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Truth Table Tasks: Directionality and Negation-Type

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Abstract
Two types of truth table task are used to examine people’s mental representation of conditionals: possibilities tasks and truth tasks. Despite their high degree of resemblance, the two task types not only differ regarding their number of answer alternatives, but also regarding their directionality: The truth task concerns the evaluation of the given rule on the basis of situations, while the possibilities task concerns the assessment of situations with respect to the given rule. The aim of the present study is to assess whether participants’ answer patterns depend on the difference in directionality when the difference in number of answer alternatives is controlled for, by presenting both the extended possibilities task and the truth task in both directions, i.e. from rule to situation and from situation to rule. Moreover, we make use of both implicit and explicit negations. Concerning the negation type, we find more three-valued patterns with implicit than with explicit negations. This is in line with the robust phenomenon of ‘matching bias’. It was replicated that possibilities tasks yield more two-valued answer patterns than truth tasks, which in turn yield more three-valued patterns than possibilities tasks. No effect of task directionality was observed.

Introduction
The interest in the linguistic, psychological and logical meaning of ‘if’ has provided us with a long history of research on thinking and reasoning about conditionals, designed in order to externalize people’s understanding and mental representation of conditionals.

The meaning of ‘if’
Traditionally, there are four different meanings ascribable to conditional ‘if Antecedent then Consequent’ sentences. According to standard logic, the connective ‘if’ is represented as the truth table for the material implication, meaning that only the TF falsifies the conditional. An alternative logical possibility for the meaning of ‘if’ is the truth table of the material equivalence: ‘C if and only if A’. This is the situation in which the antecedent implies the consequent and the consequent also implies the antecedent. Material implication and material equivalence are the two truth tables for conditionals under standard logic. Psychologically however, there is quite a lot of evidence that, next to ‘true’ or ‘false’, people make use of a third truth value representing conditionals: ‘irrelevant’. Wason (1966) was the first to introduce the ‘defective truth table’ (which we will call three-valued, following de Finetti, 1967, 2008; Politzer, Over & Baratgin, 2010), in which false antecedent cases (FT and FF) are considered to be irrelevant with respect to the conditional rather than making it true. The defective implication has a truth table of the form TFII and the defective equivalence of the form TFFI.

The truth table tasks: About possibility and truth
Conditional reasoning research has been conducted largely within three main experimental paradigms: the four card selection task, the conditional inference task and the truth table task, the latter being the focus of the present manuscript. Throughout psychological reasoning literature, the truth table tasks takes two forms, know as the possibilities task and the truth task. In the classical possibilities task, participants indicate for each of the four possible antecedent-consequent cases whether that specific combination is either possible or impossible with respect to the given rule. In the truth task, participants are asked to evaluate for each of the four cases whether the combination makes the given rule true, false or is irrelevant with respect to the truth of the rule.

Mental models theory vs. Suppositional theory
There has been substantial debate in reasoning literature concerning the processes and representations underlying people’s understanding of conditional assertions. The two main theories accounting for the mental representation of conditionals are the mental models theory (MMT) (Johnson-Laird & Byrne, 1991, 2002; Johnson-Laird, Byrne & Schaeken, 1992) and the suppositional theory (ST) (Evans, Over & Handley, 2003a; Evans & Over, 2004; Evans, Handley, Neilens & Over, 2007), making different predictions about the ‘core meaning’, the mental representation of conditionals. According to the MMT, people reason with representations resembling two-valued truth tables and according to the ST they reason with representations matching with three-valued truth tables. The starting point for much of the debate between the ST and the MMT has been the diverging results on the two kinds of truth table task. Classically it has been criticized that each theory makes use of that type of truth table task that satisfies their predictions the best: the possibilities task is used by
followers of the MMT and the truth task by followers of the ST.

