach is based on the analytical expression shown in eqn (10) that \( \lambda^2 \sim (x/M_U - x_0/M_U) \). This expression is valid for all decay exponents, \( n \), and has been verified experimentally by Comte-Bellot & Corrsin (1971), Batchelor (1947), George (1992), and Antonia et al. (2003).

Specifically, Antonia et al. (2003) note that \( x_0 \) can be determined by finding the value of \( x_0 \) that leads to 
\[
\frac{\lambda^2}{(M_U (x - x_0))} = \text{Constant}
\]
for the maximum range of downstream positions. Antonia et al. (2003) also shows that eqn (10) can be manipulated to also determine the decay exponent \( n_\lambda \) where 
\[
n_\lambda = \frac{(10M_U)/(\lambda^2 \nu)(x/M_U - x_0/M_U)}{x_0}
\]
Thus with a known \( x_0 \), the power law equation for \( \lambda^2 \) can be used to determine the decay exponent, \( n_\lambda \). It should be noted that Antonia et al. (2003) did not use one of the indicators of isotropy first (either \( S(\partial u/\partial x) \approx \text{Constant} \) or \( S(\partial u/\partial x)R_2 \approx \text{Constant} \)) to determine the range over which \( \lambda^2 \sim (x/M_U - x_0/M_U) \) but, rather, assumed that the appropriate range where the power decay is applicable is determined by finding the value that maximizes the downstream range over which 
\[
\frac{\lambda^2}{(x/M_U - x_0/M_U)} = \text{Constant}
\]
An alternative approach that obviates the need for this assumption is to first determine the downstream positions, using either \( S(\partial u/\partial x) \) or \( S(\partial u/\partial x)R_2 \) as indicators as to where the flow is isotropic and then use a slight modification of the approach used by Antonia et al. (2003) to determine \( x_0 \) and \( n_u^2 \).

The first step in the modified approach proposed herein to determine \( x_0 \) is to identify the downstream position where the flow is isotropic. The second step is based on a least-square fit of 
\[
\frac{\lambda^2 U}{\nu M_U}
\]
as a function of \( x/M_U \). This approach is based on a manipulation of eqn (10) that leads to the following expression:

\[
\frac{\lambda^2 U}{\nu M_U} = \frac{10}{n_\lambda^2 M_U} \frac{x}{x_0} - \frac{10}{n_\lambda^2 M_U} \frac{x_0}{x_0}
\]

or
\[
\frac{\lambda^2 U}{\nu M_U} = A_\lambda \frac{x}{M_U} - B_\lambda
\]

The method of least squares can be applied to find \(A_\lambda\) and \(B_\lambda\). Once those coefficients are determined, the process to determine \(x_0\) and \(n_{\lambda^2}\) becomes straightforward.

Thus, the first step is to determine the range of downstream positions where the flow is isotropic. As discussed in the preceding, one indicator of the position where the flow is isotropic is based on determining the downstream positions where \(S(\partial u/\partial x) \approx Constant\). Those positions are shown in Table 8. Downstream position where \(|S(u)| \leq 0.1\). The experimental values for \(\lambda^2 U/\nu M_U\) starting at the downstream positions listed in Table 9. Downstream position where \(S(\partial u/\partial x) \approx Constant\) are shown in fig. 10. The corresponding linear least square fit to those data correspond to the solid lines on that figure.
Figure 10. $\lambda^2 U/\nu M_U$ as a function of normalized downstream position for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x) \approx$ Constant.
The data seem to be well described by a linear fit.

It is also possible to find the isotropic range using \( S(\partial u/\partial x)R_{\lambda} = \text{Constant} \) as the indicator. In contrast to the isotropy indicator, \( S(\partial u/\partial x) \approx \text{constant} \), where only the starting location of the isotropic region is indicated, this indicator yields both a starting and ending location for the isotropic region and that are listed in Table 10. Downstream position where \( S(\partial u/\partial x)R_{\lambda} \approx \text{Constant} \)
Figure 11. $\lambda^2 U / \nu M_U$ as a function of normalized downstream position for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u / \partial x) R_\lambda \approx \text{Constant.}$
These data also seem to be well described by a linear fit. The corresponding values of the virtual origin and the decay exponent \( n_{\lambda z} \) are shown in Table 11. Values of the normalized virtual origin \( (x/M_U) \) and \( n_{\lambda z} \) for 8, 6, and 4 m s\(^{-1}\) using either \( S(\partial u/\partial x) \approx \text{Constant} \) or \( S(\partial u/\partial x)R_\lambda \approx \text{Constant} \), with both having \( S(u) \approx 0 \) for both \( S(\partial u/\partial x) \) and \( S(\partial u/\partial x)R_\lambda \) isotropic indicators.

