Title
Search for $b \rightarrow u$ transitions in $B^{\pm} \rightarrow [K^{\pm} \pi^{\pm} \pi^{0}]DK^ {\pm}$ decays

Permalink
https://escholarship.org/uc/item/0r44b7v8

Journal
Physical Review D - Particles, Fields, Gravitation and Cosmology, 84(1)

ISSN
1550-7998

Authors
Lees, JP
Poireau, V
Tisserand, V
et al.

Publication Date
2011-07-06

DOI
10.1103/PhysRevD.84.012002

License
CC BY 4.0

Peer reviewed
Search for $b \to u$ transitions in $B^+ \to [K^+ \pi^+ \pi^0]D_K^0$ decays

J. P. LEES et al.

PHYSICAL REVIEW D 84, 012002 (2011)


(The BABAR Collaboration)

1Laboratoire d'Annecy-le-Vieux de Physique des Particules (LAPP), Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France
2Universitat de Barcelona, Facultat de Fisica, Departament ECM, E-08028 Barcelona, Spain
3aINFN Sezione di Bari, I-70126 Bari, Italy
3bDipartimento di Fisica, Università di Bari, I-70126 Bari, Italy
4University of Bergen, Institute of Physics, N-5007 Bergen, Norway
5Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA
6Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany
7University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
8Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom
9Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia
10University of California at Irvine, Irvine, California 92697, USA
11University of California at Riverside, Riverside, California 92521, USA
12University of California at Santa Barbara, Santa Barbara, California 93106, USA
13University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA
14California Institute of Technology, Pasadena, California 91125, USA
15University of Cincinnati, Cincinnati, Ohio 45221, USA
16University of Colorado, Boulder, Colorado 80309, USA
17Colorado State University, Fort Collins, Colorado 80523, USA
18Technische Universität Dortmund, Fakultät Physik, D-44221 Dortmund, Germany
19Technische Universität Dresden, Institut für Kern- und Teilchenphysik, D-01062 Dresden, Germany
20Laboratoire Leprince-Ringuet, Ecole Polytechnique, CNRS/IN2P3, F-91128 Palaiseau, France
21University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
22aINFN Sezione di Ferrara, I-44100 Ferrara, Italy
22bDipartimento di Fisica, Università di Ferrara, I-44100 Ferrara, Italy
23INFN Laboratori Nazionali di Frascati, I-00044 Frascati, Italy
24aINFN Sezione di Genova, I-16146 Genova, Italy
24bDipartimento di Fisica, Università di Genova, I-16146 Genova, Italy
25Indian Institute of Technology Guwahati, Guwahati, Assam, 781 039, India
26Harvard University, Cambridge, Massachusetts 02138, USA
27Harvey Mudd College, Claremont, California 91711
28Universität Heidelberg, Physikalisches Institut, Philosophenweg 12, D-69120 Heidelberg, Germany
29Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, D-12489 Berlin, Germany
30Imperial College London, London, SW7 2AZ, United Kingdom
31University of Iowa, Iowa City, Iowa 52242, USA
32Iowa State University, Ames, Iowa 50011-3160, USA
33Johns Hopkins University, Baltimore, Maryland 21218, USA
34Laboratoire de l’Accélérateur Linéaire, IN2P3/CNRS and Université Paris-Sud 11, Centre Scientifique d’Orsay, B. P. 34, F-91898 Orsay Cedex, France
35Lawrence Livermore National Laboratory, Livermore, California 94550, USA
36University of Liverpool, Liverpool L69 7ZE, United Kingdom
37Queen Mary, University of London, London, E1 4NS, United Kingdom
38University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom
39University of Louisville, Louisville, Kentucky 40292, USA
40Johannes Gutenberg-Universität Mainz, Institut für Kernphysik, D-55099 Mainz, Germany
41University of Manchester, Manchester M13 9PL, United Kingdom
42University of Maryland, College Park, Maryland 20742, USA
43University of Massachusetts, Amherst, Massachusetts 01003, USA
We present a study of the decays $B^+ \rightarrow DK^0$ with $D$ mesons reconstructed in the $K^+\pi^-\pi^0$ or $K^-\pi^+\pi^0$ final states, where $D$ indicates a $D^0$ or a $\bar{D}^0$ meson. Using a sample of $474 \times 10^6$ $B\bar{B}$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at SLAC,
we measure the ratios \( R^\pm = \frac{\Gamma(B^+ \to f K^0)}{\Gamma(B^+ \to [f]K^+)} \). We obtain \( R^+ = (5^{+12}_{-10}(\text{stat})^{+5}_{-2}(\text{syst})) \times 10^{-3} \) and \( R^- = (12^{+10}_{-10}(\text{stat})^{+2}_{-2}(\text{syst})) \times 10^{-3} \), from which we extract the upper limits at 90% probability: \( R^+ < 23 \times 10^{-3} \) and \( R^- < 29 \times 10^{-3} \). Using these measurements, we obtain an upper limit for the ratio \( r_B \) of the magnitudes of the \( b \to u \) and \( b \to c \) amplitudes \( r_B < 0.13 \) at 90% probability.

