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Authors
Boik, A
Corts, KS

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The Effects of Platform Most-Favored-Nation Clauses on Competition and Entry

Andre Boik  
*University of California, Davis*

Kenneth S. Corts  
*University of Toronto*

Abstract

In the context of sellers who sell their products through intermediary platforms, a platform most-favored-nation (PMFN) clause is a contractual restriction requiring that a particular seller will not sell at a lower price through a platform other than the one with which it has the PMFN agreement. Contractual restrictions observed in markets for e-books and travel services, among other settings, can be viewed as examples of this phenomenon. We show that PMFN clauses typically raise platform fees and retail prices and curtail entry or skew positioning decisions by potential entrants pursuing low-end business models.

1. Introduction

Recent interest from competition authorities in contracts that reference rivals has dovetailed with interest in platforms and two-sided markets to draw significant attention to the effects of a type of contract known variously as a platform parity agreement or platform most-favored-nation (PMFN) agreement. In situations in which a seller sets a price and transacts with a buyer through an intermediary platform (which may collect a fee or a commission from the seller), such contracts restrict the seller not to sell through any alternative platform at a lower price. Most-favored-nation (MFN) contracts and other contracts that reference rivals have recently been the subject of a US Department of Justice Antitrust Division workshop (Baker and Chevalier 2013), a UK Office of Fair Trading (OFT) report (Laboratory of Economics, Antitrust, and Regulation 2012),\(^1\) and a speech by the deputy assistant attorney general of the US Department of Justice’s Antitrust Division (Scott Morton 2012). These PMFN agreements in particular have played a key role in recent antitrust cases involving e-books and travel-booking

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\(^1\) One of the authors of the present paper (Corts) was retained by the Laboratory of Economics, Antitrust, and Regulation to coauthor the cited report.
sites (see Salop and Scott Morton [2013] for an overview). The policy-oriented literature conjectures that these agreements can raise prices for consumers and profits for platforms and may limit entrants with low-end business models. However, there exists little theoretical work to support or qualify these assertions. Analyzing these agreements in an explicit model, we find support for some of these claims but with important caveats.

To fix ideas, consider an example of such a PMFN policy, which comes from a 2012 class-action suit filed in the United States, *Turik v. Expedia* (No. 3:12-cv-4365 [N.D. Cal. August 12, 2012]; see Cernak and Chaiken 2013). It is argued that the alleged damages (higher airfares and hotel prices, for example) arise from the suppression of competition and the foreclosure of entry in the platform market. Most popular travel-booking sites (Expedia, Travelocity, Orbitz, and so on) are platforms that connect buyers (travelers) and sellers (airlines, hotels, and car rental agencies); they are not resellers that buy from suppliers and resell to consumers. The platforms provide a forum in which the sellers can offer their products to potential buyers at prices that the sellers themselves determine, and the platform collects a fee from the sellers for this service. In this context, a PMFN clause is an agreement between a platform and a seller (for example, between a travel-booking website and an airline) that commits that seller (the airline) not to offer a lower price for the same item through any other platform (another travel-booking website). This institutional arrangement, and these PMFN contracts, are quite common in many categories of online commerce, including websites specializing in e-books, music and video content, and travel services, as well as websites with broader coverage, such as Amazon Marketplace. This particular case, which was brought under section 1 of the Sherman Act, was dismissed by a US District Court in 2014 on grounds that the complaint failed “to plausibly allege a price-fixing conspiracy” (*Online Travel Company Hotel Booking Antitrust Litig.*, 3:12-cv-3515-B [N.D. Texas February 18, 2014]). The legal standing of such policies remains in question and is widely discussed in the policy literature.

Another high-profile case that focused attention on PMFN agreements concerned Apple’s sales of e-books. The US Department of Justice and the European Commission both brought actions against Apple and a set of publishers for the coordinated transition of those publishers to the platform or agency model of bookselling, with PMFN agreements, from the reseller or wholesale model that had prevailed when Amazon dominated the market. These cases included discussions of the effects of PMFN agreements, among many other issues, including price coordination and coordination in the adoption of the new business model. Consent decrees were reached in 2013 in both jurisdictions (in the US case separately for Apple and for the publishers); these consent decrees contain among other provisions a 5-year ban on the use of PMFN clauses in the contracts governing e-book sales. While Apple reached a settlement with the European Commission, it appealed in US federal court but lost in 2015. Apple’s subsequent request to be heard by the Supreme Court was denied in 2016.

The conventional wisdom about these agreements—which appears with vary-
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ing degrees of clarity or explicitness in Schuh et al. (2012), Salop and Scott Morton (2013), Laboratory of Economics, Antitrust, and Regulation (2012, chap. 6), the expert testimony of economists in select cases, and the investigative documents, complaints, and decisions that have come out of antitrust enforcement activity relating to these policies—is simple. These policies create an incentive for the platform to raise fees because a PMFN clause limits the ability of the seller to pass through higher fees in the form of higher retail prices on that platform alone.

Since other platforms face the same incentives, these higher fees ultimately lead to higher retail prices across all platforms and potentially to higher profits for platforms. In addition, such policies disadvantage potential platform entrants—especially those with low-end business models—by eliminating an entrant’s ability to win customers away from incumbent platforms through lower prices. The incongruity of the arguments that these policies both raise profits and deter entry is not generally addressed.

We explore these arguments and demonstrate a number of important qualifications and nuances. With respect to price and profit effects, we find that PMFN agreements do tend to raise fees and prices but also that they may raise fees and prices so much that they hurt platforms’ profits. Whether this is the case depends on the elasticity of aggregate demand, which is in some situations related to the substitutability of the platforms. With respect to effects on entry, we find that a PMFN agreement may encourage or discourage entry. For an exogenously symmetric entrant, the policies obviously encourage rather than deter entry whenever the fee and price effects just described raise platforms’ profits. We extend the analysis of entry and positioning effects to exogenously and endogenously differentiated potential entrants. When the entrant’s product position is exogenous, there is a trade-off between higher equilibrium fees relative to the no-PMFN equilibrium (which arise through the mechanism described above) and the inability of a platform to increase market share by lowering fees, which disproportionally hurts the profits of the firm that is disadvantaged in demand. When the entrant’s product position is endogenous, a PMFN agreement may again encourage or deter entry; in addition, it may distort the entrant’s choice away from a lower-end business model (and toward a model more similar to that of the incumbent) even when it fails to deter entry. Our results therefore support some aspects of the conventional wisdom but add important caveats and refinements that enrich our understanding of the effects of these PMFN policies.

1.1. Relationship to Traditional Most-Favored-Nation Agreements

It is worth emphasizing that these PMFN policies are not the same as traditional MFN policies, which have been the subject of considerable theoretical inquiry. In a traditional MFN policy, one or more sellers commit to one or more buyers not to sell to other buyers at a lower price. When these policies are in place across all buyers for the adopting sellers, as is typically the case in this literature, this amounts to a commitment to uniform pricing—that is, a commitment not to price discriminate. A series of theory papers examine mechanisms through which this uniform-pricing commitment may be profitable. For example, Schnitzer (1994) shows that with sequential arrival of consumers, such policies make high early prices serve as a kind of commitment to high later prices by making discounting later prices more expensive. If a firm knows that price reductions for present buyers will also result in price reductions or rebates for earlier buyers, they will be reluctant to cut prices. This commitment to relatively unaggressive pricing may be a profitable commitment in a pricing game of strategic substitutes. Besanko and Lyon (1993) show that a similar logic applies if there are heterogeneous groups of buyers, such as contested and captive buyers. By committing to uniform pricing, each firm makes it less attractive for itself to compete aggressively for the contested buyers, since offering low prices to contested consumers also reduces prices and profits for captive buyers. This can profitably soften price competition. Cooper and Fries (1991) show that with sequential arrival of heterogeneous buyers, a traditional MFN can commit a firm that lacks price-setting power to engage in tougher bargaining in its negotiations over terms with late-arriving buyers.

In all of these cases, the mechanism at work is that a discount to some set of buyers is made less attractive to the seller with the MFN agreement because it necessarily triggers a discount to other buyers, and this is in turn a profitable strategic commitment vis-à-vis some other strategic player. This leads the seller to adopt the traditional MFN agreement to soften competition and raise profits. This depends on some sort of heterogeneity in the groups of buyers, whether that is in their choice sets, preferences, or timing of arrival. It also requires a rival seller (in all of the theoretical literature other than Cooper and Fries [1991]) or a situation in which the seller is not a price setter (as in Cooper and Fries [1991]). A traditional MFN policy would certainly not raise prices if adopted by a single seller that sells to a single population of homogenous buyers since the commitment to uniform pricing is of no consequence in the monopoly price setting case when uniform prices are already optimal.

Note that a platform setting is quite different in several ways. Most notably, a PMFN clause is an agreement between a seller and a platform about prices charged by the seller to a third party—the buyer. When there are multiple platforms, this creates an incentive for the platform to increase its fees to the seller (something entirely absent in the traditional MFN literature), which generally increases prices for reasons that have nothing to do with strategic commitment on
the part of the seller vis-à-vis other sellers and nothing to do with heterogeneous buyers. In the PMFN setting, as evident in our model, the price-raising effects of PMFN agreements arise even in the case of a price-setting monopoly seller facing homogenous consumers.

While this formal theory literature on the price effects of traditional MFN agreements exists, there has been only informal discussion of the effects of traditional MFN agreements on entry. Both the OFT report on price-relationship agreements (Laboratory of Economics, Antitrust, and Regulation 2012, para. 3.17) and Cooper and Fries (1991) suggest informally that traditional MFN agreements could limit downstream entry with sequential arrival of potential entrants. With an MFN in effect between an upstream seller of some input and an incumbent downstream firm, a subsequent downstream entrant will find it more difficult to obtain a price from the seller low enough to make entry profitable. This occurs for the same reasons that discounts to late-arriving final buyers are less attractive in Cooper (1986), Schnitzer (1994), and Cooper and Fries (1991).

