The contagious capacity of the international banking network: 1985-2009

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Abstract

Systemic risk among the network of international banking groups arises when financial stress threatens to crisscross many national boundaries and expose imperfect international coordination. To assess this risk, we use Rosvall and Bergstrom’s (PNAS, 2008, 1118-1123) information theoretic map equation to partition banking groups from 21 countries into modules. We consider a quarter of a century of data on the cross-border interbank market. We show that in the late 1980s four important financial centres formed one large super cluster that was highly contagious in terms of transmission of stress within its ranks, but less contagious on a global scale. But the expansion leading to the 2008 crisis left more transmitting hubs sharing the same total influence as a few large modules had previously. We show that this greater entanglement meant the network was more broadly contagious, and not that risk was more shared. Thus, our analysis contributes to our understanding as to why defaults in US sub-prime mortgages spread quickly through the network.

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1. Introduction

An astonishing feature of the 2008 financial crisis was how quickly and extensively initial losses in US subprime mortgages spread such that two years later, governments worldwide had to provide massive support to their banking systems and with write-downs estimated to have reached over 2.5 trillion US dollars (International Monetary Fund (2009)). In this paper we apply network analysis to help understand the transmission of stress in the complex financial system that had built up before the crisis. In the years prior to the crisis, large banking groups had become highly interdependent across national borders through a complex web of direct claims on each other, ownership structures and other risk transfers and also through participation in common markets. Because the system was so intertwined, the financial crisis was transmitted rapidly through default chains, funding squeezes, fire sale externalities and a rash of counterparty fear.

Our focus is on the international banking network. The international banking network is a set of bilateral claims (links) of different banking entities (nodes) on each other. Nodes are defined by separating banking groups (all the banks operating in a particular country) into their funding and credit arms; each node is a funding or credit arm of a particular banking group. This separation of banking groups into funding and credit arms allows us to distinguish between two different channels of contagion. Banks defaulting on loans transmit stress to their creditors via a credit channel. This is a situation where a problem at one banking group’s funding arm is transmitted to another banking group’s credit arm. However, it was also observed during the crisis that banks got in trouble because their creditors refused to keep lending to them — a funding channel. This is a situation where stress flows from the credit arm of one banking group to the funding arm of another.

The first objective of the analysis is to cluster the funding and credit nodes of all the different banking groups together in a way that accurately reflects areas of concentration of financial stress. In particular, we wish to cluster nodes into modules so that stress travels between the members of a module with a greater intensity than it does to the nodes outside the module. For this purpose we use a network clustering technique developed by Rosvall and Bergstrom (2008), henceforth RB. RB’s map equation determines the most parsimonious yet accurate description of the network that can be used to map the movements of an imaginary traveller, taking account of how likely he is to visit each node. Groups of nodes with long persistence times are clustered together. Because the approach clusters the network using information about flow, it has an advantage over generalised modularity approaches (for example Newman (2006), Girvan and Newman (2002) or Blondel et al. (2008)) that focus on pairwise aspects of the link structure.

Success depends upon the proper specification of a transition probability matrix that governs the flow of stress through the system. We define this matrix using data on financial claims between banking entities. Our approach emphasises mismatches between assets and liabilities. Under our specification, the prestige of each module, which is measured by the frequency with which shocks visit the module in a steady state, depends not only on the sum of the gross assets and liabilities of each banking group in the module, but also on the mismatch between liabilities and assets. The

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4For more on the evolution of financial markets from a network perspective see Haldane (2009).
modules where financial stress visits the most are those with large and mismatched balanced sheets.

Clustering is in general a difficult numerical problem because of the vast number of modular permutations possible in even a small network. A crucial advantage of RB's approach is that it uses advances in information theory, in particular a generalization of Shannon’s source coding theorem (Shannon (1948)), to simplify the computational burden associated with evaluating all possible clustering arrangements. For this reason, RB's approach is well suited to determine a revealing map of the flow of stress through the international financial network. While not yet well known to economists, this method has been found to have the best performance among the many network clustering algorithms used in other scientific applications (Lancichinetti and Fortunato (2009)).

Describing the system at a modular level is an important part of our analysis of systemic risk in the international banking network. Given our modular description of the network we can see which countries belong to the same module and hence are most heavily impacted by each other in times of financial stress. We also examine the flow within and between modules. In a safer network, the most important modules will have a lower capacity to transmit financial stress; those modules will act as absorbers. If instead the important modules have a high propensity to transmit contagion, then financial stress is more likely to crisscross many national boundaries and become truly systemic.

When financial stress crosses many national boundaries it is more problematic. This is in part because different legal systems and political preferences have to be compromised. For example, London School of Economics Law and Financial Markets Project (2009) explain that Lehman Brothers’ global business operated with over a 100 data systems that were owned and managed by some of the 6,000 legal entities within the group worldwide. Once the global firm failed, administrators in each country where the firm operated needed to co-operate over sharing the very high cost of running these data systems. Claessens et al. (2010) and Tucker (2010) emphasise difficulties in international co-ordination over crisis resolution. A corollary is that a network where financial stress can move rapidly across many national boundaries should feature greater risk that a small shock can lead to a systemic crisis.

Our results describe a dramatic evolution of the interconnectedness in the international banking network over time: in the late 1980s, the network is dominated by one or two very large modules comprising the US, Japan, UK and the Cayman Islands. But over the 1990s and 2000s, the larger modules reconfigure to create a network led by multiple important modules now also including continental European banks. By combining modularity with measures of the probability of contagion at that modular level, we show that the expected movement between modules increased over the 2000s, and reached a peak just before the crisis struck. This suggests that these multiple larger modules acted as transmitting hubs rather than absorbing centres. The international banking network featured a great capacity to be contagious because stress could then rapidly cross many borders and was less likely to be corralled within a few countries.

But could this different pattern of interconnectedness have implied more risk-sharing and not more contagion? To test this we also carry out a simulation on our data set, adapting the method of Battiston et al. (2009) that allows for both contagion and risk diversification. We show that for a wide range of parameters, there was indeed a greater likelihood of transmitting contagion,
especially just before the crisis. In no calibration can the later network be described as better at diversifying risk. This corroborates the modular patterns described by the map equation.

2. Previous Work

Models of networks for the purpose of analysing systemic risk fall into two categories. One class of models are those aimed at simulating financial stress across the network. These are reviewed in International Monetary Fund (2009). The latest generation of these simulation network models incorporate the lessons of the crisis and feature sophisticated transmission through funding and fire-sale externalities and not just through chains of credit tightening (Gai and Kapadia (2010), Guthierc et al. (2010)). Naturally they require quite a few calibrations and detailed modelling of the behaviour of each node. And the results they report are more in the form of specific experiments.

Our paper falls into another strand of the literature which, rather than simulating particular experiments, aims to summarise relevant features of the network without imposing too many assumptions. Within this subgenre, there are no other papers which allow for both funding and credit channels. Moreover, the other papers which have carried out network-measure based analysis on the international banking network (von Peter (2007), Minoiu and Reyes (2011), Kubelec and Sá (2010), Čihák et al. (2011) and Hale (2011)) do not consider modular structures. Nor do they test for greater contagion as opposed to greater risk sharing.

The relevant features of the network that we seek to summarise relate to connectedness, because that tells us how stress can flow around the system. However, standard measures of interconnectedness do not show large variation over time when applied to financial networks. Measures of the prestige of each banking group do not change very much either during the build up or in the aftermath to this global banking crisis. If we were to take these standard measures of network interconnectedness at face value, we may be led to conclude that systemic risk in the network is more a question of scale (the total value of claims in the network) rather than about the cross sectional aspects (the distribution of claims across the matrix of bilateral exposures). We might also be led to conclude that developments across these two dimensions are quite independent.

This paper departs from the literature on summarizing systemic risk in the international interbank network by allowing for funding and credit channels for the transmission of stress and by deriving modularity from an analysis of the movement of stress across the network. For these reasons, our measure is sensitive to changes in the cross sectional distribution of claims, as we think it should be. This gives us the power to track when systemic risk in the network is particularly elevated.

3. Funding and Credit Risk

The current crisis was transmitted between banks both because borrowing banks defaulted and also because lender banks cut funding; credit and funding transmission were intertwined. Thus there should be four possible channels between any two different banking groups.