DOI: 10.1103/PhysRevD.84.012002

I. INTRODUCTION

CP violation effects are described in the standard model of elementary particles with a single phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix \( V_{ij} \) [1]. One of the unitarity conditions for this matrix can be interpreted as a triangle in the plane of Wolfenstein parameters [2], where one of the angles is \( \gamma = \arg\{-V_{ub}V_{ud}/V_{cb}V_{cd}\} \). Various methods to determine \( \gamma \) using \( B^+ \to D K^+ \) decays have been proposed [3–5]. In this paper, we consider the decay channel \( B^+ \to D K^+ \) with \( D \to K^- \pi^+ \pi^0 \) [6] studied through the Atwood-Dunietz-Soni (ADS) method [4]. In this method, the final state under consideration can be reached through \( b \to u \) and \( b \to c \) processes as indicated in Fig. 1 that are followed by either Cabibbo-favored or Cabibbo-suppressed \( D^0 \) decays. The interplay between different decay channels leads to a possibility to extract the angle \( \gamma \) alongside with other parameters for these decays.

Following the ADS method, we search for \( B^+ \to [K^- \pi^+ \pi^0]_D K^+ \) events, where the favored \( B^+ \to D^0 K^+ \) decay, followed by the doubly-Cabibbo-suppressed \( D^0 \to K^- \pi^+ \pi^0 \) decay, interferes with the suppressed \( B^+ \to D^0 K^+ \) decay, followed by the Cabibbo-favored \( D^0 \to K^- \pi^+ \pi^0 \) decay. These are called “opposite-sign” events because the two kaons in the final state have opposite charges. We also reconstruct a larger sample of “same-sign” events, which mainly arise from the favored \( B^+ \to D^0 K^+ \) decays followed by the Cabibbo-favored \( D^0 \to K^- \pi^+ \pi^0 \) decays. We define \( f \equiv K^+ \pi^- \pi^0 \) and \( \bar{f} \equiv K^- \pi^+ \pi^0 \). We extract

\[
R^+ = \frac{\Gamma(B^+ \to [f]_D K^+)}{\Gamma(B^+ \to [\bar{f}]_D K^+)}, \tag{1}
\]

from the selected \( B^+ \) and \( B^- \) samples, respectively. While our previous analysis [7] used another set of observables:

\[
R_{ADS} = \frac{\Gamma(B^+ \to [\bar{f}]_D K^+)}{\Gamma(B^+ \to [f]_D K^+)}, \tag{3}
\]

\[
A_{ADS} = \frac{\Gamma(B^+ \to [f]_D K^+)}{\Gamma(B^+ \to [\bar{f}]_D K^+) + \Gamma(B^- \to [\bar{f}]_D K^-)}, \tag{4}
\]

we prefer to use observables defined in Eqs. (1) and (2) since their statistical uncertainties, which dominate in the final error of this measurement, are uncorrelated.