<table>
<thead>
<tr>
<th>( U ) (m/s)</th>
<th>Measure of isotropy</th>
<th>( S(u) \approx 0 )</th>
<th>( S(\partial u/\partial x) \approx \text{Constant} )</th>
<th>( S(u) \approx 0 )</th>
<th>( S(\partial u/\partial x)R_\lambda \approx \text{Constant} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_0/M_U )</td>
<td>( n_{\lambda z} )</td>
<td>( x_0/M_U )</td>
<td>( n_{\lambda z} )</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-39.049</td>
<td>-1.640</td>
<td>-5.7565</td>
<td>-1.294</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-83.973</td>
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<td>-27.958</td>
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<tr>
<td>4</td>
<td>-52.097</td>
<td>-2.253</td>
<td>-13.284</td>
<td>-1.735</td>
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</tbody>
</table>

**TABLE 11.** Values of the normalized virtual origin \( (x/M_U) \) and \( n_{\lambda z} \) for 8, 6, and 4 m s\(^{-1}\) using either \( S(\partial u/\partial x) \approx \text{Constant} \) or \( S(\partial u/\partial x)R_\lambda \approx \text{Constant} \), with both having \( S(u) \approx 0 \).

Of note is the fact that the values of the virtual origins depend on both the mean speed and the parameters used to determine the downstream positions where the flow is isotropic. The variation of the virtual origin and the decay exponent is consistent with George’s (1992) assertion that the value of the virtual origin and the decay exponent depends on initial conditions. One other observation are that the use of \( S(\partial u/\partial x) \approx \text{Constant} \) as an indicator of the isotropic region leads to relatively large values for the virtual origin in comparison to those found based on the indicator \( S(\partial u/\partial x)R_\lambda \approx \text{Constant} \).

### 5.3. Power law determination and assessment

This section will be divided into four subsections. The first two subsections discuss the normalized power-law fits of \( \overline{u^2}/\overline{U}^2 \), \( \epsilon M_U/\overline{U}^3 \), \( n_{\lambda z}^4/\nu^3 M_U \), \( l_U/M_U \), and \( R_\lambda^2/Re_M \), and the \( A_\theta \) and \( n_\theta \) from each power law are compared with \( A_{u^2} \) and \( n_{u^2} \). In addition, the ratio of the dissipation, \( \epsilon/\epsilon^* \), is calculated and plotted as a function of the normalized downstream position, \( x/M_U \), with the ratio expected to be near unity. In §5.3.1 the analysis is performed using the virtual origin shown in Table 11 for \( S(\partial u/\partial x) \approx \text{Constant} \) and then repeated in §5.3.2 but using the data corresponding range of
downstream positions consistent with \((\partial u/\partial x)R_\lambda \approx Constant\). The third subsection, §5.3.3, compares the two isotropic ranges found from \(S(\partial u/\partial x)\) and \(S(\partial u/\partial x)R_\lambda \approx Constant\) and assesses which measure leads to the isotropic range for active-grid-generated turbulence. Finally, the fourth subsection, §5.4, evaluates the power-law relationships of \(\eta\) and \(l_u\) with \(\lambda\) for the isotropic data sets found in the previous section.

5.3.1. Decay Power Law using \(S(\frac{\partial u}{\partial x}) \approx Constant\) as the isotropy indicator

The downstream decay of the turbulence intensity squared, \(\frac{u^2}{U^2}\), is an important measure of turbulence and has been predicted to follow a power-law decay in decaying homogenous–isotropic turbulence in multiple studies (cf. Kolmogorov 1941; Saffman 1967; and George 1992). However, those analysis lead to different predicted values of the decay exponent, \(n\), seen in eqn. (3). Shown on fig. 12 is the decay of \(\frac{u^2}{U^2}\) as a function of \(x_0/M_U - x_0/M_U\). The solid line corresponds to a least-square fit for each mean speed (8, 6, and 4 m s\(^{-1}\)) for the range of downstream position as reported in Table 10 where \(S(\partial u/\partial x) \approx Constant\).
FIGURE 12. $\overline{u^2}/U^2$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x) \approx $ Constant on a log–log scale.
As can be noted the downstream decay of the turbulence intensity squared follows a power law (solid lines in fig. 12) for all mean speeds.

The next decay of importance is the decay of the dissipation, $\epsilon$. The dissipation is also shown to follow a power law, as expressed by eqn (5). Normalized dissipation as a function of downstream distance is shown in fig. 13. In this figure, the dissipation is calculated using the temporal derivative, Taylor hypothesis and the assumption of local isotropy and eqn. (1), which leads to the following expression for the dissipation rate:

$$\epsilon = \frac{15\nu}{T^3} \left( \frac{\partial u^2}{\partial t} \right).$$  \hfill (26)
Figure 13. $\epsilon M_U/U^3$ as a function of normalized downstream position, $x/M_U - x_0/M_U$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x) \approx$ Constant on a log–log scale.
The downstream decays of the dissipation calculated from the temporal derivative are shown in fig. 13 for (A) 8 m s\(^{-1}\), (B) 6 m s\(^{-1}\), and (C) 4 m s\(^{-1}\). The solid lines represent the least-square power–decay-law fits applied to the normalized dissipation. For all mean speeds, the downstream decay is well described by a power law.