Our solution is to split each banking group into two nodes, one for each side of the balance sheet, so that there are separate funding and credit channels between different banking groups. To allow for contagion to pass from a banking group’s creditors to its funders and vice versa, it must
be that the two bi-nodes of the same banking group are also connected to each other, as if they were two departments in the same bank, but with an implicit contract. Indeed from the Bank of International Settlements data we know that cross-border intragroup claims account for about a third of all external claims between international banking groups. For some multinational banks, these claims are even priced in an internal market.

The modular structure of the network with funding and credit channels should also depend upon the intrabank links. If a banking group’s has both large gross assets but also a lot of liabilities, then contagion can be trapped within that banking group and its important trading partners. While this is bad for that group, it also means that contagion is discouraged from spreading to the less related parts of the system. So close nodes are pulled together and distant nodes are pulled apart. In this way, allowing for both credit and funding channels inevitably implies modelling the intra-bank contract, which in turn leads to a relevant and interesting modular structure for the interbank network.

Denote the set of banking groups (or countries) by $G$. Formally, there are two types of nodes for each country: bank funding departments and bank credit departments, defined respectively as $\alpha_F$ and $\alpha_C$ for $\alpha \in G$. Let $x_{\alpha_F \beta_C}$ represent the money value of the claim that the credit arm of banking group $\beta$ holds on the funding arm of banking group $\alpha$; this term represents the value of loans that banks in country $\beta$ have made to banks in country $\alpha$. What follows is a scheme for translating these money values into weighted links that indicate the ability of risk to flow through the financial network, and hence determine the path of contagion.

We assume that weights for when contagion travels between any two banking groups are given by

$$v_{\alpha_F \beta_C} = v_{\beta_C \alpha_F} = x_{\alpha_F \beta_C}, \text{ for } \alpha \neq \beta,$$

and

$$v_{\alpha_F \beta_F} = v_{\alpha_C \beta_C} = 0, \text{ for all } \alpha, \beta \in G.$$

(1) and (2)

Contagion can go up or downstream. The value $v_{\alpha_F \beta_C}$ is the weight on the directed link from the credit arm of banking group $\beta$ to the funding arm of banking group $\alpha$. This is the pathway for funding risk because it relates to the event that banking group $\beta$ stops lending to banking group $\alpha$. While the trigger for the financial crisis was the poor performance of securities and loans backed by US mortgages in early 2007, a main channel of transmission was the pressure that this put on bank funding markets. The spread between the 3-month LIBOR and central bank repo rates increased from about 5-15 basis points to above 70.

The value $v_{\beta_C \alpha_F}$ is the weight on the directed link from the funding arm of banking group $\alpha$ to the credit arm of banking group $\beta$. This is the pathway for credit risk because it relates to the event that banking group $\alpha$ defaults on its loan from banking group $\beta$. Since both risks relate to the same nominal contract, the claim that banking group $\beta$ holds on banking group $\alpha$, these weights are the same. Of course, this assumes that creditors and funders suffer the same blow when there is a problem with a contract, which may be debatable.

There is no clear-cut way to define the absorptive capacity of risk across funding and credit arms of the same banking group. For now we leave this question open and define the weights on
links across arms of banking group $\alpha$ by the parameter $w_\alpha$:

$$v_{\alpha F\alpha C} = v_{\alpha C\alpha F} = w_\alpha, \text{ for all } \alpha \in G. \quad (3)$$

Later on we justify a particular choice of weights for these terms.

Using equations 1 to 3, the matrix of contagion frequency in our network (the source along the columns, destination along the rows) is the matrix

$$V = \left( v_{\alpha j \beta k} \right)_{\alpha j, \beta k}, \quad (4)$$

where $\alpha, \beta \in G$ and $J, K \in \{C, F\}$ and the ordering is by the credit arm of banking group 1, the credit arm of banking group 2, etc., for all countries, and then the funding arm of banking group 1, the funding arm of banking group 2, etc., for all countries. Suppose $|G| = n$. Then $V$ is a $2n \times 2n$ symmetric matrix.

In order to describe the path of contagion we need to convert these weights into probabilities. Recall that each element $v_{ij}$ of the matrix $V$ describes the directional weight from $j$ to $i$; i.e., the capacity for stress to be transmitted from $j$ to $i$. Our premise is that contagion flows out of node $j$ according to probabilities that are proportional to these capacities. So, for example, if $v_{ij} = 2v_{kj}$, then (conditional on its moving out of node $j$) contagion is twice as likely to pass to $i$ as it is to $k$. Hence, we convert the weights in $V$ into probabilities by transforming the matrix $V$ into the column-stochastic Markov transition matrix

$$\Pi = \left( \pi_{\alpha j \beta k} \right)_{\alpha j, \beta k} = \left( \frac{v_{\alpha j \beta k}}{\sum_{i=1}^{2n} v_{ij \beta k}} \right)_{\alpha j, \beta k} \quad (5)$$

where $\alpha, \beta \in G$ and $J, K \in \{C, F\}$ and the ordering copies $V$.

4. Methodology

How do we decide on the best modular structure that fits the network that intertwines funding and credit transmission? At the core of RB’s approach is a formula that tells us how efficient any particular modular structure is at describing the path of an imaginary traveller, whom we call Mr Contagion, around the network, given information about the stochastic process that determines his movements. This is their map equation.

RB’s idea was to consider the dual problem of compressing data and finding patterns. We want a good map of how risk travels through the network. A good map simplifies away unnecessary details and highlights important ones. In this case the important details are the modules where Mr Contagion is likely to stay confined for extended periods of time once he enters them. The path of Mr Contagion is described by a set of codebooks, one high level index codebook and then a series of low level module codebooks. To describe travel between modules always requires use of the index codebook, and traveling within or to a module requires that module’s codebook. Each map book contains its own set of names which can be repeated across books but never within. Short names are used first for the nodes most visited.

The advantage of using two codebooks is that it can economise on the amount of information (bits) needed to describe the path of Mr Contagion. By introducing a new module, it is possible to
reuse short code names, which require less bits. However, an additional codeword is needed in the index codebook to identify the new module. There is a trade-off: if there are too few modules then the modular codebooks will need long names, but if there are too many then the index codebook will include too many names. The most efficient balance depends on the frequency with which contagion visit nodes. The important point is that the problem of dividing the nodes into modules is dual to the problem of designing an efficient map.

To derive the map equation formula explicitly, let \( p_{\alpha_J} \) be the frequency with which the traveler visits node \( \alpha_J \), for \( \alpha \in G \) and \( J \in \{ C, F \} \), and let \( q_{i\circ} \) be the frequency with which module \( i \) is exited. These would be measured after the traveller has been moving around the system for a long enough duration that his initial starting point becomes irrelevant. Mathematically, the values \( p_{\alpha_J} \) are computed as the dominant (right) eigenvector of the Markov transition matrix of contagion, \( \Pi \):

\[
p = \Pi p,
\]

where \( p = [p_1, \ldots, p_{2n}]' \) and are normalised to sum to one. This measure of eigenvector centrality can be calculated if the Markov transition matrix is irreducible.\(^5\)

We show later on that, as the matrix of contagion on which the Markov matrix is calculated, equation 4, is symmetric, the eigenvector centrality of each node is equal to the share of that node’s column sum in the total weight of the matrix. For that reason, the prestige of each node \( \alpha_J \) can easily be calculated as the ratio of the sum of the weights in column \( \alpha_J \) to the sum of all weights in the matrix \( V \). As we shall see later on, the prestige of a banking group can be related to its share of total assets and of liabilities, with the liability share being the more important.

Given the prestige parameters \( p \) from equation 6, and an arbitrary modular structure \( M \), with \( i = 1, \ldots, m \) modules, the modular exit frequency is given by

\[
q_{i\circ} = \sum_{\beta_C \in i} \sum_{\alpha_F \notin i} \pi_{\alpha_F} \beta_C p_{\beta_C} + \sum_{\beta_F \in i} \sum_{\alpha_C \notin i} \pi_{\alpha_C} \beta_F p_{\beta_F}.
\]

Given equations 6 and 7, or any other appropriate expression for these two frequencies, we can follow the procedure outlined in RB and calculate the frequency with which the traveller would need to use module \( i \)'s codebook. Here, this expression is given by

\[
p_{i\circ} = q_{i\circ} + \sum_{\alpha_C \in i} p_{\alpha_C} + \sum_{\alpha_F \in i} p_{\alpha_F}.
\]

Likewise, the value

\[
q_{\circ} = \sum_{i=1}^{m} q_{i\circ}
\]

is the frequency with which the traveller would exit any module, and therefore need the index codebook.