1.2. The Literature on Platform Most-Favored-Nation Agreements

There exists a small literature that addresses the price effects of PMFN agreements (as opposed to traditional MFN agreements) in explicit theory models. The study most related to ours is Johnson (2013), which studies an environment in which multiple sellers sell through multiple intermediaries under either the wholesale model (in which sellers set wholesale prices and resellers set retail prices, as in traditional brick-and-mortar retailing) or the agency model (in which sellers set retail prices and platforms set commissions paid by the retailer, as in many online marketplaces such as Amazon Marketplace, the e-books market, and most online travel-booking sites). Johnson (2013) is primarily concerned with a comparison of these two models; however, one section addresses the effect of PMFN agreements on the equilibrium under the agency model. Johnson finds, as do we, that PMFN agreements raise platform fees and retail prices; however, he also shows, in contrast to our results, that PMFN agreements always raise industry profits and are always adopted by platforms in equilibrium. These differences arise because Johnson’s (2013) model assumes perfectly inelastic aggregate demand; we discuss in more detail how this demand setting leads to these results. In addition, Johnson (2013) does not consider asymmetric firms and does not address effects on entry.

One other paper considers PMFN agreements in explicit theory models. In a study of the dissemination of mobile applications, Gans (2012) examines a model in which the firm controlling the mobile platform can offer direct access to app purchases on the platform, while app developers can also sell directly to consumers. He is primarily focused on the difficulties platforms have in charging for the platform in the presence of holdup by app developers, and he shows that a PMFN policy can help solve this problem.

While these studies formally consider price effects of PMFN agreements, to
our knowledge no formal theoretical analysis of the effects of PMFN agreements on entry or choice of product position exists. Informal discussions in the policy literature suggest that PMFN agreements can deter or limit downstream entry. For example, Laboratory of Economics, Antitrust, and Regulation (2012, paras. 6.49–6.50) suggests that by eliminating the possibility that a seller will charge a comparatively lower price on an entrant platform, PMFN clauses limit the ability of entrants—and, in particular, entrants with low-end business models—to establish themselves and increase market share by charging lower fees to the seller than incumbent platforms charge.

Thus, the literature on PMFN agreements is quite limited, and the literature on traditional MFN agreements does not apply directly to this different institutional context. We make a significant contribution by explicitly considering the fee and price effects of PMFN policies in a more general setting in which aggregate demand can be downward sloping and by being the first to analyze formally the effects of PMFN agreements on entry and positioning decisions.

This paper is organized as follows. Section 2 presents the model. Section 3 considers the effects of PMFN agreements on competition between two symmetric incumbent platforms. Section 4 considers the equilibrium adoption of such agreements. Section 5 analyzes the effect of such policies on incentives for entry and endogenous choice of competitive position for an entrant platform. Section 6 concludes.

2. Model

A single seller $S$ sells its products to buyers through one or both of two platforms (or marketplaces) $M_i, i = \{1, 2\}$. Each platform $i$ incurs a fixed cost $K_i$ and a constant marginal and average production cost $c_i$. The seller incurs three kinds of costs: fixed cost $K_S$, constant marginal and average production cost $c_S$, and a per-unit transaction fee $f_i$ charged by each platform $i$. The seller sets a price $p_i$ on each platform $i$. Buyers’ demand through a particular platform $i$ is given by $q_i(p)$, where $p$ is a vector of the seller’s prices on each platform.

Note that we restrict payments between platforms and sellers and between sellers and final buyers (fees $f_i$ and prices $p_i$, respectively) to be linear in quantity. This simplifies the analysis greatly and is consistent with much of the literature on vertical contracts. It does rule out nonlinear pricing schemes, including those in which sellers pay fixed fees to platforms or fixed fees coupled with linear fees that recoup platforms’ marginal costs. The latter scheme would in fact lead to pricing by the single seller that maximizes the total joint profit of all platforms and the seller. Such nonlinear schemes may be impractical or suboptimal in many real settings because of typical contracting considerations. For example, it

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3 In many applications, platforms charge a commission proportional to retail price rather than a fixed per-unit fee. We expect that our qualitative results will apply to both types of fees. In general, in these kinds of models, a proportional commission has the effect of raising the seller’s perceived marginal cost (Johnson 2013) because of the divergence between the taxed revenue and the maximized profit, whereas in our model the fixed per-unit fee directly raises that marginal cost.
may be prohibitively costly to estimate sales of each product through each platform to calculate and negotiate appropriate fixed fees. In addition, such schemes that imply no marginal profit to the platform may fail to provide adequate incentive for the platform to expend effort on sales. For these reasons and for analytical tractability, we restrict attention to linear pricing.

The timing is as follows. The platforms simultaneously choose whether to require PMFN policies. They then simultaneously choose transaction fees \( f_i \). The seller sets prices \( p_i \), abiding by the terms of any PMFN policies. The seller earns profits \( \pi_i = \sum_{i=1,2} [p_i - f_i - c_i \hat{q}_i(p)] \); each platform \( i \) earns profits \( \pi_i = [f_i - c_i \hat{q}_i(p)] \). For the analysis of competition between incumbent platforms (Sections 3 and 4), we ignore fixed costs, which will not affect pricing or fee-setting incentives. Fixed costs are introduced in Section 5, where we focus on the effect of PMFN policies on entry decisions.

Because the final stage involves only the single seller’s pricing decision, it is convenient to suppress this stage of the game in the analysis by writing platform-level demand functions as a function of the transaction fees \( f_i \) rather than prices \( p_i \) where these demand functions indicate demand at the seller’s optimal prices given the transaction fees. Note that the seller is effectively a simple multiproduct monopolist (where the underlying product sold through each of the platforms is thought of as a distinct product) facing demand \( \hat{q}_i(p) \) and with potentially different marginal costs \( (c_s + f_i) \) and \( (c_s + f_j) \) for its two products. However, the seller may also face a constraint imposed by the presence of one or more PMFN agreements. Therefore, this implied demand function varies with the PMFN regime. We denote this implied demand function \( \hat{q}_i(f) \), where \( k = \{0, 2\} \) denotes how many PMFN agreements are present and \( f \) is a vector of the platform’s fees. (The case of a single PMFN agreement is analyzed separately and does not require its own implied demand functions for reasons that will become apparent later.) Assume that demand takes the familiar linear differentiated-products form \( \hat{q}_i(p) = a - bp_i + dp_j \), where \( a, b, d > 0 \), and \( b > d \). We also assume that demand is strictly positive with marginal cost pricing; that is, the sum of the marginal costs is less than the symmetric choke price for each firm: \( c_i + c_s < a/(b - d) \).

It is straightforward to determine the optimal pricing rule for the two-product

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1 Boik and Corts (2014) shows that in fact the main results of Section 2 also hold in a more general demand model. In particular, we show that propositions analogous to propositions 1–4 in this paper hold for more general demand structures as long as underlying demand \( \hat{q}_i(p) \) is differentiable, additively separable in prices, not too nonlinear, and symmetric and satisfies the typical assumptions that the own-price derivative is negative, the cross-price derivative for the substitute is positive, and the absolute value of the own-price derivative exceeds that of the cross-price derivative. We show in Boik and Corts (2014) that these assumptions on underlying demand suffice to show that the seller’s pass-through of its own platform fee is positive regardless of whether platform most-favored-nation (PMFN) agreements are in place but is larger if they are not (\( dp_1^i / df_i > dp_2^i / df_i > 0 \)) and that the seller’s pass-through of the rival platform’s fee is weakly negative in the absence of PMFN agreements but positive (and in fact equal to own-fee pass-through) in the presence of PMFN agreements (\( dp_2^i / df_i > 0 \geq dp_1^i / df_i \)). These properties suffice, as shown in Boik and Corts (2014), for results analogous to propositions 1–4 in the present paper.
monopoly seller under both the 0PMFN (neither platform has a PMFN agreement) and 2PMFN (both platforms have a PMFN agreement) regimes. Maximizing the seller’s profit yields optimal prices that are linear in the platform fees. These are given by

\[ p_i^0 = \frac{a + (b - d)(c_i + f_i)}{2(b - d)} \]

and

\[ p_i^2 = \frac{2a + (b - d)(2c_i + f_i + f_j)}{4(b - d)}. \]

The pricing rules give the seller’s pass-through of the platforms’ fees. It is useful to examine these expressions closely to develop some intuition about the role of the price-setting seller in translating fee changes into final prices. Note that when there are no PMFN agreements in effect, the multiproduct seller reacts to a fee increase from one platform by raising that platform’s price, which diverts demand to the other, now relatively higher-margin platform (note that the price on the other platform remains unchanged). When there are two PMFN agreements in effect, the seller is constrained to set a uniform price across platforms. As a result, it has reduced flexibility in diverting sales to the other platform when one platform raises its fee. Raising the price on one platform means raising the price on both platforms. While the higher fee on one platform induces the seller to raise the price on that platform (and on the other platform), this is now more costly in lost demand on both platforms, and the seller optimally chooses to raise the price on the fee-raising platform less than it would have absent the PMFN agreements (in particular, the coefficient on own fee in these pricing equations is \( \frac{1}{2} \) for the 0PMFN case but only \( \frac{1}{4} \) for the 2PMFN case). It is useful to note that the seller’s pricing behavior implies that PMFN agreements are irrelevant to price setting when fees are symmetric: these two pricing rules yield the same final price for any set of symmetric fees.

