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NUCLEAR SPINS OF (45 MIN) Cs125 AND (13 DAY) Cs136; HYPERFINE-STRUCTURE SEPARATIONS AND NUCLEAR MAGNETIC MOMENTS OF CsI25, CsI36 AND (6.2 HR) Cs127

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NUCLEAR SPINS OF (45 MIN) Cs $^{125}$ AND (13 DAY) Cs $^{136}$; HYPERFINE-STRUCTURE SEPARATIONS AND NUCLEAR MAGNETIC MOMENTS OF Cs $^{125}$, Cs $^{136}$, and (6.2 HR) Cs $^{127}$
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NUCLEAR SPINS OF (45 MIN) $^{125}$Cs AND (13 DAY) $^{136}$Cs; HYPERFINE-STURCTURE SEPARATIONS AND NUCLEAR MAGNETIC MOMENTS OF $^{125}$Cs, $^{136}$Cs, AND (6.2 HR) $^{127}$Cs

Osama Bashir Dabbousi
(Ph. D. Thesis)

July 19, 1966
To The Memory of My Father

MOHAMMAD BASHIR DABBOUSI

without whose help and encouragement I would not have reached
the stage where I could write this thesis
NUCLEAR SPINS OF (45 MIN) Cs$^{125}$ AND (13 DAY) Cs$^{136}$; HYPERFINE-STRUCTURE SEPARATIONS AND NUCLEAR MAGNETIC MOMENTS OF Cs$^{125}$, Cs$^{136}$, AND (6.2 HR) Cs$^{127}$

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HYPERFINE-STRUCTURE SEPARATIONS AND NUCLEAR MAGNETIC
MOMENTS OF Cs$^{125}$, Cs$^{136}$, AND (6.2 HR) Cs$^{127}$

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ABSTRACT

The nuclear spin and hyperfine-structure separation of (45 min) $^{125}$Cs and (13 day) $^{136}$Cs were measured. A more accurate value for the hyperfine-structure separation of (6.2 hr) $^{127}$Cs was obtained by a least-squares fit of the previously observed resonances and the resonances we observed. The nuclear-magnetic moments of these isotopes were calculated from the hyperfine-structure separation with the use of the Fermi-Segrè formula. The results are

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half-life</th>
<th>Spin</th>
<th>$\Delta\nu$ (MHz)</th>
<th>$\mu_I$(uncorr) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{125}$Cs</td>
<td>(45 min)</td>
<td>$1/2$</td>
<td>8.754(40)</td>
<td>+1.40(2)</td>
</tr>
<tr>
<td>$^{127}$Cs</td>
<td>(6.2 hr)</td>
<td>$(1/2)$</td>
<td>9.109(45)</td>
<td>+1.45(2)</td>
</tr>
<tr>
<td>$^{136}$Cs</td>
<td>(13 d)</td>
<td>$5$</td>
<td>12.702(28)</td>
<td>+3.68(4)</td>
</tr>
</tbody>
</table>

The isotopes $^{125}$Cs and $^{127}$Cs were obtained by the reactions $^{127}$I($\alpha$,6n)$^{125}$Cs and $^{127}$I($\alpha$,4n)$^{127}$Cs on elemental iodine. Barium iodide was also used as a target material in the early runs. Iodine or the barium iodide were separated out chemically after bombardment. Plots of the ratios of the activities obtained by bombarding iodine with $\alpha$-particles are given for the $\alpha$-particle energy range from 63 to 118 MeV. The isotope $^{136}$Cs was produced by the reactions $^{136}$Xe(d,2n)$^{136}$Cs and $^{136}$Xe(p,n)$^{136}$Cs on xenon gas with natural isotopic abundance.

The activities collected on the buttons during a run were normalized by one of two methods: (1) a hot wire monitored the resonance of the stable Cs carrier, and (2) a side button collected the nonresonance atoms. Both procedures give a relative integrated flux of the beam from the oven during the exposure of a button.
Explanation of the nuclear spins and magnetic moments of the cesium isotopes studied is attempted in the light of present nuclear theories. A brief survey of the nuclear shell model, its extension to deformed nuclei, and the use of pairing plus quadrupole forces for the interaction between the nucleons in the framework of superconductivity theory is given.

The theory of the experiment, experimental procedure, modification of the apparatus, targets used for the production of the isotopes, and chemical separation of the isotopes are also discussed.
I. INTRODUCTION

Few elements have been studied more accurately or more often by atomic-beam methods than the cesium isotopes have. The stable isotope $^{133}\text{Cs}$, for example, was one of the earliest isotopes to be studied through the use of the atomic-beam method. Our measurements of $^{125}\text{Cs}$ and $^{136}\text{Cs}$ bring the total number of cesium isotopes studied by this method to twelve. The ground-state spins and nuclear magnetic moments of the nuclei, determined through these measurements, and some later information about the excited levels of some of the cesium isotopes present a wealth of information for the study of nuclear structure.

In Sec. II we attempt to explain the ground-state spins and magnetic moments of the cesium isotopes studied. We start by giving a brief survey of the nuclear models that were used in the study of cesium nuclei similar to the ones we are considering. These models are applied to our isotopes with a brief discussion of discrepancies. Section III is devoted to the theory of the experiment, the experimental apparatus, and the modifications thereof. In Section IV the isotope production, preparation, data reduction, and the experimental results are treated. Section V is devoted to a general discussion of the sign of the quadrupole moment for the neutron-deficient isotopes.

The value for the hyperfine-structure separation of $^{127}\text{Cs}$ quoted here was obtained by a least-square fit of the resonances we observed and the previously observed resonances—which were used in the determination of the previously quoted value.$^{1,2}$
II. NUCLEAR STRUCTURE

Our measurement of the ground-state spin and magnetic moments of $^{125}$Cs and $^{136}$Cs brings the total number of cesium isotopes studied to twelve. These isotopes have 55 protons and between 70 and 83 neutrons. The $^{125}$Cs and $^{136}$Cs fall near opposite ends of this chain. The explanation and possibly the prediction of their spins and magnetic moments fall into two seemingly distinct categories—those of $^{136}$Cs can be explained rather easily through the use of a simple shell model and proton-neutron residual interaction. Those of $^{125}$Cs, on the other hand, require the use of Nilsson's version of the unified model. A model that has the capability of reproducing the shell model and Nilsson's model can treat the isotopes studied on an equal footing. Such a model is usually constructed through the use of pairing plus quadrupole interactions in a shell-model picture. This model uses the treatment and notation of the theory of superconductivity; thus we shall refer to it as the superconductivity model.

A short survey of the models that have been used in the study of cesium nuclei similar to the ones we measured—namely $^{127}$Cs, $^{129}$Cs, and $^{131}$Cs—is given in this section. We hoped that this survey would clarify the terms used in explaining the observed spins and magnetic moments, and make it possible for us to see what could be expected of any of these models.

A. Shell Structure of the Nucleus

A theory describing the nucleus would be capable of finding (Ref. 3): (a) the energy eigenvalues and eigenstates of the nucleus—particularly the ground state, and (b) such properties of these states as the spin, multipole moments, and transition probabilities. Many models have been formulated to perform either or both of these tasks. The nuclear-shell model as formulated by Mayer and Jensen, its extension to deformed nuclei, and lately the inclusion of interactions between the nucleons, have been most successful in correlating theory with experimental data.

The nuclear-shell model's treatment of the nucleons follows in many respects the treatment of the electrons in the theory of atomic structure.
Thus in the atomic case the Hamiltonian is known, yet the Schrödinger equation

\[ \hat{H}\psi = E\psi \]  \hspace{1cm} (II.1)

cannot be solved for more than one electron. In the nuclear case, on the other hand, only some properties of the Hamiltonian are known. Thus in both cases approximate procedures have to be used in solving the Schrödinger equation. The usual procedure is to choose a set of orthonormal functions

\[ \psi_a, \psi_b, \psi_c, \ldots \]  \hspace{1cm} (II.2)

which are as near as possible to the eigenfunctions of the Schrödinger equation (II.1). The perturbation procedure requires that we find the matrix elements

\[ \langle a|\hat{H}|b \rangle = \int \psi_a^{\ast} \hat{H} \psi_b \, d\tau . \]  \hspace{1cm} (II.3)

In the atomic case, the basis functions (II.2) are obtained by approximating the interaction with a central potential. The difference between the two Hamiltonians is taken as the perturbing Hamiltonian. It includes terms accounting for the interaction between the electrons and spin-orbit interaction, as well as smaller terms. A Hartree-Fock (self-consistent field) calculation is then performed to obtain the solutions to the Schrödinger equation.

The shell model also assumes the nucleons to be moving in a central potential, due to the other nucleons. This assumption is justified by the fact that the Pauli exclusion principle prevents local fluctuation in the potential.\(^9\)\(^,\)\(^10\) Since two nucleons cannot occupy the same state at the same time, a nucleon has to move rather independently because there are not many states it can scatter into. However, no self-consistent field calculation has been performed for the nucleus due to the difficulty of treating the hard repulsive core which is necessary to avoid the collapse of the nucleus.
1. The Nuclear-Shell Model

The shell-model assumptions reduce the usual Hamiltonian for a system of particles interacting with a two-body force

\[ H = \sum_i T_i + \sum_{i>j} V_{ij} \]  

(ii.4)

into the form

\[ H = \sum_i \left[ T_i + U(r_i) \right] - \sum_i C(r_i) \cdot \vec{s}_i + \sum_{i>j} V_{\text{resid}}^{i,j} \]  

(ii.5)

where \( T_i = P_i^2/2M \) is the kinetic energy of a nucleon of effective mass \( M \). The term \( U(r_i) \) is an isotropic central potential in which the nucleons move. This potential is usually taken to be between a three-dimensional attractive square well and an isotropic harmonic oscillator--a square well with rounded corners. The residual interaction \( V_{\text{resid}}^{i,j} \) represents the interactions that cannot be accounted for by the average potential.

The total wave function for a nucleus with \( n \) particles, with the particles chosen as of one kind, is a product of single-particle states

\[ |a_1(1)|a_2(2)| \ldots |a_n(n) \rangle , \]  

(ii.6)

where \( |a_i(k)\rangle \) represents the state vector of a nucleon in the state specified by the quantum numbers \( a_i \) and at the coordinates \( x_k \). The single-nucleon quantum numbers appropriate to the central potential used and the strong spin-orbit coupling are \( n_i l \parallel j_i \). The degeneracy of such a configuration is \( 2j_i + 1 \). The different states in this configuration can be specified by the projection of the individual \( j_i \)'s on a z-axis, \( m_j \), or \( a_i = n_i l \parallel j_i m_j \). The Pauli exclusion principle requires that the total wave function be antisymmetric under the exchange of the spatial and spin coordinates of any two particles. Or

\[ \Psi = \sum_{\mathcal{P}} (-)^{\mathcal{P}} \mathcal{P}|a_1(1)a_2(2)\ldots a_n(n)\rangle , \]  

(ii.7)

\( \mathcal{P} \) being a permutation operator. The normalized antisymmetrized product is a "Slater determinant": 
The matrix element in Eq. (II.3) will have to be taken between these determinantal products. This can be done easily since all terms in the Hamiltonian, except the residual interaction, are single-particle operators, of the form \( F = \sum \frac{p_i^2}{2M} \). The residual interaction is a two-particle operator -- \( G = \sum_{i>j} \beta_{ij} \). The matrix elements can be expressed in terms of single-particle matrix elements. The expectation value of the first two terms in the Hamiltonian (II.5) depends on the set of quantum numbers \( \{n_i \ell_i\} \):

\[
\langle \sum \frac{p_i^2}{2M} + U(r_i) \rangle = \sum \frac{p_i^2}{2M} + U(r_i) \rangle_{n_i \ell_i} = \sum E_o(n_i \ell_i).
\] (II.9)

The inclusion of spin-orbit interaction renders the expectation values dependent on \( j \). The expectation value is

\[
\langle \sum \frac{T_i \cdot s}{2} + U(r_i) - C(r_i)\vec{r}_i \cdot \vec{s}_i \rangle = \sum E_o(n_i \ell_i j_i) = \sum \{E_o(n_i \ell_i) - \xi(n_i \ell_i)\langle n_i \ell_i j_i \mid \vec{r}_i \cdot \vec{s}_i \mid n_i \ell_i j_i \rangle \}.
\] (II.10)

In this representation

\[
2 \vec{l} \cdot \vec{s} = (\vec{l} + \vec{s})^2 - \vec{l}^2 - \vec{s}^2
\]

\[
= j^2 - \vec{l}^2 - \vec{s}^2
\]

\[
= j(j+1) - \ell(\ell+1) - s(s+1)
\]

\[
= \begin{cases} 
- (\ell+1) & \text{for } j = \ell - \frac{1}{2} \\
\ell & \text{for } j = \ell + \frac{1}{2}
\end{cases}
\] (II.11)

The splitting of the levels due to spin-orbit interaction is proportional to \((2\ell + 1)\). This causes the level with the larger \( j \), \( j = \ell + \frac{1}{2} \), to lie
lower than the level with \( j = \ell - \frac{1}{2} \). Figures II.1 and II.2 show the shell-model energy levels before and after the inclusion of spin-orbit interaction.

A full configuration, \( j^{2j+1} \), has a total angular momentum \( J = 0 \). The interaction of a single particle with a closed shell can be shown to be equivalent to an interaction with a central field.\(^{12}\) Thus, we need concern ourselves only with particles outside closed shells in studying the properties of the nucleus.

The residual interaction term in the Hamiltonian has been treated in one of two ways: (a) by replacing it with coupling rules for the nucleons (single-particle shell model), and (b) by assuming different forces for the interaction and solving the problem (many-particle shell model).

The coupling conditions set by the single-particle model can be presented as: (a) the spins of the ground state of even-even nuclei are equal to zero, (b) the properties of odd-\( A \) nuclei are determined by the group of odd nucleons (the spin of an odd-\( A \) nucleus equals the angular momentum of the last odd nucleon), and (c) the pairing energy has a magnitude that increases with angular momentum \( j \). The conditions were postulated to establish agreement with empirical data.

In the many-particle shell model, the residual interaction is expected to account for the coupling rules of the single-particle model and the observed pairing energies. This interaction should, therefore, have a pairing character while being able to mend some of the deficiencies of the single-particle model.

The residual interaction, being a two-body interaction, can have nonvanishing matrix elements between particles in different orbits, thus causing configuration mixing. Arima and Horie\(^{13,14}\) have treated the problem of residual interaction and configuration mixing to account for the deviations of the nuclear multipole moments from single-particle values. In their treatment, the residual interaction is a sum of singlet and triplet interactions of zero range.

The ground-state wave function is taken as the single-particle shell-model ground state with a small admixture of higher states:
Fig. II.1 Level scheme for the nuclear-shell model, intermediate between a three-dimensional isotropic harmonic oscillator (left) and an infinite square well (right); spin-orbit interaction not included (after Mayer and Jensen, Ref. 7).
Fig. II.2 Shell-model single-particle energy levels (spin-orbit and Coulomb energy terms included).
The mixing parameter $\alpha_n$ can be obtained from perturbation theory,

$$\alpha_n = - \langle n | \sum_{i>j} v_{ij}^{\text{resid}} | 0 \rangle / \Delta E_n ,$$

where $\Delta E_n$ is the energy difference between the states. The expectation value of a Hermitian operator, $\mathcal{O}$, to first order in $\alpha_n$, can be written as

$$\langle \mathcal{O} \rangle = \langle 0 | \mathcal{O} | 0 \rangle - 2 \sum_n \frac{\langle 0 | \mathcal{O} | n \rangle \langle n | v_{ij}^{\text{resid}} | 0 \rangle}{\Delta E_n} .$$

Noya, Arima, and Horie listed some of the configurations that contribute most to the corrections, and with these configurations they got rather good agreement with the observed moments, subject to some limitations.