The probabilities \( p_{i\circ} \) and \( q_{\circ} \) tell us how often the modular and index code books are used. Next we need to know how costly (in terms of bits) it is to access these code books. These costs must be based on the optimal assignment of code names with respect to the usage frequencies of

\(^5\)This follows from the Perron-Frobenius theorem for irreducible matrices; see, for example, Seneta (1981).
the names in the various books. RB do not need to actually produce optimum code names for each code book under every possible partition. Rather, they calculate the theoretical limits for all of the different partitions using Shannon’s coding theorem and pick the one that gives the most efficient description of the network. Shannon’s coding theorem tells us that when $N$ codewords are used to describe the $N$ states of a random variable $z$ that occur with frequency $p_i$, the average length of the codeword can be no less than the entropy of $z$, defined as

$$H(z) = -\sum_{i=1}^{N} p_i \log(p_i). \quad (10)$$

Thus, the minimum description lengths for the index and modular code books are given by

$$H(Q) = -\sum_{i=1}^{m} q_i \log\left(\frac{q_i}{q_\sim}\right) \quad (11)$$

and

$$H(P^i) = -\frac{q_i}{P^i} \log\left(\frac{q_i}{P^i}\right) - \sum_{\alpha \in i} \frac{p_\alpha}{P^i} \log\left(\frac{p_\alpha}{P^i}\right) \quad (12)$$

where $H$ is the entropy function and $Q$ and $P^i$ denote distributions of usage frequencies of the names in their respective code books.

The minimum description length of the random path followed by Mr Contagion when the whole system is organised into a particular structure $M$ with $m$ modules is thus given by RB’s map equation:

$$L(M) = q \log(Q) + \sum_{i=1}^{m} p^i \log(P^i). \quad (13)$$

$H(Q)$ is the frequency weighted average of the minimum length of names in the index guide and $H(P^i)$ is the frequency weighted average of the minimum length of names in the guide to module $i$. Thus the map equation is the weighted average of minimum name length of the index map book and the module map book.

By following RB’s technique and minimizing $L(M)$ across all possible structures, we can identify the most efficient description of the network, which may identify potential ‘hotspots’ of contagion. This does not rule out the possibility that the best description is that that all nodes are in one module. Once we know the modular structure of the network, we can use the components of RB’s map equation to produce an estimate of the propensity for each module to transmit, or conversely to contain contagion.

The prestige of a module is given by the sum of the prestiges of all the nodes contained in the module. This prestige can be thought of as the frequency with which shocks visit the module. One can think of a single shock or thousands of shocks, each of which can originate at different places and at different times, but which move through the system according to the Markov transition matrix specified in equation 5. Some of these visits will leave the module in the next step while others will remain. In other words, the prestige of a module can be divided up into two portions, the part that relates to travel within the module and the part that relates to exiting the module.

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6RB have developed an ingenious greedy algorithm for minimizing 13 over large networks. This analysis was conducted using software provided by Martin Rosvall and available at http://www.tp.umu.se/rosvall/.
The exit frequency of module $i$ is given by $q_i \cdot \mathcal{N}$ (see equation 7). If a module has a relatively high exit probability (versus non-exit) then the interpretation is that shocks to the module will be transmitted to the rest of the system with high frequency. Whereas, a low exit probability suggests that much of the damage from shocks that reach the module will be absorbed within the module; i.e., it is more likely that the damage will be contained.\footnote{Because the system is irreducible there are no ergodic sets, other than the whole system. Thus, Mr Contagion will never be completely trapped in a module.}

5. Modelling intrabanking group transmission

In this section, we explain our choice for the weight of intrabanking group transmission, which up until now has been specified as the term $w_\alpha$ in equation 3.

We begin by confirming our earlier claim that the prestige of each node is equal to the shares of each column (or row) sum in the total weight, since that is a key step in our argument. Consider the vector $\Pi z$, where $z$ is the $1 \times 2n$ vector of column sum shares in the total weight, with the $k^{th}$ element given by

$$\frac{\sum_{i=1}^{2n} v_{jk}}{\sum_{i=1}^{2n} \sum_{j=1}^{2n} v_{ij}}.$$ (14)

Given the definition of $\Pi$ in equation 5, the $k^{th}$ element of $\Pi z$ is

$$\frac{\sum_{j=1}^{2n} v_{kj}}{\sum_{i=1}^{2n} v_{ij}} = \frac{\sum_{i=1}^{2n} v_{ij}}{\sum_{j=1}^{2n} \sum_{i=1}^{2n} v_{ij}} = \frac{\sum_{i=1}^{2n} v_{ij}}{\sum_{i=1}^{2n} \sum_{j=1}^{2n} v_{ij}}.$$ (15)

The last equality follows from the previous one because the row and column sums of the symmetric matrix $V$ are identical. We have thus shown that $z = \Pi z$ and hence $z$ solves this equation and is the unique vector of prestiges.

Given assumptions 1 to 3, and using formula 14, the prestige of the credit arm of the banking group $\alpha$ can be written as

$$p_{\alpha C} = \frac{1}{2} \frac{\sum_{\beta \in G} x_{\beta C} + w_\alpha}{\sum_{\gamma \in G} x_{\gamma C} + \sum_{\beta \in G} w_\beta},$$

and that of its funding arm is

$$p_{\alpha F} = \frac{1}{2} \frac{\sum_{\beta \in G} x_{\alpha F \beta C} + w_\alpha}{\sum_{\gamma \in G} x_{\gamma F C} + \sum_{\beta \in G} w_\beta}.$$

The total prestige of the banking group $\alpha$ is the sum of the two, or

$$p_\alpha = \frac{1}{2} \left( \frac{\sum_{\beta \in G} x_{\beta C} + \sum_{\beta \in G} x_{\alpha F \beta C}}{\sum_{\gamma \in G} x_{\gamma C} + \sum_{\beta \in G} w_\beta} \right) + w_\alpha.$$ (15)

If $w_\alpha = 0$ for all banking groups $\alpha$, then the prestige of any banking group $\alpha$ is simply the equally weighted share of assets and liabilities of that banking group of the total values in the system. As prestige measures the share of visits that Mr Contagion makes to a node, it does not seem realistic
that prestige should depend purely on the relative share of a banking groups’ assets and liabilities. That would mean for example that a banking group with 80 billion dollars of gross interbank assets and 10 billion dollars of liabilities will have the same prestige as a banking group whose assets were 10 billion dollars and liabilities, 80 billion. In reality we would expect the second banking group to have more lure for stress because it had large net interbank liabilities.

The role of the term \( w_\alpha \) in the more expanded expression 15 is to improve on the benchmark by shifting prestige from nodes where intrabanking transmission is low to nodes where intrabanking transmission is high. But much depends on what exactly determines \( w_\alpha \). Our starting point is that the extent of balance sheet mismatch should matter in determining prestige, so that a banking group which has a large interbank funding requirement relative to its interbank assets receives more contagion. For consistency, we choose to make each pair of intrabanking group contracts equal to the total liabilities of that banking group in the whole system:

\[
 w_\alpha = \sum_{\beta \neq \alpha} x_{\alpha \beta} C. \tag{16}
\]

Then the prestige of banking group \( \alpha \) is

\[
 p_\alpha = \frac{\left( \frac{1}{4} \sum_{\beta \in G} x_{\alpha \beta} C + \frac{3}{4} \sum_{\beta \in G} x_{\beta \alpha} C \right)}{\sum_{\beta \in G} \sum_{\gamma \in G} x_{\beta \gamma} C} = \frac{1}{2} \left( \frac{\sum_{\beta \in G} x_{\alpha \beta} C + \sum_{\beta \in G} x_{\beta \alpha} C}{\sum_{\beta \in G} \sum_{\gamma \in G} x_{\beta \gamma} C} + \frac{1}{4} \left( \frac{\sum_{\beta \in G} x_{\alpha \beta} C - \sum_{\beta \in G} x_{\beta \alpha} C}{\sum_{\beta \in G} \sum_{\gamma \in G} x_{\beta \gamma} C} \right) \right),
\]

implying that frequency of Mr Contagion visiting is greater, the more gross assets and liabilities the banking group has compared to the other groups, and over and above that, if it has large net liabilities. The weight on the mismatch component is a half of the weight on the gross position benchmark. In a network of just the two banking groups in our example of the previous paragraph, this formula would give prestige of about 70% to the net borrower, and 30% to the net lender.