Substituting these optimal pricing rules into the demand function yields implied demand as a function of transaction fees: \( q_i^0(f) = \frac{a - b(c_i + f_i) + d(c_i + f_i)}{2} \) and \( q_i^2(f) = \frac{2a - (b - d)(2c_i + f_i + f_j)}{4} \). Note here that in the absence of PMFN agreements, the seller’s price response creates a situation in which a fee increase by one platform decreases sales for that platform and increases sales for the non-fee-raising platform. With two PMFN agreements, the seller’s price response leads one platform’s fee increase to reduce sales for both platforms; however, the sales reduction for the fee-raising platform is less than half of what it would have been in the absence of PMFN agreements (the coefficient on own fee is \( -(b - d)/4 \) rather than \( -b/2 \).
3. Competitive Effects of Platform Most-Favored-Nation Agreements

This section analyzes a model with two symmetric incumbent platforms: the platforms have the same cost structure, and demand is symmetric \((\hat{q}_i(p_i = y, p_j = z) = \hat{q}_j(p_j = y, p_i = z))\). We analyze the best-response functions and equilibrium transaction fees that arise in the stage 2 subgame in which platforms simultaneously set fees. This allows us to characterize the impact of PMFN policies on competition, comparing the cases with and without PMFN policies. As described in note 5, Boik and Corts (2014) develops the results in this section for a more general model. While we present the analysis only for the linear model here, we maintain portions of the argument in a more general form (using more general calculus-based expressions rather than the algebraic expressions that result from the closed forms available in the linear model) to emphasize the generality of the logic and intuition developed. We begin with the main model as presented in Section 2, which assumes that the buyer pays the full price set by the seller, the seller does business with both platforms, and there is only one seller. We then consider the robustness of our main results to the possibilities that platforms can rebate some portion of their fees directly to buyers, the seller may decide to do business with only a single platform, and there are multiple sellers. We argue that these alternative assumptions would not change our qualitative results.

3.1. Main Model

We begin by deriving best-response fee-setting functions under each of the PMFN regimes. Rearranging the first-order conditions (FOCs) for platforms’ profit maximization yields the following best-response functions for fees:

\[
\begin{align*}
 f_i^{\text{ORR}} &= \frac{a - (b - d) c_s + b c_j + df_i}{2b} \\
 f_i^{\text{2MR}} &= \frac{2a - (b - d) (2c_s - c_i + f_j)}{2(b - d)}
\end{align*}
\]

It is straightforward to check that the second-order conditions are satisfied. In the case of the 0PMFN regime, the second-order condition reduces to \(b > 0\), and in the case of the 2PMFN regime, it reduces to \(b - d > 0\).

Simultaneously solving the best-response functions under the assumption of symmetric costs suffices to determine equilibrium fees, thus proving the existence and uniqueness of equilibrium in each regime. Denote the equilibrium fees \(f_i^{**}\):

\[
 f_i^{**} = \frac{a + b(c_i - c_s) + dc_s}{2b - d}
\]

and
Comparing these expressions yields our first result for the competitive effects of PMFN agreements: $2\text{PMFN}$ equilibrium fees are higher than $0\text{PMFN}$ fees ($f_{i}^{**} > f_{i}^{*}$). Substituting these expressions into this inequality yields an expression that reduces to $a - (b - d)(c_{i} + c_{i}) > 0$, which is precisely the assumption we made on the sum of costs lying below the choke price.

It is instructive to consider the more general intuition behind this result, which is apparent in a comparison of the FOCs under the two regimes. Recalling that the implied demand $q_{i}^{0}(f)$ is indexed for the PMFN regime, the FOC in both regimes is given by $\frac{\partial \pi_{i}}{\partial f_{i}} = (f_{i} - c_{i})[\partial q_{i}^{0}(f)/\partial f_{i}] + q_{i}^{0}(f) = 0$. Recall that the responsiveness of implied demand to own fee is smaller in absolute value (less negative) under the $2\text{PMFN}$ regime than under the $0\text{PMFN}$ regime ($\partial q_{i}^{0}(f)/\partial f_{i} < \partial q_{i}^{0}(f)/\partial f_{i} < 0$): in essence, implied demand is made more inelastic by the adoption of PMFN agreements since it limits the seller’s incentive to pass through fee increases. Finally, recall that for symmetric fees $f_{i}, q_{i}^{0}(f) = q_{i}^{0}(f)$.

Comparison of the FOCs yields this result. Note that the $0\text{PMFN}$ FOC implies that $(f_{i}^{**} - c_{i})[\partial q_{i}^{0}(f)/\partial f_{i}] + q_{i}^{0}(f^{**}) = 0$. Since $\partial q_{i}^{0}(f)/\partial f_{i} < \partial q_{i}^{0}(f)/\partial f_{i} < 0$, the analogous $2\text{PMFN}$ FOC holds only if $f_{i}^{**} > f_{i}^{*}$: the less negative first term in the $2\text{PMFN}$ FOC evaluated at $f^{*}$ makes the entire FOC positive, which is restored to 0 only through an increase in fees, which decreases the positive term and increases the negative term.

The fact that equilibrium fees must be higher under $2\text{PMFN}$ regimes implies that equilibrium final prices are higher under $2\text{PMFN}$ regimes; this is because fees are symmetric, and the seller’s pricing rules derived earlier yield the same prices for any symmetric fees and are increasing in those symmetric fees. The above analysis is summarized in the following proposition:

**Proposition 1.** There exists a unique symmetric equilibrium in transaction fees if no platforms have PMFN agreements or if both platforms have PMFN agreements. Equilibrium fees and prices are higher when both platforms have PMFN agreements.

We can also compare the $2\text{PMFN}$ equilibrium fees and prices to those that would arise under collusive platform fee setting absent PMFN agreements. While this is perhaps surprising at first, PMFN agreements necessarily lead to fees and prices that are even higher than those chosen by colluding platforms. To see this, note first that under either symmetric collusive fees or the symmetric $2\text{PMFN}$ equilibrium, the seller optimally chooses a symmetric price. In the $0\text{PMFN}$ equilibrium, the seller’s variable profit following collusive symmetric fee setting reduces to $\sum_{i=1,2}[p - c_{i} - f_{i}]q_{i}^{0}(p)$. In the $2\text{PMFN}$ equilibrium, the seller’s variable profit reduces to $\sum_{i=1,2}[p - c_{i} - f_{i}]q_{i}^{0}(p) = 2[p - c_{i} - (f_{i} + f_{i})/2]q_{i}^{0}(p)$. Importantly, in both of these cases (and unlike the noncollusive $0\text{PMFN}$
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The seller’s optimal pricing rule can be reduced to a function of the average fee \((f_i + f_j)/2\). Therefore, these situations generate the same implied demand function, \(q_{SYM}(\bar{f})\). It immediately follows from this that the collusive fee is the same regardless of whether PMFN agreements are adopted. Finally, compare the collusive fee-setting FOC under 2PMFN agreements with the equilibrium fee-setting FOC under 2PMFN agreements. The former yields \((f_i - c_i)(\partial q_i^f_i/\partial f_i) + q_i^f_i(f_i) + (f_j - c_j)(\partial q_j^f_j/\partial f_j) = 0\), while the latter yields \((f_i - c_i)(\partial q_i^f_i/\partial f_i) + q_i^f_i(f_i) = 0\). The first two elements in each expression are identical. The fourth term in the former, \((f_j - c_j)(\partial q_j^f_j/\partial f_j)\), is negative. This implies that with collusive fees (which are the same with no or two PMFN agreements), the noncollusive 2PMFN fee-setting FOC (the latter) is positive, which in turn implies that 2PMFN equilibrium fees are higher than collusive fees. The 2PMFN equilibrium final prices are therefore also higher than those that arise under collusive fee setting. This yields the next proposition:

**Proposition 2.** The unique symmetric equilibrium fees and prices when both platforms have PMFN agreements are higher than the symmetric equilibrium fees and prices that would arise under collusive fee setting by platforms absent PMFN agreements.

Another way to understand the price-raising effects of PMFN agreements in this context is to consider the nature of competition under strategic substitutes and strategic complements. It is always the case that competition in strategic complements leads to choices of strategic variables that are too low (for example, differentiated-product equilibrium pricing is lower than joint-monopoly pricing) and that competition in strategic substitutes leads to choices of strategic variables that are too high (for example, Cournot quantities are in aggregate larger than the monopoly output). Note from the above best-response functions that platform fees absent PMFN agreements are strategic complements (the coefficient in the best-response function is \(\frac{1}{2} > 0\); a rival platform’s higher fee increases the final price on that platform, which increases implied demand and leads the other platform to set a higher fee as well. Under a 2PMFN agreement, platform fees are strategic substitutes (the coefficient in the best-response function is \(-\frac{1}{2} < 0\); a rival platform’s higher fee raises both final prices, which reduces implied demand and leads the other platform to cut its fee. Thus, PMFN agreements transform this from a game of strategic substitutes with platform fees that are lower than a monopolist would set to a game of strategic complements with platform fees that are higher than a monopolist would set.\(^5\)

Note from the expressions derived above that this higher-than-collusive result follows directly from an aggregate demand effect. The \(\partial q_i^f_i/\partial f_j\) term—which is the difference between the equilibrium and collusive FOCs under a 2PMFN regime—is negative because an increase in \(f_i\) under two PMFN agreements leads

\(^5\) The PMFN agreements are therefore an example of an extensive-form game involving both strategic complements and substitutes as studied more broadly in the game-theoretic literature (for example, Monaco and Sabarwal, forthcoming).
the seller to raise the price on both platforms. This increase in the common price in turn causes a loss of quantity for the other platform whenever there is any amount of aggregate demand elasticity. It is only aggregate demand elasticity that can account for this term being negative because shares of demand are fixed by the restriction to a common price under symmetric demand. What one can see from this analysis is that the higher-than-collusive result would not arise in the absence of aggregate demand elasticity or in a situation in which there was no pass-through of a platform’s fee to the common price by the seller (which cannot happen with smooth demand). Indeed, this is precisely why this higher-than-collusive result does not arise in Johnson (2013). That study features perfectly inelastic demand at all prices up to the point at which all buyers switch to the outside option. At that price, the seller would no longer pass through further increases in fees as higher prices. Thus, in Johnson’s model the 2PMFN fees coincide with the collusive fees, which are fees that induce the seller to price precisely at the choke point where the outside option is binding.

