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SUPERSYMMETRIC MODELS OF PARTICLE PHYSICS AND THEIR PHENOMENOLOGY*

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1. Introduction: The Need for Supersymmetry

Over the last few years there has been a remarkable synthesis in the theories of elementary particle interactions. An elegant standard model has emerged which describes the interactions of the known quarks and leptons. The model is consistent with all available data. It describes the strong interactions which bind quarks into hadrons in terms of a non-Abelian gauge theory called quantum chromodynamics (QCD) (Bardeen, Fritzsch, and Gell-Mann, 1973; Gross and Wilczek, 1973; Weinberg, 1973), based on the group SU(3).

The strong (color) force between the quarks is carried by eight massless spin-one gluons which transform according to the octet representation of SU(3). This theory has the remarkable property of asymptotic freedom (Gross and Wilczek, 1973; Politzer, 1973) according to which the interactions of quarks become weak when probed at short distances or high momentum transfers. This property enables one to explain the success of the naive parton model, according to which electron-proton scattering at large momentum transfer can be described in terms of electrons scattering from pointlike, free partons (quarks) which exist as constituents of the proton. A review of the basic properties of QCD can be found in (Wilczek, 1982).

The weak and electromagnetic interactions of quarks and leptons are described in the standard model by a theory based on the group SU(2) × U(1) (Glashow, 1961; Weinberg, 1967; Salam, 1968). This theory has four, spin-one, gauge bosons (three in the adjoint representation of SU(2) and one corresponding to the U(1) group). Unlike the case of strong interactions, this symmetry is not exact and all the bosons are not massless. Only one, the photon, remains massless while the other three (W⁺, Z) have masses of order 100 GeV. The relative feebleness and very short range of the weak interactions is due to the exchange of these heavy bosons between the interacting fermions.

This standard model is capable of explaining a large amount of data in terms of rather few fundamental parameters. There are three gauge coupling constants (g₁, g₂, and g₃) corresponding to each of the group factors. There are masses for each of the six known quarks (up, down, strange, charm, top and bottom)* and the three charged leptons (the electron, the muon, and the tau). We have no indication that the neutrino masses are non-zero. There is one other parameter which sets the scale for the masses of the W and Z, or for the size of the Fermi constant GF = g₃²/4√(2)M². This mass scale is generated dynamically. The theory contains a scalar (spin 0) field, the Higgs field, which transforms under SU(2) × U(1). The state of minimum energy occurs when this field acquires a non-zero value, its vacuum expectation value or vev, of order 250 GeV. This vev breaks the SU(2) × U(1) symmetry down to U(1), corresponding to electromagnetism with its massless photon, and generates a mass for the W and Z bosons. A physical neutral scalar, the Higgs boson, whose mass is not determined is a residue of this breakdown (Higgs, 1964; Weinberg, 1967). The Higgs boson is the only particle in the standard model for which we have no experimental evidence.

The very success of the standard model has pointed out its limitations. There is no explanation of the relative values of the three coupling constants, of why there are six quarks, or of the pattern of quark and lepton masses. Most serious, however, is the problem of the mass scale of weak interactions. We are aware of two other mass scales in physics, the Planck mass (MP = 1/√GN where GN is Newton’s constant) which is the mass scale at which gravitational interactions become relevant for particle physics, and the scale of hadron masses in QCD (~ 1GeV).

A qualitative explanation of one of these hierarchies, the ratio of hadron masses to the Planck mass is possible. As indicated earlier the QCD coupling constant becomes large at long distances, or low momentum transfers. Quarks are bound into hadrons by QCD. This binding is difficult to understand so long as the coupling is weak and perturbation theory is applicable. If the coupling is strong, non-perturbative effects can generate the binding, so that one might expect that the scale of hadronic binding and hence of the hadron masses should be close to the scale at which the QCD coupling constant becomes of order one. The coupling constant varies only logarithmically with energy (Gross and Wilczek, 1973; Politzer, 1973).

\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2f) \log(Q^2/A^2)} \]

Here Q is the scale at which \( \alpha_s \) is evaluated, A is a constant and f is the number of quark flavors (6 in the standard model). If \( \alpha_s(M_P) \) has a value of order 1/100 then \( \alpha_s(Q^2) \) will become of order one when \( Q^2 \approx 1 GeV^2 \). This idea can be made more quantitative in theories where all the interactions are unified at some large scale (Georgi and Glashow 1974; Ramond, 1983). Notice that it is the slow evolution of \( \alpha_s(Q^2) \) which enables the hierarchy \( M_P/m_{hadron} \) to be explained.

*The top quark is not yet well established. There are some experimental indications that it may have a mass of around 50 GeV (Rubbia, 1985)
There is no such natural explanation for the hierarchy $M_W/M_P$. The difficulty is caused by the behaviour of scalar masses. If we consider a radiative correction to the Higgs mass we find contributions which are quadratically divergent. Figure 1 shows a contribution at one loop which arises from the Higgs self interaction. The loop has only one propagator and so the integral over the loop momentum has the following form

$$I \sim \int \frac{d^4\ell}{(\ell^2 - m^2)}$$

which is quadratically divergent. In the case of corrections to the coupling constants we encounter only logarithmic divergences. These quadratic divergences imply that the Higgs mass or alternatively the ratio $M_W/M_P$ is unstable with respect to radiative corrections and that the parameters of the model have to be very delicately tuned (Wilson, 1971; 't Hooft, 1980; Gildener, 1976; Weinberg, 1979). This instability can be avoided if the the quadratic divergences can be eliminated. Supersymmetry (Gol'fand and Likhtman, 1971; Volkov and Akulov, 1973; Wess and Zumino, 1974; Salam and Strathdee, 1974; Fayet and Ferrara, 1977; Wess and Bagger, 1983) provides a natural solution to this problem.

The symmetries that were traditionally used in particle physics such as isospin or the gauge symmetries of the standard model are bosonic symmetries; they relate fermions to fermions or bosons to bosons. For example, the extent that isospin is a good symmetry it implies that the proton and the neutron are degenerate in mass. Supersymmetry relates fermions to bosons. In a supersymmetric theory particles occur in supermultiplets which have both bosonic and fermionic components. Each quark or lepton is in a scalar multiplet along with a spin zero particle with the same quantum numbers, and mass as the fermion. The gauge bosons have spin 1/2 fermionic partners. The Higgs boson has a spin 1/2 partner. The presence of these extra particles results in additional contributions to Higgs masses as indicated in figure 2. These contributions cancel the quadratic divergences, so providing a solution to the hierarchy problem.

We know that supersymmetry cannot be an exact symmetry since there is no spin zero particle degenerate with the electron. Fortunately we can set an approximate upper bound on the mass of the partners. There is a contribution to the Higgs mass from graphs of the type shown in figures 1 and 2, which is proportional to

$$(m_f - m_j)^2$$

where $m_f - m_j$ is the mass difference between the Higgs and its fermionic partner. If we require that this correction not be much larger than the Higgs mass itself we conclude that the mass of the superpartner is lighter than a TeV or so. In most supersymmetric models the masses of all the superpartners are comparable; they are expected to be observed in the not too distant future.

There is another perhaps more fundamental reason why theorists are so excited about supersymmetry. Supersymmetric theories are the only ones which hold a hope of being able to unify gravitational interactions with the strong, weak and electromagnetic interactions. In such supergravity (Van Nieuwenhuizen, 1981) theories the supersymmetry is realised as a local symmetry. I shall make a few remarks about such unifications of all the interactions later. Let us now consider the features of supersymmetric models. More details can be found in several more extensive and technical review articles (Nilles, 1984; Haber and Kane, 1984; Nanopoulos and Savoy-Navarro, 1983; Ellis, 1984).
2. Characteristics of Supersymmetric Models

I shall only discuss models based on $N = 1$ supersymmetry, in which case the minimal supersymmetric model must have three generations of quarks and leptons, and their superpartners, the squarks and sleptons, each of which contains the following representations under $SU(2) \times U(1)_Y$:

- $L = (\nu)_{L}$
- $E^c = e_R$
- $Q = (u)_{L}$
- $d_R$
- $u_R$
- $Y = -1$
- $2$
- $1/3$
- $-4/3$
- $2/3$

$$H_1 = \begin{pmatrix} H_1^- \\ H_1^+ \end{pmatrix}_L, \quad H_2 = \begin{pmatrix} H_2^- \\ H_2^+ \end{pmatrix}_R.$$

The subscripts L and R refer to helicity states and Y is normalized in the usual manner so that the particle's electric charge is given by

$$Q = T_3 + \frac{Y}{2},$$

where $T_3$ is the weak isospin. In a supersymmetric model each of these fields is a superfield which has a fermionic component and a scalar component. I will usually suppress indices when writing the couplings and will use the same label for a superfield as for its scalar component. The fermionic component of a superfield A will be indicated by $\psi_A$.

The gauge fields are contained in supermultiplets which contain the spin 1 gauge fields themselves as well as a spin 1/2 Majorana gauginos.

In the minimal Weinberg-Salam model (Weinberg, 1967; Salam, 1968) the gauge symmetry is broken to $U(1)_{em}$, and quark and lepton masses generated, via the vacuum expectation value of a single Higgs doublet. This is not possible in a supersymmetric model where at least two doublets are required.

The superpotential which contains the interactions between the quarks, leptons and Higgs multiplets ($H_1$ and $H_2$) must contain the following terms.

$$W_1 = \lambda_L L E^c H_1 + \lambda_d Q H_1 d^c + \lambda_u Q H_1 u^c.$$  \hspace{1cm} (1)

The second term, which contains the Yukawa interaction $\psi_q \psi_u H_1^0$, generates a mass for the down quark once $H_1^0$ obtains a vacuum expectation value (vev) $v_1$. In the non-supersymmetric model, the up quark's mass is generated from $\psi_q \psi_u H_2^0$. This term is not available in a supersymmetric model since $H_1^0$ cannot appear in the superpotential (Wess and Bagger, 1982), hence the need for $H_2$ whose vev $v_2$ will generate the appropriate mass.