The calculation of matrix elements entailed the use of a sum of single-particle matrix elements—Slater's diagonal sum method. This method has been superseded by the method of fractional parentage originated by Bacher and Goudsmit and developed by Racah. This latter method calls for the construction of an antisymmetric state of the configuration $j^n(J)$ as a sum of antisymmetric states of smaller configuration $j^{n-1}j$ or $j^{n-2}j^2$. The states representing a configuration $j^n(J)$ will be labeled by another quantum number $\alpha$ to account for other possible antisymmetric states, of the configuration $j^n$, that have the same value $J$; a state can then be represented by $| j^n \alpha \rangle$.

$$| j^n \alpha \rangle = \sum_{\alpha_j} \langle j^{n-1} \alpha_j J_1, j, J | j^n \alpha \rangle | j^{n-1} \alpha_j J_1, j, J \rangle$$

$$| j^n \alpha \rangle = \sum_{\alpha_j} \langle j^{n-2} \alpha_j J_2, j^2 J', J | j^n \alpha \rangle | j^{n-2} \alpha_j J_2, j^2 J', J \rangle .$$

The coefficients, called fractional parentage coefficients in Eq. (II.15a) and grand-parentage coefficients in Eq. (II.15b), describe how the state can be built up from its possible parents. The transformation equations (II.15a,b) can be thought of as projection operations similar to those
encountered in vector algebra. The coefficients in Eq. (II.15) are related by:

$$\langle j^{-n}a_1^-j_1^2,j^2j',j|j^n\alpha j \rangle = \sum_{j_1^1} \langle j^{-n}a_2^-j_2^2,j,j|j^{n-1}\alpha_1^-j_1^1 \rangle$$

$$\times \langle j^{n-1}\alpha_1^-j_1^1,j,j|j^n\alpha j \rangle \sqrt{(2j_1^1+1)(2j'+1)} \{-\}^{j_2^-j^-2j} \left\{ \begin{array}{ccc} j_2^- & j & j_1^1 \\ j & j & j' \end{array} \right\} . \quad (II.16)$$

The matrix elements of a two-body interaction can be written with the use of Eq. (II.15b) (Ref's. 12, 17) as

$$\langle j^n\alpha j| \sum_{i<k} g_{ik} |j^n\alpha' j'M \rangle = \frac{n(n-1)}{2} \sum_{j_2^2} \langle j^n\alpha j|j^{-n}a_2^-j_2^2,j^2j',j \rangle$$

$$\times \langle j^{n-2}a_2^-j_2^2,j^2j',j|j^n\alpha' j\rangle |j^2j'M'|e_{n-1,n}|j^2j'M' \rangle . \quad (II.17)$$

Equation (II.17) reduces the calculation of the matrix elements in an n-particle configuration to these in a two-particle configurations, and we are left with the problem of evaluating the coefficients of fractional parentages (cfp) used in the equation.

A state of the configuration can be constructed by adding \( \frac{1}{2}(n-v) \) "saturated pairs" (pairs coupled to zero angular momentum) to the configuration \( j^v \). Should the configuration \( j^v \) not contain any saturated pairs, the new state is said to have a "seniority" \( v \). Saturated pairs have no odd-order multipole moments and their effect on even-order multipole moments can be accounted for by a depolarization factor---\( -\langle j^2j-n \rangle/(j^2j-v) \). The concept of seniority simplifies the calculation of cfp, since only parentages of the states with \( v = n \) need be calculated--the others are easily deducible from them. A very extensive treatment of the many-particle shell model with emphasis on tensor forms of the residual interaction can be found in Ref. 17. Finally, note that there are some objections concerning the concept of seniority---"its use is not dictated by internal necessity."18

The above treatment dealt with particles of one type only. Thus, it was bound by the Pauli exclusion principle. The extension of this treatment to different particles relaxes some of the limitation of the
Pauli principle. Isotopic-spin quantum numbers can be used to describe a state in a similar manner to the use of angular momentum. This such use enables us to treat protons and neutrons in the same manner. However, for the heavier nuclei such as cesium the isotopic spin quantum number is not a constant of the motion due to the Coulomb interaction.

2. Deformed Nuclear Shape

The observed quadrupole moments, of odd-A nuclei in the rare-earth region, and electric quadrupole radiation (E2) transition rates, in even-even nuclei, are considerably larger than those predicted by the shell model. The discrepancy is larger for nuclei farther away from closed shells. Models were formulated to counteract this deficiency. One such model, postulated by Rainwater,\textsuperscript{19} pictures the nuclear core as being deformed by an anisotropic pressure exerted on it by one or more of the loosely bound nucleons. The angular dependence of the pressure is the same as that of the wave function of the loose nucleons. The deformation of the nucleus is then used to account for the large quadrupole moments.

A refinement of Rainwater's idea can be found in the unified model. The model was formulated by Bohr,\textsuperscript{20} Bohr and Mottelson,\textsuperscript{21} and Hill and Wheeler\textsuperscript{22} who called it the collective model. The model is a hybrid between the shell and the liquid-drop model. It considers the interplay between the intrinsic motions of nucleons and the collective modes of nuclear motion. The nucleons are assumed to be moving rather independently in a slowly changing potential. The frequency of the changes in the nuclear potential is assumed to be smaller than the intrinsic frequency of the nucleons. This assumption allows distinction between two different modes of nuclear motion, (a) intrinsic, representing the motion of the nucleons in a fixed potential, and (b) collective, associated with changes in nuclear shape.

Nilsson\textsuperscript{23} noted that the nuclear properties resulting from the interplay noted above depend essentially on the magnitude of the nuclear deformation. The nuclear deformation, in turn, depends on the nucleonic configuration: (a) Near closed shells the nuclear shape is spherically symmetric--a shell model picture with possible collective vibrational motion. (b) As we move away from closed shells, the nucleus acquires a slight deformation...
and the coupling between collective and intrinsic modes renders the nuclear states a rather complicated structure. (c) The situation simplifies as we move still further from closed shells, the deformation becoming large with resulting stability in shape and orientation. In this last case, it becomes possible to separate the nuclear motion into an intrinsic (in the deformed field), a rotational, and a vibrational motion. Or the total wave function can be written as

\[ \psi = \chi \cdot \varphi_{\text{vib}} \cdot \Omega_{\text{rot}} \]  

(II.18)

where \( \chi \) represents the intrinsic motion of the nucleons in a static potential, \( \varphi_{\text{vib}} \) represents the vibrational motion of the nucleus, and \( \Omega_{\text{rot}} \) represents the rotational motion of the nucleus. The quantum numbers representing the intrinsic motion, for the case of cylindrically symmetrical deformed potential, are \( \Omega_p (= \Lambda + \Sigma_p) \) the projection of the individual particle angular momentum, \( \mathbf{j}_p \), on the nuclear symmetry axis, and \( \Lambda_p \) and \( \Sigma_p \) are the projections of the orbital and spin angular momenta of the individual particle on the symmetry axis. The states characterized by \( \Omega_p \) are doubly degenerate, corresponding to \( \pm \Omega_p \). The wave function representing the intrinsic motion \( \chi_{\Omega_p} \) is an antisymmetric product of the individual particle wave function \( \chi_{\Omega_p} \).

The total angular momentum of the nucleus, \( \mathbf{I} \), is taken as the sum of the rotational angular momentum \( \mathbf{R} \) and the intrinsic angular momentum \( \mathbf{j} \). The projection of \( \mathbf{I} \) on the symmetry axis is designated by \( K \), and on a space-fixed axis is designated by \( M_i \). Here \( I, K \), and \( M_i \) are constants of the motion for deformed nuclei. The ground state of a nucleus will be for \( \mathbf{R} = 0 \) or \( \Omega = K \). The particles fill the levels pair-wise, with no net contribution to \( K \). Figure II.3 illustrates the coupling scheme for a deformed nucleus. For an odd-A nucleus, the odd particle occupies an unpaired orbit with \( K = \Omega_p \) of that orbit. Odd-odd nuclei can be thought of as superpositions of two odd-A nuclei.

The Hamiltonian representing a system of nucleons, in a deformed potential, coupled to a symmetrical rotor, can be written as

\[ \mathcal{H} = \mathcal{H}_p + \mathcal{H}_{\text{int}} + T_R \]  

(II.19)
Fig. II.3 Ground-state coupling of angular momenta in a deformed nucleus; \( z' \) is the nuclear symmetry axis, \( z \) a space-fixed axis.
where $\mathcal{H}_p$ is the single-particle Hamiltonian, $\mathcal{H}_{\text{int}}$ represents the interaction of the particle with the nuclear deformation, and $T_R$ is the rotational energy of a rigid rotor. The single-particle Hamiltonian is usually written in the form

$$\mathcal{H}_p = \mathcal{H}_0 + C \mathbf{J} \cdot \mathbf{S} + D \mathbf{J}^2$$

(II.20)

$$\mathcal{H}_0 = -\frac{\hbar^2 \nabla^2}{2M} + \frac{M}{2} \left( \omega_1 x_1^2 + \omega_2 x_2^2 + \omega_3 x_3^2 \right) ,$$

(II.21)

where $\omega_\kappa (\kappa = 1, 2, 3)$ correspond to quantum energy along Cartesian coordinates fixed in the nucleus. The $\mathbf{J}^2$ term is used to decrease the energy of the higher angular momentum states from their oscillator values.

Incompressibility of the nucleus is represented by the condition:

$$\omega_1 \omega_2 \omega_3 = \text{constant} ;$$

(II.22)

a transformation of coordinates changes $\mathcal{H}$ into the form (according to Nilsson's notation$^{23}$)

$$\mathcal{H}^0 = \mathcal{H}_0 + \mathcal{H}_6$$

$$\mathcal{H}_0 = \frac{\hbar^2}{2} \left[ -\nabla^2 + r^2 \right]$$

$$\mathcal{H}_6 = - \delta \kappa \omega_0 \frac{\hbar^2}{3} \sqrt{\pi/5} \ r^2 Y_0^2 ,$$

(II.23)

where

$$\omega_1^0 = \omega_2^0 = \omega_3^0 (1 + \frac{2}{3} \delta) \quad \text{and} \quad \omega_3^0 = \omega_0^2 (1 - \frac{4}{3} \delta) .$$

(II.24)

A representation in which $\mathcal{H}_0$, $\mathbf{J}^2$, $\mathbf{L}_z$, and $s_z$ are diagonal can be characterized by the quantum numbers $N$, $\ell$, $\Lambda$, and $\Sigma$, where $N$ is the total number of nodes in the wave function. Here $\ell$ is taken as the magnitude of a pseudo-angular momentum operator which is constructed in terms of the new coordinates in the usual manner--$\mathbf{\ell} = -i\hbar \mathbf{x} \times \nabla$; it differs from the original angular momentum by an order of $\delta$. Nilsson has shown that there is very little difference in the energies calculated with the use of either angular momentum (Ref. 23, Appendix A). The single-particle
wave function for a particle in the state $\Omega$ can be written in terms of the basis functions as:

$$\psi_{\alpha\Omega} = \sum_{\ell \Lambda} a_{\ell \Lambda} |N \ell \Lambda \Sigma\rangle .$$  \hspace{1cm} (II.25)

Mottelson and Nilsson\textsuperscript{25} have tabulated the quantities $a_{\ell \Lambda}$ which would render $\psi_{\alpha\Omega}$ as solutions to the Schrödinger equation:

$$(H_0 + H_0 + C \vec{r} \cdot \vec{s} + D \vec{r}^2)\psi_{\alpha\Omega} = E_{\alpha\Omega} \psi_{\alpha\Omega} .$$  \hspace{1cm} (II.26)

Plots of the single-particle energy levels as a function of deformation, with C and D chosen so as to reproduce the shell model in the limit of spherical nuclei, are known as the "Nilsson Diagrams." Figure II.4 has these single-particle levels for the region appropriate to cesium. The Nilsson levels are usually characterized by the spin and parity, and by a set of asymptotic quantum numbers, $[N n_3 \Lambda]$, appropriate for large deformations. The term $n_3$ is the number of nodal planes perpendicular to the nuclear symmetry axis.

The rotational energy part, for the case of cylindrical symmetry, can be treated by the methods used in treating homonuclear diatomic molecules (e.g., Ref. 26)

$$T_R = \sum_{\kappa} \frac{R_\kappa^2}{\mathbf{\mathcal{J}}_\kappa^2} ,$$  \hspace{1cm} (II.27)

where $R_\kappa$ are the components of the rotation around a body-fixed axis. The $\mathbf{\mathcal{J}}_\kappa$ are the components of the moment of inertia associated with the collective response of the nucleons to variations in the nuclear field. Since the field is invariant under an arbitrary rotation about the symmetry axis, the 3-component of the moment of inertia equals zero, while the other two components are equal ($= \mathbf{\mathcal{J}}_1$). Representing $R$ in terms of the total angular momentum, $\mathbf{I}$, and the intrinsic angular momentum, $\mathbf{J}$, i.e., $\mathbf{R} = \mathbf{I} - \mathbf{J}$, Eq. (II.27) reduces to:
Fig. II.4 Nilsson's single-particle levels for protons in the region $50 < z < 82$ (from Mottelson and Nilsson, Ref. 25).
\[ T_R = \sum_{k} \frac{\hbar^2}{2J_k} (I_k - J_k)^2 \quad (\text{II.28}) \]

\[ = \frac{\hbar^2}{2J} \left[ (I - J)^2 - (I_3 - J_3)^2 \right] \]

\[ = \frac{\hbar^2}{2J} \left[ (I^2 + J^2) - (I_3 - J_3)^2 \right] + \frac{\hbar^2}{2J} (-2I \cdot J) \quad (\text{II.29}) \]

The first set of terms in the last equation are the ordinary rotational terms. The last term represents the coupling between the intrinsic and the rotational motions. It contributes to the energy of the states with \( K = \frac{1}{2} \) only. Its contribution is usually represented by a "decoupling parameter," \( \alpha \). Thus, the rotational energy spectrum can be represented by

\[ E_{\text{rot}} = \frac{\hbar^2}{2J} \left[ I(I+1) + (-1)^I \frac{1}{2}(I+\frac{1}{2}) \alpha \right] \quad (\text{II.30}) \]

the decoupling parameter equals \( \alpha \)

\[ a = (-1)^I_1 \sum_{k} \left[ a_{k0} + 2[I(I+1)]^{\frac{1}{2}} a_{k0} a_{k1} \right] \quad (\text{II.31}) \]

where the \( a_{k0} \) are the ones defined in Eq. (II.25).

A generalization of Nilsson's idea to the case of an asymmetric rotor was performed by Davydov and Filippov \( 27, 28 \) and later by Newton. \( 29 \) It entails the use of different oscillator frequencies along the three major axes of the nucleus. These frequencies are subject to the conditions of Eq. (II.22). The components of the moment of inertia are chosen so as to agree with the hydrodynamical model--

\[ \mathcal{J}_k = 4B\beta^2 \sin^2(\gamma - \frac{2}{3} \kappa) \quad (\text{II.32}) \]

where \( \beta \) and \( \gamma \) are the usual parameters specifying a general ellipsoidal deformation, and \( B \) is the inertial parameter for quadrupole surface oscillation. \( 20 \) Some success has been achieved with this model. It was applied to some of the cesium isotopes by Person and Rasmussen, \( 30, 31 \) who point out that for the neutron-difficient isotopes the ellipsoidal shape might be favored.
3. **Pairing Plus Long-Range Correlations**

A possible derivation of the unified model from the shell model can give a clearer picture of nuclear structure and possibly explain the properties of nuclei in the transition region. Elliott\(^{32}\) showed that shell-model particles interacting with a long-range force of the form \(P_2(\cos \theta)\) would reproduce the unified model. Later work\(^{33}\) showed that this is a general property of the coupling scheme arising from slow angular dependence such as that of \(P_2(\cos \theta)\). [A Legendre polynomial \(P_k(\cos \theta)\) has its maximum value at \(\theta = 0\) and its half width is \(\approx 1/k\). Thus, if \(r_{12}\) is the distance between two nucleons interacting with a force having a \(P_k\) dependence, and \(R\) is the mean value of their radii, we have the condition \(r_{12} \lesssim R/k\) for the range of the interaction.]