In our model there is always some elasticity to aggregate demand. Because this is the source of the divergence between collusive and equilibrium fee setting, it is clear that as aggregate demand approaches perfect inelasticity, the equilibrium 2PMFN fees will approach the collusive fees. This also suggests that PMFN agreements may reduce profits when aggregate demand is sufficiently elastic. In fact, both of these conjectures can be proved to be true by analyzing \( \pi^2 - \pi^0 \). These expressions for profit are derived through substitution of expressions derived earlier and are given by

\[
\pi^0 = \frac{b(a - (b - d)(c_i + c_j))^2}{2(d - 2b)^2}
\]

and

\[
\pi^2 = \frac{(a - (b - d)(c_i + c_j))^2}{9(b - d)}.
\]

The sign of the difference \( \pi^2 - \pi^0 \) can be shown to be the same as that for \( 2(2b - d)^2 - 9b(b - d) \). By substituting \( d = \alpha b \) in this expression, it is straightforward to show that the expression is negative if \( \alpha < \frac{1}{2} \) and positive if \( \alpha > \frac{1}{2} \). This yields the following proposition.

**Proposition 3.** Equilibrium profits under 2PMFN regimes are higher than 0PMFN equilibrium profits if aggregate demand is sufficiently inelastic (that is, if \( b - d \) is sufficiently small—specifically, if \( d > b/2 \)) and lower than 0PMFN equilibrium profits if aggregate demand is sufficiently elastic (that is, if \( b - d \) is sufficiently large—specifically, if \( d < b/2 \)).

In this particular linear model, aggregate demand elasticity goes hand in hand with product substitutability. When the price on one platform increases, that platform loses sales of \( b \); of those \( b \) units, some portion \( d \) are purchased from the other platform. The remainder, \( b - d \), are lost to some outside option. When \( b = \)
no buyers are lost to the outside option and aggregate demand is perfectly inelastic with aggregate quantity equal to $2a$. As $d$ approaches $b$, more of the buyers who leave one platform switch to the other and fewer drop out of the market altogether. Thus, an increase in $d$ toward $b$ is an increase in product substitutability; it is also by the algebra of this demand function a decrease in aggregate demand elasticity. This relationship between substitutability and aggregate elasticity does not hold across all demand models, of course. Nonetheless, the results of this section can be read—literally in the context of this linear model or suggestively in a broader sense—with the interpretation that PMFN agreements are more likely to be attractive and increase platforms’ profits when platforms are closer substitutes.

3.2. Robustness

While we find that platforms under a 2PMFN regime will choose fees even greater than those that would exist under platform fee collusion, it is possible that platforms may rebate some fraction of those fees directly to consumers and thus face a two-sided pricing problem. Unfortunately, introducing rebates to our model creates a difficult and intractable setting with multiple equilibria with or without PMFN agreements. This intractability is common in the literature on two-sided markets. Nevertheless, in such a model with rebates we show in Appendix A that under a 2PMFN regime, platforms have an incentive to increase both fees and the fee-rebate markup beyond the collusive level.⁶

Our main model also assumes that the seller sells through both platforms, even when it is charged (or can anticipate being charged) a very high fee because of PMFN agreements. It is natural to expect that if PMFN agreements raise fees sufficiently, the seller may wish to drop one platform and sell instead through a single platform, which would render the PMFN agreement moot and therefore potentially reduce fees (though it now pays fees to a monopolist platform, so this is by no means certain). We show in Appendix B that even with PMFN agreements in place, there exist parameter values for which the 2PMFN equilibrium is preferred by the seller over dealing with a monopoly platform, which implies that the seller would in those cases not drop a platform even though it anticipated high 2PMFN fees. The intuition is that the seller may accept high platform fees if having multiple platforms sufficiently broadens the customer base. If it does not, then the seller may indeed drop a platform if expected 2PMFN fees are excessively high.

Finally, our main model features a single seller doing business through multiple platforms. In our examples, typically many sellers sell through a similar set of multiple platforms. We believe that our results apply qualitatively to this setting as well, though of course the modeling of multiproduct, multiplatform demand would make formal modeling of this case much more difficult.

⁶ Demand for each platform with rebates takes the form $q_i = a - b(p_i - r_i) + d(p - r_i)$. Under a 0PMFN regime, platforms choose a unique markup $f^* - r^*$ so that the 0PMFN equilibrium is qualitatively unchanged, while under a 2PMFN regime there is no equilibrium: platforms choose ever-increasing fees and rebates, but fees rise at a rate faster than rebates. See Appendix A.
The fundamental mechanism at work in raising fees and prices in our model is that PMFN agreements reduce the elasticity of implied demand for a platform when considering its fee; again, this is because the fee increase will be subject to less pass-through by a multiplatform seller. With multiple sellers, the same logic applies to the platform’s decision about fees with respect to any particular seller (and we are not studying any kind of restraint that relates fees to each other across sellers). When the platform considers a fee increase for a given seller, it will consider the pass-through of that fee increase on that seller’s pricing and in turn the effect on platform-level demand (of all sellers’ products). From the seller’s point of view, it does not matter whether price increases divert sales to an outside option or to other sellers; regardless, tying the seller’s price increase across platforms will reduce the seller’s incentive to pass through a platform-specific price increase. This is fundamentally the mechanism that induces higher fees from platforms, and it should persist regardless of whether there is a single seller or multiple sellers.

4. Endogenous Adoption of Platform Most-Favored-Nation Policies

This section considers stage 1 of the full game, in which firms simultaneously decide whether to endogenously adopt PMFN policies. The above results for whether PMFN agreements raise profits for the platforms do not suffice to demonstrate whether PMFN agreements will be adopted in equilibrium when chosen by the platforms simultaneously; rather, we must characterize the outcome when only one firm adopts a PMFN policy and compare the profits under that equilibrium to 0PMFN and 2PMFN profits. This section continues to employ the symmetric-duopoly model.

The results for equilibrium fees and best-response functions can be graphed to develop further intuition about competition under PMFN agreements and, in particular, about incentives to adopt PMFN agreements in the first stage of the full game. Figure 1 presents two sets of best-response functions—those that prevail under 0PMFN and 2PMFN regimes—in a single graph in $f_1 \times f_2$ space. We denote platform $i$’s best-response curve under a scenario with $k$ PMFN agreements $b^k_i$. The two points along the 45-degree line at which $b^1_i$ and $b^2_i$ cross define the 0PMFN and 2PMFN equilibria. The primary value of Figure 1 is the analysis of competition in the scenario in which only one platform (which we assume to be platform 1) has a PMFN policy in place. We therefore proceed to construct the best-response functions in this scenario.

First, note that for a particular platform, pricing incentives are determined under either the 0PMFN best-response calculation (there is no PMFN binding and $q^0(f)$ is relevant) or the 2PMFN best-response calculation (there is a PMFN binding and $q^2(f)$ is relevant). Which of these calculations is relevant depends on the relative prices of the two platforms. In particular, the PMFN agreement is irrelevant if $f_1 < f_2$, and the 0PMFN incentives apply. Alternatively, when $f_1 > f_2$, the PMFN agreement binds; the fact that platform 2 does not have a PMFN policy is
irrelevant, and the 2PMFN incentives apply. When 0PMFN incentives apply, the game is one of strategic complements, and fee undercutting is prescribed as the platform diverts sales away from the rival platform through lower prices; when 2PMFN incentives apply, the game is one of strategic substitutes, and overpricing is prescribed (similar to Cournot firms overproducing relative to the collusive benchmark) since higher platform fees must be passed through by higher retail prices on both platforms.

Consider platform 2, the platform without the PMFN agreement. At low $f_2$, platform 2 prices off its $b_2^0$ curve; since this calls for overpricing the platform with the PMFN agreement, the presence of the PMFN is irrelevant. Once the $b_2^0$ curve falls below the 45-degree line (at the 0PMFN equilibrium fee), however, this best-response curve is no longer relevant, as the price it dictates will trigger 2PMFN pricing by a seller. Considering this, platform 2 prefers to price off its $b_2^1$ curve. However, any price above the 45-degree line renders the PMFN agreement nonbinding, which triggers 0PMFN pricing by a seller. As a result, the best response by the nonadopting platform 2 is to match the fee of platform 1 for all fees between the 0PMFN equilibrium and 2PMFN equilibrium fees. (Put another way, over this range of $f_1$, profits under the PMFN-binding regime are increasing in $f_2$ below $f_2^*$, and profits under the PMFN-not-binding regime are decreasing in $f_2$ above $f_2^*$. ) Once $f_1$ exceeds the 2PMFN equilibrium fee, $b_2^1$ of platform 2 is relevant since it prescribes undercutting platform 1, which triggers the PMFN policy and makes the 2PMFN best response the relevant curve.