The superpotential can also contain the term $\mu H_1 H_2$. If this term is not present then the theory contains a Peccei-Quinn symmetry (Peccei and Quinn, 1977) under which $H_1$ and $H_2$ can have independent phase rotations. This symmetry will be broken when the Higgs fields obtain vevs and a phenomenologically unacceptable axion (Wilczek, 1983; Sikivie, 1985) may result. If $\mu \neq 0$, the axion is eliminated.

The potential for the scalar fields will have the following contributions from $W$

$$V \equiv \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 = |\mu H_2 + \lambda_L L E^c + \lambda_d Q d| + |\mu H_1 + \lambda_u Q u|^2$$

$$+ \lambda_q^2 \left( |E^c H_1| + |L H_1|^2 \right) + |\lambda_u H_1 d + \lambda_d H_2 u|^2$$

where $\phi_i$ is any field. The potential will also contain the following $D$ terms from the gauge interactions of $SU(2) \times U(1)_{Y}$. (I have suppressed that from SU(3)$_{color}$ which plays no role).

$$V \equiv \frac{D^* D}{2} + \frac{D'^* D'}{2}$$  \hspace{1cm} (2)

with

$$D^* = \frac{g_2}{2} \left[ H_1^* r^a H_1 + H_2^* r^a H_2 + Q^* r^a Q + L^* r^a L \right]$$

and

$$D' = \frac{g'}{2} \left[ H_1^* r^a H_1 - H_2^* r^a H_2 + y d^e Q' Q + y_u u^e u^e + y_d d^e d^e - L^* L + 2 E^e E^e \right].$$

Here $y_i$ is the hypercharge of the representation $i \ r^a$ is a Pauli matrix and $g_2$ and $g'$ are the $SU(2)$ and $U(1)_Y$ coupling constants.

Supersymmetry must be broken in order to lift the degeneracy between quarks and squarks. I will assume that it is broken via the appearance of soft operators.
(Girardello and Grisaru, 1982) in this potential which do not break \( SU(3) \times SU(2) \times U(1) \). These can take the form of masses for all the scalars:

\[
m_i^2 \phi_i^2
\]

and pieces in the scalar potential proportional to the terms in the superpotential itself (in this expression the superfields are replaced by their scalar components)

\[
A W_1 + B \mu H_1 H_2.
\]

These supersymmetry breaking terms have a natural origin in theories of supergravity. Suppose that we have some fields which do not couple to \( SU(3) \times SU(2) \times U(1) \) (a 'hidden sector') but interact with each other in such a way that their ground state spontaneously breaks supersymmetry, just as the ground state of the Higgs sector breaks the \( SU(2) \times U(1) \) symmetry. In a supergravity model this spontaneous breaking generates a mass for the gravitino (Deser and Zumino, 1977), the spin 3/2 partner of the graviton. This is analogous to the generation of the W and Z masses when \( SU(2) \times U(1) \) is broken. The coupling to supergravity the quark and lepton multiplets (Cremmer et al., 1982) enables this supersymmetry breaking to be fed through to these multiplets (Barbieri, Ferrara and Savoy, 1983; Hall, Lykken, and Weinberg, 1983). The supersymmetry breaking manifests itself in terms of the expressions of equations 4 and 5. Since gravitational interactions do not depend in the \( SU(3) \times SU(2) \times U(1) \) quantum numbers all the masses \( m_i \) are equal to each other, and, in the simplest cases, to the gravitino mass \( M_{3/2} \). This mechanism is independent of the details of the 'hidden sector'.

Supersymmetry breaking can also be manifested in mass terms for the gauginos

\[
M_{\tilde{g}_i} \tilde{g}_i.
\]

These mass terms can also arise from the effects of some hidden fields (Cremmer et al., 1982). Again, one would expect the partner of the W (wino) and gluon (gluino) to have the same mass.

The real situation is slightly more complicated. Mass parameters, like coupling constants, are dependent upon the momentum scale at which they are used. In the case of current phenomenology the relevant scale is that of the W mass. Since the masses are generated by gravitational effects, their equality will apply at scale where the gravitational interactions are important. This scale is the same order as the Planck mass \( (M_p \approx 10^{19} \text{GeV}) \) and, therefore, renormalization effects will be important and the masses will not be equal when they are evaluated at low energy \( (0(M_w)) \). The relevant renormalization group equations are given in the Appendix. The most important renormalization effects are due to gaugino masses and any large Yukawa couplings present in the superpotential. (Inoue et al., 1982, 1984.)

If the gaugino masses are comparable to, or larger than, scalar masses at \( M_p \) then, since over most of the range between \( M_p \) and \( M_w \) the strong coupling \( (\alpha_s) \) is larger than the weak and electromagnetic couplings, squark masses will be affected more than slepton or Higgs masses by radiative corrections and will be larger at low energy. This renormalization effect is so strong that it prevents the gaugino masses from being much greater than the squark masses.

At first it would appear that the renormalization effects are an annoyance since they spoil some of the simplest features of the supersymmetric model. However, there is an unexpected bonus. It can be seen from the appendix (see equation A11) that the effect of the Yukawa couplings in the superpotential is to reduce the masses of some of the scalars as the momentum scale is lowered. The Yukawa couplings are proportional to quark and lepton masses

\[
\lambda_{e,\tau} \frac{M_{3/2}}{M_w}.
\]

Consequently the only large coupling is that of the top quark whose mass is known to exceed 22 GeV. We have the possibility that

\[
m_{H_2}^2(M_w) < 0.
\]

The ground state where the vev of \( H_2 \) is zero becomes unstable, and the breaking of \( SU(2) \times U(1) \) will be triggered. (Alvarez-Gaumé, Claudson, and Wise, 1982; Alvarez-Gaumé, Polchinski and Wise, 1983; Ellis et al, 1983; Ibáñez and Lopez, 1984). In the simplest models this effect suffices to determine \( M_w \) in terms of the gravitino mass; we have traded off one unknown parameter for another one. In more complicated models it is possible to generate the weak signal dynamically (Ellis, 1985; Ellis, Enqvist and Nanopoulos, 1985; Ellis, Hagelin, and Nanopoulos, 1983) (Recall that the scale of hadronic binding is generated in this way). In order for the mechanism to work to the top quark Yukawa coupling (and hence its mass) is constrained to lie in a certain range. Unfortunately this range is model
dependence. Let us now turn to a more detailed discussion of the mass spectrum of the superpartners.

After the two neutral members of the Higgs doublets have obtained vevs $v_1$ and $v_2$, the slepton mass matrix will have the following form

$$
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}^{\dagger}
\begin{pmatrix}
L^2 \tilde{m}_L^2 + m_1^2 & A m_t + \mu m_t v_1 \\
A m_t + \mu m_t v_1 & R^2 \tilde{m}_R^2 + m_1^2
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}.
$$

(9)

Note that there are two charged scalars $\tilde{\ell}_L$ and $\tilde{\ell}_R$ which are the partners of the left and right handed electrons. The off-diagonal terms which cause mixing between the partners of the left and right handed leptons arise from the terms in the scalar potential coming from $W_1$. Hence the dependence on the lepton mass ($m_\ell$). The mixing is therefore likely to be small for the partners of all known leptons and it is reasonable to assume that the eigenstates are the partners of the left and right handed leptons. I have introduced a mass scale $\tilde{m}$ so that $L$ and $R$ are dimensionless.

The terms $L^2$ and $R^2$ arise from two sources. Firstly, there are the soft masses. In the renormalization to low scales $L^2$ evolves more slowly than $R^2$ due to the presence of Yukawa couplings (see Appendix) leading to $L > R$ at low energy. However, these effects are proportional to the lepton's Yukawa coupling and are therefore small for the known leptons. The effects of gaugino masses are larger for $L$ than $R$ since the winos can act in the former case. Again this tends to make $L^2$ larger than $R^2$ at low energy if they are equal at $M_\nu$.

Secondly there are the contributions to $L$ and $R$ from the $D$ terms of equation (3)

$$
\pi \alpha_{em} \left(v_2^2 - v_1^2\right) \left(\frac{2 \tilde{\alpha}_R \tilde{\alpha}_R}{\cos^2 \theta_W} - \tilde{\alpha}_L \tilde{\alpha}_L \left(\frac{1}{\cos^2 \theta_W} - \frac{1}{\sin^2 \theta_W}\right)\right).
$$

(10)

If the weak interaction breaking is triggered by a large t-quark Yukawa coupling then $m_{\tilde{t}_L}^2 < m_{\tilde{t}_R}^2$ and it is likely that $v_2/v_1$. Hence $R$ is greater than $L$. This effect is likely to overwhelm the effect from the renormalization group scaling unless the gaugino masses are large, so it is reasonable to expect $R > L$ in the slepton mass matrix. Notice that these splittings are quite small unless $v_1/v_2$ is much different from one, so that one may expect approximate degeneracy between the left and right partners of all the sleptons.

In the case of squark masses, the situation is slightly more complicated owing to the presence of Yukawa couplings which connect different generations. After diagonalization of the quark mass matrix these off diagonal couplings are responsible for the Kobayashi-Maskawa (Kobayashi-Maskawa, 1963) mixing angles. The mixing between partners of left and right handed quarks is similar to that discussed above for sleptons.

