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Guided Wave Monitoring of Prestressing Tendons

A thesis submitted in partial satisfaction of the requirements
for the degree Master of Science

in

Structural Engineering

by

Claudio Nucera

Committee in Charge:
Professor Francesco Lanza di Scalea, Chair
Professor Hyonny Kim
Professor P. Benson Shing

2010
The Thesis of Claudio Nucera is approved and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2010
DEDICATION

To my family Salvatore, Giuseppina e Michelangelo, and my fiancée Annalisa for their constant love and encouragement.
Engineering problems are under-defined, there are many solutions, good, bad and indifferent. The art is to arrive at a good solution. This is creative activity, involving imagination, intuition and deliberate choice.

Ove Arup
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Lastly, I must thank my loving family Salvatore, Giuseppina, Michelangelo and my fiancée Annalisa for their unwavering support and confidence in me.
Monitoring load levels in multi-wire steel strands is crucial to ensuring the proper structural performance of post-tensioned concrete structures, suspension and cable-stayed bridges. The post-tensioned box-girder structural scheme is widely used in several bridges, including 90% of the California inventory. Prestressing tendons are the main load-carrying components. Therefore loss of prestress as well as the presence of structural defects (e.g. corrosion and broken wires) affecting these elements are critical for the performance of the entire structure and may conduct to catastrophic failures. Unfortunately, at present there is no well-established methodology for the monitoring of prestressing (PS) tendons able to provide simultaneous and continuous information about the presence of defects as well as prestress levels.

In this thesis a methodology to assess the level of load applied to the strands is developed through the use of ultrasonic nonlinearity. Since an axial load on a multi-wire strand generates
proportional contact stresses between adjacent wires, ultrasonic nonlinearity from the inter-wire
contact must be related to the level of axial load. The work presented shows that the higher-
harmonic generation of ultrasonic guided waves propagating in individual wires of the strand
varies monotonically with the applied load, with smaller higher-harmonic amplitudes with
increasing load levels. This trend is consistent with previous studies on higher-harmonic
generation from ultrasonic plane waves incident on a contact interface under a changing contact
pressure. The thesis presents the results of experimental researches on free strands and embedded
strands. Finally numerical studies (nonlinear Finite Element Analysis) on free strands are
presented.
Chapter 1

Introduction

1.1 Ultrasonic Waves in Nondestructive Evaluation

For over sixty years, the Nondestructive Evaluation (NDE) of materials has been an area of continued growth. The need for NDE has increased dramatically in recent years for various reasons such as product safety, in-line diagnostics, quality control, health monitoring, security testing, etc. Today, in fact, the ultrasonic signal is being used for predicting material behavior, characterizing (detecting internal anomalies in) a variety of engineering structures, as well as for inspecting human body parts like tumors, bones, and unborn fetuses. Because of the ever-increasing popularity of the ultrasonic techniques in a wide range of applications, this technology has received a lot of attention from the research community.

Besides the practical demands, the progress in NDE has a lot to do with its interdisciplinary nature. NDE is an area closely linked to aerospace engineering, civil engineering, electrical engineering, material science and engineering, mechanical engineering, nuclear engineering, petroleum engineering and physics among others. There are at least two dozen NDE methods in use. In fact any sensor that can examine the inside of material nondestructively is useful for NDE. However the ultrasonic methods are still most popular because of its capability, flexibility, and relative cost effectiveness.

In addition to this, although ultrasonic techniques and theory can be quite complex, the basic concepts behind ultrasonic NDE are simple. Ultrasonic waves (sound waves vibrating at a
frequency too high to hear) can propagate in solids, i.e. travel through them (of course, such waves can also travel through liquids or air). As the waves travel, they interact with solids in ways that we can predict and represent mathematically. Armed with this kind of understanding, we can create our own ultrasonic waves with a transducer, and thereby use ultrasonics as a way of finding out the nature of a solid material (its thickness, its flaws, its elasticity, and more).

### 1.1.1 Applications

Ultrasonic NDE has nearly innumerable applications in the aircraft, piping, semiconductor, fabrication, railroad, power, and medical industries. For example, in the transportation industry (where a catastrophic failure can lead to the deaths of hundreds of people), ultrasonic NDE is used to detect cracks and fatigue damage on safety-critical parts. The rotors of military helicopters, which used to be replaced at regular intervals whether they needed it or not, can now be tested with ultrasonics and replaced only when necessary (retirement for cause), saving both time and money. Ultrasonics can also detect ice formation on aircraft wings, to determine if deicing is necessary before takeoff or in midair. In addition to detecting flaws and determining properties in materials, ultrasonics can also be used for imaging. In medicine, ultrasonic imaging methods, such as the B-scan and tomography, help evaluate fetal development and allow diagnostic imaging of soft body tissue. Ultrasonic imaging is also used during surgery to allow for less invasive surgical techniques (by determining the best place to cut). These imaging techniques are equally widespread in the other industries where NDE is used.

### 1.2 Ultrasonic Waves - General Aspects

Ultrasonic waves are high ("ultra") frequency sound ("sonic") waves: they vibrate at a frequency above 20000 vibrations per second, or 20000 Hertz (Hz) - too fast to be audible to
humans. A human ear can detect frequencies between about 20 Hz and 17 kHz. Any sound at a higher frequency is ultrasonic and then inaudible. In NDE applications, ultrasonic frequencies typically range from the tens of kilohertz (long ranges and/or highly damped materials) to a few Megahertz (short ranges and/or lightly damped materials).

Sound waves can propagate in air and in fluids and solids too. In particular, waves travel at higher velocity, and with lower attenuation (loss of energy), in fluids and solids than in air. This behavior in sound waves - faster in solids, slower in air - is the opposite of what we know about electromagnetic (light) waves. Electromagnetic waves propagate optimally in a vacuum and minimally in solids. Sound waves, on the other hand, cannot propagate in a vacuum at all. Ultrasonic wave propagation in solids is the fundamental phenomenon underlying ultrasonic NDE. We can use the features of an ultrasonic wave (velocity, attenuation) to characterize a material’s composition, structure, elastic properties, density, and geometry.

Furthermore, it is possible to use ultrasonics to detect and describe flaws in a material. Flaws cause scattering of ultrasonic waves, in the same way a stationary rock reflects a water wave (Figure 1.1). This scattering can be detected as an echo. Using the properties of the echo, we can determine the position, size, and shape of a flaw. In other words, thanks to ultrasonic detection, we know not only that a flaw exists, but also the severity of the damage.

Figure 1.1 - Ultrasonic reflection off of a circular defect [1].
1.2.1 Wave Types

Elastic waves in all frequency ranges – ultrasonic, sonic and subsonic – can be classified under two groups: body waves or bulk waves and surface waves or guided waves. Body waves propagate through a bulk material, while the surface waves propagate along the surface of a body as shown in Figure 1.2. Surface waves are often called guided waves because the boundary of the body guides them.

![Figure 1.2 - Body waves and surface waves generated by an ultrasonic source [2].](image)

Depending on the application each type of wave is preferred. In particular bulk waves are used for through-thickness inspections such as impact-echo or ultrasonic tomography techniques. Guided waves are instead used for long-range monitoring of structural components with waveguide geometries including plates, cables, pipes, etc. More details on these topics can be found in [1].

1.2.1.1 Bulk Waves: Longitudinal and Shear Waves

Bulk waves are of two types: *Longitudinal Waves* and *Shear Waves*. Only normal stress is generated in the medium when longitudinal waves propagate through an infinite medium, while
only shear stresses are generated by the shear waves. Longitudinal waves are also known as compressional waves or extensional waves since the material goes through compression and extension when the wave passes through it. In the longitudinal wave mode, particles move parallel to the direction of wave propagation while in the transverse wave mode, particles move perpendicular to the same direction.

Because the transverse particle motion has an associated shear stress, low-viscosity fluids (such as water or air), cannot support this type of waves since they are not able to support shear stresses. In terms of the elastic constants, the wave speeds of these two types of waves are given by:

\[ c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \]  

(1.1)

\[ c_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2\rho(1+\nu)}} \]  

(1.2)

where \( c_p \) and \( c_s \) are longitudinal and shear wave speeds, respectively, \( \rho \) is the density, \( \lambda \) is Lame’s first constant, \( \mu \) is the shear modulus or Lame’s second constant, \( E \) is the Young’s modulus and \( \nu \) is the Poisson’s ratio.

It is important to note that \( c_p \) is always greater than \( c_s \). During an earthquake both longitudinal and shear waves are generated at the same time by the earthquake event (dislocation) at the hypocenter. However, since the longitudinal wave propagates at a faster speed it strikes the ground surface first; then the shear wave arrives. For this reason longitudinal waves are also called primary waves or P-waves and the shear waves are called secondary waves or S-waves.

In the most used materials:

\[ c_s \approx 0.5c_p \]  

(1.3)

Displacement deformations for typical longitudinal and transverse waves are shown in Figure 1.3:
1.2.1.2 Guided Waves

The term guided wave is used to describe waves that require boundaries for their existence. Guided waves can travel on the surface of semi-infinite solids (Rayleigh waves), on the interface between two different media (Stoneley waves), along plates or layers of plates, in generic cross section beams, axial symmetric rods and cylinders. All the described structural components are commonly referred to as waveguides. They are uniform in one direction, along the longitudinal axis of the waveguide, therefore their cross-section has the same physical and geometric properties at all points along the waveguide’s axis. Ultrasonic guided waves are generated by the constructive interference of longitudinal and shear bulk waves (Figure 1.4).

The bulk stress waves, generated by a generic transducer, interact with the boundaries of the waveguide. Multiple reflections and mode conversions take place until their superposition form wave packets i.e. ultrasonic guided waves. The groundwork for studying guided waves in cylindrical and flat waveguides was laid at the end of the 19th century. Pochhammer (1876) and later Chree (1889) developed the solution of mechanical wave equation for the case of cylindrical symmetry, however, owing to its complexity, detailed calculation of the roots did not appear until the middle of the 20th century. In the same period, Gazis (1959) explored the propagation phenomena for hollow, single layer, elastic circular cylinders in vacuum. Papers on this subject
are numerous. Guided waves in cylindrical elastic waveguides have been studied theoretically, numerically and also experimentally. For instance Zemanek (1972) was one of the first authors to present a complete analytical and experimental study.

The basis for the comprehension of guided waves in flat layered waveguides was principally given by Rayleigh (1887) and Lamb (1917). Lord Rayleigh derived the equation for waves traveling along the free surface of a semi-infinite elastic half space. Its derivation yields a third order expression whose roots determine the velocity of the propagating surface wave. Stoneley (1924) carried out a generalization of the above single interface problem. He performed a study of interface waves propagating without leakage that exist at the boundary between two solid half spaces. The ranges of existence of free wave solutions (in which the wave propagates indefinitely without loss of energy) were explored by Scholte (1947). Pilant (1972) extended the study of Stoneley and examined the leaky wave solutions representing waves that attenuate as they travel. Love (1911) and Lamb (1917) added another interface to the problem studied by Rayleigh and introduced the notion of flat layer of finite thickness. The derivation by Lamb (1917) consists of two distinct expressions (Rayleigh-Lamb equations) which roots represent

![Guided plate waves in an isotropic homogeneous plate.](image)

**Figure 1.4** - Guided plate waves in an isotropic homogeneous plate.
symmetric and antisymmetric plate modes (see section 2.3). A plot of the roots in the frequency domain gives the well known Lamb wave dispersion curves.

Love (1911) showed that transverse modes were also possible in a half-space covered by a layer of finite thickness and different elastic properties. His modes involve shearing motion in the plane of the layer. Details on the calculation and theory of guided waves in isotropic media can be found in classical text books as Viktorov (1967), Auld (1973), Achenbach (1984), Graff (1991) and Rose (1999).

1.3 Advantages and Disadvantages

Ultrasonic NDE is a very flexible and robust technique, with applications in a wide range of industries. Only two NDE methods can reveal substantial subsurface flaws in materials: ultrasonic and X-ray methods (Other techniques, such as magnetic particle and eddy current, can detect some subsurface features, but only near the surface of the material). Unlike X-ray techniques, however, ultrasonic methods pose no environmental or health risks. In addition, ultrasonic methods offer contacting as well as noncontacting approaches. (Laser generated-detected ultrasound allows standoff distances of up to several meters).

Ultrasonic probes can also be designed to test complex geometries. Finally, ultrasonic detection can be used for all types of materials from biological to metal to ceramics. But ultrasonic techniques do have some disadvantages. First, they require a highly experienced technician. In addition, although noncontacting methods exist, in the majority of cases the transducer must be in contact with the object, through a water- or gel-coupling layer. Also, ultrasonic waves typically cannot reveal planar flaws (cracks) whose length lies parallel to the direction of wave travel. Finally, ultrasonic methods can be expensive to operate. Table 1.1 depicts a review of advantages and disadvantages
1.4 Research Motivation

Multi-wire steel strands are widely used in civil engineering as the tensioning components of prestressed concrete structures and in cable systems of cable-stayed and suspension bridges. In California, for instance, 90% of the inventory is post-tensioned box-girders concrete structures. Prestressing tendons are the main load-carrying components of these and other post-tensioned structures. The loss of prestress as well as the presence of defects or the tendon breakage can be catastrophic for the entire structure. Some of the documented collapses of post-tensioned structures caused by tendon failures have occurred in Wales [4], Palau [5], North Carolina [6], and Berlin [7]. Monitoring of the applied loads and assessment of the structural integrity of strands are engineering tasks that remain a challenge in the NDE and SHM communities.

Many techniques have been applied to the defect detection and load monitoring of prestressing tendons in prestressed concrete structures but, despite their criticality, much research is needed to develop and deploy techniques able to provide real-time information on the level of prestress in order to detect dangerous stress losses. In this context Guided Ultrasonic Waves (GUWs) Testing represents definitely a technique that shows promises for the simultaneous
detection of defects and monitoring of prestress levels in PS tendons. GUWs propagate along the tendon itself by exploiting its waveguide geometry. As better described in next Section 2.10.4, the advantages of this technique include the possibility of using transducers permanently attached to the strand for continuous structural monitoring, providing simultaneous defect detection and stress monitoring capabilities for the strands with the same sensing system.