Now consider platform 1. At any $f_2$ equal to or below the 0PMFN equilibrium fee, the best response of platform 1 is given by $b_1^0$, which prescribes overpric-
ing platform 2, which makes the PMFN agreement bind. Since even its 0PMFN best response involves overpricing platform 2, the PMFN agreement will certainly be binding; given this, \( b^0_i \) reflects the correct incentives. Thus, \( f^{**} \) cannot be the equilibrium fees under a 1PMFN regime. For any fee equal to or above the 2PMFN equilibrium fee, the best response of platform 1 is given by \( b_i^1 \). Since even the PMFN-binding incentives (reflected in the 2PMFN best response) imply a best-response fee at which the PMFN agreement is not binding (such as above the 45-degree line), the PMFN agreement will not be binding, and \( b_i^0 \) gives the correct best response. Thus, \( f^{**} \) also cannot be the equilibrium fees under a 1PMFN regime; it follows that there can be no intersection of the relevant best-response functions and no pure-strategy equilibrium in fees under a 1PMFN regime.

Somewhere between the 0PMFN and 2PMFN equilibrium fees there lies a fee \( \hat{f}_2 \) at which platform 1 is indifferent between undercutting and overpricing the fee of platform 2. Since platform 1 is indifferent between these two strategies, it is part of a mixed-strategy equilibrium for platform 1 to randomize between \( b_i^1(f_i) \) and \( b_i^0(f_i) \) with any probabilities \( \sigma \) and \( 1 - \sigma \), respectively. In addition, there is a unique \( \hat{T} \) for which \( \hat{f}_2 \) is the best response of platform 2 to the mixing strategy of platform 1; more formally, there exists a \( \hat{T} \) such that

\[
\hat{f}_2 = \arg \max_{f_2} \left[ \sigma \pi_2 [b_i^1(f_i), f_2] + (1 - \sigma) \pi_1 [b_i^0(f_i), f_2] \right].
\]

This follows from the continuity of the profit function. If \( \hat{T} = 0 \), then the arg max is \( f_2^* \), and if \( \hat{T} = 1 \), then the arg max is \( f_2^0 \). There is some \( \hat{T} \) in between that is the best response of platform 2 to the mixing strategy \( \hat{T} \) of platform 1. This \( \hat{T} \) and \( \hat{T} \) constitute a mixed-strategy equilibrium to the simultaneous-pricing subgame when only platform 1 has adopted a PMFN policy. This yields Figure 1 (for an arbitrary and illustrative \( \hat{T} \) between the two equilibrium fees). The following proposition follows from this analysis:

**Proposition 4.** There is no pure-strategy equilibrium in the fee-setting subgame when exactly one firm has a PMFN agreement, and there is a mixed-strategy equilibrium in such a subgame in which platform 2 sets \( \hat{f}_2 \) (such that \( f_2^{**} < \hat{f}_2 < f_2^* \)) and platform 1 randomizes with appropriate probabilities between \( b_i^1(f_i) \) and \( b_i^0(f_i) \).

Before proceeding to further analyze this case of potential asymmetric PMFN adoption, it is possible to assess the endogenous adoption of PMFN policies in an alternative timing structure, which is instructive. In particular, this analysis is sufficient to characterize the pure-strategy equilibria of the game in which PMFN agreements are adopted or not simultaneously with the setting of transaction fees.

**Proposition 5.** Consider a game with the alternative timing in which platforms simultaneously set fees and adopt PMFN agreements in the same stage. Then there are exactly two pure-strategy equilibria: one in which both firms adopt PMFN agreements and set fees \( f^{**}_i \) and one in which both firms do not adopt PMFN agreements and set fees \( f^{*}_i \).
Depending on the equilibrium selection mechanism, PMFN agreements may or may not be adopted in this game. It remains true as in the earlier propositions that either of these equilibria may be the more profitable one for the platforms, depending on the characteristics of demand. Note that this implies that in this game with alternative timing it is possible to experience a coordination trap in two forms: platforms might fail to adopt PMFN agreements when it is profitable, and platforms might also adopt PMFNs when they raise prices so high as to lower profits. Finally, note that proposition 5 immediately implies a corollary:

**Corollary 1.** Consider a game with the alternative timing in which platforms simultaneously set fees and adopt PMFN agreements in the same stage, and assume an equilibrium selection rule that eliminates equilibria that are Pareto dominated from the point of view of the platforms. Then in the unique Pareto-undominated pure-strategy equilibrium, both platforms adopt PMFN agreements if aggregate demand is sufficiently inelastic.

Returning to the original timing posited for the game, we see that what is required for the characterization of the conditions for equilibrium mutual adoption of PMFN policies is an understanding of the 1PMFN equilibrium profits. In what follows, asterisks indicate profits under the fees that arise in equilibrium of the ensuing fee-setting subgame, superscripts indicate the number of platforms with PMFN policies, and subscripts indicate the platform, where platform 1 is the adopter in the 1PMFN subgame. If \( \pi_i^{0*} < \pi_i^{1*} \) (a single PMFN policy adopter finds the policy profitable) and \( \pi_i^{1*} < \pi_i^{2*} \) (a single PMFN policy nonadopter finds it profitable also to adopt the policy), then mutual adoption is the unique equilibrium in the full game. We proceed by showing that both of these are in fact true when aggregate demand is sufficiently inelastic.

That the first inequality holds is easy to see. The sole adopter’s profit is \( \pi_i^{1*} = \pi_i^{0*}(b_i^i(f_i^i), f_i^i) > \pi_i^{1*}(f_i^{1*}) \). That is, the sole adopter’s 1PMFN equilibrium profit is the profit at that firm’s best response to a rival’s higher price, compared with the 0PMFN equilibrium. This is clearly higher since higher rival’s prices directly raise profits under 0PMFN pricing by a seller.

The second inequality is much more complicated to assess, as it requires the nonadopter’s profit under the 1PMFN regime, which is a weighted average of being undercut and overpriced by the adopting firm while maintaining the price \( f_i^i \). First, note that for platform 2, being overpriced in the mixed-strategy equilibrium is always worse than being in the 2PMFN equilibrium. To see this, note that \( \pi_2^{0*}(b_2^1(f_2), f_i^i) < \pi_2^{1*}(b_2^1(f_2), b_2^2(f_i^i)) \) \( \pi_2^{2*}(f_2^{2*} [b_2^2(f_i^i)], f_i^i) < \pi_2^{1*}(f_2^{2*} [b_2^1(f_i^i)], f_i^i) \) \( \pi_2^{1*}(f_2^{2*}, f_i^i) < \pi_2^{1*}(f_2^{1*}, f_i^{1*}) \). The first of these inequalities follows from the fact that platform 2 would certainly rather be providing a best response to the adopter’s price (in Figure 1, platform 2 would rather be on \( b_2^1 \), directly above the point at which platform 1 overprices against \( f_i^i \)). The second inequality follows from the fact that profits under the 2PMFN equilibrium are decreasing in the rival’s fee, and the third inequality follows from the fact that platform 2 would rather be at the 2PMFN equilibrium where \( f_2^{2*} \) is a best response to \( f_i^{1*} \) than at the lower fee of \( b_2^1(f_i^i) \).
In addition, we show, as a sufficient condition, that platform 2 also prefers the 2PMFN equilibrium to being undercut. This seems natural in the sense that the situation in which platform 1 is able to best respond to $\hat{f}_2$ with an undercutting fee and in which a seller, in turn, is unconstrained by any PMFN agreements in altering prices to reflect these relative prices seems very grim indeed for the nonadopting platform 2. However, it does not immediately follow that the nonadopter prefers the 2PMFN equilibrium to this, since it is at least conceptually possible that the 2PMFN pricing is so high that it is preferable to be undercut at some price intermediate to the 0PMFN and 2PMFN equilibrium pricing. The earlier results suggest that this will not be the case when aggregate demand is sufficiently inelastic, so the 2PMFN fee equilibrium is not so high as to be terribly destructive of platforms’ profits. This is true, although the algebra to prove the result is extremely tedious; it is therefore presented in Appendix C.

**Proposition 6.** If platforms simultaneously choose whether to adopt PMFN agreements prior to simultaneously setting fees, then both firms adopt PMFN policies in equilibrium if aggregate demand is sufficiently inelastic.

5. The Effects of Platform Most-Favored-Nation Agreements on Incentives for Entry

Obviously, for symmetric firms, whether PMFN agreements induce additional entry or curtail entry depends on how they affect equilibrium profits. This follows directly from the results proved for when adoption of PMFN agreements raises equilibrium platform profits. What is of interest in this section, therefore, is the effect that PMFN agreements might have on the entry of firms with different characteristics in demand or cost or on the endogenous selection of those characteristics. We consider the sequential entry of a firm facing different demand or cost parameters against an incumbent firm with a PMFN policy in place. Given that PMFN policies explicitly rule out a low-price entry strategy, and given that such a strategy is likely to be especially important for an entrant who has a lower-cost or a lower-value platform, it is natural to assume (as in the conventional wisdom described in the introduction) that a PMFN policy by an incumbent inhibits entry by lower-cost, lower-value platforms. For example, one might expect that adoption of a PMFN agreement by a full-service platform would make entry by a platform with a bare-bones, low-cost, and (potentially) low-price business model much more difficult, given the constraint it places on a seller’s ability to pass through those lower costs or to offer a discounted price for transactions through the lower-quality platform. Similarly, one could argue that these same forces would lead an entrant endogenously determining its cost and value characteristics to choose a higher-cost, higher-value position or business model than it might have done otherwise.

