INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI
Sediment Transport by Turbidity Currents

by

Kyle Thomas Winslow

B.S. (University of California, Berkeley) 1995
M.S. (University of California, Berkeley) 1996

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy
in
Engineering - Civil Engineering
in the GRADUATE DIVISION
of the

UNIVERSITY OF CALIFORNIA, BERKELEY

Committee in charge:

Professor Rodney J. Sobey, Chair
Professor Robert L. Wiegel
Professor William E. Dietrich

Spring 2001
Abstract

Sediment Transport by Turbidity Currents

by

Kyle Thomas Winslow

Doctor of Philosophy in Civil Engineering

University of California, Berkeley

Professor Rodney J. Sobey, Chair

Complete field equations have been established for turbidity current flow. A Modified Three Equation Model has been developed to predict the fate of turbidity currents. Particular attention has focused on the impacts of increased sediment concentration on the field equations and the numerical predictions. Functional relationships for closure hypotheses have been chosen after careful review.

Previous investigations have invoked the dilute suspension approximation for sediment concentrations up to a few percent by volume. Experimentally derived relationships for the effect of sediment concentration on settling velocity show that the dilute suspension approximation is not necessarily valid, even at these low concentrations.

Results show that initial conditions, such as current height, velocity, and sediment concentration, have a short-lived effect on model predictions. Channel properties, such as slope and bed friction, control the long-term evolution of turbidity currents. The Modified Three Equation Model is highly sensitive to the closure relationships. Properties of the
fluid-sediment mixture, such as kinematic viscosity and sediment grain size, can lead to predictions of contrary fates for turbidity currents.

In application to Scripps Submarine Canyon, the Modified Three Equation Model predicts a turbidity current with a smaller acceleration than the current predicted by the Parker Three Equation Model.

Turbidity current flow is most sensitive to the sediment entrainment function. The steep nature of this function leads to extreme changes in the predicted sediment entrainment rate for minor changes in the input parameters. Unfortunately, these input parameters include the settling velocity and the shear velocity, two properties of the fluid mixture that are difficult to ascertain with a high level of accuracy. Continued effort needs to focus on this relationship.

The Modified Three Equation Model does not account for shear at the upper interface of the turbidity current. This needs to be addressed. Future laboratory investigations could also provide much needed data against which to test formulations for sediment entrainment, near-bed sediment concentration, and water entrainment, and the effects of ambient currents on turbidity currents.

R. J. Sobey
22 March 01
Acknowledgments

First and foremost I would like to thank Prof. Rod Sobey for his guidance and support throughout this project. Although progress was frequently halted by my desire to find other activities with which to amuse myself and thus prolong this project, Prof. Sobey was always there with a gentle reminder that this dissertation was not going to write itself. His constant admonition not to believe everything I read, just because it is published in a respected journal, is a lesson I will not soon forget.

The assistance of Prof. Robert Wiegel and Prof. William Dietrich is greatly appreciated. Their comments and suggestions have improved this dissertation.

I would also like to thank my parents for their continued love and support. I’m sure there have been times when they were unsure if this chapter of my life would ever come to an end, but they never let on to it. Perseverance is a beautiful thing.

Brendan DeTemple was constantly available for consultation on all matters relating to fluid mechanics. His assistance in experiments on the fastest way to remove the contents from a bottle of beer proved both insightful and ingenious. Janet Ekstrom provided valuable assistance in editing this manuscript. Her assistance is greatly appreciated.

This project was funded in part by the Civil Engineering Department at the University of California at Berkeley through the Chevron Fund, the Faustman Fund, and the Ira Abraham, Sr. and Georgina Koenig Abraham Scholarship Fund. Financial support from the Environmental Water Resources Group is acknowledged and appreciated.
# Table of Contents

Acknowledgments iii  
List of Figures vii  
List of Tables x  

Chapter 1. Introduction  
1.1 Definition and Description of Turbidity Currents 1  
1.2 Historical Work and Engineering Significance 5  
1.3 Causation of Turbidity Currents 8  
1.4 Present Study 12  

Chapter 2. Development of Field Equations 15  
2.1 Conservation Equations 16  
2.1.1 Density of Mixture 16  
2.1.2 Settling Velocity 18  
2.1.3 Conservation of Mixture Mass 18  
2.1.4 Conservation of Mixture Momentum 19  
2.1.5 Conservation of Sediment 22  
2.2 Vertically-Integrated Governing Equations 23  
2.3 Cross-Section-Integrated Equations 27  
2.4 Definition of Integral Variables 32  
2.5 Constrained Flow 33  
2.6 One Dimensional Flow 37  
2.7 Rectangular Channel with Slope Constraints 40  
2.8 Steady State 43  
2.9 Three Equation Model of Parker et al. (1986) 45  
2.10 Matrix Representation of Governing Equations 47  
2.11 Numerical Algorithm 50  

Chapter 3. Existing Closure Schemes 52  
3.1 Water Entrainment 53  
3.2 Sediment Entrainment 54  
3.3 Bed Shear Stress 59  
3.4 Bed Sediment Concentration 73  
3.5 Summary of Closure Schemes 73  

Chapter 4. Approximations Used in Turbidity Current Modeling 76  
4.1 Dilute Suspension Approximation 76  
4.1.1 Adjustment of Settling Velocity through Viscosity 81  
4.1.2 Direct Adjustment of Settling Velocity 84  
4.2 Channel Containment Approximation 86  
4.3 One-Dimensional Flow Approximation 90  
4.4 Channel Geometry Approximations 91  

4.5 Quiescent Ambient Fluid Approximation
4.6 Steady State Flow Approximation
4.7 Boundary Layer Assumption
4.8 Summary of Approximations

Chapter 5. Numerical Model Results
5.1 Sensitivity to Initial Conditions
  5.1.1 Initial Sediment Concentration
  5.1.2 Initial Current Height
  5.1.3 Initial Current Velocity
5.2 Sensitivity to Channel Properties
  5.2.1 Bed Slope
  5.2.2 Bed Friction
5.3 Sensitivity to Closure Relationships
  5.3.1 Updated Closure Relationships
  5.3.2 Sediment Entrainment Relationship
  5.3.3 Near-bed Sediment Concentration
  5.3.4 Water Entrainment Relationship
5.4 Sensitivity to Fluid Mixture Properties
  5.4.1 Kinematic Viscosity
  5.4.2 Grain Size Diameter
5.5 Limitations of the Dilute Suspension Approximation
  5.5.1 Run Comparisons With and Without the Dilute Suspension Approximation
  5.5.2 Sensitivity to Settling Velocity
5.6 Conclusions

Chapter 6. Comparison with Previous Investigations
6.1 Comparisons with Parker et al.’s (1986) Three Equation Model
6.2 Ignition
6.3 Phase-Plane Diagrams
6.4 Hydraulic Jumps
6.5 Parker et al. (1986) Four Equation Model

Chapter 7. Summary and Conclusions
7.1 Summary
7.2 Model Results
7.3 Directions for Future Research

References

Appendix A. Cross-Section-Integrated Conservation Equations
A.1 Conservation of Mixture Mass
A.2 Conservation of Mixture Momentum
A.3 Conservation of Sediment
Appendix B. Transformation of Field Equations to Matrix Representation 224
  B.1 Conservation of Mixture Mass 224
  B.2 Conservation of Mixture Momentum 225
  B.3 Conservation of Sediment 227
  B.4 Matrix Equation 227

Appendix C. Transformation to Field Equations to First Order ODE System 230

Appendix D. Graphical Model Output 235
  D.1 Series F600 236
  D.2 Series F200 288
  D.3 Series D800 305
  D.4 Series P6100 315
  D.5 Series P6200 335

Appendix E. Garcia (1985) Laboratory Data and Empirical Relationship for Sediment Entrainment 365
  E.1 Introduction 365
  E.2 Attempted Reproduction of Garcia’s Results 367
  E.3 Influence of Measuring Equipment on Experimental Results 377
  E.4 Conclusions 379
List of Figures

1.1 Physical Processes in Turbidity Currents 3

2.1 Channel Geometry 17
2.2 Vertical Limits of Integration 24
2.3 Channel Geometry for Cross-Section-Integrated Flow 28
2.4 Channel Geometry for Constrained Flow 34
2.5 Channel Geometry for One Dimensional Flow 38
2.6 Channel Geometry for Wide, Rectangular Channel 41

3.1 Entrainment of Ambient Water 56
3.2 Comparison of Ambient Water Entrainment Functions with Selected Experimental Data 60
3.3 Exchange of Sediment with Erodible Bed 62
3.4a Comparison of Sediment Entrainment Formulations \( Z_1 \) 64
3.4b Comparison of Sediment Entrainment Formulations \( Z_2 \) 65
3.4c Comparison of Sediment Entrainment Formulations \( Z_3 \) 66
3.5a Initial Value of \( Z \) for Given \( U_0 \) and \( D_i \) 69
3.5b Initial Value of \( Z \) for Given \( U_0 \) and \( D_i \) - Settling Velocity Corrected for \( S = 0.1 \) 71
3.6 Parker's Formulation for Bed Sediment Concentration 74

4.1 Setting Velocity According To Dietrich (1982) 80
4.2 Setting Velocity From Dietrich with Corrections for Concentration Effect 83
4.3 Effect of Concentration on Fall Velocity - Comparison of Formulations to Laboratory Data 87
4.4 Detailed Topography of Scripps Submarine Canyon and Shelf 93
4.5 Edge Waves Near Scripps Canyon 99

5.1a Effects of Suspended Sediment Concentration - Runs F598a-F605a 109
5.1b Effects of Suspended Sediment Concentration - Runs F598a-F605a - Sediment Entrainment Variables 110
5.1c Effects of Suspended Sediment Concentration - Extended Calculations 111
5.2a Effects of Initial Current Height - Runs F600c-F605c 114
5.2b Effects of Initial Current Height - Extended Calculations 115
5.3a Effects of Initial Current Velocity - Runs F600d-F605d 117
5.3b Effects of Initial Current Velocity - Extended Calculations 118
5.4 Effects of Bottom Slope - Runs F610-F617 120
5.5a Effects of Bed Drag Coefficient - Runs F629-F634 123
5.5b Effects of Bed Drag Coefficient - Extended Calculations 124
5.5c Effects of Bed Drag Coefficient - Runs F629-F634 Sediment Entrainment Variables 125
5.6a Effects of Closure Schemes - Runs F660 - F661 129
5.6b Effects of Closure Schemes - Runs F660 - F661 - Sediment Entrainment Variables
5.7a Effects of Closure Schemes - Runs F660-F664
5.7b Effects of Closure Schemes - Closure Parameters \( e_w, E_w \) and \( r_o \)
5.8a Effects of Kinematic Viscosity - Runs F674b-F667b
5.8b Effects of Kinematic Viscosity - Runs F674b-F667b - Sediment Entrainment Variables
5.8c Effects of Kinematic Viscosity - Extended Calculations
5.9 Effects of Sediment Size - Runs F679-F684
5.10a Effects of Dilute Suspension Approximation: \( S_o = 0.0005 \)
5.10b Effects of Dilute Suspension Approximation: \( S_o = 0.0005 \) Extended Calculations (to 5000 m)
5.11a Effects of Dilute Suspension Approximation: \( S_o = 0.001 \)
5.11b Effects of Dilute Suspension Approximation: \( S_o = 0.001 \) Extended Calculations (to 5000 m)
5.12a Effects of Dilute Suspension Approximation: \( S_o = 0.003 \)
5.12b Effects of Dilute Suspension Approximation: \( S_o = 0.003 \) Extended Calculations (to 5000 m)
5.13a Effects of Dilute Suspension Approximation: \( S_o = 0.005 \)
5.13b Effects of Dilute Suspension Approximation: \( S_o = 0.005 \) Extended Calculations (to 5000 m)
5.14a Effects of Dilute Suspension Approximation: \( S_o = 0.008 \)
5.14b Effects of Dilute Suspension Approximation: \( S_o = 0.008 \) Extended Calculations (to 5000 m)
5.15a Effects of Dilute Suspension Approximation: \( S_o = 0.01 \)
5.15b Effects of Dilute Suspension Approximation: \( S_o = 0.01 \) Extended Calculations (to 5000 m)
5.16a Effects of Dilute Suspension Approximation: \( S_o = 0.03 \)
5.16b Effects of Dilute Suspension Approximation: \( S_o = 0.03 \) Extended Calculations (to 5000 m)
5.17a Effects of Dilute Suspension Approximation: \( S_o = 0.05 \)
5.17b Effects of Dilute Suspension Approximation: \( S_o = 0.05 \) Extended Calculations (to 5000 m)
5.18a Effects of Settling Velocity - Runs F224-F229
5.18b Effects of Settling Velocity - Runs F224-F229 - Extended Calculations

6.1 Current Development from Ignition (from Parker et al. 1986)
6.2 Reproduction of Figure 6.1 with the Three Equation Model
6.3 Current Development - Parker's Test Case Calculated with Three Equation Model (Parker et al. 1986)
6.4a Current Development - Parker's Test Case to \( x = 230 \) meters Calculated with Updated Closure Schemes
6.4b Current Development - Parker's Test Case to \( x = 1000 \) meters Calculated with Updated Closure Schemes - Extended Calculations
6.5 Comparison of Effects of Closure Schemes on Predictions of the Three Equation Model
6.6 Sediment Entrainment Function (Garcia, 1989)
6.7 Current Development - Parker's Test Case Calculated with the Modified Three Equation Model
6.8 Comparison of Effects of Closure Schemes on Predictions of the Three Equation Model and the Modified Three Equation Model
6.9 Phase-plane Diagram (from Parker et al. 1986)
6.10a Reproduction of Phase-plane Diagram from Parker et al. (1986)
6.10b Phase-plane Diagram with $U, \psi$
6.11 Phase-plane Diagram - Extended from Figure 6.10a
6.12a Normalized Phase-plane Diagram - Updated Closure Schemes - Series P6200
6.12b Phase-plane Diagram - Updated Closure Schemes - Series P6200
6.13 Normalized Phase-plane Diagram - Extended Calculations Series P6200
6.14a Run P6212 (through 1000 meters)
6.14b Run P6212 (through 4000 meters)
6.15 Investigation of Equation 6.1 Applied to Figure 6.14b
6.16 Long Term Current Development (50 km)

E.1 Garcia's 1985 Turbidity Current Data: $R_p$ vs. $D_t$
E.2 Garcia's 1985 Turbidity Current Data: $D_t$ vs. $w_t$
E.3 Attempted Reproduction of Garcia's Results: $E_t$
E.4 Potential Error in $E_t$ Calculations Due to Velocity Measurements
E.5 Log Representation of Potential Error in $E_t$ Calculations Due to Velocity Measurements
List of Tables

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Modified Three Equation Model</td>
<td>45</td>
</tr>
<tr>
<td>2.2</td>
<td>Matrix Representation of Modified Three Equation Model</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Closure Schemes Used in the Modified Three Equation Model</td>
<td>75</td>
</tr>
<tr>
<td>4.1</td>
<td>Richardson and Zaki Settling Velocity Formulation (1954)</td>
<td>86</td>
</tr>
<tr>
<td>4.2</td>
<td>Summary of Measured Ambient Currents in Submarine Canyons</td>
<td>102</td>
</tr>
<tr>
<td>5.1</td>
<td>Initial Conditions for Figures 5.1a, 5.1b, and 5.1c</td>
<td>108</td>
</tr>
<tr>
<td>5.2</td>
<td>Initial Conditions for Figures 5.2a and 5.2b</td>
<td>113</td>
</tr>
<tr>
<td>5.3</td>
<td>Initial Conditions for Figures 5.3a and 5.3b</td>
<td>116</td>
</tr>
<tr>
<td>5.4</td>
<td>Initial Conditions for Figure 5.4</td>
<td>121</td>
</tr>
<tr>
<td>5.5</td>
<td>Initial Conditions for Figures 5.5a, 5.5b, and 5.5c</td>
<td>122</td>
</tr>
<tr>
<td>5.6</td>
<td>Investigation of the Influence of Closure Schemes Used in Figures 5.6 and 5.7</td>
<td>127</td>
</tr>
<tr>
<td>5.7</td>
<td>Initial Conditions for Figures 5.8a, 5.8b, and 5.8c</td>
<td>139</td>
</tr>
<tr>
<td>5.8</td>
<td>Initial Conditions for Figure 5.9</td>
<td>140</td>
</tr>
<tr>
<td>5.9</td>
<td>Initial Conditions for Figures 5.10 through 5.17</td>
<td>143</td>
</tr>
<tr>
<td>5.10</td>
<td>Effects of the Dilute Suspension Approximation</td>
<td>145</td>
</tr>
<tr>
<td>6.1</td>
<td>Closure Schemes Used in Run 0506h and Series P6100</td>
<td>166</td>
</tr>
<tr>
<td>6.2</td>
<td>Closure Schemes Used in Run R0507 and Series P6200</td>
<td>173</td>
</tr>
<tr>
<td>6.3</td>
<td>Sensitivity of Ignition Parameters to Closure Schemes</td>
<td>180</td>
</tr>
<tr>
<td>6.4</td>
<td>Summary of Initial Conditions for Figures 6.10 and 6.11 (Series P6100)</td>
<td>187</td>
</tr>
<tr>
<td>6.5</td>
<td>Summary of Initial Conditions for Figures 6.12 and 6.13 (Series P6200)</td>
<td>192</td>
</tr>
<tr>
<td>B.1</td>
<td>Summary of Transformed Partial Differential Equations</td>
<td>228</td>
</tr>
<tr>
<td>B.2</td>
<td>Matrix Representation of Modified Three Equation Model</td>
<td>229</td>
</tr>
<tr>
<td>C.1</td>
<td>Summary of Simultaneous Ordinary Differential Equations</td>
<td>234</td>
</tr>
<tr>
<td>D.1</td>
<td>Summary of Input Variables for Run Series F600</td>
<td>237</td>
</tr>
<tr>
<td>D.2</td>
<td>Summary of Input Variables for Run Series F200</td>
<td>289</td>
</tr>
<tr>
<td>D.3</td>
<td>Summary of Input Variables for Run Series D800</td>
<td>306</td>
</tr>
<tr>
<td>D.4</td>
<td>Summary of Input Variables for Run Series P6100</td>
<td>316</td>
</tr>
<tr>
<td>D.5</td>
<td>Summary of Input Variables for Run Series P6200</td>
<td>336</td>
</tr>
<tr>
<td>E.1</td>
<td>Inlet Conditions from Garcia's Experiments</td>
<td>366</td>
</tr>
<tr>
<td>E.2</td>
<td>Layer-Averaged Values from Garcia's Experiments</td>
<td>368</td>
</tr>
<tr>
<td>E.3</td>
<td>Parameters Evaluated in Garcia's Experiments</td>
<td>369</td>
</tr>
<tr>
<td>E.4</td>
<td>Reported vs. Calculated Values for Particulate Reynolds Number $R_p$</td>
<td>372</td>
</tr>
<tr>
<td>E.5</td>
<td>Reported vs. Calculated Values for Settling Velocity $w_s$</td>
<td>374</td>
</tr>
</tbody>
</table>
Chapter 1. Introduction

1.1 Definition and Description of Turbidity Currents

Turbidity currents are sediment laden currents driven by the gravitational force acting on the sediment fraction. They are part of a broad class of flows defined as gravity currents, the general term for any flow governed by a gravitational force acting on a difference in density between two fluids. This density difference can be the result of a difference in temperature between the two fluids or a difference in the concentration of a solute, such as salt. Other examples of gravity currents include thermal underflows and overflows, methane gas flows in mine tunnels, the motion of weather fronts, pyroclastic flows, salt water flows into fresh water estuaries, and debris flows.

Turbidity currents are complex flows which share traits with suspended sediment flows, two-phase flows, and stratified flows; however, these flow types alone cannot successfully describe turbidity current flow. General suspended sediment transport does not aptly describe turbidity current flow, for the sediment fraction in turbidity currents drives the flow, whereas in traditional channelized suspended sediment transport, the concentrations of sediment are generally much lower and the sediment is advected by the flow.

Although results from stratified flow theory have contributed to the understanding of certain types of mixing in turbidity currents, the flow of turbidity currents is not simple stratified flow. The substances which contribute to the density differences seen in most stratified flows, namely heat and salt, are conserved. The total amount of salt remains constant in a two-layer system involving salt water. Turbidity currents, on the other hand,
are non-conservative with respect to the sediment; they are capable of exchanging sediment with the bed through erosion and deposition. Another way to describe this is in terms of the buoyancy flux. Stratified flows generally have a constant buoyancy flux, whereas the buoyancy flux of a turbidity current is proportional to the sediment fraction, which is variable.

Sediment interaction between the flow and the bed is the primary factor differentiating turbidity currents from other flows. Quantification of this interaction is crucial in their successful modeling.

Stratified flow problems typically involve fluids with similar viscosities. Changes in viscosity are ignored in the development of the theory governing such flows. The differences in viscosities between turbidity currents and the ambient fluid, however, may approach levels where viscosity effects must be taken into account.

The complexity of turbidity current flow stems from the array of processes contributing to the flow, as outlined in Figure 1.1. The primary driving force for turbidity current flow is the sediment fraction, which varies throughout the current. Erosion or deposition may increase or decrease the sediment fraction, thus changing the driving force. In fact, a turbidity current may be eroding sediment in one location while depositing sediment in another. Furthermore, as a turbidity current erodes sediment, the gravitational force increases, thus increasing the current's velocity and its ability to erode more sediment. This self-reinforcing cycle has been the focus of much attention, and will be discussed in considerable detail.
Figure 1.1 Physical Processes in Turbidity Currents
The balance of flow processes may differ from location to location, so that a single set of equations cannot describe the entire current. For example, the head of the current may be accelerating or decelerating, while the body of the current is at steady flow. Such difficulties have required experimental and numerical investigations of turbidity currents to focus on specific portions of the flow (Middleton 1966a, 1966b, 1966c). A single model covering the complete spectrum of turbidity current flow is not yet available.

Turbidity currents are generally classified into two types of currents: accelerating or decelerating, depending on the evolution of the velocity of the current (Parker et al. 1986). A current that is initially accelerating may eventually transition to a decelerating current. These two types of currents can be sub-classified as either erosional or depositional, depending on whether the current is gaining or losing sediment.

Turbidity currents can also be classified into two groups according to the duration of their existence: continuous and surge-type (Hay 1987a). Continuous currents flow for periods from several hours to several days. They are driven by a sediment source that is near constant for a considerable length of time. Examples include currents initiated by rivers carrying a large sediment load, and those initiated by mine tailings discharges. Heezen (1964) reported on cable breaks in the Congo Submarine Canyon that correlated with periods of large sediment discharge in the Congo River. He suggested that the river discharge continues as a turbidity current, which is supported by the lack of a characteristic delta at the river mouth.
Surge-type turbidity currents are generally much shorter in duration. They may pass a given location in a matter of minutes. Surge currents are commonly generated by slope failures (Hay 1987a). These can be small failures, such as the localized slump of a levee channel, or larger failures stemming from tectonic activity (Heezen and Ewing 1952).

The easiest turbidity currents to study are those caused by a continuous inflow of sediment. These currents occur during known periods, such as during increased flows associated with spring runoff. Consequently, more data exists on turbidity flows generated by these factors. Furthermore, it is generally easier to collect data in environments where these types of flows tend to occur than it is to collect data in submarine canyons or other sites on the continental slope.

Turbidity currents can also be classified according to the topography over which they flow. Unconfined flows occur over broad slopes of relatively constant inclination. Confined turbidity currents flow in some type of channel, whether it be a submarine (ocean) or sublacustrine (lake) canyon, or a channel formed by a tailings discharge. The channel constrains the flow from spreading laterally, allowing the flow to reach a higher velocity and travel further from its point of origin.

1.2 Historical Investigations and Engineering Significance

Turbidity currents, a phrase coined by Johnson (1938), have been observed in oceans, lakes, and reservoirs, and are of interest to geologists, fluid mechanicians, civil engineers, coastal engineers, and petroleum engineers. They play a vital role in near-shore sediment transport, and provide a mechanism for the transport of sediment to the deep ocean.
(Chamberlain 1964, Shepard 1951). Turbidity currents can assist in the disposal of waste, as evidenced in the discharge of mine tailings (Normark and Dickson 1976, Carstens and Tesaker 1972). They can adversely affect the operation of reservoirs by spoiling water releases, decreasing storage capacity, and burying water supply structures (Gould 1951). The design of underwater structures must take into account the potentially destructive forces levied by turbidity currents (Heezen 1963). The factors governing the generation and sustenance of turbidity currents must be understood.

Forel, in 1885, was the first known observer of turbidity currents (Heezen 1963). He observed undercurrenct generated by the inflow of the Rhône River into Lake Geneva. Forel's observations of a large sublacustrine canyon led him to the conclusion that the canyon was created by the sediment-laden Rhône River; he attributed the formation of the canyon to both erosional forces and the deposition of sediment at the lateral edges of the current.

In Lake Mead, Nevada, turbidity currents were first observed in March, 1935 (Grover and Howard 1938, Gould 1951). During the 14 years following construction of the Hoover Dam, sediment had accumulated immediately behind the dam to a thickness of over 30 meters. This was staggering considering that the Hoover Dam is located 200 kilometers from the inflow of the Colorado River into Lake Mead. These observations supported the idea that turbidity currents could carry sediment over considerable distances.

The recognition of the potentially harmful effects of these currents on reservoirs led to a field study by the U. S. Bureau of Reclamation (1941, 1947). This effort produced a
twelve-year history of changes in reservoir sediment levels. Additional studies by the U.S. Geological Survey (winter 1947 to spring 1949) investigated lake sediments and the conditions affecting their deposition and compaction.

Daly (1936) renewed interest in turbidity currents with the claim that they were responsible for the erosion of submarine canyons. Interest among geologists soon focused on the capacity for high concentration currents to transport sand and gravel into deep water and form graded beds.

Heezen et al. (1952) reported on breaks in a series of submarine telegraph cables following an earthquake in the Grand Banks, off Newfoundland, on November 18, 1929. He gave evidence suggesting the theory that a large turbidity current, initiated from a submarine slump, was responsible for the cable breaks. He estimated the current speed at over 20 meters per second! Heezen's article (1952) drew attention to high speed turbidity currents, and offered an explanation for gravel found on the continental shelf. Heezen et al. (1954) later reported on the discovery of a graded bed of silt south of the Grand Banks, proving further evidence that earthquake induced slides could generate turbidity currents.

Stow (1986) classified turbidity currents by sediment concentration. High density turbidity currents carry 25-250 g/l, while low density currents carry 0.025-2.5 g/l. These limits correspond to volume concentrations of 0.01 to 0.10 for high density currents, and 0.00001 to 0.001 for low density currents. Field observations have been limited to low density turbidity currents (Hay 1987a). Observations of high density, catastrophic currents, such as the hypothesized 20 meters-per-second current generated by the Grand
Banks Earthquake (Heezen et al. 1952) do not yet exist; it is nearly impossible to be “in the right place at the right time” to measure a current of this nature. Attempts to record turbidity current velocities in submarine canyons have resulted in damaged or lost instrumentation (Inman et al. 1976). Further discussion of measured currents appears in Chapter 4.

1.3 Causation of Turbidity Currents

Turbidity currents can be generated by a variety of factors. Most of these involve the initiation of motion of a large submarine sediment mass, which subsequently moves downslope, mixes with the surrounding water, and evolves into a turbidity current. The following discussion focuses on mechanisms of mass movement, such as earthquake-induced landslides, sediment slumps, and current-induced mass movements. Turbidity currents can also be generated by the discharge of suspended sediment into a body of water. Examples include discharges of sediment-laden rivers into lakes or reservoirs, and mine tailings discharges.

Submarine mass movements caused by earthquakes are perhaps the easiest to comprehend. These mass movements are generally large, so that their effects are more visible. The Grand Banks Earthquake of 1929 (Richter magnitude 7.2) generated a submarine slide which traveled downslope, breaking numerous telegraph cables in its path. The timing of the cable breaks, evidenced by the sudden loss of service, provided a record from which the speed of the current could be estimated (Heezen and Ewing 1952).
Similar events have occurred in other locations. After the Orleansville earthquake of 1954, several submarine cables were destroyed in the Mediterranean Sea off the coast of Algeria (Heezen 1963). Additional cable breaks occurred in the western New Britain Trench, off the coast of New Guinea, following earthquakes in 1966 and 1968 (Krause et al. 1970). In each case, records of the interruptions in service over the submarine cables, coupled with the known location of the cable breaks, have enabled researchers to estimate the speed of the turbidity currents blamed for the destruction of the cables.

Mass movements can also be generated in the absence of earthquakes. Shepard and Emery (1941) report on a sediment slump which occurred in the Redondo Submarine Canyon in Southern California during a period of calm weather. The slump was observed by fishermen on a pier extending over the head of the canyon. The fishermen reported that the episode lasted about 30 minutes, during which “swirls of sediment” surfaced. Subsequent investigation revealed that the sediment had deepened by as much as 25 feet.

Morgenstern (1967) noted an increase in submarine sliding in locations where rapid sediment deposition occurs. For example, slides are frequent in the heads of submarine canyons, near river mouths, and near mine tailings discharges. In the latter case, deposition is so rapid that surge-type turbidity currents can occur every few days (Hay 1987b).

Rapid deposition can yield slope failure for two main reasons. First, deposition may cause sediment to accumulate with slopes greater than the angle of repose of the sediment. This is termed over-steepening, and is probably the most common cause of submarine
slumping. Second, rapid deposition produces underconsolidated sediment, which can be more prone to slumping (Morgenstern 1967).

Pratson et al. (1996) attribute an increase in excess pore pressure, and subsequent seafloor destabilization, to such rapid sedimentation. Excess pore pressure is due to a lag between the deposition and the associated consolidation, which is increased by more rapid deposition. This lag leads to a build up of excess pore pressure, which decreases the effective stress of the sediment deposit, thus lowering its resistance to sliding.

Slumping in subaqueous sediments can also be caused by the presence of organic matter (Dill 1964). In the near-shore coastal environment, seaweed and sea grass are transported and deposited along with the sediment in the heads of submarine canyons. The long, thin seaweed and sea grass become intertwined with the deposited sediment, providing additional shear strength to the deposit. Dill, after observing conditions in Scripps Canyon utilizing SCUBA equipment, reports that bales of this “organic mat” can be picked up by hand. The mat’s ability to resist slump failure must be considerably greater than that of sedimentary deposits without this binding mechanism.

As organic material decays, it loses strength and becomes a plane on which the sediment can fail. Chamberlain (1964) reports on the rapid rates of decomposition of organic detritus in Scripps Submarine Canyon in Southern California, claiming that the coefficient of friction of the sediment must be lowered by the slippery, decomposing detrital material. Furthermore, Chamberlain mentions that this organic decomposition occurs at the down-canyon end of accumulated sediment masses in the heads of submarine canyons. Failure at
this location would mean a loss of support for the sediment immediately upstream from the failure, possibly causing that to fail as well. This concept is termed retrogressive slumping.

Organic matter can influence sediment deposit stability in a second manner. As organic matter decays, methane and carbon dioxide are produced. These gas bubbles, rising from decaying collections of organic matter (Dill 1964), may effect soil stability through an influence on the pore pressure. Others (Shepard 1951, Dill 1964, Chamberlain 1964, Morgenstern 1967, Pratson et al. 1996) have mentioned that methane gas, escaping from decaying organic matter, can decrease sediment strength, leading to significant creep movement. Monroe (1969) reported on slumping structures caused by the production of gases associated with decomposing organic material. He performed experiments in a pressure tank where the interstitial water in a bed of stratified sand was replaced with carbon dioxide. He observed slumping structures when the pressure in the tank was reduced, which caused the gas to leave solution.

Water currents can contribute to the generation of turbidity currents. At sufficient velocities, depending on sediment particle size and relative density, water currents can erode sediment. These sediment-laden currents may then progress into turbidity currents, just as sediment-laden rivers can. Ocean currents and currents in submarine canyons can be generated by wind, tides, waves, and suspended sediment. These mechanisms tend to act in certain depth regimes. Shallow water currents (depths less than 50 meters) induce setup, providing a hydraulic gradient for flow away from the shore. Deep water currents, (depths greater than 100 meters) usually oscillate, flowing both up and down the canyon.
Tides and internal waves influence these currents. Currents in submarine canyons are often intensified during passing storms, which produce larger than average wind setup and surface waves. Inman et al. (1976) discussed the influence of passing storms on the occurrence of turbidity currents in Scripps Canyon. Section 4.5 discusses the generation and importance of ambient currents in and around submarine canyons.

1.4 Present Study

The present study was motivated by an investigation conducted on the submarine disposal of mine tailings. There were many gaps in the turbidity current literature. Further research revealed an apparent unquestioned reliance on certain assumptions, among those being that turbidity currents are dilute (Chu et al. 1979, Parker 1982, Fukushima et al. 1985). Investigation after investigation relied on previous research justifying the use of several approximations. A careful review indicated that there was as much information supporting these approximations as there was refuting it (McNown and Lin 1952, Oliver 1961). Such was the motivation of the present investigation of the approximations used in the turbidity current literature and their impacts on the numerical prediction of turbidity current evolution.

The purpose of this dissertation is as follows:

- To establish complete equations describing turbidity current flow.
- To review current laboratory and field data on turbidity current flow.
- To compare various closure relations for the processes governing turbidity current flow.
- To review common approximations used in turbidity current modeling, and identify when these approximations may not be appropriate.
- To summarize numerical efforts at understanding and predicting turbidity currents.
- To quantify the effects of the Boussinesq approximation on turbidity current modeling.
- To compare model predictions incorporating new developments to previous modeling investigations.

In Chapter Two, the field equations for turbidity current flow are introduced. This section is structured so that the effect of common approximations to the conservation equations are readily identified. Section Two concludes with a matrix representation of the equations that forms the basis for the “Modified Three Equation Model”.

Chapter Three presents existing closure schemes for processes such as sediment entrainment and water entrainment. Weaknesses in several existing closure schemes are identified, and a final set of closure schemes are adopted for the modeling portions of this dissertation.

In Chapter Four, common approximations used in turbidity current modeling are identified and critiqued. Potential shortcomings associated with each approximation are discussed. The range of validity for each approximation is also identified.

Chapter Five presents numerical results from the turbidity current model. Sensitivity to initial conditions, boundary conditions, empirical closure relationships, and especially the dilute suspension approximation are investigated in detail.

Chapter Six compares predictions of the “Modified Three Equation Model” to previous models, including those of Fukushima et al. (1985), Parker et al. (1986), Akiyama and Stefan (1987), Parker et al. (1987), and Garcia (1993).
Chapter Seven offers conclusions and recommendations for future work.

The full derivations of the equations and complete descriptions of the boundary conditions are given in Appendix A.

Appendix B presents the complete algebraic steps taken to transform the field equations into a matrix equation. The matrix form is easily coded and more importantly, easily modified to reflect changes either in the model system, or in the underlying assumptions used in the model derivation.

Appendix C presents the First Order Ordinary Differential Equation version of the Modified Three Equation Model.

Appendix D contains a tabulation of all variables used for model results presented in this thesis, as well as a complete set of graphical results for each individual model run.

In Appendix E, the laboratory experiments conducted by Garcia (1985) and reported in Parker et al. (1987) are investigated in detail. Attempts to reproduce the reported results are discussed, and potential problems with reporting sediment entrainment results are outlined.
Chapter 2. Development of Field Equations

This chapter introduces the conservation equations describing turbidity current flow. These equations will first be presented in complete form, without any approximations that would limit their application to particular turbidity currents. Limitations might be dilute sediment concentrations or rectangular channels.

Successive presentations will introduce the equations for different stages of development of the cross-section-averaged model. Several common simplifying approximations, such as the constrained flow approximation and the wide rectangular channel approximation, will be introduced. While the approximations limit applicability and generality, they often are necessary to obtain a system of equations that is mathematically closed. Without certain approximations, additional closure schemes for complicated processes would be necessary. These would contribute to the uncertainty associated with the numerical model, possibly to the point that the numerical predictions are unreliable.

Finally, the field equations, complete with listed approximations, will be presented in a fashion similar to the “Three Equation Model” of Parker et al. (1986). The purpose of this style of presentation is to facilitate comparison to existing models, and to identify the impact of the approximations on the full conservation equations.

The conservation equations are phrased in terms of integral variables, which conveniently integrate over some aspects of the complicated dynamics within the flow field. The use of integral variables requires only that conditions at the flow boundaries be known; information on local detail is unnecessary in this solution scheme.
2.1 Conservation Equations

The equations of state for density and settling velocity, and the complete conservation
equations for mixture mass, mixture momentum, and sediment mass are presented below.
The geometry for the turbidity currents is sketched in Figure 2.1.

2.1.1 Density of Mixture

In general, an equation of state defines a fluid property in terms of the thermodynamic
state variables pressure, temperature, and concentration. For density, this would take the
form:

\[ \rho = f(P, T, c) \]  \hspace{1cm} (2.1)

For turbidity currents, the contribution of suspended sediment is assumed to be
controlling, reducing the above to:

\[ \rho = f(c) \]  \hspace{1cm} (2.2)

The density of the turbidity current is

\[ \rho = c\rho_s + (1 - c)\rho_s = \rho_s(1 + Rc) \]  \hspace{1cm} (2.3)

where \( \rho_s \) is the density of the water, \( \rho_s \) is the sediment density, \( c \) is the concentration of
sediment by volume, and \( R \) is the submerged specific gravity of the sediment:

\[ R = \frac{\rho_s - \rho_o}{\rho_o} \]  \hspace{1cm} (2.4)
Figure 2.1  Channel Geometry

Channel Cross-section

Plan view
2.1.2 Settling Velocity

Again, the contribution of suspended sediment is assumed to be dominant:

\[ w_s = f(c) \]  \hspace{1cm} (2.5)

The complete form of this relationship is presented in Section 4.1.

2.1.3 Conservation of Mixture Mass

The instantaneous conservation of mass in the turbidity current follows (Batchelor 1967)

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0, \quad i = 1, 2, \text{ and } 3 \]  \hspace{1cm} (2.6)

where \( \rho(x, t) \) is the instantaneous density of the mixture and \( u_i(x, t) \) is the instantaneous velocity of the mixture.

To quantify the effects of turbulence, the variables are split into mean (overbars) and fluctuating (prime superscript) parts:

\[ \rho = \bar{\rho} + \rho', \quad u_i = \bar{u}_i + u'_i, \quad \rho u_i = \bar{\rho} u_i + (\rho u_i)' \]

Substituting these into Equation 2.6 yields:

\[ \frac{\partial (\bar{\rho} + \rho')}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} u_i + (\rho u_i)') = 0 \] \hspace{1cm} (2.7)

The classical Reynolds rules for time averaging are adopted. These are:

1) The time average of \( A \) is: \( \bar{A} = \bar{A} + A' = \bar{A} + \bar{A}' = \bar{A} \) \hspace{1cm} (2.8)

This equation summarizes two rules: the time average of a mean quantity is equal to the mean quantity, and the time average of a fluctuating quantity is equal to zero.

2) The time average of a product is \( \bar{AB} = \bar{A}B + A'B' \) \hspace{1cm} (2.9)
3) The time average of a triple product is:

\[ \overline{ABC} = \overline{AB'C'} + \overline{A'B'C'} + \overline{A'B'C'} \]  \hspace{1cm} (2.10)

Time averaging Equation 2.7 yields:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\overline{\rho u_i}) = 0 \]  \hspace{1cm} (2.11)

where \( \overline{\rho u_i} = \overline{\rho u_i} + \overline{\rho' u'_i} \)  \hspace{1cm} (2.12)

is the total time-averaged momentum which includes both mean flow and turbulent contributions.

From left to right, these terms in Equation 2.11 represent:

1) the rate of accumulation of mixture mass within the local control volume, and
2) the net rate of outflow of mixture mass through the control surface.