According to the MMT (Johnson-Laird & Byrne, 1991, 2002; Johnson-Laird, Byrne & Schaeken, 1992), people tend to list all logical possibilities compatible with the conditional rule when they have to judge whether a situation is possible given a conditional rule, as is the case in the possibilities task. In line with the ‘truth principle’, reasoners construct mental models of the possibilities compatible with the premises, but they initially and by default do not represent what is false. Therefore, their conclusion is based on the initial model:

\[
[A] \, C \quad \ldots \quad 1
\]

Individuals do not represent what is false by default, but under certain circumstances they make ‘mental footnotes’ about the falsity of clauses (represented by the ellipsis). If they are able to retain these footnotes, people can flesh out the implicitly represented information into fully explicit models, which represent clauses even when they have false antecedents:

\[
\text{A C} \quad \neg \text{A} \quad \text{C} \quad \neg \text{A} \quad \neg \text{C}
\]

These possibilities correspond to the three rows of the truth table in which the material implication is true, including the false-antecedent cases FT and FF. This in turn leads to a two-valued truth table pattern. According to Johnson-Laird (2006), states of affairs in which the antecedent is false (FT or FF) are judged as irrelevant to the truth or falsity of the rule in the truth task, because these situations correspond to a model without explicit content (…), leading to the conclusion that nothing seems to follow from the premises and that the correct answer is ‘irrelevant’. So the MMT is able to explain three-valued answer patterns if it is assumed that people base their answer in this task-type solely on their initial model: '[A] C' in which the false-antecedent cases are not explicitly represented.

According to the ST (Evans & Over, 2004; Evans, Handley, Neilens & Over, 2007; Over & Handley, 2003a), people evaluate conditionals by means of the Ramsey test. That is, they “hypothetically add \( p \) to their stock of knowledge and evaluate their degree of belief in \( q \) given \( p \)” (Ramsey, 1931/1990 p.247). So they first estimate the probability of the consequent and the antecedent occurring together (the TT case), and then estimate the probability of the antecedent together with the non-occurrence of the consequent (the TF case). The combination of these two stages then leads to an estimation of the probability of occurrence of the consequent, given the antecedent. Running this Ramsey test, participants disregard the false-antecedent cases and only focus on those cases in which the antecedent is true, resulting in a three-valued truth table pattern. False-antecedent cases however are possible assuming any conditional because they have no influence on the believability of the conditional (P(CIA)). This implies that the ST has no problem accounting for the two-valued answer patterns yielded by the possibilities task in which false-antecedent cases are judged to be possible according to the conditional.

Number of answer alternatives There is quite a lot of evidence that the possibilities task and the truth task do not yield the same pattern of results (for a review, see Evans, Newstead & Byrne, 1993). However, as pointed out by Evans and Over (2004), with the dichotomy possible/impossible used in the classical possibilities task, one cannot point out the difference between true and irrelevant responses as the truth task does. Therefore, Sevenants, Schroyens, Dieussaert, Schaeken and d’Ydewalle (2008) developed an extended possibilities task in which ‘irrelevant’ is one of the three answer alternatives, similar to the truth task. In this way an explicit comparison between the results yielded by both truth table tasks is enabled, since the addition of ‘irrelevant’ as a third answer alternative to the possibilities task allows for a three-valued answer pattern in both tasks, and not only in the truth task.

Task directionality The possibilities task and the truth task differ not only regarding their number of answer alternatives, but also regarding their directionality: The truth task concerns the evaluation of a given conditional rule on the basis of situations (“Does this situation make the rule true/false or is it irrelevant with respect to the truth of the rule?”), while the possibilities task concerns the assessment of situations with respect to the given rule (“Is this situation possible/impossible/irrelevant according to the rule?”). Nevertheless, this is the directionality in which the tasks have been administered in the conditional reasoning research tradition for many years. Barrouillet, Gauffroy and Lecas (2008) claim that the distinction between reasoning about possibilities as in the possibilities task and reasoning about truth values as in the truth task delineate two different kinds of reasoning. In the former, people reason from assertions they consider as true and try to find out what must, can, or cannot occur in the world described by these assertions. In the latter task, people start from a given state of affairs they consider as existing, and they judge if a given assertion is true or false in this case. According to Johnson-Laird & Byrne (2002), the former is psychologically basic, whereas the latter is more complex and difficult since it requires a meta-ability that requires a notion of the relations between assertions and the world through the predicates true and false (Johnson-Laird & Byrne, 2002). So for logically untrained individuals, tasks involving reasoning about possibilities and tasks involving reasoning about truth values elicit different processes and should be distinguished. In daily life, there are numerous situations in which we are confronted which conditionals. For example, there are situations in which we learn new rules/ regulations/ instructions (if you press this button, you have to wait 10sec before the application loads on your phone) - and examine