Using the decay constant, \(A\), and the decay exponent, \(n\), calculated from the least-square fit applied to \(\overline{u^2}/\overline{U^2}\) in fig. 12, the dissipation, \(\epsilon^*\), from eqn (5) can be calculated. Thus, the ratio of \(\epsilon/\epsilon^*\) can be calculated and those values are shown as a function downstream position in Figure 14. Note that the value of \(\epsilon/\epsilon^*\) should have a constant value of unity.
Figure 14. $\epsilon/\epsilon^*$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$ for (A) $8 \text{ m s}^{-1}$, (B) $6 \text{ m s}^{-1}$, and (C) $4 \text{ m s}^{-1}$ for $S(\partial u/\partial x) \approx \text{Constant.}$
As can be seen on fig. 14, for three mean speeds, the ratio of $\epsilon/\epsilon^*$ is approximately a constant and there appears to be no trend the value of $\epsilon/\epsilon^*$ with downstream position. However it should be noted that $\epsilon/\epsilon^*$ does not equal one. The scatter is about $\pm0.025$ at 8 and 6 m s$^{-1}$ and $\pm0.04$ at 4 m s$^{-1}$. This variation is on the order of that suggested by the uncertainty due to lack of stationarity and electronic noise, which is about $\pm5\%$. The approximate average values are 0.88, 0.93, and 1.14 for 8, 6, and 4 m s$^{-1}$, respectively. For all 3 mean velocities the averages of the values of $\epsilon/\epsilon^*$ do not fall within $\pm5\%$ of unity.

The Kolmogorov microscale, $\eta$, as shown in eqn (11), also follows a power law. The normalized values of $\eta^4$ as a function of $x/M_U - x_0/M_U$ are shown in fig. 15.
FIGURE 15. $\frac{\tau u^3}{v^3 M_U}$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$ for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x) \approx$ Constant on a log–log scale.
The solid line represents the least-square power-law fit. This indicates that for all 3 mean velocities, the downstream variation of $\eta$ is well described by a power law.

The integral scale is a measure of the largest scales in turbulence where most of the energy is contained. The integral scale, $L_U$, is also predicted in eqn (12) a power-law behavior. The normalized integral scale is shown as a function of downstream position in fig. 16. The solid line in that figure corresponds to a least square fit. As can be seen the integral length scale is reasonably well described by a power law.
Figure 16. \( L_U/M_U \) as a function of normalized downstream position, \( \frac{x}{M_U} - \frac{x_0}{M_U} \), for (A) 8 m s\(^{-1}\), (B) 6 m s\(^{-1}\), and (C) 4 m s\(^{-1}\) for \( S(\partial u/\partial x) \approx \) Constant on a log-log scale.
Note that the scatter in $L_U/M_U$ is large compared to the change in $L_U/M_U$ corresponding the values predicted by the power law fit at the start and end points of the isotropic region.

The Taylor Reynolds number, $R_A$, as a function of downstream position indicated by the solid line in Figure 17 also is well described by a power law which is consistent with eqn (13). The normalized Taylor Reynolds number squared is showed as a function of downstream distance in Figure 17. The solid line shown on that figure corresponds to the power law fit determined using the method of least squares. It can be observed that, consistent with eqn (13) the normalized Taylor Reynolds number squared is well described by a power law.
Figure 17. $R_A^2 / R_{MU}$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u / \partial x) \approx \text{Constant}$ on a log–log scale.
For each power-decay law, the decay exponent and decay constant can be converted to the power-decay law of the normalized downstream velocity variance. The values for the virtual origins, \( n \) and \( A \), corresponding to using \( S(\partial u/\partial x) \approx \text{Constant} \) as the indicator of isotropy, are shown in Table 12.

Power Law for \( S(\partial u/\partial x) \approx \text{Constant} \) as indicator of isotropy.

<table>
<thead>
<tr>
<th>Figure label</th>
<th>( U )</th>
<th>( (A) )</th>
<th>( (B) )</th>
<th>( (C) )</th>
</tr>
</thead>
<tbody>
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<td>-39.049</td>
<td>-83.973</td>
<td>-52.097</td>
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</tr>
<tr>
<td>( x/M_{u\text{end}} )</td>
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<tr>
<td>( S(u)_{\text{start}} )</td>
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</tr>
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<td>-1.9416</td>
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</tr>
</tbody>
</table>

Table 12. Power Law for \( S(\partial u/\partial x) \approx \text{Constant} \) as indicator of isotropy

(1) the end position corresponds to the furthest downstream position

For mean velocities of 6 and 8 m s\(^{-1}\) the exponent values obtain from the power law fits to \( \bar{u}^2/\bar{U}^2 \), \( \varepsilon \), \( \eta \), \( L_U \) and \( R_\lambda \) vary by at most 1.2%. At 4 m s\(^{-1}\), the corresponding variation is 6%. The value of the power law exponent obtained from the power law fit to \( \lambda \) also differs from the average values obtained from \( \bar{u}^2/\bar{U}^2 \), \( \varepsilon \), \( \eta \) and \( L_U \) and 8, 6 and 4 m s\(^{-1}\) \( n_\lambda \) is respectively about 11% less than, 8% less than and 12% higher than the average of the corresponding four values. Unless the exponents determined from the least
square fit to $\lambda$ and $u^2$ are equal, the ratio of $\epsilon/\epsilon^*$ will not equal 1.0 which is required if the power law is indeed applicable to the downstream decay of the velocity variance.