\footnote{Note that this condition is sufficient but not necessary. What is necessary is that the weighted average of the nonadopter’s 1PMFN profits under the mixed-strategy equilibrium is lower than its 2PMFN profit. For tractability, we focus instead on conditions for which each component of that weighted average is smaller.}
To analyze these questions, we allow two kinds of asymmetry. We allow $c_1 < c_2$, where firm 1 refers to the incumbent. We also permit the possibility that the entrant has a lower-value offering, which results in a reduction in demand of $x > 0$ for any given prices: $\hat{q}_1(p) = a - bp_1 + dp_2$ and $\hat{q}_2(p) = a - x - bp_2 + dp_1$. Note that lower $x$ need not reflect an inferior platform in a general sense; it is a platform that faces lower demand at given prices, but this may be accompanied by lower variable or fixed costs that make the entrant quite a viable competitor and a potential contributor to total welfare. Similarly, a lower cost need not make a firm a superior creator of value if it is accompanied by a demand disadvantage.

Given the results on equilibrium PMFN adoption above, we assume that the 2PMFN regime will prevail after entry. This is basically an assumption that the entrant either adopts a PMFN agreement along with the incumbent or is asymmetric enough that the fee-setting equilibrium behaves as if there are two PMFN agreements. It is evident in Figure 1 that if the nonadapting platform has a much lower best-response function, there will come to be an intersection of the 1PMFN best responses where both firms are on their 2PMFN portions of the best responses; in this case, the (incumbent’s) single PMFN agreement is binding because platform 2 is undercutting platform 1, and whether platform 1 (the entrant) in fact has adopted a PMFN policy is irrelevant. We present in Appendix D the expressions for the relevant optimal pricing rules, implied demand functions, and equilibrium fees for the asymmetric case.

5.1. The Effects on Implied Demand

The basic logic of this argument that PMFN agreements skew entry away from lower-cost, lower-value business models and toward higher-cost, higher-value business models can be seen directly from the implied demand functions. Again, the basic intuition is that a firm seeking to compete on the basis of low price (typically, a demand-disadvantaged or marginal-cost-advantaged firm) has a hard time competing when the possibility of undercutting the higher-value, or higher-cost incumbent is precluded.

For the case of $x$, this is evident in the implied demand functions if $\partial q_1^*/\partial x < \partial q_2^*/\partial x < 0$—that is, if increases in $x$ reduce demand more quickly in the presence of two PMFN agreements. This reflects the seller’s inability to discount the lower-value platform in order to attract customers to it. It is easy to check from the (linear) implied demand functions that this is true: $\partial q_1^*/\partial x = -\frac{1}{4} < -\frac{1}{2} = \partial q_2^*/\partial x < 0$.

For the case of $c_2 < c_1$, this is evident in the implied demand functions if $\partial q_{2}^{*}/\partial f_{1} < \partial q_{2}^{*}/\partial f_{2} < 0$—that is, if lowering its fees in response to its lower marginal cost has a smaller effect on a platform’s sales in the presence of two PMFN agreements. It is easy to check from the (linear) implied demand functions that this is true: $\partial q_{2}^{*}/\partial f_{1} = -b/2 < -(b - d)/4 = \partial q_{2}^{*}/\partial f_{2} < 0$.

Thus, with respect to choices in both willingness to pay and marginal cost, the entrant’s residual demand more quickly diminishes as its position deviates from
the incumbent’s (toward lower costs or lower value) when the incumbent has adopted a PMFN policy. In this sense, the incumbent’s PMFN policy can be said to skew incentives for choice of business model or inhibit entry of low-cost, low-value business models.

5.2. The Effects on Profits

Of course, a full analysis of the incentives for entry are more complex. The analysis thus far suggests that PMFN agreements may raise levels of profits, even as they increase the absolute value of the slope of profits in quality or costs (that is, making profits decrease more quickly as a platform becomes more downward differentiated). It seems entirely possible that the former effect might outweigh the latter, which would cause PMFN agreements to encourage the entry of competing platforms even as they skew incentives for competitive positioning. To make progress in understanding these competing effects, we need to characterize the relationship of profits to competitive position across regimes both with and without PMFN agreements. For tractability, we pursue this for the case of differentiated products \((x > 0)\) with no costs throughout the model \((c_1 = c_2 = c_3 = 0)\). We are interested in the entrant’s profits as a function of \(x\) and as a function of whether the incumbent has adopted a PMFN policy. Because we are interested in entry, we are interested in net profits, accounting for fixed entry costs, which we allow to vary with \(x\). The entrant will enter if

\[
Q_2^* (x) \geq k_2(x) \geq 0, \text{ where } j = \{0, 2\} \text{ indicates whether PMFN policies are adopted. (Recall that we assume that the outcome is as if the entrant follows suit if the incumbent has already adopted a PMFN policy.)}
\]

We can establish three facts about the relationship between \(Q_2^* (x)\) and \(Q_2^* (x)\), which form the basis for this analysis.

First, from the results proved thus far, we know that as \(x \to 0\) and \(d \to b\),\n\(\pi_2^* (x) > \pi_1^* (x)\). Second, for \(x\) not too large relative to \(a\) (specifically, \(x < 2a/7\)), both profit functions are downward sloping in \(x\) \((\partial \pi_j^* (x) / \partial x < 0, \text{ for } j = 0, 2\)). This follows from straightforward algebraic manipulation of the derivatives of \(Q_j^* (x)\) with respect to \(x\). This condition is the one that ensures negativity of \(\partial \pi_j^* (x) / \partial x\), which is the stronger of the two conditions. Third, for small \(x\), PMFN agreements make profits diminish more rapidly in the demand disadvantage \((\partial \pi_j^* (x) / \partial x < \partial \pi_1^* (x) / \partial x < 0)\). This is intuitive given the earlier result that PMFN agreements make implied demand decrease more rapidly in the demand disadvantage. This follows from the straightforward comparison of the derivatives of \(\pi_j^* (x)\).

5.3. The Effects on Entry When the Entrant’s Quality Is Exogenous

Together, these facts yield the scenario depicted in Figure 2.\(^8\) For small demand disadvantages, PMFN policies raise equilibrium postentry profits. However, be-

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\(^8\) It is easy to graph a numerical example corresponding to this scenario. For example, for \(a = 10, b = 4, d = 3\), and \(x \in [0, 1]\), the graph would look much like Figure 2, with a slight convexity to both profit curves and an intersection at about \(\frac{1}{2}\).
cause PMFN agreements also make profits more sensitive to the demand disadvantage, this relationship may reverse for large enough $x$. As the demand disadvantage increases, the presence of PMFN agreements causes the entrant’s profit to fall more quickly, which implies that the ordering of profits $\pi^*_2(x)$ and $\pi^*_1(x)$ may potentially reverse. As a result, whether the incumbent’s PMFN policy encourages or discourages entry depends on the exogenous demand disadvantage $x$ of the entrant and its associated fixed cost $k_2(x)$.

Figure 2 depicts the effect of PMFN agreements on entry for any pair of exogenous $x$ and $k_2(x)$. At the top, fixed entry costs are so high that the entrant does not enter regardless of whether the incumbent adopts a PMFN policy. At the bottom, fixed entry costs are so low that the entrant enters regardless of whether the incumbent adopts a PMFN policy. At left is a region in which the profit-increasing effects of PMFN agreements encourage the entry of the relatively similar entrant. To be clear, here the entrant would not enter absent a PMFN policy but does enter when the incumbent adopts a PMFN policy. At right is a region in which the augmentation of the demand disadvantage by the PMFN policy is so strong that it outweighs the profit-increasing effects of a PMFN policy, and entry of the more demand-disadvantaged entrant is deterred. Again, in this region the entrant would have entered absent the incumbent’s PMFN policy but is deterred by that policy. Figure 2 clearly demonstrates both the legitimacy and the limits to the conventional wisdom that PMFN agreements curtail entry by low-end platforms. The conventional wisdom applies in the shaded region, but only there, when the entrant contemplates entry with an exogenous competitive position. These arguments are summarized in the following proposition (which, in addition, relies only on continuity arguments):
Proposition 7. Assume that all costs are approximately 0 ($c_1, c_2, c_3 \approx 0$) and that a potential entrant has an exogenous differentiated position ($x > 0$). Then the incumbent’s adoption of a PMFN policy encourages entry (raises postentry profits relative to those that arise absent a PMFN policy) if the entrant is not too differentiated; if the policy discourages entry (lowers postentry profits relative to those that arise absent a PMFN policy), it is only for entrants with a sufficiently large difference in position.

5.4. The Effects on Entry When Entrant’s Quality Is Endogenous

We can also consider the effect on entry by an entrant that endogenously chooses its competitive position $x$, by evaluating $\pi_j^*(x) - k_j(x) \geq 0$ for an endogenously chosen $x_j^* = \arg \max_x \pi_j^*(x) - k_j(x)$. For $k_j(x)$ convex enough, the net profit will be concave for $j = \{0, 2\}$, and we maintain this assumption throughout this section. We also restrict $x$ to some compact interval $x \in X$. Because increases in $x$ correspond to lower quality, it is natural to model $k_2$ as decreasing. Convexity of $k_j(x)$ then implies that the largest cost savings come from the first departures from symmetry ($x = 0$), with these cost savings becoming smaller at the margin as the platform becomes more downward differentiated ($x$ increases). Note that the third fact above (that the slope of profit in $x$ is greater under PMFN policies) means that PMFN policies will bias the entrant’s optimal $x$ downward (toward more similar platforms). This is most easily seen by considering the fact that the first-order condition under PMFN policies at the no-PMFN optimal $x$ must be negative. As a result, if there is an interior optimal $x$ under either regime, then $x_j^* < x_2^*$ (that is, regardless of whether the other regime has an interior or corner optimum).