A short-range attractive force should be included in the residual interaction to account for the observed pairing between nucleons. The pairing force usually chosen is a generalization of an operator formulated by Racah in connection with the concept of seniority. It separates the states with different seniority. An expression of a simple pairing force is\(^{34,35}\)

\[
(j^2(I)|v_{\text{pairing}}|j'^2(I)) = -\delta_{I0} \Gamma \sqrt{j + \frac{1}{2}} \sqrt{j' + \frac{1}{2}} (-)^{\delta + \delta'}.
\]  

This pairing force lowers further the energy of the lowest level, thus creating the gap observed in the intrinsic structure of nuclei. It is similar to a \(\delta\)-function force by having its strength proportional to the degeneracy.

The success of the Bardeen-Cooper-Schrieffer (BCS) theory\(^{36}\) in explaining superconductivity and the similarity between nuclear structure and the superconducting state of a metal\(^{37}\) suggested a possible application of the BCS treatment to nuclei. Using this approach, Belyaev\(^{38}\) showed that a nuclear model with a pairing plus a long-range interaction accounts for the gradual transition between the shell model and deformed nuclei (with their associate vibrational and rotational spectra). Explanation of some observed nuclear properties were attempted with this picture (e.g., Refs. 39, 40). Kisslinger and Sorensen\(^ {34,41}\) studied the properties of nuclei with a single closed shell, using the aforementioned forces.
and later extended their work to other nuclei—including some of the odd-A cesium nuclei.

The procedure of these treatments calls for a diagonalization of the Hamiltonian by a canonical transformation to a mixture of particle and hole states—"quasi particles." The quadrupole force is then treated by an approximation procedure.

The treatment of this model is simplified by the use of the notations of second quantization. Thus, \( \alpha_a ^+ \) and \( \alpha_a \) are creation and annihilation operators of a nucleon in the state \( a \), and \( \alpha_a ^+ \alpha_a \) is the number operator.\(^{42}\) In this notation, a single-particle state is represented by

\[
| j m \rangle = \alpha_j ^+ | 0 \rangle , \tag{II.34}
\]

where \( | 0 \rangle \) is the vacuum state. A single-particle operator, \( F = \sum_i f_i \), acquires the form

\[
F = \sum_{a,b} \alpha_a ^+ \alpha_a \langle a | f | b \rangle , \tag{II.35}
\]

and a two-body operator, \( V = \sum_{i > j} v_{ij} \),

\[
V = \sum_{(ab)(cd)} \alpha_a ^+ \alpha_b ^+ \alpha_c ^+ \alpha_d \langle ab | v | cd \rangle , \tag{II.36a}
\]

where the sum is taken over pairs of states. Thus,

\[
V = \frac{1}{4} \sum_{abcd} \alpha_a ^+ \alpha_b ^+ \alpha_c ^+ \alpha_d \langle ab | v | cd \rangle , \tag{II.36b}
\]

when the sum is taken over all single-particle states. The matrix elements in Eq. (II.36a,b) acquire a particularly simple form for the pairing interaction of Eq. (II.33), namely,

\[
\langle ab | v | cd \rangle = - \frac{1}{4} G \delta_{ab} \delta_{cd} \tag{II.37}
\]

where \( \delta (\bar{d}) \) is the time-reversed state of \( b (\bar{a}) \). Or, if \( b \) stands for \( jm \), \( \bar{b} \) will correspond to \( j-m \).\(^{43}\)

The Hamiltonian representing the nucleus will be the sum of three parts: (a) a simple shell-model Hamiltonian, \( H_s \); (b) a pairing-interaction
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Hamiltonian, $\hat{H}_p$; and (c) a long-range, quadrupole, interaction Hamiltonian, $\hat{H}_q$. The protons and neutrons will be represented by a superscript $\xi$ ($\xi = p, n$),

$$\hat{H}_s = \sum_{a,\xi} \varepsilon_a \alpha_a^\dagger \alpha_a^\xi,$$

where $\varepsilon_a$ are the shell-model energy levels.

$$\hat{H}_p = -\frac{1}{2} \sum_{a,b} G_{ab} \alpha_a^\xi \alpha_b^\xi \alpha_a^\dagger \alpha_b^\dagger,$$

where different strengths for the protons and neutrons were considered.

$$\hat{H}_q = -\frac{1}{2} \sum_{\xi} \hat{Q}_\xi \hat{Q}_\xi^\dagger,$$

where we have taken the quadrupole force constant to be of the same strength for protons and neutrons. Here $\hat{Q}_\xi$ is a quadrupole operator--

$$\hat{Q}_\mu = \sum_{ab} \langle a | r^2 y^2 | b \rangle \alpha_a^\dagger \alpha_b^\xi.$$

The terms $\hat{H}_s + \hat{H}_p \equiv \hat{H}_o$ are diagonalized by the canonical transformation

$$\alpha_{jm} = U_j \eta_{jm} V_j \eta_{j-m}^\dagger,$$

$$\alpha_{j-m} = U_j \eta_{j-m} V_j \eta_{jm}^\dagger$$

with

$$U_j^2 + V_j^2 = 1.$$  

This transformation is due to Bogolyubov and Valatin. The terms $V_j^2 (U_j^2)$ are the probability of occupancy (nonoccupancy) of the $j$-level. As this transformation mixes states with different mass numbers, a chemical potential $\lambda$, serving as a Lagrange multiplier, is added to the Hamiltonian. This adjusts the average number of protons and neutrons to correspond to the isotope under study.

$$\hat{H}_o = \hat{H}_o - \lambda \sum_{j,m,\xi} \alpha_{jm}^{\xi \dagger} \alpha_{jm}^\xi.$$
Substituting (II.42) into (II.44) and choosing the coefficients $U_j, V_j$ so that the Hamiltonian does not contain terms of the form $\eta^\dagger \eta^\dagger$ or $\eta \eta$, gives the following equations:

$$\frac{1}{4} G \sum_j \frac{2j+1}{E_j} = 1 \quad (\text{II.45})$$

where $E_j$ is the transformed single-particle energy;

$$E_j = [(\epsilon_j - \lambda)^2 + \Delta^2]^{1/2} \quad (\text{II.46})$$

where $\Delta$ is approximately half the gap in even-even nuclei and is equal to

$$\Delta = \frac{1}{2} G \sum_j (2j+1)U_j V_j \quad (\text{II.47})$$

and $U_j$ and $V_j$ are equal to:

$$U_j^2 = \frac{1}{2} [1 + \frac{\epsilon_j - \lambda}{E_j}] \quad (\text{II.48})$$

$$V_j^2 = \frac{1}{2} [1 - \frac{\epsilon_j - \lambda}{E_j}] \quad (\text{II.49})$$

The Hamiltonian in terms of the occupation number of the quasi-particle states acquires the form

$$H = \sum_{\xi, jm} E_j \eta^{\dagger}_j \eta_j + \text{const} + \text{terms containing } \eta^\dagger \eta^\dagger \text{'}s \quad (\text{II.50})$$

Thus, the Hamiltonian is approximately diagonal, since the last terms are small. The quasi-particle states can be thought of as excitations with respect to a new vacuum. This new vacuum is usually taken as the ground state of the even-even nucleus, $\Psi_o$, or in the case of odd-$A$ or odd-odd nuclei, it is the ground state of the even-even core.

$$\Psi_o = \prod_{\xi, jm > 0 \xi_j} (U_j + V_j \alpha_j^{\dagger} \beta_j^{\dagger} \beta_j \alpha_j) |0\rangle \quad (\text{II.51})$$

It is easy to show that

$$\eta_j^{\dagger} \Psi_o = 0 \quad (\text{II.52})$$
an odd-A nucleus will have a one quasi-particle wave function

\[ \eta_{JM}^{\uparrow} \psi_o \]  

(II.53)

while an odd-odd nucleus will have two quasi-particles

\[ \eta_{JM}^{\uparrow} \eta_{JM'}^{\uparrow} \psi_o \]  

(II.54)

The quadrupole term is usually treated by a variety of approximating methods. It suffices, for our purposes, to mention that double quasi-particle creation operators, vector coupled to form a tensor of rank \( L \), appear in the form

\[ A_{JJ'}^{LM} = \left[ \eta_{J}^{\uparrow} \eta_{J'}^{\uparrow} \right] M (-)^{L+L'} \]  

(II.55)

The modes corresponding to \( L = 2 \) are called phonons.

The final form of the Hamiltonian resembles to a large extent Nilsson's Hamiltonian, Eq. (II.23). Figure II.5 is an energy-level diagram obtained by Kumar and Baranger through the use of this procedure.

B. Spins and Magnetic Moments

1. The Isotope \( ^{136}\text{Cs} \)

The ground-state spin and magnetic moment of \( ^{136}\text{Cs} \) are explainable by the shell model. This stems from the fact that with 81 neutrons and 55 protons, this nucleus is rather close to major closed shells (82 and 50 for the neutrons and the protons, respectively) and its shape should be close to spherical.

The shell model predicts the protons outside the core to be in the \( 1g_{7/2} \) shell and the neutron hole to be in the \( 2d_{3/2} \) shell, or the configuration is \((\pi \, 1g_{7/2})^5 (\nu \, 2d_{3/2})^3\). This configuration is equivalent to the configuration \((\pi \, 1g_{7/2})^3 (\nu \, 2d_{3/2})^1\) for the purpose of calculating the spin and the magnetic moment.

Nordheim proposed two rules for the coupling of the proton and neutron angular momenta in odd-odd nuclei. These rules were based on the empirical observation that the spins of the odd neutron and the odd proton tend
Fig. II.5 Single-particle energy levels obtained by Kumar and Baranger (Ref. 47) by solving a diagonalized pairing plus quadrupole-interactions Hamiltonian.
to line up. de-Shalit\textsuperscript{49} used a mixture of Bartlett and Wigner forces of zero range for the interaction between a proton and a neutron to establish a foundation for Nordheim's rules. Schwartz extended de-Shalit's work to include an arbitrary number of nucleons in the configuration.\textsuperscript{50} The interaction between the protons and the neutrons was taken to be of the form

\[ V_{ij} = - \left[ (1 - \alpha) + \alpha \vec{\sigma}_i \cdot \vec{\sigma}_j \right] \delta(\vec{r}_i - \vec{r}_j) . \]  

(II.56)

In using Schwartz' calculations, we assumed that the neutrons and protons were coupled to states of seniority one. The configuration can be represented by the state \[ |j_1^n,(J_1=j_1),j_2^n,(J_2=j_2),I\rangle . \] A plot of the energy levels corresponding to different values of \( \alpha \) (Fig. II.6) showed that the \( I = 5 \) level is favored for values of \( \alpha > 0.05 \). This prediction is in agreement with the rules proposed by Brennan and Bernstein,\textsuperscript{51} which were, in a sense, based on the theoretical calculations of Schwartz. (Thus, the agreement is not surprising.) The rule that is pertinent to our case is

\[ I = |J_1 \pm J_2| \quad \text{for} \quad j_1 = l_1 \pm \frac{1}{2} \quad \text{and} \quad j_2 = l_2 \pm \frac{1}{2} , \]  

(II.57)

where the notation is the same as that used to represent the configuration. Qualitative agreement with the Brennan and Bernstein rules was obtained by inspection of the properties of Racah coefficients for a mixture of attractive Wigner and Bartlett forces.\textsuperscript{52,53} The agreement has little dependence on the range and details of the force.

The nuclear magnetic moment is related to the spin by the appropriate g-factors:

\[ \vec{\mu} = g_p \vec{J}_p + g_N \vec{J}_N \]  

(II.58a)

\[ \vec{\mu} \cdot \vec{J} = g_p \vec{J}_p \cdot \vec{I} + g_N \vec{J}_N \cdot \vec{I} . \]  

(II.58b)

Using Lande's formula for a vector \( \vec{I} \)

\[ \langle JM | \vec{I} | JM' \rangle = \frac{\langle JM | \vec{T} \cdot \vec{J} | JM \rangle}{J(J+1)} \langle JM | J | JM' \rangle , \]  

(II.59)
Fig. II.6 Predicted energy levels for $^{136}$Cs, including a mixture of Wigner and Bartlett forces of zero range (from calculations by Schwartz, Ref. 50).
we get
\[ \mu = \frac{1}{2(J+1)} \mathcal{G}_p [j_p(j_p+1) + I(I+1) - j_N(j_N+1)] \\
+ g_N[j_N(j_N+1) + I(I+1) - j_p(j_p+1)] \]  \hspace{1cm} (II.60)

However, since \( I = 5 \) implies that the angular momenta of the neutron and the proton are lined up, the magnetic moment is the sum of the magnetic moments of the proton and neutron configurations
\[ \mu = \mathcal{G}_p j_p + g_N j_N \]  \hspace{1cm} (II.61)

Using the quenched g-factors (g-factors calculated from neighboring odd-A nuclei) in the above formula, we get \( \mu_{\text{calc}} = 3.72 \) nm. The measured value for the magnetic moment \( \mu_{\text{exp}} = 3.68(4) \) nm is in good agreement with the calculated value. The difference between the Schmidt value of the magnetic moment, \( \mu_{\text{Schmidt}} = 2.87 \) (the magnetic moment obtained with the free nucleon g-factors), and the observed value can be accounted for by configuration mixing. The corrections can be divided into (a) corrections to the single-particle magnetic moments, the same as those used for odd-A nuclei, and (b) terms inherent to odd-odd nuclei. Noya, Arima, and Horie \textsuperscript{14} treated the similar problem of \( ^{137}\text{Cs} (I = 4) \) to bring the calculated magnetic moment, from the Schmidt value of 2.2 nm, into agreement with the observed value \( \approx 3.0 \) nm.

The superconductivity model has not been used for the study of odd-odd nuclei until recently. Kisslinger and co-workers \textsuperscript{54,55} have studied odd-odd nuclei in search of the form of the neutron-proton residual interactions. In their treatment they considered one of the cesium isotopes, \( ^{132}\text{Cs} \). Because this isotope is further removed from closed shells than is \( ^{136}\text{Cs} \), the use of the quadrupole interaction terms and their interaction with quasi-particles is necessitated. Finally, for a force of the form considered in discussing the spin, Eq. (II.56), they found the term \( \vec{s}_i \cdot \vec{s}_j \) to be dominant in cesium and indium isotopes \textsuperscript{55}--in agreement with the simple picture treated above.
2. The Isotopes $^{127}\text{Cs}$ and $^{125}\text{Cs}$

The spins and nuclear magnetic moments of the neutron-deficient isotopes, $^{125}\text{Cs}$ and $^{127}\text{Cs}$, on the other hand, cannot be explained easily in terms of the simple shell model. The use of the single-particle shell model for the cesium isotopes led to the wrong assignment of $I = 5/2$ to the ground-state spin of $^{127}\text{Cs}$ and $^{129}\text{Cs}$ (Refs. 56 and 57). The shell model picture of j-j coupling of five protons in the configuration

$$(1 \, \text{d}_{5/2})^{n_1} \, (2 \, \text{d}_{5/2})^{n_2},$$

with $n_1 + n_2 = 5$, cannot give $I = 1/2$. Other possibilities such as: (a) coupling five protons in the $1\text{h}_{11/2}$ level to give $I = 1/2$, or (b) assigning the last proton to the level $3\text{s}_{1/2}$ (Ref. 58), are not consistent with energy considerations. The magnetic moment of the isotopes studied are $\approx 1.4$ $\mu_n$; thus they lie midway between the Schmidt limits. Configuration mixing can account for this deviation.