If there were no renormalization effects, all the soft squark masses would be equal and the squark mass matrix would have the following form (Nilles, 1984)

$$
\tilde{m}_{ij}^2 = (m_{q_i}^+ m_q)_{ij} + \tilde{m}^2 1_{ij}
$$

(11)

where the indices $i$ and $j$ label the quark flavors down, strange, and bottom, $m_q$ is the quark mass matrix and 1 is a unit matrix. The squark mass matrix is then diagonalized by the same rotation among flavors which diagonalizes the quark mass matrix. The mixing angles appearing in the couplings of the squarks to the $W$ ('Skobayashi-Maskawa' angles) will be equal to the usual Kobayashi-Maskawa angles.

The gaugino mass is controlled by its value at $M_\nu$ and by renormalization effects. If the wino $(M_\tilde{w})$ and gluino $(M_\tilde{g})$ mass terms are equal at $M_\nu$ and the theory is grand unified so that $\alpha_s(M_\nu) = \alpha_s(M_\nu)$ then

$$
\frac{M_\tilde{w}}{M_\tilde{g}} = \frac{\alpha_s(M_\nu)}{\alpha_s(M_\nu)}
$$

(12)

where $\alpha_s$ is the strong coupling constant. One can also expect that the gluino will be much heavier than the photino.

If the gaugino masses are zero at $M_\nu$, they can arise through graphs of the type shown in figure 3. The result is proportional to the mixing between the left and right handed squarks in the loop, i.e. to the off diagonal terms in equation (9); (Barbieri, Girardello, and Masiero, 1983) The dominant contribution for gluinos will come from the top squarks, where this term is expected to be largest.

The mass matrix of the gaugino partners of the $W$, $Z$ and photon is complicated by the breaking of electro-weak symmetry. The charged partners of the $W$ boson (winos) can mix with the fermionic partners of the charged Higgs bosons (Higgsinos) and the resulting masses are model dependent (Haber and Kane, 1984). The partners of the $Z$ (zino) photon (photino) and neutral Higgsinos are mixed and the mass eigenstates called neutralinos are model dependent.
The properties of the lightest neutralino (\(\chi_0\)) are very important phenomenologically. This is because, in most supersymmetric models, the lightest sparticle is absolutely stable. There is a R parity (Fayet, 1975; Salam, and Strathdee 1975; Fayet and Ferrara, 1977) which is preserved by all the interactions that I have so far discussed. Under this parity, all the ‘old’, standard model, particles are even and all the superpartners are odd, hence the lightest sparticle is absolutely stable.

It is possible for this R parity to be broken, in which case the lightest sparticle will decay; (Hall and Suzuki, 1984; Ross and Valle, 1985) this option is rather ugly and I shall neglect it. The most likely candidate for the lightest sparticle is one of the neutralinos.

The gravitino can play an important role phenomenologically so we should discuss its interactions. The interaction of the gravitino (\(\tilde{\psi}_e\)) with a scalar A and its fermion partner is given by

\[
\frac{1}{2M^2} \tilde{\psi}_e \gamma^\mu \psi_A \beta^\nu + h.c. \]  

(13)

A gauge field (\(F_{\mu}^a\)) and gaugino \(\lambda_a\) interact according to

\[
\frac{1}{4M^2} \tilde{\lambda}_a \gamma^\mu \gamma^\nu F_{\mu\nu} \psi_\mu + h.c. \]  

(14)

Here \(M = M_p/\sqrt{8\pi}\) is the reduced Planck mass. These interactions are very weak as a consequence of the factors of \(1/M\). The gravitino can decay to a photon and a photino with a lifetime

\[
\tau(\tilde{G} \rightarrow \gamma\tilde{\gamma}) \approx \frac{4M^2}{M_{3/2}^2(1 - M_{3/2}^2/m_{3/2}^2)} \approx 4 \times 10^{8} \left(\frac{100\text{GeV}}{m_{3/2}}\right)^3 \text{ sec}. \]  

(15)

Consequently it can be regarded as a stable particle for the purpose of discussing terrestrial experiments, unless it is extremely heavy.

The minimal Weinberg-Salam model has one neutral physical Higgs boson whose mass is not predicted by the theory. In the minimal supersymmetric model there are three neutral and one charged physical Higgs bosons. The coupling of the neutral boson to fermions in the non-supersymmetric model is proportional to the fermion mass viz.

\[
\frac{g_m m_f}{2M_W} \tilde{\psi}_f \psi_f H^0. \]  

(16)

The three neutral bosons in the supersymmetric model consist of two scalars (\(H_u\) and \(H_d\)) and a pseudoscalar \(H_c\). The couplings to charge \(-1/3(\psi_d)\) and to charge \(+2/3(\psi_u)\) quarks are as follows (Haber and Gunion, 1985).

\[
igw \frac{m_u}{2M_W} \tilde{\psi}_u \left[ -H_u \sin \alpha \sin \beta - H_b \cos \alpha + i\gamma_5 \cot \beta H_c \right] \psi_u \]  

(17)

\[
igw \frac{m_d}{2M_W} \tilde{\psi}_d \left[ -H_u \cos \alpha \sin \beta - H_b \sin \alpha + i\gamma_5 \tan \beta H_c \right] \psi_d \]

where \(\tan \beta = v_2/v_1\) and

\[
\tan 2\alpha = \tan 2\beta \left(\frac{m_{H_u}^2 + m_{H_d}^2}{m_{H_c}^2 - m_{H_d}^2}\right). \]  

(18)

Notice that, if the vevs, \(v_1\) and \(v_2\) are significantly different from each other, then some of the couplings are enhanced; an effect which could make the observation of a neutral Higgs via the decay toponium \(\rightarrow H_0 + \gamma\) much easier (Wilczek, 1977).

The interactions of the sparticles are prescribed by those of the standard model, e.g. the squark-squark-gluon vertex has the same strength as the quark-quark-gluon vertex. The only unknown quantities are their masses. In order to discuss the decays of the sparticles, I shall assume that the photino is the lightest one (i.e. \(\chi_0 = \tilde{\gamma}\)). The principal decay modes and lifetimes are then given in Table 1. All the lifetimes are probably too short to leave visible tracks inside detectors unless the phase space for a decay is extremely restricted. The only possible exception is the gluino decay to a photino and a quark anti-quark pair.

If the \(\chi_0\) is a Higgsino then the lifetimes of some of the supersymmetric particles will become very long since the decay rates are then controlled by small Yukawa couplings rather than by gauge couplings. For example the selectron lifetime for the decay to electron and Higgsino is

\[
\sim 4 \times 10^{-12} \frac{m_3^3}{(m_{3/2} - m_{H_d}^2)^2} \text{ sec}. \]  

(19)

where all masses are given in GeV. If \(\chi_0\) is

---

*I have assumed that CP is conserved.
\[ a\tilde\gamma + b\tilde Z + c\psi H^0 + d\psi H^0. \]  

(20)

then a reasonable estimate of the lifetimes can be gotten by assuming that only the photino component is important. The modifying factors in this approximation are given in the table.

The \( \chi_0 \) will only interact weakly with matter, via the diagram shown in figure 4. The cross-section behaves roughly as

\[ \sigma \propto \frac{1}{M_X} E m_p \]  

(21)

where \( E \) is the energy of the \( \chi_0 \) impinging on a target particle of mass \( m_p \). \( M_X \) is the mass of an exchanged particle which is either a squark or a slepton. The cross-section is small enough so that a \( \chi_0 \) produced in the decay of another sparticle is likely to exit from a detector without interacting.

One other circumstance is worth considering. Suppose that the photino and a Higgsino are mass eigenstates and \( m_\gamma > m_\mu \). The production of a photino, for example via selectron decay \( \tilde e \rightarrow e + \tilde \gamma \), will be followed by the decay

\[ \tilde \gamma \rightarrow \tilde H^0 + f^+ f^- \]

or

\[ \tilde \gamma \rightarrow \tilde H^0 + \gamma. \]

Here \( f \) is a fermion of mass \( m_f \). The latter process is likely to dominate unless the photino is able to decay into heavy quark pairs. We have now sufficient information about the supersymmetric interactions to be able to discuss the phenomenology.

3. Supersymmetric Phenomenology

(a) Low energy experiments.

I will begin with a discussion of processes in which the effects of supersymmetry are observed indirectly, and the new particles predicted by supersymmetry are not produced. One of the most accurately known, and predicted, quantities in physics is the anomalous magnetic moment of the muon. Calculations in quantum electrodynamics (Kinoshita, 1984) agree with the measured value (J. Bailey et al., 1979) so well that

\[ |(g - 2)_{QED} - (g - 2)_{\text{experiment}}| < 2 \times 10^{-9}. \]  

(22)

In a supersymmetric model there are contributions to the magnetic moment from the graphs shown in figure 5. If \( \tilde \mu_L \) and \( \tilde \mu_R \) are degenerate then the contribution is proportional to the square of the muon mass. The effective vertex has the form

\[ \frac{e}{2m_\mu} F(q^2) \bar{u}L \sigma \mu u \]

(23)

where \( q \) is the momentum of the photon. The contribution to \( g - 2 \) is proportional to \( F(0) \), which contains one power of the muon mass as a consequence of the definition (23). The second power arises since the contribution must violate chirality. Consequently we can get no useful constraint from the measurement of \( g - 2 \) for the electron. Equation (22) translates into the following constraint (Barbieri and Maiani, 1982; Fayet, 1979a)

\[ m_\mu, m_\gamma \geq 15 \text{GeV}. \]  

(24)

The partners of left and right handed quarks and leptons may not be degenerate in mass. In this case we can get parity violation which may be observable. In the case of selectrons the best limit comes from the measurements of asymmetry in the scattering of polarized electrons from deuterium (see figure 6). The cross-section difference is sensitive to the difference in the slepton masses, (Hinchliffe and Littenberg, 1982)

\[ \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \propto \frac{1}{m_L^2} \frac{1}{m_R^2} \]

(25)

where \( \sigma_L(\sigma_R) \) is the cross-section for the scattering of left (right) handed electrons from deuterium. The measurement of the asymmetry (Prescott et al., 1978) is consistent with the value expected from electro-weak theory (Cahn and Gilman, 1978) and can be used to set the bound.
\[ \left| \frac{1}{m_{\tilde{u}}^2} - \frac{1}{m_{\tilde{d}}^2} \right| \lesssim 10^{-2} \text{GeV}^{-2}. \]  

For simplicity I have assumed that \( e_L \) and \( e_R \) are mass eigenstates, i.e., that the off-diagonal elements in equation (9) are zero. Notice that this constraint is weak in view of the limit discussed in the previous sections.