In terms of the Cartesian coordinate system, Equation 2.11 becomes:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \rho u^+ \frac{\partial}{\partial y} \rho v^+ \frac{\partial}{\partial z} \rho w^+ = 0 \]  \hspace{1cm} (2.13)

2.1.4 Conservation of Mixture Momentum

The instantaneous conservation of momentum of the mixture follows Hunt (1954)

\[ \frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = \rho g_i - \frac{\partial p^*}{\partial x_i} + \frac{\partial \tau_y}{\partial x_j} \]  \hspace{1cm} (2.14)

where \( \rho(x_i, t) \) is the instantaneous density of the mixture, solved with Equation 2.3 and the instantaneous volumetric suspended sediment concentration \( c(x_i, t) \), \( u_i(x_i, t) \) is the instantaneous velocity of the mixture, \( P^*(x_i, t) \) is the instantaneous pressure, \( \tau_y \) is the instantaneous viscous stress, and \( g_i \) is the gravitational acceleration:
\[ g_i = (0,0,-g) \]  \hspace{1cm} (2.15)

where \( x, y \) or \( x_i, x_2 \) are in the horizontal plane and \( z \) or \( x_3 \) is directed upward in opposition to gravity.

The hydrostatic pressure balance associated with the ambient fluid can be subtracted by letting

\[ P^* = P_h + P \]  \hspace{1cm} (2.16)

where \[ \frac{\partial P_h}{\partial x_i} = \rho_e g_i \]  \hspace{1cm} (2.17)

and \( P \) is pressure measured to a local hydrostatic datum.

The momentum equation reduces to:

\[ \frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) = \rho e Rg_i - \frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \]  \hspace{1cm} (2.18)

To quantify the effects of turbulence, the variables are again split into mean and fluctuating parts:

\[ \rho = \bar{\rho} + \rho', \ u_i = \bar{u}_i + u'_i, \ P = \bar{P} + P', \ c = \bar{c} + c', \ \tau_{ij} = \bar{\tau}_{ij} + \tau'_{ij} \]

Substituting into the momentum equation gives

\[ \frac{\partial}{\partial t} (\bar{\rho} + \rho')(\bar{u}_i + u'_i) + \frac{\partial}{\partial x_j} \left( (\bar{\rho} + \rho')(\bar{u}_i + u'_i)(\bar{u}_j + u'_j) \right) = \]

\[ \rho_e (\bar{c} + c') Rg_i - \frac{\partial (\bar{P} + P')}{\partial x_i} + \frac{\partial (\bar{\tau}_{ij} + \tau'_{ij})}{\partial x_j} \]  \hspace{1cm} (2.19)

Time averaging this equation yields:

\[ \frac{\partial}{\partial t} (\bar{\rho} u_i) + \frac{\partial}{\partial x_j} (\bar{\rho} u_i u_j) = \rho_e \bar{c} Rg_i - \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial (\bar{\tau}_{ij} - \bar{\rho} u'_i u'_j)}{\partial x_j} \]  \hspace{1cm} (2.20)
where
\[
\langle \rho u, u \rangle = \langle \rho \bar{u}, \bar{u} \rangle + \langle \rho' \bar{u}, u' \rangle + \langle \rho u', \bar{u} \rangle + \langle \rho u', u' \rangle
\] (2.21)

is the balance of the time averaged momentum flux tensor after separation of the familiar “Reynolds Stress” contribution:
\[
\langle \rho u, u \rangle = \langle \rho u, u \rangle + \langle \rho u', u' \rangle
\] (2.22)

The term \( \langle \rho u, u \rangle \) represents advection of mean flow momentum by the mean flow, together with contributions from advection of turbulent momentum by the mean flow and advection of turbulent momentum by the turbulence.

One final notational simplification involves the shear term. The final local equation for conservation of mixture momentum is:
\[
\frac{\partial}{\partial t} \langle \rho u_i \rangle + \frac{\partial}{\partial x_j} \langle \rho u_i u_j \rangle = \rho \bar{e} R_{g,i} - \frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}
\] (2.23)

where \( \tau_{ij} = \langle \tau_{ij} - \rho u'_i u'_j \rangle \) (2.24)

From left to right, these terms in Equation 2.23 represent:

1) the rate of accumulation of mixture momentum within the local control volume
2) the net rate of outflow of mixture momentum through the control surface,
3) the gravitational force acting on the sediment fraction in the control volume,
4) the pressure force acting on the control volume, and
5) the total shear force acting on the control volume.

2.1.5 Conservation of Sediment

Instantaneous mass conservation for suspended sediment is (Parker et al. 1986)
\[
\frac{\partial \bar{c}}{\partial t} + \frac{\partial}{\partial x_i} \left[ (\bar{u}_i - w_i \delta_{i3}) \bar{c} \right] = 0
\] (2.25)

where \( w_i \) is the settling velocity of the suspended sediment and \( \delta_{ij} \) is the Kronecker delta, or unit tensor.

To quantify the effects of turbulence, the variables are again split into mean and fluctuating parts: \( u_i = \bar{u}_i + u'_i \), \( c = \bar{c} + c' \), \( w_i = \bar{w}_i + w'_i \).

Substituting these into Equation 2.25 yields

\[
\frac{\partial (\bar{c} + c')}{\partial t} + \frac{\partial}{\partial x_i} \left[ \left( (\bar{u}_i + u'_i) - (\bar{w}_i + w'_i) \delta_{i3} \right) (\bar{c} + c') \right] = 0
\] (2.26)

Time averaging over the turbulence yields:

\[
\frac{\partial \bar{c}}{\partial t} + \frac{\partial}{\partial x_i} \left[ (\bar{u}_i \bar{c} + u'_i c') \right] = \frac{\partial}{\partial x_i} \left[ (\bar{w}_i \bar{c} + w'_i c') \delta_{i3} \right]
\] (2.27)

Following Parker (1978), Equation 2.27 is rearranged as:

\[
\frac{\partial \bar{c}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{u}_i \bar{c}) = -\frac{\partial}{\partial x_i} \left( F_i - \bar{w}_i \bar{c} \delta_{i3} \right)
\] (2.28)

where \( F_i = u'_i c' \) is the volumetric Reynolds flux of sediment into the turbidity current, and \( \bar{w}_i \bar{c} = \bar{w}_i c + w'_i c' \)

accounts for both the mean and turbulent transport of suspended sediment due to settling.

The Reynolds flux is primarily relevant in the \( z \)-direction:

\[
F = F_z = \bar{w}' c' .
\]

From left to right, these terms in Equation 2.28 represent:

1) the rate of accumulation of sediment within the local control volume
2) the net rate of outflow of sediment through the control surface by the mean flow, and
3) the net rate of outflow of sediment through the control surface by the turbulence and by settling.

In terms of the Cartesian coordinate system, Equation 2.28 becomes:

\[
\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial wc}{\partial z} = - \frac{\partial}{\partial z} \left( F - \langle w c \rangle \right)
\]  

(2.30)

2.2 Vertically-Integrated Equations

The conservation equations may be integrated through the depth of the turbidity current (Figure 2.2). The local control volume is a column of water. The vertical limits of integration extend from the sediment bottom \( z_b \), which is a function of both position along the current \( x \) and across the current \( y \), to the top of the turbidity current, defined at infinity for integration purposes. Using infinity as the upper limit of integration enables the integration over the turbidity current without having to precisely locate the top of the turbidity current. Since the vertical profiles of velocity and sediment concentration asymptotically approach zero, the extension of the integral of velocity or sediment from the top of the current to infinity is completely adequate.

The “top” of the turbidity current is generally defined to be the location at which either the velocity or the sediment concentration drops to some fraction (say 1%) of the area-integrated value.

The elevation of the sediment bed will change with erosion, but the time scales associated with such a change are sufficiently slow that the sediment bed can be considered stationary.
Figure 2.2 Vertical Limits of Integration
Appropriate boundary conditions at the surface and the bottom of the current are imposed and the Leibniz rule has been invoked. The full derivations of the equations and complete descriptions of the boundary conditions are given in Appendix A.

Conservation of Mixture Mass

Integrating Equation 2.13 through the depth of the turbidity current gives:

\[
\frac{\partial}{\partial t} \int_{z_b(x,y)}^{z_s(x,y)} \bar{\rho} \, dz + \frac{\partial}{\partial x} \int_{z_b(x,y)}^{z_s(x,y)} \rho \bar{u} \, dz + \frac{\partial}{\partial y} \int_{z_b(x,y)}^{z_s(x,y)} \rho \bar{v} \, dz + \frac{\partial}{\partial t} \int_{z_b(x,y)}^{z_s(x,y)} \rho h \, dz = \rho_s w_e + \rho_s (F - \bar{w} \cdot \nabla) 
\]  

(2.31)

where \( w_e \) is the velocity of entrainment of ambient water into the turbidity current (defined as positive for flow into the turbidity current from above).

The physical interpretation of the individual terms in Equation 2.31 is (from left to right):

1) the rate of accumulation of layer-integrated mixture mass within the local control volume,
2) the downstream change in mass transport by mean flow in the \( x \)-direction,
3) the lateral change in mass transport by mean flow in the \( y \)-direction,
4) the change in mixture mass due to the change in height of the turbidity current
5) the change in mixture mass due to the entrainment of ambient water from above the turbidity current
6) the change in mixture mass due to the exchange of sediment with the erodible bed.

Conservation of Momentum (\( x \)-direction)

Integrating the \( x \)-component of Equation 2.23 through the depth of the turbidity current yields:

\[
\frac{\partial}{\partial t} \int_{z_b(x,y)}^{z_s(x,y)} \rho \bar{u} \, dz + \frac{\partial}{\partial x} \int_{z_b(x,y)}^{z_s(x,y)} (\rho \bar{u}^2) \, dz + \frac{\partial}{\partial y} \int_{z_b(x,y)}^{z_s(x,y)} (\rho \bar{uv}) \, dz = 
\]

25
\[-\rho_{o}gR \left( \frac{\partial}{\partial x} \int_{s_{y}(x)}^{z} \bar{c}dz' dz + \frac{\partial \bar{c}}{\partial x} \int_{s_{y}(x)}^{z} \bar{c} dz \right) - \frac{\partial}{\partial y} \int_{s_{x}(x)}^{z} \bar{c} dz - \tau_{yx} \int_{s_{x}(x)}^{z} \frac{\partial \bar{c}}{\partial y} - \tau \int_{s_{x}(x)}^{z} \right) \]

The pressure distribution has been obtained from the z-momentum equation (Appendix A).

The physical interpretation of the individual terms in Equation 2.35 is (from left to right):

1) the rate of accumulation of layer-integrated x-momentum within the local control volume,
2) the downstream change in transport of x-momentum by the mean flow in the x direction,
3) the cross-stream change in transport of x-momentum by mean flow in the y-direction,
4) the gravitational force acting on variations in the sediment fraction in the x-direction,
5) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and
6) the shear forces acting on the control volume (terms 6, 7, and 8).

**Conservation of Momentum (y-direction)**

Integrating the y-component of Equation 2.23 through the depth of the turbidity current gives:

\[ \frac{\partial}{\partial t} \int_{s_{y}(x,y)}^{z} \bar{c} dz + \frac{\partial}{\partial x} \int_{s_{y}(x,y)}^{z} \bar{c} dz + \frac{\partial}{\partial y} \int_{s_{y}(x,y)}^{z} \bar{c} dz = \]

\[-\rho_{o}gR \left( \frac{\partial}{\partial y} \int_{s_{y}(x,y)}^{z} \bar{c} dz' dz + \frac{\partial \bar{c}}{\partial y} \int_{s_{y}(x,y)}^{z} \bar{c} dz \right) - \frac{\partial}{\partial x} \int_{s_{y}(x,y)}^{z} \bar{c} dz - \tau_{yx} \int_{s_{y}(x,y)}^{z} \frac{\partial \bar{c}}{\partial y} - \tau \int_{s_{y}(x,y)}^{z} \right) \]

Again, the x-momentum equation has been used to solve for the pressure distribution.

The physical interpretation of the individual terms in Equation 2.33 is (from left to right):

1) the rate of accumulation of layer-integrated y-momentum within the local control volume,
2) the downstream change in transport of y-momentum through the control surface by mean flow in the x-direction,
3) the cross-stream change in transport of y-momentum through the control surface by mean flow in the y-direction,
4) the gravitational force acting on variations in the sediment fraction in the y-direction,
5) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current,
6) the shear forces acting in the $y$-direction on the $x$-faces of the control volume, including terms accounting for the lateral spreading of the turbidity current,
7) the shear forces acting in the $y$-direction at the sediment bed (terms 7 and 8).

**Conservation of Sediment**

Integrating Equation 2.30 through the depth of the turbidity current yields:

$$
\frac{\partial}{\partial t} \int_{z_b(x,y)}^{z_t(x,y)} \bar{c} \, dz + \frac{\partial}{\partial x} \int_{z_b(x,y)}^{z_t(x,y)} \bar{uc} \, dz + \frac{\partial}{\partial y} \int_{z_b(x,y)}^{z_t(x,y)} \bar{vc} \, dz = \left[ F - \bar{w_c} \right]_{z_b}
$$

(2.34)

From left to right, the terms in Equation 2.34 indicate:

1) the rate of accumulation of the layer-integrated sediment concentration within the local control volume,
2) the change in layer-integrated sediment transport in the $x$-direction by the mean flow (in the $x$-direction),
3) the change in layer-integrated sediment transport in the $y$-direction by the mean flow (in the $y$-direction),
4) the net rate of inflow of sediment through the bottom of the control surface by erosion and settling.

**2.3 Cross-Section-Integrated Equations**

The conservation equations may be integrated both vertically and then laterally over the width of the turbidity current. The local control volume is now a thin cross-sectional slice. These cross-section-integrated equations will form the base of the model subsequently proposed. The geometry for these currents is presented in Figure 2.3. The limits of integration in the lateral (cross-channel) direction are $b_1(x,z)$ and $b_2(x,z)$. The width of the current is $B(x,z)$. These three variables vary with both position along the channel and height of the turbidity current.
Figure 2.3 Channel Geometry for Cross-section Integrated Flow

Channel Cross-section

Top view

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Conservation of Mass

Integrating Equation 2.30 over the width of the turbidity current yields:

\[
\frac{\partial}{\partial t} \int_{b_1(x,z)}^{b_2(x,z)} \rho \, dz \, dy + \frac{\partial}{\partial x} \int_{b_1(x,z)}^{b_2(x,z)} \rho u \, dz \, dy - \frac{\partial b_2}{\partial x} \int_{z_b(x,y)}^{b_2(x,z)} \rho u \, dz \bigg|_{b_1(x,z)}^{b_2(x,z)} + \frac{\partial b_1}{\partial x} \int_{z_b(x,y)}^{z_b(x,z)} \rho u \, dz \bigg|_{b_1(x,z)}^{b_2(x,z)}
\]

\[
+ \int_{z_b(x,y)}^{b_2(x,z)} \rho v \, dz - \int_{z_b(x,y)}^{b_1(x,z)} \rho v \, dz + \int_{b_1(x,z)}^{b_2(x,z)} \rho \frac{\partial h}{\partial t} \, dy = \int_{b_1(x,z)}^{b_2(x,z)} \rho \, \frac{w_s}{\rho} \, dy + \int_{b_1(x,z)}^{b_2(x,z)} \rho_s (F - \vec{w}_s) \, dy
\]

(2.35)

The physical interpretation of the individual terms in Equation 2.35 is (from left to right):

1) the rate of accumulation of cross-section integrated mixture mass within the local control volume,
2) the downstream change in mass transport by mean flow in the x-direction,
3) the change in mass transport in the x-direction due to the lateral spreading of the turbidity current (terms 3 and 4),
4) the change in mass due to transport through the lateral edges of the turbidity current by mean flow in the y-direction (terms 5 and 6),
5) the change in mixture mass due to the change in height of the turbidity current
6) the change in mixture mass due to the entrainment of ambient water from above the turbidity current
7) the change in mixture mass due to the exchange of sediment with the erodible bed.

Conservation of Momentum (x-direction)

Integrating Equation 2.32 over the width of the turbidity current gives:

\[
\frac{\partial}{\partial t} \int_{b_1(x,z)}^{b_2(x,z)} \rho u \, dz \, dy + \frac{\partial}{\partial x} \int_{b_1(x,z)}^{b_2(x,z)} \rho u^2 \, dz \, dy - \frac{\partial b_2}{\partial x} \int_{z_b(x,y)}^{b_2(x,z)} \rho u^2 \, dz \bigg|_{b_1(x,z)}^{b_2(x,z)} + \frac{\partial b_1}{\partial x} \int_{z_b(x,y)}^{b_1(x,z)} \rho u^2 \, dz \bigg|_{b_1(x,z)}^{b_2(x,z)}
\]

\[
+ \frac{\partial b_1}{\partial x} \int_{z_b(x,y)}^{b_2(x,z)} \rho v u \, dz \, dy =
\]

29
\[
-\rho_o g R \frac{\partial}{\partial x} \left( \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \xi dz' dz \, dy \right) + \rho_o g R \frac{\partial b_1}{\partial x} \left( \int_{s_1(x,y)}^{s_2(x,y)} \xi dz' \, dz \right) \bigg|_{b_3(x,z)}
\]

\[
-\rho_o g R \frac{\partial b_1}{\partial x} \left( \int_{s_1(x,y)}^{s_2(x,y)} \xi dz' \, dz \right) \bigg|_{b_3(x,z)} - \rho_o g R \int_{b_1(x,z)}^{b_2(x,z)} \frac{\partial b_1}{\partial x} \int_{t_1(x,y)}^{t_2(x,y)} \xi \, dz \, dy
\]

\[
-\frac{\partial}{\partial y} \left. \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \tau_{xy} \, dz \, dy \right|_{b_1(x,z)} - \left. \int_{b_1(x,z)}^{b_2(x,z)} \tau_{xy} \frac{\partial b_1}{\partial y} \bigg|_{s_2(x,y)} \, dy \right|_{b_1(x,z)} - \int_{b_1(x,z)}^{b_2(x,z)} \tau_{xy} \bigg|_{t_2(x,y)} \, dy
\]

(2.36)

The physical interpretation of the individual terms in Equation 2.36 is (from left to right):

1) the rate of accumulation of x-momentum within the local control volume,
2) the downstream change in transport of x-momentum by the mean flow in the x direction,
3) the change in transport of x-momentum due to the lateral spreading of the turbidity current (terms 3 and 4),
4) the cross-stream change in transport of x-momentum by mean flow in the y-direction,
5) the gravitational force acting on variations in the sediment fraction in the x-direction,
6) the gravitational force acting on variations in the sediment fraction due to the lateral expansion of the turbidity current (terms 7 and 8),
7) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and
8) the shear forces acting on the control volume (terms 10, 11, and 12).

**Conservation of Momentum (y-direction)**

Equation 2.33 is integrated across the width of the turbidity current, yielding:

\[
\frac{\partial}{\partial t} \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \rho \nu \, dz \, dy + \frac{\partial}{\partial x} \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \langle \rho \nu \rangle \, dz \, dy - \frac{\partial b_1}{\partial x} \left( \int_{s_1(x,y)}^{s_2(x,y)} \langle \rho \nu \rangle \, dz \right) \bigg|_{b_2(x,z)}
\]

\[
+ \frac{\partial b_1}{\partial x} \left( \int_{s_1(x,y)}^{s_2(x,y)} \langle \rho \nu \rangle \, dz \right) \bigg|_{b_3(x,z)} + \frac{\partial}{\partial y} \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \langle \rho \nu^2 \rangle \, dz \, dy = \]

\[
-\rho_o g R \frac{\partial}{\partial y} \int_{b_1(x,z)}^{b_2(x,z)} \int_{s_1(x,y)}^{s_2(x,y)} \xi dz' \, dz \, dy - \rho_o g R \int_{b_1(x,z)}^{b_2(x,z)} \frac{\partial b_1}{\partial y} \int_{s_1(x,y)}^{s_2(x,y)} \xi \, dz \, dy
\]

30

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
\[-\frac{\partial}{\partial x} \int_{b_1(x,z)}^{b_2(x,z)} \tau_{yy} \, dz \, dy + \frac{\partial b_3}{\partial x} \left( \int_{s_2(x,y)}^{s_1(x,y)} \tau_{yy} \, dz \right) \bigg|_{b_3(x,z)}^{b_2(x,z)} - \frac{\partial b_4}{\partial x} \left( \int_{s_2(x,y)}^{s_1(x,y)} \tau_{yy} \, dz \right) \bigg|_{b_4(x,z)}^{b_2(x,z)}\]

\[-\int_{b_1(x,z)}^{b_2(x,z)} \tau_{yy} \frac{\partial b_5}{\partial x} \, dy - \int_{b_1(x,z)}^{b_2(x,z)} \tau_{yy} \bigg|_{s_3} \, dy \quad (2.37)\]

The physical interpretation of the individual terms in Equation 2.37 are (from left to right):

1) the rate of accumulation of cross-section integrated $y$-momentum within the local control volume,
2) the downstream change in transport of $y$-momentum through the control surface by mean flow in the $x$-direction,
3) the change in transport of momentum due to the lateral spreading of the turbidity current (terms 3 and 4),
4) the cross-stream change in transport of $y$-momentum through the control surface by mean flow in the $y$-direction,
5) the gravitational force acting on variations in the sediment fraction in the $y$-direction,
6) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current,
7) the shear forces acting in the $y$-direction on the $x$-faces of the control volume, including terms accounting for the lateral spreading of the turbidity current (terms 8 - 10),
8) the shear forces acting in the $y$-direction at the sediment bed (terms 11 and 12).

**Conservation of Sediment**

Integrating Equation 2.34 across the width of the turbidity current gives:

\[\frac{\partial}{\partial t} \int_{b_3(x,z)}^{b_4(x,z)} \bar{c} \, dz \, dy + \frac{\partial}{\partial x} \int_{b_3(x,z)}^{b_4(x,z)} \bar{u} \bar{c} \, dz \, dy - \frac{\partial \bar{b}_3}{\partial x} \int_{s_4(x,y)}^{s_3(x,y)} \bar{u} \bar{c} \, dz \bigg|_{b_3(x,z)}^{b_4(x,z)} + \frac{\partial \bar{b}_4}{\partial x} \int_{s_4(x,y)}^{s_3(x,y)} \bar{u} \bar{c} \, dz \bigg|_{b_4(x,z)}^{b_3(x,z)} \]

\[+ \int_{s_3(x,y)}^{s_4(x,y)} \bar{u} \bar{c} \, dz \bigg|_{b_3(x,z)}^{b_4(x,z)} - \int_{s_3(x,y)}^{s_4(x,y)} \bar{u} \bar{c} \, dz \bigg|_{b_4(x,z)}^{b_3(x,z)} = \int_{b_3(x,z)}^{b_4(x,z)} \left( F - w_i \bar{c} \right) \, dy \quad (2.38)\]

From left to right, the terms in Equation 2.38 indicate:

1) the rate of accumulation of the cross-section integrated sediment concentration within the local control volume,
2) the change in cross-section integrated sediment transport in the $x$-direction by
the mean flow (in the $x$-direction),
3) the change in cross-section integrated sediment transport in the $x$-direction due to
changes in the width of the turbidity current (terms 3 and 4),
4) the loss of sediment through the lateral edges of the turbidity current by flow in the $y$-
direction (terms 5 and 6), and
5) the net rate of inflow of sediment through the bottom of the control surface by erosion
and settling.

2.4 Definition of Integral Variables

In the solution of the cross-section integrated equations, the following integral variables
are defined:

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} dz \ dy = A \quad \text{(Cross Sectional Area)} \quad (2.39) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} \rho \ dz \ dy = AM_f \quad \text{(Total Fluid Mass per unit length)} \quad (2.40) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} \tilde{p} \ dz \ dy = AM \quad \text{(Total Mixture Mass per unit length)} \quad (2.41) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} \rho u \ dz \ dy = AMU \quad \text{(Mass Flux in $x$-direction)} \quad (2.42) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} \rho v \ dz \ dy = AMV \quad \text{(Mass Flux in $y$-direction)} \quad (2.43) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} \tilde{c} \ dz \ dy = AS \quad \text{(Total Sediment Volume Fraction)} \quad (2.44) \]

\[ \int_{b_1(x,z)}^{b_2(x,z)} \int_{z_0(x,y)}^{\infty} uc \ dz \ dy = AUS \quad \text{(Sediment Flux in $x$-direction)} \quad (2.45) \]
\[
\int b_j(x,z) \int \langle \rho u^2 \rangle \, dz \, dy = AMUU \quad \text{(Flux of x-momentum in x-direction)} \quad (2.46)
\]

\[
\int b_j(x,z) \int \langle \rho v^2 \rangle \, dz \, dy = AMUV \quad \text{(Flux of x-momentum in y-direction and y-}
\text{momentum in x-direction)} \quad (2.47)
\]

\[
\int b_j(x,z) \int \langle \rho v^2 \rangle \, dz \, dy = AMVV \quad \text{(Flux of y-momentum in y-direction)} \quad (2.48)
\]

The relationship between fluid density and overall turbidity current density is

\[
AM = \int b_j(x,z) \int \bar{\rho} \, dz \, dy = \int b_j(x,z) \int \left( \rho_o (1 + R \bar{e}) \right) \, dz \, dy
\]

\[
= \rho_o \int b_j(x,z) \int dz \, dy + \rho_o R \int b_j(x,z) \int \bar{e} \, dz \, dy
\]

\[
= \rho_o A + \rho_o ARS \quad (2.49)
\]

Subsequent presentations of the equations will use this notation.

### 2.5 Constrained Flow

In literature discussions of turbidity currents, certain simplifying approximations have been introduced. These approximations will be presented one at a time, so that their effects on the complete equations can be identified.

For turbidity current flow constrained within a channel (Figure 2.4), the terms associated with the lateral flow of both fluid and sediment into and out of the turbidity current are omitted. With this simplification, the conservation equations take the following form:
Figure 2.4 Channel Geometry for Constrained Flow
Conservation of Mass

\[
\frac{\partial}{\partial t} (AM) + \frac{\partial}{\partial x} (AMU) + \frac{\partial h_2}{\partial x} \int_{z_h(x,y)}^{z_b(x,y)} \rho u \, dz + \frac{\partial h_1}{\partial x} \int_{z_h(x,y)}^{z_b(x,y)} \rho u \, dz \\
+ \int_{z_h(x,y)}^{z_b(x,y)} \rho \frac{\partial h}{\partial t} \, dy = \int_{z_h(x,y)}^{z_b(x,y)} \rho \omega \, dy + \int_{z_h(x,y)}^{z_b(x,y)} \rho (F - w_c) \, dy
\]

(2.50)

The physical interpretation of the individual terms in Equation 2.50 is (from left to right):

1) the rate of accumulation of cross-section integrated mixture mass within the control volume,
2) the change in mass transported by mean flow in the x-direction,
3) the change in transport in the x-direction of mixture mass due to the lateral spreading of the channel (terms 3 and 4),
4) the change in mixture mass due to the change in height of the turbidity current
5) the change in mixture mass due to the entrainment of ambient water from above the turbidity current
6) the change in mixture mass due to the exchange of sediment with the erodible bed.

Conservation of Momentum (x-direction)

\[
\frac{\partial}{\partial t} (AMU) + \frac{\partial}{\partial x} (AMUU) + \frac{\partial}{\partial y} (AMUV) \\
- \frac{\partial h_2}{\partial x} \left( \int_{z_h(x,y)}^{z_b(x,y)} (\rho u^2) \, dz \right) \bigg|_{b_2(x,y)}^{L_2(x,y)} + \frac{\partial h_1}{\partial x} \left( \int_{z_h(x,y)}^{z_b(x,y)} (\rho u^2) \, dz \right) \bigg|_{b_1(x,y)}^{L_1(x,y)} = \\
- \rho g R \frac{\partial h_2}{\partial x} \left( \int_{z_h(x,y)}^{z_b(x,y)} \bar{c} \, dz \right) \\
- \rho g R \frac{\partial h_1}{\partial x} \left( \int_{z_h(x,y)}^{z_b(x,y)} \bar{c} \, dz \right) \\
\int_{z_h(x,y)}^{z_b(x,y)} \frac{\partial \bar{c}}{\partial y} \, dy - \int_{z_h(x,y)}^{z_b(x,y)} \frac{\partial \bar{z}}{\partial y} \, dy \\
\int_{z_h(x,y)}^{z_b(x,y)} \bar{r}_y \, dy - \int_{z_h(x,y)}^{z_b(x,y)} \bar{r}_y \, dy
\]

(2.51)
The physical interpretation of the individual terms in Equation 2.51 is (from left to right):

1) the rate of accumulation of x-momentum within the control volume,
2) the downstream change in transport of x-momentum by the mean flow in the x-direction,
3) the cross-stream change in transport of x-momentum by mean flow in the y-direction,
4) the change in transport of x-momentum due to the lateral spreading of the turbidity current (terms 4 and 5),
5) the gravitational force acting on variations in the sediment fraction in the x-direction,
6) the gravitational force acting on variations in the sediment fraction due to the lateral expansion of the turbidity current (terms 7 and 8),
7) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and
8) the shear forces acting on the control volume (terms 10, 11, and 12).

**Conservation of Momentum (y-direction)**

\[
\frac{\partial}{\partial t}[AMV] + \frac{\partial}{\partial x}[AMUV] + \frac{\partial}{\partial y}[AMV^2] = \\
-\rho_g R \frac{\partial}{\partial y} \int_{\phi(x,z)}^{\phi(x,z)} \int_{\phi(y,z)}^{\phi(y,z)} \phi dz dy - \rho_g R \int_{\phi(x,z)}^{\phi(x,z)} \frac{\partial}{\partial y} \int_{\phi(y,z)}^{\phi(y,z)} \phi dz dy \\
- \frac{\partial}{\partial x} \int_{\phi(x,z)}^{\phi(x,z)} \int_{\phi(y,z)}^{\phi(y,z)} \tau_{yx} dy dx - \frac{\partial}{\partial x} \left( \left. \int_{\phi(y,z)}^{\phi(y,z)} \tau_{yx} dz \right|_{\phi(y,z)} \right) \\
- \int_{\phi(x,z)}^{\phi(x,z)} \phi \frac{\partial}{\partial x} \left|_{\phi(x,z)} \right. dy - \int_{\phi(x,z)}^{\phi(x,z)} \phi \frac{\partial}{\partial x} \left|_{\phi(x,z)} \right. dy
\]  

(2.52)

The physical interpretation of the individual terms in Equation 2.52 are (from left to right):

1) the rate of accumulation of cross-section integrated y-momentum within the control volume,
2) the downstream change in transport of y-momentum through the control surface by mean flow in the x-direction,
3) the cross-stream change in transport of y-momentum through the control surface by mean flow in the y-direction,
4) the gravitational force acting on variations in the sediment fraction in the y-direction,
5) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current,

36
6) the shear forces acting in the \( y \)-direction on the \( x \)-faces of the control volume, including terms accounting for the lateral spreading of the turbidity current (terms 6 - 8), and

7) the shear forces acting in the \( y \)-direction at the sediment bed (terms 9 and 10).

**Conservation of Sediment**

\[
\frac{\partial}{\partial t} (AS) + \frac{\partial}{\partial x} (ASU) - \left. \frac{\partial b_s}{\partial x} \right|_0^b \int_{z_s(x,y)}^z u c \, dz \left|_{b_h(x,z)}^{b_h(x,z)} \right. + \left. \frac{\partial b_h}{\partial x} \right|_0^b \int_{z_s(x,y)}^z u c \, dz \left|_{b_h(x,z)}^{b_h(x,z)} \right. = \left. \int (F - w_c) \, dy \right|_0^b
\]

(2.53)

From left to right, the terms in Equation 2.53 indicate:

1) the rate of accumulation of the cross-section integrated sediment concentration within the control volume,

2) the change in cross-section integrated sediment transport by the mean flow in the \( x \)-direction,

3) the change in cross-section integrated sediment transport in the \( x \)-direction due to changes in the width of the turbidity current (terms 3 and 4), and

4) the net rate of inflow of sediment through the bottom of the control surface by erosion and settling.

### 2.6 One Dimensional Flow

For flow contained within a channel of constant width (Figure 2.5), the terms associated with the lateral spread of the channel (i.e. those containing \( \frac{\partial b_s}{\partial x} \) or \( \frac{\partial b_h}{\partial x} \)) are zero. This 1-dimensional flow approximation also removes term containing the cross stream (\( y \)-direction) velocity, and terms with lateral variations across the current. With these approximations, the conservation equations become:
Figure 2.5 Channel Geometry for One Dimensional Flow

Channel Cross-section

Plan View
Conservation of Mass

\[
\frac{\partial}{\partial t}(AM) + \frac{\partial}{\partial x}(AMU) + \int_{b_1(x,z)}^{b_2(x,z)} \rho \frac{\partial h}{\partial t} dy = \int_{b_1(x,z)}^{b_2(x,z)} \rho w_x dy + \int_{b_1(x,z)}^{b_2(x,z)} \rho \left(F - \frac{w_x}{c}\right) dy \quad (2.54)
\]

The physical interpretation of the individual terms in Equation 2.54 are (from left to right):

1) the rate of accumulation of cross-section integrated mixture mass within the control volume,
2) the change in mass transported by the mean flow in the x-direction,
3) the change in mixture mass due to the change in height of the turbidity current
4) the change in mixture mass due to the entrainment of ambient water from above the turbidity current
5) the change in mixture mass due to the exchange of sediment with the erodible bed.

Conservation of Momentum (x-direction)

\[
\frac{\partial}{\partial t}(AMU) + \frac{\partial}{\partial x}(AMUU) =
\]

\[-\rho_o gR \frac{\partial}{\partial x} \int_{b_1(x,z)}^{b_2(x,z)} \int_0^\infty \tau_{sz} dz' dz \ dy - \rho_o gR \int_{b_1(x,z)}^{b_2(x,z)} \frac{\partial}{\partial x} \int_0^\infty \tau_{sz} dz \ dy - \int r_x \bigg|_{x_2}^{x_3} dy \quad (2.55)
\]

The physical interpretation of the individual terms in Equation 2.55 is (from left to right):

1) the rate of accumulation of x-momentum within the control volume,
2) the downstream change in transport of x-momentum by the mean flow in the x-direction,
3) the gravitational force acting on variations in the sediment fraction in the x-direction,
4) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and
5) the shear force at the sediment bed.

Conservation of Sediment

\[
\frac{\partial}{\partial t}(AS) + \frac{\partial}{\partial x}(ASU) = \int_{b_1(x,z)}^{b_2(x,z)} \left(F - \frac{w_x}{c}\right) \bigg|_{x_2}^{x_3} dy \quad (2.56)
\]

From left to right, the terms in Equation 2.56 indicate:
1) the rate of accumulation of the cross-section integrated sediment concentration within the control volume,
2) the change in cross-section integrated sediment transport in the x-direction by the mean flow (in the x-direction), and
3) the net rate of inflow of sediment through the bottom of the control surface by erosion and settling.

2.7 Rectangular Channel with Slope Constraints

Turbidity current flow may be further constrained within a wide rectangular channel (Figure 2.6), with a flat bottom in the y-direction but a variable bottom slope in the x-direction. The notation on the stress terms now reflects the stress at the bottom \( \tau_{xy} \).

Integrals in y across the width of the current simplify as follows:

\[
\int_{b_1(x,z)}^{b_2(x,z)} X \, dy = XB
\]

where \( B \) is the constant width of the channel, and of the confined turbidity current. For a rectangular channel with a flat bottom, the cross-sectional area is simply the width \( B \) times the height \( h \).

The cross-sectional integrated mass variable (Equation 2.49) becomes:

\[
AM = \rho_o A + \rho_o ARS = \rho_o Bh(1 + RS)
\]

With these approximations and Equation 2.58, the integral equations become:

**Conservation of Mass**

\[
\frac{\partial}{\partial t} (AM) + \frac{\partial}{\partial x} (AMU) + \rho_o \frac{\partial h}{\partial t} B = \rho_o w_x B + \rho_i \left(F - w_c \right) B
\]
Figure 2.6 Channel Geometry for Wide, Rectangular Channel

Channel Cross-section

Plan View
Dividing through by the density yields:

\[
\frac{\partial}{\partial t} [A(1 + RS)] + \frac{\partial}{\partial x} [A(1 + RS)U] + \frac{\partial h}{\partial t} B = w_x B + (R + 1)(F - \bar{w}c)B
\]  

(2.60)

The physical interpretation of the individual terms in Equation 2.60 is (from left to right):

1) the rate of accumulation of cross-section integrated mixture mass within the control volume,

2) the net rate of transport in the x-direction of mixture mass through the control surface by mean flow in the x-direction,

3) the change in mixture mass due to the change in height of the turbidity current

4) the change in mixture mass due to the entrainment of ambient water from above the turbidity current

5) the change in mixture mass due to the exchange of sediment with the erodable bed.

**Conservation of Momentum (x-direction)**

\[
\frac{\partial}{\partial t} (AMU) + \frac{\partial}{\partial x} (AMUU) = -\frac{1}{2} \rho_o g R \frac{\partial}{\partial x} (ASh) - \rho_o g R S \frac{\partial z_b}{\partial x} - \tau_z B
\]  

(2.61)

Substituting Equation 2.49 gives:

\[
\frac{\partial}{\partial t} [A(1 + RS)U] + \frac{\partial}{\partial x} [A(1 + RS)U^2] = -\frac{1}{2} g R \frac{\partial}{\partial x} (ASH) - g R S \frac{\partial z_b}{\partial x} - \frac{\tau_z}{\rho_o} B
\]  

(2.62)

The physical interpretation of the individual terms in Equation 2.62 is (from left to right):

1) the rate of accumulation of x-momentum within the control volume,

2) the downstream change in transport of x-momentum by the mean flow in the x-direction,

3) the gravitational force acting on variations in the sediment fraction in the x-direction,

4) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and

5) the shear force at the sediment bed.

**Conservation of Sediment**

\[
\frac{\partial}{\partial t} (AS) + \frac{\partial}{\partial x} (ASU) = (F - \bar{w}c) B
\]  

(2.63)

From left to right, the terms in Equation 2.63 indicate:
1) the rate of accumulation of the cross-section integrated sediment concentration within the control volume,
2) the change in cross-section integrated sediment transport in the $x$-direction by the mean flow (in the $x$-direction), and
3) the net rate of inflow of sediment through the bottom of the control surface by erosion and settling.

2.8 Steady State

At steady state, the time derivatives are zero. This assumption may be appropriate for continuous turbidity currents away from the point of origin. The complicated dynamics associated with the generation of turbidity currents are not addressed. With this assumption, the integral equations become:

**Conservation of Mass**

\[
\frac{d}{dx} \left[ A(1 + RS)U \right] = w_s B + (R + 1)(F - \overline{w_c})B 
\]

(2.64)

The fluid and sediment components can be separated, yielding:

\[
\frac{d}{dx} (AU) + R \frac{d}{dx} (ASU) = w_s B + (R + 1)(F - \overline{w_c})B 
\]

(2.65)

The physical interpretation of the individual terms in Equation 2.65 are (from left to right):

1) the net rate of transport in the $x$-direction of cross-section integrated fluid mass within the control volume,
2) the net rate of transport in the $x$-direction of cross-section integrated sediment concentration within the control volume
3) the change in fluid mass due to entrainment of ambient water from above, and
4) the change in sediment mass due to the exchange of sediment with the erodible bed.
Conservation of Momentum (x-direction)

\[
\frac{d}{dx} \left[ A(1 + R)U^2 \right] = -\frac{1}{2} gR \frac{d}{dx} (ASh) - gRSA \frac{dz_b}{dx} \frac{\tau_n}{\rho_o} B
\]  (2.66)

Separating the fluid and sediment components yields:

\[
\frac{d}{dx} (AU^2) + R \frac{d}{dx} (ASU^2) = -\frac{1}{2} gR \frac{d}{dx} (ASh) - gRSA \frac{dz_b}{dx} \frac{\tau_n}{\rho_o} B
\]  (2.67)

The physical interpretation of the individual terms in Equation 2.67 is (from left to right):

1) the downstream change in transport of fluid by the mean flow in the x-direction,
2) the downstream change in transport of sediment by the mean flow in the x-direction,
3) the gravitational force acting on variations in the sediment fraction in the x-direction,
4) the gravitational force acting on variations in the sediment fraction due to changes in the height of the turbidity current, and
5) the shear force at the sediment bed.