Note that the observed magnetic moments of the $3\text{s}_{1/2}$ level, as measured in the thallium isotopes, are $\approx 10\%$ larger than those observed for $^{125}\text{Cs}$ and $^{127}\text{Cs}$.

A more consistent approach to the explanation of the spins of these isotopes is offered by the Nilsson model. In this model, a deformation of the nucleus causes the level with the lowest $\Omega$ ($\Omega = 1/2$) to lie lowest on the prolate side and highest on the oblate side. Conversely, the level with the highest $\Omega$ ($\Omega = j$) lies lowest on the oblate side and highest on the prolate side. The deformation of the nucleus can be estimated by summing the energies of the single-particle levels for a given configuration (of protons and neutrons) as a function of the deformation. The configuration that has the lowest energy for some deformation, $\delta$, would be the ground configuration. The deformation could then be used to calculate the intrinsic quadrupole moment of the nucleus, and the possible ground-state spin (Ref. 25, Sec. IV), which can in turn be compared with the observed values.

The magnetic moment of a deformed nucleus can be written in the form

$$\mu = g_s \, \mathbf{S}^2 + g_\ell \, \mathbf{J}^2 + g_R \, \mathbf{R}^2, \quad \text{(II.62)}$$

where $g_s = 5.585 \, (-3.826)$, $g_\ell = 1 \, (0)$ for protons (neutrons), and $g_R$ is taken (under the assumption of irrotational flow of nuclear matter) to
be \( \equiv \frac{Z}{A} \). For \( \Omega = 1/2 \), the decoupling parameter of Eq. (II.30) appears in the magnetic moment

\[
\mu = \frac{1}{I + \frac{1}{2}} \left[ \left( g_{\ell} - g_{l} \right) \left[ \frac{1}{2} \sum_{\ell} \left( a_{\ell 0}^2 - a_{\ell 1}^2 \right) + \left( - \right)^{I - \frac{1}{2} + \frac{1}{2}} \frac{1}{2} \left( I + \frac{1}{2} \right) \sum_{\ell} a_{\ell 0}^2 \right] + \left( g_{\ell} - g_{R} \right) \left[ \frac{1}{2} + \left( - \right)^{I - \frac{1}{2}} \frac{1}{2} \left( I + \frac{1}{2} \right) a + g_{R}\left( I + 1 \right) \right] \right].
\]  

(II.63)

The spin 1/2 of \(^{125}\text{Cs}\) and \(^{127}\text{Cs}\) can be explained by assigning the last proton to the 34th level, which has the asymptotic quantum numbers \([420]1/2^+\) in Nilsson's diagram, with a deformation \( \eta \approx 2 \). However, the magnetic moment calculated for this level, with Eq. (II.63) and the values of \( \Delta a \) tabulated in Ref. 23, is \( \mu \approx 1.95 \) nm, which is \( \approx \) 35\% larger than the observed value. Another possibility is the assignment of the last proton to the 30th level with \( \eta \approx -4 \) (oblate deformation); this value gives the correct magnetic moment \( \approx 1.4 \) nm. Should we consider the agreement between the observed and the calculated magnetic moments as a measure of how good an assignment is, we see that the latter assignment is preferable. However, in the Nilsson diagram \( \Delta a \) the \( d_{5/2} \) level was taken to be lower than the \( g_{7/2} \) level; this in turn allowed the 30th level to be sufficiently low to be the ground state.

The inclusion of pairing plus long-range correlation in the superconductivity model modifies the above procedure slightly. Bes and Szymanski \(^{60}\) generalized Nilsson's procedure of calculating the equilibrium deformation by the inclusion of a pairing force. Kisslinger and Sorensen \(^{41}\) calculated the effect of a quadrupole force on the relative motion of the \( 5/2 \) and 7/2 levels. They found that the forces used tend to bring the 1/2 level down. The 1/2 level arises to zeroth order from the coupling of a phonon to 5/2 quasi-particle state. The calculated magnetic moment, properly corrected for configuration mixing, for this state is \( \approx 0.45 \) nm, which is rather far off from the measured value. However, it is postulated that this state, being sensitive to the mixing parameters, can result in the correct value for the magnetic moment, with the appropriate slight addition of the 5/2 quasi-particle, having a 3.6 nm magnetic moment.
III. ATOMIC-BEAMS MAGNETIC RESONANCE

A. Theory

1. Atomic Structure

The nonrelativistic Hamiltonian representing a system of $N$ electrons in the field of a nucleus having $Z$ protons can be written as:

$$
\hat{H} = \sum_{i=1}^{N} \left( \frac{\mathbf{p}_i^2}{2\mu} - \frac{2e^2}{r_i} \right) + \sum_{i>j=1}^{N} \frac{e^2}{r_{ij}} + \hat{H}_{\text{fs}} + \hat{H}_{\text{hfs}},
$$

where $\mu$ is the reduced mass of an electron, $r_i$ is the radial coordinate of the $i$th electron from the center of mass of the atom, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, $\hat{H}_{\text{fs}}$ is the fine-structure Hamiltonian (accounting for the spin-orbit interaction of the electrons), and $\hat{H}_{\text{hfs}}$ is the hyperfine-structure Hamiltonian representing the higher multipole interaction between the electrons and the nucleus. Smaller terms in the Hamiltonian—such as spin-spin and spin-other-orbit interactions—are neglected.

The Schrödinger equation, when used with this Hamiltonian [Eq. (III.1)], is solved by approximating the interactions with a central potential of

$$
\hat{H}_0 = \sum_{i=1}^{N} \left( \frac{\mathbf{p}_i^2}{2\mu} - U(r_i) \right);
$$

the difference between the two Hamiltonians is taken as the perturbing Hamiltonian. The terminology and the degeneracies of the energy levels that result upon the inclusion of the different perturbing terms, are, to a large extent, the same as those treated in considering the shell model (Sec. II.A.1). Figure III.1 represents the effect of the inclusion of these terms. The hyperfine-structure Hamiltonian and the terms that result from the introduction of an external magnetic field are the only terms involved in our experiments.

The electrostatic part of the hyperfine-structure Hamiltonian can be written as

$$
\hat{H}_{\text{hfs}}^{e} = \int_{\tau_e} \int_{\tau_n} \frac{\rho_e(r_e)\rho_n(r_n)}{|\mathbf{r}_e - \mathbf{r}_n|} d\tau_e d\tau_n,
$$
Fig. III.1 The effect of the inclusion of different terms in the Hamiltonian.
where \( \rho_e \) and \( \rho_n \) are the electronic and nuclear charge distributions, respectively, and \( r_e \) and \( r_n \) are the radial coordinates. Expanding the term \( 1/|\vec{r}_e - \vec{r}_n| \) in terms of spherical harmonics we get

\[
\frac{1}{|\vec{r}_e - \vec{r}_n|} = \sum_{k=0}^{\infty} \frac{r_n^k}{r_e^{k+1}} \sum_{q=-k}^{k} \frac{4\pi}{r_n} (-q)^q Y^k_{-q}(\theta, \phi) Y^k_{q}(\theta, \phi) \qquad (III.4)
\]

\[
\mathcal{H}_{\text{hfs}} = \sum_{k=0}^{\infty} \sum_{q=-k}^{k} (-q)^q q_k F_{-q}^k . \qquad (III.5)
\]

where

\[
q_k^E = \sqrt{\frac{4\pi}{2k+1}} \int_{\tau_n} \rho_n \frac{r_n^k}{r_e} Y^k_{q}(\theta, \phi) \qquad (III.6a)
\]

\[
p_k^E = \sqrt{\frac{4\pi}{2k+1}} \int_{\tau_e} \rho_e r_e^{-k-1} Y^k_{q}(\theta, \phi) . \qquad (III.6b)
\]

The same treatment could be used for the magnetostatic interaction, through the substitution

\[
\rho_e \rightarrow \nabla \cdot \vec{m}_e \qquad \rho_n \rightarrow -\nabla \cdot \vec{m}_n . \qquad (III.7)
\]

Equation (III.5) implies that the hyperfine-structure Hamiltonian can be written as the scalar product of two spherical tensor operators, or as

\[
\mathcal{H}_{\text{hfs}} = \sum_k T^k_{\tau}(e) \cdot T^k_{\tau}(n) . \qquad (III.8)
\]

The quantum numbers representing the state of a free atom are \( I, J, F, \) and \( M_F \), where \( J \) is the total electronic angular momentum, \( I \) is the nuclear spin, and \( F \) is the total angular momentum of the atom:

\[
\vec{F} = \vec{I} + \vec{J} . \qquad (III.9)
\]

The diagonal matrix elements of \( \mathcal{H}_{\text{hfs}} \) in the \( I J F M_F \) representation are

\[
W_{F,M_F} = \langle I J F M_F | \mathcal{H}_{\text{hfs}} | I J F M_F \rangle = \sum_k \langle I J F M_F | T^k_{\tau}(e) \cdot T^k_{\tau}(n) | I J F M_F \rangle = \sum_k W_{F,M_F}(k) . \qquad (III.10)
\]
The application of a theorem due to Racah gives

$$W_{P,M_P}(k) = (-1)^{I+J-F} \binom{P}{J,I} \binom{I}{k} \binom{J}{I} T^k(n) I^I T^k(n) J^J .$$  \hspace{1cm} (III.11)

We can see from Eq. (III.11) that $W_{P,M_P}$ is independent of $M_P$. The triangular condition of the $6-j$ symbols \(^{64}\) gives the condition $k \leq (2I,2J)$, or the series terminates for $k = 2I$ or $2J$, whichever is smaller.

Cesium has a $^2S_{1/2}$ ground state ($J = 1/2$). Thus the hyperfine-structure Hamiltonian, $\mathcal{H}_{\text{hfs}}$, has one term, \(^63\)

$$\mathcal{H}_{\text{hfs}} = h \alpha \mathbf{J} \cdot \mathbf{J} .$$  \hspace{1cm} (III.12)

The constant, $h \alpha$, was calculated by Fermi \(^65\) for an $s$-electron and a point nucleus; it has the form \(^{66,67}\)

$$h \alpha = - \frac{(8 \pi)}{3} g_I g_J \mu_0^2 |\psi(0)|^2 ,$$  \hspace{1cm} (III.13)

where $g_I$ and $g_J$ are the nuclear and the electronic $g$-factors, respectively, $\mu_0$ is the Bohr magneton, and $\psi(0)$ is the wavefunction of an $s$-electron at the nucleus. From Eq. (III.13), and under the same assumptions, we see that for two isotopes of the same element we can write the equation

$$\frac{a_1}{a_2} = \frac{g_{11}}{g_{12}} ,$$  \hspace{1cm} (III.14)

which is known as the Fermi-Segrè formula. The discrepancy between this formula and experimental observations is known as the hyperfine-structure anomaly. This discrepancy can be accounted for by postulating a finite distribution of: (a) nuclear magnetism (the Bohr-Weisskopf effect \(^68\)), and (b) nuclear charge (the Breit-Rosenthal effect \(^69\)). The disagreement is usually of the order of 1% for cesium.

2. The Breit-Rabi Equation

The introduction of a magnetic field to an atom can be represented formally by the addition of two terms, accounting for the electronic and nuclear dipole interactions with the external field, to the hyperfine-structure Hamiltonian:
\[ H' = H_{\text{hfs}} - g_J \mu_0 H \cdot J - g_I \mu_0 H \cdot I. \]  

(III.15)

This Hamiltonian is easily solved for \( J = 1/2 \) (or \( I = 1/2 \)). In this case only two \( F \)-levels are present, namely \( F = I \pm 1/2 \). The energy matrix for the state \( F,M_F \), with the zero field representation used, is of the form

\[
\begin{align*}
|F=I+\frac{1}{2},M_F\rangle &= \frac{\hbar a}{2} - \frac{\mu_0 H M_F}{2I+1} (2g_I + g_J) - \frac{\mu_0 H (g_I - g_J)}{2I+1}[(I+\frac{1}{2})^2 - M_F^2]^{\frac{1}{2}} \\
|F=I-\frac{1}{2},M_F\rangle &= \frac{\hbar a}{2} - \frac{\mu_0 H M_F}{2I+1}[(I+\frac{1}{2})^2 - M_F^2]^{\frac{1}{2}} - \frac{\mu_0 H (g_I - g_J)}{2I+1} [2g_\perp (I+\frac{1}{2}) - g_J].
\end{align*}
\]

(III.16)

The energy levels resulting from the above formula have the form:

\[ \Delta W_{F,M_F} = -\frac{\Delta W}{2(2I+1)} - g_I \mu_0 M_F \frac{\Delta W}{2} (1 + \frac{4M_F X}{2I+1} + X^2)^{\frac{1}{2}}, \]

(III.17)

\[ \Delta W = \hbar a \frac{2I+1}{2} \equiv \hbar \Delta \nu, \quad X = \frac{(g_I - g_J) \mu_0 H_0}{\Delta W}, \]

(III.18)

where \( \Delta W \) is the separation between the two levels; \( \Delta \nu \) is usually called the hyperfine-structure separation and is measured in cycles per second (Hz units). The positive sign in Eq. (III.17) refers to the state \( |F=I+\frac{1}{2},M_F\rangle \), the negative sign to \( |F=I-\frac{1}{2},M_F\rangle \). Equation (III.17), first derived by Breit and Rabi, is known as the Breit-Rabi formula. It forms the basis of atomic-beams research on alkali atoms. Figures III.2 and III.3 are plots of the energy levels, Eq. (III.17), appropriate to \(^{125}\)Cs, \(^{127}\)Cs (\( I = 1/2 \)), and \(^{136}\)Cs (\( I = 5 \)), as a function of the applied magnetic field.