In principle this could be refined. For example the transmission between the two halves of the bank could be made to depend on more specific properties of each banking group. But that would place undue emphasis on our ability to measure the true structure of each country banking group’s assets, liabilities, equity and liquidity. Our assumption should be seen in the spirit of an uninformed prior. This is the kind of structure that could be imposed by a regulator designing a system for the optimal distribution of contagion, if that regulator did not want to rely on any other data on each banking group other than their total asset and liabilities to other banking groups.

6. The data

We measure the claims held by each country’s resident banks on each other country’s resident banks as reported in the Bank for International Settlements (BIS) locational by residency statistics. In this data set, both domestically-owned and foreign-owned banking offices in the reporting countries record their on-balance sheet positions on other countries. These data are consistent with the residency concept of national accounts. Later on we report a comparison with the BIS consolidated banking statistics. The locational data set features all banks in each country with significant external claims, and includes the US investment banks that were protagonists of the crisis. The BIS
guide to these statistics (Bank for International Settlements (2010)) contains a list of the reporting entities in each country in the annex. Another advantage of this bilateral data set is that it contains many types of financial claims that carried contagion during this crisis. As well as standard loans and deposits, banks report on sale and repurchase transactions, certificates of deposits, financial leases, promissory notes, subordinated loans, debt securities, equity holdings and participations. Debt securities would include funding through trust preferred securities and asset backed securities, as long as issuer and holder were reporting banks residing in different countries. As far as we know, there is no publicly available bilateral data set at the level of individual banking entities or even consolidated banks that includes a wide range of interbank transactions. One exception is a recent paper by Hale (2011) that uses a specially constructed bilateral bank-level data set that includes only syndicated lending flows of long maturity. As we shall see, where our findings overlap, they are consistent.

As comprehensive as the BIS data are, they may not capture all the channels of contagion that mattered for this crisis. For example banks’ exposure to risk through derivatives such as credit instruments or swaps and futures (Segoviano and Singh (2008)), are difficult to measure. Also absent are off-balance sheet positions through special bank-sponsored vehicles. Finally we do not have complete data on other non-bank financial institutions that became commingled with banks in the build up to the crisis, the so-called shadow banking system. These missing links mattered in transmitting the crisis. In the build up to the crisis, European banks funded themselves through purpose-built, and supposedly independent off-balance sheet vehicles that lent long term back to their bank creators. These conduits funded these investments by issuing and rolling over shorter maturity paper, sometimes using a complex repackaging of the securitised assets of their creators as collateral (Longstaff (2010)). Even though losses only appeared in some of the components of composite assets, these instruments were sufficiently difficult to disentangle that investors’ confidence in the whole market was shattered across the board (Coval et al. (2009)). As the banks that created these damaged vehicles took them back on to their own balance sheets, they acknowledged their exposure to this correlated risk. Other non-bank financial companies, such as US money market mutual funds, had purchased many of these assets (Bertaut and Pounder (2009)) and another set of non-bank financials, such as monoline insurers, had guaranteed payments and both came to be embroiled in the crisis.

These are then the channels of contagion between banks through common participation in markets with shadow banking institutions rather than through direct claims held on each other. They cannot be incorporated because we do not have definitive data on these conduits’ links with their bank creators, their residency, a clear idea of what entities they ultimately borrowed from and who underwrote the risks in these transactions. Although we cannot fully account for off-balance sheet derivatives and the shadow banking sector, our analysis retains its validity provided the patterns described by the data we do have would resemble the complete network with the missing data closely enough.

We include the following 21 reporting countries in our network: Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excluding Faeroe Islands and Greenland), Spain, Finland, France (including Monaco), United Kingdom (excluding Guernsey,
Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden, and the United States. Among these are those countries which are the most important to the network and many of the countries excluded do not have complete series. All together our subsample captures about 73% of total reporting banks’ claims on banks in all vis a vis countries and the growth rate of the total claims in our subset is very similar to the growth rate in the total available to the BIS.

We apply our analysis to a sample starting in 1985 Q1 and ending in 2009 Q3. There are a few missing claims in the early data but we filled those in using the same proportion as the most complete data set that we have (2000 Q1). No one claim we filled in was more than 0.4% of the total value of all claims, and most were smaller than 0.1%. There were only eight claims filled in any year at most.

7. The international interbank market from 1985 to 2009

Before we go on to analyze the cross-sectional distribution of the interbank network, it is worth reviewing the important developments in this market. Figure 1 plots the annual growth rate of all claims in our data over this period.

![Figure 1: BIS external claims of banks on banks in a sample of 21 countries](image)

Percentage change on a year earlier

Source: Bank for International Settlements, Locational by Residence data. The countries are Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excluding Faeroe Islands and Greenland), Spain, Finland, France (including Monaco), United Kingdom (excluding Guernsey, Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden, and the United States.

The international interbank market was growing in 1985 when our sample begins. The market grew strongly until 1987, and then after a brief pause following the stock market crash in 1987, picked up speed again to finish the decade in strength. According to Bernard and Bisignano (2000), an important driver of this expansion were Japanese banks, which were channelling surplus domestic funds onto world markets. European banks also became increasingly active in cross-border
lending over this period as foreign exchange controls were removed in France and Italy, and as the prospect of greater financial and trade integration in the region loomed. It might have also mattered that many countries independently placed lower capital requirements on interbank lending than on commercial credits during the 1980s and that many banking centres liberalised their domestic financial regulations.

This first boom petered out by the end of the decade. The United States, Japan, Sweden and Finland suffered a sequence of domestic banking crises and world growth slowed down. There was also turbulence associated with the speculative attacks on the European Exchange Rate mechanism. International interbank flows remained subdued until 1994. Japanese banks in particular began to withdraw lending to other major international banks from 1989 Q4, although as we shall see they did soon start to lend to other Asian country banks.

The international interbank market revitalised again from 1994 until 1997. Bernard and Bisignano (2000) explain how this second boom was related to an excess of liquidity and low market interest rates, just as in the build up to the recent crisis. European banks, especially Swiss banks, increased their share of this market. Many funds were channelled through offshore centres, in another parallel with the more recent build up (Dixon (2001)). Some of these funds went to Asian economies, such as South Korea. Although Asian borrowers are not in our sample, Kaminsky and Reinhart (2000) reports contagion effects in the Asian crisis through shared lenders.

The second growth spurt in our data came to an end with the Asian crisis (1997 and 1998) and the collapse of Long Term Capital Management (1998). Once growth was halted, the rise in US interest rates in 1997, fears over the costs of European economic monetary union and deleveraging of earlier excesses combined to enforce a slow down. The interbank market grew at low or negative quarterly rates until 2002.

When the market picked up again, it grew fast and for a long time. This was the great boom which led up to the current crisis. Various Bank for International Settlements Quarterly Reviews over this period cite and analyse the investments of Asian economies, petro-dollars, the role of offshore financial centres and hedge funds, and more generally excess liquidity as causal factors (McGuire and Tarashev (2006)).

Importantly, many countries’ banks participated in these later two expansions compared to the late 1980s spurt. Hale (2011) also identifies the early 1990s and 2002 to 2006 as phases of rapid expansion, in terms of the number of banks and countries participating and the connections between them. Figure 2 shows the increase in each countries’ claims as a proportion of the increase in claims of the whole network over two different periods, so that the bars depict the relative contribution in the total increase. It shows that the increase in Japanese banks’ cross-border claims between 1985 Q1 and 1989 Q3 was much more concentrated (on the US and UK primarily) than were the increases in cross-border claims by the German, French, Swiss, UK and US banking groups from 2000 Q1 to 2008 Q2. The great entanglement in core cross-country exposures during this last boom plays a crucial part in our understanding of systemic risk that lead to the crisis.
As we know, this third boom ended some time between 2007 and 2008. Once the international banks in our groups began to suffer losses on investments with third parties (in the US subprime market), they cut back from lending to each other. While this had happened at least twice before since the mid 1980s, the drop in interbank lending since 2008 is remarkably abrupt: in just over a year annual growth decelerated from 30% to -20%!