Recent applications of the GUW technique were demonstrated for the evaluation of stress levels in post-tensioning rods and multi-wire strands ([8], [9] and [10]), as well as for the detection of isolated defects in these components ([11] and [12]). In particular, the present research focuses on experimental and numerical analyses based on nonlinear ultrasonic guided waves in the 100 kHz – 1 MHz range for monitoring prestress levels in seven-wire PS tendons. The technique relies on the fact that an axial stress on the tendon generates a proportional radial stress between adjacent wires (interwire stress). In turn, the interwire stress modulates nonlinear effects in ultrasonic wave propagation through both the presence of finite strains and the interwire contact.

The nonlinear ultrasonic behavior of the tendon under changing levels of prestress is monitored by tracking higher-order harmonics at \( n\omega \) arising under a fundamental guided-wave excitation at \( \omega \). Theoretical, experimental and numerical studies will be presented to identify ranges of fundamental excitations at \( \omega \) producing maximum nonlinear response, wave modes (axial, flexural or torsional) which are most sensitive to prestress levels, and optimum lay-out of the transmitting and the receiving transducers within the test tendons in order to monitor prestress level in strands. Compared to alternative methods based on linear ultrasonic features, the proposed nonlinear ultrasonic technique appears more sensitive to prestress levels and more robust against changing excitation power at the transmitting transducer or changing transducer/tendon bond conditions.
1.5 Outline of the Thesis

The thesis is structured as follows. In Chapter 2 a review of existing methods for health monitoring of concrete structures (PS tendons in particular) is presented.

In Chapter 3 the theory of wave propagation in rods and the governing equations for this problem are described. In this context longitudinal, torsional and flexural waves are considered.

Chapter 4 shows the analytical approach to describe the response of a strand to axial and torsional displacements. Firstly explicit expressions are derived for the determination of axial force, bending and twisting moments in the helical wires. Then some synthetic tables concerning the stress and strain distribution as well as resultant forces on the strand cross section in terms of forces acting on the cross section of each of the seven wires are illustrated. Later in the chapter the attention is focused on Hertz theory of contact interaction and on some numerical results developed using a commercial finite element code (ABAQUS) confirming this theoretical approach.

Chapter 5 begins presenting the main aspects of nonlinear structural systems and the principal causes for the onset of nonlinearities. The attention is then focused on nonlinearities in guided ultrasonic waves and on the generation of higher-order harmonics. It is also derived the expression of the nonlinear parameter $\beta$ and it is showed the potential in using this parameter in order to monitor applied loads in steel strands.

Chapter 6 describes the experimental results achieved from the tests performed at UCSD NDE/SHM Lab and at UCSD Powell Lab. The goal is the development of a method able principally to monitor applied loads (and defects as well) in multi-wire steel strands typically used in suspended bridges and in prestressed concrete structures.

Chapter 7 is dedicated to the description of developer numerical analyses and obtained results. ABAQUS was used in order to numerically simulate wave propagation in seven-wire
steel strand and, especially, analyze nonlinear characteristics of the response mostly related to contact and friction phenomena. Explicit dynamic analysis was used.

Chapter 8 presents the conclusions of the thesis and recommendations for future areas of research.
Chapter 2

Literature Review on Health Monitoring of Prestressing Cables

2.1 Visual Inspection

The Visual Inspection technique is the simplest and oldest form of inspection that can provide information on the state of degradation of post-tensioned structures. This is often the only method available to bridge engineers [13]. Even though routine inspections are typically completed using only the visual inspection technique, they rely heavily on the subjective judgments of bridge inspectors [14]. Furthermore, the infrequent collection of data makes the establishment of trends very difficult. Finally, the visual method is only efficient when the degradation is visible at the surface of the structure; this may be too late for a cost-effective repair strategy.

2.2 Radiography, Radioscopy and Computed Tomography

Radiography is a relatively common technique to image the interior of objects, and it is used for both medical and industrial applications. The technique generally uses X-rays discovered by Rontgen in 1895. X-rays are a form of electromagnetic waves with a wavelength in the range of 10 to 0.01 nanometers, corresponding to frequencies in the range 30 to 30000 PHz (1015
Gamma rays can also be used in radiography by means of radioactive isotopes as radiation sources. A radiographic imaging system consists of three main components: the radiation source, the object to image and the detector, usually a photographic film (see Figure 2.1). A radioactive source generates the X-rays which penetrate the object to be inspected. Thicker and denser areas of the object will absorb more of the radiation, resulting in a decreased exposure of the detector film (white).

Alternatively, low-density areas, such as those with voids or other defects, will produce an increased exposure of the film (dark). Figure 2.2 shows an example of a radiographic image of a reinforced concrete section. Here the tendons (higher density) appear lighter than the concrete (lower density). A rectangular void is also visible as a particularly dark area.

**Figure 2.1** - Essential scheme of radiography to image the interior of an object.

**Figure 2.2** - Radiographic image of reinforced concrete. The duct and cables can be seen in the middle of the image, together with a rectangular void.
In 1985, researchers at the Laboratoire Regional des Ponts et Chaussees in France developed an X-ray radiographic system called the Scorpion [15] which could provide high quality images of elements up to 1.1 m thick.

Radiographic inspections of post-tensioned concrete structures [16] can in principle provide the following advantages:

- relative ability to penetrate concrete;
- potential for high-resolution images of ducts, voids, broken wires;
- ability to accurately locate reinforcements.

However, the following limitations also exist:

- radiation hazard requiring an exclusion zone around the test site and highly-trained operators;
- long inspection times due to exposure requirements;
- requirement of accessibility to both sides of the structure;
- difficulty in imaging a tendon if screened by other surrounding tendons.

Radioscopy is a different form of radiography in which the transmitted radiation is converted into visible light and recorded by a video camera. Radioscopy has been used in France for the detection of grouting defects [15]. Although it provides lower-quality images than those obtained by conventional film radiography, radioscopy offers the important advantage of real-time imaging.

Computed tomography is used to produce 3-D images instead of the conventional 2-D radiographic images. The intensity of the transmitted X- or gamma-ray signals is a function of the attenuation coefficient over the travel path. The attenuation coefficient depends on the position inside the object. Computed tomography processes the measured intensities over different paths; based on the known ratio between incident and transmitted intensities, it reconstructs the distribution of the attenuation coefficient in many planes thereby producing several cross-
sectional images of the object. Since computed tomography is even more expensive than traditional radiography and it presents the same safety risks, the technique remains today largely limited to laboratory studies.

2.3 Infrared Termography

Infrared (IR) Thermography is based on the principle that subsurface anomalies in a material result in localized differences in surface temperature caused by different rates of heat flow through the defected zones. Thermography senses the emission of thermal radiation from the material surface, and produces a visual image of this thermal signal which can be related to the presence and the size of an internal defect. Most infrared thermography applications use a thermographic camera (infrared-sensitive detector, Figure 2.3). Thermographic imaging may involve active or passive sources, e.g. a flashlamp or solar radiation. A popular configuration is impulse thermography, where a transient heat flux is induced and the surface temperature change is monitored as a function of time.

![Figure 2.3 - A thermal camera.](image)

Thermography is well suited for the detection of voids in concrete (up to ~10 cm thick), as well as for the location of delaminations in multi-layered systems (CFRP-laminates on concrete, asphalt on concrete). Ref. [18] used active, impulse thermography to detect voids inside tendon ducts of post-tensioned concrete structures. These tests focused on the higher portions of
the tendons, where the concrete covering is usually small and the likelihood of grouting voids is high (Figure 2.4). Reinforced concrete beams containing tendon ducts with partially grouted wires were constructed in this study. Transient temperature distributions were obtained with three different approaches: (a) using the heat during the hydration process of the cement inside the ducts, (b) applying an electrical current to the wires inside the duct, and (c) heating up the outside concrete surface by means of infrared radiators. An infrared camera was employed to record the temperature changes. The poorly grouted areas were successfully detected in the two approaches (a) and (b) above, whereas approach (c) was only effective to detect defects close to the beam surface.

![Figure 2.4 - A reinforced concrete beam for the detection of voids in tendon ducts by infrared thermography.](image)

This and other similar studies showed that the thermographic detection of voids in grouted tendons has the following advantages:

- low danger involved in the use of equipment;
- large speed of the inspection because of the large coverage of the cameras;
- ability to identify the horizontal extent of large defects.
However, the following limitations restrict the widespread application of this technique to prestressed and post-tensioned concrete structures:

- inability to detect defects within the tendons such as corrosion or broken wires;
- inability to detect void defects under a concrete cover thicker than ~10 cm;
- long measurement time required for locating defects at larger depths;
- sensitivity of the results to weather and surface conditions.

### 2.4 Ground Penetrating Radar

Ground Penetrating Radars (GPR) send short microwave pulses (wavelengths between 1 to 300 mm) into an object (Figure 2.5). The pulses are reflected at interfaces between media of differing dielectric properties. A receiver antenna is used to record the echoes. The signals measured can be plotted in a distance-versus-time display. As the radar antenna is slowly towed across the surface, a continuous cross-sectional “picture” of subsurface conditions is generated.

![Figure 2.5 - Basic scheme of a GPR system: transmitted microwaves and waves reflected from a void.](image)

Holographic (wavefront) reconstruction of GPR measurements can make the task of data interpretation easier. Depending on the application, specific frequencies of the waves are used. For example, higher frequencies are preferred to image subsurface features with high spatial resolutions; lower frequencies are, instead, preferred to image deeper features while sacrificing
spatial resolution. This method, initially used to locate buried objects such as cables or pipes, has become an important tool for geophysical explorations and archeological surveys. Today, GPR is commonly used for inspecting concrete pavement (ASTM D4748), buildings, and bridge decks.

As a general conclusion, GPR seems well suited to locate reinforcement (rebars and tendons) and ducts (Figure 2.6). For detect detection, its effectiveness is limited to voids in the grout [18]. However, Ref. [19] showed that the orientation of the receiving antennas can significantly affect the detectability of the duct voids.

![Figure 2.6](image.png)

**Figure 2.6** - Processed GPR data showing the configuration of steel reinforcement in a $1\text{m} \times 1\text{m}$ panel of a concrete chimney stack at a UK refinery ([www.benthamgeoconsulting.co.uk/geophysics/radar.htm](http://www.benthamgeoconsulting.co.uk/geophysics/radar.htm)).

Microwave imaging of concrete structures has the following advantages:

- ability to penetrate deep members without posing a hazard;
- limited scattering thereby providing excellent contrast between concrete and steel;
- lower cost compared with radiography;
- requirement for single-side access only;
- fast data collection;
- ability to scan large areas rapidly;
- insensitivity to atmospheric conditions.

Disadvantages in the application to PS tendons include:

- resolution not high enough to detect corrosion or broken wires;
- high attenuation of waves in the presence of moisture;
inability to resolve individual tendons in the presence of multiple, closely-spaced tendons.

2.5 Fiberoptic Strain Sensors

Fiberoptic Strain Sensors are fiber-based devices capable of sensing temperature and/or mechanical strain. The most common fiber optic strain sensor is the Bragg grating, Figure 2.7(a). A Bragg grating is a periodic perturbation of the refractive index of the fiber core. It operates as a selective mirror reflecting only a specific wavelength of the incident light spectrum. The reflected wavelength depends on the Bragg grating period, which changes if mechanical strain is applied to the fiber, Figure 2.7(b). If the relationship between Bragg wavelength shift and applied strain is known, a frequency spectrum analyzer can measure strain values, Figure 2.7(c). Fiberoptic strain sensors have been proposed in recent years for load measurement in bridge stay cables [20] and for structural health monitoring of post-tensioned concrete bridges [21]. The layout of the sensors used in the latter work is shown in Figure 2.8. This surface technique provides only localized measurements of strain. No information is given on the presence of small defects such as corrosion.

Figure 2.7 - Fiber Bragg grating strain sensor: (a) undeformed fiber, (b) deformed fiber, (c) spectra of input, reflected and transmitted light signals.
Compared to traditional electrical resistance strain gages, fiberoptic strain sensors offer the following advantages:

- immunity from electromagnetic interference and corrosion;
- minimum number of cables needed;
- multiplexing capabilities (multiple sensing points along the same fiber channel).

The general limitations are:

- inability to detect any defect, unless resulting in a measurable strain change;
- localized measurement of strain; difficulty to average over finite lengths.

New developments in fiberoptic sensors have included their use as detectors of stress waves such as acoustic emission signals [22]. In this work the sensors were able to detect acoustic emissions caused by fractures of the individual wires in tendons of post-tensioned concrete beams. It was shown that such a fiberoptic system could be embedded inside the monitored concrete structure once particular attention is paid in the design of relevant protective devices able to guarantee the durability and sensitivity of the fragile fiber. Applications of acoustic emission monitoring to detect fractures in tendons and cables are discussed further in Section 2.10.3.

### 2.6 Modal Analysis

A global approach that has been employed for the measurement of stay cable loads relies on Modal Analysis techniques [23-25]. In this case the cable natural frequencies are correlated to
the applied tension following the vibrating chord theory. The cable vibrations can be measured either by traditional surface-mounted accelerometers or by non-contact laser vibrometers. The uncertainties of the values of parameters such as mass, length and cross sectional area can introduce significant errors in the estimated load levels. In addition, the technique is generally not sensitive to small defects such as corrosion and it is not applicable to embedded PS tendons directly. Changes in environmental conditions (such as temperature) and the uncertainties in the boundary conditions contribute to making damage detection based on modal data at least challenging [26].

A few studies have attempted to correlate the prestress load loss in post-tensioned concrete beams to the change in their dynamic stiffness. The major problem with this technique is that prestress loss does not necessarily reflect into changes of global dynamic properties of the entire structure until the problem is too severe.