The solution to the Hamiltonian, \( H' \), acquires a particularly simple form in the limits \( X \gg 1 \) and \( X \ll 1 \). In the weak-field limit, \( X \ll 1 \), the magnetic field contribution is:

\[
\langle I J F M_F | H_{\text{mag}} | I J F M_F \rangle = - g_J \mu_0 H M_F \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} - g_I \mu_0 H M_F \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \approx - g_I \mu_0 H M_F.
\]

(III.19)
Fig. III.2 Breit-Rabi diagram for $I = \frac{1}{2}$ and $J = \frac{1}{2}$.
Fig. III.3 Breit-Rabi diagram for $^{136}$Cs, with $I = 5$, $J = 1/2$, and $\Delta \nu = 12,702.128$ MHz.
or,

\[
\varepsilon_F = \varepsilon_J \frac{F(F+1) - I(I+1)}{2F(F+1)} + \varepsilon_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} - \frac{g_J \mu_o}{h} H_o 2F(F+1)
\]

\[
\approx \varepsilon_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}. \quad (\text{III.20})
\]

The approximation in the last equation stems from the fact that \(g_I\) is smaller than \(g_J\) by the ratio of the proton mass to that of the electron, \(\approx 2000\). The frequency of a transition between two adjacent magnetic sublevels, \((\Delta F = 1)\), of a given \(F\) level is

\[
\nu_F \approx \varepsilon_J \frac{\mu_o}{h} H_o \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}. \quad (\text{III.21})
\]

In treating cesium, \(J = 1/2\), we use the level \(F = I + 1/2\), in which case Eq. (III.21) becomes

\[
\nu_F \approx \varepsilon_J \frac{\mu_o}{h} H_o \frac{1}{2I+1}. \quad (\text{III.22})
\]

Equation (III.22) forms the basis for the spin search in the Zeeman region. The resonance frequency of the isotope under study is related to that of the calibration isotope (labeled by a zero index) by

\[
\nu = \frac{2I_o + 1}{2I + 1} \nu_o. \quad (\text{III.23})
\]

In the strong-field limit, \(x >> 1\). The electronic and nuclear angular momenta decouple, each becoming a good quantum number. They precess independently about the magnetic-field direction. For cesium, fields of the order of 10,000 G are sufficient to decouple the electronic and nuclear angular momenta. The appropriate quantum numbers for this case are \(IM, J\). The diagonal matrix elements of \(H_i\) [Eq. (III.15)] for this case are:

\[
W_{M_I, M_J} = aM_i M_J - \varepsilon_J \frac{\mu_o}{h} M_J - \varepsilon_I \frac{\mu_o}{h} M_I. \quad (\text{III.24})
\]

An effective magnetic moment of the atom is defined as

\[
\mu_{\text{eff}} = -\frac{\partial W}{\partial H_o}. \quad (\text{III.25})
\]
In this limit

\[
\frac{\partial W_{\mu M^J}}{\partial H_0} = -g_J^\mu_o M^J - g_I^\mu_o M^I
\]

\[
\equiv -g_J^\mu_o M^J
\]  \hspace{1cm} (III.26)

Figure III.4 shows the coupling of \( \vec{I} \) and \( \vec{J} \) for the strong- and weak-fields limits. For fields intermediate between these two limits, shifts from the linear relation [Eq. (III.23)] set in, due to the repulsion between levels with the same \( M^F \) value. The shift depends on the magnitude of the hyperfine-structure separation, \( \Delta \nu \). A measurement of this shift can be used to get the magnitude of the \( \Delta \nu \). For a "standard" flop-in transition, \( (F = I + 1/2, M^F = -I + 1/2) \leftrightarrow (F = I + 1/2, M^F = -I - 1/2) \), the hyperfine-structure separation \( \Delta \nu \) is related to the observed resonance frequency and the applied magnetic field by

\[
\Delta \nu = \frac{(\nu + \mu_o g_J H_0 / \hbar)(-\mu_o g_J H_0 / \hbar - \nu)}{\nu + \mu_o g_J H_0 / [\hbar(2I+1)] + 2\mu_o g_J H_0 / [\hbar(2I+1)]} \]  \hspace{1cm} (III.27)

The sign of \( g_I^\mu \) can be deduced from the fit of the observed resonances to the above formula (Ref. 65, p. 376).

3. Radio-Frequency Transitions

Transitions between hyperfine-structure energy levels are induced by the application of an oscillating magnetic field. The effect of this field can be represented by the addition of

\[
\hat{H}_{rf} = -\mu \cdot \vec{H}_1(t)
\]  \hspace{1cm} (III.28)

to the hyperfine structure Hamiltonian \( \hat{H}_1 \) (III.15). Here \( \mu \) is the magnetic moment of the atom

\[
\vec{\mu} = -g_J^\mu_o \vec{J} - g_I^\mu_o \vec{I}
\]

\[
\equiv -g_J^\mu_o \vec{J}
\]  \hspace{1cm} (III.29)

and the oscillating field

\[
H_1(t) = (H_{1x} \hat{x} + H_{1y} \hat{y} + H_{1z} \hat{z}) \sin \omega t
\]  \hspace{1cm} (III.30)
Fig. III.4 The precession of $\vec{I}, \vec{J}$ in the presence of (a) a weak field, and (b) a strong field.
The transition probability in the Zeeman region of the hyperfine structure, 
P_{FM_F,F'M'_F}, is approximately given by
\[ \hbar^2 P_{FM_F,F'M'_F} \approx \pi^2 t^2 \mu_0 \mu \left| \langle FM_F | \hat{J} \cdot \hat{H}_1 | F'M'_F \rangle \right|^2, \]

where \( t \) is the time spent by the atom in the oscillatory field. Equation (III.31) holds if the transition probability is small compared with one.

If \( J \) does not change, the matrix elements in (III.31) are proportional to the matrix elements of \( J \). Thus, the nonvanishing matrix elements of \( J \) in the \( FM_F \) representation correspond to possible transitions, and reproduce the selection rules.

The computer routine TEE PEE, written by Professor W. A. Nierenberg, calculates these matrix elements as a function of \( H_0 \) for small \( H_1 \).

B. Apparatus

1. General Description

An atomic-beam machine consists of an oven (serving as the source of the atoms), three magnetic-field regions, and a detector (Figs. III.5 through III.10). The magnetic-field regions are labeled as A, B, and C regions (Fig. III.5). The magnetic fields in the A and B regions are inhomogeneous and strong enough to cause decoupling of the electronic and nuclear angular momenta. Figure III.9 shows the B-magnet of the atomic beam machine II. An atom in these regions experiences a force proportional to the gradient of the field and to the effective magnetic moment of the atom,

\[ F = -\nabla W = -\frac{\partial W}{\partial H_0} \nabla H_0 = \mu_{\text{eff}} \nabla H_0, \]

or from Eqs. (III.25) and (III.26), the force is proportional to \( \mu_{\text{J}} \), \( \mu_{\text{J}}' \), and the gradient of the field. The magnetic field of the C region (Fig. III.8) is uniform. A radio-frequency "hair pin" (Fig. III.10) in this region causes transitions between magnetic sublevels of the atom.

In a "flop-in" beams apparatus, the atoms effusing out of the oven are deflected upon passing through the A magnet. Should an atom undergo a transition in the C region, which changes the sign of its \( M_j \) in the
Fig. III.5 Schematic of atomic beam machine II:

O = Oven
S = Stop wire
D = Detector

1 = Path of nonresonance atoms
2 = Path of resonance atoms
Fig. III. 6 Atomic beam machine II.
Fig. III.7 The A magnet.
Fig. III. 8 The C magnet.
Fig. III. 9 The B magnet.
Fig. III.10 The hair pin and radio-frequency shield.
strong-field limit, the force exerted on the atom in the B region would counteract that of the A region. Thus the atom would be refocused on to the detector and appear as a signal (Fig. III.5).

Our study of the cesium isotopes was, by and large, performed on the atomic beam machine II (Fig. III.6). Few experiments were performed on the more accurate atomic beam machine VI. The dimensions, design considerations, and details of the atomic beam machine II were described in many theses (particularly Refs. 1 and 74).

2. Modifications

Few changes and repairs were performed on the apparatus (AB II) to improve its efficiency. The changes which might directly affect its performance are considered here:

a. The Oven Loader. Although a new oven loader was built, its general shape and dimensions were the same as those of the previous one. However, the oven platform was milled out of copper and cooled on one side only. A shoulder was added to the oven platform to increase its strength and the heat conduction area. Figure III.11 shows the new oven loader after two years of use, with a tantalum oven used in the $^{136}$Cs experiments placed on it. The oven was insulated from the oven loader by 1/8-in. alumina plate covering the oven platform and of the same dimensions.

b. The C Magnet. Figure III.8 shows the new C magnet, the design of which was constrained by the dimensions of the available space. Thus the pole tips were of the same dimensions as the previous magnet. These tips were 3/4-in. thick, polished Permandur. A 10-mil copper foil was placed between the pole tips and the rest of the magnet. A copper envelope with 1/2-in. spacers for the gap held the pole tips and the copper foils to the rest of the magnet. The C-magnet assembly was held together with plates of soft iron that served as shields. Metal dowels fastened the C-magnet to the shields. The close proximity of the shield and the size of the gap (1/2 in.) rendered the magnetic field leakage to the shields very large. Thus, the magnet saturated for fields of $\approx 300$ G. This could be remedied easily by moving the shield $\approx 1/4$ in. away from the
Fig. III.11 The oven loader with a tantalum oven in place.
C magnet, thereby increasing the reluctance of the stray circuit and thus increasing the field in the gap.

The electrical windings were chosen so as to get a maximum number of turns with a resistance of 1000 ohms. The windings were placed on a brass spool cooled on the sides. The 1000 ohms resistance was motivated by the limitation of the C magnet power supply (John Fluke Model No. 921A) which has a maximum regulated current output of 100 mA at 100 V.

c. The Hair Pin. The radio-frequency hair pin and housing were replaced by ones that fit the new magnet. The rf housing was made of two matching pieces milled out of copper (Fig. III.10). The two pieces were fastened together by two sets of screws on the bottom (to hold the hair pin) and two on the top. The hair pin sat on the envelope of the C magnet. The idea of having the hair pin movable from the outside was retained.

d. The A Magnet. The magnetic circuit of the A magnet shorted to the flange of the atomic beam machine casing. We thus cut off the pieces that were shorting and replacing them with aluminum supports (Fig. III.7). The A and C magnets were moved further apart and away from the B magnet to reduce the effect of the fringing fields.

e. The Monitor Hot Wire. In the study of the cesium nuclei, we attempted to utilize the ease of ionization of the stable cesium carrier for the purpose of normalizing the counting rates on the different buttons and continuously calibrating the magnetic field. This purpose was achieved by the use of small (0.003-in. Ir) hot wires in front of the button loader. This hot wire detected the stable cesium resonance, which in turn was recorded on a chart.

To prevent the hot wire from buckling we made the lower connection a brass rod, allowing it to slide freely inside a cylinder. Thus, the weight of the brass rod kept the hot wire taut. This tension was \( \approx 3 \times 10^3 \text{ psi} \), which the Ir wire was able to support when heated.
IV. EXPERIMENTAL PROCEDURE

A. The Isotopes $^{125}$Cs and $^{127}$Cs

1. Production Considerations

A variety of reactions may be employed in the production of $^{125}$Cs and $^{127}$Cs. The reactions $\alpha$(kn)Cs and $\alpha$(kn)Cs have both been used in the preparation of some of the neutron-deficient cesium isotopes. Either of these procedures could be used to produce $^{125}$Cs and $^{127}$Cs. It is interesting to note also that Lander and Yaffe produced $^{125}$Cs and $^{127}$Cs by bombarding uranium metal with 28-GeV protons, which gave $^{125}$Ba and $^{127}$Ba, which in turn decayed into their respective Cs isotopes. Since these isotopes are detected by the decay of their radioactivity, a desirable aspect of a sample is to have a minimum number of other radioactive isotopes present. Xenon gas has nine stable isotopes with neutron numbers between 70 and 82. Thus, a Xe(p,kn)Cs reaction will produce many isotopes and make the analysis unnecessarily difficult. Iodine, on the other hand, has but one stable isotope, $^{127}$I, making it more desirable if $\alpha$-particle energies required for the ($\alpha$,6n) reaction on iodine are available. The 88-inch cyclotron can furnish $\alpha$-particles with energies up to 120 MeV. Thus, the energy needed for an ($\alpha$,6n) reaction is readily available—as a result the reaction $\alpha$(kn)Cs was used.

At the energies needed for the $^{127}$I($\alpha$,6n)$^{125}$Cs reaction, the reactions $^{127}$I($\alpha$,4n)$^{127}$Cs and $^{127}$I($\alpha$,2n)$^{129}$Cs have finite cross sections. The isotopes (1.6 min) $^{126}$Cs and (3 min) $^{128}$Cs resulting from the reactions $^{127}$I($\alpha$,5n)$^{126}$Cs and $^{127}$I($\alpha$,3n)$^{128}$Cs are also produced. However, their half-lives are so short, decaying into stable xenons, that they do not appear in the analysis.

a. Cross-Section Determination. Since no experimental information about the cross section for the reaction $^{127}$I($\alpha$,6n)$^{125}$Cs was available, we decided to get some idea about its magnitude as a function of incident $\alpha$-particle energy. Attempts were made, at first, to find the absolute value for the production cross section. The procedure consisted of depositing a known amount of elemental iodine on an aluminum disk, and
bombarding the assembly with a known current of $\alpha$ particles of a certain energy. We dissolved 100 mg of $I_2$ in 25 cc of alcohol, and placed 1 cc of the solution in a cylindrical chamber (see next paragraph) of circular cross section 1-1/8 in. i.d. and 1/2 in. thick. The target was cooled to dry ice temperature and the alcohol was pumped away, leaving the iodine. We did not carry the attempt further, since after several trials we found that some of the iodine was deposited in the O-ring grooves that isolate the chamber and subsequently was not available for bombardment. On the other hand, we found that a measurement of the relative production cross sections for the available isotopes avoids this problem and it is, in fact, all we need, so we chose this method instead.

A seven-cell target was built for this purpose. The chambers were separated by aluminum foils acting as degraders of the $\alpha$-particles energies. The target was cylindrical with circular cross sections. The dimensions were 1-1/8 in. i.d., and the height of each chamber was 1/2 in. (see Fig. IV.1). The aluminum foil was 0.015 in. thick. The first foil, nearest to the cyclotron beam pipe, was of 0.002 in. Dural. A lump of iodine ($\approx 1/2$ gm) was placed in each of the chambers. The air in the target was driven out by placing the target on a hot plate until iodine vapors came out of the ports of each chamber, at which time the chambers were sealed with Lucite plugs. (The plugs were later changed to 1/8 in. Rad Lab fittings.) These Rad Lab fittings and O-rings on both sides of the aluminum foil prevented the iodine from escaping.

The 86-inch cyclotron provided us with a beam of 120 MeV $\alpha$ particles. After passing a cyclotron vacuum isolation foil and our 0.002 in. Dural foil, the $\alpha$-particle energy in the first chamber (No. 7) was 118.5 MeV, and was degraded to 63.1 MeV in the last chamber (No. 1). In calculating the energy of the $\alpha$-particles in each of the chambers, we made no allowance for possible degradation of the energy by the iodine gas. There is also the possibility that some of the $\alpha$ particles passed through the iodine crystal, thus losing more energy than the rest of the beam. This effect was avoided, hopefully, by focusing the beam away from the lower part of the target. The target was bombarded with a 1.5 $\mu$A current and the integrated beam was 0.1 $\mu$A-hr. Two cc of water were added to each chamber and after the target was shaken, one cc of the solution was drawn out and the activity of the samples was found to be:
Fig. IV.1 Seven-cell gaseous target.
<table>
<thead>
<tr>
<th>Chamber No.</th>
<th>Activity (mR/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (high energy)</td>
<td>36 - 38</td>
</tr>
<tr>
<td>6</td>
<td>19 - 20</td>
</tr>
<tr>
<td>5</td>
<td>26 - 28</td>
</tr>
<tr>
<td>4</td>
<td>76 - 78</td>
</tr>
<tr>
<td>3</td>
<td>46 - 48</td>
</tr>
<tr>
<td>2</td>
<td>10 - 12</td>
</tr>
<tr>
<td>1 (low energy)</td>
<td>8 - 10</td>
</tr>
</tbody>
</table>

The samples were then diluted with water, and fractions of the solutions with a counting rate of $\approx 10^4$ counts/min were placed on buttons. A decay analysis on each of the samples showed that chamber No. 4, with $\alpha$-particle energy 97.5 MeV, had the largest amount of $^{125}$Cs, in agreement with the activity measurement. Figure IV.2 shows the decay scheme for samples from chamber No. 1 (63.1 MeV), chamber No. 4 (97.5 MeV), and chamber No. 7 (118.5 MeV). Figure IV.3 has plots of the decay of samples from the intermediate chambers.