The amplitude of the two-body \( B \) decay can be written as

\[
A(B^+ \to D^0 K^+) = |A(B^+ \to D^0 K^+)| r_B e^{i\delta_B}, \tag{5}
\]

where \( r_B = \frac{|A(B^- \to D^0 K^-)|}{|A(B^+ \to D^0 K^+)|} \) is the ratio of the magnitudes of the \( b \to u \) and \( b \to c \) amplitudes, \( \delta_B \) is the CP conserving strong phase, and \( \gamma \) is the CP violating weak phase. For the three-body \( D \) decay we use similarly defined variables:

\[
r_D^2 \equiv \frac{\Gamma(D^0 \to f)}{\Gamma(D^0 \to \bar{f})} = \int \frac{d\tilde{m}A_{\text{DCS}}(\tilde{m})}{\sqrt{\int d\tilde{m}^2 A_{\text{DCS}}^2(\tilde{m})}}, \tag{6}
\]

\[
k_D e^{i\delta_D} = \frac{\int d\tilde{m}A_{\text{DCS}}(\tilde{m}) A_{\text{CF}}(\tilde{m}) e^{i\delta(\tilde{m})}}{\sqrt{\int d\tilde{m}^2 A_{\text{DCS}}^2(\tilde{m})}}, \tag{7}
\]

where \( A_{\text{CF}}(\tilde{m}) \) and \( A_{\text{DCS}}(\tilde{m}) \) are the magnitude of the Cabibbo-favored (CF) and doubly-Cabibbo-suppressed (DCS) amplitudes, respectively, \( \delta(\tilde{m}) \) is the relative strong phase, and \( \tilde{m} \) indicates a position in the \( D \) Dalitz plot of squared invariant masses \( [m_{K \pi^+}^2, m_{K \pi^0}^2] \). The parameter \( k_D \), called the coherence factor, can take values in the interval [0, 1].

Neglecting \( D \)-mixing effects, which in the standard model give negligible corrections to \( \gamma \) and do not affect the \( r_B \) measurement, the ratios \( R^+ \) and \( R^- \) are related to the \( B^- \) and \( D \)-mesons’ decay parameters through the following relations:

\[
R^+ = r_B^2 + r_D^2 + 2 r_B r_D k_D \cos(\gamma + \delta), \tag{8}
\]

\[
R^- = r_B^2 + r_D^2 + 2 r_B r_D k_D \cos(\gamma - \delta), \tag{9}
\]

with \( \delta = \delta_B + \delta_D \). The values of \( k_D \) and \( \delta \) measured by the CLEO-c Collaboration [8], \( k_D = 0.84 \pm 0.07 \) and
The measurements of ratios $R^{\pm}$ are mainly sensitive to $r_B$. For the same reason, the sensitivity to $\gamma$ is reduced, and therefore the main aim of this analysis is to measure $R^{+}$, $R^{-}$, and $r_B$. The current high precision on $r_B$ is based on several earlier analyses by the BABAR [7,11–13], BELLE [14–16], and CDF [17] Collaborations. This paper is an update of our previous analysis [7] based on $226 \times 10^6 B\bar{B}$ pairs and resulting in a measurement of $R_{ADS} = (13_{-10}^{+17}) \times 10^{-3}$, which was translated into the 95% confidence level limit $r_B < 0.19$.

The results presented in this paper are obtained with 431 fb$^{-1}$ of data collected at the $Y(4S)$ resonance with the BABAR detector at the PEP-II $e^+e^-$ collider at SLAC, corresponding to $474 \times 10^6 B\bar{B}$ pairs. An additional “off-resonance” data sample of 45 fb$^{-1}$, collected at a center-of-mass (CM) energy 40 MeV below the $Y(4S)$ resonance, is used to study backgrounds from “continuum” events, $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, or c$).