Conservation of Sediment

\[
\frac{d}{dx} (ASU) = \left( F - \bar{w_c} \right)|_{n_o} B
\]  (2.68)

From left to right, the terms in Equation 2.68 indicate:

1) the change in cross-section integrated sediment transport in the x-direction by the mean flow (in the x-direction), and
2) the net rate of inflow of sediment through the bottom of the control surface by erosion and settling.

Equations 2.65, 2.67, and 2.68 are the reduced integral conservation equations for mass, momentum, and sediment. They will be referred to as the “Modified Three Equation Model” (Table 2.1). This model will later be compared to the three equation model of Parker et al. (1986), which will be referred to as the “Parker Three Equation Model”.

44
2.9 Three Equation Model of Parker et al. (1986)

In order to compare the Modified Three Equation Model to the Parker Three Equation Model, a few adjustments must be made. First, the Modified Three Equation Model must be presented on a unit width basis, which is accomplished by assigning $B$ as 1. With this simplification, and using the height $h$ of the turbidity current ($h = \eta - z_h$), the equations in Table 2.1 are recast in a form that can be directly compared to the Parker Three Equation Model.

<table>
<thead>
<tr>
<th>Table 2.1 Modified Three Equation Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservation of Mass:</td>
</tr>
<tr>
<td>[ \frac{d}{dx} (AU) + R \frac{d}{dx} (ASU) = w_s B + (R + 1) (F - w_c) B ]</td>
</tr>
<tr>
<td>Conservation of Momentum:</td>
</tr>
<tr>
<td>[ \frac{d}{dx} (AU^2) + R \frac{d}{dx} (ASU^2) = -\frac{1}{2} gR \frac{d}{dx} (ASH) - gRS \frac{d}{dx} \frac{\tau_{sh}}{\rho_o} - B ]</td>
</tr>
<tr>
<td>Conservation of Sediment:</td>
</tr>
<tr>
<td>[ \frac{d}{dx} (ASU) = (F - w_c) B ]</td>
</tr>
</tbody>
</table>

45
Conservation of Mass

\[
\frac{d}{dx}(Uh) + R \frac{d}{dx}(SUh) = w_e + (R + 1)(F - \bar{w}_e c)
\]  

(2.69)

Conservation of Momentum

\[
\frac{d}{dx}(U^2h) + R \frac{d}{dx}(SU^2h) = -\frac{1}{2} gR \frac{d}{dx}(Sh^2) - gRS_h \frac{dz_b}{dx} - \frac{\tau_{zb}}{\rho_o}
\]  

(2.70)

Conservation of Sediment

\[
\frac{d}{dx}(SUh) = \left[F - \bar{w}_e c\right]_{i_o}
\]  

(2.71)

Slight modifications have been made in the variable notation of Parker et al. to clarify the relationship of the two models. Parker's sediment variable has been changed from \( C \) to \( S \), and Parker's bottom slope from \( S \) to \(-\frac{dz_b}{dx}\). With variable definitions consistent with those defined above, the Parker Three Equation Model (at steady state) is:

\[
\frac{d}{dx}(Uh) = w_e
\]  

(2.72)

\[
\frac{d}{dx}(U^2h) = -\frac{1}{2} Rg \frac{d}{dx}(Sh^2) - RgS_h \frac{dz_b}{dx} - \frac{\tau_{zb}}{\rho_o}
\]  

(2.73)

\[
\frac{d}{dx}(SUh) = \left[F - \bar{w}_e c\right]_{i_o}
\]  

(2.74)
Certain differences between the two model formulations are immediately evident. In particular, the Parker Three Equation Model excludes:

1. Two sediment terms in the mass balance equation (2\textsuperscript{nd} and 4\textsuperscript{th} terms in Equation 2.69): This is a direct result of Parker adopting the dilute suspension approximation in the derivation of the Parker Three Equation Model. The Modified Three Equation Model does not adopt this approximation. For vanishing $S$, the mass balance equations of both models are equivalent.

2. The sediment term in the momentum equation (2\textsuperscript{nd} term in Equation 2.70): This too is a direct result of Parker adopting the dilute suspension approximation. The second term in the momentum equation accounts for the momentum of the sediment. Again, for vanishing $S$, this term is negligible.

2.10 Matrix Representation

The Modified Three Equation model has been coded into a series of MATLAB programs. Steps have been taken to structure the code so that it may be quickly adapted to model turbidity currents with varying forms of complexity. One of these steps was to transform the three equations presented in Table 2.1 to matrix form. The matrix form is easily coded and more importantly, easily changed to reflect changes either in the model system, or in the underlying assumptions adopted in the model derivation. The complete algebraic steps are presented in Appendix B. The full matrix equation, representing the Modified Three Equation Model, is presented in Table 2.2. A variable shift from $U$, $S$, and $A$ to $Q$, $S$, and $\eta$ was introduced for interpretive clarity. The new variable $Q$ is simply the volumetric...
Table 2.2 Matrix Representation of Modified Three Equation Model

\[
\begin{pmatrix}
(1 + RS) & 0 & RQ \\
2(1 + RS) \frac{Q}{A} & RSgA - (1 + RS) \frac{Q^2}{A^2} B & R \frac{Q^2}{A} + \frac{1}{2} (\eta - z_b) RgA \\
S & 0 & Q
\end{pmatrix}
\begin{pmatrix}
dQ \\
dx \\
d\eta \\
dx \\
dS \\
dx
\end{pmatrix}

\begin{align*}
&= \begin{pmatrix}
w_e B + (R + 1) \left( F - w_e c \right) \bigg|_{z_b} B \\
- (1 + RS) \frac{Q^2}{A^2} B \frac{dz_b}{dx} - \frac{\tau_{z_b}}{\rho_o} B \\
\left( F - w_e c \right) \bigg|_{z_b} B
\end{pmatrix}
\end{align*}

flow; for rectangular channels the flow is equal to the cross-section-averaged velocity times the cross sectional area: \( Q = UA \).

The approximations used thus far in the derivation of the above matrix equation are:

- the turbidity current flow is constrained within a wide, rectangular channel;
- the channel has a variable down-canyon slope, but zero slope across the channel;
- the flow is nearly horizontal; and
- the flow has reached steady state.

Note that the dilute suspension approximation has not been used. This is a fundamental difference between the present model and previous analyses of turbidity currents. Section 2.9 presented the effect of the dilute suspension approximation on the field equations.

This approximation will be discussed in detail in Section 4.1.
The matrix equation above still contains more unknown variables than can be solved by
the three equations. Chapter 3 will present the equations providing mathematical closure.

It is of interest to note the ease in which the matrix formulation can be changed from the
full version presented above to represent a much simpler case, for example the case of a
conservative density current. In such a current, the excess density and the flow remain
constant; there is no exchange of sediment with the bed and there is no entrainment of
water through the upper surface of the turbidity current. For this case, the matrix
equation from Table 2.2 becomes:

\[
\begin{pmatrix}
(1 + RS) & 0 & RQ \\
2(1 + RS) \frac{Q}{A} & RSGA - (1 + RS) \frac{Q^2}{A} B & R \frac{Q^2}{A} + \frac{1}{2}(\eta - z_s)RgA \\
2(1 + RS) \frac{Q}{A^2} & 0 & \frac{\eta}{Q}
\end{pmatrix}
\begin{pmatrix}
\frac{dQ}{dx} \\
\frac{d\eta}{dx} \\
\frac{dS}{dx}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
-(1 + RS) \frac{Q^2}{A^2} B \frac{dz_s}{dx} - \frac{\tau_{ss}}{\rho_o} B
\end{pmatrix}
\]

(2.75)

This matrix equation reduces to uniform flow,

\[
RgSA \frac{d\eta}{dx} = -\frac{\tau_{ss}}{\rho_o} B
\]

(2.76)

demonstrating the balance between the gravitational force and friction.
2.11 Numerical Algorithm

The matrix equation is solved with a classic 4th order Runge-Kutta method. This simple yet accurate method utilizes a set of initial conditions and the differential equations to calculate the solution forward in time, or in this case downstream, from the point of origin.

The 4th order Runge-Kutta solution takes the following form:

\[ y_{n+1} = y_n + \Delta x \left( \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} + O(\Delta x^5) \right) \]  \hspace{1cm} (2.77)

where \( y_n \) is the known value at position \( n \), \( \Delta x \) is the distance between calculations,

\[ k_1 = f(x_n, y_n), \]  \hspace{1cm} (2.78)

\[ k_2 = f\left(x_n + \frac{\Delta x}{2}, y_n + \frac{k_1 \Delta x}{2}\right), \]  \hspace{1cm} (2.79)

\[ k_3 = f\left(x_n + \frac{\Delta x}{2}, y_n + \frac{k_1 \Delta x}{2}\right), \]  \hspace{1cm} (2.80)

\[ k_4 = f(x_n + \Delta x, y_n + k_3 \Delta x), \]  \hspace{1cm} (2.81)

\( O(\Delta x^5) \) is an estimate of the truncation error.

The 4th order Runge-Kutta algorithm is far superior to other approximations, such as the common Euler's method, which is a first order approximation represented by

\[ y_{n+1} = y_n + k_1 \Delta x + O(\Delta x^2) \]  \hspace{1cm} (2.82)

where \( k_1 \) is as defined above. The truncation error associated with Euler's method is of the order of \( \Delta x^2 \), where the 4th order Runge-Kutta algorithm has a truncation error of the order \( \Delta x^5 \). The numerical calculations of Garcia (1985) and Parker et al. (1987) utilized Euler's method.
The accuracy of any numerical approximation is proportional to the step size $\Delta x$.

Sensitivity runs were made with variations in step size from 0.01 meters to 1.0 meters. In general, model runs made with a first order approximation required a smaller step size, while those utilizing the 4th order Runge-Kutta algorithm were capable of accurate calculations with step sizes of 1 meter.

The numerical solution outlined above progresses downstream from the origin, which consists of a defined set of initial conditions. This solution is valid for either supercritical flows or subcritical flows, but not for flows as they transition from supercritical to subcritical.

The demarcation between supercritical and subcritical flow is defined by a critical Richardson number; which happens to be the location of a singularity in the system of field equations. Numerical predictions cannot proceed through this singularity for it defines the point at which the assumptions made in deriving the model, namely the nearly-horizontal flow assumption, are no longer valid. The implications of this and its effects on the numerical model will be discussed in Chapters Five and Six.
Chapter 3. Literature Closure Schemes

The matrix representation in Table 2.2 has a total of nine unknowns: $S$, $U$, $\eta$, $w_e$, $\tau_n$, $F_n$, $c_n$, $w_z$, and $\rho$. Five equations have already been introduced, three conservation equations (Equations 2.65, 2.67, and 2.68) and two equations of state (Equations 2.3 and 2.5). Model closure requires four additional independent equations among the unknowns. These additional equations are provided by the closure relationships for:

1. shear stress,
2. water entrainment, and
3. sediment entrainment.

These mathematical closure equations are the weakest link in the model formulation. The relationships are often empirical curve fits to small data sets that were compiled for a small range of input parameters. The sensitivity of model predictions to these closure relationships is highly uncertain. Small variations may lead to very large differences in model predictions. Care is necessary in applying such closure schemes to broad ranges of input parameters.

The nature of numerical modeling lends equal weight to all equations in a system of equations. The closure schemes have equal mathematical status to the equations of state and the conservation equations for fluid mass, momentum, and sediment. When choosing closure schemes, serious consideration must be given to their applicability to the current situation, as they can “make or break” a modeling effort.
One complete set of closure schemes for sediment entrainment and water entrainment
comes from Parker and colleagues at the University of Minnesota. These formulations
have been used in many numerical studies on turbidity currents. The closure schemes used
by Parker will be used as a datum for comparison so that the differences in numerical
predictions among the models presented herein and the models of Parker and his
colleagues (Parker et al. 1986, Parker et al. 1987) are not due to differences in the closure
schemes. The sensitivity of model predictions to the choice of closure schemes is
considered in Chapter 5.

Mathematical closure relationships describe processes such as water entrainment through
the body of the turbidity current and sediment entrainment into the turbidity current.
Comparative analyses identify suitable relationships for use in the numerical model.

3.1 Closure Relationship #1: Bed Shear Stress

The first closure scheme provides a relation between the bed shear stress term in the
momentum equation (2.69) and the dependent variable \( U \). The classic (Darcy-Weisbach)
approach is quadratic friction, relating the shear at the bed to the square of the cross-
section averaged velocity:

\[
\frac{\tau_b}{\rho} = \frac{f}{8} U^2
\]  

(3.1)

in which the friction factor \( f \) is a dimensionless coefficient typically estimated from the
Moody diagram. Parker et al. (1987) defined a coefficient of bed drag \( c_D \):

\[
c_D = \frac{f}{8}
\]  

(3.2)
The friction factor or coefficient of drag will depend principally on surface roughness and bed forms, with a smaller dependence on the Reynolds Number. It is not expected to vary over a wide range. Previous investigations (Akiyama and Stefan 1985, Parker et al. 1987) have used values for $c_D$ in the range 0.004 to 0.02 ($f$ in range 0.0005 to 0.0025). Parker et al. (1987) found that model predictions are similar throughout this range. However, results presented in Chapter 5 show that this is not necessarily true for all currents.

3.2 Closure Relationship #2: Water Entrainment

Turbidity currents grow in volume through the entrainment of ambient water, through both the head and body of the current. Entrainment through the body of the current, which is caused by the upward propagation of turbulence into the ambient water, is applicable here. Allen (1971) discusses entrainment and mixing at the head of a turbidity current.

Entrainment of ambient water reduces the overall density of the turbidity current and the component of the gravitational force acting in the downslope direction. This will reduce the velocity of the current, and thus its ability to transport sediment. The entrainment of ambient water is also vitally important in the transformation of sediment slumps to turbidity currents.

Experimental observations (Ellison and Turner 1959, Egashira 1980, Fukushima et al. 1981, Parker 1986) have shown that entrainment is influenced by turbidity current velocity, the relative density difference between the turbidity current and the ambient
water, and the dimensions of the turbidity current. Differences in viscosity between the
two fluids can also affect entrainment. Ambient currents are expected to affect
entrainment, but are not considered here.

The entrainment velocity \( w_e \) (Figure 3.1) is represented as

\[
w_e = e_u U
\]

(3.3)

where \( U \) is the cross-section averaged velocity and \( e_u \) is a dimensionless coefficient of
entrainment, which can be represented:

\[
e_u = \frac{w_e}{U} = f(U, \rho_r - \rho_s, h, S)
\]

(3.4)

Conveniently, these influences are all present in the Richardson number:

\[
Ri = \frac{R g S h}{U^2}
\]

(3.5)

where \( R \) is the submerged specific gravity of the sediment, \( g \) is the gravitational constant,
\( S \) is the layer averaged sediment concentration, \( h \) is the height of the current, and \( U \) is the
layer-averaged current velocity. The coefficient of entrainment is represented as:

\[
e_u = f\left(\frac{R g S h}{U^2}\right) = f(Ri)
\]

(3.6)

Ellison and Turner (1959) suggested a relationship between the Richardson number and
the rate of entrainment of ambient water into conservative density currents. Ashida and
Egashira (1977), Fukushima et al. (1981), Fukushima (1985), and Parker (1986) have
proposed variations on this relationship.
Figure 3.1  Entrainment of Ambient Water

\[ h(x,y) \]

\[ z_0(x,y) \]

\[ w_e \]

"Edge" of Turbidity Current

\[ U \]
The Richardson number quantifies the relative importance of the dominant forces acting on density currents, the inertial force and the gravitational force. A more familiar form of the Richardson number is the densimetric Froude number $F_d$:

$$F_d^2 = \frac{1}{Ri}$$  \hspace{1cm} (3.7)

The Richardson number is also used as a way to classify a flow as either subcritical or supercritical. A flow is subcritical ($Ri > Ri_c, F_d < F_{dc}$) if the depth of flow is greater than the critical depth and the flow velocity is less than the critical velocity. Supercritical flow ($Ri < Ri_c, F_d > F_{dc}$) occurs when the depth of flow is less than the critical depth and the flow velocity is greater than the critical velocity. For open channel flows, the critical Richardson number and the critical Froude number are equal to 1.0.

The transition between supercritical flow and subcritical flow is important in turbidity current modeling. This transition occurs through a hydraulic jump, where the depth of flow increases abruptly and the energy of the flow is significantly reduced. The critical Richardson number is an indicator for a transition between flow regimes. However, the location of the critical Richardson number does not pinpoint the location of the hydraulic jump. Generally, a hydraulic jump will have occurred before the location at which the Richardson number reaches the critical value. Hydraulic jumps in turbidity currents will be discussed in Chapter 6.

Five similar relationships describing the dependence of the entrainment coefficient on the Richardson number are listed below.
(1) Ellison and Turner (1959):

\[ e_w(Ri) = \frac{a_1 - a_2 Ri}{a_3 + a_4 Ri} \text{ for } Ri < 0.8, \]  

where \(a_1 = 0.08\), \(a_2 = 0.1\), \(a_3 = 1\), and \(a_4 = 5\).

This relationship fits data on conservative density currents reasonably well only for Richardson numbers smaller than 0.8. Otherwise, it predicts negative values for the entrainment coefficient.

(2) Ashida and Egashira (1977):

\[ e_w(Ri) = \frac{a_1}{Ri} \]  

where \(a_1 = 0.0015\).

(3) Fukushima (1981):

\[ e_w(Ri) = \frac{a_1}{Ri^{a_2}} \]  

where \(a_1 = 0.0028\) and \(a_2 = 1.2\).


\[ e_w(Ri) = \frac{a_1}{a_2 + Ri} \]  

where \(a_1 = 0.00153\) and \(a_2 = 0.0204\). This formulation approaches that of Ashida and Egashira (Equation 3.9) for large values of the Richardson number. Parker et al (1986) also used this relationship.

(5) Garcia (1985):
\[ e_w(Ri) = \frac{a_1}{(a_2 + a_3 Ri^{a_4})^{a_5}}, \]  

(3.12)

where \( a_1 = 0.075, a_2 = 1, a_3 = 718, a_4 = 2.4, \) and \( a_5 = 0.5. \) For large values of the Richardson number, this formulation converges to Equation 3.10. Despite the apparent sophistication, this relationship is a visual curve fit, not a least squares fit, to laboratory data. This formulation is used in Parker et al. (1987), Choi and Garcia (1995), and Bradford and Katopodes (1999).

Figure 3.2 compares these relationships to several data sets from experiments conducted by Lofquist (1960) and Garcia (1985, 1993). Note the log-log scale. The portion of the figure most relevant here lies to the left of \( Ri = 1, \) since the numerical investigations focus on the supercritical flow regime.

Equations 3.11 and 3.12 have been used in most previous numerical modeling studies of turbidity currents (Fukushima et al. (1985), Parker et al. (1986, 1987)). Both have been used in comparative analyses in Chapter 5, where they are shown to lead to fundamentally different predictions for the fate of a turbidity current.

Closure hypothesis for water entrainment remain an active issue. Equation 3.12 is subsequently adopted as the default relationship for ambient fluid entrainment.

3.3 Closure Relationship #3: Sediment Entrainment

The quantification of sediment entrainment is perhaps the most difficult component of turbidity current modeling. It is a key issue as the gravitational force acting on the suspended sediment drives the flow.
Figure 3.2
Comparison of Ambient Water Entrainment Functions with Selected Experimental Data
Experimental investigations (Akiyama and Fukushima 1985, Garcia 1985, Parker et al. 1987) have shown that sediment entrainment is influenced by turbidity current velocity, boundary shear, sediment size, sediment density, fluid density, fluid viscosity, and particle settling velocity. It is also suspected that the sediment fraction \( S \) will influence the sediment entrainment.

The volumetric rate of sediment entrainment into suspension (Figure 3.3) is represented as

\[
F|_{z=0} = E_s w_s
\]

where \( w_s \) is the settling velocity and \( E_s \) is a dimensionless coefficient of sediment entrainment (Garcia 1985), which can be represented:

\[
E_s = \frac{F|_{z=0}}{w_s} = f(U, u_*, D_s, \rho_s, \rho_o, \nu, w_s)
\]

These influences can be grouped into two dimensionless numbers, the first being a normalized shear velocity \( \frac{u_*}{w_s} \), and the second being the particulate Reynolds number \( R_p \):

\[
R_p = \left( \frac{RgD_s}{\nu} \right)^{0.5} \frac{D_s}{\nu}
\]

where \( \nu \) is the kinematic viscosity of clear water, \( D_s \) is the grain diameter, and

\[
R = \frac{\rho_s - \rho_o}{\rho_o}
\]

The coefficient of sediment entrainment is now represented as:

\[
E_s = \frac{F|_{z=0}}{w_s} = f\left( \frac{u_*}{w_s}, \frac{\sqrt{RgD_s D_s}}{\nu} \right)
\]
Figure 3.3 Exchange of Sediment with Erodible Bed
The settling velocity is chosen to normalize the entrainment rate so that the gain and loss of sediment can be grouped into a single term (Equation 2.28).

A similarity variable $Z$, grouping both dimensionless parameters on the right hand side, was introduced by Akiyama and Fukushima (1985). Parker et al. (1987) and Garcia (1989) have proposed formally similar sediment entrainment formulations. The differences between the three formulations are the definition of the grouping and especially the exponent of the similarity variable $Z$. These formulations are enumerated below, and compared in Figures 3.4a through 3.4c. The data sets included in the figures are from open channel flow (Garcia 1990) and turbidity current experiments (Garcia 1985). Note the similarity in the exponent to which the particulate Reynolds number is raised in these formulations; and also the log-log scales on these three plots.

(1) Akiyama and Fukushima (1985):

$$E_s = \begin{cases} a_1, & Z_1 > a_2, \\ a_3 Z_1^{a_4} \left( a_5 - \frac{a_6}{Z_1} \right), & a_6 < Z_1 < a_2 \\ a_7, & Z_1 < a_6 \end{cases}$$

(3.18)

where $Z_1 = \frac{u_*}{w_s} R_p^{0.5}$.

(3.19)

$a_1 = 0.3$, $a_2 = 13.2$, $a_3 = 3 \times 10^{-12}$, $a_4 = 10$, $a_5 = 1$, $a_6 = 5.0$, and $a_7 = 0$. This formulation was based on field and lab data for open channel flow suspensions, but Garcia (1985) has argued that turbidity current data follow a similar trend.

(2) Parker et al. (1987):
Figure 3.4a
Comparison of Sediment Entrainment Formulations

for $Z_1 = u^*/w_s^* R_p^{0.5}$

$Z_1 = u^*/w_s^* R_p^{0.5}$

- Turbidity Current Data (Garcia, 1985)
- Open Channel Data (Garcia, 1990)
- Akiyama and Fukushima (1985)
- Parker et al (1987)
- Garcia (1989)
Figure 3.4b
Comparison of Sediment Entrainment Formulations
for $Z_3 = \frac{u^*}{w_s} \cdot R_p^{0.6}$

$E_s$

$1.0E+00$

$1.0E+01$

$1.0E+02$

$1.0E+03$

$1.0E+04$

$Z_3 = \frac{u^*}{w_s} \cdot R_p^{0.6}$

- Turbidity Current Data (Garcia, 1985)
- Open Channel Data (Garcia, 1990)
- Akiyama and Fukushima (1985)
- Parker et al (1987)
- Garcia (1989)
Figure 3.4c
Comparison of Sediment Entrainment Formulations
for $Z_2 = u^*/w_s \cdot R_p^{0.75}$

$Z_2 = u^*/w_s \cdot R_p^{0.75}$

- Turbidity Current Data (Garcia, 1985)
- Open Channel Data (Garcia, 1990)
- Akiyama and Fukushima (1985)
- Parker et al (1987)
- Garcia (1989)
\[ E_s = \frac{a_1 Z_3^{a_2}}{(a_3 + a_4 Z_3^{a_5})} \]  \hspace{1cm} (3.20)

where \( Z_2 = \frac{u_*}{w_s} R_p^{0.75} \) \hspace{1cm} (3.21)

\( a_1 = 3 \times 10^{-11}, \ a_2 = 7, \ a_3 = 1, \) and \( a_4 = 1 \times 10^{-10}. \) This is a visual curve fit to a set of open channel flow data collected by Akiyama (1985) and to a set of turbidity current flow collected by Parker et al. (1987). It is a continuous function, which removes some of the operational difficulties associated with the mathematical discontinuities in Equation 3.18.

(3) Garcia (1989):

\[ E_s = \frac{a_1 Z_3^{a_2}}{(a_3 + a_4 Z_3^{a_5})} \]  \hspace{1cm} (3.22)

where \( Z_3 = \frac{u_*}{w_s} R_p^{0.6} \) \hspace{1cm} (3.23)

\( a_1 = 1.3 \times 10^{-7}, \ a_2 = 5, \ a_3 = 1, \) and \( a_4 = 0.3.\)

This formula was established from a least squares fit to an extensive set of open channel flow data. Garcia (1989) presents modifications to this equation for mixtures of non-uniform sediment, for sediment with low particulate Reynolds numbers, and for the effects of drag attributable to bedforms. These modifications can be ignored for the present purposes, as the sediment used in the numerical model is uniform in size and large enough so that the particulate Reynolds number correction is not necessary. Finally, the currents are assumed to be sufficiently large that the height of the current is much larger than the height of the bedforms, so that bedforms can thus be ignored without consequence.
There are several potential problems with these sediment entrainment functions. All are extremely steep functions of the similarity variable, with powers ranging from 5 to 10. Slight variations in the similarity variable lead to very considerable variations in the sediment entrainment coefficient. For example, Garcia’s formulation (or Equation 3.22) predicts that a 50% increase in \( Z \), from 10 to 15 causes a 500% increase in the predicted sediment entrainment rate (0.0125 to 0.0743). Because of the nature of the empirical sediment entrainment equations, it is imperative that all facets of the equation be completely understood.

The relationship of Akiyama and Fukushima (1985) requires additional attention. A major difficulty in using this formulation in a numerical model is the step function nature of the formulation, which leads to discontinuities in the numerical predictions. Furthermore, the maximum value of \( E_r = 0.3 \) is predicted for a very wide range of conditions, as seen in Figure 3.4a. A sediment entrainment rate of 0.3 is reached at \( Z = 13.2 \) according to Equation 3.18; alternate formulations don’t predict entrainment rates at such levels until \( Z \) reaches approximately 40. This difference is significant. For a given sediment size and bed friction, \( Z \) becomes a function of velocity only. Equation 3.18 predicts the maximum entrainment rate at approximately one-third of the velocity which the other two formulations require to reach the same entrainment rate.

Figure 3.5a shows values of \( Z \) for a range of velocities and sediment sizes. For certain combinations of sediment size and initial current velocity, the \( Z \) value is above the maximum of 13.2 (the dashed line) from the onset of numerical calculations. The turbidity current entrains sediment at the maximum rate and thus begins to accelerate, with no
Figure 3.5a
Initial Value of $Z$ for given $U_0$ and $D_s$

- $U_0 = 0.50$ (m/s)
- $U_0 = 0.75$ (m/s)
- $U_0 = 1.00$ (m/s)
- $U_0 = 1.25$ (m/s)
- $U_0 = 1.50$ (m/s)
- $U_0 = 1.75$ (m/s)
- $U_0 = 2.00$ (m/s)
- $Z = 13.2$
mechanism to stop it. Any combination of sediment size and initial current \( (U_o) \) above the line \( Z_o = 13.2 \) will cause this to occur. For example, a medium sand with a mean diameter of 0.025 cm in a 1.25 m/s current will combine to give a \( Z \) value above 13.2, and thus an entrainment rate of 0.30.

The problem with the maximum of \( Z = 13.2 \) is further compounded when concentration effects on the settling velocity are incorporated. Figure 3.5b is a similar presentation to Figure 3.5a, the only change is that the settling velocity has been modified with Oliver's Equation (Equation 4.9) to reflect a sediment concentration \( S \) of 10% by volume.

Using the same example as the paragraph above, the 0.025 cm sand now reaches the critical \( Z \) value with a current of only 0.75 m/s. In addition, Figure 3.5a predicts that a 1.75 m/s current coupled with all sediments smaller than course sand will produce a \( Z \) value above the maximum of 13.2. Taking sediment concentration into account lowers the velocity required from 1.75 m/s to 1.0 m/s. This is a significant decrease. There is further discussion of the influence sediment concentration on settling velocity in Chapter 4.

The similarity variable \( Z \) contains the sediment settling velocity, a parameter which is influenced by the sediment concentration (as will be discussed in Section 4.1). Because of this influence, the settling velocity is variable and difficult to measure. Errors in measuring the settling velocity, or in assuming that measured settling velocity at a given concentration is applicable to a range of concentrations, will propagate throughout the calculation of sediment entrainment, compromising the entire solution. A reduction in settling velocity of 50% is quite possible (Section 4.1) for sediment concentrations as little as 10% by volume.
Figure 3.5b
Initial Value of $Z$ for given $U_o$ and $D_s$
$Z$ Calculated with Settling Velocity
Corrected for $S = 0.1$

![Graph showing the relationship between $Z$ and Sediment Size ($D_s$) for different $U_o$ values.](image)
Other complications relating to the use of empirical equations for sediment entrainment arise from the inherent difficulty in conducting laboratory experiments with suspended sediment. A common result is small, somewhat scattered data sets, as shown by the scatter of data presented by Garcia (1985) and reproduced in Figure 3.2.

The measurement of suspended sediment concentration is notoriously difficult. Calculations of sediment transport variables often rely on the assumption of steady, uniform flow. This flow condition is often difficult to ensure, even in a controlled laboratory environment. The experimental flume is often not large enough to allow development of steady state conditions. Constant sediment concentration in the discharge is also difficult to achieve. Changing bedforms may further influence the flow, and downstream sorting of sediment may be another complication. Despite the myriad problems with the quantification of sediment transport, it remains of fundamental importance in turbidity current modeling.

Equation 3.22 appears to be the best fit for all data. It was used in the numerical modeling efforts that follow, unless otherwise noted. The formulation of Parker et al. (1987) is adequate, but not preferable. The formulation of Akiyama and Fukushima should not be used; it overestimates sediment entrainment for a large range of conditions, leading to self-accelerating turbidity currents. Alternate formulations predict lower sediment entrainment rates, which in turn provide a smaller driving force for the flow, thus decreasing the possibility of self-acceleration.
3.4 Closure Relationship #4: Bed Sediment Concentration

The concentration of sediment at the bed ($c_b$) is required to calculate the amount of sediment removed from the turbidity current through settling. Using the Rouse (1938) distribution for open channel suspensions, Parker (1982) related the sediment concentration at the bed to the sediment concentration averaged through the thickness of the turbidity current:

$$r_0 = \frac{c_b}{c},$$  \hspace{1cm} (3.24)

where

$$r_0 = 1 + 31.5 \frac{u_s}{w_s}^{-1.46} \quad \text{for} \quad \frac{u_s}{w_s} > 0.5$$  \hspace{1cm} (3.25)

$u_s = \sqrt{\frac{r_b}{\rho}}$ is the shear velocity, and $w_s$ is the settling velocity of the sediment.

Garcia (1985) compared this formula with three sets of laboratory turbidity currents, as shown in Figure 3.6. He noted that $r_0$ does not seem to depend strongly upon $\frac{u_s}{w_s}$ and takes values typically near 2.0. Parker (1982) characterized the agreement as acceptable, but also points out that a constant value of $r_0 = 1.6$ would also be a reasonable fit to the data. A constant value for $r_0$ seems to be the current consensus; a constant of 2.0 will be used as the default relationship herein, unless otherwise noted.

3.5 Summary of Closure Schemes

The literature includes a range of alternative closure relationships for sediment entrainment, water entrainment, and near-bed sediment concentration. Intercomparisons have been rare. This chapter has presented existing alternative versions of each closure
Figure 3.6
Parker (1982) Formulation for Bed Sediment Concentration

Shear Velocity / Settling Velocity
u/w_s

- Parker (1982)
- ro = 1.6
- ro = 2.0
- Garcia (1985)
relationship, along with relevant data, in order to identify the most appropriate relationship for the range of conditions relevant to this study.

Table 3.1 summarizes the closure schemes chosen as the default relationships for the Modified Three Equation Model. Model runs will use these relationships unless otherwise noted. Chapter 5 will demonstrate the sensitivity of the Modified Three Equation Model to these relationships.

<table>
<thead>
<tr>
<th>Closure Scheme</th>
<th>Reference</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed friction</td>
<td>Parker et al. (1986)</td>
<td>$f = 0.0005$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($c_D = 0.004$)</td>
</tr>
<tr>
<td>sediment entrainment</td>
<td>Garcia (1989)</td>
<td>Equation 3.22</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Garcia (1985)</td>
<td>$r_o = 2.0$</td>
</tr>
</tbody>
</table>
Chapter 4. Approximations Used in Turbidity Current Modeling

The complete field equations for turbidity current flow were presented in Chapter 2. Previous efforts (Pantin 1979, Chu et al. 1979, Fukushima et al. 1985, Parker et al. 1986, Akiyama and Stefan 1987) at delineating the integral conservation equations for this flow imposed some very specific approximations, considerably simplifying the conservation equations. In this section, these specific approximations and their impact on the mathematical formulation are discussed. The impacts of certain approximations on the numerical predictions are discussed in Section 5.

4.1 Dilute Suspension Approximation

The primary assumption invoked in the majority of existing turbidity current models is the dilute suspension approximation. This approximation assumes that the suspended sediment concentrations are small, and impacts the total mass conservation equation, the momentum conservation equation, and the sediment mass conservation equation. From the dilute suspension approximation, it follows that the turbidity current density can be treated as a constant, equal to the density of the ambient water. This removes the sediment terms from the mass conservation equation, as demonstrated in Section 2.9. The dilute suspension approximation reduces Equation 2.69 to Equation 2.72.

Conservation of Mass

\[
\frac{d}{dx}(Uh) + R \frac{d}{dx}(SUh) = w_e + (R + 1)(F - \overline{w_c}) \tag{2.69}
\]

\[
\frac{d}{dx}(Uh) = w_e \tag{2.72}
\]
The Boussinesq approximation, that density differences are only important in the gravity term of the momentum equation, is one direct consequence of the dilute suspension approximation. The effects of this approximation on the momentum conservation equation are shown below; Equation 2.70 is reduced to Equation 2.73 by the dilute suspension approximation.

Conservation of Momentum

\[
\frac{d}{dx}(U^2h) + R \frac{d}{dx}(SU^2h) = -\frac{1}{2} gR \frac{d}{dx}(Sh^2) - gRSh \frac{dz_b}{dx} - \frac{\tau_s}{\rho_o} \tag{2.70}
\]

\[
\frac{d}{dx}(U^2h) = -\frac{1}{2} Rg \frac{d}{dx}(Sh^2) - RgSh \frac{dz_b}{dx} - \frac{\tau_s}{\rho_o} \tag{2.73}
\]

The dilute suspension approximation also implies that both the particulate settling velocity \( w_s \) and the mixture viscosity \( \nu \) are constant. In reality, both of these vary with the sediment concentration.

The constant settling velocity approximation, which seems to have been used in all previous models, is not appropriate for turbidity currents, even those previously considered dilute. Seemingly small concentrations can significantly decrease the settling velocity. McNown and Lin (1952) observed that the fall velocity was reduced by up to 20% for volumetric sediment concentrations of only 1%. This reduction in fall velocity is significant, as will be shown in the presentation of results from the numerical model. The implications of this on turbidity current prediction have been overlooked in the literature.

Stokes (1851) developed a theoretical prediction for the terminal velocity of a sphere falling in a fluid. The so-called Stokes’ velocity follows from a balance between the drag
force and the gravitational force on a spherical particle falling through a homogeneous fluid:

\[ \frac{C_D A \rho w_s^2}{2} = \rho R g \pi \frac{d^3}{6} \]  \hspace{1cm} (4.1)

where \( C_D \) is a drag coefficient, \( A \) is the projected area of the sphere, \( \rho \) is the fluid density, \( w_s \) is the settling velocity, \( R \) is the submerged specific gravity of the sediment, \( g \) is the gravitational constant, and \( d \) is the sediment diameter.

For Reynolds Numbers \( Re \) less than 0.1, termed the Stokes' region, the drag coefficient is

\[ C_D = \frac{24}{Re}, \]  \hspace{1cm} (4.2)

where \( Re = \frac{w_s d}{v} \) and \( v \) is the kinematic viscosity of the fluid. With these equations, the Stokes' velocity can be written:

\[ w_s = \frac{R g d^3}{18 v} \]  \hspace{1cm} (4.2)

This relationship can be used with good results for Reynolds numbers up to 1. For larger Reynolds numbers, the drag coefficient is no longer a simple function of the Reynolds number, and there is no analytical relationship for the settling velocity.

Dietrich (1982) developed an empirical equation for the settling velocity of natural particles through a least squares fit to over a thousand data points from 14 separate experiments. His fourth order polynomial (Equation 4.3) is presented in terms of a dimensionless settling velocity and a dimensionless particle size, equal to the square of the particulate Reynolds number:

\[ \log W_s = a_0 + a_1 (\log D_s) + a_2 (\log D_s)^2 + a_3 (\log D_s)^3 + a_4 (\log D_s)^4 \]  \hspace{1cm} (4.3)
where \( W_c = \frac{\rho w_s^3}{(\rho_s - \rho) g \nu} \) \hspace{1cm} (4.4)

\( D^* = \frac{(\rho_s - \rho) g D_n^3}{\rho \nu^2} \) \hspace{1cm} (4.5)

\( a_0 = -3.76715, a_1 = 1.92944, a_2 = -0.09815, a_3 = -0.00575, a_4 = 0.00056 \), \( w_s \) is the particulate settling velocity, \( \rho_s \) is the density of the particle, \( \rho \) is the density of the fluid, \( \nu \) is the kinematic viscosity of the fluid, \( g \) is the gravitational acceleration, and \( D_n \) is the nominal diameter of a particle.

This equation can be used to solve for settling velocity, given sediment size and density, fluid density, and kinematic viscosity, and it fits data over an extensive range (11 log cycles) of sediment characteristics. Figure 4.1 shows the fit of Equation 4.3 to data; this curve is for spherical particles. Angularity shifts the curve slightly to the right (Dietrich 1982). Equation 4.3 agrees well with Stokes’ velocity for small sediment sizes, and thus small Reynolds numbers, as shown in Figure 4.2.

Dietrich’s equation for settling velocity is a function of viscosity, which is in turn a function of sediment concentration. Equation 4.3, derived with dilute suspensions, is appropriate for estimating particle settling velocity; as the sediment concentration increases, a correction to this equation must be included. It is expected that the Dietrich curve will be shifted as sediment concentrations vary.
Figure 4.1 Settling Velocity According to Dietrich (from Dietrich 1982)

\[ W_s = \frac{\rho v_s^3}{(\rho_s - \rho)gD_s^2} \]

\[ D_s = (\rho_s - \rho)gD_s^3 / (\rho v_s^2) \]

Settling velocity of spheres plotted as a function of \( W_s \) and \( D_s \). Curve is a least squares fit of a fourth order polynomial (Equation 4.3).
Adjustment of Settling Velocity Through Viscosity

One method of adjusting the settling velocity of sediment particles is to recognize that viscosity is a function of concentration, indirectly linking the settling velocity with the sediment concentration. Einstein (1941) and Bagnold (1954) have linked fluid viscosity to sediment concentration.

Einstein (1941) measured the kinematic viscosity of clay suspensions of various concentrations. His results are summarized as:

$$\frac{\nu_m}{\nu} = (1 + 2.5c_v) \tag{4.6}$$

where $\nu_m$ is the kinematic viscosity of the mixture, $\nu$ is the kinematic viscosity of clear water, and $c_v$ is the sediment concentration by volume.

Bagnold (1954) conducted a series of experiments to measure the increase in fluid viscosity associated with changes in particulate concentration. He used neutrally buoyant particles in high concentrations ranging from 13 to 62 percent by volume. Bagnold measured the shear stress on two concentric drums and the difference in velocity of the two drums, and found that the data fit a semi-empirical relationship of the form:

$$\tau = (1 + \lambda) \left(1 + \frac{\lambda}{2}\right) \mu \frac{dU}{dy}, \tag{4.7}$$

where $\tau =$ shear stress, $\lambda = \frac{1}{\left(\left(c_o \left(\frac{c}{c_o}\right)^{\frac{1}{3}}\right) - 1\right)}$, $\frac{dU}{dy} =$ the velocity shear, $c_o =$ the maximum static volume concentration (when spherical particles are placed touching one another to maximize the concentration; $c_o$ is 0.74 for spherical particles), $c =$ the volumetric...
concentration of the particles, and $\mu =$ the dynamic viscosity of the fluid.