8. The results

As a first step we use the data from 1985 Q1 to demonstrate the process by which the original network is split into credit and funding arms and then modularised. Figure 3 shows the network...
before splitting the funding and credit arms of each banking node and before allocating country banking systems into modules. This is the matrix of international banking exposures, straight from the BIS database. Figure 4 plots the network after splitting along the lines of equations 1 to 2, where XX.A and XX.L indicate the credit and funding arms respectively of banks resident in country XX. The connections between different banking groups are balanced against strong connections across the balance sheet of the same banking group. The forces that transmit funding risk take equal place alongside credit risk channels. With this richer interaction, naturally, the picture becomes more complicated.

Figure 3: Nodes before split (1985 Q1)

Source: Bank for International Settlements, Locational by Residence data. Austria (AT), Australia (AU), Belgium (BE), Canada (CA), the Cayman Islands (KY), Switzerland (CH), Germany (DE), Greece (GR), Denmark (DK), Spain (ES), Finland (FI), France (FR), United Kingdom (GB), Ireland (IE), Italy (IT), Japan (JP), Luxembourg (LU), Netherlands (NL), Portugal (PT), Sweden (SE), and the United States (US).
The benefit of modularity is shown in Figure 5 where we see the network structures after applying the map equation algorithm to the split system. In Figure 5 the area of each vertex reflects the prestige of that module, which is the sum of the prestige of its members ($\sum_{\alpha_j \in i} p_{\alpha_j}$ for each module $i$). The prestige of a module is the probability that Mr Contagion will be found in that module, but that does not tell us if he is then about to stay or go. That information is conveyed by the area of the outer ring which shows the probability of leaving ($q_{i \rightarrow}$ for each module $i$) and the area of the inner circle which shows the likelihood of staying ($\sum_{\alpha_i \in i} p_{\alpha} - q_{i \rightarrow}$). The arrows out of each module further divide up the probability of leaving among the different destinations — the thickness of the arrow going from module $i$ to module $j$ ($i \neq j$) is proportional to the probability that Mr Contagion travels from module $i$ to module $j$. In all cases, a country’s funding arm and credit arm are clustered together into the same module and hence only country labels, without the .A and .L designations, are used in the figure (this is also true in all of the figures that follow).

The diagrams presented in figures 5, 6 and 9 were formed using Martin Rosvall’s Map Generator tool, which is available at http://www.mapequation.org. The options we chose were as follows: placement/links (125); type (circle tree); colour and size tools (absolute scale); label size (root, min 15, and max 20); link size (root, min 0, and max 25); module size (root, min 0, and max 50); module colour range (linear); link colour range (linear); absolute scale module reference (20%) and link reference (3%). If the difference between the prestiges of the two largest modules was less than 5 percentage points, more than one module is in the centre of the circle.
In 1985 Q1, the United States formed the most prestigious module with the Cayman Islands. Mr Contagion spends about 25% of his time there. As we shall see these two banking groups remain together for the whole sample, reflecting the fact that the Cayman Islands is an offshore centre for US banking. The IMF recently estimated that 57% of the assets of the Cayman Islands banking system are overnight sweep accounts in branches of US banks (International Monetary Fund (2009)). But in this crisis, contagion could well have traversed this apparently innocuous route — Cayman Islands residents were large foreign holders of private-label US mortgage-backed securities leading up to the crisis (Lane and Milesi-Ferretti (2009)).

Then there is another module containing only the United Kingdom, which is nearly as prestigious and is where Mr Contagion spends about 23% of the time. It turns out that the UK banking group is always in the most prestigious module or the second most prestigious module for the entire sample, no doubt given its role as a host to many foreign-owned banks as well as the international nature of its own banks. After these two large modules, come four others with much smaller prestige. Japan, Belgium, France and Germany and Luxembourg together, have prestige scores ranging from 9 to 7%. At the other extreme, there are seven modules that have prestige under 1%. Apart from the US-Caymans and Germany-Luxembourg modules, all countries are in their own modules.

It is perhaps not surprising that Luxembourg forms a module with Germany. Distance can matter it seems in international banking as it does in trade networks (Rauch (1999)). But the modular structure also reveals for example that though they held important claims on each other, Canada and the United States were not in the same module in 1985 Q1. Presumably at that moment in time US banks and Canadian banks were less linked together than the United States is with the rest of the network. As we shall see also in later periods, modules are sometimes formed by banking groups with geographical or historical links, but not always. Canada and the United States were briefly in the same module during the late 1990s.

Note also that more prestige does not necessarily imply more contagion. The UK module has
less prestige than the US-Caymans module. Yet the UK module is the more contagious of the two: the UK module has a larger outer ring and on average larger arrows flowing out than the US module.

Bearing this in mind, we can now survey similar diagrams for the whole sample. We provide diagrams for quarters where there is significant change in the modular structure in the panel in Figure 6. The scaling of the arrows and the circles are fixed across all of the diagrams so that their respective areas are comparable across time. If at one time, a prestigious module is absorbing, the circle will be large and within that, the inner circle will dominate and the outer ring will be narrow. If at another moment, the module is as prestigious but more contagious, the outer ring will be thicker taking up more of the area of the circle of the same size. The smallest arrows and circles are not shown. Table 1 in the Appendix gives the numbers for prestiges along with the exit frequencies of each module for each diagram in Figure 6. This is the total area of all the outer rings or of all arrows. Numbers for the complete sample are available upon request. Table 2 summarises the changes in modular structure over the whole sample period.
Figure 6: Modular networks

Source: Bank for International Settlements, Locational by Residence data and own calculations.
The first story to highlight is the internationalisation of Japanese banks over the second half of the 1980s and the affect this had on the network. Japanese banks start off in their own module in 1985 with a prestige of just over 9%. Their prestige score increases until 1986, when Japanese and UK banks coalesce into a single module with the most prestige. By the end of 1987 this module has combined with the US-Caymans module to form a giant hub in the network. That hub continues to grow until its prestige reaches 63% in 1989 Q3. Crucially for us, that module is quite absorbing. The outer ring takes up 10% of the frequency of the whole network and so once Mr. Contagion arrives in this megamodule he is five times more likely to stay there than leave.

As we pointed out earlier, this great drive of the Japanese banks into the international banking market peaked in 1989 Q3. The impact of their retreat on our network is to shatter the large module; See Figure 6, panel 1989 Q4.

The next phase of growth began in the early 1990s when European banks, especially Swiss banks, played a key role. Switzerland, which previously had only been attached to Luxembourg, enters into a module with the United Kingdom. See Table 2a, column 1992 Q3. Also in this period, Germany and Luxembourg form a union, which as we shall see persists through the crisis. In 1997 Q1, Finland and Sweden combine. A few years later in 2000 Q3, Denmark joins them to form a Scandinavian bloc. In 2005 Q1, Belgium and Netherlands merge, although briefly. And although France does not form a module with another country, its prestige steadily rises over this period.

The widespread nature of these later expansions meant that prestige became more evenly distributed among the larger modules. In 2000 Q1, the US-Caymans module has nearly as much prestige as the UK and Swiss module (around 25%), and there are now eight modules with prestige above 3% compared to five in 1989 Q3. In general, the modular network is more interconnected, with large arrows from prestigious modules, and relatively larger outer rings (See also Table 1).

A further, bigger shift towards this pattern happens around 2006 Q1 when Switzerland drops out of the largest module with the United Kingdom. As a consequence, the two most prestigious modules lose out to the chasing pack of slightly smaller hubs. There are now eleven modules with prestige above 3%. As we later demonstrate, the network now has less propensity to absorb contagion when compared to the turn of the century and certainly when compared to 1989. In this contagious network, the United Kingdom is singled out as the central hub in this metropolis where stress arrives and is likely to be sent out again to many destinations. A year or so later, stresses from the US subprime market began to make themselves felt on international banks. Tellingly, the network remained in this contagious state right up until 2008 Q2, just before the collapse of Lehman Brothers.