The amplitudes of the different possible half lives of the producible Cs isotopes were fitted by a computer least-squares routine. It was concluded that the (31 hr) $^{129}$Cs, (6.2 hr) $^{127}$Cs, (45 min) $^{125}$Cs, and a longer-lived activity, whose magnitude depended on the total counting rate, were the only components present. The (8 min) $^{123}$Cs also appeared in chambers 4 through 7, though in very small quantities. The fit of the decay curves to these half lives got progressively worse with increase in energy. This could be the result of fissioning of the iodine nucleus. Table IV.1 lists the results of the decay fit routine to the different samples. Figure IV.4 has a plot of the different ratios of activities as a function of energy.

It is worth noting that the energies we obtained agree with estimates made using Cameron's mass table and assuming the average kinetic energy of each emitted neutron to be 4 to 5 MeV.

b. Barium Iodide Target. Elemental iodine reacts with most desirable target materials and sublimes if heated. Thus, until these difficulties are overcome, it is necessary to use iodine compounds rather than pure.
Fig. IV.2 Decay scheme of the activity resulting from $^{127}$I(α, kn)Cs at different energies. (a) $E_\alpha = 63$ MeV, (b) $E_\alpha = 97.5$ MeV, and (c) $E_\alpha = 118.5$ MeV.
Fig. IV.3 Decay schemes of the activities resulting from $^{127}$I($\alpha$,kn)Cs at different energies. (a) $E_\alpha = 80$ MeV, (b) $E_\alpha = 90$ MeV, (c) $E_\alpha = 105$ MeV, and (d) $E_\alpha = 112$ MeV.
<table>
<thead>
<tr>
<th>Chamber</th>
<th>Energy (MeV)</th>
<th>$^{125}_{\text{Cs}}$</th>
<th>$^{127}_{\text{Cs}}$</th>
<th>$^{129}_{\text{Cs}}$</th>
<th>($\infty$)</th>
<th>$^{127}_{\text{Cs}}$</th>
<th>$^{129}_{\text{Cs}}$</th>
<th>($\infty$)</th>
<th>$^{129}_{\text{Cs}}$</th>
<th>($\infty$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63.1</td>
<td>437.7</td>
<td>650</td>
<td>30.3(1.3)</td>
<td>9.8(3)</td>
<td>0.67</td>
<td>14.4</td>
<td>21.5</td>
<td>44.6</td>
<td>66.3</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>786</td>
<td>312</td>
<td>29.4(1.1)</td>
<td>6.9(3)</td>
<td>2.52</td>
<td>26.7</td>
<td>10.6</td>
<td>114</td>
<td>45.3</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>5020</td>
<td>683</td>
<td>106.0(1.9)</td>
<td>19.1(5)</td>
<td>7.35</td>
<td>47.3</td>
<td>6.4</td>
<td>263</td>
<td>35.8</td>
</tr>
<tr>
<td>4</td>
<td>97.5</td>
<td>5092</td>
<td>440</td>
<td>80.5(1.6)</td>
<td>14.7(4)</td>
<td>11.57</td>
<td>63.2</td>
<td>5.5</td>
<td>346</td>
<td>29.9</td>
</tr>
<tr>
<td>5</td>
<td>105</td>
<td>3520</td>
<td>399</td>
<td>70.1(1.6)</td>
<td>15.5(4)</td>
<td>8.82</td>
<td>50.2</td>
<td>5.7</td>
<td>227</td>
<td>25.7</td>
</tr>
<tr>
<td>6</td>
<td>112</td>
<td>1728</td>
<td>233</td>
<td>41.7(1.2)</td>
<td>9.6(3)</td>
<td>7.42</td>
<td>41.4</td>
<td>5.9</td>
<td>180</td>
<td>24.3</td>
</tr>
<tr>
<td>7</td>
<td>118.5</td>
<td>3644</td>
<td>620</td>
<td>93.7(1.9)</td>
<td>23.1(1)</td>
<td>5.88</td>
<td>38.9</td>
<td>6.6</td>
<td>158</td>
<td>26.8</td>
</tr>
</tbody>
</table>

**Table IV.1. Summary of multichamber target decays.**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Half life</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{125}_{\text{Cs}}$</td>
<td>45 min</td>
<td>$^{127}<em>{\text{I}}(\alpha,6n)^{125}</em>{\text{Cs}}$</td>
</tr>
<tr>
<td>$^{127}_{\text{Cs}}$</td>
<td>6.2 hours</td>
<td>$^{127}<em>{\text{I}}(\alpha,4n)^{127}</em>{\text{Cs}}$</td>
</tr>
<tr>
<td>$^{129}_{\text{Cs}}$</td>
<td>31 hours</td>
<td>$^{127}<em>{\text{I}}(\alpha,2n)^{129}</em>{\text{Cs}}$</td>
</tr>
</tbody>
</table>
Fig. IV.4  Ratios of the different activities resulting from the reaction $^{127}$I($\alpha$,kn)Cs as a function of incident-particle energy. (a) Ratio of the production of $^{125}$Cs to $^{127}$Cs; (b) ratio of the activity of $^{125}$Cs, $^{127}$Cs, and $^{129}$Cs to a constant background of a long-lived activity ($\infty$) that appeared in the decay of the sample; (c) ratio of the productions of $^{125}$Cs and $^{127}$Cs to that of $^{129}$Cs.
iodine as the bombarded material. One such compound that was used is barium iodide. Barium iodide is available in the form BaI₂·6H₂O; if heated to 60 to 80°C, it evolves 4H₂O molecules. It is also unstable, decomposing with time and acquiring a brownish color characteristic of the iodine. This iodine is lost upon heating. Hence, under the best circumstances only about one third of the bombarded material is iodine.

Two barium iodide assemblies were tried: (i) a water-cooled target which consisted of a backing plate with grooves 3/32 in. wide, 0.050 in. deep, and 1/32 in. apart, milled out inside a circle one-inch in diameter, a cover plate with a matching geometry, and a 0.005 in. Dural cover foil. The target arrangement is similar to that of Fig. IV.6. The barium iodide was placed in the backing plates' grooves, heated to drive the water out, the cover foil and plate then placed over the material, and the whole assembly fastened to a standard 88-in. cyclotron water-cooled target block. This target suffered from poor heat transfer out of the barium iodide, resulting in local hot spots and subsequent burning through of the cover foil with loss of material. (ii) Another water-cooled target assembly with better heat-transfer properties was also attempted. It consisted of a 0.010 to 0.020 in. aluminum backing dish, made of 0.005 in. aluminum or Dural cover foil with barium iodide powder pressed by 16,000 psi pressure into a 1-in. diameter briquette sandwiched in between the two foils. The assembly was fastened, by a cover plate with one-inch circular hole in it, to a standard 88-in. cyclotron target block.

c. Elemental Iodine Targets. The short half life of ¹²⁵Cs makes the use of barium iodide, or other iodine compounds, undesirable for two basic reasons: (a) a chemical separation of the cesium out of the barium iodide and other produced activities is time consuming, and (b) only a fraction of a target's volume, designed for a certain α-particle energy drop in the material, is occupied by iodine atoms.

To overcome the difficulties encountered with elemental iodine, two approaches were tried: (i) to cool the target with liquid nitrogen, indirectly or directly, and (ii) to employ a platinum-plated water-cooled target.
i. Nitrogen-Cooled Target. A target that uses nitrogen gas as the primary coolant was first attempted. The procedure was to have liquid nitrogen flow into a reservoir connected to the target. The heat would cause the liquid nitrogen to boil, with the resultant nitrogen gas used to cool the target. The temperature of the target was monitored during bombardment with a thermocouple connected to the backing plate. It was found that to keep the target temperature below 0°C, an upper limit on the current for a beam of 95 MeV α-particles is 8 μA.

A qualitative idea about the speed of the reaction between iodine and aluminum was obtained by placing iodine in an aluminum dish for a period of time, at different temperatures, and looking for tarnishing of the aluminum surface. At liquid nitrogen and at dry ice temperatures, no reaction was observed after 20 min. However, at 20°C some tarnishing of the aluminum was observed after 20 min. Thus, our nitrogen gas-cooled target was discarded in favor of a liquid-nitrogen-cooled target.

ii. Liquid-Nitrogen-Cooled Target. An elemental iodine target that uses liquid nitrogen for cooling was attempted. Figure IV.5 shows the target assembly. It consisted of the target, liquid nitrogen reservoir, and a jacket that housed the reservoir and the target; it helped to reduce heat losses and to keep ice from forming on the reservoir and the target. Teflon O-rings--placed around the target, the open end of the reservoir, and the jacket, and pressed with the lid--isolated the reservoir and jacket atmospheres from each other and from the outside. The lid had ports for a liquid-nitrogen-level sensor gauge, a liquid-nitrogen-intake tube, and spent-nitrogen-gas exhaust. The target assembly was clamped to the cyclotron beam pipe.

The target consisted of a backing plate with grooves 3/32 in. wide, 0.050 in. to 0.055 in. deep and 1/32 in. apart, milled inside a 1-1/2 in. diameter circle. The target cover had the same geometry with the grooves milled out, leaving 0.010 in. to 0.012 in. of the aluminum to serve in place of a cover foil. A Teflon O-ring between the backing plate and the cover foil prevented the iodine from escaping. Fins on the back of the target plate increased the area in contact with the liquid nitrogen--liquid nitrogen can take up to 6 watts/cm² before going into a rapid nucleate
Fig. IV.5 Liquid-nitrogen-cooled target assembly. (a) The components of the target assembly (left to right): target, liquid nitrogen reservoir, and the jacket that houses the reservoir. (b) The target and the target assembly.
boiling. The total area of the fins submerged in liquid nitrogen was 380 cm\(^2\). Thus, the fins could carry \(\approx 2280\) watts of power should the heat be generated uniformly throughout the aluminum. Power carried out radially to the reservoir walls was \(\approx 500\) watts. Since most of the heat was generated in the iodine, the power carried out by the fins depended on the thermal conductivity of aluminum. Thus, we placed an upper limit of 1500 watts on the power put into the target.

This target enjoyed limited success due to the limited amount of liquid nitrogen available inside the bombardment area--25 liters. [The latent heat of vaporization of \(N_2\) is 47.6 cal/gm (at \(-195.55^\circ C\)). Thus, a 10 \(\mu\)A beam of 100-MeV \(\alpha\) particles would use up 25 liters in 1.1 hours.] Consequently nitrogen dewars had to be changed twice during a three-hour run.

Powdered iodine was placed in the backing plates grooves. To grind the iodine, we added liquid nitrogen to the iodine to reduce its vapor pressure and make the grinding procedure simpler. After filling, the target was placed in liquid nitrogen until bombardment time.

We scheduled a run using this arrangement. The target temperature was monitored with a thermocouple. A 10-\(\mu\)A beam of 95-MeV \(\alpha\) particles was used with steady-state temperature of about \(\approx -150^\circ C\). A rapid rise in temperature indicated that the liquid nitrogen supply was exhausted, and a new dewar had to be put in. This occurred twice and the second time it occurred the temperature was allowed to go above 20\(^\circ C\) before the beam was shut off. At the end of bombardment the target blew up upon its removal from the housing. A possible explanation for what happened is that the iodine could not leak out of the target and the rise in temperature could have increased the iodine pressure inside the target to the extent that it blew up. Run No. 799A was made with the remainder of the iodine in the target. Several attempts to have more L.N. available in the bombardment area were contemplated. During this time we tried using a water-cooled target. The success of this target put an end to the use of the liquid nitrogen cooled target.

iii. Water-Cooled Target. The last target assembly tried was a water-cooled target. A thin platinum coating on the target reduced chemical interaction. We also used a 0.002 in. Pt cover foil for the target. A
0.001 in. Teflon gasket between the backing plate and the cover foil prevented the iodine from escaping. Figure IV.6 shows the target assembly. The beam current, increased gradually with each run, reached 15 µA in the last run.

2. Chemical Separation

The barium iodide chemistry is the same as that of Ref. 1. It consists of precipitating the barium in the form of BaCO$_3$ by adding ammonium carbonate, and checking the supernatant solution for any remaining barium iodide by adding a drop of the (NH$_4$)$_2$CO$_3$ solution. The supernatant solution is boiled to dryness, and the ammonium iodide and excess ammonium carbonate are then sublimed in a cone heater at temperatures of approximately 600°C, leaving the Cs in the form of a halide. This sample is then transferred to an oven. The processes of filtering the barium carbonate out and the subliming of the ammonium iodide are time consuming, and could be eliminated with the use of elemental iodine as a target material.

The iodine chemistry consists of washing the bombarded iodine with alternating jets of alcohol, or some good iodine solvent such as benzene or ether, and water. The solution is collected in a beaker that has a measured amount (≈ 20 to 30 mg) of CsCl carrier in it. The iodine solvents facilitate extracting the iodine from the grooves of the target, and also reduce the total amount of liquid needed to wash the target clean.

Coupled with a lower boiling temperature and smaller latent heat of vaporization than water, the time needed to boil the solvent away is hence shortened if we use an iodine solvent. Benzene was tried in the first run, but its fire hazard curtailed its use. Hence ethyl alcohol was used in most runs and the chemistry went as expected. The latent heat of vaporization of the different solvents and the corresponding temperatures are:

<table>
<thead>
<tr>
<th>Solvent</th>
<th>Temperature (°C)</th>
<th>Heat of vaporization (cal/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>100</td>
<td>539.55</td>
</tr>
<tr>
<td>Benzene</td>
<td>80.2</td>
<td>94.3</td>
</tr>
<tr>
<td>Ether</td>
<td>34.6</td>
<td>89.3</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>78.3</td>
<td>204</td>
</tr>
<tr>
<td>Methyl alcohol</td>
<td>64.7</td>
<td>262.8</td>
</tr>
</tbody>
</table>
Fig. IV. 6 Water-cooled iodine target assembly.
Boiling the solution to dryness and waiting till the iodine has sublimed away leave the radioactive cesium with the stable carrier in the form of a chloride or iodide on the walls of the beaker. The beaker is then washed with water and the activity is transferred to a test tube. The solution is boiled down in a cone heater until one drop remains. This drop is then taken with a micropipette and placed in the oven. The oven is heated to 80° to 90°C by a hot plate. A hot air jet directed at the drop keeps the solution from boiling and hence reduces loss of activity. By monitoring the activity of the beaker and test tube, we found that for very clean equipment 60 to 80% of the activity in the test tube is transferred to the oven with each washing. Most of the remaining activity could be recovered from the walls of the test tube and the pipette. Thus, repeated washings, though time consuming, assured better recovery of the sample. For $^{129}$Cs, we attempted to speed up the chemistry and hence our recovery was less than 60% in most runs.

The recovery of the sample was estimated by one of two methods: (i) by monitoring the activity of the sample till it reached the oven and (ii) by seeing how long the sample lasted in the oven and comparing that to the time it takes an equal amount of carrier to effuse out of the oven under the same oven power conditions.

Calcium metal filings are put in the oven with the sample. Calcium reduces cesium halides (at temperatures of 400°C); giving out cesium which forms our beam.

3. Counting and Data Reduction

In Sec. IV.A.1 we mentioned that while producing (45 min) $^{125}$Cs by $^{127}$I$(\alpha,6n)^{125}$Cs, we also produced (6.2 hr) $^{127}$Cs by $^{127}$I$(\alpha,4n)^{127}$Cs and a small amount of (31 hr) $^{129}$Cs by $^{127}$I$(\alpha,2n)^{129}$Cs. This necessitated a decay analysis of each button to establish the composition of the activity on it. A relative enrichment of one of the isotopes was then taken as an indication of a resonance.