Given Newton’s Law of Viscosity,

$$\tau = \mu \frac{dU}{dy}$$  \hspace{1cm} (4.8)

the additional factor in Bagnold’s equation must account for all effects of the sediment.

Thus, Bagnold’s additional factor is equal to the ratio of dynamic viscosity of the mix to the dynamic viscosity of the fluid without the particles:

$$\frac{\mu_m}{\mu} = (1 + \lambda) \left(1 + \frac{\lambda}{2}\right).$$  \hspace{1cm} (4.9)

Equations 4.6 and 4.9, taking into account the change in fluid viscosity due to the addition of particles, can now be used in Dietrich’s equation (Equation 4.3) as an approximation to quantify the effect of increased concentration on settling velocity. Figure 4.2 shows Dietrich’s settling velocity for varying particle sizes with modifications for viscosity. For comparative purposes, the modified viscosity of both Einstein and Bagnold are included. Also included is the equation of Oliver (1961), which is presented in Section 4.1.2.

The two approaches presented for the correction of fluid viscosity due to the presence of sediment agree in trend but not in magnitude. The Einstein experiments used clay suspensions, which are inherently more complicated in that clay particles have electrostatic interactions as well as physical interactions. For sediment mixtures containing above 10% clay, these electrostatic forces can be more significant than the gravitational forces acting on the particles (van Rijn 1989). These electrostatic forces can cause the clay particles to
bind together, a process termed flocculation. Flocculated particles will settle at rates much higher than the individual clay particles.

The range of sediment concentrations also varied in the experiments. Einstein used sediment concentrations with a volume fraction ranging from 0.01 to 0.1 (1 to 10 percent sediment by volume), while Bagnold used much higher concentrations (13 to 62 percent sediment by volume). At the higher concentrations used in Bagnold's experiments, it is quite feasible that inter-particle collisions play a more important role in influencing the local flow field and thus the particulate settling velocity.

4.1.2 Direct Adjustment of Settling Velocity

A second method of adjusting the settling velocity of sediment particles is to assign the settling velocity a function of concentration. This method is similar to the one described in Section 4.1.1, but it is not as complete a solution. Adjusting the settling velocity for varying concentrations, while neglecting to adjust the fluid viscosity, ignores the possible effects of increased viscosity on other processes governing turbidity current flow. For example, the particulate Reynolds number, which governs the sediment entrainment, is a function of fluid viscosity. By neglecting to consider changes in fluid viscosity with increased concentration, the sediment entrainment calculations, and ultimately the entire numerical model predictions, are compromised. With this qualification, previous research on the direct effect of increased concentration on settling velocity is examined below.

Kuenen (1951) reported experiments showing the effect of concentration on settling velocity. These experiments were conducted with sand grains in various clay suspensions.
While the quantitative results of these experiments are not directly applicable to turbidity currents due to the inclusion of the clay suspensions, the general trend observed is notable. Increased concentrations of sediment serve to hinder particulate settling. For a certain elevated concentration, settling is no longer possible. Kuenen found that this critical concentration is a function of particle size.

McNown and Lin (1952) report two sets of experiments, one with a homogeneous suspension of sand particles, and another with commercially-graded glass beads. The objective was to test a theory relating settling velocity to particle spacing, which in turn can be related to sediment concentration. The full equation is quite involved. Equation 4.10 is a first approximation supplied by McNown and Lin to their full equation.

\[ \frac{w_{s,m}}{w_s} = \frac{1}{1 + 1.3 \frac{d}{s}} \]  \hspace{1cm} (4.10)

where \( w_{s,m} \) is the settling velocity of particles in the sediment mixture, \( w_s \) is the setting velocity of an individual particle in clear fluid, \( d \) is the particle diameter, \( s \) is the particle spacing. The ratio of the particle diameter to the particle spacing is related to the volumetric sediment concentration \( c_v \) by:

\[ c_v = \frac{1}{6} \pi \left( \frac{d}{s} \right)^3 \]  \hspace{1cm} (4.11)

Richardson and Zaki (1954) proposed the following relationship for fall velocity in a mixture:

\[ \frac{w_{s,m}}{w_s} = (1 - c_v)^r \]  \hspace{1cm} (4.12)
Table 4.1 Richardson and Zaki Settling Velocity Formulation (1954)

<table>
<thead>
<tr>
<th>Re</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re &lt; 0.2</td>
<td>4.65</td>
</tr>
<tr>
<td>0.2 &lt; Re &lt; 1.0</td>
<td>4.35Re(^{-0.3})</td>
</tr>
<tr>
<td>1.0 &lt; Re &lt; 200</td>
<td>4.45Re(^{-0.1})</td>
</tr>
<tr>
<td>Re &gt; 200</td>
<td>2.39</td>
</tr>
</tbody>
</table>

where \( \gamma \) varies with the Reynolds number \( \text{Re} = \frac{w_D D_s}{v} \) according to Table 4.1.

Oliver (1961) also presented experimental results adjusting the settling velocity due to increased concentration of suspended sediment. His experiments used sediment concentrations up to 35 percent sediment by volume. Oliver’s relationship (Equation 4.13) follows data collected by Oliver (1961) and by McNown and Lin (1952), as shown in Figure 4.3.

\[
\frac{w_{s,m}}{w_s} = (1 - 2.15c_s)(1 - 0.75c_s^{0.33})
\]

(4.13)

Figure 4.3 compares the equations of Oliver, Richardson and Zaki, Bagnold, and McNown and Lin. The relationship of Richardson and Zaki severely underestimates the effect of sediment on settling velocity, while the relationship of Bagnold seems to overestimate the effect. Oliver’s relationship appears to be the best fit; this relationship is adopted in the Modified Three Equation Model.

4.2 Channel Containment Approximation

The assumption that turbidity currents remain contained in channels without possible overflow is another common assumption in turbidity current modeling. Constraining the flow to a well defined channel considerably simplifies the conservation equations. Without
Figure 4.3
Effect of Concentration on Fall Velocity
Comparison of Formulations to Laboratory Data

Clear water settling velocity as predicted by Dietrich (Equation 4.3); modified for concentration effects by the various authors.
this approximation, boundary flow conditions must be provided to close the cross-section integrated equations. This assumption is appropriate for well developed, continuous turbidity currents away from the upper reaches of submarine canyons, but generally not for surge-type turbidity currents close to the origin of the surge.

Komar (1972) investigated the relative significance of channel spill from the head of a turbidity current and the body of a turbidity current. He proposed a relationship between the Froude number $F_d^2 = \frac{1}{R_d}$ and the portion of the flow (body or head) that contributes more to spill over the channel banks.

For high Froude number flows, Komar argued that the head of the turbidity current is thicker than the body, and would thus contribute more to spill over the channel banks. Low Froude number flows, on the other hand, would tend to produce a current whose body is much thicker than its head, and thus the body would contribute more to spill over the channel banks.

Komar pointed out that high Froude number flow (supercritical flow) is more likely to occur on steeper slopes, near the upper regions of submarine canyons and on the continental slope. Low Froude number flow (subcritical flow) is more likely to occur in portions of submarine canyons with more gentle slopes, such as those which intersect a shelf or plain, forming a fan. Komar's work does not relate the occurrence of channel overspill to the Froude number, but assigns a greater potential for channel overspill to a certain portion of the turbidity current, based on the Froude number of the flow. Komar
explicitly acknowledges that only "exceptionally large flows would spill" over the channel banks in the lower reaches of a submarine canyon.

Hay (1987b) collected acoustic images of the continuous turbidity current composed of mine tailings discharged from the Island Copper Mine in Rupert Inlet, British Columbia. These images show the discharge plume spilling over the outside bank in a channel meander. His show that the flow does not necessarily have to fill the entire channel before overspill can occur. Hay attributed the overspill to an inertial effect due to the meandering nature of the channel and the relative curvature of the channel levees with respect to the channel axis. If the outer levee has a radius of curvature smaller than that of the channel axis, the flow will gradually rise up the outer bank as the flow progresses through a turn in the channel. Given a meander of sufficient length, the flow will eventually reach the top of the levee and spill out of the channel. Hay coined the term "inertial overspill" to describe this process.

Hay discussed two types of overflow: "inertial overflow" and "flow stripping". Flow stripping refers to the removal (by a sharp bend in the channel) of that portion of a turbidity current that grows to a height exceeding the depth of the confining channel. Flow stripping essentially cleaves a turbidity current into two parts, the part remaining in the channel and the part flowing outside the channel in the same direction as the channel immediately upstream of the channel bend that stripped the flow. Bowen et al. (1984) also address the issue of flow stripping.
Hay (1987b) proposed a set of equations for turbidity current flow allowing for current overspill from the confining channels. His model is based on the cross-stream momentum balance between Coriolis and centrifugal accelerations and the cross-stream pressure gradient. Certain assumptions in Hay's model limit its applicability to the upper reaches of the channel in Rupert Inlet.

4.3 One Dimensional Flow Approximation

Once the assumption has been made that the flow remains within a channel, an almost automatic assumption is that the flow is predominantly in the downstream direction. This approximation neglects the cross-stream velocity and all variations in the cross-stream direction, effectively removing the y-momentum equation from the set of conservation equations. The consequence is a reduction of a three-dimensional problem to a two-dimensional problem. This approximation, coupled with the boundary layer approximation (discussed in Section 4.7), reduces the three-dimensional set of complete turbidity current equations to a one-dimensional set of equations in the downstream direction.

For turbidity currents confined in submarine or sublacustrine channels, the one-dimensional flow approximation is appropriate. By removing the cross-current (y) momentum equation from the system of equations, this approximation removes the need to rely on one additional closure scheme. The removal of reliance on closure schemes, especially those that are considered the weakest link of the numerical model, generally serve to strengthen confidence in the model.
The one-dimensional flow approximation would not be valid for turbidity currents that are not channelized. For example, a turbidity current discharging into a newly constructed reservoir will spread laterally until enough time has passed for the current to either carve out a channel in the underlying sediment or deposit sediment in the form of levees which then act to constrain the flow. Modeling of the initial stages of such a discharge would need to include the cross-stream momentum equation. Additionally, turbidity currents being discharged above the sediment bed have an additional vertical component of momentum that must be included in the system of conservation equations. Fietz and Wood (1967) and Fukushima and Hayakawa (1995) present work on three-dimensional density currents.

4.4 Channel Geometry Approximations

Approximations regarding channel geometry are also prevalent in the turbidity current literature. Among these approximations are:

(1) channels are much wider than they are deep,
(2) channels are rectangular in cross section, and
(3) channels have a constant bottom slope in the downstream direction.

The origin of the wide channel approximation is open channel flow, where it enables the hydraulic radius to be approximated by the flow depth. The main consequence is the omission of boundary shear associated with the channel sides. The rectangular cross section approximation further simplifies the conservation equations, relating the flow and the current height as:
\[ Q = UA = UhB \]

where \( Q \) is the flow, \( U \) is the cross-section averaged velocity, \( A \) is the area of the cross-section, \( h \) is the flow height, and \( B \) is the flow width.

The approximation of constant slope may be suitable for limited reaches of submarine canyons.

The wide rectangular channel approximation is appropriate for many open channel flow applications. It may not be routinely appropriate for submarine canyons, however. Inman et al. (1976) present detailed topography of the Scripps and La Jolla Submarine Canyons in Southern California which shows that the upper reaches of typical submarine canyons are not always wide. Cross sections presented in the first kilometer of the channel show that the width to depth ratio is only 2:1 (Figure 4.4).

Komar (1969) shows cross-channel profiles of the Monterey Submarine Canyon obtained from a precision depth recorder (PDR). Twelve channel cross sections were presented, each showing a V-shaped cross section with varying degrees of steepness. Cross sections closer to the origin of the submarine canyon have width to depth ratios of approximately 3:1 or 4:1, while cross sections further down the canyon have a milder side slope and a width to depth ratio closer to 10:1.

Hay (1987a) also provided information on submarine channel geometry from acoustic surveys of a channel in Rupert Inlet, British Columbia. One particular channel cross section showed a width to depth ratio of approximately 3:1 (Figure 4, Hay, 1987b). A wide rectangular channel approximation is not appropriate throughout.
Figure 4.4 Detailed Topography of Scripps Submarine Canyon and Shelf (from Inman et al. 1976)
The submarine channel in Rupert Inlet is unique in several respects. First, the channel was developed by a discharge of mining tailings from Island Copper Mine. Second, the development of the submarine channel was well documented by annual surveys during the first few years following the instigation of mine disposal operations. This is a valuable record of the first few years of development of a submarine channel. Finally, discharge records from the mining operation have enabled detailed sediment budgets to be compiled. These records provide valuable information, in contrast to general flows in submarine canyons where calculating the sediment budget is difficult if not impossible. Hay also pointed out several features common in submarine channels, including levee asymmetry and a downstream decrease in channel width and relief.

Another problem with the wide rectangular channel approximation is that it makes it difficult to compare numerical predictions with laboratory data. In laboratory experiments involving turbidity currents, the geometry of the flume is usually such that the side walls contribute significant frictional force that is not necessarily accounted for in the model. Data taken from these experiments must be carefully adjusted to account for this additional retarding force before comparison with numerical models derived using the wide rectangular channel approximation.

4.5 Quiescent Ambient Fluid Approximation

Another common approximation used in the formulation of turbidity current models is the approximation that a turbidity current travels beneath a quiescent ambient fluid. The consequences of this approximation are:
(1) a potential underestimation of mixing through the interface of the turbidity current and the ambient fluid, as mixing at the interface generally depends on the velocity difference between the turbidity current and the ambient water, and

(2) omission of interfacial shear at the interface of the turbidity current and the ambient water.

Since this frictional force is proportional to the square of the velocity difference between the two moving fluids, it becomes increasingly significant with relatively higher velocity currents. The omission of interfacial friction in the modeling work by Parker et al (1986) may be one reason why Parker's Three Equation Model predicts continually accelerating turbidity currents. Parker's modeling work will be discussed further in Chapter 6.

Ambient currents in canyons can be induced by such mechanisms as wind, tides, and storms. Ambient currents in submarine canyons are often oscillatory and can reach velocities over 0.50 meters per second (Reimnitz 1971, Shepard and Marshall 1973, Kelley and Lambert 1973, Inman et al. 1976). Ambient currents of this magnitude traveling up the canyon, against the turbidity current flow, must affect the entrainment of ambient water into the turbidity current; a complete turbidity current model should account for this increased mixing.

Ambient currents in submarine canyons often originate over adjacent shelf areas, and then travel towards and down the canyons. These currents are driven by longshore gradients in mean surface water level. Several mechanisms can cause the mean water surface level over adjacent shelf areas to be higher than the mean water level over the submarine canyon. This in turn can cause ambient currents over shelf areas to be larger than those
over submarine canyons. Inman et al. (1976) reported on measurements that showed
ambient currents were higher adjacent to Scripps Canyon than in the head of Scripps
Canyon. There appear to be three mechanisms that can generate such longshore gradients
in water level.

One explanation for the larger setup and resulting larger currents over the adjacent shelf is
wave refraction, a process in which the underlying topographical features of a near-shore
region influence the direction of incoming waves. Waves traveling towards shore move
faster in deeper water than they do in shallower water, forcing incident wave crests into
alignment with local depth contours. Near-shore regions containing submarine canyons
affect incident waves in much the same fashion; waves tend to diverge over the canyon
area and converge over the adjacent shelf areas. Inman et al. (1976) report that surface
waves breaking over the shelf adjacent to Scripps Canyon may be two to four times higher
than those breaking over the canyon. The result is a differential wave set-up along the
shoreline, which causes local circulation patterns directed towards the submarine canyon.
These circulation patterns are important in the transport of sand to the heads of submarine
canyons.

A second cause of larger currents over adjacent shelf areas than over the submarine
canyons themselves is that wind-induced setup is higher over shelf areas than over
canyons. A simple steady state model (Inman et al. 1976) of the area demonstrates this
concept. The steady, vertically integrated, cross-shore momentum equation is

\[ \rho g (h + \eta_w) \frac{\partial \eta_w}{\partial x} = \tau_w - \tau_o \]  

(4.14)
where \( \rho \) is the density of the water, \( g \) is the acceleration of gravity, \( h = x \tan \beta \) is the water depth, \( x \) is the offshore coordinate, \( \tan \beta \) is the constant bottom slope, \( \overline{\eta}_w(x) \) is the mean free-surface set-up due to the wind, \( \tau_w \) is the wind stress, and \( \tau_o \) is the bottom stress.

Assuming that \( \tau_w \) is much greater than \( \tau_o \) and that \( h \) is much greater than \( \overline{\eta}_w \), integration yields:

\[
\overline{\eta}_{\text{max}} \sim \frac{\tau_w}{\rho g \tan \beta}
\]  

(4.15)

The wind-induced setup is inversely proportional to the bottom slope for a given wind stress. Thus, since the canyons have a steeper bottom slope than the adjacent slopes, the wind-induced setup above the canyon will be smaller than above the adjacent slopes. This difference in setup will induce a current towards the canyon from both sides. Although it is difficult to measure variations in wind-induced setup, this effect has been measured indirectly. During a two-day period of onshore winds, persistent down-channel currents were measured in Scripps Canyon. Visual observations and temperature measurements confirmed that these currents were due to differential setup.

Visual observations showed the existence of foam lines, which occur when one body of water flows under another. Buoyant debris and foam will float to the surface as the water carrying it sinks under a second water body. The line formed by this material serves to outline the location where the underflow occurs. The foam lines observed during the above mentioned period were aligned over the rims of the canyon.

Temperature measurements taken in and around the head of the canyon showed that temperatures in the canyon were generally a constant 15 °C to a depth of 18.5 meters,
while the water over the adjacent shelf was 15 °C at the surface, but quickly dropped to
11 °C just 6 meters below water. This situation could only occur if surface water from the
adjacent shelf was being transported down into the canyon.

Edge waves, trapped longshore-periodic motions over a plane sloping beach, are a third
mechanism (Inman et al. 1976) leading to elevated currents in submarine canyons. Wave
crests in an edge wave are aligned perpendicular to shore, whereas incident surface waves
are generally aligned more or less parallel to shore. Edge waves can either move up or
down coast, or exist as standing waves. Edge waves become standing edge waves when
they are trapped by such barriers as rock headlands and submarine canyons, which offer an
impedance to flow (Figure 4.5). Such barriers must be located at certain intervals based
on the period of the edge wave and the nearshore bed slope in order to for the edge wave
to become trapped. An integer number of half-wavelengths must fit between the barriers.

Although it is easy to see how a headland forces an anti-node, it is not immediately
obvious that a submarine canyon can accomplish the same feat. The sudden transition
from shallow to steep water causes currents flowing past these canyons in a longshore
direction to separate, form vortices, and loose large amounts of energy. Field
measurements (Inman et al. 1976) substantiate this claim. A similar phenomenon is visible
when a river or similar discharge enters a large lake or reservoir.

Decomposing the edge wave motion is vital to understanding the influence edge waves
have on currents in submarine canyons. When edge waves are trapped, be it by submarine
canyons or headlands, the longshore motion is arrested at these points. These anti-nodes,
Longshore dependence of trapped edge waves in La Jolla embayment, which are presumed to drive strong currents in the submarine canyons. The schematic edge wave shows the positions along the coast of amplitude nodes and antinodes; the latter are also points of maximum run-up. Circled numbers and letters indicate measurement sites.
however, are the locations of maximum cross-shore (on/off-shore) velocity. Thus, for standing edge waves, local on/off-shore currents are highest above submarine canyons. Furthermore, the magnitude of the velocity of currents produced by these edge waves is proportional to the amplitude of the edge wave, which generally increases with the passage of storm fronts. Edge wave-induced currents were calculated by Inman et al. (1976) to be 0.3 m/sec and 0.6 m/sec for edge waves with amplitudes of 0.15 and 0.30 meters, respectively. This cross-shore velocity, in the case where submarine canyons are involved, can be an important component in the instigation of turbidity currents.

These three mechanisms which generate ambient currents in submarine canyons are amplified during storm events when the winds and surface water waves are commonly higher. Inman et al. (1976) collected current data during storm conditions, and it is these data that are most relevant to turbidity current modeling. Currents of 0.25 m/sec were measured flowing from the shelf towards Scripps Canyon after a two-day period of sustained onshore wind. On a separate occasion (April 24, 1964), storm-induced down-canyon currents were measured at 0.5 m/sec. It is possible that the current in question achieved velocities larger than 0.5 m/sec, but the instrument failed at this velocity. The largest recorded down-canyon current occurred on November 24, 1968; sensors recorded a current of 1.9 m/sec for a period of 2.5 hours, before the sensors were lost. It is highly probably that this large current was a turbidity current, as opposed to an ambient current in the submarine canyon. Inman noted that following sustained down-canyon flows, divers inspecting the canyon head report that all sand had been removed. He concluded that the turbidity currents were limited by the local sand supply.
While diving in the Rio Balsas submarine canyon, Reimnitz (1971) measured ambient bottom currents of 0.54 m/sec. In a separate location, the currents were too strong to measure with the instrumentation at hand; Reimnitz estimated the current velocity at greater than 2 m/sec. These currents were attributable to a storm event, and are indicative of the significant magnitude of ambient currents in and near submarine canyons.

Further measurements of submarine canyon currents were summarized by Shepard and Marshall (1973). Canyon currents were measured at La Jolla Canyon, Newport Canyon, Redondo Canyon, Monterey Canyon, Scripps Canyon, and San Lucas Canyon, off the coasts of California and Baja California, Mexico. Savonius rotor current meter systems were deployed at various depths, and positioned 3.6 meters above the bottom. Current measurements are summarized in Table 4.2 below. These currents may not be associated with any storm activity, perhaps explaining the relatively low magnitude of some of the measurements. These currents are not turbidity currents, but rather ambient currents.

Keller and Lambert (1973) presented current measurements taken in Hudson Canyon off the coast of New York. Measurements were taken from both a bottom-mounted current meter (at 471 meters depth) and from a submersible, which measured currents with a Savonius meter deployed 0.12 meters above the canyon floor. The submersible took measurements throughout the canyon, at depths of up to 1800 meters. Measurements from the submersible showed that currents were highest in the central portion of the canyon, at depths ranging from 400 to 1000 meters. Here, currents averaged 0.10 to 0.15
Table 4.2 Summary of Measured Ambient Currents in Submarine Canyons

<table>
<thead>
<tr>
<th>Canyon</th>
<th>Depth (meters)</th>
<th>Highest Velocity (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>La Jolla</td>
<td>46</td>
<td>0.26</td>
</tr>
<tr>
<td>La Jolla</td>
<td>78</td>
<td>0.17</td>
</tr>
<tr>
<td>La Jolla</td>
<td>167</td>
<td>0.29</td>
</tr>
<tr>
<td>La Jolla</td>
<td>206</td>
<td>0.29</td>
</tr>
<tr>
<td>La Jolla</td>
<td>375</td>
<td>0.22</td>
</tr>
<tr>
<td>Scripps</td>
<td>125</td>
<td>0.34</td>
</tr>
<tr>
<td>San Lucas</td>
<td>137</td>
<td>0.17</td>
</tr>
<tr>
<td>San Lucas</td>
<td>216</td>
<td>0.095</td>
</tr>
<tr>
<td>San Lucas</td>
<td>328</td>
<td>0.33</td>
</tr>
<tr>
<td>Newport</td>
<td>101</td>
<td>0.17</td>
</tr>
<tr>
<td>Newport</td>
<td>252</td>
<td>0.115</td>
</tr>
<tr>
<td>Redondo</td>
<td>92</td>
<td>0.27</td>
</tr>
<tr>
<td>Redondo</td>
<td>283</td>
<td>0.19</td>
</tr>
<tr>
<td>Monterey</td>
<td>156</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Adapted From Shepard and Marshall (1973)

m/sec and reached a maximum of 0.27 m/sec. The bottom mounted current meter failed shortly after deployment, but did record down-canyon currents as high as 0.22 m/sec.

Additional measurements by Komar et al. (1974) include velocities of up to 0.20 m/sec at a depth of 73 meters on the continental shelf off the coast of Oregon. These measurements were taken at a height of two meters above the sea floor. Komar proposed that these measured currents were composed of density currents of about 0.05 m/sec superimposed on ambient currents of about 0.15 m/sec.

Also of interest to turbidity current modeling is the magnitude of ambient currents in the deep ocean, well away from the steep, upper reaches of submarine canyons. Heezen (1963) reported on a set of measurements made by Swallow (1957), where neutrally buoyant floats were deployed at depths of up to 4000 meters in the western Atlantic Ocean. Velocities reportedly reached magnitudes of 0.43 m/sec at these great depths.
Other measurements of deep sea currents approach this magnitude. Isaacs et al. (1966) reported measured currents at depths of 3700 to 4300 meters off the coast of Baja California. These currents exhibited semidiurnal fluctuations that were coherent with the tides at La Jolla, California. The recorded mean velocities achieved magnitudes of up to 0.04 m/sec and maximum velocities of up to 0.12 m/sec.

The above data shows that ambient currents, whether storm induced or not, are sufficiently large that they should not be ignored in turbidity current modeling. These ambient currents can reach the same order of magnitude as the turbidity currents themselves. It should also be noted that the above mechanisms can act together to produce even larger currents. They can also be superimposed on ambient currents caused by periodic tidal fluctuations.

4.6 Steady State Flow Approximation

The steady state approximation assumes that there are no variations in time throughout the turbidity current; all terms in the conservation equations with time derivatives are set to zero. It is assumed that the portion of the turbidity current flow being modeled is fully developed and is no longer varying with time. For models focusing on the main body of the turbidity current, this approximation is appropriate away from the origin of the turbidity current. This approximation is less appropriate the closer one gets to the origin of the turbidity current, whether it be the head of a submarine canyon or a discharge pipe in a mine-tailings disposal operation.
One compromise with this approximation is that all boundary conditions, input
parameters, and closure schemes have to be steady once time is removed from the system
of field equations. For example, both ambient currents and input variables such as initial
velocity must remain steady when using this approximation.

4.7 Boundary Layer Assumption

One final assumption often used in turbidity current modeling is that the flow in the
turbidity current can be represented as a boundary layer flow. The boundary layer
assumption decouples the vertical momentum equation which can be solved for the
pressure distribution, which is then used in the x momentum equation. This assumption
adopted in this thesis.

4.8 Summary of Approximations

The approximations described above limit the applicability of numerical models that adopt
them. The dilute suspension approximation is obviously applicable to dilute suspensions.
This approximation, however, seems to have been invoked routinely. Data from McNown
and Lin (Figure 4.3), and formulations by McNown and Lin (Equation 4.10), Oliver
(Equation 4.13), and Bagnold (Equation 4.9) strongly question the general applicability of
this approximation. Previous investigations have invoked the dilute suspension
approximation at concentrations of 1 to 2 percent sediment by volume (Garcia, 1994).
Figure 4.3 shows that at these concentrations, the settling velocity may be reduced by 25
to 30%. The impact of a change in settling velocity of this magnitude can be ascertained
through the sediment entrainment formulation (Equation 3.21), where a 25% reduction in
settling velocity yields a 250% increase in the sediment entrainment rate. This is significant, and cannot be overlooked.

The Modified Three Equation Model uses Oliver’s relationship (Equation 4.13) to predict the settling velocity dependence on sediment concentration. The kinematic viscosity is modified with the relationship of Einstein (Equation 4.6), which was developed with a range of concentrations more applicable to turbidity currents than that of Bagnold. Settling velocity is adjusted from Dietrich’s clear water settling velocity (Equation 4.3), which is calculated with the kinematic viscosity for clear water so as not to “double count” the effect of sediment concentration.

The channel containment approximation is adopted by the Modified Three Equation Model. This approximation may break down if a hydraulic jump occurs within the channel; if the resulting height after the jump is larger than the channel depth, then flow over the channel banks may occur. Since the Modified Three Equation Model does not continue calculations for currents once they encounter a hydraulic jump, this approximation is appropriate.

The one-dimensional flow approximation is adopted by the Modified Three Equation Model. For current contained in canyons, this is appropriate. The channel geometry approximations are also adopted, more so for purposes of comparing predictions to previous models.
The quiescent ambient fluid approximation is adopted, in order to use the chosen integration scheme. The influence of this approximation on turbidity currents needs additional investigation. Currents presented herein are assumed to have reached steady state; for developed currents away from the point of origin, this approximation is appropriate.

In Chapter 5, numerical results will be presented showing the impact of certain approximations. The dilute suspension approximation will be investigated in detail.
Chapter 5. Presentation of Results of Modified Three Equation Model

Predictions from the Modified Three Equation Model are presented in this chapter. Section 5.1 explores model predictions for variations in initial conditions of the turbidity current, including changes in initial velocity, initial sediment concentration, and initial height. Section 5.2 investigates predictions for variations in channel properties such as channel slope and bed friction coefficient. Section 5.3 reviews model predictions for changes in adopted closure schemes. Section 5.4 addresses the effect of mixture viscosity and sediment grain size on model predictions. Section 5.5 explores the dilute suspension approximation. A tabulation of all variables used for model results presented in this chapter, as well as a complete set of graphical results for each individual model run, is included as Appendix D.

Results are graphically presented with two separate templates. The first contains the cross-section-integrated turbidity current velocity $U$, height $h$, sediment concentration $S$, and Richardson number $Ri$. The second template contains variables specific to the sediment entrainment formulations: velocity $U$, sediment parameter $Z$, sediment entrainment rate $E_n$, and shear velocity $u_*$. Initial Conditions for velocity, height, and sediment concentration, as well as the channel slope, are included at the bottom of each plot. The range of the individual variable under review is included in the bottom right corner of each figure. Unless otherwise noted, the runs were made for those approximations summarized in Section 4.8.
5.1 Sensitivity to Initial Conditions

5.1.1 Initial Sediment Concentration

Initial conditions for this series of model runs are summarized in Table 5.1. Figures 5.1a through 5.1c present model predictions for a range of initial sediment concentrations. Figures 5.1a and 5.1b focus on variations closer to the origin (up to 200 meters), while Figure 5.1c expands the range to 1000 meters. These results should not be extended, without further investigation, to initial sediment concentrations outside of the range presented; such currents may evolve differently than those presented here.

Figure 5.1a demonstrates that the initial sediment concentration has a short-lived effect on the development of the currents. Even with a range in initial concentration of two orders of magnitude, all currents are predicted to reach approximately the same sediment concentration after only 200 meters.

The explanation for this paradox, that the initial conditions do not seem to matter, lies in the sediment entrainment formulation. Figure 5.1b presents the evolution of the variables \( U, Z, E_z, \) and \( u^* \) for this series of model runs. Notice the values of \( E_z \) converge to the

<table>
<thead>
<tr>
<th>Table 5.1 Initial Conditions for Figures 5.1a, 5.1b, and 5.1b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run Number</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>F598a</td>
</tr>
<tr>
<td>F599a</td>
</tr>
<tr>
<td>F600a</td>
</tr>
<tr>
<td>F601a</td>
</tr>
<tr>
<td>F602a</td>
</tr>
<tr>
<td>F603a</td>
</tr>
<tr>
<td>F604a</td>
</tr>
<tr>
<td>F605a</td>
</tr>
</tbody>
</table>
Figure 5.1a Effects of Suspended Sediment Concentration - Runs F598a-F605a

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>So</th>
</tr>
</thead>
<tbody>
<tr>
<td>F598a</td>
<td>0.0005</td>
</tr>
<tr>
<td>F599a</td>
<td>0.001</td>
</tr>
<tr>
<td>F600a</td>
<td>0.003</td>
</tr>
<tr>
<td>F601a</td>
<td>0.005</td>
</tr>
<tr>
<td>F602a</td>
<td>0.008</td>
</tr>
<tr>
<td>F603a</td>
<td>0.01</td>
</tr>
<tr>
<td>F604a</td>
<td>0.03</td>
</tr>
<tr>
<td>F605a</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 5.1b Effects of Suspended Sediment Concentration - Runs F598a-F605a

Sediment Entrainment Variables

<table>
<thead>
<tr>
<th>Slope</th>
<th>0.08 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Speed</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>So</th>
</tr>
</thead>
<tbody>
<tr>
<td>F598a</td>
<td>0.0005</td>
</tr>
<tr>
<td>F599a</td>
<td>0.001</td>
</tr>
<tr>
<td>F600a</td>
<td>0.003</td>
</tr>
<tr>
<td>F601a</td>
<td>0.005</td>
</tr>
<tr>
<td>F602a</td>
<td>0.008</td>
</tr>
<tr>
<td>F603a</td>
<td>0.01</td>
</tr>
<tr>
<td>F604a</td>
<td>0.03</td>
</tr>
<tr>
<td>F605a</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 5.1c Effects of Suspended Sediment Concentration - Extended Calculations

Slope: 0.08 (m/s)
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)

Run | So
---|---
F598a | 0.0005
F605a | 0.05
maximum allowed value of 0.30 after roughly 150 meters. This is a direct result of the predicted evolution of the sediment entrainment variable \( Z \), which develops downstream in the same fashion as the velocity \( U \).

The initial velocity (1.0 m/s) begins to erode sediment, regardless of the initial sediment concentration. Since the gravitational force acting on the sediment concentration drives the flow, the currents containing higher sediment concentrations have a greater tendency to accelerate (Runs F604a and F605a). This, coupled with continuity concerns associated with the entrainment of ambient water, is responsible for the trend reversal in \( U, h, \) and \( Ri \) near the origin in Figure 5.1a.

As long as the decelerating currents in Figure 5.1a are above a threshold value, they still have the ability to entrain sediment, due to their initial velocity. As long as a decelerating current maintains the minimum velocity required to entrain sediment at a rate greater than that lost by settling, the sediment concentration in these decelerating currents will increase and the currents will eventually accelerate.

Figure 5.1c shows the extended effects of initial sediment concentration on turbidity current development. For interpretive clarity, the number of cases shown in Figure 5.1c has been reduced. The two cases retained encompass the full range of runs presented in Figure 5.1a; the predicted solution convergence makes the inclusion of all model runs unnecessary. Later, it will be demonstrated that the line to which these currents converge is a function of the bed slope, the bed drag coefficient, and the choice of closure relationships.
The Richardson numbers presented in Figure 5.1c have stabilized at approximately 200 meters, implying that the water entrainment has become constant, and the sediment entrainment rate has reached its maximum value. At this stage, all of the currents are roughly the same size, contain the same sediment, and are under the influence of the same boundary conditions. These factors lead to the convergence of the various predictions.

5.1.2 Initial Current Height

Initial conditions for this series of model runs are summarized in Table 5.2. Figures 5.2a and 5.2b present model predictions for a range of initial current heights. As in Figures 5.1a and 5.1c, the two plots are identical except for scale. The model reacts to changes in initial height in a similar fashion to changes in initial sediment concentration. As with sediment concentration, the influence of different initial heights is short-lived. Figure 5.2b shows that for initial heights ranging from 0.5 to 3.0 meters, the currents are predicted to approach similar values for speed, height, and sediment concentration at approximately 1000 meters.

The near-field effects (Figure 5.2a) of initial height are primarily driven by the water entrainment relationship, which is in turn forced by the Richardson number. Since the currents in this series have the same initial velocity and the same initial sediment concentration.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Initial Velocity $U_o$ (m/s)</th>
<th>Initial Height $h_o$ (m)</th>
<th>Initial Sediment Concentration $S_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F600c</td>
<td>1.0</td>
<td>0.5</td>
<td>0.01</td>
</tr>
<tr>
<td>F601c</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F603c</td>
<td>1.0</td>
<td>2.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F605c</td>
<td>1.0</td>
<td>3.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 5.2a Effects of Initial Current Height - Runs 600c-605c

<table>
<thead>
<tr>
<th>Run</th>
<th>ho</th>
</tr>
</thead>
<tbody>
<tr>
<td>F600c</td>
<td>0.5</td>
</tr>
<tr>
<td>F601c</td>
<td>1.0</td>
</tr>
<tr>
<td>F603c</td>
<td>2.0</td>
</tr>
<tr>
<td>F605c</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Slope: 0.08 (m/s)

Initial Sediment Concentration: 0.01

Initial Speed: 1.0 (m/s)
Figure 5.2b Effects of Initial Current Height - Extended Calculations

<table>
<thead>
<tr>
<th>Run</th>
<th>$h_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F600c</td>
<td>0.5</td>
</tr>
<tr>
<td>F601c</td>
<td>1.0</td>
</tr>
<tr>
<td>F603c</td>
<td>2.0</td>
</tr>
<tr>
<td>F605c</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Slope: 0.08 (m/s)

Initial Sediment Concentration: 0.01

Initial Speed: 1.0 (m/s)
concentration, these variables develop along similar tracks. The height, however, is more strongly influenced by the Richardson number. Smaller values of the Richardson number (Runs F600c and F601c) produce larger values for water entrainment, thus causing the height of the current to increase more rapidly than those currents (Runs F603c and F605c) with a larger Richardson number, which yield smaller values for entrainment. Once the Richardson number stabilizes, the currents develop in a similar fashion (Figure 5.2b).

5.1.3 Initial Current Velocity

Initial conditions for this series of model runs are summarized in Table 5.3. Figures 5.3a and 5.3b present model predictions for a range of initial current velocities. Figure 5.3a demonstrates the near-field influence of initial velocity on the development of the current, while Figure 5.3b shows the far-field influence.

The influence of initial current velocity manifests itself though the Richardson number, which controls the water entrainment. The currents with greater initial velocities have smaller Richardson numbers (Runs F604d and F605d). These currents will thus entrain ambient water at a higher rate, causing the height of the current to increase relatively rapidly. Through continuity, this rapid increase in height leads to a decrease in the current velocity. This is visible in Figure 5.3a.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Initial Velocity $U_0$ (m/s)</th>
<th>Initial Height $h_0$ (m)</th>
<th>Initial Sediment Concentration $S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F600d</td>
<td>0.5</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F601d</td>
<td>0.75</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F602d</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F604d</td>
<td>1.5</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>F605d</td>
<td>2.0</td>
<td>1.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 5.3a Effects of Initial Current Velocity - Runs 600d-605d

<table>
<thead>
<tr>
<th>Run</th>
<th>Uo</th>
</tr>
</thead>
<tbody>
<tr>
<td>F600d</td>
<td>0.5</td>
</tr>
<tr>
<td>F601d</td>
<td>0.75</td>
</tr>
<tr>
<td>F602d</td>
<td>1.0</td>
</tr>
<tr>
<td>F604d</td>
<td>1.5</td>
</tr>
<tr>
<td>F605d</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Slope: 0.08 (m/s)

Initial Sediment Concentration: 0.01

Initial Height: 1.0 (m)
Figure 5.4 Effect of Bottom Slope - Runs 610-617

<table>
<thead>
<tr>
<th>Run</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>F610</td>
<td>0.03</td>
</tr>
<tr>
<td>F611</td>
<td>0.04</td>
</tr>
<tr>
<td>F612</td>
<td>0.05</td>
</tr>
<tr>
<td>F613</td>
<td>0.06</td>
</tr>
<tr>
<td>F614</td>
<td>0.07</td>
</tr>
<tr>
<td>F615</td>
<td>0.08</td>
</tr>
<tr>
<td>F616</td>
<td>0.10</td>
</tr>
<tr>
<td>F617</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Initial Sediment Concentration: 0.01
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)
The far-field influence of initial current velocity on current predictions is minor. Figure 5.3b shows that once the Richardson number begins to stabilize, all currents are predicted to develop along similar tracks.

At first glance, the short-lived influence of initial conditions on the far-field development of these turbidity currents seems illogical, but logical explanations can be found for this behavior by studying the complex interactions in more detail. One common theme in the results presented in this section is that the Richardson numbers eventually converge to a common value. This will be investigated more in Section 5.2.