II. EVENT RECONSTRUCTION AND SELECTION

The BABAR detector is described in detail elsewhere [18]. Charged-particle tracking is performed by a five-layer silicon vertex tracker and a 40-layer drift chamber. In addition to providing precise position information for tracking, the silicon vertex tracker and drift chamber measure the specific ionization, which is used for identification of low-momentum charged particles. At higher momenta, pions and kaons are distinguished by Cherenkov radiation from a common vertex. The probability distribution of the cosine of the $B$ polar angle with respect to the beam axis in the CM frame, $\cos \theta_B$, is expected to be proportional to $(1 - \cos^2 \theta_B)$. We require $|\cos \theta_B| < 0.8$.

We measure two almost independent kinematic variables: the beam-energy substituted (ES) mass $m_{ES} = \sqrt{(s/2 + \vec{p}_0 \cdot \vec{p}_B)^2/E_0^2 - p_B^2}$, and the energy difference $\Delta E = E_B - \sqrt{s}/2$, where $E$ and $\vec{p}$ are the energy and momentum, the subscripts $B$ and $0$ refer to the candidate $B$ meson and $e^+e^-$ system, respectively, $\sqrt{s}$ is the center-of-mass energy, and $E_B$ is measured in the CM frame. For correctly reconstructed $B$ mesons, the distribution of $m_{ES}$ peaks at the $B$ mass, and the distribution of $\Delta E$ peaks at zero. The $B$ candidates are required to have $\Delta E$ in the range $[-23, 23]$ MeV ($\pm 1.3$ standard deviations). We consider only events with $m_{ES}$ in the range $[5.20, 5.29]$ GeV/$c^2$.

In less than 2% of the events, multiple $B^+$ candidates are present, and in these cases we choose that with a reconstructed $D$ mass closest to the nominal mass value [9]. If more than one $B^+$ candidate share the same $D$ candidate, we select that with the smallest $|\Delta E|$. In the following, we refer to the selected candidate as $B_{sig}$. All charged and neutral reconstructed particles not associated with $B_{sig}$, but with the other $B$ decay in the event, $B_{other}$, are called the rest of the event.

III. BACKGROUND CHARACTERIZATION

After applying the selection criteria described above, the remaining background is composed of nonsignal $B\bar{B}$ events and continuum events. Continuum background events, in contrast to $B\bar{B}$ events, are characterized by a jetlike topology. This difference can be exploited to discriminate between the two categories of events by means of a Fisher discriminant $\mathcal{F}$, which is a linear combination of six variables. The coefficients of the linear combination are chosen to maximize the separation between signal and continuum background so that $\mathcal{F}$ peaks at 1 for signal and at $-1$ for continuum background. They are determined with samples of simulated signal and continuum events,
and validated using off-resonance data. In the Fisher discriminant, we use the absolute value of the cosine of the angle between $B_{\text{sig}}$ and $B_{\text{other}}$ thrust axes, where the thrust axis is defined as the direction maximizing the sum of the longitudinal momenta of all the particles. Other variables included in $\mathcal{F}$ are the event shape moments $L_0 = \sum_i p_i$, and $L_2 = \sum_i p_i |\cos\theta_i|^2$, where the index $i$ runs over all tracks and energy deposits in the rest of the event; $p_i$ is the momentum; and $\theta_i$ is the angle with respect to the thrust axis of the $B_{\text{sig}}$. These three variables are calculated in the CM system. We also use the distance between the decay vertices of $B_{\text{sig}}$ and $D$, the distance of closest approach between $K$ meson tracks belonging to signal decay chain, and $|\Delta l|$, the absolute value of the proper time interval between the $B_{\text{sig}}$ and $B_{\text{other}}$ decays [22]. The latter is calculated using the measured separation along the beam direction between the decay points of $B_{\text{sig}}$ and $B_{\text{other}}$ and the Lorentz boost of the CM frame. The $B_{\text{other}}$ decay point is obtained from tracks that do not belong to the reconstructed $B_{\text{sig}}$, with constraints from the $B_{\text{sig}}$ momentum and the beam-spot location. We use $m_{\text{ES}}$ and $\mathcal{F}$ to define two regions: the fit region, defined as $5.20 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $-5 < \mathcal{F} < 5$, and the signal region, defined as $5.27 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $0 < \mathcal{F} < 5$.