In the case of squarks, the best limit arises from nuclear parity violation (Suzuki, 1982; Duncan, 1983). Graphs of the type shown in figure 7 result in short range, parity violating, four-fermion interactions. Some parity violating nuclear transitions have been seen. For example

\[ ^{18}F(J^P = 1^-, I = 0) \rightarrow ^{18}F(J^P = 1^+, I = 0) + \gamma \text{ (circularly polarized)}. \]

Or the back forward asymmetry (\( \Delta \)) in the decay

\[ ^{19}F(J^P = 1/2^-) \rightarrow ^{19}F(J^P = 1/2^+) + \gamma \]

These rates are connected with standard model expectations. In order to get a constraint, require that the contributions to parity violation from supersymmetry be smaller than the corresponding ones from the standard model.

If \( m_{\tilde{q}} \approx m_{\tilde{u}} \) then the constraint becomes

\[ \frac{1}{m_{\tilde{u}}^2} - \frac{1}{m_{\tilde{d}}^2} \approx \frac{1}{m_{\tilde{d}}^2} - \frac{1}{m_{\tilde{d}}^2} < \left( \frac{1}{100 \text{GeV}} \right)^2. \]  

or if \( m_{\tilde{q}} >> m_{\tilde{u}} \) we have

\[ \log\left( \frac{m_{\tilde{u}}^2}{m_{\tilde{d}}^2} \right) - \frac{1}{m_{\tilde{d}}^2} < \frac{1}{m_{\tilde{d}}^2} - \frac{1}{m_{\tilde{d}}^2} < \left( \frac{1}{80 \text{GeV}} \right)^2. \]  

As indicated in section two, the splitting \( m_{\tilde{u}} - m_{\tilde{d}} \) is expected to be proportional to the up quark mass. The constraints are therefore easily satisfied provided that the squarks and sleptons are heavier than 20 GeV or so.

There are very tight limits on the existence of flavour changing neutral currents. The most restrictive data come from the kaon system (Campbell, 1983). There are contributions to the \( K_L - K_S \) mass matrix from the processes shown in figure 8. If the exchanged gauginos are winos, this graph is simply the supersymmetric analog of the usual contribution involving W bosons and the charm and top quarks. The contribution to the mass mixing implies (Ellis and Nanopoulos, 1982)

\[ \frac{g^4}{64 \pi^2} \left( \frac{\Delta m_{ij}^2}{m_{\tilde{A}}^2} \right) \frac{\Gamma_{ij}}{M^2} < 5 \times 10^{-15} \text{GeV}^{-2}. \]  

Here \( g \) is the appropriate coupling constant and \( M \) is the larger of the squark and gaugino masses. \( \Delta m_{ij}^2 \) is the mass difference between squarks of flavors i and j. This mass difference is assumed to be much smaller than the average value \( m_{\tilde{u}}^2 \). The quantity \( \Gamma_{ij} \) depends upon the mixing angles appearing at the vertices in figure 8.

If, in the case of the wino diagrams, we assume that the mixing angles are equal to the Kobayashi–Maskawa angles then we get

\[ \frac{1}{M^2} \frac{\Delta m_{ij}^2}{m_{\tilde{A}}^2} < 10^{-7} \text{GeV}^2 \]

for squarks of the first two generations. Here \( M \) is the larger of the squark and wino masses. In the context of the model discussed in section two, the contribution to \( \Delta m_{ij} \) arises from the Yukawa couplings. Hence \( \Delta m_{ij}^2 \approx m_c^2 \) where \( m_c \) is the mass of the charm quark. The resulting constraint on squark and wino masses is similar to that obtained from direct searches which will be discussed below.

The contribution from gluino exchange cannot be discussed without reference to a specific model. If we neglect radiative corrections then the \( \Gamma_{ij} \) are all zero. This occurs since the quark-squark-gluino coupling is diagonal in flavor as the squark and quark mass matrices are diagonalized by the same rotation. (See equation 11.) The radiative corrections can produce non-zero values and constraints are obtained which are comparable to those discussed below from direct searches for superparticles (Nilles, 1986; Donohue, Nilles and Wyler, 1983).

Rare decays of the kaons can also provide constraints. An analysis of \( K \rightarrow \pi^0 \gamma \) missing neutrals can be sensitive to the existence of very light photinos (Gaillard et al., 1983; Ellis and Hagelin, 1983a).

\[ BR(K \rightarrow \pi^0 \gamma) = 7 \times 10^{-11} \left( \frac{20 \text{GeV}}{m_{\tilde{A}}} \right)^4 \left[ 1 + 0.43 \log\left( \frac{m_{\tilde{A}}}{20 \text{GeV}} \right) \right] \]
where \( m_3 \) is the mass of the charm squark. Experiments underway at Brookhaven (Littenberg, 1984) can expect to be sensitive to branching ratios of order \( 10^{-10} \), so that they are unlikely to make a significant contribution. If the photino is sufficiently light then

\[
K \rightarrow \pi^+\pi^0 \\
\gamma\gamma
\]

which occurs with a branching ratio of

\[
2 \times 10^{-11} \left( \frac{20\text{GeV}}{m_3} \right)^4 \left( \frac{m_3}{1\text{MeV}} \right)^2
\]

may be observable. A photino light enough to be produced in K decay is excluded by cosmological arguments (see below) unless its mass is less than 100 eV or it is unstable.

It is unfortunate that these indirect searches are not more potent. Some alternatives to the standard model such as technicolor models are seriously jeopardized by them (Farhi and Susskind, 1981). In the case of supersymmetry we must look for most direct evidence.

(b) Supersymmetry in \( e^+e^- \) Annihilation

In this section I shall discuss the supersymmetric phenomenology of \( e^+e^- \) annihilations. The cross section for the production of a pair of squarks or sleptons is due to the exchange of the photon or Z in the s channel and is given by

\[
\sigma = \frac{\pi \alpha^2}{38} k \beta^2 \left( Q_3^2 - 2 \chi Q_4 p(1 - 4 \sin^2 \theta_W) + p^2 \chi^2 (1 - 4 \sin^2 \theta_W)^2 \right)
\]

where \( k = 3 \) for squarks and 1 for sleptons and

\[
\chi = \frac{s}{(s - M_\chi^2)} \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W}.
\]

For the partners of left handed fermions

\[ p = 4Q_i \sin^2 \theta_W - 2I_3 \]

and for the partners of right handed fermions

\[
p = 4Q_i \sin^2 \theta_W - 2I_3
\]

where \( Q_i \) is the fermion charge and \( I_3 \) is its weak isospin. The sparticles are produced with a \( \sin^2 \theta \) angular distribution (\( \theta \) is the angle between the beam and the sparticle).

An exception to this formula occurs when the sparticle is a selectron. In this case there is a contribution from Zino and photino exchange in the t channel (Fayet and Ferrar, 1980). The cross section peaks in the forward direction, but close to threshold has the same order of magnitude as that for smuon production.

The final state from a pair of sleptons will be a lepton pair and two x’s arising from the decays \( \tilde{\ell} \rightarrow \ell + \chi \). Since the Yukawa couplings of the known leptons are small the decay will be into the x state which is dominantly photino or zino even if this state is not the \( \chi_0 \). The lifetime of the sleptons will be too short for the decay vertex to be visible, see table 1. If the \( \chi \) is the lightest sparticle it will leave the detector without interacting so that the final state will consist of a lepton anti-lepton pair with unbalanced momenta.

The only backgrounds arise from two-photon production of a fermion antifermion pair and from tau pair production. The former can be eliminated by taking events where the missing momentum vector does not point along the beam direction. The latter produces lepton antilepton pairs which are back to back, since the tau is light, and \( \mu \) final states which cannot be produced by the slepton pair decay. Sleptons of mass less than \(~ 20\text{GeV}\) are ruled out by such searches (Brandelik et al., 1982; Behrend et al., 1982; Adeva et al., 1985; Bartel et al., 1985).

If the \( \chi \) is not the lightest sparticle it may decay inside the detector. If it decays to a photon and \( \chi_0 \), which then exits, the final state will consist of a lepton anti-lepton pair and two photons with unbalanced momenta. A search for this channel has also been carried out and the resultant limits are similar to those in the case of the stable \( \chi \) (Bartel et al., 1984).

If the photino is a mass eigenstate and is stable (i.e. it is the \( \chi_0 \)), then a search for the final state \( \gamma + \gamma + \gamma \) can be carried out. The differential cross section for the production of a photon of energy \( E_\gamma \) at angle 0 to the beam is given by (Ellis...
where search will have to be sensitive to soft photons. The principal background is due and Hagelin, 1983b, Grassie and Pandita, 1984) to radiative Bhabha scattering, where the transverse momentum of the photon is balanced by an electron. A dedicated experiment at SLAC (Barta et al., 1985) has set a limit on this process which translates into the constraint on photino and selectron masses shown in figure 9. The limit is extremely model dependent; the experiment produces no constraint if $\chi_0$ is a Higgsino.