However, both $^{127}$Cs and $^{129}$Cs have nuclear spins of 1/2, and after the first few runs it was found that $^{125}$Cs too has a nuclear spin of 1/2. It was also found that the hfs separations, $\Delta\nu$, for these isotopes were
of comparable magnitude. As a result the component resonances will not be resolved until a high enough field has been reached. This fact required extra care in performing a run and necessitated a method that would give us a time integral of the beam during the exposure of a button. Two normalization procedures were used: (i) to record the stable beam through the use of a hot wire whose output was put on a chart recorder, and (ii) to collect the radioactive atoms that did not go through a resonance transition.

The first of these procedures was done in two different ways. On beam machine II, a small hot wire was put in front of the button to monitor the resonance of the stable carrier; on beam machine VI, the hot wire was placed at the position of one of the Stern-Gerlach peaks. The output of these hot wires was then recorded on a chart recorder. The area under the graph with the background subtracted was then cut out and weighed for each button. These weights are, therefore, directly proportional to the time-integrated beam at the detector.

The normalization of the counting rate of each isotope and their uncertainties was performed as follows: the button with the largest integrated beam was given the normalization factor unity. For other buttons, the activities and their uncertainties were multiplied by the ratio of the weight for the button with highest integrated beam to the weight corresponding to the integrated beam for that button.

The second procedure consisted of collecting the radioactive atoms in one of the Stern-Gerlach peaks. Two buttons were placed next to each other, the center one scanning the resonance and the side button collecting the atoms in one of the Stern-Gerlach peaks. A decay analysis of the activity of both buttons was performed. A plot of the ratio of the activity of an isotope on the resonance button to that of the side buttons as a function of frequency would indicate whether a resonance was observed or not. A bell fit, Lorentzian line shape, of the plot would locate the center of the resonance and its width. The resonance observed by this method would be an enhanced resonance, since we are taking the ratio of a flop-in signal to a flop-out signal. However, only the height of the resonance is affected—the width being the same for both flop-in and flop-out resonances.
On the beam machine VI, both procedures can be used simultaneously—a side button collected one of the Stern-Gerlach peaks and a hot wire was placed on the other Stern-Gerlach peak. By normalizing to the hot wire, we can observe both flop-in and flop-out resonance; both procedures yield essentially similar results.

The buttons were cycled in four Geiger counters and counted for the same period of time in each counter to eliminate consideration of counter efficiency. The counting time was chosen so that an uncertainty of less than 1% in the counting rate could be obtained. The advantages of the use of more than one counter for short-lived isotopes has been discussed by Hubbs and Mierenberg. 78

A computer routine (OMNIBUS II) was used to analyze the results of the runs. The decay rates of a button is the input and the output is the initial activity of each isotope. The activities are normalized and plotted as a function of frequency. Due to the short half life of $^{125}\text{Cs}$, the activities of all of the buttons are referred to the same starting time. The routine could also normalize the counting rates and fit the data to a bell-shaped curve as a function of frequency if desired. The input in this case would be the normalization factors for each button, its frequency, and the decay rates. The output is the location of the resonance, its half width, and the chi-square of the fit.

4. Results

In discussing the production of $^{125}\text{Cs}$ (Sec. IV.A.1), we pointed out that a small, yet measurable, portion of the activity in our sample was due to (6.2 hr) $^{127}\text{Cs}$ and (31 hr) $^{129}\text{Cs}$. The spins of these isotopes are both $1/2$, and their hfs separations, $\Delta \nu$, are of comparable magnitude (Ref. 2)--8900 (150) and 9229 (20) MHz, respectively. Using the normalization procedure discussed later in Sec. IV.A.3, we found $^{125}\text{Cs}$ to have a spin of $1/2$ also, and with increasing magnetic field, the hfs separation to be of comparable magnitude to those of $^{127}\text{Cs}$ and $^{129}\text{Cs}$. Since very little $^{129}\text{Cs}$ was produced, most of our work was concentrated on $^{125}\text{Cs}$ and $^{127}\text{Cs}$ only.

Fourteen runs were performed for the determination of the $^{125}\text{Cs}$ and $^{127}\text{Cs}$ constants. The first six of these runs were spent in determining
spin and in improving production procedures. Six runs were attempted at sweeps of resonances due to both isotopes. The overlap between the resonances disappeared at a field of $\approx 100$ G. The last two runs were performed at a field of $\approx 300$ G on the separate resonances of $^{125}$Cs and $^{127}$Cs.

The early runs in which we attempted a sweep of both resonances did not achieve much success for one or more of the following reasons: (i) an accurate estimate of $H_1$ [Eq. (III.28)] for a given field $H_0$ was not available; (ii) the formation of aluminum iodide, which turned into aluminum oxide during chemical separation, made the chemistry both lengthy and inefficient; this was later overcome by plating the target with platinum. Figure IV.7(a) shows an example of a run in which $H$ was too large; however, we can see that the $^{125}$Cs resonance has been shifted by $\approx 15$ kHz from that of $^{127}$Cs. We calculated where the $^{127}$Cs resonance should be from the Khan value $^2$ for $^{127}$Cs $\Delta \nu$, and the $^{125}$Cs resonance would be at the frequency of $^{127}$Cs + 20 kHz. Figure IV.7(b) is an example of where the chemistry was so inefficient that only three buttons were obtained from the sample. However, they show the tails of the $^{125,127}$Cs resonances.

The difficulty of determining the appropriate $H_1$ stems mostly from the lack of knowledge of the impedance of the hair pin and the transmission lines. The resonance frequency of $^{125}$Cs and $^{127}$Cs is almost four times that of the $^{133}$Cs resonance at the same field. This, coupled with the limit on the field $H_0$ attainable, makes cumbersome the use of the resonance of the stable isotope in the deduction of the impedance of the hair pin and the line. This procedure, however, was used successfully for $^{136}$Cs.

The hyperfine-structure separation for $^{125}$Cs was found to be 8754.40 MHz. The uncertainty was taken to be twice the standard deviation of a least-squares fit of five data points. The use of the Fermi-Segrè formula [Eq. (III.14)] gives a nuclear magnetic moment of $\mu_I = + 1.40(2)$ for $^{125}$Cs. The hyperfine-structure separation obtained for $^{127}$Cs was just outside the error of the previously quoted value.$^2$ Recalculation of the $\Delta \nu$ from the resonances observed previously indicated that the discrepancy was due to an error in the calculation of the value of $\Delta \nu$. Hence, we incorporated the previously observed resonances of $^{127}$Cs into a least-
Fig. IV.7 Difficulties encountered in the study of a short-lived isotope. (a) Large rf power (Run 810). (b) Poor recovery in the chemical separation (Run 812).
squares-fit routine (HYPERFINE 4-94) with the resonances we observed. The fit of the data was good and the result was $\Delta v = 9109.45$ MHz giving a nuclear magnetic moment of $+1.45(2)$ for $^{127}$Cs. Tables IV.2 and IV.3 list the resonances observed and their fit for $^{125}$Cs and $^{127}$Cs. Figures IV.8 through 11 show the resonances observed for $^{125}$Cs and $^{127}$Cs.

B. The Isotope $^{136}$Cs

1. Production and Chemical Separation

The difficulty in producing sufficient amounts of this isotope could be the main reason that early (1956) attempts in this laboratory were inconclusive. The half life of $^{136}$Cs is 13 days. For such a half life, the use of mass spectroscopy or decay analysis of the activity, as means of detecting the isotope, are of comparable usefulness. The difficulty in producing $^{136}$Cs atoms arises mostly from its position in the nuclidic chart: (i) It has 81 neutrons, 1 less than a neutron "magic" shell, making the cross section for the evaporation of a neutron smaller than that for neighboring nuclei. (ii) It is shielded against $\beta$ decay; hence, it is not a fission product, $^{136}$Xe being stable. (iii) It has three more neutrons than the stable cesium, $^{133}$Cs, making neutron capture an unfeasible method of producing $^{136}$Cs. The $^{135}$Cs ($2.0 \times 10^6$ years) is highly radioactive in the quantities needed for targets. Sufficient amounts of its parent, $^{135}$Xe (9.2 hours), are produced in reactors and could be collected from a liquid reactor. Cesium-135 has a neutron capture cross section of 8.7 barns, but its biological activity makes its use as a means of producing $^{136}$Cs undesirable. Also the background of the $^{135}$Cs would be a nuisance. Hence, the production of $^{136}$Cs requires either the use of high-energy particles, which usually have low fluxes, or the use of reactions of small cross section. Thus, a usable sample would require a prolonged bombardment—at high expense.

Three reactions were considered for the production of $^{136}$Cs: (i) The 1.3-mb cross section of $^{139}$La(n,\(\alpha\))$^{136}$Cs at a neutron energy of 14.5 MeV necessitates the use of a Cockroft-Walton generator. Though the cross section is small, the reaction seemed attractive. The 99.91% abundance of $^{139}$La ensures a pure $^{136}$Cs activity after chemical separation of the
### Table IV.2. Summary of $^{133}$Cs results.

$I = 1/2$ \hspace{1cm} $J = 1/2$ \hspace{1cm} State $^2\!g_1/2$

<table>
<thead>
<tr>
<th>Run</th>
<th>$^{133}$Cs calibration frequency (MHz)</th>
<th>Field (G)</th>
<th>Observed$^a$ Freq. (MHz)</th>
<th>Residual $v_{\text{obs}} - v_{\text{calc}}$ (kHz)</th>
<th>$g_I &gt; 0$</th>
<th>$g_I &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>799A</td>
<td>0.882(6)</td>
<td>2.520(17)</td>
<td>3.259(6)</td>
<td>- 1.4</td>
<td>- 6.8</td>
<td></td>
</tr>
<tr>
<td>801</td>
<td>0.807(6)</td>
<td>2.307(17)</td>
<td>3.228(6)</td>
<td>- 3.1</td>
<td>- 8.0</td>
<td></td>
</tr>
<tr>
<td>810</td>
<td>21.318(6)</td>
<td>59.960(17)</td>
<td>84.770(30)</td>
<td>+ 0.2</td>
<td>- 102.7</td>
<td></td>
</tr>
<tr>
<td>813</td>
<td>51.944(12)</td>
<td>142.837(32)</td>
<td>204.628(20)</td>
<td>+ 29.9</td>
<td>- 127.5</td>
<td></td>
</tr>
<tr>
<td>815</td>
<td>117.715(12)</td>
<td>309.033(29)</td>
<td>454.142(17)</td>
<td>- 5.6</td>
<td>+ 30.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$g_I$</th>
<th>$\Delta v$ (MHz)</th>
<th>$\mu_I$ (nm)</th>
<th>$g_I \times 10^{-4}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>8754.(19)</td>
<td>+ 1.40</td>
<td>+ 15.2</td>
<td>0.38</td>
</tr>
<tr>
<td>Negative</td>
<td>9025.(20)</td>
<td>- 1.44</td>
<td>- 15.7</td>
<td>14.16</td>
</tr>
</tbody>
</table>

**Calibration data**

<table>
<thead>
<tr>
<th>Name $^{133}$Cs</th>
<th>$I$</th>
<th>$g_J$</th>
<th>$g_I \times 10^{-4}$</th>
<th>$\Delta v$ (MHz)</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{133}$Cs</td>
<td>7/2</td>
<td>- 2.002543</td>
<td>3.98994</td>
<td>9192.6317</td>
<td>$(4,-3) \leftrightarrow (4,-4)$</td>
</tr>
<tr>
<td>$^{85}$Rb</td>
<td>5/2</td>
<td>- 2.002332</td>
<td>2.93700</td>
<td>3035.7324</td>
<td>$(3,-2) \leftrightarrow (3,-3)$</td>
</tr>
</tbody>
</table>

---

a. Transitions are between $(1,0) \leftrightarrow (1,-1)$. 

Table IV.3. Summary of $^{137}$Cs results.

$I = \frac{1}{2}$  
$J = \frac{1}{2}$  
State $^2S_{1/2}$

<table>
<thead>
<tr>
<th>Run</th>
<th>Calibrating isotope</th>
<th>Calibration freq. (MHz)</th>
<th>Field (G)</th>
<th>Observeda freq. (MHz)</th>
<th>Residual $v_{obs} - v_{calc}$ (kHz)</th>
<th>$g_I &gt; 0$</th>
<th>$g_I &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>799A</td>
<td>$^{133}$Cs</td>
<td>0.882(6)</td>
<td>2.520(17)</td>
<td>3.259(6)</td>
<td>-1.2  - 6.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>801</td>
<td>$^{133}$Cs</td>
<td>0.807(6)</td>
<td>2.307(17)</td>
<td>3.228(6)</td>
<td>-2.9  - 8.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0231b</td>
<td>$^{133}$Cs</td>
<td>50.787(75)</td>
<td>139.773(199)</td>
<td>200.020(89)</td>
<td>+ 83.5  - 94.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0232b</td>
<td>$^{133}$Cs</td>
<td>50.811(75)</td>
<td>139.837(199)</td>
<td>200.119(67)</td>
<td>+ 89.6  - 88.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>813</td>
<td>$^{133}$Cs</td>
<td>141.948(12)</td>
<td>142.848(32)</td>
<td>204.450(20)</td>
<td>+ 20.7  - 158.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>820A</td>
<td>$^{133}$Cs</td>
<td>124.259(20)</td>
<td>324.761(48)</td>
<td>477.430(90)</td>
<td>- 40.2  - 41.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>820B</td>
<td>$^{133}$Cs</td>
<td>124.238(20)</td>
<td>324.710(48)</td>
<td>477.379(70)</td>
<td>- 13.8  - 15.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>820C</td>
<td>$^{133}$Cs</td>
<td>124.230(20)</td>
<td>324.691(48)</td>
<td>477.340(80)</td>
<td>- 23.3  - 24.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>115c</td>
<td>$^{85}$Rb</td>
<td>528.100(80)</td>
<td>649.570(63)</td>
<td>1000.200(600)</td>
<td>+ 409.9  + 18052.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ g_I = \frac{\Delta \nu}{2 \pi \hbar} \]

\[ \Delta \nu \text{ (MHz)} \]

\[ \mu_I \text{ (nm)} \]

\[ g_I \times 10^{-4} \]

\[ \chi^2 \]

Positive  
9109.23  
+ 1.45  
+ 15.8  
0.99

Negative 
9382.24  
- 1.50  
- 16.3  
19.21

a. Transitions are between \((1,0) \leftrightarrow (1,-1)\).

b. Resonances observed by Shugart (Ref. 1).

c. Resonance observed by Khan (Ref. 2).
Fig. IV.8 Resonances of $^{125}\text{Cs}$ and $^{127}\text{Cs}$ (Run 813).
Fig. IV.9 Decay analysis of a half-beam and of $^{125}$Cs and $^{127}$Cs resonance buttons (Run 813).
Fig. IV.10 Resonance of $^{125}$Cs. The counting rates were normalized on the basis of the hot-wire reading (Run 815).
Fig. IV.11 Resonance of $^{127}\text{Cs}$. (a) Run 820A, (b) Run 820B, and (c) Run 820C.
Cs. However, the use of large quantities of lanthanum, and a lanthanum chemistry to separate the cesium were a drawback in this reaction.

(ii) The reaction $^{136}\text{Ba}(n,p)^{136}\text{Cs}$ has a 38-mb cross section at neutron energy of 14.5 MeV (Ref. 79). However, $^{136}\text{Ba}$ has a natural abundance of 7.81% and an enriched isotope would be too expensive. A chemistry sample sent to the Sandia Cockroft-Walton generator at Livermore was bombarded for 90 min. The target was a 3 in. diameter cylinder 3-1/2 in. high of 1/4 in. Plexiglass with a removable 0.017 in. Al cover foil. The sample was 350 gm of BaO in a sealed plastic bag. The Cs could be gotten out of the BaO by a chemistry similar to that used for BaI$_2$. However, the large amount of BaO used made the chance of recovering 20 to 30 mg carrier of CsCl very small, due to its co-precipitation with Ba(OH)$_2$·8H$_2$O. Figure IV.12 shows schematically the target and the integrated flux at both ends after the bombardment. The difficulty encountered in recovering the Cs from the sample and the fact that the sample did not show much $^{136}\text{Cs}$ activity caused us to resort to other means of producing $^{136}\text{Cs}$.