1 Square brackets [ ] are the notation for an exhaustive representation. [A] C means that the antecedent is represented exhaustively.
2 The symbol ‘¬’ denotes an abstract mental symbol representing negation.
the validity of the rule on the basis of different examples experienced (slower than 10 sec, 10 sec, loaded, failed to load), vs. situations where the focus is on the particular instances and their implications for the rule (e.g., it took 30 sec to load).

Sevenants et al. (2008) investigated whether participants’ answer patterns depend on the difference in directionality when the difference in number of answer alternatives is controlled for, by presenting both the extended possibilities task and the truth task in both directions, i.e. from rule to situation and from situation to rule. So task directionality was manipulated in order to find out whether the frequently observed difference between the possibilities task and the truth task is due to the difference in task directionality and whether the difference in answer patterns dilutes when this difference in directionality is controlled for. It was observed that more three-valued answer patterns were yielded by the situation to rule than by the rule to situation tasks so task directionality indeed accounted for a part of the difference between both task-types. That was the first time however that task directionality was manipulated in a truth table task experiment. So in the present study we aim to replicate the result that tasks with a rule-to-situation directionality yield more two-valued answer patterns than do tasks with a situation-to-rule directionality and this in addition to the effect that possibilities tasks yield more two-valued answer patterns than do truth tasks.

**Negation-type** Task-type (possibilities vs. truth) and directionality (situation-to-rule vs. rule-to-situation) however are not the only factors that can account for the difference in results that have in the past been observed in truth table task experiments. Negation-type is also a variable that needs to be taken into consideration when interpreting the results of a truth table task. Considering a conditional rule, e.g. “If there is a B, then there is a 7”, there are two ways in which negations can be accomplished: explicitly or implicitly. In the present study we therefore add negation-type as a between subjects variable, making use of both implicit and explicit negations. With explicit negations, permuting the truth value of both the antecedent and the consequent leads to the following situations:

<table>
<thead>
<tr>
<th></th>
<th>TT:</th>
<th>TF:</th>
<th>FT:</th>
<th>FF:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a B and a 7</td>
<td>a B and not a 7</td>
<td>not a B and a 7</td>
<td>not a B and not a 7</td>
</tr>
</tbody>
</table>

With implicit negations, the same four situations look as follows:

<table>
<thead>
<tr>
<th></th>
<th>TT:</th>
<th>TF:</th>
<th>FT:</th>
<th>FF:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a B and a 7</td>
<td>a B and a 5</td>
<td>a T and a 7</td>
<td>a T and a 5</td>
</tr>
</tbody>
</table>

It is clear that with explicit negations, even in the FF case, there is still some link with what is described in the rule (not a B and not a 7). But with implicit negations, the rule is about a B and a 7, while the situation in the implicit task is about a T and a 5. In that case, people are much more inclined to judge a situation as irrelevant with respect to the conditional. This phenomenon is called matching-bias and is defined as a tendency for people to see implicitly negated cases as irrelevant to the truth of the rule (Evans, 1998). Evans states that the phenomenon is almost entirely dependent on the use of implicit negation in the logical cases to which rules are applied: Reasoners allocate selective attention towards matching cases and away from mismatching cases. It is concluded that matching bias is a highly robust effect which largely or entirely disappears when negations are made explicit. However, in most selection task and truth table experiments described to date, negation within the cases is implicit.

Since most of the truth table tasks in the literature make use of implicit negations, and since those implicit negations make the task vulnerable to matching bias, it is hard to disentangle what proportion of the observed three-valued answer patterns is due just to this matching bias and what proportion is caused by the underlying mental representation of conditionals and the extent to which false antecedent cases are suppositionally processed. In the present study we aim to shed light on this issue by making a direct comparison between both negation-types, keeping all other variables equal. The difference in three-valued answer patterns observed between the tasks using different negation-types is a measure of the amount of three-valued answer patterns in an implicit negations task that is due to matching bias.