Indeed the system remains in a broadly contagious state even until the latest date (2009 Q3). In Figure 6, we can see that the areas taken up by the blue arrows, or the outer rings, in the panel for 2009 Q3 are relatively large compared to a decade or two decades earlier. Despite the massive retrenching during these crisis years, the network at this date had not returned to how it was in 2000 and is not that different to its state when the subprime crisis struck. Abstracting from the question of the average quality of investments, that is on cross sectional grounds alone, one could conclude that financial stress could still be transmitted rapidly around the international banking network.

To see this pattern of change in a more concise form, Figure 7 plots the density of prestige
across modules. The figure shows clearly how that density goes from having a steep slope in 1989 Q3 to what looks more like a mountain with a flat cliff by the end of the sample. The flattening happens since 2000 and especially in 2006 reflecting the emergence of multiple important modules in the build up to the current crisis — further evidence of greater entanglement. Using a country by country data set, Schiavo et al. (2009) also find that the leading financial centres intermediate a large share of asset trade in 2004, much more than they do for goods trade.

Figure 7: The density of modules prestige

![Chart 6. The density of modules' prestige](chart6)

Source: Bank for International Settlements Locational by Residence data and own calculations.

8.1. Tracking contagion over time

Comparison of the maps in Figure 7 give us a visual indication of how the contagious properties of the international banking network change over time. However, it would be useful to have a simple, quantitative measure. For a given modular structure, the measure $q\rightarrow$ defined in equation 9 tells us the fraction of time that a shock travels between modules. This gives us a sense of how broadly contagious shocks are. However, values of this measure are not easily comparable across time periods with different modular structures. Increases in clustering associated with a different optimal modular structure necessarily result in reductions in $q\rightarrow$ as broader system-wide contagion is internalised into a module. However, it is not appropriate to say the new network is less contagious.

In order to determine whether the international banking network is becoming more or less contagious over time one needs to select a benchmark modular structure and compute $q\rightarrow$ over time holding the benchmark structure fixed. This tells us whether the amount of system-wide contagion increases or decreases over time and gives additional insight into why changes in modular structure are produced by the map equation.
A natural candidate for this benchmark modular allocation is the one selected by the map equation algorithm in 1989 Q3. The GB-US-KY-JP module had about 63% prestige and an exit probability of 10% in 1989 Q3. This is the largest module of the sample period. By applying this modular structure to the rest of the sample we can see how much of the contagion between these major financial centres spread to other countries over time.

The results are shown in Figure 8. There we can see evidence of the increased capacity of shocks that originated in the GB-US-KY-JP module to be pandemic. The amount of contagion flowing outside the fixed modules increased by over 10 percentage points from 1989 to peak in 2008 Q2, just before the default of Lehman Brothers. The contagiousness of the benchmark structure has fallen since the default of Lehman Brothers, but still remains at a high level relative to the late 80s.

As a final illustration, Figure 9 compares the 1989 Q3 diagram to the same 1989 Q3 modular structure applied to the data on 2008 Q2. This counterfactual modular description was rejected by the map equation for that later date. Note first that the volume of traffic leaving this counterfactual module is greater relative to the internal traffic than in 1989 Q3: the outer ring of the large module in 2008 Q2 occupies a larger share of the area of the total circle compared to the share that it takes in the 1989 Q3 data. This large module would have had an exit probability of 11% but with much less prestige (42%), meaning that if Mr Contagion were to arrive at the 2008 Q2 module, then he is about three times as likely to stay as to leave. Remember that in 1989 Q3 he was five times more likely to stay than leave. The total area of all blue arrows is also greater in the counterfactual 2008 Q2 case, 37% greater in fact, implying that there would be more contagion in the network as a whole also. For all these reasons, the map equation rejects the possibility of the large GB-US-KY-JP module that it selected twenty years earlier for 2008 Q2 because that grouping would have not
been able to contain contagion sufficiently. In summary, the network was more broadly contagious in 2008 Q2 than in 1989 Q3, and that is revealed to us by this shift in optimal modularity.

Figure 9: The 1989 Q3 modules imposed on the 1989 Q3 data (left) and on the 2008 Q2 data (right).

Anecdotal evidence of the great entanglement of the 2000s is provided by Gillian Tett (Tett (2009), page 273), who describes the thoughts of a banker attending the meeting at the Fed on Lehmans in September 2008: “As he arrived at the Fed, he was struck by a historical irony: almost a weekend exactly a decade earlier Calello had rushed down to the Fed at the weekend to discuss the fate of the Long Term Capital Management hedge fund. This time, Calello noted there were far more European bankers assembling for crisis talks in relation to Lehman,..”

9. Contagion or risk sharing?

The map equation revealed that there was a great entanglement leading up to 2008 Q2 and our interpretation was that this implied an increased capacity for broad contagion. But an alternative view is that what we are seeing is more risk sharing and less cross-sectional systemic risk. After all, banks borrow and lend from each other to pool individual risks, for example from the unpredictable liquidity needs of their depositors. However, as we have seen interbank trade also yields each bank to contagion: the risk of failure of its bank counterparties. Clearly the fact that an international banking crisis happens lends support to the contagion viewpoint. Nevertheless it is worth revisiting this classical dilemma in our data.

In a landmark paper, Allen and Gale (2001) asked whether connectivity implies more risk-sharing or more contagion in a stylised network of four identical banks with unweighted links. They found that there were fewer cases of contagious bank failures when all four banks were connected to each other, compared to, for example, a chain pattern where each bank was connected to only one other. Gai and Kapadia (2010) working with a more general configuration confirmed that greater connectivity reduced the likelihood of widespread default, though once they occurred, shocks could have a significantly larger impact on a connected financial system. Recently Battiston et al. (2009) established that a mechanism in which individual financial fragility feeds back on
itself, called a financial accelerator, can act as a countervailing force against diversification. In our model, a financial accelerator would mean that a visit by Mr Contagion in any node implies that he is more likely to visit the same node the next round. Battiston et al. (2009) demonstrated in a network of identical banks that as connectivity increases, a strong enough financial accelerator can overcome risk diversification and imply a greater risk of fully fledged bankruptcy cascades. As in Gai and Kapadia (2010), connectivity can imply a smaller risk of an individual collapse but a greater risk of a systemic event.

To validate our interpretation, we simulate a model of contagion that nests both risk-sharing and financial accelerator mechanisms and can be calibrated to our data. We adapted Battiston et al. (2009)’s model of contagion. Adaptation was necessary because in our real world data set the number of links can vary between banks and each link can have value between zero and one. As a result, diversification can no longer be measured by the number of links of each bank. In our case also, it matters where the shocks start from and where they hit as banks are not the same. In the absence of extensive information on the balance sheet strength of all participants, the sources of likely losses and the costs of liquidating different types of assets over time, the extended model has to be kept fairly simple.

The state variables in the model are the pressure on a banking group’s capital. This can vary from zero when there is no equity pressure to one when the banking group is under maximum pressure. To capture the fact that capital can be strained from the funding side (liquidity) and the credit side (solvency), we let $e^{r}_{\alpha J}$ denote the capital pressure on the credit or funding arm of banking group $\alpha$ for each round of contagion, $r$ (for $J, K \in \{C, F\}$).