5.3.2. Decay power law using $S\left(\frac{\partial u}{\partial x}\right) R_\lambda \approx \text{Constant}$ as the indicator of isotropy

In this section, the power law exponent and constant are determined for the range of downstream positions where $S(\partial u/\partial x) R_\lambda$ is found to be nearly a constant. The corresponding beginning and end positions where $S(\partial u/\partial x) R_\lambda \approx \text{Constant}$ are shown in Table 10. Downstream position where $S(\partial u/\partial x) R_\lambda \approx \text{Constant}$ and the virtual origin is provided in Table 11. Values of the normalized virtual origin $(x/M_U)$ and $n_{\lambda^2}$ for 8, 6, and 4 m s$^{-1}$ using either $S(\partial u/\partial x) \approx \text{Constant}$ or $S(\partial u/\partial x) R_\lambda \approx \text{Constant}$, with both having $S(u) \approx 0$ Fig. 18. shows the variation of $\overline{u^2}/\overline{U^2}$ with downstream position. The solid line is a least square fit to the data where $S(\partial u/\partial x) R_\lambda \approx \text{Constant}$. The variation appears to be well described by a power law.
Figure 18. $\overline{u^2}/U^2$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$ for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ on a log–log scale.
The dissipation of the turbulent kinetic energy, $\epsilon$, calculated using the temporal derivative with the use of Taylor's hypothesis and the assumption of local isotropy also follows a power law as a function of downstream distance, as indicated by the as solid lines fig. 19.
Figure 19. $\epsilon M_U / U^3$ as a function of normalized downstream position, $x / M_U - x_0 / M_U$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u / \partial x) \mathcal{R}_\lambda \approx \text{Constant}$ on a log–log scale.
Using the decay constant, \( A \), and the decay exponent, \( n \), calculated from the least-square fit applied to \( \overline{u^2}/U^2 \) in fig. 18, the dissipation, \( \epsilon^* \), from eqn (5) can be calculated. Then, the ratio of \( \epsilon/\epsilon^* \) can be calculated and should have a value of 1 (see fig. 20). The values of \( \epsilon/\epsilon^* \) as a function of \( x/M_U - x_0/M_U \) are shown in fig. 20. The ratio, \( \epsilon/\epsilon^* \) which should have a value of unity and where \( \epsilon \) is determined from the time derivative of the velocity and \( \epsilon^* \) is determined from eqn. (5). The average values for \( \epsilon/\epsilon^* \) for mean velocities of 8, 6 and 4 m s\(^{-1}\) are respectively 1.0, 1.03 and 1.18. The estimated uncertainty in \( \epsilon/\epsilon^* \) for the three mean velocities are about \( \pm 5\% \), the values of \( \epsilon/\epsilon^* \). The values of \( \epsilon/\epsilon^* \) at 8 and 6 m s\(^{-1}\) are consistent with the power law form of the decay of the velocity variance. However the average value of \( \epsilon/\epsilon^* \) of 1.18 at 4 m s\(^{-1}\) is not consistent with the value expected and suggests that, at this velocity, the flow at the furthest downstream positions may not be sufficient homogenous.
FIGURE 20. $\epsilon/\epsilon^*$ as a function of normalized downstream position, $x/M_U - x_0/M_U$ for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$. 
As shown in eqn (11), the Kolmogorov microscale, $\eta$, also follows a power law. The normalized value of $\eta$ is expressed as a function of the normalized downstream position on fig. 21. The solid line is the power law fit and it can be seen that variation of the Kolmogorov microscale is well described by a power law.
FIGURE 21. $\frac{\frac{4}{3}U^3}{v^3M_U}$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$ for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ on a log–log scale.
The integral scale, $L_U$, also is predicted to be well described by a power law as a function of downstream position. The normalized $L_U$ as a function of normalized downstream position is shown in fig. 22. The solid lines correspond to the power law. As can be seen, the downstream variation of $L_U$ is reasonably well described by a power law.
FIGURE 22. $L_U/M_U$ as a function of normalized downstream position, $\frac{x}{M_U} - \frac{x_0}{M_U}$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx Constant$ on a log–log scale.
The variation of Taylor Reynolds number, $R_A$, with downstream distance should also follow a power law according to eqn (13). $R_A$ is shown on fig. 23 as a function of downstream distance. The solid line is the power law fit. As can be observed, the downstream variation of $R_A$ is well described by a power law fit. Note that, while a decrease in value of $R_A$ with downstream distance is observed, the variation of $R^2_A/R_{MU}$ over the isotropic range varies from about 6.325 at 8 m s$^{-1}$ to 3.25 at 4 m s$^{-1}$. The uncertainty in the measured values of $R^2_A/R_{MU}$ is estimated to be about 6%. For that reason, variation in the exponent value determined for $R^2_A/R_{MU}$ and the velocity variance decay could be expected to difference.
Figure 23. $R^2 / R_{MU}$ as a function of normalized downstream position, $\frac{x}{MU} - \frac{x_0}{MU}$, for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$ for $S(\partial u / \partial x)R_\lambda \approx \text{Constant}$ on a log–log scale.
Using eqn.(5) and eqns. (10)-(13) the exponent and coefficients for power-decay laws for \( \varepsilon, \lambda^2, \eta, L_U, \) and \( R_\lambda \) can be related to the decay exponent and coefficient for the velocity variance. Those values are shown in Table 13. Tabulated data in nearly homogenous–isotropic decaying turbulence for \( S \left( \frac{\partial u}{\partial x} \right) R_\lambda \).