Summarizing the aims of the present study, we firstly want to shed light on the difference between possibilities tasks an truth tasks, taking into account that both tasks differ not only regarding their wording (possibility vs. truth) but also regarding their directionality (situation-to-rule vs. rule-to-situation). Do both tasks still yield different results when this difference in directionality is controlled for? We aim to replicate Sevenants et al. (2008) in which it was observed that next to wording directionality had an impact on the resulting answer patterns. Secondly, with the present study we aim to explore on the difference between implicit and explicit negations, keeping all other variables (task-type and task-directionality). Is it the case, as is to be expected from Evans (1998) that tasks with implicit negations are more prone to matching bias than tasks with explicit negations, yielding therefore more irrelevance answers? Finally, we seek to explore the interaction between task-type, task-directionality and negation-type.

**Method**

**Participants and design**

In total 385 last-year high-school students (17-29 years of age, M = 17.6), all unfamiliar with logic, participated on voluntary basis. Participants were run in eight groups,
respectively completing the R-S possibilities task, the S-R possibilities task, the R-S truth task and the S-R truth task. Task-Type (possibilities vs. truth) and Negation-Type (implicit vs. explicit) were between subjects variables, implying that each group of participants received only one version of the tasks.

We used the three-option version of both the possibilities and the truth task, and both tasks were constructed in both directions. For the explicit negations, 45 participants received the three-option possibilities task in a rule-to-situation directionality (RS-PT), 46 participants completed the possibilities task from situation to rule (SR-PT), 47 participants completed the situation-to-rule truth task (SR-TT) and while finally 47 participants received the rule-to-situation truth task (RS-TT). For the implicit negations versions of the experiment, the number of participants was 50 for the RS-PT, 50 for the SR-PT, 49 for the SR-TT and finally 50 for the RS-TT. The students were randomly assigned to the different task-types.

**Materials and Procedure**

Tasks were constructed with ‘E-prime’ software (Psychological Software Tools, Pittsburgh, PA) and presented to the participants on individual PCs in a self-paced manner. Responses were given with the arrow-keys on an AZERTY keyboard. The experiment lasted between 3 and 7 minutes. All parts of the experiment were administered in Dutch.

All participants received the same instructions, appearing on the first screen and explaining that the purpose of the experiment was to examine how people reason with conditionals. The instructions also contained the description of a machine producing cards with a letter on the front side and a number on the back, always doing so following a certain rule, for example ‘If there is an T on the front, there is a 5 on the back of the card’. Participants could read that in the upcoming task they were going to see four cards presented to the participants on individual PCs in a self-paced manner. Responses were given with the arrow-keys on an AZERTY keyboard. The experiment lasted between 3 and 7 minutes. All parts of the experiment were administered in Dutch.

In all conditions, participants were successively presented with four items. Each item consisted of the abstract conditional rule followed by one of the four combinations of occurrence and non-occurrence of A and C. Both task directionality and negation type were manipulated between subjects. In the R-S possibilities task, participants had to evaluate for each situation whether that combination was possible, impossible or irrelevant according to the rule. In the S-R possibilities task, they had to evaluate for each of the four possible antecedent-consequent combinations whether the combination made the given rule either possible or impossible, or whether it was irrelevant with respect to the possibility of the rule. Concerning the S-R truth task, for each of the four combinations participants had to judge whether the combination made the given rule either true or false, or whether it was irrelevant with respect to the truth of the rule. For the R-S truth task, finally, participants had to evaluate whether each of the four possible situations was true, false or irrelevant according to the rule. In the explicit-negation versions of the experiment, negations of for example the rule ‘If B then 7’ were formulated as ‘a B’ and ‘not a 7’ (TF), ‘not a B’ and ‘a 7’ (FT) and ‘not a B and ‘not a 7’ (FF). In the implicit-negation conditions, the negations were: ‘a B’ and ‘a 5’ (TF), ‘a T’ and ‘a 7’ (FT) and ‘a T’ and ‘a 5’ (FF).