2.7 Magnetic Flux Leakage

Magnetic Flux Leakage (MFL) has been used with some success to detect defects in cables and tendons. Tendons embedded in concrete are first magnetized by applying a magnetic field from the outside of the concrete with a yoke magnet, Figure 2.9(a). The magnetic field resulting from a magnetized tendon or a magnetized steel wire is comparable to the magnetic field of a bar magnet. In the vicinity of a fracture, a magnetic dipole-distribution is formed, Figure 2.9(c) and, accordingly, a magnetic leakage field is produced in the surrounding region. The transverse component of the magnetic flux density, measured at the concrete surface, is shown in Figure 2.9(b). The technique is usually carried out by means of a scanning yoke magnet and sensor which are moved across the concrete surface in the direction of the tendon.
The magnetic flux leakage (stray field) measurement is either conducted during magnetization by the exciting field (active field) or as a residual field measurement. In the latter case, the stray field is caused by the residual magnetization of the steel after switching off the field [27]. In the active field measurement, ruptures of the longitudinal tendons appear as local maxima in the recorded signal.

In order to draw unequivocal conclusions on the status of the PS tendon from the magnetic flux density measured at the concrete surface, the external magnetization of the steel has to generate a magnetic state where all irreversible magnetization processes are completed. This is necessary in order to erase the unknown magnetic history of the steel. The magnetization is a time consuming procedure, typically requiring several passes along the test tendon.

Although the MFL technique shows great promise for detecting corrosion in PS steel tendons, fundamental difficulties exist when probing tendons located deep within the concrete member, particularly when looking for early stages of corrosion. In this case, besides the problems with detecting the magnetic leakage itself, proximate rebars (e.g. stirrups) can “mask” the magnetic field associated with the test tendon.

Figure 2.9 - Magnetic flux leakage: (a) application of the magnetic field and measurement of leakage; (b) transverse component of the magnetic flux density due to a broken wire; (c) magnetic dipole distribution due to the local fracture.
2.8 Magnetic Permeability

An alternative method to the MFL was explored in Refs. [28-31] with the objective of measuring the applied stress in tendons and cables. This technique is based on the inverse magnetostrictive effect. Some ferromagnetic materials such as high strength steels obey magnetostriction, i.e. the material exhibits a mechanical deformation under an applied magnetic field. Conversely, the magnetic permeability (the degree of magnetization) of such a material changes as a function of the applied mechanical stress. A measure of the permeability thus allows the estimation of the applied stress (Figure 2.10).

![Figure 2.10 - Magnetic permeability versus stress in magnetostrictive materials.](image)

The magnetic permeability can be evaluated indirectly, for example, by measuring the inductance of a coil placed around or near the tendon or cable [28]. A typical scheme of an elasto-magnetic sensor is represented in Figure 2.11. It consists of a primary coil and a secondary coil mounted in a protective steel shield and sealed with insulating materials. The coils surround the tendons. A DC current applied to the primary coil produces a magnetic field (H). Consequently, a magnetic flux density (B) is generated within the tendon. The amplitude permeability is defined by the ratio B/H and it is then related to the applied stress level.
Magnetic permeability sensors can be embedded in concrete structures, and they can be realized to be waterproof and resistant to highly corrosive environments. Such characteristics, in addition to the capacity to withstand large mechanical stresses for periods comparable to the life of the host structure, make the technique suitable for estimating stress levels in the PS tendons. Complications exist due to the effect of temperature changes. Also, since the response of the sensors depends on material properties, a calibration must be performed to account for the specific steel under test. The magnetic permeability technique generally does not provide any information on localized defects such as corrosion or broken wires.

![Figure 2.11 - Schematic description of an elasto-magnetic sensor.](image)

### 2.9 Time-domain Reflectometry

Time-domain Reflectometry (TDR) is a technique developed for locating discontinuities in electrical transmission lines. The method consists of sending an electric pulse through the transmission line using a step-pulse generator, and detecting any reflections of the signal back to the source point. If the conductor is of uniform impedance and properly terminated, the entire transmitted pulse will be absorbed in the far-end termination and no signal will be reflected back.
Since defects generate an impedance discontinuity, they can produce a reflection. The location of the discontinuity can be also estimated using the elapsed “time of flight.” The method is the electrical equivalent of the known pulse-echo method of ultrasonic testing.

TDR was used to characterize and locate faults in suspended bridge cables [32] and, more recently, to detect voids and corrosion in grouted PS tendons of post-tensioned bridges [33]. In the latter work, an optimized configuration for void detection was obtained by employing a sensing line in conjunction with the tensioning cable to form a two-wire transmission line (Figure 2.12). A typical TDR signal recorded in a 1-meter long strand is shown in Figure 2.13. The reflection observed at 13 ns is produced by the simulated void (see Figure 2.12).

It should be noted that this configuration can be adopted only for new structures, since for existing structures the second line must be external.

![Figure 2.12 - Two-wire transmission line formed by the strand and the transmission cable for detection of voids in grouted PS tendons by TDR. A rubber ball is used to simulate a void in the grouted specimen.](image)

The TDR approach is economically advantageous. For example, the sensing line coupled to the strand can be a commercially available transmission line, such as a lamp cord or standard TV cable that provide uniform geometry over their length. Although the potential exists for detecting voids in PS grouts, the signal-to-noise ratio of the measurements can be adversely
affected by several factors including water, varying moisture levels, and potential by-product in the voids [33].

The TDR technique cannot be used to monitor prestress levels in the tendons.

![Figure 2.13](image)

**Figure 2.13** - Typical TDR signal recorded in a 1-meter strand. The reflection at 13ns is produced by the simulated void defect.

### 2.10 Ultrasonic Testing

Ultrasonic Testing (UT) is one of the most widely used methods today for Non-Destructive Evaluation (NDE) and Structural Health Monitoring (SHM) purposes [1]. Although stress wave theory can be quite complex, ultrasonic NDE popularity descends from the simplicity of the basic concepts behind it. Traditional ultrasonic testing, based on shear and longitudinal (bulk) waves, is performed locally on specimens of small dimensions or by scanning the ultrasonic sources and receivers across large structural components. As a consequence, while the sensitivity of ultrasonic waves to small flaws and defects is unanimously recognized, traditional ultrasonic inspections are quite time consuming.

As mentioned in Chapter 1, ultrasonic waves are elastic perturbances above ~20 kHz in frequency. Depending on the test structure, the probing frequency can range from the tens of
kilohertz (long ranges and/or highly damped materials) to a few Megahertz (short ranges and/or lightly damped materials). Various properties of the waves, including amplitude, velocity, arrival time and frequency, can be used to detect structural defects and also monitor applied stresses.

Ultrasonic signals, depending on the inspected materials, can be detected dozens of meters away from the source, particularly in elongated structures such as pipes, railroad rails, plates and beams. Waves propagating in such structures are called “guided waves” (see Section 1.2.1.2).

Guided wave NDE/SHM techniques, owing to extensive research in the 1990’s, have grown to be a reality outside the confines of academic research (Ditri and Rose 1992, Cawley and Alleyne, 1996). For example, in the pipeline industry, ultrasonic guided waves represent a fast and practical method of conducting inspection. Principal advantages, besides allowing for long range inspection, are the complete coverage of the waveguide cross-sections and an increased sensitivity to small defects when compared to global vibrations owing to the ~kHz probing frequencies.

Guided waves are complex because of the existence of multiple modes at any given frequency, the frequency-dependent velocities (dispersion) and the frequency-dependent attenuation. In order to avoid errors in the interpretation of the guided wave signals and to fully exploit their potential, various signal-processing techniques have been used as Short Time Fourier Transform (Gabor, 1946), Continuous Wavelet Transform (Daubechies, 1992) and Two-Dimensional Fourier Transform (Alleyne and Cawley, 1991). Yet, accurate prediction of guided wave modal and forced solutions is still indispensable. Dispersion properties as phase velocities and group/energy velocities are important for mode identification. Similarly, the knowledge of the mode attenuation helps maximizing the inspection range by exploiting modes associated to minimum energy attenuation.
2.10.1 Impact Echo

The Impact-Echo (IE) technique uses bulk waves to measure slab thickness in concrete and masonry materials, as well as to detect flaws in these structures including delaminations, large cracks, and voids in PS ducts [34-36]. Concrete is an acoustically damped material, which requires probing frequencies lower than what traditionally used, for example, in metals (tens of kilohertz rather than megahertz).

In IE, a short duration mechanical impact at a free surface of the slab generates low-frequency stress waves (<50 kHz) that propagate through the specimen thickness; cracks or voids are detected by early unexpected wave echoes that appear in the signals recorded by the accelerometers located at the free surface. The IE results are normally analyzed in the frequency domain on the basis of resonance (spectral) principles.

In the typical configuration of Figure 2.14, an impact provided by an instrumented hammer produces both surface and bulk waves. Both waves are recorded by the accelerometer. The first arrival in Figure 2.14(b) is associated to surface waves while the second arrival is due to the longitudinal bulk waves reflected by the defect.

Recently IE has been combined with Spectral Analysis of Surface Wave (SASW) principles to provide both detection of voids in grouts of PS tendons as well as deficiencies in the concrete layer above the ducts [37]. In this work two accelerometers were used in place of the traditional single accelerometer to detect both through-thickness bulk waves and surface (Rayleigh) waves in the post-tensioned concrete slab. It was concluded that the IE detectability of voids decreases with decreasing diameter of the duct; the void detectability was zero for concrete cover layers thicker than 5.5” above the ducts.

Other complicacies of IE detection of voids in grouted conduits for post-tensioned concrete members are the presence of stirrups and longitudinal/transverse steel which can “mask” the indications of small voids. Because of the low probing frequencies, IE is ineffective for
detecting small discontinuities such as corrosion and broken wires. Also no information is given on the level of prestress in the tendons.

Finally, IE is a scanning technique which requires point-by-point inspection rather than continuous monitoring. For this reason it is most effective when used in conjunction with global inspection techniques that can pinpoint areas of concern within a structure.

![Impact Echo test method: basic setup and measurement.](image)

**Figure 2.14** - Impact Echo test method: basic setup and measurement.

### 2.10.2 Ultrasonic Tomography

Ultrasonic Tomography provides information on a two-dimensional section of the test specimen. This “slice” is constructed using measurements of ultrasonic arrival times through the specimen carried out from the specimen’s surface with conventional through-transmission ultrasonic testing. The mathematical theory behind tomographic reconstruction demonstrates that the internal characteristics of an object can be exactly recovered by a complete set of projections through the object. This is being used extensively in modern radiographic medicine. Unfortunately, concrete and steel are solids and, therefore, they transmit shear waves in addition
to longitudinal waves which propagate in human tissue. This makes the task of image reconstruction more difficult.

The simplest form of tomography is performed by measuring the transit times of a series of stress pulses propagating along different paths through the specimen (Figure 2.15). Each pulse travels through the specimen and interacts with its internal construction [31]. Variations in the internal conditions result in different times-of-flight being measured. The tomographic software reconstructs the section by combining the information contained in a series of these projections, obtained at different angles (Figure 2.15). The greater the number of measurements, the more accurate the results. A flowchart of operational steps is provided in Figure 2.16.
Time-of-flight tomography has the potential to detect voids in grouted ducts of prestressed and post-tensioned concrete structures. However, the technique is rather time-consuming. Furthermore, depending on the complexity of the structure, the position of transmitters and receivers can be limited to certain areas. For example, the approach becomes impractical if only one surface is available.

The images obtained using ultrasonic tomography have less resolution than those of radiographic methods (typically ~5-20 cm compared to ~1 mm). As a consequence, the interpretation of the results is more complex and can lead to misinterpretation. Conversely, the equipment is more economic than the radiographic equivalent, and it does not require extensive training and particular safety precautions.

2.10.3 Acoustic Emission

Acoustic Emission (AE) testing is a passive-only version of UT. AE is very effective for detecting and locating active flaws or impacts in structures in real-time. The principle of the technique is that active flaws (initiating or growing) generate elastic waves that can be detected away from the source by conventional ultrasonic receivers. An imperative condition of a successful monitoring is a permanent data acquisition with a sufficient high acquisition rate.

Several properties (features) of the AE signals, including amplitude, arrival time and energy, are analyzed in real-time and related to the presence of the flaw. More advanced spectral analysis techniques have also been proposed to identify the type of flaw. Location of the flaw is a relatively straight-forward manner, based on simple time-of-flight information collected at multiple detection points (two for a 1-D location, three for a 2-D location, etc...). Figure 2.17 shows an example of location of a sudden failure in a strand using the difference in arrival times of the AE waves detected at two separate locations.
Because of its effectiveness for real-time damage detection of waveguide-like structures, AE has been successfully used for monitoring both steel and composite cables [40-42]. Successful demonstrations of AE monitoring have also been given for the detection of wire failure in grouted and ungrouted tendons of both prestressed and post-tensioned concrete structures [43, 44].

Due to the leakage of the ultrasonic wave into the surrounding concrete, AE of grouted tendons requires a higher number of sensors located in close proximity to each other. A general
limitation of the AE approach is its inability to detect “inactive” flaws, i.e. pre-existing conditions of the structure. AE is also ineffective in monitoring prestress levels; in order to provide both defect detection and prestress level monitoring capabilities, the “passive” AE approach can be combined with the “active” guided ultrasonic testing discussed in the next section.

2.10.4 (“Active”) Guided Ultrasonic Testing

This represents a very effective technique that shows promises for the simultaneous detection of defects and monitoring of prestress levels in PS tendons. The “Active” Guided Ultrasonic Wave (GUW) method is based on the traditional UT approaches (through-transmission or pulse-echo [1]), but it is conducted using waves propagating in waveguide geometries such as plates, cables or rods. Considering the particular case of a strand, GUWs propagate along the tendon itself by exploiting its waveguide geometry.

The term “active” derives from the fact that this way of testing involves external generation and detection of waves in contrast to the “passive” Guided Ultrasonic Testing where the real-time monitoring is obtained using acoustic emission principles.