The upstream and downstream positions where \( S(\partial u/\partial x)R_\lambda \approx \text{constant} \) are also indicated in that table.

<table>
<thead>
<tr>
<th>Figure label</th>
<th>( (A) )</th>
<th>( (B) )</th>
<th>( (C) )</th>
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<td>( U )</td>
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<td>4</td>
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<td>( x_0 )</td>
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<td>-27.958</td>
<td>-13.284</td>
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<td></td>
<td></td>
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<tr>
<td>( \chi )</td>
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<td>123</td>
<td>110</td>
</tr>
<tr>
<td>( \chi_{M_U}^{(1)} )</td>
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<td>140</td>
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</tr>
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<td>( \chi_{M_{U_{end}}} )</td>
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<td>0.02</td>
<td>0.088</td>
</tr>
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<td>300</td>
<td>204</td>
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<tr>
<td>( R_\lambda_{start} )</td>
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<tr>
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<td>( n_{\mu^2} )</td>
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<td>29.44087</td>
<td>6.95122</td>
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<td>7.475778</td>
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<td>-1.7178</td>
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<td>( A_{R_\lambda} )</td>
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<td>28.02847</td>
<td>4.720757</td>
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<tr>
<td>( n_{R_\lambda} )</td>
<td>-1.3072</td>
<td>-1.713</td>
<td>-1.4645</td>
</tr>
</tbody>
</table>

**Table 13.** Tabulated data in nearly homogenous–isotropic decaying turbulence for \( S(\partial u/\partial x)R_\lambda \) as the isotropy indicator

(1) the end position is corresponds to the furthest downstream position.

From Table 13 the decay exponents for all three mean velocities obtained from the power law fits to \( \bar{u}^2/\bar{U}^2, \varepsilon, \eta, L_U \) and \( R_\lambda \) vary by at most 3% for each respective mean speeds. The power law fit to decay exponent for \( \lambda \) for 8 and 6 m s\(^{-1}\) are within the average decay exponents while in contrast 4 m s\(^{-1}\) the decay exponent varies by 20%.
5.3.3. Comparison of $S(\partial u/\partial x)$ and $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ as indicators of the isotropic range.

The downstream variation of $\bar{u}^2$, $\varepsilon$, $\eta$, $L_U$ and $R_\lambda$ for both indicators and all mean speeds are well described by a power law, the decay exponents varies by no more than 6%. $\lambda$ is also shown to follow a power law. However the decay exponents obtain from the least square fit to $\lambda^2$ vary from less than 1% to 20%. For both indicators $\varepsilon/\epsilon^*$ is nearly constant but the constant value varies from 0.87 to 1.15 with $S(\partial u/\partial x)$ as the indicator and 1.0 to 1.175 with $S(\partial u/\partial x)R_\lambda$ as the indicator. For both indicators $\varepsilon/\epsilon^*$ increases monotonically as $\bar{U}$ decreases from 8 to 4 m s$^{-1}$. There is no noticeable trend with mean speed with either indicators for both the decay exponents and the virtual origin. However, the decay exponent decreases and the virtual origin increases for $S(\partial u/\partial x)$ relative to $S(\partial u/\partial x)R_\lambda$ for each mean speed.