**Results and Discussion**

Given the focus of this study on the nature of the mental representation of conditionals, we first focus on participants’ individual answer patterns, that is the combination of judgments per individual on the four different situations presented in the truth table tasks. The ‘two-valued’ answer patterns include the conjunctive answer pattern, the material implication and the material equivalence responses (respectively TFFF, TFTF and TFTT), whereas ‘defective’ or three-valued answer patterns include the defective implication, the defective equivalence, the X-pattern and the Y-pattern (respectively TFII, TFFI, TFTI and TFTI; see Table 1 and Table 2).

| Table 1: Answer Patterns (% and (n)) for the Possibilities Tasks (PT) and the Truth Tasks (TT) with Explicit Negations. |
|---|---|---|---|---|---|
|   | RS-PT | SR-PT | RS-TT | SR-TT | Total |
| TFFF | 4 (2) | 9 (4) | 13 (6) | 2 (1) | 7 (13) |
| TFFT | 13 (6) | 28 (13) | 13 (6) | 21 (10) | 19 (35) |
| TFTT | 40 (18) | 17 (8) | 11 (5) | 4 (2) | 18 (33) |
| TFFI | 2 (1) | 9 (4) | 26 (12) | 30 (14) | 17 (31) |
| TFII | 13 (6) | 9 (4) | 19 (9) | 19 (9) | 15 (28) |
| TFFT | 13 (6) | 4 (2) | 4 (2) | 6 (3) | 7 (13) |
| TFTI | 7 (3) | 7 (3) | 2 (1) | 4 (2) | 5 (9) |
| Other | 7 (3) | 17 (8) | 13 (6) | 13 (6) | 12 (23) |
| Total | 100 (45) | 100 (46) | 100 (47) | 100 (47) | 100 (185) |
Focussing on the effect of Task-Type on the answer patterns, we replicated the effect of previous studies (Sevenants et al., 2008): Chi-square tests revealed that more two-valued answer patterns were yielded by the possibilities tasks (RS-PT and SR-PT) than the by the truth tasks (SR-TT and RS-TT). This goes both for the explicit negations ($\chi^2(1) = 29.31, p < .01$) and for the implicit negations ($\chi^2(1) = 31.79, p < .01$). Moreover, the truth tasks yielded a higher proportion of three-valued answer patterns than the possibilities tasks both with explicit ($\chi^2(1) = 19.58, p < .01$) and implicit negations ($\chi^2(1) = 41.8, p < .01$). This relates to the claim of Barrouillet et al. (2008) that reasoning about truth or falsity of assertions given possibilities is a different kind of reasoning than reasoning about possibilities given the truth of assertions. According to Barrouillet et al., the former requires a higher level of cognitive development and might therefore be more difficult. However, there is also a considerable amount of three-valued patterns in the possibilities task and of two-valued patterns in the truth task. This leads us to the interpretation that irrespective of the task, some people have a three-valued truth table representation (~ST) and others have a possibilities-based, two-valued MMT-representation. The mental representation of this kind of conditionals can thus be seen as an individual difference.

Regarding the effect of Negation-Type, we observed that more two-valued answer patterns were yielded by the truth table tasks with explicit negations than by the truth table tasks with implicit negations. This goes both for the possibilities tasks ($\chi^2(1) = 24.52, p < .01$) and for the truth tasks ($\chi^2(1) = 24.37, p < .01$). Moreover, a higher proportion of three-valued answer patterns was yielded by the truth table tasks making use of implicit negations compared to the tasks with explicit negations. This was the case for the truth tasks ($\chi^2(1) = 5.53, p < .05$) and the possibilities tasks ($\chi^2(1) = 11.32, p < .01$). These findings to be expected from the phenomenon of matching-bias: Especially for the false antecedent cases, with implicit negations there is no direct link between the conditional rule and the situations that have to be evaluated. Still, with the explicit negations there is a wealth of three-valued answer patterns, even in the possibilities tasks, bearing evidence to the suppositional processing of the false antecedent cases (Evans, 2004).