Ultrasonic guided waves provide a highly efficient method for the non-destructive evaluation (NDE) and the structural health monitoring (SHM) of solids with finite dimensions. Principal advantages in using guided ultrasonic waves (GUW) in non-destructive evaluation and structural health monitoring are: 1) possibility of using transducers permanently attached to the strand for continuous structural monitoring, providing simultaneous defect detection and stress monitoring capabilities for the strands with the same sensing system, 2) possible to couple long ranges due to waveguide propagation (~meters) with high sensitivity to small discontinuities due to the high probing frequencies (~hundreds of kilohertz), 3) long range inspection since guided waves propagate for long distances in waveguide like structures, 4) complete coverage of the waveguide cross-section, and 5) increased sensitivity to small defects when compared to global
vibrations. Drawbacks of guided waves are the existence of multiple modes (more than one propagating mode is generally excited by real transducers) and their dispersive nature that is represented by frequency-dependent velocities and frequency-dependent attenuation. Consequently, guided wave signals are very complicated.

In order to avoid errors in the interpretation of the guided wave propagation phenomena and to fully exploit their potential, an accurate knowledge of their dispersive properties is crucial. Phase velocities and group/energy velocities are important for mode identification. Similarly, the knowledge of the mode attenuation helps maximizing the inspection range by exploiting modes associated to minimum energy attenuation. Finally, an accurate prediction of the mode shapes can be exploited to selectively excite and detect only specific modes that are sensitive to critical defects of the structural component. Recent applications of the GUWs technique were demonstrated for the evaluation of stress levels in post-tensioning rods and multi-wire strands [45-50], for the detection of isolated defects in these components [40,45,50,51], and for estimation of corrosion damage in steel reinforced concrete structures [52,53].

2.10.4.1 Magneto-Strictive Sensor as Guided Wave Transducers

Whether GUW testing is carried out in a passive or in an active manner, it requires suitable transducers able to generate and/or detect the ultrasonic wave propagating in the test structure. For structures with cylindrical geometries such as PS tendons, Magneto-Strictive Sensors (MsSs) are good candidates.

Such sensors consist of electrical-wire coils that can be wrapped around the tendon. Figure 2.18 shows a typical “active” configuration using a coil transmitter and a coil receiver. A bias magnetic field must be provided to enhance the transduction efficiency. Also shown is the picture of one such coil constructed at UCSD and installed on a 0.6-in, seven-wire strand.
In magnetostriction an alternate electrical current in the transmitter coil induces a variation of magnetic field that, in turn, produces a change of magnetization within the ferromagnetic test material; the subsequent deformation (Joule’s effect) produces a stress wave. The inverse mechanism (Villari’s effect) is used in wave detection. Magnetostrictive sensors for exciting GUWs in cables were employed in Refs. [40, 45, 48-50, 54]. These applications were limited to free cables.
Chapter 3

Basic Aspects of Wave Propagation in Rods

3.1 Introduction

As mentioned in section 1.2.1.2, Guided Waves are those waves that require a boundary for propagation. Typical examples include wave propagation along a surface or in a rod, plate, tube, or multilayer structure. Guided waves are really made up of a superposition of bulk longitudinal and shear waves, but, because of boundary conditions, wave interference patterns are developed so that nicely formed guided wave packet scan propagate in the structure. Bulk longitudinal and shear waves are those considered to propagate in either a half space or infinite space with no boundary disturbances whatsoever.

In last decades, varied theoretical approaches, approximations, analyses and experiments on the subject of wave propagation in rodlike structures have been reported by many investigators and researchers.

In this chapter a basic approach to this problem is presented and governing equations for longitudinal, torsional and flexural wave are derived.

3.2 Longitudinal Waves in Thin Rods

This section firstly presents a brief introduction to longitudinal waves in thin rods and later focuses the attention on the general problem of wave propagation in a rod.
We shall use a “strength of materials” approach with approximations that neglect lateral inertia. This assumption is valid for long wavelengths, but, as described in [55], for small wavelength and with lateral inertia, the results are dispersive in nature. Herein the simplified case is considered.

Considering Figure 3.1 and neglecting lateral inertia we have:

$$\sum F_i = ma_i$$

and

$$-\sigma A + \left( \sigma + \frac{\partial \sigma}{\partial x} dx \right) A + q A dx = \rho A dx \frac{\partial^2 u}{\partial t^2}.$$ 

![Figure 3.1 – Differential element of a rod.](image)

By Hooke’s Law in one dimension, $$\sigma = E \varepsilon$$, where $$\varepsilon$$ denotes axial strain and so $$\varepsilon = \partial u / \partial x$$. Therefore,

$$\frac{\partial \sigma}{\partial x} + q = \rho \frac{\partial^2 u}{\partial t^2}$$ and $$\frac{\partial}{\partial x} \left( E \frac{\partial u}{\partial x} \right) + q = \rho \frac{\partial^2 u}{\partial t^2}$$

If the rod is homogeneous then neither $$E$$ nor $$\rho$$ is a function of $$x$$; hence

$$E \frac{\partial^2 u}{\partial x^2} + q = \rho \frac{\partial^2 u}{\partial t^2}$$
Thus,

\[ \frac{\partial^2 u}{\partial x^2} = \frac{1}{c_o^2} \frac{\partial^2 u}{\partial t^2} \]  

(3.1)

where \( c_o = \sqrt{\frac{E}{\rho}} \)

\( c_o \) is known as the traditional bar velocity.

Equation (3.1) is applicable for long wavelengths - that is, for wavelengths that are greater than the diameter of the rod. This bar velocity for long wavelengths is valid for rods of any cross-sectional area. Note that the basic concepts for a string (including D'Alembert's solution) are also applicable when there is no dispersion in a thin rod, so we have \( u \) and \( \sigma \) propagation (where \( \sigma = E(\partial u/\partial x) \)).

### 3.3 Waves in an Infinite Rod

We may now investigate the propagation of elastic harmonic waves in an infinite rod. It is most convenient to solve this problem using cylindrical coordinates (Figure 3.2), where the \( z \)-axis is along the axis of the rod.

![Figure 3.2 – Cylindrical coordinates for a solid cylindrical rod.](image)

The equation of motion can be written in full as follows, using aspects of Navier’s equation in cylindrical coordinates [56]:

\[ (\lambda + 2\mu) \left( \frac{\partial \phi}{\partial r} \right) \frac{2\mu}{r} \frac{\partial \omega}{\partial \theta} + 2\mu \frac{\partial \omega}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}, \]  

(3.2)
\[(\lambda + 2\mu) \frac{1}{r} \frac{\partial \phi}{\partial \theta} - 2\mu \frac{\partial \omega_r}{\partial z} + 2\mu \frac{\partial \omega_{r\theta}}{\partial r} = \rho \frac{\partial^2 u_r}{\partial t^2}, \tag{3.3}\]

\[(\lambda + 2\mu) \frac{\partial \phi}{\partial z} - \frac{2\mu}{r} \frac{\partial}{\partial r} (r\omega_r) + \frac{2\mu}{r} \frac{\partial \omega_r}{\partial \theta} = \rho \frac{\partial^2 u_z}{\partial t^2}. \tag{3.4}\]

where \(\phi\) is the dilatation in cylindrical coordinates and \(\omega_r, \omega_{r\theta}, \omega_z\) represent elements of the rotation tensor. Hence:

\[
\phi = \frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{\partial u_{r\theta}}{\partial z} + \frac{\partial u_z}{\partial z},
\]

\[2\omega_r = \frac{1}{r} \frac{\partial}{\partial \theta} (ru_r) - \frac{\partial u_{r\theta}}{\partial z},\]

\[2\omega_{r\theta} = \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial z},\]

\[2\omega_z = \frac{1}{r} \left[ \frac{\partial}{\partial r} (ru_\theta) - \frac{\partial u_r}{\partial \theta} \right].\]

At the surface of the rod, three stress components \((\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz})\) must vanish. From the stress-deformation relation, we find:

\[\sigma_{rr} = \lambda \phi + 2\mu \frac{\partial u_r}{\partial r},\]

\[\sigma_{r\theta} = \mu \left[ \frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{u_{r\theta}}{r} \right) \right],\]

\[\sigma_{rz} = \mu \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right).\]

We now consider propagation of harmonic waves along a rod. As follows from the equation of motion \([57]\), for the general case of vibration we have the following displacements:

\[u_r = U(r) \cos nt e^{i(kz - \omega t)}, \tag{3.5}\]

\[u_{r\theta} = V(r) \sin nt e^{i(kz - \omega t)}, \tag{3.6}\]
\[ u_z = W(r) \cos n \theta e^{(kz-\omega t)}. \]  \hspace{1cm} (3.7)

where \( n \) can be zero or an integer. We shall examine three types of vibration in a cylindrical rod: longitudinal, torsional, and flexural.

### 3.3.1 Longitudinal Modes in a Solid Cylindrical Rod

Considering a solid circular cylindrical rod (see Figure 3.2), longitudinal waves are axially symmetric, with displacement components in the radial and axial directions. Longitudinal waves correspond to the case \( n = 0 \) in Equations (3.5) - (3.7); the modes are represented schematically in next Figure 3.3.

![Figure 3.3](image)

**Figure 3.3** – Longitudinal modes in a solid cylindrical rod.

It is convenient to employ the potentials \( \Phi \) and \( \psi \) that satisfy the wave equations:

\[ \nabla^2 \Phi = \frac{1}{c_L^2} \frac{\partial^2 \Phi}{\partial t^2}, \]  \hspace{1cm} (3.8)

\[ \nabla^2 \psi = \frac{1}{c_T^2} \frac{\partial^2 \psi}{\partial t^2}, \]  \hspace{1cm} (3.9)

where \( c_L^2 = \frac{\lambda + 2\mu}{\rho} \), \( c_T^2 = \frac{\mu}{\rho} \)

Because of symmetry, the solution with respect to the \( z \)-axis is

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \]  \hspace{1cm} (3.10)
The scalar components of the displacement vector $\vec{u} = (u_r, 0, u_z)$ are given by

$$u_r = \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Psi}{\partial r \partial z},$$  \hspace{1cm} (3.11)

$$u_z = \frac{\partial \Phi}{\partial z} + \frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{r} \frac{\partial \Psi}{\partial r}.$$  \hspace{1cm} (3.12)

The stresses are given by Hooke's law as

$$\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r} + \lambda \left( \frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{\partial u_r}{\partial z} \right),$$  \hspace{1cm} (3.13)

$$\sigma_{rz} = \mu \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right).$$  \hspace{1cm} (3.14)

The boundary conditions for the problem will be given by

$$\sigma_{rr} = \sigma_{rz} = 0 \text{ at } r=a.$$  \hspace{1cm} (3.15)

The harmonic waves propagate in a cylinder along the $z$-axis. Thus, we consider solutions of (3.8) and (3.9) to be of the general form

$$\Phi = G_1(r) e^{i(kz-\omega t)},$$  \hspace{1cm} (3.16)

$$\Psi = G_2(r) e^{i(kz-\omega t)}.$$  \hspace{1cm} (3.17)

When (3.16) and (3.17) are substituted into the wave equations (3.8) and (3.9), respectively, we obtain ordinary differential equations for $G_j(r)$ ($j=1,2$):

$$\frac{d^2 G_j}{dr^2} + \frac{1}{r} \frac{d G_j}{dr} + \left( \frac{\omega^2}{c_j^2} - k^2 \right) G_j = 0 \quad (j = 1, 2).$$  \hspace{1cm} (3.18)

Assume that

$$\alpha^2 = \frac{\omega^2}{c_j^2} - k^2$$  \hspace{1cm} (3.19)

and
\beta^2 = \frac{\omega^2}{c^2_r} - k^2. \quad (3.20)

Equation (3.18) is Bessel’s equation, whose solutions are

\begin{align*}
G_1 (r) &= AJ_0 (\alpha r), \quad (3.21) \\
G_2 (r) &= BJ_0 (\beta r). \quad (3.22)
\end{align*}

The second solution – \( Y_0(\alpha r) \) and \( Y_0(\beta r) \), where \( Y_0 \) is the Bessel function of the second kind – has been discarded because of its singular behavior at the origin. Substituting (3.21) and (3.22) into (3.16) and (3.17), we discover that

\begin{align*}
\Phi &= AJ_0 (\alpha r)e^{i(kz-\omega t)}, \quad (3.23) \\
\Psi &= BJ_0 (\beta r)e^{i(kz-\omega t)}. \quad (3.24)
\end{align*}

Substituting (3.23) and (3.24) into (3.11) and (3.12) then yields

\begin{align*}
u_r &= \left[ AJ_1 (\alpha r) + Bj J_0 (\beta r) \right] e^{i(kz-\omega t)}, \\
u_z &= \left[ AikJ_0 (\alpha r) + \beta^2 BJ_0 (\beta r) \right] e^{i(kz-\omega t)},
\end{align*}

where \( J_0' (\alpha r) = (d/dr)[J_0 (\alpha r)] \); hence,

\[ J_0' (x) = -J_1 (x). \quad (3.27) \]

From (3.27), (3.25) and (3.26) it follows that

\begin{align*}
u_r &= \left[ -\alpha AJ_1 (\alpha r) - ik \beta BJ_1 (\beta r) \right] e^{i(kz-\omega t)}, \\
u_z &= \left[ ikAJ_0 (\alpha r) + \beta^2 BJ_0 (\beta r) \right] e^{i(kz-\omega t)},
\end{align*}

Let \( C = \beta B \). Then (3.21) and (3.22) lead to

\begin{align*}
u_r &= \left[ -\alpha AJ_1 (\alpha r) + ik C J_1 (\beta r) \right] e^{i(kz-\omega t)}, \\
u_z &= \left[ ikAJ_0 (\alpha r) + \beta C J_0 (\beta r) \right] e^{i(kz-\omega t)},
\end{align*}
At the cylindrical surface \( r=a \), the stresses must be zero. Substituting (3.30) and (3.31) into (3.13), and setting the resulting expression for \( \sigma_r \) equal to zero at \( r=a \), we find that

\[
\left[ -\frac{1}{2} \left( \beta^2 - k^2 \right) J_0(\alpha a) + \frac{\alpha}{a} J_1(\alpha a) \right] A + \left[ -ik \beta J_0(\beta a) + \frac{ik}{a} J_1(\beta a) \right] C = 0.
\] (3.32)

From the condition \( \sigma_r=0 \) at \( r=a \),

\[
[2ik\alpha J_1(\alpha a)] A - \left( \beta^2 - k^2 \right) J_1(\beta a) C = 0.
\] (3.33)

The requirement that the determinant of the coefficients must vanish presents us with the frequency equation as follows:

\[
\frac{2\alpha}{a} \left( \beta^2 + k^2 \right) J_1(\alpha a) J_1(\beta a) - \left( \beta^2 - k^2 \right)^2 J_0(\alpha a) J_1(\beta a) - 4k^2 \alpha \beta J_1(\alpha a) J_0(\beta a) = 0
\] (3.34)

This expression is known as the Pochhammer frequency equation for the longitudinal modes. It was first published in 1876 but, owing to its complexity, detailed calculations of the roots did not appear until the 1940s. Next Figure 3.4 depicts the phase velocities of longitudinal modes in terms of \( fd \), where \( f \) is frequency and \( d = 2a \) is the diameter of the rod. (The axially symmetric longitudinal modes for the rod of diameter \( d = 2a \) and symmetric modes for the plate of thickness \( d \) are quite similar).