5.2 Sensitivity to Channel Properties

Results in this section explore the effect of channel properties on model predictions. In the previous section, it was shown that over a reasonable range, initial conditions had no major lasting effect on the development of turbidity currents. Results presented in this section will demonstrate that this is not the case for properties such as channel slope and bed friction.

5.2.1 Bed Slope

Initial conditions for this series of model runs are summarized in Table 5.4. Figure 5.4 demonstrates the influence of bed slope on turbidity current evolution. Runs are made for a series of slopes ranging from 0.03 to 0.12 (1.72° to 6.84°). As expected, currents on larger slopes accelerate faster. Faster currents have a lower Richardson number, and thus entrain more water from above. This is in the cause of the faster increase in turbidity current height for currents on larger slopes.
Figure 5.3b  Effects of Initial Current Velocity - Extended Calculations

Slope: 0.08 (m/s)
Initial Sediment Concentration: 0.01
Initial Height: 1.0 (m)

Run   Uo
F600d 0.50
F601d 0.75
F602d 1.00
F604d 1.50
F605d 2.00
Table 5.4 Initial Conditions for Figure 5.4

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Initial Velocity $U_o$ (m/s)</th>
<th>Initial Height $h_o$ (m)</th>
<th>Initial Sediment Concentration $S_o$</th>
<th>Channel Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>F610</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>F611</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>F612</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>F613</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>F614</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>F615</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>F616</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>F617</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The predicted evolution of sediment concentration may seem counter-intuitive, for the faster currents end up with a smaller sediment concentration. This can be explained by observing the sediment entrainment rates associated with these currents. Currents on larger slopes accelerate faster, thus reaching the maximum sediment entrainment rate more quickly. Once this maximum is reached, the water entrainment dilutes the sediment concentration. Since faster currents entrain more water, there is more dilution and a lower sediment concentration. The volumetric sediment transport, however, is larger for currents on steeper slopes.

In Figure 5.4, note that the Richardson number increases with decreasing slope. For slopes smaller than 0.03, the Richardson number approaches values that indicate a transition from supercritical to subcritical flow.

5.2.2 Bed Friction
Initial conditions for this series of model runs are summarized in Table 5.5. Figure 5.5a shows model predictions for variations in the bed friction coefficient \( f \) from \( 3.75 \times 10^{-4} \) to \( 1.875 \times 10^{-3} \) \( (c_D \text{ from } 0.003 \text{ to } 0.015) \). Bed friction serves to retard the flow, and is proportional to the square of the current velocity (Equation 3.1). Model runs with smaller values of \( f \) will have a smaller retarding force; decelerating currents will be less likely to decelerate and accelerating currents will be more likely to accelerate as \( f \) decreases. In Figure 5.5a, Run F629, which has the smallest value for \( f \), initially decelerates the least of all the runs in the series.

Table 5.5 Initial Conditions for Figures 5.5a, 5.5b, and 5.5c

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Initial Velocity ( U_0 ) (m/s)</th>
<th>Initial Height ( h_0 ) (m)</th>
<th>Initial Sediment Concentration ( S_0 )</th>
<th>Bed Friction Factor ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>F629</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 3.75 \times 10^{-4} )</td>
</tr>
<tr>
<td>F630</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 5.00 \times 10^{-4} )</td>
</tr>
<tr>
<td>F631</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 7.50 \times 10^{-4} )</td>
</tr>
<tr>
<td>F632</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 1.00 \times 10^{-3} )</td>
</tr>
<tr>
<td>F633</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 1.25 \times 10^{-3} )</td>
</tr>
<tr>
<td>F634</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>( 1.875 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Figure 5.5b extends the calculations presented in Figure 5.5a. In this figure, the Richardson number is shown to increase with increasing drag coefficient. For \( f = 1.875 \times 10^{-3} \), the current is predicted to reach a Richardson number above 0.9. From here, further increase in \( f \) will cause the Richardson number to reach the critical value, which signifies that a hydraulic jump has occurred. This conflicts with Parker's assertion (Parker et al. 1987) that the numerical predictions are not sensitive to the value of the drag coefficient, for an accelerating current can be shown to decelerate simply by adjusting the drag coefficient slightly upward.
Figure 5.5a  Effects of Bed Drag Coefficient - Runs 629-634

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>F629</td>
<td>3.75E-04</td>
</tr>
<tr>
<td>F630</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>F631</td>
<td>7.50E-04</td>
</tr>
<tr>
<td>F632</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>F633</td>
<td>1.25E-03</td>
</tr>
<tr>
<td>F634</td>
<td>1.88E-03</td>
</tr>
</tbody>
</table>

123

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 5.5b Effects of Bed Drag Coefficient - Extended Calculations

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0  (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0  (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>f</th>
<th>c₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>F629</td>
<td>3.75E-04</td>
<td>0.003</td>
</tr>
<tr>
<td>F630</td>
<td>5.00E-04</td>
<td>0.004</td>
</tr>
<tr>
<td>F631</td>
<td>7.50E-04</td>
<td>0.006</td>
</tr>
<tr>
<td>F632</td>
<td>1.00E-03</td>
<td>0.008</td>
</tr>
<tr>
<td>F633</td>
<td>1.25E-03</td>
<td>0.010</td>
</tr>
<tr>
<td>F634</td>
<td>1.88E-03</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Figure 5.5c Effects of Bed Drag Coefficient - Runs 629-634
Sediment Entrainment Variables

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.1</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>F629</td>
<td>3.75E-04</td>
</tr>
<tr>
<td>F630</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>F631</td>
<td>7.50E-04</td>
</tr>
<tr>
<td>F632</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>F633</td>
<td>1.25E-03</td>
</tr>
<tr>
<td>F634</td>
<td>1.88E-03</td>
</tr>
</tbody>
</table>
The Richardson number is shown to stabilize at different values depending on the friction factor. Notice that like the runs presented in Figure 5.4, the runs with the larger velocities have the smaller Richardson numbers, which leads to larger entrainment and a larger increase in the height of the current as it travels downstream.

The predicted evolution of sediment concentration presented in Figure 5.5b again seems counter-intuitive, since the slowest current (F634) has the highest sediment concentration. This can be explained by observing the sediment entrainment rates associated with these currents. As before, the sediment entrainment rate is controlled by the parameter \( Z \), which in this series of plots develops downstream in a similar fashion to \( u_e \).

Figure 5.5c shows the downstream development of \( U \), \( Z \), \( E_s \), and \( u_e \) for this series of model runs. Notice that Run F634, which initially decelerates the fastest due to its value for \( f \), has the largest calculated \( u_e \). This causes Run F634 to reach \( E_s \) before the other runs, and thus gives it the largest sediment concentration. From this point on (roughly 100 meters), the dilution by entrainment from above is smallest for Run F634, which thus maintains the largest sediment concentration of the runs in this series.

It is important to note that while results presented in Section 5.1 show that the Richardson number approaches the same value despite changes in initial velocity, height, and sediment concentration, the same is not true for changes in channel properties such as bed slope and bed friction. The runs presented in Section 5.1 converge to the same Richardson number because they were all calculated with the same bed friction coefficient and the same channel slope. The results presented in this section show that the stable Richardson...
number is indeed a function of these two properties. The next section will present results that show how the stable Richardson number is affected by changes in the adopted closure relationships.

5.3 Sensitivity to Closure Relationships

As expected from the discussion in Chapter 4, The Modified Three Equation Model is very sensitive to the adopted closure relationships. The differences are extreme; results show that one set of closure schemes predicts an accelerating current, while another set predicts a decelerating, or subsiding current. The closure schemes are investigated first as a set and then individually to ascertain their impacts on model predictions. Table 5.6 summarizes the closure relationships used in these runs; initial conditions for these runs are listed at the bottom of the table.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>$E_s$</th>
<th>$e_w$</th>
<th>$r_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F660</td>
<td>G85</td>
<td>G89</td>
<td>G85</td>
</tr>
<tr>
<td>F661</td>
<td>A&amp;F</td>
<td>F85</td>
<td>P82</td>
</tr>
<tr>
<td>F662</td>
<td>A&amp;F</td>
<td>G89</td>
<td>G85</td>
</tr>
<tr>
<td>F663</td>
<td>G85</td>
<td>G89</td>
<td>P82</td>
</tr>
<tr>
<td>F664</td>
<td>G85</td>
<td>F85</td>
<td>G85</td>
</tr>
</tbody>
</table>

Key:
- $E_s$: A&F = Akiyama and Fukushima (1985); G85 = Garcia (1985)
- $e_w$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $r_o$: P82 = Parker (1982); G85 = Garcia (1985)

Calculations performed with $U_o = 0.5$ m/s, $h_o = 2$ m, $S_o = 0.002$, slope = 0.05, $D_i = 0.10$ mm, and viscosity = $1 \times 10^{-6}$ m²/s (20°C).
5.3.1 Updated Closure Relationships

Figure 5.6a shows model predictions for two runs. The first case (F660) uses the closure relationships selected in Chapter 3, and the second case (F661) uses those adopted by Parker et al. (1986). The predictions of the two runs are contrary; the updated closure relationships predict an accelerating current while the Parker et al. (1986) closure relationships predict a decelerating current. The differences between these model predictions are controlled by the different sediment entrainment formulations. The sediment entrainment rate of Garcia (Equation 3.22) predicts a larger value than that of Akiyama and Fukushima (Equation 3.18). This in turn causes Run F660 to accelerate and Run F661 to decelerate.

Figure 5.6b presents variables controlling the sediment concentration, namely $U$, $r_o$, $Z$, and $E_s$. Notice first the difference (over an order of magnitude) in the initial value of $E_s$, and the log-log scale used in this plot. This shows the potential difference between versions of the sediment entrainment formulation. This difference is so severe that it alters the initial velocity development in the numerical predictions. Note the initial response of the velocity for the two model runs: Run F660 initially decelerates while Run F661 initially accelerates.

Equation 5.1 is the Ordinary Differential Equation for turbidity current velocity, taken from the complete set of field equations presented in Appendix C.
Figure 5.6a Effects of Closure Schemes - Runs F660 and F661

Slope: 0.05
Initial Sediment Concentration: 0.002
Initial Speed: 0.5 (m/s)
Initial Height: 2.0 (m)
Figure 5.6b Effects of Closure Schemes - Runs F660 and F661
Sediment Entrainment Variables

Slope: 0.05
Initial Sediment Concentration: 0.002
Initial Speed: 0.5 (m/s)
Initial Height: 2.0 (m)
\[
\frac{dU}{dx} = U \left[ \frac{R_i \frac{dz_h}{dx} + c_d + \left( \frac{1}{2} R_i + 1 \right) e_w + \left( \frac{1}{2} R_i + 1 + R \right) S + \frac{1}{2} R_i}{(1 + R S - R_i)} \right] \frac{w_s}{U} r_o \left( \frac{\psi_e}{\psi} - 1 \right)
\]

(5.1)

Notice the final ratio on the right,

\[
\left( \frac{\psi_e}{\psi} - 1 \right)
\]

(5.2)

where \( \psi_e = \frac{E_s A U}{r_o} \) represents the suspended sediment transport at equilibrium and

\( \psi = A S U \) represents the general suspended sediment transport (after Parker et al., 1986). This normalized transport variable is positive if the sediment transport is below its equilibrium value, and negative if it is above its equilibrium value. A positive value means that the current has the capability to entrain and carry more sediment, while a negative value means that the current is carrying too much sediment and that the excess load will be lost through settling. The importance of this term (Equation 5.2) is that it tends to control the sign of Equation 5.1, and thus the initial development of the velocity of the turbidity currents. This is precisely the case demonstrated in Figure 5.6b.

5.3.2 Sediment Entrainment Relationship

The closure schemes were then investigated on an individual basis; the results are presented in Figure 5.7a. The sediment entrainment formulation is investigated by comparing Run F660 to Run F662. A change in the sediment entrainment formulation
Figure 5.7a Effects of Closure Schemes - Runs F660-F664

Slope: 0.05
Initial Sediment Concentration: 0.002
Initial Speed: 0.5 (m/s)
Initial Height: 2.0 (m)
from Equation 3.22 (used in Run F660) to Equation 3.18 (used in Run F662) leads to contrary predictions for the fate of the turbidity current. The reasons for this have been discussed at length above.

5.3.3 Near-bed Sediment Relationship

The near-bed sediment concentration relationship is investigated by comparing Run F660 to Run F663. Figure 5.7a shows that this too has the same ability to change the fate of a current; use of a constant value \( r_0 = 2.0 \) leads to an accelerating current (Run F660), while use of Equation 3.25 (Parker 1982) leads to a subsiding current (Run F663). The reason for this parallels the explanation above regarding the ratio of the equilibrium sediment transport to the actual sediment transport (Equation 5.2).

The term \( r_0 \) is capable of reversing the sign of Equation 5.2, in the same fashion as the term \( E_n \), described above. Since the sign of Equation 5.2 generally controls the sign of Equation 5.1 and thus the development of the velocity of the turbidity current, it must be given appropriate consideration. Figure 5.7b presents the values used for \( r_0 \) in the runs presented in this section. Note that Runs F660, F662, and F664 all use \( r_0 = 2.0 \), while Run F663 uses Equation 3.25, the predictions of which can be orders of magnitude higher than 2.0. The importance of this parameter, coupled with Figure 3.6, indicate that Equation 3.25 needs additional data support.

5.3.4 Water Entrainment Relationship

The numerical predictions are also sensitive to the formulation chosen for water entrainment. The effects of changes in this relationship are seen by comparing Run F660
Figure 5.7b Effects of Closure Schemes - Runs F660-F664
Closure Parameters $e_w$, $E_a$, and $r_o$

Slope: 0.05
Initial Sediment Concentration: 0.002
Initial Speed: 0.5 (m/s)
Initial Height: 2.0 (m)
to Run F664. Figure 5.7a shows that while the change in the water entrainment formulation is not enough to reverse the fate of this particular current, the effects are still significant.

Figure 5.7b demonstrates that the water entrainment relationship of Fukushima et al. (1985), used in Run F664, predicts a smaller rate of entrainment than that of Garcia (1985), used in Run F660. As a result, Run F664 maintains a higher sediment concentration, due to a smaller amount of dilution, and thus a higher velocity than Run F660.

The results presented in this section do not imply that changes in the closure schemes will change the nature of all turbidity currents. Rather, the results identify the potential impact of variations in closure schemes. The results presented in Figures 5.7a and 5.7b show impacts relative to an accelerating current (F660). Impacts may be different for decelerating, or subsiding currents.

5.4 Sensitivity to Fluid Mixture Properties

5.4.1 Kinematic Viscosity

Predictions of the Modified Three Equation Model can be very sensitive to kinematic viscosity. This variable influences the particulate Reynolds number, the sediment entrainment parameter $Z$, and ultimately the sediment entrainment rate $E_t$. Figures 5.8a, 5.8b, and 5.8c demonstrate the potential impacts of changes in the kinematic viscosity of the ambient water from $0.8 \times 10^{-6}$ m$^2$/s to $1.5 \times 10^{-6}$ m$^2$/s ($30^\circ$C to $5^\circ$C). Initial conditions for this series of runs are summarized in Table 5.7.
Figure 5.8b Effects of Kinematic Viscosity - Runs 674b-677b
Sediment Entrainment Variables

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.002</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>0.5  (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>2.0  (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>ν</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>F674b</td>
<td>0.8 x 10^{-6}</td>
<td>30</td>
</tr>
<tr>
<td>F675b</td>
<td>1.0 x 10^{-6}</td>
<td>20</td>
</tr>
<tr>
<td>F676b</td>
<td>1.3 x 10^{-6}</td>
<td>10</td>
</tr>
<tr>
<td>F677b</td>
<td>1.5 x 10^{-6}</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 5.8c Effects of Kinematic Viscosity - Extended Calculations

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.002</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>0.5 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>2.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>$\nu$</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>F674b</td>
<td>$0.8 \times 10^{-6}$</td>
<td>30</td>
</tr>
<tr>
<td>F675b</td>
<td>$1.0 \times 10^{-6}$</td>
<td>20</td>
</tr>
<tr>
<td>F676b</td>
<td>$1.3 \times 10^{-6}$</td>
<td>10</td>
</tr>
<tr>
<td>F677b</td>
<td>$1.5 \times 10^{-6}$</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 5.8a shows the near-field response to changes in kinematic viscosity. Run F677b has the highest value for kinematic viscosity, corresponding to a temperature of 5 °C.

This current has the largest initial acceleration, causing it to be the first run in the series to reach the maximum sediment entrainment rate, as demonstrated in Figure 5.8b. This entrainment rate is responsible for the large initial increase in sediment concentration. The Richardson number associated with this run is the first to reach a peak value. The subsequent reduction in the Richardson number implies an increase in the entrainment of ambient water, which in turn leads to an increase in the dilution and ultimately a decrease in the sediment concentration. This is visible in Figure 5.8c, which extends the calculations presented in Figure 5.8a.

Run F674b, which has a kinematic viscosity corresponding to a temperature of 30 °C, illustrates the sensitivity of the Modified Three Equation Model to this fluid property. This particular run is predicted to reach the critical Richardson number (Figure 5.8a); this current will undergo a hydraulic jump in its transition to subcritical flow. Accompanying this hydraulic jump is an increase in depth and a decrease in velocity. This subcritical current will continue to decelerate, drop its suspended sediment, and eventually cease to
exist. Not that this is contrary to the predicted behavior of Run F675b, which has a kinematic viscosity only 25% greater than that used in Run F674b.

These results suggest that kinematic viscosity must be carefully considered in turbidity current modeling, for a current predicted to accelerate at one viscosity (or temperature) may be predicted to decelerate (after going through a hydraulic jump) at another. A current predicted to decelerate at one water temperature may accelerate at a colder temperature. This is relevant to modeling submarine canyons, since the ocean cools considerably with depth.

5.4.2 Sediment Grain Size

Figure 5.9 shows how sediment grain size influences the evolution of turbidity currents. For the initial conditions used in this series of runs, which are summarized in Table 5.8, the Modified Three Equation Model predicts accelerating currents for a narrow range of sediment size ranging from coarse silt to fine sand (0.065 mm to 0.15 mm).

Note that the Richardson number for Run F684 nearly reaches the critical value. For the particular conditions used in this run, 0.065 mm is the smallest sediment that will lead to an accelerating current. If smaller sediment is used, the current will reach the critical

| Table 5.8 Initial Conditions for Figure 5.8a, 5.8b, and 5.89c |
|-------------|-------------|-------------|-----------------|-----------------|
| Run Number  | Initial Velocity \( U_0 \) (m/s) | Initial Height \( h_0 \) (m) | Initial Sediment Concentration \( S_0 \) | Grain Size (mm) |
| F679        | 0.5         | 2.0         | 0.002           | 0.20            |
| F680        | 0.5         | 2.0         | 0.002           | 0.15            |
| F681        | 0.5         | 2.0         | 0.002           | 0.10            |
| F684        | 0.5         | 2.0         | 0.002           | 0.065           |
Figure 5.9 Effects of Sediment Size - Runs 679-684

Slope: 0.05
Initial Sediment Concentration: 0.002
Initial Speed: 0.5 (m/s)
Initial Height: 2.0 (m)

Run | $D_s$ (mm)
---|---
F679 | 0.20
F680 | 0.15
F681 | 0.1
F684 | 0.065

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Richardson number, signifying that it has undergone the transition from supercritical to subcritical flow, and decelerate until all the suspended sediment settles out and the current ceases to exist.

Figure 5.9 also shows the maximum sediment size leading to an accelerating current for the given initial and boundary conditions. The Richardson number for Run F680 approaches the critical value. Any sediment larger than 0.15 mm will lead to a decelerating current. Run F679, run with a 0.20 mm sediment, is an example of one such current.

5.5 Limitations of the Dilute Suspension Approximation

The dilute suspension approximation was discussed in Section 4.1. Detailed analysis will establish the levels of sediment concentration at which the dilute suspension approximation is no longer valid.

Three separate models were used to investigate the effects of the dilute suspension approximation:

1. The first is the Modified Three Equation Model. Results from this model are presented as Series F600.

2. The second is a version of the author’s replica of the Parker Three Equation Model, with updated closure schemes to match those adopted by the Modified Three Equation Model. Results from this model are presented as Series D800. This model contains the dilute suspension approximation.
3. The third model, presented as Series F200, is a version of the Modified Three Equation Model coded with a constant settling velocity. This model was developed to separate the effects of changes in settling velocity from other effects associated with the dilute suspension approximation.

These three models allow investigation of the impacts of both the dilute suspension approximation and the constant settling velocity approximation.

5.5.1 Run Comparisons With and Without the Dilute Suspension Approximation

Figures 5.10 through 5.17 present predictions made with and without the dilute suspension approximation, and with and without the constant settling velocity approximation. Runs were made with sediment concentrations ranging from 0.0005 to 0.05 (0.05 to 5% sediment, by volume). In each plot, three runs are included, one from each of the models described above. Two plots are included for each initial sediment concentration, showing results to 1000 meters and 5000 meters, respectively. The initial conditions used in this series of runs are summarized in Table 5.9.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Initial Velocity $U_0$ (m/s)</th>
<th>Initial Height $h_0$ (m)</th>
<th>Initial Sediment Concentration $S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 5.10</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0005</td>
</tr>
<tr>
<td>Figure 5.11</td>
<td>1.0</td>
<td>1.0</td>
<td>0.001</td>
</tr>
<tr>
<td>Figure 5.12</td>
<td>1.0</td>
<td>1.0</td>
<td>0.003</td>
</tr>
<tr>
<td>Figure 5.13</td>
<td>1.0</td>
<td>1.0</td>
<td>0.005</td>
</tr>
<tr>
<td>Figure 5.14</td>
<td>1.0</td>
<td>1.0</td>
<td>0.008</td>
</tr>
<tr>
<td>Figure 5.15</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>Figure 5.16</td>
<td>1.0</td>
<td>1.0</td>
<td>0.03</td>
</tr>
<tr>
<td>Figure 5.17</td>
<td>1.0</td>
<td>1.0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

143
Figures 5.10a and 5.10b show that for an initial sediment concentration of 0.0005, the Modified Three Equation Model (Run 598a) predicts an accelerating current while the Parker Three Equation Model predicts a decelerating current. Notice that the predictions for Run F208 match those of the Parker Three Equation Model. This is expected due to the relatively dilute nature of the suspension. At this concentration, the differences in the model predictions are due to the concentration dependent settling velocity used in Run F598a. Figures 5.11a and 5.11b show similar results for an initial sediment concentration of 0.001. At larger concentrations, the effects of the dilute suspension approximation will be visible, and the three models will predict distinctly different results.

Figure 5.12b demonstrates that for an initial concentration of 0.003, predictions from Series D800 and Series F200 begin to diverge. Notice that the divergence is related to the sediment concentration; for low values of the sediment concentration, the results are similar, but as the concentration increases, so does the difference in the model predictions. These differences are very slight, and remain so for the rest of the plots in the series (through concentrations as high as 0.05). The differences between the predictions from Series D800 and Series F200 are minor because both models used to produce these results adopt the constant settling velocity approximation.

The largest impacts of the dilute suspension approximation involve the adjustment of the settling velocity for increased concentrations, and this single adjustment is sufficient to change the predicted evolution of turbidity currents. These effects are visible by comparing the plots from Series F600 to those from Series D800 in Figures 5.10 through 5.17. There is a common pattern in this comparison (Figures 5.12b to 5.17b):
- Series F600 predicts a larger initial sediment concentration, since the settling velocity is reduced.
- This causes a greater initial acceleration in the turbidity current.
- This acceleration causes the current to quickly reach the maximum sediment entrainment rate.
- From this point, entrainment of ambient fluid dilutes the current, thus lowering the sediment concentration.
- The decrease in sediment concentration reduced the driving force and the acceleration is reduced.

Table 5.10 summarizes the effects of the dilute suspension approximation on velocity, height, and sediment concentration. Note for small concentrations (below 0.1 percent by volume), the models predict contrary results for the fate of turbidity currents at 5000 meters. Also note that as $S_o$ increases above 0.003, the Modified Three Equation Model predicts lower velocities at 5000 meters than the Three Equation Model.

### Table 5.10  Effects of the Dilute Suspension Approximation

<table>
<thead>
<tr>
<th>$S_o$ (m)</th>
<th>Distance (m)</th>
<th>Velocity U (m/s)</th>
<th>Height h (m)</th>
<th>Sediment Concentration $S$ (by volume)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>1000</td>
<td>4.32</td>
<td>9.0</td>
<td>3.71E-02 7.30E-06</td>
</tr>
<tr>
<td>0.001</td>
<td>1000</td>
<td>4.32</td>
<td>9.1</td>
<td>3.71E-02 1.27E-05</td>
</tr>
<tr>
<td>0.003</td>
<td>1000</td>
<td>4.35</td>
<td>9.2</td>
<td>3.69E-02 1.58E-04</td>
</tr>
<tr>
<td>0.005</td>
<td>1000</td>
<td>4.36</td>
<td>9.2</td>
<td>3.69E-02 2.87E-02</td>
</tr>
<tr>
<td>0.008</td>
<td>1000</td>
<td>4.37</td>
<td>9.3</td>
<td>3.68E-02 5.51E-02</td>
</tr>
<tr>
<td>0.01</td>
<td>1000</td>
<td>4.38</td>
<td>9.3</td>
<td>3.68E-02 5.41E-02</td>
</tr>
<tr>
<td>0.03</td>
<td>1000</td>
<td>4.40</td>
<td>9.5</td>
<td>3.67E-02 5.17E-02</td>
</tr>
<tr>
<td>0.05</td>
<td>1000</td>
<td>4.42</td>
<td>9.6</td>
<td>3.66E-02 5.14E-02</td>
</tr>
<tr>
<td>0.0005</td>
<td>5000</td>
<td>7.96</td>
<td>45.7</td>
<td>2.45E-02 1.00E-07</td>
</tr>
<tr>
<td>0.001</td>
<td>5000</td>
<td>7.97</td>
<td>45.8</td>
<td>2.45E-02 1.00E-07</td>
</tr>
<tr>
<td>0.003</td>
<td>5000</td>
<td>7.97</td>
<td>45.9</td>
<td>2.45E-02 4.03E-02</td>
</tr>
<tr>
<td>0.005</td>
<td>5000</td>
<td>7.98</td>
<td>46.0</td>
<td>2.45E-02 3.57E-02</td>
</tr>
<tr>
<td>0.008</td>
<td>5000</td>
<td>7.98</td>
<td>46.0</td>
<td>2.45E-02 3.49E-02</td>
</tr>
<tr>
<td>0.01</td>
<td>5000</td>
<td>7.98</td>
<td>46.0</td>
<td>2.45E-02 3.47E-02</td>
</tr>
<tr>
<td>0.03</td>
<td>5000</td>
<td>7.99</td>
<td>46.2</td>
<td>2.45E-02 3.43E-02</td>
</tr>
<tr>
<td>0.05</td>
<td>5000</td>
<td>8.00</td>
<td>46.3</td>
<td>2.44E-02 3.42E-02</td>
</tr>
</tbody>
</table>

Series F600 contains results from the Modified Three Equation Model
Series D800 contains results from the Three Equation Model
Figure 5.10a Effects of Dilute Suspension Approximation: So = 0.0005
Extended Calculations (to 5000 m)

Slope: 0.08  
Initial Sediment Conc.: 0.0005  
Initial Speed: 1.0 (m/s)  
Initial Height: 1.0 (m)  

Run Approximations
D808a Dilute Suspension  
Constant \( w_s \) and viscosity \( v \)  
F208 Constant \( w_s \) and viscosity \( v \)  
F598a None of the above
Figure 5.10b Effects of Dilute Suspension Approximation: \( \text{So} = 0.0005 \)

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.0005</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

Run | Approximations |
--- | ----------------|
D608a | Dilute Suspension |
F208 | Constant \( w_s \) and viscosity \( \nu \) |
F598a | None of the above |
Figure 5.11a Effects of Dilute Suspension Approximation: So = 0.001

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.001</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D809a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant ( w_s ) and viscosity ( \nu )</td>
</tr>
<tr>
<td>F209</td>
<td>Constant ( w_s ) and viscosity ( \nu )</td>
</tr>
<tr>
<td>F599a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>
Figure 5.11b Effects of Dilute Suspension Approximation: So = 0.001
Extended Calculations (to 5000 m)

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.0005</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D809a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td>F209</td>
<td>Constant ( w_e ) and viscosity ( \nu )</td>
</tr>
<tr>
<td>F599a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>
Figure 5.12a Effects of Dilute Suspension Approximation: So = 0.003

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.003</td>
<td>D810a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
<td>F210</td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
<td>F600a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 5.12b Effects of Dilute Suspension Approximation: So = 0.003

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.003</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
<tr>
<td>Run</td>
<td>Approximations</td>
</tr>
<tr>
<td>D810a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant ( \omega ) and ( \nu )</td>
</tr>
<tr>
<td>F210</td>
<td>Constant ( \omega ) and ( \nu )</td>
</tr>
<tr>
<td>F600a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 5.13a Effects of Dilute Suspension Approximation: $So = 0.005$

Slope: 0.08
Initial Sediment Conc.: 0.005
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)

Run Approximations
D811a Dilute Suspension
Constant $w_s$ and viscosity $\nu$
F211 Constant $w_s$ and viscosity $\nu$
F601a None of the above
Figure 5.13b Effects of Dilute Suspension Approximation: So = 0.005

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D811a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F211</td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F601a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

Slope: 0.08
Initial Sediment Conc.: 0.005
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)
Figure 5.14a Effects of Dilute Suspension Approximation: $S_o = 0.008$

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.008</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D812a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td>F212</td>
<td>Constant $w_e$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F602a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

154
Figure 5.14b  Effects of Dilute Suspension Approximation: $S_o = 0.008$

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D812a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F212</td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F602a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

Slope: 0.08
Initial Sediment Conc.: 0.008
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)
Figure 5.15a Effects of Dilute Suspension Approximation: So = 0.01

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

Run Approximations

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D813a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant $w_a$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F213</td>
<td>Constant $w_a$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F603a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>
Figure 5.15b Effects of Dilute Suspension Approximation: So = 0.01

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D813a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant ( w_e ) and viscosity ( \nu )</td>
</tr>
<tr>
<td>F213</td>
<td>Constant ( w_e ) and viscosity ( \nu )</td>
</tr>
<tr>
<td>F603a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>
Figure 5.16a Effects of Dilute Suspension Approximation: So = 0.03

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.03</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0  (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0  (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D814a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F214</td>
<td>Constant $w_s$ and viscosity $\nu$</td>
</tr>
<tr>
<td>F604a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 5.16b Effects of Dilute Suspension Approximation: So = 0.03
Extended Calculations (to 5000 m)

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.03</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>Approximations</th>
</tr>
</thead>
<tbody>
<tr>
<td>D814a</td>
<td>Dilute Suspension</td>
</tr>
<tr>
<td></td>
<td>Constant ( w_u ) and viscosity ( v )</td>
</tr>
<tr>
<td>F214</td>
<td>Constant ( w_u ) and viscosity ( v )</td>
</tr>
<tr>
<td>F604a</td>
<td>None of the above</td>
</tr>
</tbody>
</table>

159
Figure 5.17a Effects of Dilute Suspension Approximation: $S_0 = 0.05$

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Conc.:</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0 (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0 (m)</td>
</tr>
</tbody>
</table>

Run   | Approximations                     |
------|------------------------------------|
D815a | Dilute Suspension                  |
      | Constant $w_0$ and viscosity $\nu$ |
F215  | Constant $w_0$ and viscosity $\nu$ |
F605a | None of the above                  |

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 5.17b Effects of Dilute Suspension Approximation: So = 0.05
Extended Calculations (to 5000 m)

Slope: 0.08
Initial Sediment Conc.: 0.05
Initial Speed: 1.0 (m/s)
Initial Height: 1.0 (m)

Run Approximations
D815a Dilute Suspension
F215 Constant w_s and viscosity \nu
F605a None of the above
5.5.2 Settling Velocity

Figures 5.18a and 5.18b present results from Series F200 demonstrating the effects of small changes in settling velocity on the numerical predictions. Recall that this version of the Modified Three Equation Model does not contain the dilute suspension approximation, but does hold the settling velocity constant despite changes in concentration. All other modifications in the Modified Three Equation Model are contained in this version.

Note the wide range in predictions for a narrow range in settling velocity. Figure 5.18b shows that a settling velocity of 0.016 m/s leads to an accelerating current, while a settling velocity of 0.017 m/s leads to a decelerating current. The increased settling velocity removes sediment from suspension faster for Run F229 than for the other runs. This reduces the driving force on the flow, and it decelerates. As it decelerates, the erosion of bed sediment is reduced. Once the rate of erosion is smaller than the rate of sediment lost through settling, the current will decelerate until there is no sediment and thus no current.
Figure 5.18a Effects of Settling Velocity - Runs F224-F229

<table>
<thead>
<tr>
<th>Slope:</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sediment Concentration:</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial Speed:</td>
<td>1.0  (m/s)</td>
</tr>
<tr>
<td>Initial Height:</td>
<td>1.0  (m)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>(w_s) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F224</td>
<td>0.014</td>
</tr>
<tr>
<td>F225</td>
<td>0.015</td>
</tr>
<tr>
<td>F226</td>
<td>0.016</td>
</tr>
<tr>
<td>F229</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Figure 5.18b: Effects of Settling Velocity - Runs F224-F229

- **Slope:** 0.08
- **Initial Sediment Concentration:** 0.01
- **Initial Speed:** 1.0 (m/s)
- **Initial Height:** 1.0 (m)

<table>
<thead>
<tr>
<th>Run</th>
<th>$w_s$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F224</td>
<td>0.014</td>
</tr>
<tr>
<td>F225</td>
<td>0.015</td>
</tr>
<tr>
<td>F226</td>
<td>0.016</td>
</tr>
<tr>
<td>F229</td>
<td>0.017</td>
</tr>
</tbody>
</table>

164
5.6 Conclusions

The Modified Three Equation Model is sensitive to properties of the sediment/fluid mixture, such as grain size and kinematic viscosity, and to channel properties, such as slope and bed friction. The sensitivity to initial conditions such as velocity, height, and concentration is relatively small.

The dilute suspension approximation is often adopted without question in turbidity current models. The constant settling velocity approximation is included in the dilute suspension approximation. The settling velocity is reduced significantly for sediment concentrations as low as 1 percent by volume. These concentrations are easily attainable in both laboratory experiments (Garcia 1994) and in the field (Gould 1951). Results show that the change in settling velocity associated with these concentrations, previously thought of as dilute, is sufficient to reverse the predicted fate of turbidity currents.
Chapter 6. Comparison with Previous Investigations

Chapter 5 demonstrated the sensitivity of the Modified Three Equation Model to several parameters, including initial conditions and the choice of closure relationships used in the model. This chapter extends the model investigation to include comparisons to previous numerical investigations. Phase-plane analysis is introduced, and hydraulic jumps in turbidity currents are discussed.

6.1 Comparisons with Parker et al.’s (1986) Three Equation Model

The Three Equation Model of Parker et al. (1986), presented in Chapter 2, predicts the evolution of the turbidity current presented in Figure 6.1. It shows the development of a current modeled using representative conditions for potential currents in submarine canyons: $D_r = 0.1 \text{ mm},$ slope $= 0.05,$ $c_D = 0.004$ ($f = 0.0005$), and $h_o = 2 \text{ m}$.

In order to investigate the effects of closure schemes on Parker et al.’s predictions, their Three Equation Model has been coded. Figure 6.2 confirms the ability of this re-coded model to reproduce Parker et al.’s results. The initial conditions used in the calculations are summarized in the figure. Note that the calculations have been stopped at $Ri = 1$. The closure schemes used for these calculations are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Closure Scheme</th>
<th>Reference</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>water entrainment</td>
<td>Fukushima et al. (1985)</td>
<td>Equation 3.2.7</td>
</tr>
<tr>
<td>sediment entrainment</td>
<td>Akiyama and Fukushima (1985)</td>
<td>Equation 3.3.4</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Parker (1982)</td>
<td>Equation 3.4.2</td>
</tr>
</tbody>
</table>

166
Figure 6.1 Current Development from Ignition (from Parker et al. 1986)
Figure 6.2 Reproduction of Figure 6.1 with the Three Equation Model

---

Slope = 0.05

$S_a = 0.0029$

$h_o = 2 \text{ m}$

$U_o = 0.652 \text{ m/s}$

$W_b = 0.00845 \text{ m/s}$

$Ri = 1$

---
Figure 6.3 shows the evolution of all variables and parameters used in the Figure 6.2 calculations. The nine plots include the development with distance of:

- the layer averaged current velocity $U$
- the layer averaged sediment transport $\psi$ ($\psi = USh$)
- the layer averaged volumetric suspended sediment concentration $S$
- the current height $h$
- the variable $Z$
- the coefficient of sediment entrainment $E_s$
- the Richardson number $Ri$
- the normalized water entrainment velocity $e_w$

The final plot in the bottom right-hand corner is a plot of $U$ vs. $\psi$. This is termed a phase-plane, and is discussed in Section 6.3.

Figure 6.3 shows that the Richardson number reaches a value of 1.0 at approximately 220 meters ($x/h_o = 110$ in Figure 6.2). Unfortunately, Parker et al. do not present results past 200 meters ($x/h_o = 100$ in Figure 6.1). This will be discussed further in Section 6.5. Note that the sediment entrainment coefficient has not reached the maximum value of 0.3, and that the sediment concentration is 0.02.

Chapter 5 demonstrated the sensitivity of the Modified Three Equation Model to the choice of closure schemes. The Parker Three Equation Model is expected to react in a similar fashion. Calculations were made with the Parker Three Equation Model using initial conditions identical to those presented above; the closure schemes were changed as summarized in Table 6.2. Results of this run (Run number R0507) are presented in Figure 6.4a, which presents calculations through 230 meters (as in Figure 6.3), and Figure 6.4b, which extends the calculations until the current reaches a nearly constant Richardson number.
Figure 6.3  Current Development - Parker's Test Case
Calculated with Three Equation Model (Parker et al. 1986)
Table 6.2 Closure Schemes Used in Run R0507 and Series P6200

<table>
<thead>
<tr>
<th>Closure Scheme</th>
<th>Reference</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>water entrainment</td>
<td>Garcia (1985)</td>
<td>Equation 3.2.8</td>
</tr>
<tr>
<td>sediment entrainment</td>
<td>Garcia (1989)</td>
<td>Equation 3.3.7</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Garcia (1985)</td>
<td>$r_o = 2.0$</td>
</tr>
</tbody>
</table>

The two model predictions are compared in Figure 6.5, which shows that the change in closure schemes has a severe impact on the predictions. In particular with the updated closure schemes, the current no longer reaches the critical Richardson number; this is primarily due to the difference in the sediment entrainment function.

Garcia's (1989) sediment entrainment function has an inflection point at $Z = 17.28$ (Figure 6.6), found by setting the second derivative of Equation 3.21 to zero. The second derivative of the sediment concentration ultimately controls the Richardson number. For $Z$ values less than 17.28, the rate of sediment entrainment increases, but as soon as $Z$ reaches the inflection point, the rate of change of sediment concentration begins to decrease.

The Modified Three Equation Model was also run for comparison, with the same initial conditions used in the above runs and the closure schemes outlined in Table 6.2. The results of this run (Run number C6004) are presented in Figure 6.7. Since the sediment concentration in R0507 reaches 0.06 (Figure 6.4b), it is expected that the Modified Three Equation Model will predict somewhat different results, since it includes all effects of elevated sediment concentrations.
Figure 6.5 Comparison of Effects of Closure Schemes on Predictions of the Three Equation Model

| Slope: | 0.05 |
| Settling velocity: | 0.00845 (m/s) |
| Initial Sediment Concentration: | 0.0029 |
| Initial Speed: | 0.652 (m/s) |
| Initial Height: | 2.0 (m) |

Run: R0506h
Run: R0507

Figure 6.3
Figure 6.4
Figure 6.6 Sediment Entrainment Function (Garcia, 1989)

Inflection point at $Z = 17.28$
Figure 6.7 Current Development - Parker's Test Case
Calculated with Modified Three Equation Model

- U (m/s) vs Distance (m)
- ψ (m^3/s) vs Distance (m)
- h (m) vs Distance (m)
- z vs Distance (m)
- u^* vs Distance (m)
- Rf vs Distance (m)
- ψ vs Distance (m)

176
Figure 6.8 compares the effects of closure relationships on predictions from the Modified Three Equation Model (C6005, Figure 6.7) and the Parker Three Equation Model (R0506h, Figure 6.3 and R0507, Figures 6.4a and 6.4b). Note the difference in the velocity plots. The Modified Three Equation Model, taking into account the mass and momentum of the sediment, predicts a velocity which initially exceeds that predicted by the Parker Three Equation Model. This occurs because in the Modified Three Equation Model, the elevated sediment concentrations decrease the settling velocity of the sediment, keeping the sediment in suspension for a greater period of time, thus increasing the gravitational force driving the flow. This in turn causes the increased acceleration seen in Figure 6.8.