Proposition 8. Assume that a potential entrant chooses its position $x \in X$ after observing the incumbent’s PMFN adoption decision and that the entrant’s optimal $x$ is interior to $X$ either with or without PMFN policies (or both). Then if entry occurs regardless of PMFN adoption, the entrant chooses a less differentiated position (strictly smaller $x$) when the incumbent adopts a PMFN policy.

Whether entry is encouraged or deterred because of the incumbent’s PMFN policy now rests on the profit that is obtainable by the entrant at its optimal competitive position, which may vary with the incumbent’s PMFN decision. We must separately determine $x_j^*$ and then evaluate $\pi_j^*(x_j^*) - k_j(x_j^*)$ for each $j$.

Two possibilities arise. It may be that the optimized net profit $\pi_j^*(x_j^*) - k_j(x_j^*)$ is higher under 0PMFN or 2PMFN regimes. When it is higher under a 0PMFN regime, the incumbent’s PMFN policy may deter entry, in the sense that it is reducing the maximal profit available to the entrant. When it is higher under a 2PMFN regime, the incumbent’s PMFN policy may encourage entry, in the sense that it is increasing the maximal profit available to the entrant. Given the analysis of the case with exogenous $x$, it seems natural that the former (entry-deterring) scenario is more likely when the optimal $x$ absent PMFN agreements is high,
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which will be the case when cost savings associated with higher $x$ are significant.
Similarly, the latter (entry-encouraging) scenario is more likely when the optimal $x$ absent PMFN agreements is low, as when cost savings are relatively small.

It is possible to use numerical examples to illustrate these possibilities. For simplicity, assume that $k(x) = F - w\sqrt{x}$, which is convex. Fixing $w$, we can then find the optimal $x$ (which will not depend on the fixed component of cost $F$) and the profits at that optimal $x$, net of all costs except $F$. This then yields the threshold $F_j$ at which entry is realized under the various scenarios. Comparison of this $F_j$ under the 0PMFN and 2PMFN scenarios then determines whether entry is encouraged or discouraged (or unaffected) by the incumbent’s PMFN policy. In both of the following examples, $a = 10$, $b = 4$, $d = 3$, and $x \in [0, 1]$.

First consider a case in which cost savings are significant enough to create an interior $x^*_2$ but still relatively small: $w = 1$. Here, $x^*_0 = .2$ and $x^*_2 = 0$. The threshold fixed costs are $F_0 = 8.2$ and $F_2 = 11.1$. Thus, for low fixed costs ($F < 8.2$), there is entry regardless of the incumbent’s adoption of a PMFN policy, and the chosen $x$ is reduced by the incumbent’s PMFN policy. For intermediate entry costs ($F \in (8.2, 11.1)$), entry occurs only if the entrant incumbent adopts a PMFN policy. For high entry costs ($F > 11.1$), there is no entry regardless of the incumbent’s PMFN decision about adoption.

Consider a case with more significant cost savings: $w = 7$. Now $x^*_0 = 1.0$ and $x^*_2 = .25$. The threshold fixed costs are $F_0 = 13.85$ and $F_2 = 12.75$. For low entry costs ($F < 12.75$), there is entry regardless of the incumbent’s adoption of a PMFN policy, and the chosen $x$ is reduced by the incumbent’s PMFN policy. For intermediate entry costs ($F \in (12.75, 13.85)$), entry occurs only if the entrant incumbent does not adopt a PMFN policy. For high entry costs ($F > 13.85$), there is no entry regardless of the incumbent’s PMFN adoption decision. This case is depicted in Figure 3. When $F$ is in region A, the incumbent’s PMFN has no effect; when $F$ is in region B, the incumbent’s PMFN deters entry; and when $F$ is in region C, the incumbent’s PMFN does not deter entry but does distort the entrant’s position.

This case, in which cost savings are sufficiently high that an entrant would choose a position that is substantially different from the incumbent’s, absent PMFN policies, illustrates precisely the conventional wisdom. Here, for low fixed costs there is entry regardless of the PMFN policy, but the presence of the policy distorts the entrant’s choice of position and leads the entrant to choose a less differentiated and higher-end business model. For intermediate fixed costs, the PMFN policy deters entry that would have occurred absent the policy, because the entrant would have maximized its profits by choosing a very differentiated position that is penalized too heavily by the PMFN policy. This illuminates the potential for both deterrence of entry of firms with low-cost business models and the distortion of choice of business model when entry does occur.

Comparing this case with the prior case, in which cost savings were more modest, also demonstrates the limitations of the conventional wisdom. When an entrant would not choose a very differentiated position absent the incumbent’s
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PMFN policy, skewing that position by the PMFN policy is unlikely to deter entry; in fact, it is quite possible that the price-raising effects of the PMFN will encourage entry that would not have occurred absent the PMFN policy. Obviously, a full analysis of whether the encouragement, deterrence, or skewing of entry increases or decreases social welfare requires much more structure regarding both demand and costs and is beyond the scope of this paper.

6. Conclusion

We study the effects on pricing and entry of PMFN policies—a type of policy not widely studied in the extant literature but one that is of increasing interest and importance in antitrust enforcement. We show that PMFN agreements tend to raise fees charged by platforms and prices charged by sellers and that these policies are adopted in equilibrium and increase platforms’ profits when aggregate demand is sufficiently inelastic. However, in other cases PMFNs may raise prices so high that industry profits fall. We also show that the adoption of a PMFN agreement by an incumbent platform can discourage entry by another platform if it is sufficiently downward differentiated; however, when the potential entrant has a business model that is relatively similar to the incumbent’s, PMFNs work to encourage entry through their price-raising effects. Moreover, when entry occurs regardless of the incumbent’s adoption of a PMFN policy, PMFNs have the effect of distorting the entrant’s choice of business model toward one more similar to that of the incumbent. These results have important implications for ongoing antitrust scrutiny of these policies in e-book markets, travel websites, and other online marketplaces.

The results for fees and prices imply that competition among platforms is softened and that fees may increase relative to those in a scenario with more un-
restricted competition, with a similar effect on the sellers’ prices. To our knowledge there is no empirical work assessing this effect, and there are obvious difficulties in formulating identification strategies that would be capable of isolating the effect of a PMFN policy from other effects at work in these markets. However, one can also see that these policies, especially in the context of escalating fees, imply a cross subsidization from consumers using high-cost or high-fee platforms to consumers using low-cost or low-fee platforms. For example, a website that offers a sophisticated interface with product reviews, recommendations, a system for users’ comments, and a large inventory for fast shipping might have a higher-cost position relative to a no-frills website or other low-cost sales channel. An interesting particular example of a lower-cost sales channel is the direct phone or Internet sales offering of the manufacturer or seller. The PMFN policy of the higher-cost website requires a common price across outlets, which implies that the seller cannot sell directly at a lower price, even though this direct sales channel might have much lower costs. As a result, consumers who would be happy to utilize the lower-cost sales channel pay the same price as those who utilize the higher-cost channel, which implies a sort of cross subsidization of the high-amenities consumer by the low-amenities consumer. To the extent that the consumer requiring a lower level of amenities is likely to be a lower-income consumer, this represents a kind of regressive income redistribution that might be especially problematic from a policy perspective.

In addition to this redistributive effect of the common price given a certain set of choices, there is also the possibility that PMFN policies limit choice by either deterring entry of low-cost platforms or skewing entry of platforms away from lower-end business models. As with the cross subsidy, this seems likely to negatively affect especially those with less willingness to pay for the product (or at least for the amenities associated with the platform). The effect we describe here is consistent with the relative paucity of no-frills alternatives to conventional travel websites or e-book stores. To the extent that there are customers who would prefer to save money by buying through a lower-cost platform with fewer amenities, PMFN agreements may be limiting options targeting those consumers, driving them in effect to purchase bundled platform amenities that they do not value. This has the effect of creating another kind of cross subsidy from low-valuation, low-income to high-valuation, high-income consumers.

Appendix A

Results When Platforms May Offer Rebates

Here we allow for the platforms to rebate to consumers a portion of the fees collected from sellers. We amend the seller’s demand function to account for these rebates: a rebate from a platform effectively lowers the seller’s price on that platform by an equivalent amount. Let $r_i$ and $r_j$ be the per-unit rebates offered by platforms $i$, $j$, which we assume are chosen at the same time that the platforms choose their fees $f_i$ and $f_j$. Demand for the seller’s product through platform
i is therefore \( \hat{\eta}_i(p, r) = a - b(p_i - r_i) + d(p_j - r_j) \). With consumer rebates, the profit function of platform \( i \) becomes \( \pi_i(f_i, r_i) = [f_i - r_i - c_i] \hat{\eta}_i(p, r) \).