Squark pair production will produce final states consisting of quarks (which materialize as jets of hadrons) and missing energy from the decays.

\[ \bar{q} \rightarrow q + \chi_0 \]
\[ \bar{q} \rightarrow \bar{g} + q \]
\[ \bar{q} \rightarrow q\bar{q} + \chi_0 \]

The latter will dominate if the gluino is lighter than the squark. In either case the final state consists of hadrons with missing energy/momentum.

If all the squark flavors were degenerate then the onset of squark pair production could be detected by a rise in the total hadronic cross-section. The rise due to the crossing of a single squark threshold could be too small to see, particularly if the squark had charge 1/3. In this case, one must look at specific final states. Searches have been carried out for the mode $\bar{q} \rightarrow q + \chi$ on the assumption that the $\chi$ is stable (its detailed properties are irrelevant). The final state consists of two jets with unbalanced momenta. Squarks of mass less than 14 GeV are excluded. The rather unlikely case of a stable squark is also excluded if its mass is in the same region (Yamada, 1983). I am aware of no search which is sensitive to the mode $\bar{q} \rightarrow q + \bar{g}$, indeed the situation with regard to squark searches in $e^+e^-$ annihilation is rather unsatisfactory.

Other charged particles such as winos can be pair produced. Searches for them have been carried out which yield $m_{\tilde{W}} > 18 GeV$ (Bartel et al., 1984). Final states involving sneutrinos are also possible (Barnett, Haber, and Lackner, 1983, 1986). They can be produced in pairs via an intermediate Z boson. This process is too small to produce a measurable rate at current energies but could be important in the forthcoming generation of $e^+e^-$ machines. The signal depends upon the decay of the sneutrino. The two body decay into a neutrino and a $\chi$ produces nothing observable unless the $\chi$ decays. The four body final states $l\tilde{q}\tilde{q}$ or $t\tilde{t}\chi_0$ are also possible. These decay rates are very model dependent but the two body mode is likely to dominate unless the gluino channel is open. A measurement of the Z width will be able to constrain sneutrino masses respective of their decay products. The contribution of each pair ($\bar{\nu}_L, \bar{\nu}_R$), assumed degenerate to the Z width is

\[ \Gamma = 80\beta^2 MeV. \] (35)

Here $\beta$ is the velocity of the sneutrino in the Z rest frame.

I have not discussed the production of gluinos in $e^+e^-$ annihilation since they have no electro-weak charges. Three jet events will arise from the final state $q\bar{q}g$, but will only be clear when the energy is far above the threshold. It may also be possible to detect a gluino from the decays onium $\rightarrow \bar{g}g$, onium $\rightarrow gg$ or onium $\rightarrow q\bar{q}$ where onium is a bound state of a heavy quark and its antiquark (Nelson and Osland, 1982; Ellis and Rudaz, 1983; Keung, 1983). None of these searches are easy and are superceded by the limits from hadronic searches to which we now turn.

(c) Supersymmetry in Hadronic Reactions.

The searches for supersymmetry in hadronic reactions are more complicated, and more model dependent than those in $e^+e^-$ annihilation. Detailed limits usually depend upon uncertainties beyond those inherent in the supersymmetric models.

I shall first discuss the searches for sleptons in hadron colliders. Fixed target experiments at CERN and FNAL have nothing to contribute in view of the limits quoted in the previous section. There are only two relevant sources of new leptons and sleptons in hadron colliders; pair production via the Drell-Yan (Drell and Yan, 1971) mechanism and the decay of W's and Z's. The luminosity of a collider must be large before the former can be exploited effectively and so we are left with the latter mechanism as the only one relevant at the SppS and Tevatron colliders. Charged...
sleptons can be pair produced in the decay of the Z (Cabbibo et al., 1983). The widths $\Gamma$ for the decay into leptons and sleptons are related by

$$\frac{\Gamma(Z \rightarrow \tilde{\ell} \tilde{\ell})}{\Gamma(Z \rightarrow e^+ e^-)} = \frac{1}{2} \left( \frac{2p}{M_Z} \right)^3$$

(36)

where $p$ is momentum of the slepton in the Z rest frame. I have assumed that $\tilde{\ell}_L$ and $\tilde{\ell}_R$ are degenerate and have summed. The sleptons will decay and will produce a final state consisting of a lepton pair with unbalanced momentum, provided $\chi$ exits or decays into unobserved particles. There is no published limit from this process since there are, as yet, insufficient produced Z's. The detection of 100 decays of the type $Z \rightarrow e^+ e^-$ should be sufficient to be sensitive to slepton masses less than 35 GeV.

Strongly interacting sparticles, squarks and gluinos are produced from the scattering of quarks and gluons which exist as constituents of the protons. The cross-section for the production of a pair of particles $xx$ is given by

$$\sigma = \sum_i \int dx_1 dx_2 f_i(x_1, Q^2) f_i(x_2, Q^2)\sigma_{ij \rightarrow xx}.$$  

(37)

Where the sum $i$ runs over quarks anti-quarks and gluons and $f_i(x, Q^2)$ is the parton distribution function for parton of type $i$, which are extracted from deep inelastic scattering (Altarelli, 1981). In order to calculate the production rate one must first calculate the partonic cross sections ($\sigma_{ij \rightarrow xx}$). Gluino pairs can be produced from initial states of quark anti-quark or gluon-gluon (Kane and Leveille, 1982; Dawson, Eichten, and Quigg, 1985; Harrison and Llewellyn Smith, 1983).

$$gg \rightarrow \tilde{g} \tilde{g},$$

$$q\bar{q} \rightarrow \tilde{g} \tilde{g}.$$  

Squarks can also be produced in quark quark scattering

$$qq \rightarrow \tilde{q} \tilde{q}.$$  

Finally a squark and a gluino can be produced from an initial state of gluon-quark (Antoniadis, Baulieu, and Delduc, 1984)

$$qq \rightarrow \tilde{q} \tilde{g}.$$  

The initial states with gluons are the most important since the cross sections are larger and the gluon distribution function is bigger than that of quarks over most of the relevant range of $x$. There is some uncertainty in these structure functions and in the value of $\alpha_s$, so it is important to check that the ones being used are reasonable. These checks are performed by comparing the expected yields for W/Z's and hadronic jets with those observed by the UA1 and UA2 collaborations. (Bagnaia et al., 1983, 1984; Arnison et al., 1983).

If the gluino is very light (of order a few GeV in mass) it can be produced in fixed target experiments at FNAL or CERN. The rate of gluino pair production is shown in figure 10. These rates are probably reliable to a factor of 5 or so. The decay of such light gluinos is unlikely to result in a clear direct signal at these low energies. However, the decay of the gluinos which are moving rapidly in the direction of the incident beam will produce a beam of $\chi$'s which may be detected by their interactions downstream. An experiment will then place a limit on the product of the gluino production cross-section and the interaction cross-section of a $\chi$ and a target nucleon. The discussion is model dependent so I will specialize to the case where $\chi$ is a photino which is also the lightest sparticle.

In this case the interaction cross-section is described by figure 4 where the exchanged particle is a squark and is given by (Fayet, 1979b).

$$\sigma_{\chi+N \rightarrow x} = \sum_{\text{quarks}} \int dx f_i(x, Q^2)\sigma_p.$$  

(38)

with

$$\sigma_p = 2 \times 10^{37} E_\chi \left( \frac{m_q}{m_\chi} \right)^4 \frac{e_q^2 x}{(1 - \frac{m_q^2}{2m_p E_\chi})(1 + \frac{m_q^2}{16m_p E_\chi})} \text{cm}^{-2}.$$  

(39)

Here $m_p$ is the proton mass $e_q$ is the charge of a quark of type $q$ and $E_\chi$ is the energy of the incoming photino beam in GeV. The events produced by this interaction will look similar to a neutral current neutrino beam events. The cross-section depends upon the squark mass, so that the experimental limit can be translated into a coupled bound on squark and gluino masses shown in figure 11 (Ball et al., 1984, Bergsma et al., 1983, Cooper-Sarkar, 1985). If the gluino is light enough so that its lifetime is long, it will be scattered in the target before it can decay and the energy of the photino will be degraded. This effect explains the loophole at small gluino masses which is indicated on the figure.
Could a very light gluino have escaped detection elsewhere? If the gluino is lighter than 2 GeV or so it could live long enough to leave a track if the shadron containing it is charged. A model of hadronic binding is required in order to decide whether the charged shadron (made up of $gud$) or the neutral shadron (made up of $gud$) is stable with respect to strong interactions. Bag model calculations indicate that the charged one is stable if the gluino mass is less than 2 GeV (Chanowitz and Sharpe, 1983). Such a charged stable particle should probably have been seen in charm searches in bubble chambers. However, no definitive statement is possible in the absence of a dedicated search. A search for contamination in a neutral beam at FNAL (Gustafson et al., 1976; Appel et al., 1974) also constrains neutral shadrons. A gluino with a lifetime of more than $2 \times 10^{-8}$ seconds and a production rate of more than $20 \mu b$ in proton nucleon collision at $\sqrt{s}$ of 28 GeV is excluded. This constraint excludes the region $m_\tilde{g} \approx 1$ GeV provided $m_\tilde{q} \geq 500$ GeV. It is difficult to believe such a very light gluino could have escaped detection, but precise limits are difficult to set.