This increased acceleration causes the turbidity current to reach the maximum sediment entrainment rate more quickly; this ultimately controls the long-term fate of the turbidity current. Once the maximum sediment entrainment value is reached, the suspended sediment concentration begins to decrease; the increasing entrainment of water from above coupled with the now constant entrainment of sediment leads to a dilution of the sediment in suspensions. This lowers the gravitational driving force, and thus the acceleration of the turbidity current. This progression of events is visible in Figure 6.8; the velocity predicted by the Modified Three Equation Model is ultimately less than that predicted by the Parker Three Equation Model.
Figure 6.8 Comparison of Effects of Closure Schemes on Predictions of the Three Equation Model and the Modified Three Equation Model

Slope: 0.05
Settling velocity: 0.00845 (m/s)
Initial Sediment Concentration: 0.0029
Initial Speed: 0.652 (m/s)
Initial Height: 2.0 (m)

Run
R0506h Figure 6.3
R0507 Figure 6.4
C6004 Figure 6.7

178

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
6.2 Ignition

"Ignition" identifies a set of initial conditions from which a turbidity current is predicted to initially accelerate. The concept was introduced by Parker (1982) who used it to define a criteria for self-sustaining turbidity currents, which are defined as currents that accelerate and entrain sediment in a self-reinforcing cycle.

The initial conditions used in the calculations presented in the previous section were chosen to correspond to the ignition conditions presented in Parker et al. (1986); these are defined as a pseudo-equilibrium state where both \( \frac{dU}{dx} \) and \( \frac{d\psi}{dx} \) are zero at \( x = 0 \). The ignition point is characterized by the lowest finite values of \( U \) and \( \psi \) which provide the required solution; these values are denoted \( U_I \) and \( \psi_I \), respectively.

The point of ignition for Parker's sample case is \( U_I = 0.652 \text{ m/s}, \psi_I = 0.00378 \text{ m}^2/\text{s}, \) and \( S_I = 0.0029 \). Parker notes that this sediment concentration is dilute, and acknowledges that the value for \( U_I \) is attainable in a coastal region. Since the value of \( S_I \) seems to justify Parker et al.'s use of the dilute suspension approximation, the potential variation in this variable is investigated.

Table 6.3 shows the effects of closure scheme choice on ignition calculations. Results are presented for initial heights ranging from 0.5 to 2.0 meters. This range encompasses reasonable values for turbidity currents in submarine canyons and currents emanating from the discharge of mine tailings (Hay 1987b). Notice the values for \( S_I \) increase with decreasing height, and that for smaller currents, the values can no longer be considered
Table 6.3 Sensitivity of Ignition Parameters to Closure Schemes

<table>
<thead>
<tr>
<th>$h_o$(m)</th>
<th>$R_l$</th>
<th>$U_l$(m/s)</th>
<th>$S_l$</th>
<th>$\psi_l$(m^3/s)</th>
<th>$r_o$</th>
<th>$E_r$</th>
<th>$e_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.221</td>
<td>0.749</td>
<td>0.01531</td>
<td>0.00573</td>
<td>P82</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>0.5</td>
<td>0.220</td>
<td>0.702</td>
<td>0.01341</td>
<td>0.00471</td>
<td>G85</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>0.5</td>
<td>0.329</td>
<td>0.780</td>
<td>0.02471</td>
<td>0.00964</td>
<td>P82</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>0.5</td>
<td>0.327</td>
<td>0.734</td>
<td>0.02177</td>
<td>0.00799</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>0.5</td>
<td>0.327</td>
<td>0.786</td>
<td>0.02494</td>
<td>0.00980</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>1.0</td>
<td>0.223</td>
<td>0.699</td>
<td>0.00672</td>
<td>0.00470</td>
<td>P82</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>1.0</td>
<td>0.221</td>
<td>0.650</td>
<td>0.00576</td>
<td>0.00375</td>
<td>G85</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>1.0</td>
<td>0.329</td>
<td>0.727</td>
<td>0.01074</td>
<td>0.00781</td>
<td>P82</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>1.0</td>
<td>0.327</td>
<td>0.679</td>
<td>0.00932</td>
<td>0.00633</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>1.0</td>
<td>0.327</td>
<td>0.597</td>
<td>0.00720</td>
<td>0.00430</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>2.0</td>
<td>0.221</td>
<td>0.652</td>
<td>0.00290</td>
<td>0.00378</td>
<td>P82</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>2.0</td>
<td>0.222</td>
<td>0.603</td>
<td>0.00250</td>
<td>0.00301</td>
<td>G85</td>
<td>A&amp;F</td>
<td>F85</td>
</tr>
<tr>
<td>2.0</td>
<td>0.328</td>
<td>0.678</td>
<td>0.00466</td>
<td>0.00631</td>
<td>P82</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>2.0</td>
<td>0.328</td>
<td>0.629</td>
<td>0.00401</td>
<td>0.00504</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
<tr>
<td>2.0</td>
<td>0.326</td>
<td>0.468</td>
<td>0.00221</td>
<td>0.00206</td>
<td>G85</td>
<td>A&amp;F</td>
<td>G89</td>
</tr>
</tbody>
</table>

Key:
- $e_w$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $r_o$: P82 = Parker (1982); G85 = Garcia (1985)

Calculations performed with slope = 0.05, $D_r = 0.10$ mm, $w_s = 0.8445$ cm/sec, and viscosity = $1 \times 10^{-6}$ m^2/s (20°C).

dilute according to results presented in Chapter 5. The effect of the change in closure schemes is not consistent with changes in height; for $h_o = 0.5$ m, the updated closure schemes increase the values of $U_l$ and $\psi_l$, while the values decrease for $h_o = 2.0$ m.

6.3 Phase-Plane Diagrams

Phase-plane analysis visually identifies the interaction of different variables. Pantin (1979) used phase-plane analysis to study the interactions between velocity and density of turbidity currents. Parker et al. (1986) also utilized this method of presentation to study the relationship between velocity and sediment transport in turbidity currents. Figure 6.9 is taken from Parker et al. (1986); it shows the development of turbidity currents in the
\((U, \psi)\) plane. There are three lines on this figure that should be noted: the autosuspension generation line (AGL), the convergence line (CL), and the line \(Ri = 1\).

The line \(Ri = 1\) (dashed line) demarcates the regions of supercritical \((Ri < 1)\) and subcritical \((Ri > 1)\) flow. This line is of considerable importance, for it represents the location of a singularity in Parker’s equations (Parker et al. 1986). Calculations cannot proceed through this singularity. Turbidity currents predicted to reach this line will undergo a hydraulic jump as they transition from supercritical flow to subcritical flow.

Parker defines an autosuspension generation line (AGL), which splits the supercritical flow region into two parts, an igniting region and a subsiding region. Currents with initial conditions lying above this AGL are said to ignite. Such currents accelerate and entrain sediment in a self-reinforcing cycle. Currents with initial conditions below the AGL are said to subside; these currents decelerate, drop their sediment load, and cease to exist.

Parker’s presentation of the AGL implies that these are the only two options for the development of a current. There is no possibility in his phase-planes of currents that undergo a transition to subcritical flow.

The convergence line (CL) represents a quasi-uniform flow condition to which the turbidity currents eventually approach. Evidence of this flow phenomenon was presented in Figures 5.1b, 5.2b, and 5.3b, which presented the effect of initial conditions on flow development. In those figures, notice that the Richardson number eventually reaches a
Figure 6.9 Phase-plane Diagram (from Parker et al. 1986)
constant value, regardless of the initial velocity, height, or sediment concentration. This convergence line is unique to several parameters, including bed slope and bed friction, as well as to the choice of closure schemes used in the model.

Figure 6.10 is a reproduction of Figure 6.9. These calculations were performed with the present replication of the Parker Three Equation Model. Notice that the traces from runs P6102, P6103, P6109, and P6118 assist in the delineation of Parker's AGL. The trace from run P6100 originates at the ignition point in Figure 6.9.

Figure 6.10b is a reproduction of Figure 6.10a, with a shift in the axes from values normalized by $U_i$ and $\psi_i$ to the values $U$ and $\psi$. This presentation is used for comparison to plots made using alternate closure schemes. Since the values for $U_i$ and $\psi_i$ are dependent on the choice of closure schemes, they are not suitable for comparing normalized phase-plane diagrams.

Figure 6.11 shows how the calculations progress for larger values of $\psi/\psi_i$. It is immediately evident that the AGL provided by Parker et al. (1986) does not accurately demarcate the regions producing igniting and subsiding flows. Several flows initiating above the AGL line, whose origins are marked with triangles, eventually reach the critical Richardson number. In fact, even Parker's ignition case reaches the critical Richardson number approximately 230 meters from the onset of calculations. Currents reaching the critical Richardson number will eventually subside after encountering a hydraulic jump.

The initial conditions used in the runs compromising Figures 6.10 and 6.11 are summarized in Table 6.4. Plots for these model runs are collected in Appendix D.4.
Figure 6.10a Reproduction of Phase-Plane Diagram from Parker et al. (1986)
Figure 6.10b Phase-Plane Diagram with $U$, $\Psi$
Figure 6.11 Phase-plane Diagram - Extended from Figure 6.10a

- $U/U_i$
- $\psi/\psi_i$
- $R_i = 1$
- Igniting Currents
- Currents Reaching $R_i = 1$
- Subsiding Currents
Table 6.4 Summary of Initial Conditions for Figures 6.10 and 6.11 (Series P6100)

<table>
<thead>
<tr>
<th>Run Number</th>
<th>U_o (m/s)</th>
<th>S_o</th>
<th>(\psi_o) (m^2/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Igniting Currents</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6104</td>
<td>0.490</td>
<td>0.00700</td>
<td>0.00686</td>
</tr>
<tr>
<td>P6105</td>
<td>0.652</td>
<td>0.00430</td>
<td>0.00561</td>
</tr>
<tr>
<td>P6106</td>
<td>0.652</td>
<td>0.00363</td>
<td>0.00473</td>
</tr>
<tr>
<td>P6115</td>
<td>0.600</td>
<td>0.00667</td>
<td>0.00800</td>
</tr>
<tr>
<td>P6116</td>
<td>0.700</td>
<td>0.00570</td>
<td>0.00798</td>
</tr>
<tr>
<td>P6117</td>
<td>0.475</td>
<td>0.00640</td>
<td>0.00608</td>
</tr>
<tr>
<td>P6118</td>
<td>0.472</td>
<td>0.00640</td>
<td>0.00604</td>
</tr>
<tr>
<td>Currents Reaching (Ri = 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6100</td>
<td>0.652</td>
<td>0.00290</td>
<td>0.00378</td>
</tr>
<tr>
<td>P6101</td>
<td>0.850</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6107</td>
<td>0.652</td>
<td>0.00350</td>
<td>0.00456</td>
</tr>
<tr>
<td>P6109</td>
<td>0.790</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6110</td>
<td>1.500</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6111</td>
<td>0.900</td>
<td>0.00220</td>
<td>0.00396</td>
</tr>
<tr>
<td>P6112</td>
<td>0.900</td>
<td>0.00440</td>
<td>0.00792</td>
</tr>
<tr>
<td>P6113</td>
<td>0.900</td>
<td>0.00660</td>
<td>0.01188</td>
</tr>
<tr>
<td>P6114</td>
<td>0.800</td>
<td>0.00500</td>
<td>0.00800</td>
</tr>
<tr>
<td>Subsiding Currents</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6102</td>
<td>0.780</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6103</td>
<td>0.470</td>
<td>0.00620</td>
<td>0.00583</td>
</tr>
</tbody>
</table>

The phase-plane presentations of Parker et al. are incomplete, as they do not account for currents that progress towards hydraulic jumps. As evidenced in Figure 6.11, a considerable portion of the phase-plane can generate currents which eventually reach the line \(Ri = 1\). A complete phase-plane should include this region in addition to the igniting and subsiding regions. Another problem associated with Parker et al.'s presentation is that their choice for the sediment entrainment closure scheme (Equation 3.18) contains a step function, which can wreak havoc with numerical predictions. The step function associated with this closure scheme is responsible for the behavior of the currents near the line \(Ri = 1\) in the above plots. All accelerating currents will reach the line \(Ri = 1\) unless the sediment entrainment rate reaches 0.3 before this occurs.
Due to the problems associated with the sediment closure scheme of Akiyama and Fukushima (1985), the closure schemes used by Parker et al. (1986) to develop Figure 6.9 and herein to develop Figures 6.10 and 6.11 have been updated to those listed in Table 6.3.

The sensitivity of the phase-plane to the choice of closure schemes is considerable. Figure 6.12a shows the differences in current development attributable to these updated closure schemes. The ignition point for Parker's sample case, calculated with the updated closure schemes, is defined by \( U_1 = 0.468 \text{ m/s} \), \( \psi_1 = 0.00206 \text{ m}^2/\text{s} \), and \( S_1 = 0.0022 \). This ignition velocity is notably smaller than the one presented above, quantifying the sensitivity of the ignition calculations to the choice of closure schemes.

Figure 6.12b is a replica of Figure 6.12a, with a change in the axes from values normalized by \( U_1 \) and \( \psi_1 \) to the values \( U \) and \( \psi \). These variables permit direct comparison to Figure 6.10b.

Figure 6.13 expands on Figure 6.12a to show that the igniting currents do not reach the line \( Ri = 1 \), which previously had been a problem. For clarity, only three representative traces are presented in the figure. The initial conditions used in the runs compromising Figures 6.12 and 6.13 are summarized in Table 6.5. Plots for these model runs are collected in Appendix D.5.
Figure 6.12a Normalized Phase-Plane Diagram
Updated Closure Schemes - Series P6200
Figure 6.12b Phase-Plane Diagram
Updated Closure Schemes - Series P6200

Igniting Field

- - - - - - Ri = 1

■ Igniting Currents

△ Currents Reaching Ri = 1

◇ Subsiding Currents

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Figure 6.13 Normalized Phase-Plane Diagram
Extended Calculations - Series P6200

---

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>Run Number</th>
<th>$U_o$ (m/s)</th>
<th>$S_o$</th>
<th>$\psi_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Igniting Currents</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6200</td>
<td>0.468</td>
<td>0.002200</td>
<td>0.002059</td>
</tr>
<tr>
<td>P6201</td>
<td>0.560</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6207</td>
<td>0.400</td>
<td>0.004375</td>
<td>0.003500</td>
</tr>
<tr>
<td>P6208</td>
<td>0.400</td>
<td>0.003750</td>
<td>0.003000</td>
</tr>
<tr>
<td>P6209</td>
<td>0.400</td>
<td>0.002500</td>
<td>0.002000</td>
</tr>
<tr>
<td>P6210</td>
<td>0.400</td>
<td>0.001250</td>
<td>0.001000</td>
</tr>
<tr>
<td>P6213</td>
<td>0.500</td>
<td>0.000500</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6214</td>
<td>0.600</td>
<td>0.000833</td>
<td>0.001000</td>
</tr>
<tr>
<td>P6215</td>
<td>0.700</td>
<td>0.001070</td>
<td>0.001498</td>
</tr>
<tr>
<td>P6216</td>
<td>0.800</td>
<td>0.001250</td>
<td>0.002000</td>
</tr>
<tr>
<td>P6217</td>
<td>0.350</td>
<td>0.002500</td>
<td>0.001750</td>
</tr>
<tr>
<td>P6224</td>
<td>0.475</td>
<td>0.000526</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6227</td>
<td>0.350</td>
<td>0.002143</td>
<td>0.001500</td>
</tr>
<tr>
<td><strong>Currents Reaching $R_i = 1$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6202</td>
<td>0.540</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6203</td>
<td>0.520</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6204</td>
<td>0.500</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6212</td>
<td>0.300</td>
<td>0.002500</td>
<td>0.001500</td>
</tr>
<tr>
<td>P6220</td>
<td>0.320</td>
<td>0.002340</td>
<td>0.001498</td>
</tr>
<tr>
<td>P6221</td>
<td>0.300</td>
<td>0.002290</td>
<td>0.001374</td>
</tr>
<tr>
<td>P6222</td>
<td>0.400</td>
<td>0.000938</td>
<td>0.000750</td>
</tr>
<tr>
<td>P6223</td>
<td>0.450</td>
<td>0.000555</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6225</td>
<td>0.425</td>
<td>0.000588</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6226</td>
<td>0.350</td>
<td>0.001429</td>
<td>0.001000</td>
</tr>
<tr>
<td><strong>Subsiding Currents</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6205</td>
<td>0.450</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6206</td>
<td>0.480</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6211</td>
<td>0.300</td>
<td>0.001670</td>
<td>0.001002</td>
</tr>
<tr>
<td>P6218</td>
<td>0.400</td>
<td>0.000625</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6219</td>
<td>0.300</td>
<td>0.002083</td>
<td>0.001250</td>
</tr>
</tbody>
</table>
Several items are evident from comparing Figures 6.10 and 6.11 to Figures 6.12 and 6.13:

1) Figures 6.12a and 6.12b show a significant increase in the size of the igniting region, from which flows are predicted to ignite. This is most easily seen when comparing Figure 6.10b to Figure 6.12b, which have identical scales.

2) Figures 6.12a and 6.12b show a critical region: currents originating in this region are predicted to reach the critical Richardson number, and eventually subside. Such flows are confined to a relatively small region in Figures 6.12a and 6.12b. The presentation of Parker et al. (1986), reproduced in Figure 6.10a, does not account for flows that reach the critical Richardson number, despite the relatively large area from which flows are predicted to do so (Figure 6.11).

3) Turbidity currents beginning at Parker's ignition state are predicted to develop differently when the closure schemes are changed. Figure 6.11 shows the current originating at the point of ignition \((U_i/U\) and \(\psi_i/\psi\) equal to 1) eventually reaches the critical Richardson number, while Figure 6.13 shows that the current originating at this point continues to accelerate. The change in closure schemes used in these calculations is responsible for this difference.

Figures 6.12a and 6.12b show a narrow band (cross-hatched in the figures) in the \(U, \psi\) plane from which currents are predicted to reach the line \(Ri = 1\). This band was developed through a systematic investigation of numerous combinations of the initial conditions; it is not defined mathematically, but through trial and error.

Considering the difficulties with the phase-plane presentations in Parker et al. (1986), it is suggested that all areas of the \(U, \psi\) plane be investigated before dividing the plane into

193
igniting and subsiding regions, and that a third region, the critical region, be included.

Even though currents originating in this region eventually subside after going through a hydraulic jump, they should not be grouped with currents that originate in the subsiding region, for those currents which reach the critical Richardson number can do so with velocities and sediment concentrations far exceeding those currents which originate in the subsiding zone. This is evident in Figure 6.11.

The phase-plane analysis can be complicated by currents such as Run P6212, in Figure 6.14a. This current initially decelerates and drops its sediment load. Notice, however, that the Richardson number is still increasing. Figure 6.14b extends the calculations and shows that the current eventually accelerates. Calculations are stopped when $Ri$ reaches 1, as the model is not valid past this point.

The concept of ignition is not readily applicable to the Modified Three Equation Model. Ignition is uniquely defined for a given set of variables, including the bottom slope, the drag coefficient, the particulate Reynolds number, the sediment size, and the settling velocity. In the Modified Three Equation Model, the settling velocity, viscosity, and particulate Reynolds number all vary with the sediment concentration, whereas in the Parker Three Equation Model they are all constant. This does not preclude the use of phase-plane diagrams. It merely means that they should not be normalized by ignition values when using the Modified Three Equation Model. Care must also be taken when using phase-planes and the Modified Three Equation Model to insure that the dividing line between supercritical and subcritical flow is represented by the line $Ri = 1 + RS$, as outlined in the following section.
Figure 6.14a Run P6212 (through 1000 meters)
Figure 6.14b Run P6212 (through 4000 meters)
6.4 Hydraulic Jumps

Hydraulic jumps are important to turbidity current modeling, though the models presented thus far cannot predict turbidity current development through a hydraulic jump. The problem stems from a singularity in the governing equations when the Richardson number reaches the critical value. This singularity occurs at $Ri = 1$ in the Parker Three Equation Model. The Modified Three Equation Model, however, has a singularity that includes the influence of the suspended sediment concentration. Appendix C presents the First Order Ordinary Differential Equation version of the Modified Three Equation Model. Equation C.21 shows that the singularity occurs when $1 + RS - Ri = 0$, or when $Ri = 1 + RS$.

Yih (1980) presents the standard conjugate depth equation for an internal hydraulic jump in a stratified flow with a stationary upper layer:

$$\frac{h_2}{h_1} = \frac{1}{2} \left( \sqrt{1 + 8Ri^{-1}} - 1 \right) \quad (6.1)$$

where $h_2$ is the height of the current after the jump, $h_1$ is the height of the current before the jump, and $Ri$ is the Richardson number of the current before the jump. It is derived from the momentum equation, with the assumption that the lower layer is conservative. For turbidity currents, this implies that there is no entrainment through the hydraulic jump and that the current is at equilibrium regarding sediment concentration.

Hydraulic jumps in turbidity currents have been discussed by Menard (1964), Komar (1971), Garcia and Parker (1989), and Garcia (1993). Most previous investigations have focused on jumps occurring at the transition of a submarine canyon and its associated submarine fan. The sudden change in slope at this transition can force a hydraulic jump.
This transition area becomes the location of sediment deposition, as the gravitational force on the current (proportional to the slope), and ultimately the current velocity and its ability to transport sediment decrease.

The canyon/fan transition is relatively easy to model in a laboratory environment. Garcia (1993) conducted experiments with such a setup and reports measurements of the flow depths before and after the hydraulic jump. He presents data supporting the application of Equation 6.1 to hydraulic jumps in turbidity currents.

Equation 6.1 provides potential for continuing turbidity current calculations after a hydraulic jump, but also has potential problems. For example, consider the current presented in Figure 6.14b. This current eventually reaches the critical Richardson number, requiring a transition from supercritical to subcritical flow. The question regarding the application of Equation 6.1 to this current involves the location at which the equation should be applied. Figure 6.15 presents the conjugate depth $h_2$ calculated from Equation 6.1; it shows that $h_2$ depends on where the calculations are performed. For cases such as this, where the Richardson number is continually evolving, the application of Equation 6.1 is not recommended. However, for a current that has reached a stable Richardson number, this equation can be applied at a break in slope, as demonstrated by Garcia (1993).

Hydraulic jumps do not occur at the exact location where the Richardson number reaches the critical value. In open channel flow, the actual location of a hydraulic jump can be back-calculated from the backwater relationships if assumptions are made about the
Figure 6.15 Investigation of Equation 6.1 Applied to Figure 6.14b
downstream control affecting the jump. This involves solving the gradually varied flow
equation from the downstream control until the height calculated matches the conjugate
depth calculated from Equation 6.1. In turbidity current flow, however, it is not clear that
this is possible. Since the Richardson number contains the sediment concentration variable
$S$, there is an additional factor influencing the flow regime in turbidity current flow. It may
be that turbidity currents can reach conditions forcing a hydraulic jump without any
influence from downstream.

Model results show that turbidity currents can reach the critical Richardson number
without a break in slope, implying that they can undergo hydraulic jumps while still in
submarine canyons. This has implications on the way submarine canyons are viewed. The
relief (height from channel bottom to levee top) in deep sea channels can range upwards of
200 to 300 meters. Calculations of supercritical turbidity currents of this height yield
seemingly unreasonable velocities (Komar 1975). However, if a large scale turbidity
current were to experience a hydraulic jump while still in the submarine canyon, it would
be able to attain such heights at a considerably lower velocity.

6.5 Parker et al. (1986) Four Equation Model

A “Four Equation Model” (Fukushima et al. 1985, Akiyama and Stephan 1985, Parker et
al. 1986, 1987) is comprised of the Parker Three Equation Model and an additional
equation, representing the balance of the turbulent kinetic energy in the turbidity current.

The Parker Three Equation Model predicts accelerating currents that form a self-
reinforcing cycle of acceleration and erosion. The mass, momentum, and sediment
entrainment equations are coupled such that the following progression of events occurs for igniting currents:

1. A sediment driven current moves down canyon under the influence of the gravitational force acting on the sediment fraction.
2. At a certain threshold velocity, this current entrains sediment from the bed.
3. The gravitational force acting on the sediment increases with the increase in sediment concentration in the current.
4. This increase in the driving force causes the current to accelerate further.
5. This acceleration leads to additional sediment entrainment, thus setting up a cyclical pattern.

This process, which Parker has termed "self-acceleration", led Parker et al (1986) to the conclusion that the Parker Three Equation Model is flawed. This leads to the introduction of the equation for the balance of the turbulent kinetic energy, and the Four Equation Model.

Parker et. al. (1986) explain the theory behind the addition of this fourth equation. The essence of the argument is that there is a finite amount of turbulent energy supplied by the mean flow. This energy is used in the entrainment and maintenance of the sediment in suspension inside the current. The turbulent energy balance effectively limits the amount of sediment that can be entrained by the flow.

The Parker Three Equation Model, for the self-accelerating current described above, predicts high values of sediment entrainment that violate the turbulent energy balance presented by Parker et al. (1986). These high levels of sediment entrainment cannot be supported by the turbulent energy produced by the mean flow. As a means to avoid such self-accelerating currents, Parker et al. introduce the Four Equation Model.
There are several issues that must be addressed regarding the transition from the Parker Three Equation Model to the Four Equation Model. The Four Equation Model attempts to compensate for a perceived flaw in the Parker Three Equation Model, while at the same time retaining each equation from the Parker Three Equation Model. Any inherent problem in the Parker Three Equation Model will also be present in the Four Equation Model.

The Parker Three Equation Model is mathematically closed, given certain approximations for such variables as sediment entrainment, water entrainment, and shear velocity. The numerical model treats each equation with equal weight. Conservation equations, such as mass and sediment conservation, are viewed as being equally important as empirical closure schemes. This point cannot be stressed enough. It is imperative that closure schemes be derived with this in mind. One weak closure scheme will undermine an entire numerical model.

The addition of the turbulent kinetic energy equation adds further to the need for closure schemes and approximations. One single triple product in the mean turbulent energy equation adds nine unknowns to the system of equations. Approximations must be made to deal with these added unknowns.

The basis for the Parker Three Equation Model, the three equations representing balances of mass, momentum, and sediment, are well understood. Any flaw or shortcoming in the model must be due to approximations made in the derivation of the model or in the selection of closure schemes. Material presented above has identified problems
associated with the sediment entrainment function of Akiyama and Fukushima (1985), which was used by Parker et al. (1986) and Fukushima et al. (1985).

Evidence of problems with the Parker Three Equation Model was presented in Figure 6.2. The sample case used by Parker et al. (1986), which is used to justify the need for the turbulent energy equation, in fact reaches the critical Richardson number shortly after it violates Parker’s energy constraint. This fact is not acknowledged by Parker et al.

The supposed tendency of the Parker Three Equation Model to predict unrealistic currents is investigated in Figure 6.16, which contains extended calculations for the currents previously presented in Figures 6.4 and 6.7. These currents were developed with the Parker Three Equation Model and the Modified Three Equation Model, respectively. In both cases, the closure schemes were changed from those originally used by Parker in his sample case. Thus, differences in the model predictions cannot be attributed to the closure schemes.

Figure 6.16 shows that after 50 kilometers, the Modified Three Equation Model predicts a velocity of 14.9 m/s, and the Parker Three Equation Model predicts a velocity of 16.8 m/s. The acceleration of the current has already decreased considerably, and the velocity is leveling off. Both of these values are possible for large scale turbidity currents. Several researchers (Heezen and Ewing 1952, Heezen 1963, and Krause et al. 1970) have used records from breaks in underwater telephone and telegraph cables to calculate turbidity current velocities ranging from 14 m/s to over 20 m/s.
Figure 6.16 Long Term Current Development

- **Slope:** 0.05
- **Settling velocity:** 0.00845 (m/s)
- **Initial Sediment Concentration:** 0.0029
- **Initial Speed:** 0.652 (m/s)
- **Initial Height:** 2.0 (m)
The calculated height of approximately 350 meters is also realistic, based on measurements of channel relief (Komar 1975). The predicted sediment concentration had been reduced to approximately 0.02 after 50 kilometers, implying a near-bed sediment concentration of 0.04. This is not an unrealistic value. The empirically derived sediment entrainment function allows for a maximum sediment concentration of 0.3.

None of the variables presented in Figure 6.16 are indicative of a "run-away" turbidity current, accelerating to unrealistic velocities and carrying unreasonable sediment concentrations. Furthermore, these calculations adopted a constant slope. In reality, the slope will generally decrease with distance down the canyon, causing the driving force, and thus the current's velocity, to decrease as well.
Chapter 7. Summary and Conclusions

7.1 Summary

Complete field equations have been established for turbidity current flow. The Modified Three Equation Model has been developed to predict the fate of a specific set of turbidity currents. In the final form, the model is applied to steady state, one-dimensional turbidity currents flowing through a quiescent ambient fluid. The currents are assumed to carry uniform sediment in wide, rectangular channels.

Modifications in this model focus on the impacts of increased sediment concentration on the field equations and the numerical predictions. Previous investigations have invoked the dilute suspension approximation for sediment concentrations up to a few percent. Experimentally derived relationships for the effect of sediment concentration on settling velocity show that the dilute suspension approximation is not necessarily valid, even at these low concentrations.

Functional relationships for closure hypotheses have been chosen after careful review. The closure schemes chosen for the Modified Three Equation Model are summarized in Table 3.1.

7.2 Model Results

Results presented in Chapter 5 demonstrate the influence of several factors on model predictions. Initial conditions, such as current height, velocity, and sediment concentration, have a short-lived effect on model predictions. Channel properties, such as
slope and bed friction, control the long-term evolution of turbidity currents. The Modified Three Equation Model is highly sensitive to the closure relationships, as demonstrated in Section 5.3. Section 5.4 presents results demonstrating the impact of changes in kinematic viscosity and sediment grains size; for changes in these properties within a given range, differences in predictions are small. Outside this range, these properties can lead to predictions of contrary fates for turbidity currents.

Limitations of the dilute suspension approximation are discussed in Section 5.5. The influence of concentration on the settling velocity is one major factor overlooked when the dilute suspension approximation is adopted. Results show that this influence cannot be disregarded in turbidity current modeling.

A four equation model is not necessary to aptly describe turbidity current flow. Since the four equation model includes all components of the three equation model, the argument that the three equation model is flawed undermines the competency of the four equation model.

Results from the Modified Three Equation Model for a turbidity current with conditions expected in Scripps Submarine Canyon show a smaller acceleration than results from the Parker Three Equation Model. Although the differences are relatively minor, the impact of modifications to the field equations are clearly visible.
7.3 Directions for Future Research

This investigation could not address all possible aspects of turbidity current flow. In hopes of motivating and directing future work, some of these items are now addressed.

As formulated, the Modified Three Equation Model does not account for shear at the upper interface of the turbidity current. This is due to the assumed vertical profiles of sediment and velocity, and the choice for the limits of integration. The quiescent ambient fluid approximation is related to these assumptions, for any interfacial shear would be proportional to the difference between the velocity at the upper edge of the turbidity current and the velocity of the ambient fluid. Both of these velocities are assumed to be zero; the upper edge of the turbidity current is defined to be the location at which the velocity is reduced to the ambient value. The impacts of non-zero ambient velocities on turbidity currents needs to be addressed.

The sediment entrainment function is the most important formulation in any model for turbidity current flow. The steep nature of this empirical function leads to extreme changes in the predicted sediment entrainment rate for minor changes in the input parameters. Unfortunately, these input parameters include the settling velocity and the shear velocity, two properties of the fluid mixture that are difficult to ascertain with a high level of accuracy. Continued effort needs to focus on this relationship.

Future laboratory investigations could provide much needed data against which to test several formulations:
• Sediment entrainment: One particular need for this formulation is an investigation into the behavior of the relationship near the maximum entrainment rate of 0.3. All accelerating turbidity currents are predicted to reach this maximum, yet there is scarcely any data supporting entrainment rates of this magnitude. A second item that needs attention is the influence of increased levels of suspended sediment on the rate of sediment entrainment.

• Near-bed sediment concentration: The relationship between the near-bed sediment concentrations and flow variables needs to be improved. Two relationships in the literature are mediocre at best.

• Ambient velocities: Ambient currents in submarine canyons have been measured exceeding 0.5 m/s. The effects of ambient velocities on developed turbidity currents are two-fold. First, they serve to increase the friction acting on the upper surface of the turbidity current. Second, they lead to increased mixing of ambient fluid into the current, thus diluting the sediment concentration and lowering the gravitational driving force. Information on the extent of these effects is lacking in the literature. Experiments investigating these effects would be beneficial.

• Concentration effects: Increased sediment concentrations lower settling velocities. Laboratory experiments could provide data on how particle size and particle shape influence the effect of concentration on settling velocity.
References

Akiyama, J. and Fukushima, Y. 1985. Entrainment of noncohesive bed sediment into suspension. External Memo No. 175, St. Anthony Falls Hydraulic Laboratory, University of Minnesota.


Appendix A. Cross-Section-Integrated Conservation Equations

The following appendix details the steps taken to integrate the partial differential equations for conservation of mixture mass, momentum, and sediment (presented in Section 2.1) over the cross section of the turbidity current.

A.1 Conservation of Mixture Mass

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} \rho u_i = 0
\]

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial z} \rho w = 0
\]  

(A.1)

First, the equations are integrated in \( z \) through the depth of the turbidity current.

\[
\int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial \bar{\rho}}{\partial t} \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial \rho u}{\partial x} \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial \rho v}{\partial y} \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial \rho w}{\partial z} \, dz = 0
\]  

(A.2)

Using Leibniz Rule:

\[
\frac{\partial}{\partial t} \int_{z_b(x,y)}^{z_t(x,y)} \bar{\rho} \, dz + \bar{\rho} \bigg|_{z_b(x,y)} \, \frac{\partial z_b}{\partial t} + \frac{\partial}{\partial x} \int_{z_b(x,y)}^{z_t(x,y)} \rho u \, dz + \rho u \bigg|_{z_b(x,y)} \frac{\partial z_b}{\partial x}
\]

\[
+ \frac{\partial}{\partial y} \int_{z_b(x,y)}^{z_t(x,y)} \rho v \, dz + \rho v \bigg|_{z_b(x,y)} \frac{\partial z_b}{\partial y} + \frac{\partial}{\partial z} \int_{z_b(x,y)}^{z_t(x,y)} \rho w \, dz = 0
\]  

(A.3)

Next, the boundary condition at the bottom is used to simplify the above equation.

\[
\bar{\rho}(z_b) \frac{\partial z_b}{\partial t} + \rho u \bigg|_{z_b} \frac{\partial z_b}{\partial x} + \rho v \bigg|_{z_b} \frac{\partial z_b}{\partial y} - \rho w \bigg|_{z_b} = -\rho_s \left( F - \bar{w}_c \right)
\]  

(A.4)

whence

\[
\int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial}{\partial t} \bar{\rho} \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial}{\partial x} \rho u \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial}{\partial y} \rho v \, dz + \int_{z_b(x,y)}^{z_t(x,y)} \frac{\partial}{\partial z} \rho w \, dz = \rho_s \left( F - \bar{w}_c \right)
\]  

(A.5)
Next, the entrainment boundary condition is applied.

\[
\begin{align*}
\frac{\partial h}{\partial t} w|_w &= w_e. \tag{A.6}
\end{align*}
\]

Applying the appropriate densities to this condition yields:

\[
\begin{align*}
\rho \frac{\partial h}{\partial t} - \frac{\partial w}{\partial t} &= \rho w_e \tag{A.7}
\end{align*}
\]

where \( w_e \) is the velocity of entrainment of ambient water into the turbidity current, and is defined as positive in the negative \( z \) direction.

This condition allows further simplification of the mass conservation equation to:

\[
\begin{align*}
\frac{\partial}{\partial t} \int_{s_0(x,y)}^{\infty} \rho \, dz + \int_{b_0(x,y)}^{\infty} \frac{\partial}{\partial x} \int_{s_0(x,y)}^{\infty} \rho u \, dz + \int_{b_0(x,y)}^{\infty} \frac{\partial}{\partial y} \int_{s_0(x,y)}^{\infty} \rho v \, dz + \rho \frac{\partial h}{\partial t} &= \rho w_e + \rho_s (F - w, c) \tag{A.8}
\end{align*}
\]

Next, the depth-integrated equation is integrated in \( y \) across the width of the turbidity current.

\[
\begin{align*}
\frac{\partial}{\partial t} \int_{b_0(x,y)}^{\infty} \rho \, dz \, dy + \int_{b_0(x,y)}^{\infty} \frac{\partial}{\partial x} \int_{b_0(x,y)}^{\infty} \rho u \, dz \, dy + \int_{b_0(x,y)}^{\infty} \frac{\partial}{\partial y} \int_{b_0(x,y)}^{\infty} \rho v \, dz \, dy + \int_{b_0(x,y)}^{\infty} \rho \frac{\partial h}{\partial t} \, dy &= \\
\int_{b_0(x,y)}^{\infty} \rho w_e \, dy + \int_{b_0(x,y)}^{\infty} \rho_s (F - w, c) \, dy \tag{A.9}
\end{align*}
\]

Using Leibniz Rule:

\[
\begin{align*}
\frac{\partial}{\partial t} \int_{b_0(x,y)}^{\infty} \rho \, dz \, dy + \int_{b_0(x,y)}^{\infty} \frac{\partial}{\partial x} \int_{b_0(x,y)}^{\infty} \rho u \, dz \, dy - \frac{\partial}{\partial x} \int_{b_0(x,y)}^{\infty} \rho u \, dz + \int_{b_0(x,y)}^{\infty} \rho \frac{\partial h}{\partial t} \, dy = \\
\frac{\partial}{\partial y} \int_{b_0(x,y)}^{\infty} \rho \, dz + \int_{b_0(x,y)}^{\infty} \rho u \, dy + \int_{b_0(x,y)}^{\infty} \rho \frac{\partial h}{\partial t} \, dy = \\
\int_{b_0(x,y)}^{\infty} \rho w_e \, dy + \int_{b_0(x,y)}^{\infty} \rho_s (F - w, c) \, dy \tag{A.10}
\end{align*}
\]
A.2 Conservation of Mixture Momentum

\[ \frac{\partial}{\partial t} \langle \rho u_i \rangle + \frac{\partial}{\partial x_j} \langle \rho u_i u_j \rangle = \rho_o \bar{c} R g_i - \frac{\partial \bar{P}}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} \]  \hspace{1cm} (A.11)

The convention on the subscripts in the shear terms is \((i,j) = \text{(direction, face)}\).