The $BB$ background is divided into two components: nonpeaking (combinatorial) and peaking. The latter consists of $B$-meson decays that have a well-pronounced peak in the $m_{\text{ES}}$ signal region. One of the decay channels which can mimic opposite-sign signal events, is the $B^+ \rightarrow D\rho^+$ decay with $D \rightarrow K^+K^-$ and $\rho^+ \rightarrow \pi^+\pi^0$. In order to reduce this contribution, we veto events for which the invariant $K^+K^-$ pair mass $m_{K^+K^-}$ is $|m_{K^+K^-} - M_D(\text{PDG})| > 20\text{ MeV}/c^2$ (with the $D$ meson invariant mass, $M_D(\text{PDG})$, taken to be 1864.83 MeV/$c^2$ [9]). Simulations indicate that the remaining background is negligible.

Another possible source of peaking $BB$ background is the decay $B^+ \rightarrow D\pi^+$ with $D \rightarrow K^+\pi^-\pi^0$, which can contribute to the signal region of the same-sign sample due to the misidentification of the $\pi^0$ as a $K^+$. The number of events is expected to be about 8% of the total same-sign signal sample (see Table I).

The charmless $B^+ \rightarrow K^+K^-\pi^+\pi^0$ decay can also contribute to the signal region. The branching fraction of this decay has not been measured. Therefore the size of this background is estimated from the sidebands of the reconstructed $D$ mass, $1.904 < M_D < 2.000$ GeV/$c^2$ or $1.700 < M_D < 1.824$ GeV/$c^2$. The result of the study is reported in Table I. In the final fit, we fix the yield of the same-sign $BB$ peaking background to the sum of charmless and open-charm events. The opposite-sign background in the final event sample is assumed to be negligible.

The overall reconstruction efficiency for signal events is $(9.6 \pm 0.1)\%$ for opposite-sign signal events and $(9.5 \pm 0.1)\%$ for same-sign signal events. These numbers are equal within the uncertainty as expected. The composition of the final sample is shown in Table I.

### IV. FIT PROCEDURE AND RESULTS

To measure the ratios $R^+$ and $R^-$, we perform extended maximum-likelihood fits to the $m_{\text{ES}}$ and $\mathcal{F}$ distributions, separately for the $B^+$ and $B^-$ data samples. We write the extended likelihood functions $\mathcal{L}^\pm$ as

$$\mathcal{L}^\pm = \frac{e^{-N^\pm}}{N^\pm!} \cdot N_{f}^{N^\pm} \cdot \prod_{j=1}^{N} f^\pm(x_j|\theta, N^\pm),$$

with

$$f^\pm(x|\theta, N^\pm) = \frac{1}{N^\pm} \left( \frac{R^\pm N_{\text{total}}}{1 + R^\pm} f_{\text{sig,os}}(x|\theta_{\text{sig,os}}) + N_{\text{total}} f_{\text{sig,ss}}(x|\theta_{\text{sig,ss}}) \right).$$