(iii) A $^{136}\text{Xe}({\alpha},2n)^{136}\text{Cs}$ reaction was also attempted. The seven-compartment gaseous target used in the $^{125}\text{Cs}$ experiment (see Sec. IV.A.1) was converted into a single cell. Xenon gas was introduced into the target after the air was pumped out. A 1-μA-hr, 30-MeV deuteron bombardment produced sufficient activity for the $^{136}\text{Cs}$ isotope to be detected by pulse-height analysis.

We next built a target for the bombardment of Xe gas (Fig. IV.13). The target was 2.5 ft long and consisted of two concentric Al cylinders with 1/16 in. walls. The inner cylinder of 1-1/8 in. inside diameter served as the Xe gas chamber. The outer cylinder was of 2 in. i.d., and the region between the two cylinders served as a water jacket. The ends of the cylinder were welded to flanges in which holes were drilled radially to the Xe chamber, and to the water jacket. Aluminum foil 0.010-in. thick sealed the Xe chamber at both ends. The target was bolted to a helium gas chamber connected to the cyclotron beam pipe. This gas cooled the front foil of the target and the cyclotron vacuum isolation foil.

The target was designed for the use of $\approx 35$ MeV deuterons. When the target contained about two atmospheres of Xe, the energy loss was $\approx 20$ MeV in the Xe gas. Before the beam left the chamber, the deuteron
Approximate integrated neutron flux (90-min bombardment)  
(as measured by the $^{27}$Al($n$, $a$)$^{24}$Na reaction)

<table>
<thead>
<tr>
<th>Foil No.</th>
<th>Integrated flux $n/cm^2$ ($\times 10^{12}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.87</td>
</tr>
<tr>
<td>2</td>
<td>5.02</td>
</tr>
<tr>
<td>3</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Fig. IV.12 Schematic and integrated neutron flux of BaO target.
Fig. IV, 13 Gaseous xenon target (a) connected to the cyclotron beam pipe, (b) paraffin shield used during deuteron bombardment.
energy covered most of the (d,2n) energy range. However, the use of deuterons made the target too hot for repeated use, and the $^{136}\text{Xe}(p,n)^{136}\text{Cs}$ reaction was chosen for the final production. We chose the Xe gas pressure in such a way that 10 to 15 MeV of the proton energy would be lost in the gas. The proton starting energies ranged from 16 to 21 MeV, with best results at 21 MeV and Xe gas pressure of 19 psi. For this case the protons entered the Xe chamber at 19 MeV and left with $\approx 8$ MeV. The protons coming out the other end were collected on a graphite block cooled with an air jet.

Xenon gas was introduced into the chamber by pumping the air out on one end, closing the pump line, and introducing the Xe from the other end. A similar procedure removed the cesium out of the chamber. The Xe gas was pumped out of the chamber and passed through a slightly acid CsCl water solution. When the vacuum was removed, the water was sucked back into the target. The target was then shaken to dissolve the Cs off of the walls. Repeating the washing (4 or 5 times) assured recovery of most of the sample. The volume of the Xe chamber was $\approx 450$ cc. We used $\approx 200$ cc of water in each washing. By monitoring the activity of the drained solution, we found that most of the activity was recovered in the first two washings. The solution was then boiled down to a few drops, which were transferred to the oven. The solution in the oven was dried by heating the oven and applying a hot air jet to the solution. The beaker, test tube, and pipette used in the boiling down and transferring processes were washed, and the solution boiled to a few drops and transferred to the oven. This process was repeated until the remaining activity was less than 1% of the starting activity. The chemistry took 8 to 10 hours. The Cs in the oven was in the form of CsCl, which is reduced by Ca at $\approx 400^\circ\text{C}$ and emits Cs metal atoms. The amount of carrier used was 15 to 30 mg with beam lasting as long as 12 hours.

2. Counting

We detected $^{136}\text{Cs}$ by decay analysis. We used the same Geiger counters as in the $^{125}\text{Cs}$, $^{127}\text{Cs}$ runs (see Sec. IV.A.3). These counters are most efficient to $\beta$ particles, but much less efficient to $\gamma$ and $X$ rays.

We ran the sample $\approx 2$ weeks after bombardment to allow short-lived isotopes to decay away, leaving only (9.7 day) $^{131}\text{Cs}$, (6.58 day) $^{132}\text{Cs}$,
and (13 day) $^{136}\text{Cs}$. The proton energies were high enough that the reaction $^{132}\text{Xe}(p,n)^{132}\text{Cs}$ had a small cross section over most of the Xe region. Thus, despite the fact that $^{132}\text{Xe}$ has a natural abundance of 26.89%, compared to 8.87% abundance for $^{136}\text{Xe}$, the shorter half life of $^{132}\text{Cs}$ and the higher proton energies made its activity much less than that of $^{136}\text{Cs}$. The $^{131}\text{Cs}$, made by $^{131}\text{Xe}(p,n)^{131}\text{Cs}$ and $^{132}\text{Xe}(p,2n)^{131}\text{Cs}$ reactions, decays only by electron capture into $^{131}\text{Xe}$. Figure IV.14 is a plot of the decay of a chemistry sample produced by Xe(d,2n)Cs. The deuteron energy was 30 MeV. Two of the four chemistry buttons were covered with a layer of Scotch tape after the 16th day to show what effect this would have on the counting rate. Thus, the decay analysis showed that by running our sample $\sim 2$ weeks after bombardment, we are assured of an almost pure sample of $^{136}\text{Cs}$.

The counting procedure for $^{136}\text{Cs}$ was in many ways the same as that for $^{125}\text{Cs}$ and $^{127}\text{Cs}$. The differences are: (i) because the activity of the sample is low, the buttons have a small counting rate, which necessitates long periods of counting (2 to 8 hours for each button in four counters) to reduce the uncertainty in the counting rates; (ii) for decay purposes, the long half lives meant counting the buttons for about a month. Fluctuations in counters background and efficiencies over the decay time limited our decay analysis to buttons that had a counting rate twice the background or larger. The counters background was $\approx 2.5$ counts/min.

The counting rates were normalized by the use of a side button that collected nonresonance atoms. The ratio of the counting rate of a spin button to that of the side button was plotted vs frequency. Normalization by monitoring the resonance of stable Cs (see Sec. IV.A.3) was used, however, in several runs. To observe the simultaneous resonance of both $^{136}\text{Cs}$ and $^{133}\text{Cs}$, we used I.F.I. 500 and 510 wide-band amplifiers on the output of each of the oscillators.

### Results

The spin of $^{136}\text{Cs}$ was found to be $I = 5$. Several attempts at determining the spin suffered from low specific activity, the subsequent long exposure time, possible drift of the field, and low counting rate.
Fig. IV.14 Decay scheme of the activity resulting from the reaction Xe(d,2n)Cs, $E_d \approx 30$ MeV. Two of the samples were covered with Scotch tape after the sixteenth day to reduce the counting rate of $^{131}$Cs.
Table IV.4 has the results of one of these runs where isotopes of spin 1, 2, 3, and 5 were exposed.

Table IV.4. Results of a $^{136}$Cs spin search experiment.

<table>
<thead>
<tr>
<th>Spin</th>
<th>Normalized counting rate $^{132}$Cs</th>
<th>Normalized counting rate $^{136}$Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0(4)</td>
<td>2.7(1)</td>
</tr>
<tr>
<td>2</td>
<td>6.6(9)</td>
<td>1.3(4)</td>
</tr>
<tr>
<td>3</td>
<td>0.5(7)</td>
<td>2.6(1.2)</td>
</tr>
<tr>
<td>5</td>
<td>0.2(6)</td>
<td>4.3(5)</td>
</tr>
</tbody>
</table>

Figure IV.15 is a plot of the decay scheme of a resonance button, a normalization button, and a chemistry sample of run 849.

Subsequent runs were performed at progressively higher fields. The $\Delta w$ obtained for $^{136}$Cs was 12702.28; the error is twice the standard deviation from a least-squares fit. With the use of the Fermi-Segré formula, Eq. (III.14), the nuclear magnetic moment is calculated to be $\mu_1 = +3.68(4)$; the 1% error in $\mu_1$ is intended to include a possible hyperfine-structure anomaly.

The problem of using the appropriate rf power for the study of $^{136}$Cs was overcome by using the same rf power needed to obtain a maximum signal for $^{133}$Cs at the same frequency as the one to be used in the search. The program TEE PEE-l (see Sec. III.A.3) calculated the ratio of the matrix elements of $J_z$ for $^{136}$Cs and $^{133}$Cs at different fields. This ratio was found to be close to one for the field region studied. In run 849 (Fig. IV.18) the peak point was taken at the same power level as that required for $^{133}$Cs; the side buttons were taken at slightly different rf power.

Figures IV.14 and IV.15 show the decay scheme of the resonance and chemistry samples of two different bombardments. Figures IV.16 through IV.21 show the resonances observed for $^{136}$Cs. Table IV.5 lists the results of a least-squares fit of the resonances.
Fig. IV.15 Decay scheme for $^{136}$Cs resonance, normalization, and chemistry buttons (Run 849).
Fig. IV.16 Resonance of $^{136}\text{Cs}$ (Run 838).
Fig. IV.17 Resonances of $^{136}$Cs observed on atomic-beam machine VI. (a) Run 840, (b) Run 841A.
Fig. IV.18 Resonance of $^{136}\text{Cs}$ at 118.42 G—the buttons about the peak were exposed at different rf powers to establish the optimum power to be used (Run 349).
Fig. IV.19 Resonance of $^{136}$Cs (Run 850).
$^{136}\text{Cs}$

$H = 268.67(3)$

Fig. IV.20 Resonance of $^{136}\text{Cs}$ (Run 851).
Fig. IV.21 Resonances of $^{136}$Cs (Run 854).
Table IV.5. Summary of $^{136}$Cs results.  
$I = 5$, $J = 1/2$, State $^2S_{1/2}$

<table>
<thead>
<tr>
<th>Run</th>
<th>Calibration isotope</th>
<th>Calibration freq. (MHz)</th>
<th>Field (G)</th>
<th>Observed$^a$ freq. (MHz)</th>
<th>Residual $v_{obs} - v_{calc}$ (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$g_I &gt; 0$</td>
</tr>
<tr>
<td>836</td>
<td>$^{133}$Cs</td>
<td>1.185(20)</td>
<td>3.384(57)</td>
<td>0.862(25)</td>
<td>+ 1.0</td>
</tr>
<tr>
<td>838</td>
<td>$^{85}$Rb</td>
<td>11.108(10)</td>
<td>23.373(21)</td>
<td>5.965(12)</td>
<td>- 6.4</td>
</tr>
<tr>
<td>840</td>
<td>$^{133}$Cs</td>
<td>25.359(6)</td>
<td>71.110(17)</td>
<td>18.340(10)</td>
<td>- 4.3</td>
</tr>
<tr>
<td>841</td>
<td>$^{133}$Cs</td>
<td>25.655(6)</td>
<td>71.925(17)</td>
<td>18.555(10)</td>
<td>- 2.3</td>
</tr>
<tr>
<td>849</td>
<td>$^{133}$Cs</td>
<td>42.776(12)</td>
<td>118.416(32)</td>
<td>30.833(7)</td>
<td>- 11.1</td>
</tr>
<tr>
<td>850A</td>
<td>$^{133}$Cs</td>
<td>58.835(11)</td>
<td>160.981(29)</td>
<td>42.307(15)</td>
<td>+ 7.4</td>
</tr>
<tr>
<td>850B</td>
<td>$^{133}$Cs</td>
<td>58.837(11)</td>
<td>160.986(29)</td>
<td>42.310(8)</td>
<td>+ 8.9</td>
</tr>
<tr>
<td>851</td>
<td>$^{133}$Cs</td>
<td>101.182(12)</td>
<td>268.672(30)</td>
<td>72.200(12)</td>
<td>+ 2.6</td>
</tr>
<tr>
<td>854A</td>
<td>$^{133}$Cs</td>
<td>149.872(11)</td>
<td>385.017(25)</td>
<td>106.043(12)</td>
<td>- 1.8</td>
</tr>
<tr>
<td>854C</td>
<td>$^{133}$Cs</td>
<td>149.876(11)</td>
<td>385.026(25)</td>
<td>106.047(12)</td>
<td>- 0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$g_I$</th>
<th>$A$ (MHz)</th>
<th>$\Delta \nu$ (MHz)</th>
<th>$\mu_I$ (nm)</th>
<th>$g_I \times 10^{-4}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>2309.4(2.5)</td>
<td>12702.1(4)</td>
<td>+ 3.68</td>
<td>+ 4.01</td>
<td>2.30</td>
</tr>
<tr>
<td>Negative</td>
<td>2432.3(2.8)</td>
<td>13378.1(15)</td>
<td>- 3.88</td>
<td>- 4.22</td>
<td>281.53</td>
</tr>
</tbody>
</table>

$^a$ Transitions are between $(11/2,-9/2) \leftrightarrow (11/2,-11/2)$. 
V. DISCUSSION

In Sec. III.B we discussed the agreement of the different models with the observed spin and magnetic moments. We felt that the discussion there would come more naturally than in a separate section. However, the question of possible prolate or oblate deformations for the neutron-deficient isotopes was not answered. We noted that the calculated magnetic moment agrees rather well for an oblate deformation. Though the spin-1/2 prediction of the Nilsson model depends on the choice of the $2d_{5/2}$ to lie lower than the $1g_{7/2}$ level, the possibility of an oblate deformation seems rather interesting in itself. The usual trend of deformed nuclei is toward prolate rather than oblate deformations; Refs. 4 and 82 have given simplified arguments to that effect.

The possibility of deformed nuclear shape of the neutron-deficient isotopes in the Ba region was first suggested by Mottelson. Experimental observation of the excited levels of some of the neutron-deficient Ba isotopes showed that these nuclei are deformed. Theoretical calculations based on the Nilsson calculation of the deformed shape (with pairing) also implied deformation for nuclei in this region. On the basis of these results, we can conclude that $^{125}$Cs and possibly $^{127}$Cs are deformed, whereas $^{129}$Cs and $^{131}$Cs are in the transition region. Kumar and Baranger, using a pairing plus quadrupole interactions, found that oblate deformations are favored.

The deformation of the nuclear shape should be primarily due to the neutron configuration. This depends critically on the single-particle levels chosen. However, for the choice of Fig. III.5 the oblate deformation for neutron number 70 is favored. Should this be the case, the assignment of the 55th proton to the 34th Nilsson level seems to be doubly desirable. It also brings out another possibility; e.g., the assignment of the 55th proton to $\Omega = 1/2$ levels belonging to lower single-particle levels (such as the $g_{9/2}$ or $p_{1/2}$ levels) which come up sufficiently for large oblate deformation. One such level is the 26th level of Nilsson's diagram, which for large oblate deformation $\eta \approx -6$ has a magnetic moment $\approx +1.49$ nm. This assignment is motivated by the fact that the energy levels in the actinide region follow those predicted by Nilsson for a certain deformation. It would be interesting to see if this agrees with
the prediction of the superconductivity model or a Nilsson's picture with a different single-particle level ordering.

In conclusion, we feel that such a long chain of studied isotopes should be utilized in terms of evaluating possible residual interactions, and as a test for a possible prediction of the nuclear properties of a nucleus when the properties of neighboring nuclei are known.
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