I will now discuss the searches in $p\bar{p}$ collisions performed at the SppS collider. The characteristic signature is that of missing energy arising from the decays

$$\tilde{g} \rightarrow q + \tilde{q} + \chi,$$  \hspace{1cm} (40a)

$$\tilde{q} \rightarrow q + \tilde{g},$$  \hspace{1cm} (40b)

$$\tilde{q} \rightarrow q + \chi.$$  \hspace{1cm} (40c)

The precise nature of $\chi$ is not critical. It is usually taken to be a photino, but provided it exits the detector without interacting or decaying the signal is unaffected. The relative branching ratio of the channels 40b and 40c is sensitive to the couplings of $\chi$. While the first channel will dominate if it is open, it produces more hadrons and less missing energy than the decay 40c. Consequently it is less likely to produce events which will pass the cuts discussed below.

In the case of a gluino decay to a quark anti-quark pair, if the gluino mass is large and its momentum small, the two quarks will be well separated and the final state will consist of two jets. As the gluino momentum is increased, the angle between the two jets will be reduced and eventually they will coalesce. The structure visible in the final state also depends critically upon the detector and in particular upon its ability to separate nearby jets and to resolve soft ones. There are a large number of theoretical papers on this subject (Ellis and Kowalski, 1984, 1985; Roy, 1983, 1985; Barger et al., 1985; Allan, Glover and Martin 1984, Tracas and Vlassopulos, 1985; Maintas and Vlassopulos, 1985; Delduc et al., 1985).

I shall base my discussion upon a theoretical analysis (Barnett, Haber, and Kane, 1985) which attempts to compare with the data from the UA1 collaboration. Although I believe that the results of this analysis are a good representation of the supersymmetric limits available from the experiment, I should emphasize that the only people who can really set limits are the experimenters themselves!

The events are required to pass the following cuts.

(a) There be a jet with transverse energy ($E_T$) greater than 15 GeV.

(b) There be at least 15 GeV of missing (unbalanced) $E_T$.

(c) There be no jet within 30° of the missing $E_T$ vector. This cut reduces the background from QCD two jet events where one jet is mismeasured, or from three jet events where one jet is missed.

(d) Nearby jets are merged according to the UA1 jet algorithm.

(e) There is no jet within 30° of a direction opposite to the leading jet. This is again helps to reduce the QCD background.

(f) The average missing energy in a two jet QCD event is determined ($\sigma$), and the event is rejected if the missing $E_T$ is less than 4$\sigma$. This cut is effective only for events which just pass the cut (b).

(g) An attempt has been made to simulate the effects of a fluctuation in the so called minimum-bias background. This is the host of hadrons which are produced with a rather flat rapidity distribution and limited transverse momentum, and are present in all events, irrespective of whether or not they contain jets. There is a problem here since this minimum bias is not well understood and there seem to be more such particles in events with jets than in events without (Rohlf, 1985).

Figure 12 shows contours of the number of events passing these cuts as a function of gluino and squark masses. All squark flavors have been taken to be degenerate. The discontinuity along the line $m_\tilde{q} = m_\tilde{g}$ is caused by the abrupt change in the allowed decay chains.

The cuts are very effective in reducing the predicted number of supersymmetric events. The UA1 collaboration (Rubbia, 1985, Rohlf, 1985) reports a small number of monojets (23 in the 1984 data which corresponds to an integrated luminosity $E_T$ is a two dimensional vector defined in the plane orthogonal to the beam direction.
of 260 nb\(^{-1}\) at 630 GeV) which pass these cuts. Of these, some are due to the decay \(W \rightarrow \tau \nu\); others to the production of jets in association with \(W\)'s or \(Z\)'s, where the \(Z\) decays to neutrinos and the \(W\) to \(e\nu\) with the lepton being missed; or others to the production of heavy quarks. The estimates of (Rohlf, 1985, Ellis, Kleiss, and Stirling, 1985) backgrounds from these sources may account for all the events. It seems that there are fewer than 5 events/100 nb\(^{-1}\) which could be due to supersymmetry.

This appears to exclude squark and gluino masses below 60 GeV. A close examination of the figure reveals the possibility of an allowed region where \(m_{\tilde{g}} \approx 3\) GeV, \(m_{\tilde{q}} \approx 100\) GeV. The possibility of this so-called window for light gluinos has been much discussed (Herrero et al., 1983, 1984; Ellis, and Kowalski 1985; De Rujula and Petronzio, 1985; Kunz and Herzog, 1985). The total cross-section for the pair production of gluinos is very large in this region but very few of the events pass the cuts.* In an event where the gluinos are back to back, they must have large energy in order that there be a jet which can pass the cut (a). On the average the missing transverse momenta cancel so that the events will fail to pass cut (b).

A significant fraction of the events in this region come from the reaction

\[ pp \rightarrow \tilde{g} + \tilde{g} \rightarrow q + \bar{q}. \]

The cross-section is small but the final state readily passes the cuts since it has one hard jet and an energetic photino from the squark decay. In view of the strong dependence upon the cuts which are imposed on theoretical calculations some caution is needed. Nevertheless I think that is extremely unlikely that this window is open in view of the additional constraints from the beam dump searches discussed above.

Once there is evidence for some signal it should be fairly easy to distinguish the sources. For example, if the gluino mass is much larger than the squark mass which is of order 60 GeV, the missing \(E_T\) events will mostly have two jets from

\[ pp \rightarrow \tilde{q} \tilde{q} \rightarrow q + \bar{q} + \chi + \chi. \]

Occasionally one jet will be lost in the beam fragments resulting in a monojet event. Events with three jets and missing \(E_T\) cannot arise directly.

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*Approximately one gluino pair event in \(10^6\) passes the cuts in this region of very small gluino mass.

On the other hand, if the gluino mass is of order 60 GeV and the squark is much heavier, the events will tend to have a higher jet multiplicity, since the decay chain will be

\[ pp \rightarrow \tilde{g} + \tilde{g} + X \]

\[ \rightarrow q + \bar{q} + q + \bar{q} + \chi + \chi. \]

If all the squarks are not degenerate then the limit quoted on the squark mass will be modified. Likewise, if the photino decays to a photon and a Higgsino the missing momentum signature will be diluted. It is not clear whether the UA1 data provide any limit in this case.

(d) Supersymmetry in Cosmology

In the early universe when the temperature \((T)\) was very high all particles with masses much less than \(T\) were in thermal equilibrium. If a stable particle of mass \(m\) remained in equilibrium as the temperature became smaller then its number density would follow the Boltzmann distribution and would became very small at low \(T\)

\[ n \propto e^{-m/4T}. \]  

It's contribution to the current mass density of the universe

\[ \rho = mn \]

will be negligible in this case. The equilibrium is maintained by the annihilation of pairs of particles. If the particle interactions are weak enough it may not be able to maintain equilibrium in which case its number density would be much higher and its contribution to the current mass density could be large. Astrophysical measurements constrain the mass density of the universe so an analysis of the above effect will be able to constrain the mass and interaction rate of any stable particle (Steigman, 1979; Lee and Weinberg, 1977).

If the photino were the lightest sparticle, it would be stable. Photino pairs can annihilate into final states of lepton anti-lepton or quark anti-quark pairs. The cross-section has the following form (Goldberg, 1983; Krauss, 1983).

\[ \sigma = \frac{e^2}{4\pi} \sum_f \frac{Q_f^4}{m_f^2} \left[ \frac{A}{3}(m_{\tilde{g}}^2 - m_f^2)(m_f^2 + m_{\tilde{g}}^2) \right]. \]
Here $m_f$ is the mass of the final state fermion of charge $Q_f$ and $m_j$ is that of the exchanged sparticle and $v$ is the velocity of the photino.

The origin of the two terms on the right hand side of equation 42 can be understood simply. In the limit of large slepton and squark masses, we can write an effective vertex which couples two photinos to a fermion anti-fermion pair as follows

$$\frac{e^2}{2m_j^2} Q_f^2(\bar{\psi}_f \gamma^\mu \gamma^5 \psi_f)(\bar{f} \gamma^\nu \gamma^5 f).$$

(43)

I have assumed that the left and right handed sparticles are degenerate. If the fermion mass is zero then helicity conservation forces the final state to have angular momentum $J = 1$. The photino is a Majorana fermion and so Fermi statistics force the initial state into a p-wave, resulting in an angular momentum barrier which generates the factor of $v^2$. This factor can be avoided if the final state fermion has mass since helicity is no longer conserved, hence the term proportional to $m_f^2$ in equation (42).

If we require that the current density of photinos is less than the density required to close the universe and assume that all squark flavors are degenerate in mass, then a region of squark and photino masses is excluded (see figure 13) (Silk and Srednicki, 1985; Ellis et al., 1984)

$$m_q \gtrsim 1 GeV \quad \text{for} \quad m_\chi \approx 20 GeV,$$

$$m_q \gtrsim 5 GeV \quad \text{for} \quad m_\chi \approx 70 GeV.$$

A very light photino is also allowed.

$$m_\chi \lesssim 100 eV.$$

This latter region corresponds to the situation where the annihilation rate is so low that all the photinos survive to the present time. As the mass increases, the annihilation rate rises and eventually few enough of the photinos survive so that the observed mass density of the universe is not compromised.

If the photino and squark masses are such that the photinos are contributing to the current mass density, then annihilations could still be occurring at a very small rate (Silk and Srednicki, 1985). Reactions of the type

$$\tilde{\chi} \tilde{\chi} \rightarrow \tau^+ \tau^- (\rightarrow e^+ e^- + X)$$

$$\rightarrow \bar{c}c (\rightarrow p + X, e^+ + X)$$

could yield reasonable cosmic ray fluxs of anti-protons, positrons or high energy gamma rays. This could produce a bound which is slightly tighter than that given above.