The \(z\)-component of the momentum equation is solved for the vertical pressure distribution, for use in the \(x\)-component of the momentum equation.

\[ \frac{\partial}{\partial t} \bar{p}_w + \frac{\partial}{\partial x} \langle \rho u w \rangle + \frac{\partial}{\partial y} \langle \rho v w \rangle + \frac{\partial}{\partial z} \langle \rho w^2 \rangle = -\rho_o \bar{c} R g - \frac{\partial \bar{P}}{\partial z} + \left( \frac{\partial}{\partial x} \bar{\tau}_{xx} + \frac{\partial}{\partial y} \bar{\tau}_{xy} + \frac{\partial}{\partial z} \bar{\tau}_{xz} \right) \]  \hspace{1cm} (A.12)

The boundary layer assumptions reduce the above equation to

\[ \frac{\partial \bar{P}}{\partial z} = -\rho_o \bar{c} R g \]  \hspace{1cm} (A.13)

Integrating this equation from a point within the turbidity current to the top of the turbidity current yields:

\[ \int_{z}^{\infty} \frac{\partial \bar{P}}{\partial z} dz = -\int_{z}^{\infty} \rho_o \bar{c} R g dz = -\rho_o R g \int_{z}^{\infty} \bar{c} dz \]  \hspace{1cm} (A.14)

\[ \bar{P}(z) = \rho_o R g \int_{z}^{\infty} \bar{c} dz \]  \hspace{1cm} (A.15)

This approximation for the pressure is then used in the \(x\)-momentum equation, yielding:

\[ \frac{\partial}{\partial t} \langle \rho u \rangle + \frac{\partial}{\partial x} \langle \rho u^2 \rangle + \frac{\partial}{\partial y} \langle \rho uv \rangle + \frac{\partial}{\partial z} \langle \rho uw \rangle = -\rho_o g R \frac{\partial}{\partial x} \int_{z}^{\infty} \bar{c} dz - \left( \frac{\partial}{\partial x} \bar{\tau}_{xx} + \frac{\partial}{\partial y} \bar{\tau}_{xy} + \frac{\partial}{\partial z} \bar{\tau}_{xz} \right) \]  \hspace{1cm} (A.16)

Schlichting (1979) presented measurements of the velocity fluctuations in a boundary
layer. The cross components (i.e. \( \overline{u'v'}, \overline{u'w'} \)) are an order of magnitude larger than the strain components (i.e. \( \overline{u'u'}, \overline{v'v'}, \overline{w'w'} \)). Only those stress components with cross terms will be retained in the equations.

\[
\frac{\partial}{\partial t} \overline{pu} + \frac{\partial}{\partial x} \langle pu^2 \rangle + \frac{\partial}{\partial y} \langle puv \rangle + \frac{\partial}{\partial z} \langle puw \rangle = -\rho_o g R \frac{\partial}{\partial x} \int_0^z \overline{c} dz - \left( \frac{\partial}{\partial y} \tau_{xy} + \frac{\partial}{\partial z} \tau_{xz} \right)
\]

(A.17)

Integration over the cross-section of the channel begins with integration in the vertical \( z \)-direction:

\[
\int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial}{\partial t} \overline{pu} dz + \int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial}{\partial x} \langle pu^2 \rangle dz + \int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial}{\partial y} \langle puv \rangle dz + \int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial}{\partial z} \langle puw \rangle dz =
\]

\[
-\rho_o g R \int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial}{\partial x} \int_0^z \overline{c} dz dz - \int_{z_t(x,y)}^{z_b(x,y)} \frac{\partial \tau_{xy}}{\partial y} dz - \tau_{xz}|_{z_b}
\]

(A.18)

Using Leibniz Rule,

\[
\frac{\partial}{\partial t} \int_{z_t(x,y)}^{z_b(x,y)} \overline{pu} \, dz + \overline{pu}|_{z_b} \frac{\partial z_b}{\partial t} + \int_{z_t(x,y)}^{z_b(x,y)} \langle pu^2 \rangle \, dz + \langle pu^2 \rangle |_{z_b} \frac{\partial z_b}{\partial x}
\]

\[
+ \frac{\partial}{\partial y} \int_{z_t(x,y)}^{z_b(x,y)} \langle puv \rangle \, dz + \langle puv \rangle |_{z_b} \frac{\partial z_b}{\partial y} - \langle puv \rangle |_{z_t} =
\]

\[
-\rho_o g R \left( \frac{\partial}{\partial x} \int_{z_t(x,y)}^{z_b(x,y)} \overline{c} dz \, dz + \int_{z_t(x,y)}^{z_b(x,y)} \overline{c} dz \right) - \frac{\partial}{\partial y} \int_{z_t(x,y)}^{z_b(x,y)} \tau_{xy} \, dz - \tau_{xz}|_{z_b} \frac{\partial z_b}{\partial y} - \tau_{xz}|_{z_t}
\]

(A.2.9)

Next, the boundary condition at the bottom is introduced.

\[
\frac{\partial z_b}{\partial t} \bigg|_{z_t} + u \bigg|_{z_b} \frac{\partial z_b}{\partial x} + v \bigg|_{z_b} \frac{\partial z_b}{\partial y} - w \bigg|_{z_b} = 0
\]

(A.19)

or:

218
\[ \rho u \left|_{s_b} \partial_z b + \left( \rho u^2 \right) \left|_{s_b} \partial_x b + \left( \rho uv \right) \right| \partial_y b = 0 \quad (A.20) \]

Hence

\[ \frac{\partial}{\partial t} \int_{s_0(x,y)} \rho u \, dz + \frac{\partial}{\partial x} \int_{s_0(x,y)} \left( \rho u^2 \right) \, dz + \frac{\partial}{\partial y} \int_{s_0(x,y)} \left( \rho uv \right) \, dz = \]

\[ - \rho g R \left( \frac{\partial}{\partial x} \int_{s_0(x,y)} \bar{c} \, dz' \, dz + \frac{\partial z}{\partial x} \int_{s_0(x,y)} \bar{c} \, dz \right) - \frac{\partial}{\partial y} \int_{s_0(x,y)} \tau_{yx} \, dz - \left. \frac{\partial z}{\partial y} \right|_{s_b} \frac{\partial z}{\partial z} - \tau_{zz} \right|_{s_b} \quad (A.21) \]

Integration in \( y \) over the width of the turbidity current completes the area integration:

\[ \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial t} \int_{s_0(x,y)} \rho u \, dz 
+ \frac{\partial}{\partial x} \int_{s_0(x,y)} \left( \rho u^2 \right) \, dz 
+ \frac{\partial}{\partial y} \int_{s_0(x,y)} \left( \rho uv \right) \, dz 
= \]

\[ - \rho g R \int_{b_1(x,y)}^{b_2(x,y)} \left( \frac{\partial}{\partial x} \int_{s_0(x,y)} \bar{c} \, dz' \, dz 
+ \frac{\partial z}{\partial x} \int_{s_0(x,y)} \bar{c} \, dz \right) \, dy 
- \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{s_0(x,y)} \tau_{yx} \, dz 
- \left. \frac{\partial z}{\partial y} \right|_{s_b} \frac{\partial z}{\partial z} - \int_{b_1(x,y)}^{b_2(x,y)} \tau_{zz} \, dy \quad (A.22) \]

Using Leibniz Rule

\[ \frac{\partial}{\partial t} \int_{b_1(x,y)}^{b_2(x,y)} \rho u \, dz 
+ \frac{\partial}{\partial x} \int_{b_1(x,y)}^{b_2(x,y)} \left( \rho u^2 \right) \, dz 
- \frac{\partial}{\partial x} \int_{s_0(x,y)} \left( \rho u^2 \right) \, dz \]

\[ + \frac{\partial}{\partial x} \int_{b_1(x,y)}^{b_2(x,y)} \left( \rho uv \right) \, dz \, dy = \]

\[ - \rho g R \frac{\partial}{\partial x} \int_{b_1(x,y)}^{b_2(x,y)} \bar{c} \, dz' \, dz 
+ \rho g R \frac{\partial}{\partial x} \int_{s_0(x,y)} \bar{c} \, dz' \, dz \quad \left. \right|_{b_1(x,y)}^{b_2(x,y)} \]

\[ - \rho g R \frac{\partial}{\partial x} \int_{s_0(x,y)} \frac{\partial z}{\partial x} \, dz \, dy - \rho g R \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial z}{\partial x} \, dz \, dy \]

219
\[ - \frac{\partial}{\partial y} \int_{b_1(x,z)}^{b_2(x,z)} \int_{b_1(x,y)}^{b_2(x,y)} \tau_{xy} \, dz \, dy - \int_{b_1(x,z)}^{b_2(x,z)} \frac{\partial \tau_{zy}}{\partial y} \, dy - \int_{b_1(x,z)}^{b_2(x,z)} \tau_{zy} \big|_{z_0} \, dy \]  

(A.23)

Now consider the y-momentum equation:

\[ \frac{\partial}{\partial t} \rho u + \frac{\partial}{\partial x} (\rho u^2) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = -\rho g R \frac{\partial}{\partial y} \int_{z_0}^{z_1} c dz - \left( \frac{\partial}{\partial x} \tau_{yx} + \frac{\partial}{\partial z} \tau_{yz} \right) \]  

(A.24)

Integration over the cross section of the current begins with integration in \( z \) over the depth of the current:

\[ \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial t} \rho v \, dz + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} (\rho u v) \, dz + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} (\rho v^2) \, dz + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial z} (\rho v w) \, dz = \]

\[ -\rho g R \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{z_0}^{z_1} c \, dz \, dz - \left. \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} \tau_{yx} \, dz \right|_{z_0} - \left. \tau_{yz} \right|_{z_0} \]  

(A.25)

Using Leibniz Rule,

\[ \frac{\partial}{\partial t} \int_{b_1(x,y)}^{b_2(x,y)} \rho v \, dz + \rho v \big|_{b_2} - \rho v \big|_{b_1} + \int_{b_1(x,y)}^{b_2(x,y)} \left( \frac{\partial}{\partial x} (\rho u v) \right) \, dz + \int_{b_1(x,y)}^{b_2(x,y)} \left( \frac{\partial}{\partial y} (\rho v^2) \right) \, dz + \int_{b_1(x,y)}^{b_2(x,y)} \left( \frac{\partial}{\partial z} (\rho v w) \right) \, dz = \]

\[ -\rho g R \left( \frac{\partial}{\partial y} \int_{b_1(x,y)}^{b_2(x,y)} c \, dz \right) + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial z_b}{\partial y} \frac{\partial z_b}{\partial y} - \left. \left( \frac{\partial}{\partial x} \tau_{yx} \right) \right|_{z_0} - \left. \tau_{yz} \right|_{z_0} \]  

(A.26)

Next, the boundary condition at the bottom is introduced.

\[ \frac{\partial z_b}{\partial t} + u \big|_{b_1} \frac{\partial z_b}{\partial x} + v \big|_{b_2} \frac{\partial z_b}{\partial y} - w \big|_{b_2} = 0 \]

(A.27)

or:

\[ \frac{\partial z_b}{\partial t} + \rho v \big|_{b_1} \frac{\partial z_b}{\partial x} + \left( \rho uv \right) \big|_{b_2} \frac{\partial z_b}{\partial y} + \left( \rho v^2 \right) \big|_{b_2} \frac{\partial z_b}{\partial z} - \left( \rho vw \right) \big|_{b_2} = 0 \]

(A.28)
Use of this condition yields:

\[
\frac{\partial}{\partial t} \int_{z_1(x,y)}^{\infty} \rho v \, dz + \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \langle \rho uv \rangle \, dz + \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \langle \rho v^2 \rangle \, dz =
\]

\[-\rho_o g R \left( \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \bar{c} dz \, dz' + \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \bar{c} dz \right) - \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz - \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \bigg|_{z_1}^{z_2} \]  
(A.29)

Integration in \( y \) over the width of the turbidity current completes the area integration:

\[
\int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial t} \int_{z_1(x,y)}^{\infty} \rho v \, dz \, dy + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \langle \rho uv \rangle \, dz \, dy + \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \langle \rho v^2 \rangle \, dz \, dy =
\]

\[-\rho_o g R \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \bar{c} dz \, dy - \rho_o g R \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \bar{c} dz \, dy
\]

\[- \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \, dy - \int_{b_1(x,y)}^{b_2(x,y)} \tau_{yx} \, dy - \int_{b_1(x,y)}^{b_2(x,y)} \tau_{yx} \, dy \bigg|_{z_1}^{z_2} \]  
(A.30)

Using Leibniz Rule:

\[
\frac{\partial}{\partial t} \int_{b_1(x,y)}^{b_2(x,y)} \int_{z_1(x,y)}^{\infty} \rho v \, dz \, dy + \frac{\partial}{\partial x} \int_{b_1(x,y)}^{b_2(x,y)} \int_{z_1(x,y)}^{\infty} \langle \rho uv \rangle \, dz \, dy - \frac{\partial}{\partial x} \left( \int_{z_1(x,y)}^{\infty} \langle \rho v^2 \rangle \, dz \right) \bigg|_{b_1(x,y)}^{b_2(x,y)}
\]

\[+ \frac{\partial}{\partial x} \left( \int_{z_1(x,y)}^{\infty} \langle \rho uv \rangle \, dz \right) \bigg|_{b_1(x,y)}^{b_2(x,y)} + \frac{\partial}{\partial y} \int_{b_1(x,y)}^{b_2(x,y)} \int_{z_1(x,y)}^{\infty} \langle \rho v^2 \rangle \, dz \, dy =
\]

\[-\rho_o g R \frac{\partial}{\partial y} \int_{b_1(x,y)}^{b_2(x,y)} \int_{z_1(x,y)}^{\infty} \bar{c} dz \, dy - \rho_o g R \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial y} \int_{z_1(x,y)}^{\infty} \bar{c} dz \, dy
\]

\[- \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \, dy + \frac{\partial}{\partial x} \left( \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \right) \bigg|_{b_1(x,y)}^{b_2(x,y)} \bigg|_{b_1(x,y)}^{b_2(x,y)} - \frac{\partial}{\partial x} \left( \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \right) \bigg|_{b_1(x,y)}^{b_2(x,y)}
\]

\[- \int_{b_1(x,y)}^{b_2(x,y)} \frac{\partial}{\partial x} \int_{z_1(x,y)}^{\infty} \tau_{yx} \, dz \, dy - \int_{b_1(x,y)}^{b_2(x,y)} \tau_{yx} \, dy \bigg|_{z_1}^{z_2} \]  
(A.31)

221
A.3 Conservation of Sediment

\[
\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial wc}{\partial z} = -\frac{\partial}{\partial z} \left( F - \overline{w}c \right) \tag{A.32}
\]

Integration in \( z \) over the depth of the turbidity current yields:

\[
\int_{z_1(x,y)}^{z_2(x,y)} \frac{\partial c}{\partial t} dz + \int_{z_1(x,y)}^{z_2(x,y)} \frac{\partial uc}{\partial x} dz + \int_{z_1(x,y)}^{z_2(x,y)} \frac{\partial vc}{\partial y} dz + \int_{z_1(x,y)}^{z_2(x,y)} \frac{\partial wc}{\partial z} dz =
\]

\[
- \int_{z_1(x,y)}^{z_2(x,y)} \frac{\partial}{\partial z} \left( F - \overline{w}c \right) dz \tag{A.33}
\]

Using Leibniz Rule and \( c|_{z_2} = 0 \) yields:

\[
\frac{\partial}{\partial t} \int_{z_1(x,y)}^{z_2(x,y)} cdz + c|_{z_1(x,y)} \frac{\partial c}{\partial t} + \frac{\partial}{\partial x} \int_{z_1(x,y)}^{z_2(x,y)} uc \ dz + u|_{z_2(x,y)} c|_{z_2(x,y)} \frac{\partial c}{\partial x} 
\]

\[
+ \frac{\partial}{\partial y} \int_{z_1(x,y)}^{z_2(x,y)} vc \ dz + v|_{z_2(x,y)} c|_{z_2(x,y)} \frac{\partial c}{\partial y} - w|_{z_2(x,y)} c|_{z_2(x,y)} = \left( F|_{z_2} - \overline{w}c|_{z_2(x,y)} \right) \tag{A.34}
\]

Application of the bottom boundary condition:

\[
c|_{z_2(x,y)} \left( \frac{\partial c}{\partial t} + u|_{z_2(x,y)} \frac{\partial c}{\partial x} + v|_{z_2(x,y)} \frac{\partial c}{\partial y} - w|_{z_2(x,y)} c = 0 \right) \tag{A.35}
\]

yields:

\[
\frac{\partial}{\partial t} \int_{z_1(x,y)}^{z_2(x,y)} cdz + \frac{\partial}{\partial x} \int_{z_1(x,y)}^{z_2(x,y)} uc \ dz + \frac{\partial}{\partial y} \int_{z_1(x,y)}^{z_2(x,y)} vc \ dz = \left( F|_{z_2} - \overline{w}c|_{z_2(x,y)} \right) \tag{A.36}
\]

Integration in \( y \) over the width of the turbidity current completes the area integration:

\[
\int_{y_1(x,z)}^{y_2(x,z)} \frac{\partial}{\partial t} \int_{z_1(x,y)}^{z_2(x,y)} \cdz dy + \int_{y_1(x,z)}^{y_2(x,z)} \frac{\partial}{\partial x} \int_{z_1(x,y)}^{z_2(x,y)} uc \ dz dy + \int_{y_1(x,z)}^{y_2(x,z)} \frac{\partial}{\partial y} \int_{z_1(x,y)}^{z_2(x,y)} vc \ dz dy = \int_{y_1(x,z)}^{y_2(x,z)} \left( F|_{y_2} - \overline{w}c|_{y_2} \right) dy \tag{A.37}
\]
Using Leibniz Rule yields:

\[
\frac{\partial}{\partial t} \int_{b_1(x,t)}^{b_2(x,t)} c \, dz \, dy + \int_{b_1(x,t)}^{b_2(x,t)} \left[ \frac{\partial}{\partial x} \right] _{z_4(x,y)}^{z_6(x,y)} \left[ \int_{b_3(x,t)}^{b_5(x,t)} uc \, dz \right] \, dy - \frac{\partial b_2}{\partial x} \left| _{z_4(x,y)}^{z_6(x,y)} \right. \left[ \int_{b_3(x,t)}^{b_5(x,t)} uc \, dz \right] \, dy + \left. \frac{\partial b_1}{\partial x} \right| _{z_4(x,y)}^{z_6(x,y)} \left[ \int_{b_3(x,t)}^{b_5(x,t)} uc \, dz \right] \, dy
\]

\[
= \int_{b_1(x,t)}^{b_2(x,t)} \left( F \right| _{z_4}^{z_6} \right) \, dy
\]
Appendix B: Transformation of Field Equations to Matrix Form

The following appendix details the steps taken to transform the three partial differential equations for mass, momentum, and sediment balance (Table 2.1) to matrix form. The matrix representation was chosen for ease of coding the numerical model.

The matrix equation takes the form \( A \frac{dX}{dx} = B \), where \( A \) is a 3x3 coefficient matrix, \( X \) is the matrix of unknowns, namely \( Q \), \( \eta \), and \( S \), and \( B \) is the solution matrix. The goal here is to transform the three equations in Table 2.1 into three differential equations containing only the following differentials: \( \frac{dQ}{dx} \), \( \frac{d\eta}{dx} \), and \( \frac{dS}{dx} \). These three equations will then be put into matrix form.

### B.1 Conservation of Mixture Mass

The conversion of the conservation of mixture mass equation (2.65) is as follows:

\[
\frac{d}{dx} (AU) + R \frac{d}{dx} (ASU) = w_e B + (R + 1)(F - w_e c) B \tag{B.1}
\]

\[
\frac{d}{dx} (Q) + R \frac{d}{dx} (QS) = w_e B + (R + 1)(F - w_e c) B \tag{B.2}
\]

\[
\frac{d}{dx} (Q) + RS \frac{dQ}{dx} + RQ \frac{dS}{dx} = w_e B + (R + 1)(F - w_e c) B \tag{B.3}
\]

\[
(1 + RS) \frac{dQ}{dx} + RQ \frac{dS}{dx} = w_e B + (R + 1)(F - w_e c) B \tag{B.4}
\]
B.2 Conservation of Mixture Momentum

The conversion of the conservation of momentum equation (2.67) is as follows:

\[
\frac{d}{dx}(AU^2) + R \frac{d}{dx}(ASU^2) = -\frac{1}{2} gR \frac{d}{dx}(AS\theta) - gRSA \frac{dz_b}{dx} - \frac{\tau_b}{\rho_o} B \tag{B.5}
\]

Recast with \( Q \), (B.5) becomes:

\[
\frac{d}{dx}(QU) + R \frac{d}{dx}(QSU) = -\frac{1}{2} gR \frac{d}{dx}(AS\theta) - gRSA \frac{dz_b}{dx} - \frac{\tau_b}{\rho_o} B \tag{B.6}
\]

First, the left hand side of the momentum equation will be expanded:

\[
\frac{d}{dx}(QU) + R \frac{d}{dx}(QSU) = \tag{B.7}
\]

\[
Q \frac{dU}{dx} + U \frac{dQ}{dx} + QSU \frac{dU}{dx} + RU \frac{d(QS)}{dx} = \tag{B.8}
\]

\[
Q \frac{dU}{dx} + U \frac{dQ}{dx} + QSU \frac{dU}{dx} + RU \left( \frac{dS}{dx} + S \frac{dQ}{dx} \right) = \tag{B.9}
\]

\[
Q \frac{dU}{dx} + QSU \frac{dU}{dx} + U \frac{dQ}{dx} + RUS \frac{dQ}{dx} + RUQ \frac{dS}{dx} = \tag{B.10}
\]

\[
(Q + QSU) \frac{dU}{dx} + (U + RUS) \frac{dQ}{dx} + RUQ \frac{dS}{dx} = \tag{B.11}
\]

Now, using

\[
\frac{dU}{dx} = \frac{d}{dx} \left( \frac{Q}{A} \right) = \frac{A \frac{dQ}{dx} - Q \frac{dA}{dx}}{A^2} = \left( \frac{1}{A} \frac{dQ}{dx} - \frac{Q}{A^2} \frac{dA}{dx} \right) \tag{B.12}
\]

and \( U = \frac{Q}{A} \) \tag{B.13}
the left hand side of the momentum equation (B.11) becomes:

\[
(1 + R S)\frac{Q}{A} \left( \frac{dQ}{dx} - \frac{Q}{A^2} \frac{dA}{dx} \right) + (1 + R S) \frac{Q}{A} \frac{dQ}{dx} + R \frac{Q}{A} \frac{dS}{dx} =
\]

(B.14)

\[
2(1 + R S) \frac{Q}{A} \frac{dQ}{dx} - (1 + R S) \frac{Q^2}{A^2} \frac{dA}{dx} + R \frac{Q^2}{A} \frac{dS}{dx} = \]

(B.15)

Assuming constant width \( B \), Equations B.15 is further transformed with

\[
\frac{dA}{dx} = B \frac{d(\eta - z_b)}{dx}
\]

(B.16)

to:

\[
2(1 + R S) \frac{Q}{A} \frac{dQ}{dx} - (1 + R S) \frac{Q^2}{A^2} B \frac{d(\eta - z_b)}{dx} + R \frac{Q^2}{A} \frac{dS}{dx} = \]

(B.17)

The right hand side of Equation (B.6) is transformed with the following algebra:

\[
\frac{d}{dx} \left[ AS(\eta - z_b) \right] = AS \frac{d(\eta - z_b)}{dx} + (\eta - z_b) \left( S \frac{dA}{dx} + A \frac{dS}{dx} \right)
\]

(B.18)

\[
\frac{d}{dx} \left[ AS(\eta - z_b) \right] = AS \frac{d(\eta - z_b)}{dx} + (\eta - z_b) \left( SB \frac{d(\eta - z_b)}{dx} + A \frac{dS}{dx} \right)
\]

(B.19)

\[
\frac{d}{dx} \left[ AS(\eta - z_b) \right] = \left( 2 AS \frac{d(\eta - z_b)}{dx} + (\eta - z_b) A \frac{dS}{dx} \right)
\]

(B.20)

to the following form:

\[
-\frac{1}{2} gR \left( 2 AS \frac{d(\eta - z_b)}{dx} + (\eta - z_b) A \frac{dS}{dx} \right) - gRSA \frac{dz_b}{dx} - \frac{\tau_{z_b} B}{\rho_o}
\]

(B.21)

and finally to:

\[
-RgAS \frac{d\eta}{dx} - \frac{1}{2} (\eta - z_b) RgA \frac{dS}{dx} - \frac{\tau_{z_b} B}{\rho_o}
\]

(B.22)

Now, the momentum equation has been transformed to:
\[ 2(1 + RS) \frac{Q}{A} \frac{dQ}{dx} - (1 + RS) \frac{Q^2}{A^2} B \frac{d(\eta - z_s)}{dx} + R \frac{Q^2}{A} \frac{dS}{dx} = \]

\[-RgAS \frac{d\eta}{dx} - \frac{1}{2} (\eta - z_s)RgA \frac{dS}{dx} - \frac{\tau_{z_s}}{\rho_o} B \]  

(B.23)

which can finally be cast as:

\[ 2(1 + RS) \frac{Q}{A} \frac{dQ}{dx} + \left( RgA - (1 + RS) \frac{Q^2}{A^2} B \right) \frac{d\eta}{dx} = \]

\[ + \left( R \frac{Q^2}{A} + \frac{1}{2} (\eta - z_s)RgA \right) \frac{dS}{dx} = -(1 + RS) \frac{Q^2}{A^2} B \frac{dz_s}{dx} - \frac{\tau_{z_s}}{\rho_o} B \]  

(B.24)

### B.3 Conservation of Sediment

The conversion of the conservation of sediment equation (2.68) is as follows:

\[ \frac{d}{dx} (ASU) = \left( F \big|_{z_s} - \overline{w_s} c \right)_s B \]  

(B.25)

\[ \frac{d}{dx} (QS) = \left( F \big|_{z_s} - \overline{w_s} c \right)_s B \]  

(B.26)

\[ Q \frac{dS}{dx} + S \frac{dQ}{dx} = \left( F \big|_{z_s} - \overline{w_s} c \right)_s B \]  

(B.27)

### B.4 Matrix Equation

The three partial differential equation from Table 2.1 have now been transformed

three partial differential equations in terms of \( Q, \eta, \) and \( S \) (B.4, B.24, and B.27).

These equations, summarized in Table B.1, are recast in matrix form in Table B.2.
Table B.1 Summary of Transformed Partial Differential Equations

Conservation of Mass

\[(1 + RS) \frac{dQ}{dx} + RQ \frac{dS}{dx} = w_x B + (R + 1)(F - \overline{w_c} c) B\]

Conservation of Momentum

\[2(1 + RS) \frac{Q}{A} \frac{dQ}{dx} + \left( R S g A - (1 + RS) \frac{Q^2}{A^2} B \right) \frac{d\eta}{dx} \]

\[+ \left( R \frac{Q^2}{A} + \frac{1}{2} (\eta - z_b) R g A \right) \frac{dS}{dx} = -(1 + RS) \frac{Q^2}{A^2} B \frac{dz_b}{dx} - \frac{\tau_b}{\rho_o} B \]

Conservation of Sediment

\[Q \frac{dS}{dx} + S \frac{dQ}{dx} = \left( f_{z_b} - \overline{w_c} c_{z_b} \right) B \]
### Table B.2 Matrix Representation of Modified Three Equation Model

\[
\begin{pmatrix}
(1 + RS)
& 0 & RQ \\
2(1 + RS) \frac{Q}{A} & RSgA - (1 + RS) \frac{Q^2}{A^2} B & R \frac{Q^2}{A} + \frac{1}{2} (\eta - z) RgA \\
\frac{S}{A} & 0 & \frac{Q}{A}
\end{pmatrix}
\begin{pmatrix}
\frac{dQ}{dx} \\
\frac{d\eta}{dx} \\
\frac{dS}{dx}
\end{pmatrix}
= \\
\begin{pmatrix}
w_B + (R + 1) \left( F|_{\eta} - \overline{w_{\xi}|_{\xi}} \right) B \\
-(1 + RS) \frac{Q^2}{A^2} B \frac{dz_B}{dx} - \frac{\tau_B}{\rho_o} B \\
\left( F|_{\eta} - \overline{w_{\xi}|_{\xi}} \right) B
\end{pmatrix}
\]
Appendix C: Transformation of Field Equations to First Order ODE System

The Modified Three Equation Model (Table 2.1) with $B = 1$ takes the following form:

$$\frac{d}{dx} (U h) + R \frac{d}{dx} (S U h) = w_e + (R + 1) \left(F \big|_{x} - \overline{w_c} \big|_{x}\right) \tag{C.1}$$

$$\frac{d}{dx} \left(U^2 h\right) + R \frac{d}{dx} \left(S U^2 h\right) = \frac{1}{2} g R \frac{d}{dx} \left(Sh^2\right) - g R S h \frac{dz_h}{dx} - \frac{\tau_s}{\rho_o} \tag{C.2}$$

$$\frac{d}{dx} (S U h) = \left(F \big|_{x} - \overline{w_c} \big|_{x}\right) \tag{C.3}$$

These equations can be transformed into a set of three explicit ordinary differential equations for $\frac{dU}{dx}$, $\frac{dS}{dx}$, and $\frac{dh}{dx}$.

First, the sediment equation (Equation C.3) is transformed as follows:

$$\frac{d}{dx} (S U h) = w_s (E_s - r_o S) \tag{C.4}$$

where $r_o = \frac{\varepsilon_s}{S}$ and $F \big|_{x} = \overline{w_s} E_s$.

Subtracting $R$ times C.3 from C.1 yields:

$$\frac{d}{dx} (U h) = w_e + w_s (E_s - r_o S) \tag{C.5}$$

which is equivalent to:

$$\frac{d}{dx} (U h) - \frac{d}{dx} (S U h) = w_e \quad \text{or} \quad \frac{d}{dx} (U h) - \frac{d}{dx} (S U h) = e_u U \tag{C.6}$$

Solving this for $\frac{dh}{dx}$ progresses as follows:

$$U \frac{dh}{dx} + h \frac{dU}{dx} - \frac{d}{dx} (S U h) = e_u U \tag{C.7}$$
\[
U \frac{dh}{dx} = e_s U - h \frac{dU}{dx} + \frac{d}{dx} (SUh) \tag{C.8}
\]

\[
\frac{dh}{dx} = e_s - \frac{h}{U} \frac{dU}{dx} + \frac{w_t}{U} (E_s - r_o S) \tag{C.9}
\]

Next, the sediment equation (Equation C.3) is transformed:

\[
\frac{d}{dx} (SUh) = w_t (E_s - r_o S) \tag{C.10}
\]

\[
U \frac{d(Sh)}{dx} + Sh \frac{dU}{dx} = w_t (E_s - r_o S) \tag{C.11}
\]

\[
\frac{d(Sh)}{dx} = - \frac{Sh}{U} \frac{dU}{dx} + \frac{w_t}{U} (E_s - r_o S) \tag{C.12}
\]

Transformation of the momentum equation (Equation C.2) begins with the chain rule:

\[
U^2 \frac{dh}{dx} + h2U \frac{dU}{dx} + RSh2U \frac{dU}{dx} + RU^2 \frac{d(Sh)}{dx} = - \frac{1}{2} gRh \frac{d(Sh)}{dx} - \frac{1}{2} gRSh \frac{dh}{dx} - gRSh \frac{dz_b}{dx} \frac{\tau_z}{\rho_o}
\]

\[
\left( \frac{1}{2} gRSh + U^2 \right) \frac{dh}{dx} + 2Uh (1 + RS) \frac{dU}{dx} + \left( RU^2 + \frac{1}{2} gRh \right) \frac{d(Sh)}{dx} = -gRSh \frac{dz_b}{dx} \frac{\tau_z}{\rho_o}
\]

\[
\left( \frac{1}{2} Ri + 1 \right) \frac{dh}{dx} + \frac{h}{U} (1 + RS) \frac{dU}{dx} + \left( R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{d(Sh)}{dx} = -Ri \frac{dz_b}{dx} - f \tag{C.15}
\]

Next, the equation is divided through by \(U^2\), the Richardson number \(Ri = \frac{RgSh}{U^2}\) is reintroduced, and Equation 3.1 is used to simplify the shear term:

\[
\left( \frac{1}{2} Ri + 1 \right) \frac{dh}{dx} + 2 \frac{h}{U} (1 + RS) \frac{dU}{dx} + \left( R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{d(Sh)}{dx} = -Ri \frac{dz_b}{dx} - f
\]

Now, substituting in for \(\frac{dSh}{dx}\), and \(\frac{dh}{dx}\) yields:

231
\[
\left( \frac{1}{2} \text{Ri} + 1 \right) \left( -\frac{h}{U} \frac{dU}{dx} \right) + 2 \frac{h}{U} (1 + \text{RS}) \frac{dU}{dx} + \left( R + \frac{1}{2} \frac{Rgh}{U^2} \right) \left( -\frac{Sh}{U} \frac{dU}{dx} \right) = \\
- \text{Ri} \frac{dz_b}{dx} - f - \left( \frac{1}{2} \text{Ri} + 1 \right) e_w - \left( \frac{1}{2} \text{Ri} + 1 \right) \frac{w_z}{U} (E_s - r_o S) - \left( R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{w_z}{U} (E_s - r_o S)
\]

(C.16)

which can be simplified to:

\[
\left( -\frac{1}{2} \text{Ri} - 1 + 2(1 + \text{RS}) - \text{RS} - \frac{1}{2} \frac{Rgh}{U^2} \right) \left( \frac{h}{U} \frac{dU}{dx} \right) = \\
- \text{Ri} \frac{dz_b}{dx} - f - \left( \frac{1}{2} \text{Ri} + 1 \right) e_w - \left( \frac{1}{2} \text{Ri} + 1 + R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{w_z}{U} (E_s - r_o S)
\]

(C.17)

and subsequently to:

\[
(1 + \text{RS} - \text{Ri}) \frac{h}{U} \frac{dU}{dx} = - \text{Ri} \frac{dz_b}{dx} - f - \left( \frac{1}{2} \text{Ri} + 1 \right) e_w - \left( \frac{1}{2} \text{Ri} + 1 + R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{w_z}{U} (E_s - r_o S)
\]

(C.18)

The final term in the equation can be recast as:

\[
- \left( \frac{1}{2} \text{Ri} + 1 + R + \frac{1}{2} \frac{Rgh}{U^2} \right) \frac{w_z}{U} r_o S \left( \frac{\psi}{\psi_e} - 1 \right)
\]

(C.19)

where \( \psi_e \frac{E_s AU}{r_o} \) and \( \psi = ASU \) represent the suspended sediment transport at equilibrium and the general suspended sediment transport, respectively (after Parker et al., 1986).

With this, the Equation C.18 can be recast as:

\[
(1 + \text{RS} - \text{Ri}) \frac{h}{U} \frac{dU}{dx} = - \text{Ri} \frac{dz_b}{dx} - f - \left( \frac{1}{2} \text{Ri} + 1 \right) e_w - \left( \frac{1}{2} \text{Ri} + 1 + R \right) S + \frac{1}{2} \text{Ri} \frac{w_z}{U} r_o \left( \frac{\psi}{\psi_e} - 1 \right)
\]

(C.20)
and finally as:

$$\frac{dU}{dx} = -\frac{U}{h} \left[ \text{Ri} \frac{dz_h}{dx} + f + \left( \frac{1}{2} \text{Ri} + 1 \right) e_w + \left[ \left( \frac{1}{2} \text{Ri} + 1 + R \right) S + \frac{1}{2} \text{Ri} \right] \frac{w_s}{U} \frac{r_o}{\psi} \frac{(\psi_s - 1)}{\psi} \right] \frac{1}{(1 + RS - Ri)}$$

(C.21)

With this, Equation C.9 becomes:

$$\frac{dh}{dx} = e_w + \frac{Ri}{\text{Ri}} \frac{dV}{dx} + \left( \frac{1}{2} \frac{Ri + 1}{Ri} \right) e_w + \left[ \left( \frac{1}{2} \frac{Ri + 1 + R}{Ri} \right) S + \frac{1}{2} \frac{Ri}{\text{Ri}} \right] \frac{w_s}{U} \frac{r_o}{\psi} \frac{(\psi_s - 1)}{\psi} \frac{1}{(1 + RS - Ri)} + \frac{w_s}{U} \frac{r_o}{\psi} S \frac{(\psi_s - 1)}{\psi}$$

(C.22)

Equation C.12 is expanded and rearranged to yield:

$$\frac{dS}{dx} = \frac{1}{h} \left( -S \frac{dh}{dx} - Sh \frac{dU}{dx} + \frac{w_s}{U} (E_s - r_o S) \right)$$

(C.23)

This is simplified with a form of Equation C.9 to:

$$\frac{dS}{dx} = \frac{1}{h} \left( (1 - S) \frac{w_s}{U} (E_s - r_o S) - S e_w \right)$$

(C.24)
Table C.1  Summary of Simultaneous Ordinary Differential Equation System

\[ \frac{dh}{dx} = e_\psi + \frac{R_i \frac{dz_\psi}{dx} + f + \left( \frac{1}{2} R_i + 1 \right) e_\psi + \left[ \left( \frac{1}{2} R_i + 1 + R \right) S + \frac{1}{2} R_i \right] \frac{w_x r_o}{U} \left( \frac{\psi_x}{\psi} - 1 \right)}{(1 + RS - R_i)} + \frac{w_x r_o}{U} r_S \left( \frac{\psi_x}{\psi} - 1 \right) \]

\[ \frac{dU}{dx} = -\frac{U}{h} \left[ \frac{R_i \frac{dz_\psi}{dx} + f + \left( \frac{1}{2} R_i + 1 \right) e_\psi + \left[ \left( \frac{1}{2} R_i + 1 + R \right) S + \frac{1}{2} R_i \right] \frac{w_x r_o}{U} \left( \frac{\psi_x}{\psi} - 1 \right)}{(1 + RS - R_i)} \right] \]

\[ \frac{dS}{dx} = \frac{1}{h} \left( 1 - S \right) \frac{w_x}{U} \left( E_s - r_o S \right) - S e_\omega \]
Appendix D. Graphical Model Output
D.1 Series F600
<table>
<thead>
<tr>
<th>Run ID</th>
<th>Initial Conditions</th>
<th>Channel/mix. variables</th>
<th>Closure schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_0$ (m/s)</td>
<td>$h_0$ (m)</td>
<td>$S_o$</td>
</tr>
<tr>
<td>F598a</td>
<td>1</td>
<td>1</td>
<td>0.0005</td>
</tr>
<tr>
<td>F599a</td>
<td>1</td>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>F600a</td>
<td>1</td>
<td>1</td>
<td>0.003</td>
</tr>
<tr>
<td>F601a</td>
<td>1</td>
<td>1</td>
<td>0.005</td>
</tr>
<tr>
<td>F602a</td>
<td>1</td>
<td>1</td>
<td>0.008</td>
</tr>
<tr>
<td>F603a</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F604a</td>
<td>1</td>
<td>1</td>
<td>0.03</td>
</tr>
<tr>
<td>F605a</td>
<td>1</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>F606c</td>
<td>1</td>
<td>0.5</td>
<td>0.01</td>
</tr>
<tr>
<td>F606d</td>
<td>0.5</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F607d</td>
<td>0.75</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F608d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F609d</td>
<td>1.25</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F610d</td>
<td>1.5</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F611d</td>
<td>2</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F612d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F613d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F614d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F615d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F616d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F617d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F618d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F619d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F620d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F621d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F622d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F623d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F624d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F625d</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>F626d</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F626f</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
<tr>
<td>F627b</td>
<td>0.5</td>
<td>2</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Key:
- $E_s$: A&F = Akiyama and Fukushima (1985); G85 = Garcia (1985)
- $a_w$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $r_s$: P82 = Parker (1982); G85 = Garcia (1985)
D.2 Series F200
<table>
<thead>
<tr>
<th>Run ID</th>
<th>Initial Conditions</th>
<th>( U_s ) ( \text{m/s} )</th>
<th>( h_0 ) ( \text{m} )</th>
<th>( S_o )</th>
<th>( R_{l_o} )</th>
<th>Slope</th>
<th>( D_s ) ( \text{m} )</th>
<th>Viscosity</th>
<th>( W_s ) ( \text{m/s} )</th>
<th>( c_D )</th>
<th>( \epsilon_w )</th>
<th>( E_s )</th>
<th>( r_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>F208</td>
<td>1</td>
<td>1</td>
<td>0.0005</td>
<td>0.01</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F209</td>
<td>1</td>
<td>1</td>
<td>0.001</td>
<td>0.02</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F210</td>
<td>1</td>
<td>1</td>
<td>0.003</td>
<td>0.05</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F211</td>
<td>1</td>
<td>1</td>
<td>0.005</td>
<td>0.08</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F212</td>
<td>1</td>
<td>1</td>
<td>0.008</td>
<td>0.13</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F213</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F214</td>
<td>1</td>
<td>1</td>
<td>0.03</td>
<td>0.49</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F215</td>
<td>1</td>
<td>1</td>
<td>0.05</td>
<td>0.81</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.0144</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F220</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.010</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F224</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.014</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F225</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.015</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F226</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.013</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F227</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.018</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F228</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.020</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
<tr>
<td>F229</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.16</td>
<td>0.08</td>
<td>0.00015</td>
<td>1.50E-06</td>
<td>0.017</td>
<td>0.004</td>
<td>G85</td>
<td>G85</td>
<td>G85</td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- \( E_s \): A&F = Akaiyama and Fukushima (1985); G85 = Garcia (1985)
- \( \epsilon_w \): F85 = Fukushima et al. (1985); G85 = Garcia (1989)
- \( r_o \): P82 = Parker (1982); G85 = Garcia (1985)
Table D.3 Summary of Input Variables for Run Series D800