where $f_{\text{sig,ss}}(x|\theta_{\text{sig,ss}})$, $f_{\text{sig,os}}(x|\theta_{\text{sig,os}})$, and $f_{\text{B}}(x|\theta)$ are the probability density functions (PDFs) of the hypotheses that the event is a same-sign signal, opposite-sign signal, or a background event ($B_i$ are the different background categories used in the fit), respectively, $N$ is the number of events in the selected sample, and $N^\pm$ is the expectation value for the total number of events. The symbol $\theta$ indicates the set of parameters to be fitted. $N_{\text{total}}$ is the total number of signal events, $R^\pm = \frac{N_{\text{sig}}}{N_{\text{total}}}$ for the decays of the $B^\pm$ meson, and $N_{B_i}^\pm$ is the total number of events of each background component. For the opposite-sign events, the background comes from continuum and $BB$ events. The peaking $BB$ background is introduced as a separate component in the fit to the same-sign sample. The fit is performed to the $B^+$ sample (consisting of 15 706 events) to determine $R^+$ and $R^-$. The errors are from the statistics of the control samples only.

### TABLE I. Composition of the final selected sample as evaluated from the MC samples normalized to data and from data for the charmless peaking background. The signal contribution is estimated using values of branching fractions from the PDG [9] and $r_B = 0.1$ [10]. The errors are from the statistics of the control samples only.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Region</th>
<th>Signal</th>
<th>$BB$ nonpeaking</th>
<th>Continuum</th>
<th>$D\pi$</th>
<th>Charmless peaking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same sign</td>
<td>Fit</td>
<td>2252 ± 20</td>
<td>459 ± 12</td>
<td>7403 ± 62</td>
<td>176 ± 14</td>
<td>28 ± 14</td>
</tr>
<tr>
<td></td>
<td>Signal</td>
<td>1921 ± 18</td>
<td>147 ± 8</td>
<td>203 ± 10</td>
<td>130 ± 14</td>
<td>21 ± 14</td>
</tr>
<tr>
<td>Opposite sign</td>
<td>Fit</td>
<td>28.7 ± 0.2</td>
<td>434 ± 12</td>
<td>21201 ± 104</td>
<td>⋮</td>
<td>−2 ± 9</td>
</tr>
<tr>
<td></td>
<td>Signal</td>
<td>24.4 ± 0.2</td>
<td>65 ± 5</td>
<td>612 ± 18</td>
<td>⋮</td>
<td>−2 ± 9</td>
</tr>
</tbody>
</table>

012002-6
to the $B^-$ sample (consisting of 15 057 events) to determine $R^+$. The PDFs for $R^+$ and $R^-$ fits are identical. The $R_{\text{ADS}}$ ratio is fitted to the same likelihood ansatz, but to the combined $B^+$ and $B^-$ data sample.

Since the correlations among the variables are negligible, we write the PDFs as products of the one-dimensional distributions of $m_{\text{ES}}$ and $\mathcal{F}$. The absence of correlation between these distributions is checked using MC samples. The signal $m_{\text{ES}}$ distributions are modeled with the same asymmetric Gaussian function for both same-sign and opposite-sign events, while the $\mathcal{F}$ distribution is taken as a sum of two Gaussians. The continuum background $m_{\text{ES}}$ distributions for the same and opposite-sign events are modeled with two different threshold ARGUS functions [23] defined as follows:

$$A(x) = \sqrt{1 - \left(\frac{x}{x_0}\right)^2} \cdot e^{(1 - \left(\frac{x}{x_0}\right))^2},$$

(10)

where $x_0$ represents the maximum allowed value for the variable $x$, and $c$ determines the shape of the distribution. The $m_{\text{ES}}$ distribution of the nonpeaking $B\bar{B}$ background components are modeled with crystal ball functions that are different for same-sign and opposite-sign events [24]. The crystal ball function is a Gaussian modified to include a power-law tail on the low side of the peak. The $\mathcal{F}$ distributions for the $B\bar{B}$ background are approximated with sums of two asymmetric Gaussians. For the peaking $B\bar{B}$ background, we conservatively use the same parameter set as for the signal.