In the case that $\chi_0$ is a Higgsino, a similar argument yields the constraint $m_{\chi_0} \gtrsim 5 GeV$ or $m_{\chi_0} \lesssim 100 eV$ (Ellis, et al., 1984). In supersymmetric models of the type that I have discussed, it is unlikely that any other sparticle could be the lightest and hence stable (Hinchliffe, 1985).

There is one other particle whose cosmological implications need to be discussed — the gravitino. Let us begin by assuming it is stable. It can annihilate into pairs of particles with a cross-section of order

$$\sigma(\tilde{G} \tilde{G} \rightarrow XX) \sim \frac{1}{M_p^2} m_{3/2}^2.$$  

(44)

This yields an interaction rate which is so slow that essentially all the gravitinos survive to the present time yielding the bound (Weinberg, 1982; Primack and Pagels, 1982)

$$m_{3/2} < 1 KeV.$$

Even if the gravitinos can decay, their lifetime is very long (see equation 15). Assuming a lifetime of $10^8$ seconds, they will decay when the temperature of the universe is of order $10^{-7}$ GeV. Their decay products will eventually result in a gas of $\chi_0$ with approximately the same density as the gravitinos. These decay products will be far from thermal equilibrium and the $\chi_0$ pairs will not be able to find one another in order to annihilate. All the $\chi_0$ will be around today, giving the bound

$$m_{\chi_0} \lesssim 1 KeV.$$

Such late decays also have the potential to disrupt the spectrum of the microwave background radiation (Ellis, Kim, and Nanopoulos, 1985).

A simple solution to this problem is provided by inflation (Guth, 1981, Linde, 1982, Albrecht and Steinhardt, 1984). The gravitinos go out of thermal equilibrium
at very early times when $T \approx 0(M_p)$. In the inflationary picture, at this time, the universe is in a phase where the cosmological constant is non-zero and there is exponential expansion so that the density of gravitinos will rapidly diminish. The universe then undergoes a phase transition to a phase where the cosmological constant is zero. During the transition the universe is reheated to temperature $T_R$ and matter is created. Provided that $T_R < M_P$ gravitinos will not be regenerated with their equilibrium density.

There are two possible sources of gravitinos in the inflationary scenario. They could be produced by the decay of the inflation field itself (Ovrut and Steinhardt, 1984). These gravitinos are not produced with a thermal distribution and have no disastrous effects. A small number of gravitinos can be produced via scatterings of other particles. The resulting density depends upon the mass of the gravitino and upon $T_R$. The gravitino will subsequently decay, but the resulting mass density of $\chi_0$ is too low to yield an interesting constraint.

A more interesting constraint comes from considering the fate of the decay products of the gravitino (Khlopov and Linde, 1984; Ellis, Nanopoulos and Sakar, 1985). If the gravitino is heavy enough it will decay into strongly interacting particles via

$$\tilde{G} \rightarrow \tilde{g} + g$$

$$\rightarrow \tilde{g} + q.$$ 

In this case, the ultimate decay products will include anti-protons. We would expect to obtain of the order of one anti-proton per decay. (Recall that there is approximately one anti-proton per hadronic event seen at PEP.) (Aihira et al., 1985.) Even if these decay modes are not available because the gravitino is too light, the decay

$$\tilde{G} \rightarrow \tilde{\gamma} + \gamma$$

$$\rightarrow e + \bar{\epsilon}$$

will generate final state photons. The number of photons per decay and their energy spectrum is not easy to obtain. A full shower Monte-Carlo is required (Ellis, Nanopoulos and Sakar, 1985).

The produced anti-protons and photons are able to initiate the break up of nuclei through reactions of the type

$$\bar{p} + ^{4}He \rightarrow d + n$$

$$\rightarrow ^{3}H + \gamma$$

$$\rightarrow ^{3}He + e^- + \nu.$$ 

The abundance of $^4He$ will be reduced while that of $^3He$ and deuterium will be increased, and the successful prediction of the Helium abundance in the hot big bang model will be lost (Schramm and Wagner, 1977). The universe is so cool that subsequent destruction of large amounts of $^3He$ is not possible. Consequently the tight limit on the observed amount of $^3He$ can be used to bound the number of decaying gravitinos. Requiring the density of $^3He$ to be less than $10^{-4}$ of that of $^4He$ gives $T_R \lesssim 10^8$ GeV for a gravitino mass of 100 GeV.

The tight constraint on the reheating temperature could be avoided if the gravitino were heavier than $10^6$ GeV so that it could have decayed before nucleosynthesis. Alternatively, if the gravitino decay released enough energy so that all the helium were destroyed and nucleosynthesis restarted, there would be no problem. This occurs if the gravitino mass is larger than $10^4$ GeV (Weinberg, 1982; Primack and Pagels, 1982). Gravitinos lighter than about 10 MeV will have survived to the present time and will dominate the mass of the universe. We can conclude, therefore, that if we require a reheat temperature greater than $10^{10}$ GeV and a successful Big Bang cosmology, gravitinos in the mass range 1 KeV to $10^4$ GeV are excluded.

Why are we so interested in the reheat temperature? The conventional mechanism for generating the baryon asymmetry (Kolb and Turner, 1983) of the universe relies upon the decay of superheavy gauge bosons and Higgs particles in a Grand Unified theory. The mass of these particles is of the same order as the unification scale, $M_G \sim 10^{14}$ GeV. As the universe cools through temperatures of this order, these particles go out of thermal equilibrium. Baryon and CP invariance are broken by their interactions so a net baryon asymmetry can be generated. It is one of the successes of Grand Unification that the required baryon to entropy ratio of order $10^{-11}$ can be generated in this way.

After the universe has inflated and reheated, the superheavy gauge bosons and Higgs bosons cannot reach thermal equilibrium unless $T_R \sim M_G$. In view of the constraints discussed above we must give up on this conventional mechanism if we wish to have gravitino mass of order $M_W$. Several alternate mechanisms for generation baryon number have been suggested. The decay of particles with masses
less than $10^8$ GeV is one option (Masiero and Yanigida, 1982; Claudson, Hall and Hinchliffe, 1984; Kosower, Hall, and Krauss, 1985). Models based on this idea have been constructed but they are very ugly. A better alternative is for the superheavy gauge bosons to be produced during the phase transition from the inflationary phase or by the decay of the scalar field responsible for inflation, the inflaton (Coughlan et al., 1985; Holmon, Ramond, and Ross, 1984; Mahajan, 1985).

4. Conclusion

The limits on the masses of supersymmetric particles are summarized in Table 2. In view of the good limits from the SppS collider, models with radiative gaugino masses in which $m_{\tilde{q}} > m_{\tilde{q}}$ are disfavored, since they would require very large squark masses. Recall that the squark masses cannot be much larger than the scale of electroweak symmetry breaking if supersymmetry is to be relevant to the hierarchy problem (Hall and Polchinski, 1985; Nandi, 1985; Ellis and Sher, 1984).

Where can we look for a significant step in the search for supersymmetry? The searches discussed in section 3(b) can be carried out at the next generation of $e^+e^-$ machines, LEP and SLC. Particles with electro-weak charges and masses less than the beam energy should be produced copiously enough for a discovery to be made. Using the same techniques as I discussed in section 3(c), the Tevatron collider with center of mass energy of 2 TeV should be sensitive to squark and gluino masses less than about 140 GeV. Squarks and selectrons can be produced in $ep$ collisions at HERA. The production rates at this machine are not large and discovery will be difficult given the existing limits (Cashmore et al., 1985; Hinchliffe, 1985). The SSC, a proposed high luminosity, proton-proton collider with center-of-mass energy of 40 TeV should be capable of searching for supersymmetric particles with masses in excess of 1 TeV (Eichten et al., 1984; Dawson and Savoy-Navarro, 1984).

Despite some false alarms, we still have no experimental evidence in favor of supersymmetry. Should we be discouraged? Probably not, since, as I indicated in section one, the natural mass scale for the superpartners is the W mass and searches have not yet reached this value. We are getting close however, and something has to show up soon. I hope that the extra energy range opened up by the Tevatron collider will prove decisive, and that we do not have to wait for the SSC. Suppose nothing is found, when should theorists give up? The mass range accessible at the SSC is so large that if it fails to find supersymmetry we can safely assume that supersymmetry is not relevant to the hierarchy problem, and that all the currently fashionable supersymmetric models are wrong.

I will conclude this review with some more theoretical remarks. In section 1 I allowed to the possibility that supersymmetric theories are motivated in part by the desire to have a unified theory of all the interactions, including gravity. It has recently been realized that the superstring theory (Schwartz, 1982; Green, 1985) may be a candidate for such a theory of everything. The theory exists in a 10 dimensional world. Six of the dimensions are compactified and have size...
$\sim 10^{-43}$ cm. They cannot therefore be probed in current (or future) experiments. When the theory is compactified it gives rise to an effective 4-dimensional theory in which, in principle, all of the coupling constants and masses as well as the number of quarks and leptons is predicted. An $N = 1$ supersymmetry remains unbroken by this compactification and hence the low energy theory ($\sim M_W$) has the same features as the supersymmetric models I have discussed. A set of fields exists which can provide a 'hidden sector' and be responsible for the breaking of the supersymmetry in the manner discussed in section 1. In addition to all the supersymmetric particles, the string theories predict the existence of new gauge bosons and quarks. As yet, none has been able to extract predictions for masses of quarks and leptons from these string theories, but they have provided more encouragement to the idea that some supersymmetric theory may really be the theory of everything!

Acknowledgments.

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Figure Captions

Figure 1 Feynman diagram showing a contribution to a scalar mass from the self interaction of a scalar field.