5.4. Relationship between $\eta$, $L_U$ and $\lambda$

As noted in §3, eqns (14) and (15) show for $n \neq 1$ that $\eta$ and $L_U$ are not directly proportional to $\lambda$ or to each other. The variation of normalized $\eta$ and $L_U$ as a function of normalized $\lambda$ raised to the power determined from the power law fit to $\bar{u}^2/\bar{U}^2$ and using eqns (14) and (15) are shown on fig. 24 and fig. 25. The solid line corresponds to the least square fit to the data and the agreement is reasonable for $\eta$ as a function of $\lambda$ but to significant scatter the variation of $L_U$ to significant scatter the variation of $L_U$ with $\lambda$ is not well described by that power law fit as predicted by eqn (15).
Figure 24. Normalized $\eta^2$ as a function of normalized $(\lambda^2)^{(n+1)/2}$ for (A) 8 m s$^{-1}$ and (B) 6 m s$^{-1}$ for $S(\partial u/\partial x) R \lambda \approx \text{Constant}$ on a semi-log scale.
Figure 25. Normalized $L_U^2$ as a function of normalized $(\lambda^2)^{2-n}$ for (A) 8 m s$^{-1}$ and (B) 6 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ on a semi-log scale.
In fig. 24 and 25, fits are applied to the data to find the optimal constant in front of the variable. The constant can then be compared with the theoretical constant from fig. 24 for $\eta$ and fig. 25 for $L_U$. This relationship is shown in Table 14. $\eta$ and $L_U$ function of $\lambda$. Note the uncertainty expressed in fig. 25 compared with the fit. Next, in fig. 26 and 27, for $\eta$ and $L_U$, respectively, the data are shown again on a linear scale.
Figure 26. Normalized $\eta^2$ as a function of normalized $\lambda^2$ for (A) 8 m s$^{-1}$ and (B) 6 m s$^{-1}$ for $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ on a linear scale.
Figure 27. Normalized $L_U^2$ as a function of normalized $\lambda^2$ for (A) 8 m s$^{-1}$ and (B) 6 m s$^{-1}$ for $S(\partial u/ \partial x)R_\lambda \approx Constant$ on a linear scale.
Fig. 26 and 27 show $\eta$ and $L_U$, respectively, as a function of $\lambda$ on a linear scale. A least square fit is applied to the data sets where the exponents are forced to be 1.152 and 1.363 for $\eta$ for 8 and 6 respectively and 0.697 and 0.282 for $L_U$ with 8 and 6 respectively and the constants are allowed float to find the optimal values. On the linear scale, both $\eta$ and $L_U$ appear to be linear function of $\lambda$. However this is only the appearance of linearity as, eqns (14) and (15) tell us that the data follow a power law except when $n = 1$, and only in this case is the relationship linear. The exponents differ from under by 15% to 30% for $\eta$ and -30% to -70% for $L_U$ for 8 and 6 m s$^{-1}$. Again, the scatter of $L_U$ as a function to $\lambda$ is observed. Table 14. $\eta$ and $L_U$ function of $\lambda$ shows the exponent for fit for both the linear scales and the nonlinear scales. All of the values are within 5% of each other from the theoretical value to the constant found from the fit, Seen below in Table 14. $\eta$ and $L_U$ function of $\lambda$ the three right most columns.

<table>
<thead>
<tr>
<th>Mean speed (m s$^{-1}$)</th>
<th>Variable</th>
<th>Exponent</th>
<th>Theoretical</th>
<th>$\lambda^n$</th>
<th>$\lambda$</th>
</tr>
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<tr>
<td>8</td>
<td>$\eta$</td>
<td>1.152</td>
<td>0.03578</td>
<td>0.0356</td>
<td>0.03577</td>
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<tr>
<td>6</td>
<td>$\eta$</td>
<td>1.363</td>
<td>0.0104</td>
<td>0.0104</td>
<td>0.0104</td>
</tr>
<tr>
<td>8</td>
<td>$L_U$</td>
<td>0.697</td>
<td>0.2321</td>
<td>0.2296</td>
<td>0.2296</td>
</tr>
<tr>
<td>6</td>
<td>$L_U$</td>
<td>0.282</td>
<td>2.5897</td>
<td>2.593</td>
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</tr>
</tbody>
</table>

**Table 14. $\eta$ and $L_U$ function of $\lambda$**

6. **Conclusion**

The motivation behind this thesis is multifaceted and includes the following aims: (a) to formulate a set of power-law relationships for different measures of decaying homogenous–isotropic turbulence that can be related back to the power-decay law of $\overline{u^2}/\overline{U^2}$; (b) to determine and assess an indicator for the nearly homogenous–isotropic range for active–grid-generated turbulence where a power decay law for $\overline{u^2}/\overline{U^2}$ would be expected; (c) to find the correct virtual origin that leads to consistency in the decay exponent calculated from different measures and yields a value of $\epsilon/\epsilon^* \approx 1$ in nearly
homogenous–isotropic range; and (d) to show that for the nearly homogenous–isotropic data sets, $L_U$ and $\eta$ are related to $\lambda$ by a power law and for $n \neq 1$ where for $n = 1 L_U$ and $\eta$ are linearly related to $\lambda$.