As \( fd \to \infty \), the phase velocity of the lowest mode approaches the velocity of Rayleigh waves, while velocities of the higher modes approach \( c_T \). Over a short range of frequencies near the cutoff frequency, the behavior of the second dispersion curve is unusual; see Figure 3.5. This branch of the dispersion curve includes a range of frequencies where the group velocity and phase velocity have opposite signs. Such wave motions carry energy in one direction but appear to propagate in the other direction: the wave troughs and crests appear to move against the energy flux. This phenomenon of backward wave transmission in rods and plates was investigated in [58]. Figure 3.6 shows graphs for these group velocity dispersion curves.
For each $k_n$ root of (3.33), from Equation (3.32) we find:

$$C_n = -\frac{2i k_n \alpha_n J_n(\alpha_n a)}{\left(\beta_n^2 - k_n^2\right) J_1(\beta_n a)} A_n \quad (n = 1, 2, \ldots) \tag{3.35}$$

Substituting (3.35) into (3.30) and (3.31) yields the following displacements:

$$u_r = \alpha_n \left[\left(\beta_n^2 - k_n^2\right) J_1(\alpha_n r) J_1(\beta_n a) + 2k_n^2 J_1(\alpha_n a) J_1(\beta_n r)\right] e^{ik_z - i\alpha t}, \tag{3.36}$$

$$u_r = i k_n \left[\left(\beta_n^2 - k_n^2\right) J_0(\alpha_n r) J_1(\beta_n a) - 2\alpha_n \beta_n J_1(\alpha_n a) J_0(\beta_n r)\right] e^{ik_z - i\alpha t}; \tag{3.37}$$

$$D_n = -\frac{A_n}{\left(\beta_n^2 - k_n^2\right) J_1(\beta_n a)}; \tag{3.38}$$

$$\alpha_n^2 = \frac{\omega^2}{c_L^2} - k_n^2, \tag{3.39}$$

$$\beta_n^2 = \frac{\omega^2}{c_T^2} - k_n^2; \tag{3.40}$$

$D_n$ denotes unknown constants.
Figure 3.5 – Magnified portion of the phase velocity dispersion curves for an aluminum rod ($c_L = 6.3$ km/s, $c_T = 3.1$ km/s).

Figure 3.6 – Group velocity dispersion curves for an aluminum rod ($c_L = 6.3$ km/s, $c_T = 3.1$ km/s).

A general representation for the displacement field can be written as follows:

$$u_r = \sum_{n=1}^{\infty} D_n u_r^n,$$  \hspace{1cm} (3.41)

$$u_z = \sum_{n=1}^{\infty} D_n u_z^n.$$  \hspace{1cm} (3.42)
### 3.3.2 Torsional Waves

Torsional waves result when \( u_r \) and \( u_z \) vanish (see Figure 3.7); from the equation of motion, it follows that \( u_\theta \) must be independent of \( \theta \). For torsional waves, the equation of motion is

\[
\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{\partial^2 u_\theta}{\partial z^2} = \frac{1}{c_t^2} \frac{\partial^2 u_\theta}{\partial t^2}
\]  
(3.43)

(see [57] for more details).

![Figure 3.7 – Torsional modes in a solid rod.](image)

We consider harmonic waves of the form:

\[
u_\theta = V(r) e^{i(kz - \omega t)}.
\]  
(3.44)

Substituting (3.44) into (3.43) and solving the differential equation for the unknown function \( V(r) \), we obtain

\[
u_\theta = \frac{1}{\beta} BJ_1(\beta r) e^{i(kz - \omega t)},
\]  
(3.45)

where \( B \) is arbitrary.

Of the three boundary conditions,

\[
\sigma_{rr} = \sigma_{rz} = \sigma_{r\theta} = 0 \quad \text{at} \quad r = a,
\]  
(3.46)

only the condition

\[
\sigma_{r\theta} = 0 \quad \text{at} \quad r = a
\]  
(3.47)

is nontrivial. This condition yields the dispersion frequency transcendental equation,
\begin{equation}
(\beta a) J_0(\beta a) - 2J_1(\beta a) = 0,
\end{equation}
where the first three roots are
\[
\beta_1 = 0, \quad \beta_2 a = 5.136, \quad \beta_3 a = 8.417.
\]
Taking the limit of (3.41) as $\beta \to 0$ yields:
\[
u_\vartheta = \frac{1}{2} B e^{i(kz-\omega t)}.
\]
This displacement represents the lowest torsional mode. In the lowest mode, the amplitude of $u_\vartheta$ is proportional to the radius, and both $u_r$ and $u_z$ are zero. The motion corresponding to this solution is a rotation of each cross-section of the cylinder as a whole about its center. Note that, since $\beta = 0$ implies that the phase velocity equals $c_T$, the lowest torsional mode is not dispersive. The higher modes are dispersive and the resulting frequency spectrum has the same shape as that of SH waves in a plate.

### 3.3.3 Flexural Waves

Flexural waves depend on the circumferential angle $\vartheta$ through the trigonometric functions shown in Equations (3.5) - (3.7). A schematic representation of flexural waves is shown in Figure 3.8.
Of the flexural modes, the family defined by \( n = 1 \) is most important. Through the use of (3.5) - (3.7), we obtain:

\[
\begin{align*}
u_r &= U(r) \cos \theta e^{j(kz - \omega t)}, \\
u_\theta &= V(r) \sin \theta e^{j(kz - \omega t)}, \\
u_z &= W(r) \cos \theta e^{j(kz - \omega t)}. 
\end{align*}
\]

Substituting these equations into (3.2) - (3.4), we find the system of three differential equations containing \( U(r), V(r), W(r) \). Without going into details of the solution, we present the final form:

\[
\begin{align*}
U(r) &= A \frac{\partial}{\partial r} J_1(\alpha r) + B \frac{\partial}{\partial r} J_1(\beta r) + ikC J_2(\beta r), \\
V(r) &= A r J_1(\alpha r) + ikC J_2(\beta r) - B \frac{\partial}{\partial r} J_2(\beta r), \\
W(r) &= ikA J_1(\alpha r) - \frac{C}{r} \frac{\partial}{\partial r} \left[r J_2(\beta r)\right] - \frac{C}{r} J_2(\beta r).
\end{align*}
\]

In order to illustrate the motions represented by displacement distributions (3.53) - (3.55), we choose the \((y,z)\)-plane (the vertical plane) as the one from which \( \theta \) is measured (see Figure 3.9). It now follows from (3.54) that for points in the vertical plane, the \( u_\theta \) component vanishes, so that these points remain in the vertical plane. In the \((x,z)\)-plane (the horizontal plane), where \( \theta = \pm \pi/2 \), the displacements \( u_r, u_z \) vanish. Points in the horizontal plane have purely vertical oscillations, which suggest the terminology "flexural" waves.

To determine the frequency equation, the displacements (3.50) - (3.52) must be substituted into the stress expressions, and \( \sigma_{rr}, \sigma_{rz}, \sigma_{r\theta} \) must all be set equal to zero at \( r = a \). This
leads to a system of three homogeneous equations for $A$, $B$, and $C$. The requirement that the determinant of the coefficients must vanish yields the frequency equation (see [57]). This frequency equation was examined in [59].

**Figure 3.9** – Cross – section of the rod.
Chapter 4

Interwire Contact Stresses in Seven-Wire Strands

4.1 Introduction

Following Machida and Durelli theory [60], this chapter describes the analytical approach to determine the load transfer and strain distribution in strands and cables subjected to axial and torsional displacements. Using geometric considerations explicit expression will be presented for the determination of axial force, bending and twisting moments in the helical wire, and for the axial force and twisting moment in the core of a seven-wire strand subjected to axial and torsional displacements.

The theory discussed below is general because, since the equations given are linear, it could be similarly developed for strands of any number of wires or for strands subjected to large displacements. This is a simplification that makes the equations extremely easier to handle by designers.

Measurements on oversize epoxy models of the strand show good correlation with the theory and support the observation that axial load has no effect on the effective torsional rigidity of the strand. In order to confirm this fact, a numerical simulation using a finite element commercial code (ABAQUS) is also presented in Section 4.8.

The problem is illustrated with graphical representations for clearer understanding of its complexity and to emphasize the frequently erratic behavior of actual cables.
4.1.1 Notation

A  Cross-sectional area of wire (used with c or h subscript).

A - D  Coefficients, functions of geometry and material properties.

d  Wire diameter (used with c or h subscript).

E  Young’s modulus.

G  Shear modulus.

I  Moment of inertia of the cross section of helical wire.

I_p  Polar moment of inertia of the cross section of wire (used with c or h superscript).

M  Bending moment acting on the helical wire in a plane containing the axis of helical wire and the principal normal (positive M is shown in Figure 4.7).

N  Axial tensile load applied to the strand.

N_c  Axial force acting in the core.

N_h  Axial force acting in the helical wire.

P  Resultant contact force acting on a helical wire (per unit length).

P_c  Contact force acting between core and a helical wire (per unit length).

P_h  Contact force acting between two adjacent helical wires (per unit length).

p  Pitch of helical wire.

R  Radius of strand measured from the centre of strand to the centre of helical wire (radius of reference cylinder shown in Figure 4.1).

r  Distance on the transverse cross section of a wire from the centroid (Figure 4.7).

s  Arc length of helical wire.

T  Torsional load applied to the strand.

T_c  Twisting moment acting on the core (positive T_c is shown in Figure 4.7).

T_h  Twisting moment acting on the helical wire (positive T_h is shown in Figure 4.7).

β  Helical wire lay angle (Figure 4.1).
Normalized rotation per original one pitch length of the strand and defined by $\gamma = \Delta / 2\pi$.

Shear strain on the transverse cross section of helical wire associated with twisting moment $T_h$ acting on the helical wire.

Shear strain on the transverse cross section of core associated with twisting moment $T_c$ acting on the core.

Rotation per original pitch length applied to the strand (positive in winding rotation) (in radians).

Axial displacement per original pitch length applied to the strand (positive in elongation) (in inches).

Axial displacement of the strand per unit length and defined by $\varepsilon = \delta / \rho$.

Axial strain in the helical wire associated with the axial force $N_h$ acting in the helical wire.

Axial strain in the helical wire associated with bending moment $M$ acting on the helical wire.

Axial strain in the core associated with the axial force $N_c$ acting in the core.

Radius of twist of the axis of helical wire.

Distance on the transverse cross section of helical wire from its neutral axis for bending (Figure 4.7).

Poisson’s ratio.

Radius of curvature of the axis of helical wire.

Normal stress corresponding to $\varepsilon_s^{ah}$.

Normal stress corresponding to $\varepsilon_z^{ac}$.

Shear stress corresponding to $\gamma_s^{th}$.
\[ \tau_{c}^{h} \]  Shear stress corresponding to \[ \gamma_{c}^{h} \].

\( c \) Used to define core as subscripts and/or superscript.

\( h \) Used to identify the helical wire as subscripts and/or superscript.

Quantities after deformation are identified by prime symbols (e.g. \( s', p', \beta', \) etc.).

### 4.2 Geometry and loading of the strand

The analysis that follows is focused on the most typical and simple strand consisting in a straight core wire wrapped around by a layer of six helical wires (Figure 4.1) but the equations to be given in the later part of this chapter can be extended to the case where the strand has an arbitrary number of helical wires.

![Figure 4.1 – Geometry of a helical wire wrapped around a core.](image)

Each helical wire is assumed to have a circular cross section in a plane normal to its axis (a helix) (Figure 4.1), the diameter of which is small in comparison with the pitch of the helix. It
is also assumed that each helical wire is in contact with the two adjacent wires, with the core, or with both core and adjacent wires.

A strand can be loaded by three principal static types of loadings: (1) *pulling* (or axial loading of the strand), (2) *torsion* and (3) *bending*. Besides these three principal types of loadings, a strand could be subjected to thermal loadings, if there are temperature gradients in its environment, impact and vibrations. In the analysis, torsion and pulling are conveniently associated and will be treated together. Bending (as produced when the strand is wrapped around a drum) may develop when the strand is subjected to different levels of prestressing (by pulling). It may also develop when the strand is used as a component of a wire rope and the wire rope is subjected to axial loading or torsional loading. The loadings considered in this section are axial tension, torsion and combined torsion with tension. These loadings cause elongation and rotation in the strand. Because of geometric restrictions, the state of stress and strain associated with these loadings is constant all the way along the axial line of each helical wire. Therefore, stresses and loads in the strand can be described by the stresses and loads on a single transverse cross section of a helical wire and core. In case of bending of the strand, the stresses and loads on the transverse cross section change along its axial line making the analysis more complicated. This case is not considered in the present section.

In the present analysis, the radial dimensions of the cross section in the unloaded strand and its position with respect to the core are assumed to remain constant under load; the interwire contact deformation and Poisson’s effect due to axial strain are neglected. In other words, the axial lines of helical wire after deformation in the strand are assumed to remain on the cylindrical surface (Figure 4.1) on which they were before deformation (this cylinder will be called ‘reference cylinder’ and its radius will be called $R$). This assumption is reasonable because each helical wire is restrained by the core and by its neighbours. This restriction makes the main difference in the deformation characteristic between a free helical wire and a helical wire in a
strand. The deformation of a helical wire in a strand will be better understood with the following statement. Consider material line segments and say $AB$ and $CD$ in a helical wire which are the principal normal and binormal to the axial line (helix) at point 0, respectively (Figure 4.2). After deformation these material line segments are assumed to displace in such a way that they still are the principal normal and binormal to the deformed axial line ($A'B'$ and $C'D'$ in Figure 4.2).