We reproduce the closed-form expressions for the seller’s optimal pricing rule, implied demand function, and platform profits under either type of PMFN regime, with rebates. Under a 0PMFN regime, these are

\[
p_i^0 = \frac{a + (b - d)(c_i + f_i + r_i)}{2(b - d)},
\]

\[
\hat{q}_i^0 = \frac{1}{2}[a - b(f_i - r_i + c_i) + d(f_j - r_j + c_j)],
\]

and

\[
\pi_i^0(f_i, r_i) = \frac{1}{2}(f_i - r_i - c_i)[a - b(f_i - r_i + c_i) + d(f_j - r_j + c_j)].
\]

Under a 2PMFN regime, these are

\[
p_i^2 = \frac{2a + (b - d)(2c_i + f_i + f_j + r_i + r_j)}{4(b - d)},
\]

\[
\hat{q}_i^2 = \frac{2a - (b - d)(2c_i + f_i + f_j) + (3b + d)r_i - (3d + b)r_j}{4},
\]

and

\[
\pi_i^2(f_i, r_i) = \frac{1}{4}(f_i - r_i - c_i)[2a - b(2c_i + f_i + f_j - 3r_i + r_j) + d(2c_i + f_i + f_j - 3r_i)].
\]

It is useful to begin by considering exogenous rebates. Under either a 0PMFN or 2PMFN regime, the rebate terms in the seller’s optimal price and implied demand functions can be rewritten as absorbed into the intercept term \( a \), and it can be easily shown that platforms view own fees and rebates as strategic complements under either PMFN regime. Since our main results relate to the special case of \( r_i = r_j = 0 \), this suggests that in any equilibrium under either PMFN regime with strictly positive rebates, fees will always be higher than when rebates are not used. Considering a case in which the exogenous rebate is the same for both platforms, \( r_i = r_j = r \), it is straightforward to show that \( \partial p_i^0 / \partial r = \partial p_i^2 / \partial r = \frac{1}{4} \). That is, prices increase faster than rebates in both PMFN regimes, which implies that net prices rise. Moreover, the equality of these expressions implies that exogenous rebates will not reverse the ranking of prices; it will remain that \( p_i^2 > p_i^0 \) for any exogenous rebate \( r \).

When rebates are endogenously determined by platforms, there exist multiple (fee, rebate) equilibria under a 0PMFN regime, and there does not exist an equilibrium under a 2PMFN regime. Under a 0PMFN regime, it can be seen directly
from the platform’s profit function that the relevant term for profit maximization is the difference $f_i - r_i$; given that the platform chooses this difference to maximize profit, any (fee, rebate) pair that satisfies that difference can exist in equilibrium. Under a 2PMFN regime, an equilibrium does not exist because any increase in rebates leads to a greater increase in fees, which in turn yields another increase in rebates, and so on. This follows from fees and rebates being strategic complements and not satisfying a stability requirement: each platform’s optimal fee and rebate reaction functions do not cross in (fee, rebate) space. While this produces no equilibrium, it does confirm that the local effect of introducing PMFNs in the presence of rebates is to raise prices and to raise prices by more than would be the case absent rebates.

These results suggest that platforms with PMFNs again have an incentive to increase fees beyond the collusive level, as in the model without rebates. Moreover, the platforms have an incentive to raise fees faster than they raise rebates under a 2PMFN regime. This suggests that the basic economic logic of how platform fees can exceed the collusive level persists with endogenous rebates and, if anything, is strengthened: the ability to raise fees without suffering the full consequence of the implied reduction in quantity sold leads platforms to set fees higher than those they would choose collusively.

Appendix B

Results When the Seller May Sell through a Single Platform

Here we allow the seller to choose to sell its product through only one platform. We add a stage to the sequential game, which results in modified timing as follows: platforms simultaneously choose whether to adopt PMFN policies, the seller decides whether to work with one or both platforms, the monopoly platform sets its fee or the two platforms simultaneously set fees, and finally the seller sets prices. This captures the idea that if the seller anticipates extraordinarily high fees because of a 2PMFN regime unfolding, it may choose instead to do business with a single platform, which renders the PMFN policies moot.

To evaluate whether the seller prefers to operate through a single platform or participate in a 2PMFN regime requires comparing its profits under a monopoly platform to those under a 2PMFN regime. This in turn requires modeling demand for a monopoly platform, which is not incorporated in an obvious way in our demand model. We model a monopoly platform as being the equivalent of a combination of the two platforms (that is, summing their demand) but suffering a quantity penalty $z$ because some consumers who would have preferred that the now-excluded platform exit the market rather than purchase through the alternative platform. For example, imagine that two e-book platforms differ in support for different technologies or in the style and function of the user interface, which yields differentiated demand. Our assumption here is that if a publisher chooses to sell through only one of these platforms, the chosen platform is unable to perfectly replace the now-excluded platform, and some of those custom-
ers who would have been reached by the now-excluded platform instead choose the outside option.

In this environment, the profits of monopoly platform 1 as a function of the seller’s price are \( \pi_1 = (f'_1 - c_i)(a - bp + dp - z) + (f'_1 - c_i)(a - bp + dp - z) \). The seller now pays a fee set by a monopolist platform, which is by no means certain to be lower than the equilibrium fees chosen by competing platforms. This platform-profit-maximizing fee can be derived in the usual way and substituted into the demand function to derive the seller’s profit through a monopoly platform with penalty \( z \). The seller’s maximized profit in the one-seller subgame is 
\[
\pi^*_1 = [a - (b - d)(c_i + c_j) - z]^2/(b - d).
\]

We must compare this with the seller’s profit under the 2PMFN fees with two platforms, which is given by 
\[
\pi^*_2 = [(b - d)(c_i + c_j + 2c_c) - 2a]^2/72(b - d).
\]

For simplicity, we evaluate the relationship of these profits when costs are 0: \( c_i = c_j = c_c = 0 \). For these parameter values, and with the penalty term \( z \) set to 0, the profit comparison shows that in fact the seller does prefer to work with a single platform rather than suffer the consequence of the high 2PMFN fees.

This in itself is a very interesting result: PMFNs put enough upward pressure on fees that the seller would prefer to pay a monopoly fee to a single (combined) platform than enjoy the benefits of competitive fee setting by multiple platforms. From this comparison, we can also determine that for larger demand penalties, in particular for \( z > a/3 \), the seller prefers to sell through both platforms, even though it anticipates that the 2PMFN regime raises fees above the collusive level. Thus, the spirit of our main results holds (that PMFN policies raise fees and prices) as long as there is a sufficient demand penalty to dealing with a single seller.

### Appendix C

The Profits of Platform 2 in the Mixed-Strategy Equilibrium

With linear demand, the profits of platform 2 at the 2MFN fee equilibrium are
\[
\pi^*_2 = \pi_2^*(f'_2, f'_2) = \frac{1}{36(b - d)}[2a - (b - d)(c_i + c_j + 2c_c)],
\]
and its profits in the 1MFN mixed-strategy fee equilibrium when undercut by platform 1 are
\[
\pi^*_2[b_2^*(\hat{f}_2), \hat{f}_2] = \frac{1}{2}(\hat{f}_2 - c_i)[a + dc_e - b(\hat{f}_2 + c_j)] + \frac{1}{2b}[d[a + b(c_i - c_j) + d(\hat{f}_2 + c_j)]],
\]
where \( \hat{f}_2 \) is given by
\[
\hat{f}_2 = \frac{b - d}{b^2 - 3bd + 2d^2}[2a - (b - d)c_j - bc_i] \pm \sqrt{5\sqrt{b[a - (c_i + c_j)(b - d)^2/(b - d)}}}.
\]
Most-Favored-Nation Clauses

Substituting \( f_2 \) into \( \pi_2^o \) and defining \( Z = \pi_2^o(f^2_1, f^2_2) - \pi_2^o[b^o(f^2_1, f^2_2)] \), any parameter values for which \( Z \geq 0 \) support PMFN policy adoption by platform 2. It is helpful to define \( h = d/b \in (0, 1) \); by substituting \( d = hb \), this simplifies the expressions greatly. Under the maintained assumption of symmetry (\( c_1 = c_2 \)), it can be shown that \( \text{sign}(Z) = \text{sign}[2(38 + h - 28h^2) + 9\sqrt{2} - 2h(-6 - 3h + 2h^3)] \). For any \( h \), \( (38 + h - 28h^2) > 0 \) and \( (-6 - 3h + 2h^3) < 0 \). Therefore, the negative root guarantees that \( Z \geq 0 \). For the positive root, it can be shown that \( Z > 0 \) for \( h \) larger than the value of a complex expression that can be shown numerically to be approximately .303.

Appendix D

The Asymmetric Linear Case

In the asymmetric linear case, we can derive closed-form expressions for the optimal pricing rule, implied demand function, and equilibrium fees under each regime. These are given by

\[
    p^i_1(f) = \frac{a(b + d) + b^2(c_s + f_i) - d[d(c_s + f_i) + x]}{2(b^2 - d^2)},
\]

\[
    p^i_2(f) = \frac{a(b + d) + b^2(c_s + f_i) - d^2(c_s + f_i) - bx}{2(b^2 - d^2)},
\]

\[
    p^i_1(f) = \frac{2a + (b - d)(2c_s + f_i + f_j) - x}{4(b - d)},
\]

\[
    q^i_1(f) = \frac{1}{2}[a - b(c_s + f_i) + d(c_s + f_i)],
\]

\[
    q^i_2(f) = \frac{1}{2}[a - b(c_s + f_i) + d(c_s + f_i) - x],
\]

\[
    q^i_1(f) = \frac{1}{4}[2a + 2dc_s + d(f_i + f_j) - b(2c_s + f_i + f_j) + x],
\]

\[
    q^i_2(f) = \frac{1}{4}[2a + 2dc_s + d(f_i + f_j) - b(2c_s + f_i + f_j) - 3x],
\]

\[
    f^i_1 = \frac{2b^2(c_s - c_i) + bd(c_s + c_i) + a(2b + d) + d(d(c_s - x))}{4b^2 - d^2},
\]

\[
    f^i_2 = \frac{2b^2(c_s - c_i) + d^2c_s + a(2b + d) + b[d(c_s + c_i) - 2x]}{4b^2 - d^2}.
\]
\[ f_1^{2*} = \frac{2a + (b - d)(2c_1 - c_2 - 2c_3) + 5x}{3(b - d)}, \]

and

\[ f_2^{2*} = \frac{2a + (b - d)(2c_2 - c_1 - 2c_3) - 7x}{3(b - d)}. \]

References