The power law equations for $\epsilon, \lambda, \eta, L_U$ and $R_\lambda$ that are consistent with the power law decay description for $\overline{u^2}/\overline{U^2}$ with downstream distance as discussed in Section 3. The relationships between the exponents and coefficients in the various power laws are elucidated. It is also shown that for $n \neq 1 \eta$ is not proportional $\lambda$ and $L_U$ is not proportional to $\lambda$. The power decay law is predicted only to be applicable in the isotropic and homogeneous portion of the flow. The isotropic range is identified using two pairs of indicators: 1) $S(u) \approx 0$ and $S(\partial u/\partial x) \approx \text{Constant}$ as suggested by Batchelor (1947); and 2) $S(u) \approx 0$ and $S(\partial u/\partial x)R_\lambda \approx \text{Constant}$ as suggested by George (1992). However, it should be noted that $R_\lambda$ in the power decay law region is shown to decrease with increasing downstream distance while the measured values of $S(\partial u/\partial x)$ are found to either decrease or approach a constant value with downstream distance. Thus, for the Reynolds number values of the present study, $S(\partial u/\partial x)R_\lambda$ cannot be a constant with downstream position but a range can be chosen where the variation is less than 4%. For both sub ranges, consistent values for the decay exponent for each of the quantities of interest, for all means speeds and for each sub range are observed. Specifically the exponents of $\overline{u^2}, \epsilon, \eta, L_U$ and $R_\lambda$ vary at most by 6%. For the range of downstream positions where the isotropic indicators $S(u) \approx 0$ and $S(\partial u/\partial x) \approx \text{constant}$, the decay exponent from $\overline{u^2}$ compared to the decay exponent of $\lambda$ vary by less than 11%, 8% and 12% for 8, 6, and 4 m s$^{-1}$ respectively. For the range of downstream positions where $S(u) \approx 0$ and $S(\partial u/\partial x)R_\lambda \approx \text{constant}$ the exponents vary by less than 3% for 8 and 6 m s$^{-1}$ and by about 20% for 4 m s$^{-1}$. Of special note is that $\epsilon/\epsilon^* \approx 1$ for only 8 and 6 m s$^{-1}$ using the isotropy criteria that $S(\partial u/\partial x)R_\lambda \approx \text{constant}$. For the range obtained using $S(\partial u/\partial x) \approx \text{const}, \epsilon/\epsilon^* \neq 1$. This could will be due to the fact that $S(\partial u/\partial x)$ is not sufficiently close to a constant for the entire range of positions. In any case an alternative approach to determine the downstream position where the power decay law applies and where $\epsilon/\epsilon^* \approx 1$ for all mean velocities must be determined.
It is shown that a linear fit to $\lambda^2$ can be used to obtain the virtual origin independently of the value of the decay exponent. This is shown for the case where $S(\partial u/\partial x)R_\lambda \approx Constant$ for 8 and 6 m s$^{-1}$ where the virtual origin yields $\epsilon/\epsilon^* \approx 1$. For all other ranges the virtual origin found from the decay of $\lambda^2$ causes $\epsilon/\epsilon^*$ to be constant but the value is not 1. Finally, for 8 and 6 m s$^{-1}$ in the range where $S(\partial u/\partial x)R_\lambda \approx Constant$, both $L_U$ and $\eta$ follow a power law with $\lambda$, which is consistent with the analysis conducted in §3. Extension of the power law.

7. References


http://journals.cambridge.org/abstract_S0022112096007562.


http://books.google.com/books?id=XgCaNwAACAAJ.

Appendix A: Calibration procedure determination of $T_w$ and $n$

The calibration to the hot wire, HW, starts with determining the $T_w$ and $n$ from eqn (23):

$$\frac{E_{HW}^{2}}{T_{w}-T_{g}} = (A_{HW} + B_{HW} \cdot \bar{U}^{n}).$$  

(23)

This procedure is achieved by taking at least 35 points at a random velocity and random temperature to ensure the velocity ranges from 4 to 20 m s$^{-1}$ and the temperature ranges from room temperature (approximately 23°C) to approximately 100°C. A linear fit is applied to $E_{HW}^{2}/(T_{w} - T_{g})$ as a function of $\bar{U}^{n}$, and a nonlinear optimization is applied to find $T_w$ and $n$ to optimize the goodness of fit of the linear fit. The nonlinear optimizer used was a Solver add-in for Excel. A standard $n = 0.45$ is shown in fig. 28, and an optimized $n$ is shown in fig. 29.
**Figure 28.** $E^2_{HW}/(T_w - T_g)$ as a function of $U^{0.45}$ for $T_w = 246.23^\circ C$, found through nonlinear optimization using Microsoft Excel. The solid line indicates a linear least-square fit applied to the data.

**Figure 29.** $E^2_{HW}/(T_w - T_g)$ as a function of $U^{0.399}$ for $T_w = 246.2^\circ C$. The $n$ and $T_w$ values were found through nonlinear optimization using Microsoft Excel.
The solid lines indicate the linear least-square fits applied to the data for both fig. 28 and 29. For \( n = 0.45 \) and optimized \( n = 0.399 \), respectively, the \( T_w \) value was found to be 246.2°C from the nonlinear optimizer. Under close inspection of Figure 28 and Figure 29, it is seen that the optimized value of \( n = 0.399 \) has a better collapse over the entire data range. Thus, \( n = 0.399 \) was chosen for the experiment data.