**Figure 4.2** – Deformation of a cross section of a helical wire.

### 4.3 Forces in a helical wire due to axial and torsional displacements

Consider a segment of strand (length $l$, which can be taken arbitrarily) as shown in the next Figure 4.3. It is assumed that, because of geometric constraints, whether the strand is pulled axially, or is twisted about its axis, the core and the wire are restricted to displace axially and rotate in contact with each other. The deformed state is shown by heavier lines in Figure 4.3. The axial line of helical wire $EF$ (undeformed helix) moves to another helix $EF'$ (deformed helix).
This deformation consists of two components: one is a displacement in the axial direction of strand. Call $\delta$ this displacement per pitch (positive in elongation). The other component is a rotation around the axis of the strand. Call this rotation per pitch $\Delta$ in radians (positive in winding direction). The forces in the helical wire associated with the above mentioned deformation can be categorized into four types: (1) axial force, (2) bending in the plane containing the axial line of the helical wire and the principal normal of the helix, (3) twisting around axis of helical wire and (4) contact force, the resultant force of which lies on the plane containing the axial line of helical wire.

Strains and stresses acting on the cross section of the helical wire and associated with each of the above four types of forces will be expressed as functions of $\delta$ and $\Delta$. The strains and stresses will be described in terms of their components in the plane normal to the axial line of the helix. Cylindrical co-ordinates ($r$, $\theta$, $s$) will be used (Figure 4.1).

Figure 4.3 – Elongation and rotation in a helical wire.
4.3.1 Axial Force in the wire

The displacement of the axial line segment from EF to EF’ (Figure 4.3) requires a change in the arc length. This is associated with an axial strain and with an axial stress and axial force (uniaxial state of stress in the wire is assumed).

The axial strain can be expressed in terms of $\delta$ and $\Delta$ as follows. Call $l$ the length of undeformed strand. The initial length $s$ of the axial line of a helical wire is given by

$$s = \frac{l}{p} \sqrt{p^2 + (2\pi R)^2}$$  \hspace{1cm} (4.1)

The final length $s'$ is given by

$$s' = \frac{l}{p} \sqrt{(p + \delta)^2 + [(2\pi + \Delta)R]^2}$$  \hspace{1cm} (4.2)

If it is assumed that the helical wire is subjected to uniform axial stress distribution, the axial strain of helical wire is

$$\varepsilon_{sh} = \frac{s' - s}{s} = \frac{p}{\sqrt{p^2 + (2\pi R)^2}} \sqrt{\left(1 + \frac{\delta}{p}\right)^2 + \left(\frac{2\pi R}{p}\right)^2\left(1 + \frac{\Delta}{2\pi}\right)^2} - 1 = \cos \beta \sqrt{(1 + \varepsilon)^2 + (1 + \gamma)^2 \tan^2 \beta} - 1$$  \hspace{1cm} (4.3)

where $\varepsilon = \delta / p$, $\gamma = \Delta / 2\pi$.

Assuming small deformation, i.e. $\varepsilon, \gamma \ll 1$, and neglecting higher order small quantities:

$$\varepsilon_{sh} = \varepsilon \cos^2 \beta + \gamma \sin^2 \beta$$  \hspace{1cm} (4.4)

Thus the quantities related to axial force acting in a helical wire become

$$\varepsilon_{sh} = \varepsilon \cos^2 \beta + \gamma \sin^2 \beta$$

$$\sigma_{sh} = E \left(\varepsilon \cos^2 \beta + \gamma \sin^2 \beta\right)$$

$$N_h = A_h E \left(\varepsilon \cos^2 \beta + \gamma \sin^2 \beta\right)$$  \hspace{1cm} (4.5)
4.3.2 Bending Moment in the wire

The deformation of the axial line of the helical wire (Figure 4.3) causes a change in the radius of curvature and this is associated with a bending moment in the plane containing the helix and the principal normal.

Using the Cartesian co-ordinates \((X, Y, Z)\), the parametric expression of the helix \(EF\) is given by

\[
X = R \cos \phi \\
Y = R \sin \phi \\
Z = \frac{p}{2\pi} \phi
\]  

(4.6)

The radius of curvature \(\rho\) can be computed from

\[
\frac{1}{\rho^2} = \left[ \left( \frac{d^2X}{d\phi^2} \right)^2 + \left( \frac{d^2Y}{d\phi^2} \right)^2 + \left( \frac{d^2Z}{d\phi^2} \right)^2 - \left( \frac{d^2s}{d\phi^2} \right)^2 \right]^{1/2}
\]  

(4.7)

where \(ds\) is a line increment of the helix defined as

\[
ds^2 = (dX)^2 + (dY)^2 + (dZ)^2
\]

Using equation (4.7) the radius of curvature of the undeformed helix \(EF\) is

\[
\rho = R + \left( \frac{p}{2\pi} \right)^2 \frac{1}{R}
\]  

(4.8)

The radius of curvature after deformation is given by:

\[
\rho' = R + \left( \frac{p'}{2\pi} \right)^2 \frac{1}{R}
\]  

(4.9)

The deformed pitch \(p'\) can be expressed in terms of the original pitch \(p\), the axial and rotational deformations of the strand.
\[ p' = (p + \delta) \left( \frac{2\pi}{2\pi+\Delta} \right) = p \frac{1+\varepsilon}{1+\gamma} \] (4.10)

Using equations (4.8) – (4.10), the following quantities related to bending of the helical wire can be determined for a given deformation of the strand (\( \delta \) and \( \Delta \), or \( \varepsilon \) and \( \gamma \)).

\[
\varepsilon_s^{bh} = \left( \frac{\rho' - \rho}{\rho \rho'} \right) \eta \\
\sigma_s^{bh} = E \left( \frac{\rho' - \rho}{\rho \rho'} \right) \eta \\
M = EI \left( \frac{\rho' - \rho}{\rho \rho'} \right)
\] (4.11)

When strains are small \( \varepsilon, \gamma \ll 1 \), then:

\[ p' = p \left( 1 + \varepsilon - \gamma \right) \] (4.12)

and equation (4.11) becomes

\[
\varepsilon_s^{bh} = 2 \left( \varepsilon - \gamma \right) \eta \cos^2 \beta \sin^2 \beta \\
\sigma_s^{bh} = 2E \left( \varepsilon - \gamma \right) \eta \cos^2 \beta \sin^2 \beta \\
M = 2EI \left( \varepsilon - \gamma \right) \cos^2 \beta \sin^2 \frac{\beta}{R}
\] (4.13)

The positive direction of the bending moment is shown in following Figures 4.7 and 4.8.

**4.3.3 Twisting Moment in the wire**

The line segments \( AB \) and \( CD \) embedded in the helical wire (Figure 4.2) can be considered as lines of geometric reference. When moving along the axial line of the helical wire, these lines rotate around the axial line. Call \( \Delta \psi \) the angle of rotation resulting from travelling through distance \( \Delta s \) along the axial line. Then the quantity
is the angle of twist, per unit length of the plane of geometric reference containing a line segment, such as \( AB \) or \( CD \) (Figure 4.4). It should be noted that this has nothing to do with twisting strain; \( \zeta \) in equation (4.14) is called the radius of twist (or the second radius of curvature) of a spatial line.

Denoting the original and final values of the radius of twist as \( \zeta \) and \( \zeta' \) respectively, a segment of helical wire \( s \) undergoes twisting deformation around its axis through an angle which is given by

\[
\Delta \psi' - \Delta \psi = \frac{\Delta s}{\zeta'} - \frac{\Delta s}{\zeta}
\]

(4.15)

Figure 4.4 – Plane of geometric reference in helical wire for consideration of twisting deformation.

Thus, the angle of twist per unit length due to the deformation is given by
\[ \lim_{\Delta \to 0} \frac{\Delta \psi' - \Delta \psi}{\Delta s} = \frac{\zeta - \zeta'}{\zeta \zeta'} \quad (4.16) \]

Further computations can be made by assuming that the elementary theory of twist of circular bar can be applied to this problem. The radius of twist \( \zeta \) of any spatial line is given by

\[ \frac{1}{\zeta^2} = \left( \frac{d\lambda}{ds} \right)^2 + \left( \frac{d\mu}{ds} \right)^2 + \left( \frac{d\nu}{ds} \right)^2 \quad (4.17) \]

where \( \lambda, \mu \) and \( \nu \) are the direction cosines of the binormal at the point considered. In case of helix \( \zeta \) is constant and is expressed as follows:

Before deformation \( \zeta = \left( \frac{p}{2\pi} \right) + R^2 \left( \frac{p}{2\pi} \right) \)

After deformation \( \zeta' = \left( \frac{p'}{2\pi} \right) + R^2 \left( \frac{p'}{2\pi} \right) \quad (4.18) \)

Using equations (4.10), (4.16) and (4.18), the following quantities related to twisting of the helical wire can be determined for a given deformation of the strand.

\[ \gamma_{sb}^h = \left( \frac{\zeta - \zeta'}{\zeta \zeta'} \right) r \]

\[ \varepsilon_{sb}^h = G \left( \frac{\zeta - \zeta'}{\zeta \zeta'} \right) r \]

\[ T_h = G I_p^h \left( \frac{\zeta - \zeta'}{\zeta \zeta'} \right) r \quad (4.19) \]

Assuming again small strain (\( \varepsilon, \gamma \ll 1 \)), equations (4.19) become

\[ \gamma_{sb}^h = \frac{(\gamma - \varepsilon) r}{4R} \sin 4\beta \]

\[ \varepsilon_{sb}^h = \frac{G(\gamma - \varepsilon) r}{4R} \sin 4\beta \]

\[ T_h = \frac{G I_p^h (\gamma - \varepsilon) r}{4R} \sin 4\beta \quad (4.20) \]

The positive direction of the twisting moment is shown in Figures 4.7 and 4.8.
4.4 Contact Forces between wires

The stress distribution is complex at the points of contact, but the effect is considered local and the details of the stress distribution could be determined theoretically using the Hertz solution (Section 4.8). Following the geometric considerations made previously, the contact between helical wires and the core under tension and torsion loadings is such that there is no relative motion along the lines of contact.

Motion is possible, however, between adjacent helical wires. Along these lines of contact, if motion occurs, it is assumed that the lubrication is sufficient to allow only a negligible frictional force. Thus, the resultant force due to contact is a force directed outward normally to the axial line of the helical wire as shown in Figure 4.5.

The resultant contact force per unit length, $P$, is

$$P = 2P_h \cos 60^\circ + P_c = P_h + P_c$$

(4.21)

From a consideration of equilibrium for a short segment of helical wire (Figure 4.6), on which all of the forces discussed in the preceding sections are acting, the following relation can be obtained with regard to contact force:

$$P = \frac{N_h}{\rho} = \frac{N_h}{\rho}$$

(4.22)

![Figure 4.5](image)

**Figure 4.5** – Resultant contact force in the transverse cross section of helical wire.
4.5 Forces in the core due to axial or torsional displacements

If the core of a strand is connected with the surrounding helical wires at both ends of the strand, the quantities representing the deformation applied to the strand, $\delta$ and $\Delta$, also represent the deformation of the core. Since a core is a straight circular rod, estimation of strains, stresses and forces can be made easily.

Where there is contact between core and helical wires, the core will act as a simple bar in tension and torsion with the addition of some loading due to six lines of spiral contact from helical wires. This suggests that the deformed cross section of the core may not remain plane, but may be warped. This effect could be neglected for the first approximation, since it is considered that the localized radial forces due to contact would have only a small contribution to overall deformation of the core.

The axial deformation $\delta$ of the strand produces an average axial strain $\delta / p$ ($= \varepsilon$) in the core. The rotational deformation $\Delta$ of the strand results in a twisting deformation per unit length $\Delta / p \left(= \frac{2\pi \gamma}{p}\right)$ of the core. Therefore, the core is subjected to axial force and twisting moment.
Strains, stresses and forces expressed in terms of $\varepsilon$ and $\gamma$ are:

\[
\begin{align*}
\varepsilon_{zc}^{ac} &= \varepsilon \\
\sigma_{zc}^{ac} &= E\varepsilon \\
N_c &= A_c E\varepsilon \\
\gamma_{zc}^{ac} &= \frac{2\pi G\gamma r}{p} \\
\tau_{zc}^{ac} &= \frac{2\pi G\gamma r}{p} \\
T_c &= GI_p \frac{2\pi \gamma}{p}
\end{align*}
\]

(4.23)

4.6 Forces acting on the strand due to applied displacements

From the preceding considerations, it follows that each helical wire is subjected to (1) axial load, (2) bending moment in a plane containing the axial line and principal normal and (3) twisting moment around the axial line. The core is subjected to (1) axial load and (2) twisting moment. The stress and strain distribution associated with these types of loading can be obtained approximately by the theory of strength of materials. The main results are summarized in Figure 4.7.

Resultant forces on the strand cross section can easily be expressed in terms of forces acting on the cross sections of each of the seven wires. This is done in Figure 4.8 and the following relations between internal forces and external forces are obtained from consideration of equilibrium.

$N$  
External axial force of the strand

\[
N = N_c + 6N_h \cos \beta'
\]

(4.24)

$T$  
External torque of the strand

\[
T = T_c + 6(T_h \cos \beta' - M \sin \beta' + N_h R \sin \beta')
\]

(4.25)

where $\beta'$ is lay angle of helical wire after deformation and is expressed in terms of $\varepsilon$ and $\gamma$ as follows:

\[
\beta' = \tan^{-1} \left( \frac{1 + \gamma \tan \beta}{1 + \varepsilon} \right)
\]

(4.26)
From the assumption of small deformation, we can put

$$\beta' = \beta$$

(4.27)
The approximation of equation (4.27) reduces equations (4.24) and (4.25) to much more simple forms which are linear functions of $\varepsilon$ and $\gamma$.