Capital pressure passes from one node to another according to the system of difference equations

$$e^{r+1}_{\alpha J} = \begin{cases} \sum_{\beta K}^{2N} \pi_{\alpha \beta \epsilon} e^{r}_{\beta K} + \epsilon^{r} & \text{if } 0 \leq e^{r}_{\alpha J} < 1 \\ 1 & \text{if } (e^{r}_{\alpha C} - 1) (e^{r}_{\alpha F} - 1) = 0 \end{cases}$$

(17)

where $\alpha, \beta \in G$, $J, K \in \{C, F\}$,

$$\epsilon^{r} = \begin{cases} \sigma \text{sgn} \left( e^{r}_{\alpha J} - e^{r-1}_{\alpha J} \right) & \text{if } \text{sgn} \left( e^{r}_{\alpha J} - e^{r-1}_{\alpha J} \right) > \frac{\mu}{I_{\alpha J}} \\ 0 & \text{if } \text{sgn} \left( e^{r}_{\alpha J} - e^{r-1}_{\alpha J} \right) \leq \frac{\mu}{I_{\alpha J}} \end{cases}$$

(18)

and

$$I_{\alpha J} = \begin{cases} \sum_{\beta=1}^{N} \left( \frac{x_{\beta \alpha}}{\sum_{\beta=1}^{N} x_{\beta \alpha}} \right)^{2} & \text{if } J = C \\ \sum_{\beta=1}^{N} \left( \frac{x_{\beta \alpha}}{\sum_{\beta=1}^{N} x_{\beta \alpha}} \right)^{2} & \text{if } J = F. \end{cases}$$

The steady-state frequencies (equation 6) used in the map equation model are derived from the equation $e^{r+1}_{\alpha J} = \sum_{\beta K}^{2N} \pi_{\alpha \beta \epsilon} e^{r}_{\beta K}$ for $\alpha, \beta \in G$ and $J, K \in \{C, F\}$. $\text{sgn}$ denotes the signum function. The difference with our simulations model (equation 17) is the drift term ($\epsilon^{r}$) which captures the financial accelerator. Because of this additional term, capital pressure can reach a maximum level in a banking group, either by the funding or the credit side, and then it is assumed that both sides of that banking group will continue to experience only maximum capital pressure thereafter. In the map equation, capital pressure will always remain between zero and one.
Equation 18 defines the drift term. Because of the financial accelerator, there could be more capital pressure this round if there was strong pressure last round. The financial accelerator is also controlled by a parameter $\sigma > 0$. But how strongly the last round’s rise has to be to trigger the accelerator also depends on risk sharing: the degree of diversification of each node ($I_{\alpha_j}$) relative to an overall parameter, $\mu$.

$I_{\alpha_j}$ is called the Herfindahl index in the industrial organisation literature and the disparity measure in the network analysis (Barthelemy et al. (2003)). If the $J = C$ and the banking group has a diffuse distribution of borrowers, then this index will be close to zero, switching off the accelerator. A similar logic applies if $J = F$; if there are many lenders with an even share of liabilities, then funding sources should be diverse, and the financial accelerator will be disabled. At the other extreme, greater concentration would imply values close to one, and more destabilising accelerator effects. In our sample, values typically range from just above zero to 0.6. The largest shift in these indices between 1989 Q3 and 2008 Q2 was a widening disparity of funders of the United Kingdom group (the index decreased from 0.18 to 0.09). All other things equal, this change in structure around an important node acts for a greater risk diversification in our simulations.

Note that the formulation in equation 17 collapses back to map equation when either $\sigma$ is zero, $\mu$ is high or all the $I_{\alpha_j}$ are low, as then $e$ is always less than one and captures the frequency of visits of Mr Contagion in each round. Thus, the map equation is a special case of this model.

The dynamics of the simulations model are set off by shocking the capital pressures of a few less prestigious nodes during each round. Specifically we shocked the credit and funding arms of Australia, Portugal, Greece, Finland, Spain and Ireland (in general the less prestigious nodes). The shocks are uniformly distributed in the range [0,1] independently of any property of that node; we do not allow the size of the shock to capital pressure on each starting node to depend on its risk-sharing capacity. The simulations end when all the most prestigious nodes (measured as the top 25% most prestigious nodes in each quarter) become subject to capital pressure above the threshold of 0.5, and we say that then contagion is complete. Initially we hold the value of $\sigma$ fixed at 0.2. There remains the question of where to initialise the simulations from: in the absence of decent measures of the capital or liquid assets of banking groups, we assume that pressures all start at zero. The exercise is repeated across 10,000 simulations and we report the mean number of rounds it takes for contagion to be complete.

This is a natural measure of contagious capacity. A slow spread of an outbreak in the outer reaches of the network can be more likely dealt with by the authorities of the countries where Mr Contagion tarries. But if instead he moves fast and puts all the main nodes under severe strain in few steps, an internationally coordinated response will have to be marshalled and the network can be said to have contagious capacity. One might imagine that the system is contagious when a prestigious banking group gets hit several times recurrently, such that its capital gets eroded. But if this happens to only one banking group, we argue that this indicates that risk is being corralled

Note that this formulation runs contrary to the notion that the resolution costs of a failed entity are greater with a wider distribution of its international creditors as in Haldane et al. (2005). If many equally important creditors means prolonged and costly legal disputes, the financial accelerator would be greater the less concentrated the weight among a banking group’s creditors. By ignoring this possibility, we are biasing our results towards risk sharing.
where it can be relatively easily absorbed and is not systemic. Contagion would be broad and systemic if the most prestigious banking groups were hit recurrently and in rapid succession.

Figure 10 reports the results across the whole sample (1985 Q1 to 2009 Q3), for different degrees of risk diversification (values of $\mu$ from 0.05 to 0.5). The figure is a contour map where the height is the number of rounds contagion takes to raise the capital of all the top 25% prestigious nodes from 0 to 0.5.

Figure 10: Estimates of the contagious capacity of the international interbank network (1985 Q1 to 2009 Q3)

Figure 10 reveals that there are dramatic differences in the contagious capacity of the international interbank network. In the late 1980s, it takes 60 to 70 rounds for contagion to infect the top nodes. From then on contagion speed builds up especially after the mid 1990s and again from 2000 onwards. By 2008 Q2, we estimate it took less than 10 rounds for contagion to complete: six or seven times faster than the late 1980s. An impressive speed for a network of 42 nodes.

This is true for a wide swathe of values of the diversification parameter $\mu$. Only for very small values of $\mu$ (below 0.0005) is there no change reported in contagious capacity across time and in this range, there is fast contagion always. It is never the case that there is less contagion in the 2000s compared to the 1980s. For higher values of $\mu$, even those not reported in the figure, the same patterns as in the top half of the figure across the whole sample persist. We also experimented with different values of $\sigma$ ranging from 0.1 to 2. The higher the value, the faster contagion spreads for all time periods and across all values while preserving the strong relative difference across time identified by the map equation.

In summary, these experiments confirm that there a heightened contagious capacity at the time
of the Lehman bankruptcy, following the great entanglement. They corroborate our interpretation of the map equation’s outputs.

10. Robustness checks

Before concluding, it is worth reviewing the robustness checks we carried out.

10.1. Consolidated data

The locational data used in this study are based on the residence principle, identifying banks with countries where they do business. There is a different BIS ultimate-risk dataset which consolidates positions based on the ultimate country-owners of banks (McGuire and von Peter (2009)). These data include local exposures of own foreign offices, but exclude cross-border inter-office positions. A first robustness check is to see if our results hold for the different dataset.

The main difficulty in making this comparison is that the ultimate-risk data are only available from 2005 Q3 to 2009 Q3. Also the Cayman Islands do not report ultimate-risk data and some other important countries, notably Germany do not publish data. In the latter case, the data has to be estimated.

Figure 11 compares the maps for the locational and consolidated ultimate-risk data for specific periods; the numbers for all quarters are available on request.\(^\text{10}\)

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\(^\text{10}\)The locational networks on the left column are depicted in a different formation to Figure 6 for the purpose of making a better comparison with the ultimate-risk data.
Figure 11: Locational and ultimate-risk data comparison

Locational data

Ultimate-risk data

The international interbank network is shown to be highly interconnected under both approaches, with a lot of movement from module to module. However, differences arise in the modules to which specific countries belong. On the one hand, cross-border flows between the US and Switzerland are not strong enough for them to be classified as one module based on locational data. And yet, when one takes into account the substantial US presence of the two Swiss banks, UBS and Credit Suisse, the two countries are clustered together. A similar picture emerges for Germany-Italy links. On the other hand, strong cross-border exposures between Germany and Luxembourg cause the two countries to be clustered together based on locational data. But once one removes the inter-office claims, the two countries stand separately.

The varying results reflect the differences in the funding models and in the properties of large financial centers versus large financial markets. The Swiss branches in the US operate mostly as stand-alone operations, both raising funds and investing locally. As a result, their activity is not reflected in large cross-border flows. The closeness of Luxembourg and Germany on the other hand reflects the status of the former as a financial centre, with money channeling through it through the foreign branches and subsidiaries.