**Appendix B: Process of finding \( S(\partial u/\partial x)R_\lambda \approx \text{Constant} \)**

To find where \( S(\partial u/\partial x)R_\lambda \) is constant within the uncertainty of the data, there are a number of steps to be taken. The first step is to find twice the uncertainty of \( S(\partial u/\partial x)R_\lambda \) calculated from stationary information. The next step is to traverse the data of interest one point at a time to determine how many proceeding points fall within twice the uncertainty found with step one, Figure 30.
Figure 30. Number of points within twice the uncertainty of $S(\partial u/\partial x)R_3$ due to temporal stationarity as a function of normalized downstream position for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$. 
It is seen from fig. 30 that there are many of possible data sets for each mean speed. Thus, the next step is to take the data ranges where at least 10 $M_U$ was obtained and find the average high change across the data. To accomplish this task, it is necessary first to find the slope across the data set. With the slope, the heights of the first data point and last data point are subtracted and then normalized by half the uncertainty. From the truncated data set, that value is within one uncertainty of $S(\partial u/\partial x)R_\lambda$ due to temporal stationary, Figure 31.
FIGURE 31. Average change in height for data ranges that are longer than 10 $M_U$ and have a height change of less than one times the $S(\partial u/\partial x)R_A$ due to temporal stationary for (A) 8 m s$^{-1}$, (B) 6 m s$^{-1}$, and (C) 4 m s$^{-1}$. 
Fig. 31 shows a further reduction of points by focusing only on the ranges with an average height change across the range within 1 times the uncertainty of $S(\partial u/\partial x)R_\lambda$ due to temporal stationary for each mean speed. The figure shows that there are still a few choices for the data range; thus, the range that was chosen was the longest $x/M_U$ range from fig. 10.

**Appendix C: Code to finding $S(\partial u/\partial x)R_\lambda \approx Constant$**

%Program finding the number points in a certain flat range
%Primary code
%s8 is the x/M_U data in s8(:,1)
%s8 is the S*R\_\_lambda data in s8(:,2)

last=length(s8);
avg8 = 0;
UNC = 0.0364936; %the uncertainty of the data For S*Re
UNC = 0.029675; %the uncertainty of the data For S(du)

%Number of UNC
n = 2;

Trun8 = 0;

for i =1 : last
    t = FRpossANDinfo(s8,i,last,UNC,n);
    avg8(i,1) = t(1);
    avg8(i,2) = t(2);
    avg8(i,3) = t(3);
    avg8(i,4) = t(4);
end

Trun8 = FlatRangeUnderDH(avg8,s8,n/2,last);

%Looks at the taken value vs to original value
hold on
figure(5);
plot(avg8(:,1),avg8(:,2),'ks','markerfacecolor','k')
grid on
title(# of Points in a Give Flat Range at 2*UNC 8m/s')
axis([20 160 0 35])
xlabel(x/M_U')
ylabel(# of Points')
hold off

hold on
figure(11)
plot(avg8(:,1),avg8(:,3),'ks','markerfacecolor','k')
grid on
title('Slope of a Give Flat Range at 2*UNC 8m/s')
axis([20 160 -2 2])
xlabel('x/M_U')
ylabel('Slope of Flat Range')
hold off

hold on
figure(12)
plot(avg8(:,1),avg8(:,4), 'ks', 'markerfacecolor', 'k')
grid on
title('Height Change Give Flat Range at 2*UNC 8m/s')
axis([20 160 -n/2 n/2])
xlabel('x/M_U')
ylabel('Delta Height / UNC')
hold off

for i = 1 : last
    E8(i) = (s8(i,2)*UNC);
end

figure(6)
hold on
plot(s8(:,1),s8(:,2), 'bs', 'markerfacecolor', 'b')
%errorbar(s8(:,1),s8(:,2), E8, 'bs', 'markerfacecolor', 'b')

figure(6)
plot(Trun8(:,1),Trun8(:,2), 'ks', 'markerfacecolor', 'k')
grid on
axis([20 160 150 550])
xlabel('x/M_U')
ylabel('S(\partial u/\partial t)*R\_\lambda')
title('Finding the constant S(\partial u/\partial t)*R\_\lambda Range For 8 m/s')
legend('Full','Flat Range');

%Find #pts(2) Flat Range @i with UNC*n error, Find Slope(3) and dH(4)
$\text{sub code called by main code}$
function avg = FRposANDinfo(s,i,last,UNC,n)

j = 0;
q = 0;
space = s(i,2)*UNC*n;
for k = i : last
    if (s(i,2) - space) < s(k,2) && s(k,2) < (s(i,2) + space)
        if q == 0
            j = j+1;
        end
    else
        q = 1;
    end
end
avg(1) = s(i,1);
avg(2) = j;
Temp = 0;
%takes the current Flat range to find the slope
if(j > 10)
for q = 1 : j
    Temp(q,1) = s(i+q-1,1);
    Temp(q,2) = s(i+q-1,2);
end
f1 = fit(Temp(:,1),Temp(:,2),’poly1’);
B = coeffvalues(f1);

%Slope
avg(3) = B(1);

%Change in height from slope/1st pt*UNC
avg(4) = B(1)*(s(i-1+j,1)-s(i,1))/(Temp(1,2)*UNC);
else
    avg(3) = 10;
    avg(4) = 100;
end

avg;

%Finds the largest Flat Range with dH across the Range less than deltaH
$\text{Sub code called by main code}$
function Trun = FlatRangeUnderDH(avg,s,deltaH,last)

for i=1:last,
    [C, I] = max(avg(:,2));
    if abs(avg(I,4)) <= deltaH
        break ; % break out of the for-loop
    else
        avg(I,2) = 0;
    end
end

if i == last
    'No range meet the requirement'
    Trun = 0;
else
    C
    xMu = s(I,1)
    SReL = s(I,2)

    for i = 1:C
        Trun(i,1) = s(i+I-1,1);
        Trun(i,2) = s(i+I-1,2);
    end
end

Trun;