### Figure 4.8 – Forces and moments on a transverse cross section of a strand.

Using equations (4.5), (4.13), (4.20), (4.23) and (4.27), equations (4.24) and (4.25) can be expressed in terms of $\varepsilon$ and $\gamma$ as follows

\[ N = A\varepsilon + B\gamma \quad (4.28) \]

\[ T = C\varepsilon + D\gamma \quad (4.29) \]

where $A$, $B$, $C$ and $D$ are constants determined by the geometry of the strand and the elastic constants of the strand material, and are given by
\[ A = A_c E + 6 A_i E \cos^3 \beta \]
\[ B = 6 A_i E \sin^2 \beta \cos 4\beta \]
\[ C = 6 A_i RE \sin \beta \cos^2 \beta - \frac{3GI_p \sin 4 \beta \cos \beta}{2R} - \frac{12EI \cos^2 \beta \sin^3 \beta}{R} \]
\[ D = 6 A_i RE \sin^3 \beta + \frac{3GI_p \sin 4 \beta \cos \beta}{2R} + \frac{12EI \cos^2 \beta \sin^3 \beta}{R} + \frac{2\pi GI_p}{p} \]

From equations (4.28) and (4.29), we have
\[ \varepsilon = \frac{D}{AD - CB} N - \frac{B}{AD - CB} T \]
\[ \gamma = \frac{C}{AD - CB} N + \frac{A}{AD - CB} T \]

Substituting equation (4.31) into equations (4.5), (4.13), (4.20), (23) and (4.24), strains, stresses, forces and moments acting in the helical wires and the core can be expressed in terms of the external axial force \( N \) and torque \( T \) which can be treated as applied force and applied torque for the strand.

### 4.7 Applications

Using the equations developed in the preceding sections, it is possible to determine the response of the strand to an applied load as a function of that load. Since the equations in the previous chapter are all expressed in linear and explicit form, the procedure of computations is easy.

In the present section just the case of axial loading with restricted ends will be discussed since it concerns the experiments and the numerical simulations developed in the present research. The description of other boundary and loading cases together with an experimental analysis aimed at validating the analytical approach described above can be found in [60].
4.7.1 Axial loading with restricted ends

When an axial load is applied to a strand whose ends are restricted from rotating, the following forces develop: axial force $N_c$ in the core wire; and axial force $N_h$, bending moment $M_h$, and twisting moment $T_h$ in the helical wires.

In this case since unwinding motion is restricted we have

$$\Delta = 0 \quad \text{or} \quad \gamma = 0 \quad \quad (4.32)$$

For a given elongation of strand $\varepsilon$, the axial tensile load $N$ and torque $T$ required from the support to restrict unwinding of strand are from equations (4.28) and (4.29)

$$N = A\varepsilon \quad \quad (4.33)$$

$$T = C\varepsilon = \frac{C}{A} N \quad \quad (4.34)$$

In this case the applied torque is linearly related to the applied tension.

Forces and moments in the core and the helical wires expressed in terms of applied axial load $N$ are:

1. Axial force in the core.

$$N_c = A_c E \frac{1}{A} N \quad \quad (4.35)$$

2. Twisting moment in the core.

$$T_c = 0 \quad \quad (4.36)$$

3. Axial force in the helical wire.

$$N_h = A_h E \frac{\cos^2 \beta}{A} N \quad \quad (4.37)$$

4. Bending moment in the helical wire.

$$M = \frac{2EI \cos^2 \beta \sin^2 \beta}{R} \frac{1}{A} N \quad \quad (4.38)$$
Twisting moment in the helical wire.

\[ T_h = \frac{G l^b}{4R} \sin^4 \beta \frac{1}{A} N \]  

(4.39)

The effective tensile rigidity of the strand with the ends restrained is given by

\[ \frac{N}{\varepsilon} = A \]  

(4.40)

The ratio between the axial force in the core wire and the axial force in a helical wire is given by

\[ \frac{N_c}{N_h} = \frac{A_c}{A_h} \frac{1}{\cos^2 \beta} \]  

(4.41)

where \( A_c \) and \( A_h \) are the cross-sectional areas of the core wire and the helical wire, respectively and \( \beta \) is the lay angle of the helical wire. For a lay angle of 7.9° in the subject strands (typical value), Equation (4.41) predicts that \( N_c \) is only 1% larger than \( N_h \). The effects of bending and twisting moments in the helical wires can be expressed as

\[ \frac{\sigma_b^{\text{max}}}{\sigma^a} = \frac{2\hat{r} \sin^2 \beta}{R} \]  

(4.42)

\[ \frac{\tau_t^{\text{max}}}{\sigma^a} = \frac{G \hat{r} \sin 4\beta}{4RE \cos^2 \beta} \]  

(4.43)

where \( \sigma_b^{\text{max}} \) is the maximum cross-sectional normal stress due to the bending moment; \( \tau_t^{\text{max}} \) is the maximum cross sectional shear stress due to twisting moment and \( \sigma^a \) represents the uniform normal stress due to the axial force.

In equations (4.42) and (4.43), \( \hat{r} \) is the radius of the helical wire, and \( E \) and \( G \) are the material Young’s modulus and shear modulus, respectively.

For the test strand (whose characteristic are described in detail in section 6.1), equations (4.42) and (4.43) yield the values of \( \frac{\sigma_b^{\text{max}}}{\sigma^a} = 1.5 \times 10^{-2} \) and \( \frac{\tau_t^{\text{max}}}{\sigma^a} = 10^{-5} \), indicating that bending and twisting stresses in the helical wires can be neglected. The test strands considered in Chapter
6 can be therefore assumed subject to an axial load distributed uniformly over the wires cross-section and producing only a state of uniform normal stress, $\sigma$, in each individual wire.

**4.8 Contact Area Analysis – Hertz Theory and Numerical Results**

Stress distribution and contact area can then be estimated according to Hertz theory that models the contact interaction between two frictionless elastic solids that are smooth and can be described locally with orthogonal radii of curvature such as a toroid. Further, the size of the actual contact area must be small compared to the dimensions of each body and to the radii of curvature (non-conforming contact). Hertz made the assumption based on observations that the contact area is elliptical in shape for such three dimensional bodies. The equations simplify when the contact area is circular such as with spheres in contact. At extremely elliptical contact, the contact area is assumed to have constant width over the length of contact such as between parallel cylinders.

Referring to Figure 4.9, directions $r$ and $y$ define, respectively, the horizontal axis (contact arc direction) and the vertical axis (cylinder radial direction). The distribution of pressure exerted between the two solids under a force per unit length $P$ is given by:

$$\sigma_z(r) = \frac{2P}{\pi a} \sqrt{1 - \frac{r^2}{a^2}}$$

(4.44)

The half length of the contact arc can be evaluated with the following expression

$$a = \sqrt{\frac{8P(1-v^2)}{\pi E} \frac{l_{c}\cdot l_{b}}{l(l_{c} + l_{b})}}$$

(4.45)

where $l$ is the length of contact along the axis of the wires. From Equation (4.45) it is observed that the contact length $a$ is proportional to the square root of the resultant $P$. 
A set of 2-D non-linear finite element simulations were performed using ABAQUS/Implicit to compute contact areas and stresses between the cross-section of the wires and to confirm theoretical results.

### 4.8.1 ABAQUS Simulation of Contact between Two Wires

A first simulation was carried out by modeling two of the seven wires comprising the 0.6-in strand, as shown in Figure 4.10. The purpose of this analysis was to determine the level of mesh refinement to properly represent the interwire contact stresses by comparison with the Hertzian theoretical results. The interwire radial force was calculated according to Equation (4.22) where the radius of curvature of the helical wire, $\rho'$, is given in Equation (4.9). For example, for the subject strand loaded at 70% of U.T.S. (Ultimate Tensile Strength), the helical wire radius of curvature is $\rho' = 0.27$ m, and the helical wire axial force is $N_h = 21$ kN, resulting in an interwire force of $P \approx 96.5$ kN. The length of the contact arc from Hertz theory in this case is $a = 0.073$ mm.

![Reference system of Hertz contact problem](image)

**Figure 4.9** – Reference system of Hertz contact problem.
The contact model was treated as a classical case of non-linear geometry using ABAQUS Newton-Raphson algorithm. Triangular, 3-node plane-strain elements were used in the model with two degrees of freedom per node. Five meshes (Meshes 1 through 5 in Table 4.1) were performed with a decreasing level of refinement in the contact area to find the convergence with the theoretical results.

It was found that Mesh 4 provided satisfactory convergence with the theoretical results. A final mesh configuration, Mesh 6 in Table 4.1, was then considered to further reduce the number of elements from Mesh 4 away from the contact area. Mesh 6 was then chosen as the optimum mesh having the minimum number of elements which satisfied convergence of results. Figure 4.11 shows the discretized model for the six different meshes considered.
Table 4.1 - Mesh considered in convergence study of two-core contact problem in ABAQUS FE simulations.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Mesh 2</th>
<th>Mesh 3</th>
<th>Mesh 4</th>
<th>Mesh 5</th>
<th>Mesh 6*</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of elements along contact arc</td>
<td>935</td>
<td>372</td>
<td>138</td>
<td>108</td>
<td>76</td>
</tr>
<tr>
<td>No. of elements along wire radius</td>
<td>81</td>
<td>63</td>
<td>25</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Total no. of elements in single wire</td>
<td>6653</td>
<td>5381</td>
<td>2253</td>
<td>1890</td>
<td>1774</td>
</tr>
</tbody>
</table>

*optimum mesh.

Figure 4.11 - Discretizations considered for convergence study of two-wire contact problem in ABAQUS FE simulations.

Next Figure 4.12 shows the values of the relevant stresses calculated from the different mesh sizes and compared with the theoretical Hertzian predictions at the final load step of 75% U.T.S. or 1.4 GPa. The corresponding unit-length contact force was $P = 7$ MN/m as discussed above. The plots in Figure 4.12 (a) and (b) show the two stresses $\sigma_z$ and $\sigma_r$ along the contact arc direction, $r$. It can be seen in these plots that mesh 6 (purple curve) provides the necessary
convergence with theory (blue curve) while minimizing the number of elements i.e. the computational expense. It can also be seen that the half-length of the contact arc predicted by the Mesh 6 model, \(a_{75\%U.T.S.} \sim 3.4 \times 10^{-4} \text{ m}\), is close to the Hertzian value of \(a_{75\%U.T.S.} = 3.10 \times 10^{-4} \text{ m}\). Mesh 5 (yellow curve) contains too few elements in the contact zone and it overestimates the stress values as well as the length of the contact arc.

Figure 4.12 (c) and (d) show \(\sigma_z\) and \(\sigma_r\) along the radial direction of the wires, \(y\). The stress \(\sigma_z\) decays past a radial depth \(\sim 3a\), whereas \(\sigma_r\) decays faster, disappearing past a depth of \(\sim a\). Set aside Figure 4.12 (c), where all FE results seem to overestimate the theoretical stresses, these plots confirm the appropriateness of Mesh 6 for the contact problem.

In next Figure 4.13 we can find the value of the Von Mises equivalent stress calculated by the Mesh 6 model for increasing levels of axial stress applied to the strand, namely 25%,
50% and 75% of the strand’s material U.T.S. (last step of contact force $P = 7$ MN/m). It can be seen in this figure that the stress concentration is confined to the contact area as expected from the theory. It can also be seen that the extent of the region affected by the contact increases with increasing applied force.

![Figure 4.13 - Von Mises equivalent stress obtained from ABAQUS two-wire contact simulation (mesh 6) at increasing levels of applied axial stress.](image)

### 4.8.2 ABAQUS Simulation of Contact between Three Wires

Once an appropriate mesh size was chosen to model the most critical area surrounding the contact arc, the attention was focused on the entire strand cross section.

The particular load conditions induced by the axial load (i.e. resultant interwire force $P$ every 60 degree angle) allowed to exploit the symmetry of the strand cross section. As shown in
next Figure (4.14), only a 60 degree circular sector comprising the central wire and two peripheral wires was analyzed and appropriate boundary conditions were assumed.

According to the theory of Machida – Durelli [60], two interwire stresses were applied as shown in Figure (4.14) corresponding to a 70% U.T.S. axial load level. The analysis was treated as a quasi-static problem and so it was developed using the ABAQUS Standard module. The results are depicted in next Figures (4.15), (4.16) and (4.17).

Figure 4.14 – Schematic representation of Three-wire contact problem studied by ABAQUS.

Figure 4.15 – Contour Plot of Von Mises equivalent stresses for axial load equal to 70% of ultimate load.
Figure 4.16 – Contour Plot of Von Mises equivalent stresses for axial load equal to 70% of ultimate load (no edges).

Figure 4.17 – Isolinear Plot of Von Mises equivalent stresses for axial load equal to 70% of ultimate load.

The results confirmed that stresses are mainly concentrated in the contact areas. As predicted by Hertz theory, stresses along the radial and normal directions decay past a radial depth \(\sim 3a\).
The ABAQUS Analysis results are presented in a detailed way in next Figures (4.12) – (4.13) in terms of contour plot and isolines plot, respectively.

**Figure 4.18** – Zoomed view on the contact zone representing a contour plot of the distribution of Von Mises equivalent stresses.

**Figure 4.19** - Zoomed view on the contact zone representing a isolinear plot of the distribution of Von Mises equivalent stresses.
Chapter 5

Nonlinearity in Ultrasonic Tests

5.1 General Aspects

Nonlinearity is a frequent visitor to engineering structures which can modify - sometimes catastrophically - the design behavior of the systems. This phenomenon affects almost every engineering fields and it is therefore desirable a better understanding of its causes and its consequences.

First, an application in civil engineering. Many demountable structures such as grandstands at concerts and sporting events are prone to substantial structural nonlinearity as a result of looseness of joints, this creates both clearances and friction and may invalidate any linear-model-based simulations of the behavior created by crowd movement. A second case comes from aeronautical structural dynamics; there is currently major concern in the aerospace industry regarding the possibility of limit cycle behavior in aircraft, i.e. large amplitude coherent nonlinear motions. The implications for fatigue life are serious and it may be that the analysis of such motions is as important as standard flutter clearance calculations. There are numerous examples from the automotive industry; brake squeal is an irritating but non-life-threatening example of an undesirable effect of nonlinearity. Many automobiles have viscoelastic engine mounts which show marked nonlinear behavior: dependence on amplitude, frequency and preload.
The distinction between linear and nonlinear systems is important; nonlinear systems can exhibit extremely complex behavior which linear systems cannot. The most spectacular examples of this occur in the literature relating to chaotic systems; a system excited with a periodic driving force can exhibit an apparently random response.