Figure 2 Feynman diagram showing a contribution to a scalar mass from the interaction of a scalar (dashed line) and a fermion (solid line).

Figure 3 Diagrams showing contributions to gluino masses from a loop of quarks and squarks, the suffices $L$ and $R$ refer to chirality states.

Figure 4 Diagram showing the scattering of $\chi_0$ states from a nucleon.

Figure 5 Feynman diagram showing a contribution to the muon (g-2) from smuon loops. There are other contributions involving winos.

Figure 6 Feynman graphs showing a contributions to electron proton scattering which results in a different cross-section for left and right and right handed electrons.

Figure 7 Diagram showing a contribution to a parity violating interaction between quarks.

Figure 8 Diagrams showing supersymmetric contributions to the $K_L - K_S$ mass matrix.

Figure 9 The excluded region in photino-selectron masses from the non-observation of the process of $e^+ e^- \rightarrow \gamma \gamma \tilde{e}_L$ and $\tilde{e}_R$ are assumed to be degenerate.

Figure 10 The cross-section for producing a pair of gluinos in proton proton collisions at low energy. The dependence upon $m_q$ is slight, it has been set to 50 GeV.

Figure 11 The excluded region in squark and gluino masses arising from the beam dump experiments discussed in the text.

Figure 12 Contour plot showing the number of missing transverse energy events per 100 nb$^{-1}$ of luminosity in $pp$ collisions at 630 GeV. The cuts are described in the text. Figure courtesy of M. Barnett. All the squark flavors are assumed to be degenerate in mass.

Figure 13 Figure showing the region of squark and photino masses which is allowed by the cosmological considerations of section 3(d). All squarks and sleptons are taken to be degenerate and the photino is assumed to be stable.
Table 1

<table>
<thead>
<tr>
<th>Particle</th>
<th>decay mode</th>
<th>lifetime sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{e} )</td>
<td>( e + \tilde{\gamma} )</td>
<td>( 2 \times 10^{-22} \frac{m_\tilde{\gamma}^3}{(m_\tilde{\gamma}^2 - m_\tilde{e}^2)^2} \left( \frac{1}{a^2} \right) )</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( \nu + \tilde{\nu} )</td>
<td>( 6 \times 10^{-24} \frac{m_\tilde{\nu}^3}{m_\tilde{\nu}^2 - m_\nu^2} \left( \frac{1}{a^2} \right) )</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( \nu + \tilde{\nu} )</td>
<td>( 3 \times 10^{-22} \frac{m_\tilde{\nu}^3}{(m_\tilde{\nu}^2 - m_\nu^2)^2} \left( \frac{1}{a^2} \right) )</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( \nu + \tilde{\nu} )</td>
<td>( 6 \times 10^{-23} \frac{m_\tilde{\nu}^3}{(m_\tilde{\nu}^2 - m_\nu^2)^2} \left( \frac{1}{a^2} \right) )</td>
</tr>
<tr>
<td>( \tilde{\gamma} )</td>
<td>( \tilde{\nu} + \tilde{\gamma} )</td>
<td>( 4 \times 10^{-23} \frac{m_\tilde{\gamma}^3}{(m_\tilde{\gamma}^2 - m_\tilde{\nu}^2)^2} \left( \frac{1}{a^2} \right) )</td>
</tr>
<tr>
<td>( \tilde{\gamma} )</td>
<td>( \tilde{\nu} + \tilde{\gamma} )</td>
<td>( 10^{-11} \frac{m_\tilde{\gamma}}{M_W} \left( \frac{1}{m_\tilde{\gamma}} \right) \left( \frac{1}{a^2} \right) )</td>
</tr>
</tbody>
</table>

Table Caption

Lifetime estimates for sparticle decays. Quarks and lepton masses are neglected. If the photino is not the lightest state a reasonable approximation is obtained by including the factor in parentheses. The term \( a \) is given by equation (20). All masses are GeV.

Table 2

<table>
<thead>
<tr>
<th>Particle</th>
<th>Excluded Mass Range</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{e}, \tilde{\nu}, \tilde{\gamma} )</td>
<td>( &lt; 20 \text{ GeV} )</td>
<td>Valid provided decay is ( \tilde{e} \rightarrow \ell + \chi_0 ) or ( \tilde{\nu} \rightarrow \ell + \gamma + \chi_0 )</td>
</tr>
<tr>
<td>( \tilde{e} )</td>
<td>( &lt; 50 \text{ GeV} )</td>
<td>Valid if ( m_\tilde{e} ) is small and ( \tilde{\gamma} ) is an eigenstate of mass.</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( &gt; 60 \text{ GeV} )</td>
<td>( \tilde{\nu} \rightarrow \tilde{\nu} + \chi ) or ( \tilde{\nu} \rightarrow \tilde{\nu} + q )</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( &gt; 60 \text{ GeV} )</td>
<td>( \tilde{\nu} \rightarrow \tilde{\nu} + \tilde{\nu} + \chi ) or ( \tilde{\nu} \rightarrow \tilde{\nu} + \tilde{\nu} + \chi ) ( \chi ) must be long lived.</td>
</tr>
<tr>
<td>( \tilde{\gamma} )</td>
<td>( 100 \text{ ev} &lt; m &lt; \text{ few GeV} )</td>
<td>Valid if ( \tilde{\gamma} ) stable</td>
</tr>
<tr>
<td>( \tilde{\tilde{H}} )</td>
<td>( 100 \text{ ev} &lt; m &lt; \text{ few GeV} )</td>
<td>Valid if ( \tilde{\tilde{H}} ) stable</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>( 100 \text{ ev} &lt; m &lt; 10 \text{ MeV} )</td>
<td>Valid if ( \tilde{\nu} ) stable</td>
</tr>
</tbody>
</table>

Table Caption

Limits on sparticle mass from the processes discussed in sections 2–5. This table assumes that all squarks masses are approximately equal, that \( R \) parity is not broken.

Appendix

This appendix gives the renormalization group equations which determine the evolution of the masses and coupling constants\textsuperscript{14,14} discussed in section one. For simplicity I will assume that the superpotential contains only two terms viz

\[ W = \lambda Q u H_2 + \mu H_1 H_2 \]  \( (A1) \)

where \( Q \) is a quark doublet, \( U \) is a right handed quark and \( H_1 \) and \( H_2 \) are Higgs doublets: I will further assume that the supersymmetry is broken in the appearance of the following terms. Scalar masses:
\[ m_Q^2 |Q|^2 + m_{H_1}^2 |H_1|^2 + m_u^2 |u|^2 + m_{\tilde{H}_3}^2 |H_2|^2. \]  
(A2)

Soft Operators:

\[ A\lambda m Qu H_2 + B_{um} H_1 H_2 + h.c. \]  
(A3)

The scale \( m \) has been introduced so that \( B \) is dimensionless.

Gaugino masses:

\[ \frac{1}{2} M_3 \tilde{\psi}_3 \tilde{\psi}_3 + \frac{1}{2} M_2 \tilde{\psi}_2 \tilde{\psi}_2 + \frac{1}{2} M_1 \tilde{\psi}_1 \tilde{\psi}_1. \]  
(A4)

Here \( B \) is the gauge boson associated with the group \( U(1)_Y \). If we assume that there are 3 generations of quarks and leptons, then the evolution of the gauge coupling constants \( \alpha_i \) is given by

\[ \frac{d}{dt} \alpha_i(t) = \frac{b_i}{2} \alpha_i^2(t) \]  
(A5)

where \( t = \log Q^2 \) and \( b_1 = \frac{33}{5}, \ b_2 = 1 \) and \( b_3 = -3 \). I have assumed that there are 3 generations of quarks and leptons. Note that the coupling constant \( \alpha_1 = g_1/4\pi \) is normalized so that if the theories grand unified at scale \( M_G \).

\[ \alpha_i(M_G) = \alpha_G \]

for all \( i \). Hence \( \frac{3}{5} g^2 = g_1^2 \). The gaugino masses evolve according to

\[ \frac{d}{dt} \left( \frac{M_i}{\alpha_i} \right) = 0. \]  
(A6)

The Yukawa coupling \( \lambda \) evolves according to

\[ \frac{d\lambda}{dt} = \frac{\lambda}{4\pi} \left[-\frac{16}{3} \alpha_2 - 3 \alpha_2 - \frac{13}{15} \alpha_1 + \frac{6\lambda^2}{4\pi}\right]. \]  
(A7)

The other superpotential parameter \( \mu \) evolves according to

\[ \frac{d\mu}{dt} = \frac{\mu}{4\pi} \left[-3 \alpha_2 - \frac{3}{5} \alpha_1 + \frac{3\lambda^2}{4\pi}\right]. \]  
(A8)

Finally the scalar masses

\[ \frac{d}{dt} \left( \frac{m_i^2}{m_Q^2} \right) = -\frac{2}{\pi} \sum_i \alpha_i T_{ai}^2 M_i^2 + \frac{\lambda^2}{8\pi^2} \left( \frac{3}{1} \right) (m_{H_3}^2 + m_u^2 + m_{\tilde{q}}^2 + A^2), \]  
(A11)

\[ \frac{d}{dt} \left( \frac{m_{H_2}^2}{m_Q^2} \right) = -\frac{2}{\pi} \sum_i T_{ai}^2 \alpha_i M_i^2. \]  
(A12)

Here \( T_{ai}^2 = \sum \Delta \Delta T_{ai}^2 \) is the quadratic Casimir of the \( ai \) scalar with respect to the \( i \)th gauge group. As a consequence of equation (A11), models which have large gaugino masses at \( M_G \) tend to have squark masses where are comparable at low energy. The equations given in this appendix are valid to lowest order in \( \alpha_i \lambda^2 \) and \( B \).

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