We did not observe an effect of directionality as was the case in Experiment 3 of Sevenants et al. (2008). A similar proportion of two-valued answer patterns was yielded by the rule-to-situation tasks (RS-PT and RS-TT) than by the tasks with a direction from situation to rule (SR-PT and SR-TT). Neither was there a difference regarding the proportion of three-valued answer patterns. This goes both for the explicit and the implicit negations. Reasoning from rule to situation might be a more natural and intuitive direction for participants than reasoning from situation to rule as was claimed by Sevenants et al. (2008), but confronting them with a task with a situation to rule directionality (while controlling for Task-Type), seemed to yield no difference in the observed answer patterns in the present study. In line with this finding however, Sevenants et al., (2008, Experiment 3) already observed that, when asked to write down a justification for their truth table task judgments, none of the participants mentioned something concerning the task directionality: Irrespective of the actual task-type or directionality, they formulated their justifications with a rule-to-situation direction (“E.g., “The situation is possible according to the rule”, even when task directionality was from situation to rule). Participants thus seem to overlook task directionality when classifying the truth table cases.

Finally, comparing the most similar tasks, i.e. RS-PT to RS-TT and SR-PT to SR-TT, Chi-square tests revealed more two-valued patterns in the R-S possibilities tasks than in the R-S truth tasks for the explicit negations ($\chi^2(1) = 3.95, p < .05$) and for the implicit negations ($\chi^2(1) = 5.85, p < .05$). Likewise, more three-valued answer patterns were yielded by the R-S truth tasks than by the R-S possibilities tasks, both for the explicit ($\chi^2(1) = 5.0, p < .05$) and the implicit negations ($\chi^2(1) = 5.63, p < .05$). Comparing SR-PT to SR-TT, we observed more two-valued patterns in the S-R possibilities task than in the S-R truth task for the explicit ($\chi^2(1) = 15.48, p < .01$) and the implicit negations ($\chi^2(1) = 17.57, p < .01$). Finally there were more three-valued answer patterns in the S-R truth task than in the S-R possibilities task, again both for the explicit ($\chi^2(1) = 5.29, p < .05$) and the implicit negations ($\chi^2(1) = 10.33, p < .01$).
General Discussion

First of all, we replicate the effect of task type we observed in all previous studies: Irrespective of task directionality, there are more two-valued answer patterns in the possibilities tasks than in the truth tasks, and more three-valued answer patterns in the truth tasks than in the possibilities tasks. This relates to our discussion that reasoning about truth or falsity requires a higher level of cognitive development than reasoning about possibilities and might therefore be more difficult, since reasoning about truth requires the understanding of the meta-linguistic meaning of ‘true’ and ‘false’. However, there is also a considerable amount of three-valued patterns in the possibilities task and of two-valued patterns in the truth task. This leads us to the interpretation that irrespective of the task, some people have a three-valued truth table representation (~ ST) and others have a possibilities-based, two-valued MMT-representation. The mental representation of this kind of conditionals can thus be seen as an individual difference.

Secondly, we were not able to replicate the directionality effect that more two-valued answer patterns were yielded by rule-to-situation tasks than by situation-to-rule tasks, and more three-valued answer patterns were observed with situation-to-rule tasks than with rule-to-situation tasks, as was observed by Sevenants et al. (2008). In the present experiment, participants seem to overlook task directionality when classifying the truth table cases, as is shown by the similarity of the justifications in both task types.

As for the negation type, we observe that tasks making use of implicit negations yield more three-valued answer patterns than the tasks with explicit negations and that two-valued answer patterns are almost non-existent when implicit negations are used. This was to be expected from the phenomenon of matching bias: Especially for the false antecedent cases, with implicit negations there is no direct link between the conditional rule and the situations that have to be evaluated. Still, with the explicit negations there is a wealth of three-valued answer patterns, even in the possibilities tasks, bearing evidence to the suppositional processing of the false antecedent cases (Evans, 2004).

References