In general the ultimate-risk clustering tends to be more fragmented. For any given quarter, the map equation identifies more stand-alone clusters in the ultimate-risk than in locational data. This reflects banks relying more on establishing own branches and subsidiaries abroad instead of engaging in direct cross-border lending as part of banks’ global operations. As a result, consolidated data shows more fragmentation.

Ultimately, locational data have advantages over the ultimate-risk data for our purpose. First, they more adequately track cross-country linkages, which is crucial for spillover analysis of capital flows. Internal capital market transfers of global banks are an important part of their liquidity management and can play a role in the international transmission of shocks. For example, Cetorelli and Goldberg (2008) and Cetorelli and Goldberg (2010) show that in the case of the US, banks with global operations are insulated from the US monetary policy in their lending decisions because they activate internal capital markets with their foreign offices in response to changes in domestic liquidity. And then for practical purposes, locational data cover more countries (42 versus 30) over a longer period of time (24 countries provide locational data starting 1983 versus only 15 countries for ultimate-risk data). With a short sample we would not have been able to observe and compare the great entanglement. And finally there are important gaps in the reported consolidated data.

10.2. Other robustness checks

We applied the map equation on the data set before splitting into funding and credit arms to see if this intrabanking mechanism was crucial to generate some interesting modular structure. Without splitting, the algorithm always only reported one large module suggesting that these new mechanisms are important to understanding contagion.

As another check, we compared the map equation with a well known modularity minimisation function, a version of Girvan and Newman (2002)’s modularity function adapted for weighted directed networks by Arenas et al. (2008):

\[ Q(C, V) = \sum_{i} \sum_{j} \left[ \frac{v_{ij}}{\sum_{i} \sum_{j} v_{ij}} \delta(C_i, C_j) - \frac{\left( \sum_{j} v_{ij} \right) \left( \sum_{i} v_{ij} \right)}{\left( \sum_{i} \sum_{j} v_{ij} \right)^2} \delta(C_i, C_j) \right] \]  

(19)
where $V = (v_{ij})_{i,j}$ is the given weighted value matrix of a weighted directed network, $C_i$ denotes the module that node $i$ belongs to and $\delta(C_i, C_j)$ is the kronecker delta which takes a value of 1 if $i$ and $j$ are in the same module, and 0 otherwise. This function can be maximized by the modularity choice $C = (C_1, \ldots, C_{2n})$. The first term in the square brackets is the value of all links inside modules divided by the value of all links in the whole matrix. The idea is that the best modular description should maximise the weight of links within modules. But there has to be a counterweight, otherwise the best description would trivially be one module. Girvan and Newman (2002)’s chosen counterweight is captured in the second term, the expected value of all links inside modules for the same modular structure, assuming that the in and out links from each node in the same module are randomly and independently reassigned.

Combining expression (5) with our earlier result that the prestige of each node is equal to the shares of each column (or row) sum in the total weight, we can rewrite (19) in terms of transition probabilities and prestige:

$$Q = \sum_{\alpha \in G} \sum_{\beta \in G} \pi_{\alpha C} \pi_{\alpha F} p_{\beta F} \delta(C_{\alpha C}, C_{\beta F}) + \sum_{\alpha \in G} \sum_{\beta \in G} \pi_{\alpha F} \pi_{\beta C} p_{\beta C} \delta(C_{\alpha F}, C_{\beta C})$$

$$- \sum_{\alpha \in G} p_{\alpha C} \sum_{\beta \in G} p_{\beta F} \delta(C_{\alpha C}, C_{\beta F}) - \sum_{\alpha \in G} p_{\alpha F} \sum_{\beta \in G} p_{\beta C} \delta(C_{\alpha F}, C_{\beta C}).$$

(20)

Using the fact that the sum of prestige is equal to one ($\sum_{\alpha \in G} p_{\alpha C} + \sum_{\alpha \in G} p_{\alpha F} = 1$),

$$Q = \sum_{\alpha \in G} \sum_{\beta \in G} \pi_{\alpha C} \pi_{\alpha F} p_{\beta F} \delta(C_{\alpha C}, C_{\beta F}) + \sum_{\alpha \in G} \sum_{\beta \in G} \pi_{\alpha F} \pi_{\beta C} p_{\beta C} \delta(C_{\alpha F}, C_{\beta C})$$

$$+ \sum_{\alpha \in G} p_{\alpha C} \left(1 - \sum_{\beta \in G} p_{\beta F} \delta(C_{\alpha C}, C_{\beta F})\right) + \sum_{\alpha \in G} p_{\alpha F} \left(1 - \sum_{\beta \in G} p_{\beta C} \delta(C_{\alpha F}, C_{\beta C})\right) - 1. (21)$$

The first two terms are the expected frequency of travel within modules, or the sum of the areas of the inner rings in Figure 6. This can be trivially maximized at a value of one by having one large module. The second two terms (minus one) are Girvan and Newman (2002)’s counterweight. Each represents the sum of the prestige of each node multiplied by one minus the sum of the prestige of other linked nodes in the same module, and as such can be considered as an estimate of the exit frequency of the modules.

The map equation also trades off internal travel frequencies against exit frequencies, using information theory to weigh up the cost of either. But the map equation crucially uses the actual exit frequencies of the whole network, whereas this function uses an estimate derived purely from the prestige of each two possible modular partners. Perhaps this is why Lancichinetti and Fortunato (2009) found that the map equation has a remarkable performance in weighted networks compared to other methods.

We used an algorithm provided by Blondel et al. (2008) to optimise equation 21 on our data set with split funding and credit arms. But the best description was always found to be the uninformative modular pattern where only the funding and credit arm of the same banking group were in each module; modules were equivalent to countries.

To sum up, we found several reasons to prefer the map equation over the most popular contender as a tool to analyse modularity on our data set. First, the map equation uses more precise
information on financial stress. Second it estimates on the basis of movement in the whole network, not just by calculating pairwise comparisons of each two nodes in a likely module. And third, it delivers informative results.

One obvious extension to our paper is to allow for country banking groups to hold claims on each other’s and their own non-bank sector. This would build in another very important aspect to the crisis, where banking groups were afflicted not because they had direct claims on each other but because they shared the same credit and funding markets with non-bank entities. For example, if one banking group sells its assets on another country’s non-banking group then that would put pressure on all other banking groups that hold assets on that same non-bank sector. In our initial experiments with introducing non-banks we only found modularity along the lines of the countries. The reason is that the claims held by a banking group on its own non-bank sector are much larger than its other links. This can be overcome by additional parameterizations that downweight links with non-banks, but we would then have to justify the choice of those parameters.

11. Concluding Remarks

If financial stress can be contained within a few countries, it can be more easily dealt with. When the network is so interconnected that stress rapidly crisscrosses many national borders, it becomes truly systemic. In these circumstances, resolution is more complicated, the probabilities of default are higher and the losses given default are larger.

The clustering analysis performed in this paper provides insight into when shocks that hit the international banking network might be expected to stay contained within a few countries and when they are likely to spread globally. In cases where stress is likely to travel within groups of countries these countries are clustered together. However, this is not an exact science. The modular structures we compute are based upon a belief that stress flows around the network in a way that is proportional to our estimates of international claims and liabilities, and this can only be approximately true. That said, it is encouraging that our clustering analysis captures well known changes in the international banking landscape that have occurred over the past quarter century.

Changes in modular structure only tell part of the story. We also examine the extent to which modules transmit stress and how this changes over time. Using a fixed modular structure that combines the major financial centres as a benchmark, we find that the international banking network became more prone to systemic risk after 1989 and peaked at the time of the Lehman Brothers’ collapse. Furthermore, it appears that the capacity of the international banking network to transmit contagion was not much less a year after the failure of Lehman Brothers.

It is important to understand that our results cannot be used to infer anything about the current riskiness of the system. The reason for this is that our contagion analysis only concerns the cross-sectional component of systemic risk and offers no insights as to changes in the average quality of banks balance sheets over time. Contagion refers to the capacity to transmit stress. And, drawing an analogy to biological viruses, how contagious a disease is and how severe it is are separate issues.
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Table 2a. Module members ordered by prestige (all quarters where there is change)

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Table 2b. Module members ordered by prestige (all quarters where there is change)

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References