The PDF parameters are derived from data when possible. The parameters for continuum events are determined from the off-resonance data sample. The parameters for the $m_{\text{ES}}$ distribution of signal events are extracted from the sample of $B^+ \rightarrow D\pi^+$ with $D \rightarrow K^+\pi^0\pi^0$, while for the parameters of the signal Fisher PDF we use the MC sample. The parameters of nonpeaking $B\bar{B}$ distributions are determined from the MC sample.

From each fit, we extract the ratios $R^+$, $R^-$, or $R_{\text{ADS}}$, the total number of signal events in the sample ($N_{B^+\text{, tot}}$) along with the nonpeaking background yields and threshold function slope for the continuum background. We fix the number of peaking $B\bar{B}$ background events.

To test the fitting procedure, we generated 10 000 pseudoexperiments based on the PDFs described above. The fitting procedure is then tested on these samples. We find no bias in the number of fitted events for any component of the fit. Tests of the fit procedure performed on the full MC samples give values for the yields compatible with those expected.

The main results of the fit to the data are summarized in Table II.

The fits to the $m_{\text{ES}}$ for $\mathcal{F} > 0.5$ and the $\mathcal{F}$ distribution with $m_{\text{ES}} > 5.27$ GeV/$c^2$ are shown in Fig. 2, for the

![FIG. 2 (color online). Distribution of (a,b) $m_{\text{ES}}$ (with $\mathcal{F} > 0.5$) and (c,d) $\mathcal{F}$ (with $m_{\text{ES}} > 5.27$ GeV/$c^2$) and the results of the maximum-likelihood fits for the combined $B^+$ and $B^-$ samples (extracting $R_{\text{ADS}}$), for (a,c) opposite-sign and (b,d) same-sign decays. The data are well described by the overall fit result (solid blue line) which is the sum of the signal, continuum, nonpeaking, and peaking $B\bar{B}$ backgrounds.](image-url)
combined $B^+$ and $B^-$ sample. These restrictions reduce the background and retain most of the signal events. Figure 3 shows the fits for the separate $B^+$ and $B^-$ samples.

V. SYSTEMATIC UNCERTAINTIES

We consider various sources of systematic uncertainties, listed in Table III. One of the largest contributions comes from the uncertainties on the PDF parameters. To evaluate the contributions related to the $m_{ES}$ and $f$ PDFs, we repeat the fit varying the PDF parameters for each fit species within their statistical errors, taking into account correlations among the parameters (labeled as “PDF error” in Table III).

To evaluate the uncertainties arising from peaking background contributions, we repeat the fit varying the peaking $B\bar{B}$ background contribution within its statistical uncertainties and the errors of branching fractions, $B$, used to estimate the contribution. For the opposite-sign events, only the positive part of the probability distribution is used in the evaluation.

Differences between data and MC (labeled as “Simulation” in Table III) in the shape of the $f$ distribution are studied for signal components using the data control samples of $B^+ \rightarrow D\pi^+$ with $D \rightarrow K^+\pi^−\pi^0$. These parameters are expected to be slightly different between the $B \rightarrow D\pi$ and $B \rightarrow DK$ samples. We conservatively take the systematic uncertainty as the difference in the fit results from the nominal parameters set (using MC events) and the parameters set obtained using the $B \rightarrow D\pi$ data sample.

The systematic uncertainty attributed to the cross feed between opposite-sign and same-sign events has been evaluated from the MC samples. The number of same-sign events passing the selection of the opposite-sign events is taken as a systematic uncertainty. The efficiencies for same-sign and opposite-sign events were verified to be the same within a precision of 3% [25]. We hence assign a systematic uncertainty on $R^\pm$ based on variations due to changes in the efficiency ratio by ±3%.

The systematic uncertainties for the ratios $R^+$, $R^-$, and $R_{ADS}$ are summarized in Table III. The overall systematic errors represent the sum in quadrature of the individual uncertainties.