<table>
<thead>
<tr>
<th>Run ID</th>
<th>Initial Conditions</th>
<th>Channel/mix. variables</th>
<th>Closure schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_0$ (m/s)</td>
<td>$h_0$ (m)</td>
<td>$S_0$</td>
</tr>
<tr>
<td>D808a</td>
<td>1</td>
<td>1</td>
<td>0.005</td>
</tr>
<tr>
<td>D809a</td>
<td>1</td>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>D810a</td>
<td>1</td>
<td>1</td>
<td>0.003</td>
</tr>
<tr>
<td>D811a</td>
<td>1</td>
<td>1</td>
<td>0.005</td>
</tr>
<tr>
<td>D812a</td>
<td>1</td>
<td>1</td>
<td>0.008</td>
</tr>
<tr>
<td>D813a</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>D814a</td>
<td>1</td>
<td>1</td>
<td>0.03</td>
</tr>
<tr>
<td>D815a</td>
<td>1</td>
<td>1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Key:
- $E_s$: A&F = Akiyama and Fukushima (1985); G85 = Garcia (1985)
- $e_w$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $r_o$: P82 = Parker (1982); G85 = Garcia (1985)
D.4 Series P6100
<table>
<thead>
<tr>
<th>Run ID</th>
<th>Initial Conditions</th>
<th>Channel/mix. variables</th>
<th>Closure schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_0$ (m/s)</td>
<td>$h_0$ (m)</td>
<td>$S_0$</td>
</tr>
<tr>
<td>P6100</td>
<td>0.652</td>
<td>2.0</td>
<td>0.00290</td>
</tr>
<tr>
<td>P6101</td>
<td>0.850</td>
<td>2.0</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6102</td>
<td>0.780</td>
<td>2.0</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6103</td>
<td>0.470</td>
<td>2.0</td>
<td>0.00620</td>
</tr>
<tr>
<td>P6104</td>
<td>0.490</td>
<td>2.0</td>
<td>0.00700</td>
</tr>
<tr>
<td>P6105</td>
<td>0.652</td>
<td>2.0</td>
<td>0.00430</td>
</tr>
<tr>
<td>P6106</td>
<td>0.652</td>
<td>2.0</td>
<td>0.00363</td>
</tr>
<tr>
<td>P6107</td>
<td>0.652</td>
<td>2.0</td>
<td>0.00350</td>
</tr>
<tr>
<td>P6109</td>
<td>0.790</td>
<td>2.0</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6110</td>
<td>1.500</td>
<td>2.0</td>
<td>0.00000</td>
</tr>
<tr>
<td>P6111</td>
<td>0.900</td>
<td>2.0</td>
<td>0.00220</td>
</tr>
<tr>
<td>P6112</td>
<td>0.900</td>
<td>2.0</td>
<td>0.00440</td>
</tr>
<tr>
<td>P6113</td>
<td>0.900</td>
<td>2.0</td>
<td>0.00680</td>
</tr>
<tr>
<td>P6114</td>
<td>0.800</td>
<td>2.0</td>
<td>0.00500</td>
</tr>
<tr>
<td>P6115</td>
<td>0.600</td>
<td>2.0</td>
<td>0.00687</td>
</tr>
<tr>
<td>P6116</td>
<td>0.700</td>
<td>2.0</td>
<td>0.00570</td>
</tr>
<tr>
<td>P6117</td>
<td>0.475</td>
<td>2.0</td>
<td>0.00640</td>
</tr>
<tr>
<td>P6118</td>
<td>0.472</td>
<td>2.0</td>
<td>0.00840</td>
</tr>
</tbody>
</table>

Key:
- $E_s$: A&F = Akiyama and Fukushima (1985); G85 = Garcia (1985)
- $e_w$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $r_0$: P82 = Parker (1982); G85 = Garcia (1985)
Run number P6101

U (m/s)

Ps (m²/s)

Distance (m)

Distance (m)

Distance (m)

h (m)

Z

L°

Distance (m)

Distance (m)

Distance (m)

Rl

Distance (m)

Distance (m)

Distance (psi)

318
Run number P6106

- $U$ (m/s) vs. Distance (m)
- $P_{ai}$ (m$^2$/s) vs. Distance (m)
- $\rho$ vs. Distance (m)
- $h$ (m) vs. Distance (m)
- $Z$ vs. Distance (m)
- $U^*$ vs. Distance (m)
- $R_l$ vs. Distance (m)
- $\theta_m$ vs. Distance (m)
- $U$ vs. psi

323
D.5 Series P6200
<table>
<thead>
<tr>
<th>Run ID</th>
<th>Initial Conditions</th>
<th>Channel/mix. variables</th>
<th>Closure schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_o$ (m/s)</td>
<td>$h_o$ (m)</td>
<td>$S_o$ (m)</td>
</tr>
<tr>
<td>P6200</td>
<td>0.468</td>
<td>2.0</td>
<td>0.002200</td>
</tr>
<tr>
<td>P6201</td>
<td>0.560</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6202</td>
<td>0.540</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6203</td>
<td>0.520</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6204</td>
<td>0.500</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6205</td>
<td>0.450</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6206</td>
<td>0.480</td>
<td>2.0</td>
<td>0.000000</td>
</tr>
<tr>
<td>P6207</td>
<td>0.440</td>
<td>2.0</td>
<td>0.004375</td>
</tr>
<tr>
<td>P6208</td>
<td>0.400</td>
<td>2.0</td>
<td>0.003750</td>
</tr>
<tr>
<td>P6209</td>
<td>0.400</td>
<td>2.0</td>
<td>0.002500</td>
</tr>
<tr>
<td>P6210</td>
<td>0.400</td>
<td>2.0</td>
<td>0.001250</td>
</tr>
<tr>
<td>P6211</td>
<td>0.300</td>
<td>2.0</td>
<td>0.001670</td>
</tr>
<tr>
<td>P6212</td>
<td>0.300</td>
<td>2.0</td>
<td>0.002500</td>
</tr>
<tr>
<td>P6213</td>
<td>0.500</td>
<td>2.0</td>
<td>0.000500</td>
</tr>
<tr>
<td>P6214</td>
<td>0.600</td>
<td>2.0</td>
<td>0.000833</td>
</tr>
<tr>
<td>P6215</td>
<td>0.700</td>
<td>2.0</td>
<td>0.001070</td>
</tr>
<tr>
<td>P6216</td>
<td>0.800</td>
<td>2.0</td>
<td>0.001250</td>
</tr>
<tr>
<td>P6217</td>
<td>0.350</td>
<td>2.0</td>
<td>0.002500</td>
</tr>
<tr>
<td>P6218</td>
<td>0.400</td>
<td>2.0</td>
<td>0.000825</td>
</tr>
<tr>
<td>P6219</td>
<td>0.300</td>
<td>2.0</td>
<td>0.002083</td>
</tr>
<tr>
<td>P6220</td>
<td>0.320</td>
<td>2.0</td>
<td>0.002340</td>
</tr>
<tr>
<td>P6221</td>
<td>0.300</td>
<td>2.0</td>
<td>0.002290</td>
</tr>
<tr>
<td>P6222</td>
<td>0.400</td>
<td>2.0</td>
<td>0.000938</td>
</tr>
<tr>
<td>P6223</td>
<td>0.450</td>
<td>2.0</td>
<td>0.000555</td>
</tr>
<tr>
<td>P6224</td>
<td>0.475</td>
<td>2.0</td>
<td>0.000526</td>
</tr>
<tr>
<td>P6225</td>
<td>0.425</td>
<td>2.0</td>
<td>0.000588</td>
</tr>
<tr>
<td>P6226</td>
<td>0.350</td>
<td>2.0</td>
<td>0.001429</td>
</tr>
<tr>
<td>P6227</td>
<td>0.350</td>
<td>2.0</td>
<td>0.002143</td>
</tr>
</tbody>
</table>

**Key:**
- $E_s$: F85 = Fukushima et al. (1985); G89 = Garcia (1989)
- $f_o$: P82 = Parker (1982); G85 = Garcia (1985)
Appendix E. Garcia (1985) Laboratory Data and Empirical Relationship for Sediment Entrainment

E.1 Introduction

Chapters 5 and 6 have emphasized the importance of the sediment entrainment relationship in turbidity current modeling. One recent set of sediment entrainment data is Garcia (1985). This appendix focuses on that data set, as it is representative of several difficulties inherent in sediment transport experiments. The purpose of this discussion is to draw attention to certain potential problems in conducting experiments with turbidity currents, so that future investigations on turbidity currents are more useful to others attempting to use the data. The importance of reporting complete experimental data cannot be underestimated, for as will be shown, seemingly inconsequential omissions can make it impossible for others to reproduce the reported experimental results.

Garcia’s data is from a series of 24 laboratory flume experiments with variations in sediment size, channel slope, and initial conditions such as velocity, height, and sediment concentration. He discharged sediment laden water down a sloped channel whose bed was covered with sediment. The initial conditions used in these experiments are summarized in Table E.1; experiments were conducted with a relatively narrow range of initial conditions. Experimental setup, operating procedures, and complete results can be found in Garcia (1985) and in Parker et al. (1987).
<table>
<thead>
<tr>
<th>run</th>
<th>$U_0$ (cm/s)</th>
<th>$h_0$ (cm)</th>
<th>$C_0$ $10^{-3}$ (cm$^2$/s)</th>
<th>$y_0$ (cm)</th>
<th>$R_{_0}$ (g/s)</th>
<th>$Q_{_w}$ (l/s)</th>
<th>$D_s$ (mm)</th>
<th>slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td>0.613</td>
<td>0.32</td>
<td>114.4</td>
<td>10.22</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td>0.613</td>
<td>0.32</td>
<td>114.4</td>
<td>10.22</td>
<td>0.06</td>
</tr>
<tr>
<td>3</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td>0.613</td>
<td>0.32</td>
<td>114.4</td>
<td>10.22</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>10</td>
<td>3.8</td>
<td>0.565</td>
<td>0.27</td>
<td>105.0</td>
<td>10.50</td>
<td>0.06</td>
</tr>
<tr>
<td>5</td>
<td>15.0</td>
<td>10</td>
<td>4.9</td>
<td>0.735</td>
<td>0.35</td>
<td>128.0</td>
<td>10.50</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>15.0</td>
<td>10</td>
<td>4.3</td>
<td>0.645</td>
<td>0.31</td>
<td>120.0</td>
<td>10.50</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>20.0</td>
<td>10</td>
<td>7.5</td>
<td>1.508</td>
<td>0.30</td>
<td>280.0</td>
<td>14.00</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>20.0</td>
<td>10</td>
<td>8.3</td>
<td>1.663</td>
<td>0.34</td>
<td>300.0</td>
<td>14.00</td>
<td>0.03</td>
</tr>
<tr>
<td>9</td>
<td>27.0</td>
<td>8</td>
<td>6.9</td>
<td>1.482</td>
<td>0.12</td>
<td>250.0</td>
<td>15.00</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>27.0</td>
<td>8</td>
<td>4.3</td>
<td>0.920</td>
<td>0.08</td>
<td>170.0</td>
<td>15.00</td>
<td>0.04</td>
</tr>
<tr>
<td>11</td>
<td>27.0</td>
<td>12</td>
<td>1.6</td>
<td>0.502</td>
<td>0.04</td>
<td>93.0</td>
<td>23.00</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>27.0</td>
<td>8</td>
<td>3.5</td>
<td>0.752</td>
<td>0.06</td>
<td>139.5</td>
<td>15.00</td>
<td>0.06</td>
</tr>
<tr>
<td>13</td>
<td>27.0</td>
<td>8</td>
<td>4.1</td>
<td>0.887</td>
<td>0.07</td>
<td>164.6</td>
<td>15.00</td>
<td>0.03</td>
</tr>
<tr>
<td>14</td>
<td>27.0</td>
<td>8</td>
<td>5.0</td>
<td>1.078</td>
<td>0.09</td>
<td>200.0</td>
<td>15.00</td>
<td>0.03</td>
</tr>
<tr>
<td>15</td>
<td>27.0</td>
<td>8</td>
<td>5.0</td>
<td>1.078</td>
<td>0.09</td>
<td>200.0</td>
<td>15.00</td>
<td>0.03</td>
</tr>
<tr>
<td>16</td>
<td>27.0</td>
<td>8</td>
<td>3.7</td>
<td>0.808</td>
<td>0.07</td>
<td>150.0</td>
<td>15.00</td>
<td>0.06</td>
</tr>
<tr>
<td>17</td>
<td>27.0</td>
<td>8</td>
<td>3.7</td>
<td>0.808</td>
<td>0.07</td>
<td>150.0</td>
<td>15.00</td>
<td>0.03</td>
</tr>
<tr>
<td>18</td>
<td>27.0</td>
<td>8</td>
<td>2.9</td>
<td>0.847</td>
<td>0.05</td>
<td>120.0</td>
<td>15.00</td>
<td>0.03</td>
</tr>
<tr>
<td>19</td>
<td>25.0</td>
<td>15</td>
<td>9.9</td>
<td>3.750</td>
<td>0.38</td>
<td>695.0</td>
<td>26.25</td>
<td>0.03</td>
</tr>
<tr>
<td>20</td>
<td>20.0</td>
<td>15</td>
<td>5.3</td>
<td>1.617</td>
<td>0.32</td>
<td>300.0</td>
<td>21.00</td>
<td>0.06</td>
</tr>
<tr>
<td>21</td>
<td>18.0</td>
<td>15</td>
<td>3.2</td>
<td>0.862</td>
<td>0.24</td>
<td>160.0</td>
<td>19.00</td>
<td>0.03</td>
</tr>
<tr>
<td>22</td>
<td>20.0</td>
<td>15</td>
<td>3.0</td>
<td>0.920</td>
<td>0.18</td>
<td>170.0</td>
<td>21.00</td>
<td>0.03</td>
</tr>
<tr>
<td>23</td>
<td>23.8</td>
<td>15</td>
<td>4.5</td>
<td>1.617</td>
<td>0.19</td>
<td>300.0</td>
<td>25.00</td>
<td>0.03</td>
</tr>
<tr>
<td>24</td>
<td>23.8</td>
<td>15</td>
<td>4.5</td>
<td>1.617</td>
<td>0.19</td>
<td>300.0</td>
<td>25.00</td>
<td>0.03</td>
</tr>
</tbody>
</table>
During the experiments, measurements of current velocity, current height, and sediment concentration were made at a series of four locations along the flume, including the origin. Measurements were taken throughout the water column at each sampling location. These values were used to calculate layer averaged velocity and sediment concentration; the results are summarized in Table E.2. Only 19 of the 24 runs contained enough data to compute these values.

Results of Garcia's calculations for such parameters as shear velocity, sediment entrainment, and water entrainment are summarized in Table E.3. These results are presented for each reach; reach 0-1 has \( x = 0 \) meters as the upstream boundary, and \( x = 1.5 \) meters as the downstream boundary.

E.2 Attempted Reproduction of Garcia's Results

Garcia's proposed relationships for such physical processes as sediment entrainment (Equation 3.21) and water entrainment (Equation 3.12) were verified with this data set. Considerable efforts were made to reproduce certain calculations reported in Garcia (1985) and subsequently in Parker et al. (1987). During the attempted reconstruction of Garcia's calculations, several factors were discovered making reproduction of his results less than satisfactory.
<table>
<thead>
<tr>
<th>Run</th>
<th>U (cm/s)</th>
<th>h (cm)</th>
<th>C*10^3</th>
<th>U (cm/s)</th>
<th>h (cm)</th>
<th>C*10^3</th>
<th>U (cm/s)</th>
<th>h (cm)</th>
<th>C*10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Section 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>14.6</td>
<td>10</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>10</td>
<td>3.8</td>
<td>10.80</td>
<td>22.24</td>
<td>2.75</td>
<td>13.18</td>
<td>21.45</td>
<td>1.30</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>10</td>
<td>4.9</td>
<td>13.85</td>
<td>19.53</td>
<td>3.30</td>
<td>13.30</td>
<td>22.23</td>
<td>1.88</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>10</td>
<td>4.3</td>
<td>11.30</td>
<td>23.75</td>
<td>3.73</td>
<td>13.30</td>
<td>22.23</td>
<td>1.88</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>10</td>
<td>7.5</td>
<td>14.63</td>
<td>23.44</td>
<td>4.52</td>
<td>17.78</td>
<td>20.93</td>
<td>4.60</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>10</td>
<td>8.3</td>
<td>14.70</td>
<td>23.05</td>
<td>5.91</td>
<td>17.24</td>
<td>23.91</td>
<td>2.62</td>
</tr>
<tr>
<td>9</td>
<td>27</td>
<td>8</td>
<td>6.9</td>
<td>14.25</td>
<td>24.35</td>
<td>3.73</td>
<td>13.48</td>
<td>26.70</td>
<td>3.55</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>8</td>
<td>4.3</td>
<td>14.72</td>
<td>24.94</td>
<td>2.58</td>
<td>14.65</td>
<td>26.39</td>
<td>1.98</td>
</tr>
<tr>
<td>11</td>
<td>27</td>
<td>12</td>
<td>1.6</td>
<td>17.70</td>
<td>27.64</td>
<td>1.55</td>
<td>14.18</td>
<td>28.80</td>
<td>1.62</td>
</tr>
<tr>
<td>12</td>
<td>27</td>
<td>8</td>
<td>3.5</td>
<td>12.39</td>
<td>23.52</td>
<td>2.20</td>
<td>14.00</td>
<td>23.03</td>
<td>1.29</td>
</tr>
<tr>
<td>13</td>
<td>27</td>
<td>8</td>
<td>4.1</td>
<td>12.49</td>
<td>23.09</td>
<td>2.37</td>
<td>12.66</td>
<td>28.53</td>
<td>1.86</td>
</tr>
<tr>
<td>14</td>
<td>27</td>
<td>8</td>
<td>5</td>
<td>10.84</td>
<td>25.80</td>
<td>3.33</td>
<td>12.19</td>
<td>26.79</td>
<td>2.40</td>
</tr>
<tr>
<td>15</td>
<td>27</td>
<td>8</td>
<td>5</td>
<td>11.60</td>
<td>16.46</td>
<td>3.05</td>
<td>11.47</td>
<td>23.70</td>
<td>2.12</td>
</tr>
<tr>
<td>16</td>
<td>27</td>
<td>8</td>
<td>3.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>27</td>
<td>8</td>
<td>3.7</td>
<td>9.75</td>
<td>23.94</td>
<td>2.00</td>
<td>13.21</td>
<td>23.38</td>
<td>1.69</td>
</tr>
<tr>
<td>18</td>
<td>27</td>
<td>8</td>
<td>2.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>25</td>
<td>15</td>
<td>9.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>15</td>
<td>5.3</td>
<td>17.60</td>
<td>13.91</td>
<td>8.00</td>
<td>16.95</td>
<td>20.56</td>
<td>3.80</td>
</tr>
<tr>
<td>21</td>
<td>18</td>
<td>15</td>
<td>3.2</td>
<td>11.20</td>
<td>26.36</td>
<td>1.85</td>
<td>12.81</td>
<td>24.62</td>
<td>1.40</td>
</tr>
<tr>
<td>22</td>
<td>20</td>
<td>15</td>
<td>3</td>
<td>9.56</td>
<td>26.90</td>
<td>2.10</td>
<td>14.22</td>
<td>20.68</td>
<td>2.55</td>
</tr>
<tr>
<td>23</td>
<td>23.8</td>
<td>15</td>
<td>4.5</td>
<td>15.34</td>
<td>23.08</td>
<td>2.98</td>
<td>19.70</td>
<td>26.28</td>
<td>2.80</td>
</tr>
<tr>
<td>24</td>
<td>23.8</td>
<td>15</td>
<td>4.5</td>
<td>16.25</td>
<td>26.00</td>
<td>3.45</td>
<td>18.20</td>
<td>26.07</td>
<td>2.80</td>
</tr>
</tbody>
</table>

Section 0 is at x = 0
Section 1 is at x = 1.5m
Section 2 is at x = 4.5m
Section 3 is at x = 8.5m
<table>
<thead>
<tr>
<th>run</th>
<th>reach</th>
<th>$u^*$ (cm/s)</th>
<th>$w_0$ (cm/s)</th>
<th>$uW_0$ (cm/s)</th>
<th>$r_s$ (mm)</th>
<th>$D_s$ (mm)</th>
<th>$R_p$</th>
<th>$Z$</th>
<th>$E_s$</th>
<th>$R_s$</th>
<th>$a_w$</th>
<th>$R_s$</th>
<th>$c_0$</th>
</tr>
</thead>
</table>
| 4   | 0-1   | 1.75        | 0.56         | 10.94       | 0.06    | 2.08    | 18.94 | 0.0061
| 5   | 0-1   | 2.23        | 0.44         | 5.07        | 0.06    | 2.40    | 9.77  | 0.0081
| 6   | 1-2   | 1.99        | 0.44         | 4.52        | 0.06    | 2.40    | 8.72  | 0.0012
| 7   | 2-3   | 2.41        | 0.44         | 3.20        | 0.06    | 2.40    | 6.16  | 0.0013
| 8   | 0-1   | 1.90        | 0.40         | 4.75        | 0.03    | 2.40    | 8.16  | 0.0020
| 9   | 1-2   | 1.32        | 0.40         | 3.00        | 0.03    | 2.40    | 8.50  | 0.0015
| 10  | 2-3   | 1.00        | 0.40         | 2.50        | 0.03    | 2.40    | 6.62  | 0.0010
| 11  | 0-1   | 2.40        | 0.10         | 24.00       | 0.03    | 0.75    | 19.34 | 0.0100
| 12  | 1-2   | 1.31        | 0.10         | 13.10       | 0.03    | 0.75    | 10.56 | 0.0157
| 13  | 2-3   | 2.83        | 0.10         | 28.30       | 0.03    | 0.75    | 22.81 | 0.0010
| 14  | 0-1   | 2.16        | 0.20         | 10.80       | 0.03    | 1.17    | 12.15 | 0.0345
| 15  | 1-2   | 1.08        | 0.20         | 5.45        | 0.03    | 1.17    | 6.13  | 0.0029
| 16  | 2-3   | 2.16        | 0.20         | 10.80       | 0.03    | 1.17    | 12.15 | 0.0017
| 17  | 0-1   | 1.22        | 0.18         | 6.78        | 0.04    | 1.17    | 7.62  | 0.0056
| 18  | 1-2   | 2.21        | 0.18         | 12.28       | 0.04    | 1.17    | 13.81 | 0.0017
| 19  | 2-3   | 1.78        | 0.18         | 8.89        | 0.04    | 1.17    | 11.12 | 0.0067
| 20  | 0-1   | 1.59        | 0.10         | 15.90       | 0.03    | 0.62    | 11.11 | 0.0198
| 21  | 1-2   | 3.63        | 0.10         | 36.30       | 0.03    | 0.62    | 25.36 | 0.30
| 22  | 0-1   | 3.54        | 0.34         | 10.41       | 0.06    | 1.81    | 18.25 | 0.0013
| 23  | 1-2   | 1.45        | 0.34         | 4.26        | 0.06    | 1.81    | 6.85  | 0.0010
| 24  | 2-3   | 0.36        | 0.34         | 1.26        | 0.06    | 1.81    | 1.97  | 0.0080
| 25  | 1-2   | 1.43        | 0.48         | 7.88        | 0.03    | 0.60    | 12.19 | 0.0040
| 26  | 2-3   | 2.08        | 0.08         | 26.00       | 0.03    | 0.60    | 17.73 | 0.0005
| 27  | 1-2   | 1.58        | 0.07         | 22.57       | 0.03    | 0.50    | 13.42 | 0.0094
| 28  | 2-3   | 1.68        | 0.07         | 24.00       | 0.03    | 0.50    | 14.27 | 0.0037
| 29  | 1-2   | 2.31        | 0.07         | 33.00       | 0.03    | 0.50    | 19.82 | 0.0038
| 30  | 2-3   | 2.50        | 0.07         | 35.71       | 0.03    | 0.50    | 21.24 | 0.0010
| 31  | 1-2   | 0.77        | 0.05         | 15.40       | 0.03    | 0.40    | 7.75  | 0.0110
| 32  | 2-3   | 2.23        | 0.05         | 44.60       | 0.03    | 0.40    | 22.43 | 0.0031
| 33  | 1-2   | 2.26        | 0.05         | 45.20       | 0.03    | 0.40    | 22.73 | 0.0012
| 34  | 2-3   | 2.09        | 0.05         | 41.80       | 0.03    | 0.40    | 21.02 | 0.0020
| 35  | 1-2   | 2.41        | 0.05         | 44.29       | 0.03    | 0.40    | 25.94 | 0.0040
| 36  | 2-3   | 2.01        | 0.07         | 28.71       | 0.03    | 0.40    | 16.82 | 0.0027
| 37  | 1-2   | 2.10        | 0.07         | 30.00       | 0.03    | 0.40    | 17.57 | 0.0010
| 38  | 2-3   | 1.54        | 0.07         | 22.00       | 0.03    | 0.40    | 12.86 | 0.0014
| 39  | 2-3   | 1.75        | 0.07         | 25.00       | 0.03    | 0.40    | 14.84 | 0.0012
| 40  | 2-3   | 0.98        | 0.07         | 14.00       | 0.03    | 0.40    | 8.20  | 0.0042
| 41  | 1-2   | 3.50        | 0.09         | 36.89       | 0.03    | 0.66    | 28.48 | 0.0090
| 42  | 2-3   | 1.41        | 0.09         | 15.67       | 0.03    | 0.66    | 11.47 | 0.0260

- Some values of $E_s$ and $a_w$ that turned out to be negative when evaluated, and therefore do not have any physical meaning, have not been included in the table.
- The second column indicates the portion of the channel where the parameters have been evaluated. The number 0 stands for the inlet ($x = 0$), and the numbers 1, 2, and 3 stand for the measuring Sections 1, 2, and 3, respectively.

(Captions from Parker et al. 1987.)
The most obvious shortcoming is that the data are often reported to only one significant figure. The sediment grain size, for example, is reported as between 0.03 and 0.06 mm (Table E.1). Problems result when these values are used to reproduce the reported values for the particulate Reynolds number ($R_p$). García's calculated values for $R_p$ cannot be duplicated.

Table E.3 shows that the runs made with the 0.03 mm diameter sediment are reported to have a particulate Reynolds number ranging from 0.04 to 2.40. Equation 3.15, however, states that for a given sediment specific gravity ($R = 1.65$) and a given fluid viscosity ($\nu$), both of which can be considered constant in García's experiments, the particulate Reynolds number is constant for a given sediment size. Figure E.1 plots García's reported values for $R_p$ against the reported values for grain size diameter. The five lines included in the figure are Equation 3.15 calculated with viscosities corresponding to water between 5 and 25 °C. Viscosity was not reported in García (1985), but subsequent reporting of these same experiments (Parker et al. 1987) use a value of $1.5 \times 10^{-6} \text{ m}^2/\text{s}$, corresponding to a water temperature of 5 °C.

Of the 19 reported values for $R_p$, only 10 are within 50% of the value obtained from Equation 3.15 and the reported values for grain size diameter, $R$, and viscosity. Table E.4 summarizes the differences between the reported values for $R_p$ and those calculated from Equation 3.15. The percent difference is with respect to the calculated value; for Run 4, the reported value is 68% higher than the value calculated with Equation 3.15. This and Figure E.1 indicate that the particulate Reynolds number reported for several runs are
Figure E.1
Garcia's 1985 Turbidity Current Data: \( R_p \) vs \( D_s \)

\[ R_p \text{ data points were taken from Table E.3. Lines were calculated from Equation 3.15. The various curves represent different viscosities corresponding to temperatures ranging from 5 \(^\circ\)C to 25 \(^\circ\)C. Data presented by Garcia that are outside a reasonable range are individually labeled by run number.} \]
Table E.4. Reported vs. Calculated Values for Particulate Reynolds Number $R_p$

<table>
<thead>
<tr>
<th>Run Number</th>
<th>$D_x$ (mm)</th>
<th>$R_p$ Reported</th>
<th>$R_p$ Calculated</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.06</td>
<td>2.08</td>
<td>1.24</td>
<td>68%</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
<td>2.40</td>
<td>1.24</td>
<td>94%</td>
</tr>
<tr>
<td>6</td>
<td>0.03</td>
<td>2.40</td>
<td>0.44</td>
<td>448%</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
<td>0.75</td>
<td>0.44</td>
<td>71%</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td>1.17</td>
<td>0.44</td>
<td>167%</td>
</tr>
<tr>
<td>9</td>
<td>0.04</td>
<td>1.17</td>
<td>0.67</td>
<td>74%</td>
</tr>
<tr>
<td>10</td>
<td>0.04</td>
<td>1.17</td>
<td>0.67</td>
<td>74%</td>
</tr>
<tr>
<td>11</td>
<td>0.03</td>
<td>0.62</td>
<td>0.44</td>
<td>42%</td>
</tr>
<tr>
<td>12</td>
<td>0.06</td>
<td>1.81</td>
<td>1.24</td>
<td>46%</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>0.60</td>
<td>0.44</td>
<td>37%</td>
</tr>
<tr>
<td>14</td>
<td>0.03</td>
<td>0.50</td>
<td>0.44</td>
<td>14%</td>
</tr>
<tr>
<td>15</td>
<td>0.03</td>
<td>0.50</td>
<td>0.44</td>
<td>14%</td>
</tr>
<tr>
<td>17</td>
<td>0.03</td>
<td>0.40</td>
<td>0.44</td>
<td>-9%</td>
</tr>
<tr>
<td>18</td>
<td>0.03</td>
<td>0.40</td>
<td>0.44</td>
<td>-9%</td>
</tr>
<tr>
<td>20</td>
<td>0.06</td>
<td>0.49</td>
<td>1.24</td>
<td>-60%</td>
</tr>
<tr>
<td>21</td>
<td>0.03</td>
<td>0.49</td>
<td>0.44</td>
<td>12%</td>
</tr>
<tr>
<td>22</td>
<td>0.03</td>
<td>0.49</td>
<td>0.44</td>
<td>12%</td>
</tr>
<tr>
<td>23</td>
<td>0.03</td>
<td>0.49</td>
<td>0.44</td>
<td>12%</td>
</tr>
<tr>
<td>24</td>
<td>0.03</td>
<td>0.66</td>
<td>0.44</td>
<td>51%</td>
</tr>
</tbody>
</table>

highly questionable. The reported data for runs 4, 5, 6, 7, 8, 9, 10, and 20 are particularly uncertain.

Additional evidence of problems in Garcia's reported data is presented in Figure E.2, where Garcia's reported values for $w_i$ are plotted against the reported values for grain size diameter. The five lines included in the figure are Dietrich's equation for settling velocity (Equation 4.3) calculated with viscosities corresponding to water between 5 and 25 °C.

The results are similar to those presented in Figure E.1. Again, only 9 runs have a reported settling velocity within 50% of that calculated with Equation 4.3, using the reported values for grain size diameter, $R$, and viscosity.

---

372
Figure E.2
Garcia's 1985 Turbidity Current Data: $D_s$ vs $w_s$

$w_s$ data points were taken from Table E.3. Lines were calculated from Equation 4.3 (Dietrich, 1982). The various curves represent different viscosities corresponding to temperatures ranging from 5 °C to 25 °C. Data presented by Garcia that are outside a reasonable range are individually labeled by run number.


<table>
<thead>
<tr>
<th>Run Number</th>
<th>$D_s$ (mm)</th>
<th>$w_z$ Reported</th>
<th>$w_z$ Calculated</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.06</td>
<td>0.16</td>
<td>0.211</td>
<td>-24%</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
<td>0.44</td>
<td>0.211</td>
<td>108%</td>
</tr>
<tr>
<td>6</td>
<td>0.03</td>
<td>0.40</td>
<td>0.054</td>
<td>646%</td>
</tr>
<tr>
<td>7</td>
<td>0.03</td>
<td>0.10</td>
<td>0.054</td>
<td>86%</td>
</tr>
<tr>
<td>8</td>
<td>0.03</td>
<td>0.20</td>
<td>0.054</td>
<td>273%</td>
</tr>
<tr>
<td>9</td>
<td>0.04</td>
<td>0.18</td>
<td>0.096</td>
<td>87%</td>
</tr>
<tr>
<td>10</td>
<td>0.04</td>
<td>0.18</td>
<td>0.096</td>
<td>87%</td>
</tr>
<tr>
<td>11</td>
<td>0.03</td>
<td>0.10</td>
<td>0.054</td>
<td>86%</td>
</tr>
<tr>
<td>12</td>
<td>0.06</td>
<td>0.34</td>
<td>0.211</td>
<td>61%</td>
</tr>
<tr>
<td>13</td>
<td>0.03</td>
<td>0.08</td>
<td>0.054</td>
<td>49%</td>
</tr>
<tr>
<td>14</td>
<td>0.03</td>
<td>0.07</td>
<td>0.054</td>
<td>31%</td>
</tr>
<tr>
<td>15</td>
<td>0.03</td>
<td>0.07</td>
<td>0.054</td>
<td>31%</td>
</tr>
<tr>
<td>17</td>
<td>0.03</td>
<td>0.05</td>
<td>0.054</td>
<td>-7%</td>
</tr>
<tr>
<td>18</td>
<td>0.03</td>
<td>0.05</td>
<td>0.054</td>
<td>-7%</td>
</tr>
<tr>
<td>20</td>
<td>0.06</td>
<td>0.07</td>
<td>0.211</td>
<td>67%</td>
</tr>
<tr>
<td>21</td>
<td>0.03</td>
<td>0.07</td>
<td>0.054</td>
<td>31%</td>
</tr>
<tr>
<td>22</td>
<td>0.03</td>
<td>0.07</td>
<td>0.054</td>
<td>31%</td>
</tr>
<tr>
<td>23</td>
<td>0.03</td>
<td>0.07</td>
<td>0.054</td>
<td>31%</td>
</tr>
<tr>
<td>24</td>
<td>0.03</td>
<td>0.09</td>
<td>0.054</td>
<td>68%</td>
</tr>
</tbody>
</table>

Detailed investigation into the reported values for $R_p$ and $w_z$ is necessary since these values are used to determine the sediment parameter $Z$, which subsequently is used to check the proposed formulation for sediment entrainment. The difficulties experienced in reproducing the reported values for $R_p$ and $w_z$ continue with the attempts to reproduce Garcia's calculations for the sediment parameter $Z$ and the sediment entrainment rate $E_z$.

The values for the sediment parameter $Z$ reported by Garcia (1985) and reproduced in Table E.3 are calculated with

$$Z = \frac{u_z}{w_z} R_p^{0.75}$$  \hspace{1cm} (E.1)
and the values in Table E.3 for all remaining variables. Since Garcia's reported values for
$R_p$ and $w_s$ appear to contain questionable data, the reported values for $Z$ are also
questionable. This is problematic since the values for the sediment entrainment rate $E_s$ are
plotted against these questionable values for $Z$.

The author attempted to reproduce the reported values for the shear velocity $u_*$, but was
unsuccessful as shape factors necessary in the calculations were not reported. Garcia
(1985) provides sample calculations for determining the shear velocity from the layer
averaged values for $U$, $C$, and $h$. The results of his sample calculations (run 12) do not
match the results reported in Table E.3 (Table 7.2 in Garcia, 1985). This in turn
constrained efforts to calculate values for $Z$. Aside from these difficulties, investigation
into the reported values for $E_s$ continued.

Figure E.3 compares Garcia's reported sediment entrainment rates (Table E.3) to those
calculated with the layer-averaged values presented Table E.2. The calculations followed
the method outlined in Garcia (1985):

$$E_s = \frac{1}{w_s} \frac{\partial \psi}{\partial x} + r_0 \overline{C},$$

(E.2)

where $\frac{\partial \psi}{\partial x} = \left( \frac{UCH_{i+1} - UCH_i}{x_{i+1} - x_i} \right)$ and $\overline{C} = \frac{C_{i+1} + C_i}{2}$

The indices $(i)$ and $(i+1)$ refer to locations where the measurements were taken. In order
to facilitate comparison with Garcia's proposed sediment entrainment formulation
(Equation 3.21), both sets of $E_s$ values are plotted against $Z$ calculated with

375
Garcia's data and the authors reproduction of Garcia's data. Both are plotted against the sediment parameter $Z$ calculated with Equation E.3 for comparison with Garcia's formulation for sediment entrainment. Agreement is better for larger values of $E_s$. 
\[ Z = \frac{u_s}{w_s} R_p^{0.6}. \]  

(E.3)

The agreement is fair, but is noticeably worse for smaller values of \( E_s \). Roughly half of the reproduced values are within 10% of the reported values.

**E.3 Influence of Measuring Equipment on Experimental Calculations**

During the attempted reproduction of Garcia's results for the sediment entrainment parameter, another issue arose that requires attention. The device used to measure velocity in Garcia's flume experiments has a stated accuracy of ±1 cm/sec (Garcia 1985). This accuracy is generally less than 10% of the measured values. While not a problem in itself, coupled with the relatively small values for sediment concentration and the similarity in sediment concentrations throughout the flume in individual experiments, this potential inaccuracy can yield values of \( \frac{\partial \psi}{\partial x} \) with opposite signs.

Figure E.4 presents error bars corresponding to values of \( E_s \) calculated with a range of velocities related to the stated accuracy of the velocity measurements. Specifically, the upper error bounds are calculated with \( (U + 0.5) \) cm/sec, and the lower error bounds are calculated with \( (U - 0.5) \) cm/sec. The purpose of this figure is to show the nature of the sediment entrainment calculations. Because of their relatively small numeric values, they are very sensitive to input parameters. Figure E.4 shows that slight inaccuracies in the velocity measurements can lead to changes in the sediment entrainment parameter that reverse the sign of the calculated term.
Figure E.4 Potential Error in $E_s$ Calculations Due to Velocity Measurements

$Z = (u/w_s) R_p^{**}$

Plot of potential error in sediment entrainment calculations due to accuracy of measuring device for current velocity. $E_s$ values are those calculated by the author. $Z$ values are those presented by Garcia (1985) and summarized in Table E.3.

Upper and lower error bars correspond to calculations made with $U+0.5$ cm/s and $U-0.5$ cm/s, respectively. Note semi-log scale.
Another way to visualize this effect is presented in Figure E.5. This figure presents the calculated values of $E_n$, complete with error bands described above, in relation to the derived sediment entrainment formulation proposed by Garcia. Since this is a log-log plot, the error bands containing negative numbers cannot be plotted. The plot shows that even allowing for the inaccuracy of the velocity measurements, there is still a rather poor fit between the calculated sediment entrainment rated and the formulation proposed by Garcia.

E.4 Conclusions

Sediment transport experiments are notoriously difficult to conduct. Reliable data sets are valuable, but only if they are complete. In order for reported experiments to be beneficial to other researchers, sufficient information should be included such that results can be reproduced. This chapter has demonstrated several errors in reporting which may deem reported results irreproducible. The concept of accuracy of measuring equipment and its effect on results was discussed.
Figure E.5 Log Representation of Potential Error in $E_s$ Calculations Due to Velocity Measurements

Author's calculations for $E_s$ using Equation E.2 and layer-averaged values form Table E.2. $Z$ values as reported by Garcia (1985) and in Table E.3. Negative lower bounds cannot be plotted on the log-log plot. Line is Garcia’s proposed sediment entrainment formulation (Equation 3.21).