VI. EXTRACTION OF $r_B$

Following a Bayesian approach [26], the probability distributions for the $R^+$ and $R^-$ ratios obtained in the fit

FIG. 3 (color online). Projections of the 2D likelihood for $m_{ES}$ with the additional requirement $f > 0.5$, obtained from the fit to the $B^+$ (left) and $B^-$ (right) data sample for opposite-sign events (extracting $R^+$ and $R^-$). The labeling of the curves is the same as in Fig. 2.

FIG. 4 (color online). Bayesian posterior probability density function for $r_B$ from our measurement of $R^+$ and $R^-$ and the hadronic $D$ decay parameters $r_D$, $\delta_D$, and $k_D$ taken from [8,9]. The dark and light shaded zones represent the 68% and 90% probability regions, respectively.

<table>
<thead>
<tr>
<th>Source</th>
<th>$R^+$</th>
<th>$R^-$</th>
<th>$R_{ADS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDF error</td>
<td>+1.1</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Same-sign peaking background</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Opposite-sign peaking background</td>
<td>+0</td>
<td>+0</td>
<td>+0</td>
</tr>
<tr>
<td>Simulation</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>$B$ errors</td>
<td>0.2</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Cross feed contribution</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Efficiency ratio</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Combined uncertainty</td>
<td>+1.2</td>
<td>+1.6</td>
<td>+1.4</td>
</tr>
</tbody>
</table>

TABLE III. Systematic errors for $R^\pm$ and $R_{ADS}$ in units of $10^{-3}$. 
are translated into a probability distribution for $r_B$ using Eqs. (8) and (9) simultaneously. We assume the following prior probability distributions: for $r_D$ a Gaussian with mean $4.7 \times 10^{-2}$ and standard deviation $3 \times 10^{-3}$ [9]; for $k_D$ and $\delta_D$, we use the likelihood obtained in Ref. [8], taking into account a 180 degree difference in the phase convention for $\delta_D$; for $\gamma$ and $\delta_B$ we assume a uniform distribution between 0 and 360 degrees, while for $r_B$ a uniform distribution in the range $[0, 1]$ is used. We obtain the posterior probability distribution shown in Fig. 4. Since the measurements are not statistically significant, we integrate over the positive portion of that distribution and obtain the upper limit $r_B < 0.13$ at 90% probability, and the range

$$r_B \in [0.01, 0.11] \text{ at } 68\% \text{ probability,}$$

and 0.078 as the most probable value.

**VII. SUMMARY**

We have presented a study of the decays $B^+ \to D^0 K^+$ and $B^\pm \to D^0 K^\pm$, in which the $D^0$ and $\bar{D}^0$ mesons decay to the $K^\pm \pi^\mp \pi^0$ final state using the ADS method. The analysis is performed using $474 \times 10^6 B\bar{B}$ pairs, the full BABAR data set. Previous results [7] are improved and superseded by improved event reconstruction algorithms and analysis strategies employed on a larger data sample.

The final results are

$$R^+ = (5.9_{-1.4}^{+1.2}(\text{stat})^{+1}_{-4}(\text{syst})) \times 10^{-3},$$

$$R^- = (12.0_{-10}^{+12}(\text{stat})^{+3}_{-4}(\text{syst})) \times 10^{-3},$$

$$R_{\text{ADS}} = (9.1_{-2.5}^{+6.2}(\text{stat})^{+1}_{-3}(\text{syst})) \times 10^{-3},$$

from which we obtain 90% probability limits

$$R^+ < 23 \times 10^{-3},$$

$$R^+ < 29 \times 10^{-3},$$

$$R_{\text{ADS}} < 21 \times 10^{-3}. $$

From our measurements, we derive the limit

$$r_B < 0.13 \text{ at 90% probability.}$$

**ACKNOWLEDGMENTS**

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the U.S. Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Énergie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Ciencia e Innovación (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union), the A.P. Sloan Foundation (USA), and the Binational Science Foundation (USA-Israel).