In contrast, a linear system always responds to a periodic excitation with a periodic signal at the same frequency. At a less exotic level, but no less important for that, the stability theory of linear systems is well understood; this is emphatically not the case for nonlinear systems. It is important to note that the subject of nonlinearity is extremely broad and extensive literature exists. In this chapter it’s firstly presented a brief discussion on linear systems and later the attention is focused on nonlinear phenomena concerning ultrasonic tests, object of the present research.

5.2 Linear Systems

Following [61], a system is any process that produces an output signal in response to an input signal. This is illustrated by the block diagram in Figure 5.1. Continuous systems input and output continuous signals, such as in analog electronics. Discrete systems input and output discrete signals, such as computer programs that manipulate the values stored in arrays.

Figure 5.1 – Schematic representation of Continuous and Discrete Linear Systems.
A system is called linear if it respects two mathematical principles: superposition (or additivity) and homogeneity. If one can show that a system has both properties, then you it has been proven that the system is linear. Likewise, if one can show that a system doesn't have one or both properties, it has been proven that it isn't linear. A third property, shift invariance, is not a strict requirement for linearity, but it is a mandatory property for most techniques (Digital Signal Processing, etc...). These three properties form the mathematics of how linear system theory is defined and used.

5.2.1 The Principle of Superposition

The principle of superposition is more than a property of linear systems; in mathematical terms it actually defines what is linear and what is not. The principle of superposition can be applied statically or dynamically and simply states that the total response of a linear structure to a set of simultaneous inputs can be broken down into several experiments where each input is applied individually and the output to each of these separate inputs can be summed to give the total response.

This can be stated precisely as follows. If a system in an initial condition $S_i = \{y_i(0), \dot{y}_i(0)\}$ responds to an input $x_i(t)$ with an output $y_i(t)$ and in a separate test an input $x_2(t)$ to the system initially in state $S_2 = \{y_2(0), \dot{y}_2(0)\}$ produces an output $y_2(t)$ then superposition holds if and only if the input $\alpha x_1(t) + \beta x_2(t)$ to the system in initial state $S_3 = \{\alpha y_1(0) + \beta y_2(0), \alpha \dot{y}_1(0) + \beta \dot{y}_2(0)\}$ results in the output $\alpha y_1(t) + \beta y_2(t)$ for all constants $\alpha, \beta$, and all pairs of inputs $x_1(t), x_2(t)$. In words, signals added at the input produce signals that are added at the output. This property is depicted in next Figure 5.2.
Despite its fundamental nature, the principle offers limited prospects as a test of linearity. The reason being that in order to establish linearity beyond doubt, an infinity of tests is required spanning all $\alpha, \beta, x_1(t)$ and $x_2(t)$. This is clearly impossible. However, to show nonlinearity without doubt, only one set of $\alpha, \beta, x_1(t)$ and $x_2(t)$ which violate superposition are needed. In general practice it may be more or less straightforward to establish such a set.

5.2.2 Homogeneity

As illustrated in Figure 5.3, homogeneity means that a change in the input signal's amplitude results in a corresponding change in the output signal's amplitude. In essence, homogeneity is an indicator of the system’s insensitivity to the magnitude of the input signal. In mathematical terms, if an input signal of $x[n]$ results in an output signal of $y[n]$, an input of $kx[n]$ results in an output of $ky[n]$, for any input signal and constant, $k$. 

**Figure 5.2** - Schematic representation of the Principle of Superposition.
This property represents a restricted form of the principle of superposition and it is undoubtedly the most common method in use for detecting the presence of nonlinearity in dynamic testing.

The important point in this last property is that added signals pass through the system without interacting. The most striking consequence of this is in the frequency domain. Considering the two input and output functions in the time domain $x(t)$ and $y(t)$ and the corresponding two representations in the frequency domain $X(\omega)$ and $Y(\omega)$ ($\text{FRF} = \text{Frequency Response Function}$), it’s possible to note that $\alpha x(t) \rightarrow \alpha y(t)$ implies $\alpha X(\omega) \rightarrow \alpha Y(\omega)$. This means that if $x(t) \rightarrow \alpha x(t)$,

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \rightarrow \frac{\alpha Y(\omega)}{\alpha X(\omega)} = H(\omega)$$

(5.1)

and the FRF is invariant under changes of $\alpha$ or effectively of the level of excitation.

Because of this, the homogeneity test is usually applied in dynamic testing to FRFs where the input levels are usually mapped over a range encompassing typical operating levels. If the FRFs for different levels overlay, linearity is assumed to hold. This is not infallible as there are
some systems which are nonlinear which nonetheless show homogeneity. The reason for this is that homogeneity is a weaker condition than superposition.

An example of the application of a homogeneity test is shown in Figure 5.4. In this case band-limited random excitation has been used but, in principle, any type of excitation signal may be employed. Although a visual check is often sufficient to see if there are significant differences between FRFs, other metrics can be used such as a measure of the mean-square error between the FRFs.

![Figure 5.4 – Application of a homogeneity test on a real structure. The close agreement of the results is an indicator that the structure is linear within the test bounds.](image)

### 5.2.3 Shift Invariance

As shown in next Figure 5.5, *shift invariance* means that a shift in the input signal will result in nothing more than an identical shift in the output signal. In more formal terms, if an input signal of \( x[n] \) results in an output of \( y[n] \), an input signal of \( x[n + s] \) results in an output of \( y[n + s] \), for any input signal and any constant, \( s \).
Shift invariance is important because it means the characteristics of the system do not change with time (or whatever the independent variable happens to be).

\[ \text{IF} \]

\[ \begin{array}{c}
\text{x}[n] \\
\downarrow \\
\text{System} \\
\downarrow \\
y[n]
\end{array} \]

\[ \text{THEN} \]

\[ \begin{array}{c}
\text{x}[n+s] \\
\downarrow \\
\text{System} \\
\downarrow \\
y[n+s]
\end{array} \]

**Figure 5.5** - Schematic representation of shift invariance property.

### 5.2.4 Static Linearity and Sinusoidal Fidelity

Homogeneity, superposition, and shift invariance are important because they provide the mathematical basis for defining linear systems. Unfortunately, these properties alone don't provide most scientists and engineers with an intuitive feeling of what linear systems are about.

The properties of static linearity and sinusoidal fidelity are often of help here. The last one, in particular, is the property widely used to identify nonlinearities in ultrasonic tests, as better described later on.

*Static linearity* defines how a linear system reacts when the signals aren't changing. The static response of a linear system is very simple: the output is the input multiplied by a constant. That is, a graph of the possible input values plotted against the corresponding output values is a straight line that passes through the origin. This is shown in Figure 5.6 for two common linear systems: Ohm's law for resistors and Hooke's law for springs. For comparison, Figure 5.7 shows the static relationship for two nonlinear systems: a pn junction diode, and the magnetic properties of iron.
All linear systems have the property of static linearity. The opposite is usually true, but not always. There are systems that show static linearity, but are not linear with respect to changing signals. However, a very common class of systems can be completely understood with static linearity alone. In these systems it doesn't matter if the input signal is static or changing. These are called memoryless systems, because the output depends only on the present state of the input, and not on its history. For example, the instantaneous current in a resistor depends only on the instantaneous voltage across it, and not on how the signals came to be the value they are. If a
system has static linearity, and is memoryless, then the system must be linear. This provides an important way to understand (and prove) the linearity of these simple systems.

The other important characteristic of linear systems is how they behave with sinusoids, a property it’s generally called *sinusoidal fidelity*; If the input to a linear system is a sinusoidal wave, the output will also be a sinusoidal wave, and at exactly the same frequency as the input. Sinusoids are the only waveforms that have this property. For instance, there is no reason to expect that a square wave entering a linear system will produce a square wave on the output. Although a sinusoid on the input guarantees a sinusoid on the output, the two may be different in amplitude and phase.

### 5.3 Nonlinear Systems

Nonlinear behavior is pervasive in mechanical systems. In fact, it is fair to state that linear behavior is just an idealization of the many nonlinear phenomena that influence dynamic response. It is equally fair and important to add that assuming linear behavior quite often leads to successful estimates of dynamic response. Moreover, the analytical methods made possible through linearization form the foundation for classical vibration analysis and companion experimental methods as well; refer, for example, to entries under Modal analysis, experimental. Thus, the central issue is not whether a phenomenon is nonlinear or not, but rather whether the nonlinearities ultimately influence the response in any significant manner.

To appreciate immediately the effect of nonlinearities, we can consider the possible responses that may be exhibited by the single-degree-of-freedom system (SDOF) illustrated in Figure 5.8 to sustained harmonic excitation assuming that the nonlinearity derives solely from the nonlinear behavior of a spring, as might be realized by either material nonlinearities or geometric nonlinearities, as discussed in the following sections.
In particular, the spring-restoring force, \( F(x) = k_1x + k_3x^3 \), is a nonlinear function of the stretch \( x \), and the stiffness of this spring, \( k(x) = dF/dx = k_1 + 3k_3x^2 \), either monotonically decreases or increases with the stretch, depending on the sign of the ratio \( k_3/k_1 \). A spring whose stiffness decreases with stretch is said to exhibit *softening* behavior, while one whose stiffness increases with stretch is said to exhibit *hardening* behavior (refer to Figure 5.9 and examples that follow). In either case, the nonlinearity can produce motions that are qualitatively distinct from those of the approximate linear system, that is, the system for which \( k_3 = 0 \).

This fact is depicted in Figure 5.10 which shows the long-term response (the response after transients have decayed) of the approximate linear system and those possible for the nonlinear system subject to the same harmonic excitation. For the linear system, harmonic input
generates a harmonic response with the very same frequency as the input. The amplitude and phase of this harmonic response are controlled by nearness to resonance, that is, how close the excitation frequency $\Omega$ is to the natural frequency $\omega_n = \sqrt{k_i/m}$.

By contrast, the response of the nonlinear system can be periodic (harmonic as an example) or aperiodic. Among possible periodic responses are the primary resonant response (a modification of the sole resonant response of the linear system), and secondary resonant responses, including subharmonic and superharmonic resonant responses. The secondary resonant responses include response frequencies that are related to the excitation frequency by $(p/q)\Omega$, where $p$ and $q$ are positive integers.

Among possible aperiodic responses are quasiperiodic responses (motions with periodically modulated amplitude and/or phase) and chaotic responses. In addition, multiple responses of the nonlinear system may coexist; that is, multiple solutions exist (in general) to the governing equation of motion. By contrast, solution uniqueness is guaranteed for the approximate linear system. Thus, even the prototypical single-degree-of-freedom nonlinear system illustrated here may exhibit significantly different responses than its linear counterpart. These differences become even greater with the inclusion of additional degrees-of-freedom.

![Figure 5.10](image_url) - A comparison of possible responses of linear and nonlinear single-degree-of-freedom systems to harmonic excitation.
5.3.1 Symptoms of Nonlinearity

As stated at the end of the last chapter, many of the properties which hold for linear structures or systems break down for nonlinear. This section discusses some of the more important ones.

5.3.1.1 Harmonic Distortion

Considering the FFT (Fast Fourier Transform) of the response of a given system, harmonic or waveform distortion is one of the clearest indicators of the presence of nonlinearity. It is a straightforward consequence of the principle of superposition. If the excitation to a linear system is a monoharmonic signal, i.e. a sine or cosine wave of frequency $\omega$, the response will be monoharmonic at the same frequency (after any transients have died out). The proof is elementary and proceeds as follows [62].

Suppose $x(t) = \sin(\omega t)$ is the input to a linear system. First of all, it is observed that $x(t) \to y(t)$ implies that $\dot{x}(t) \to \dot{y}(t)$ and $\ddot{x}(t) \to \ddot{y}(t)$. This is because superposition demands that

$$
\frac{x(t + \Delta t) - x(t)}{\Delta t} \to \frac{y(t + \Delta t) - y(t)}{\Delta t}
$$

(5.2)

and $\dot{x}(t) \to \dot{y}(t)$ follows in the limit as $\Delta t \to 0$ (it’s possible to note that there is also an implicit assumption of shift invariance here, namely that $x(t) \to y(t)$ implies $x(t + \tau) \to y(t + \tau)$ for any $\tau$). Again by superposition,

$$
x_1(t) + \omega^2 x_2(t) \to y_1(t) + \omega^2 y_2(t)
$$

(5.3)

Hence taking

$$
x_1(t) = \ddot{x}(t) \text{ and } x_2(t) = \dot{x}(t)
$$

gives:
\( \ddot{x}(t) + \omega^2 x(t) \rightarrow \ddot{y}(t) + \omega^2 y(t) \)  \hspace{1cm} (5.4)

Now, as \( x(t) = \sin(\omega t) \),

\( \ddot{x}(t) + \omega^2 x(t) = 0 \)  \hspace{1cm} (5.5)

In the steady state, a zero input to a linear system results in a zero output. It therefore follows from Equation (5.4) that

\( \ddot{y}(t) + \omega^2 y(t) = 0 \)  \hspace{1cm} (5.6)

and the general solution of this differential equation is

\( y(t) = A \sin(\omega t - \phi) \)  \hspace{1cm} (5.7)

and this establish the result.

This proof is rather interesting as it only uses the fact that \( x(t) \) satisfies a homogeneous linear differential equation to prove the result. The implication is that any such function will not suffer distortion in passing through a linear system.

It is not a corollary of this result that a sine-wave input to a nonlinear system will not generally produce a sine-wave output; however, this is usually the case and this is the basis of a simple and powerful test for nonlinearity as sine waves are simple signals to generate in practice. As mentioned before, this is the main characteristic to identify nonlinearity in experimental and numerical results presented in next Chapters 6 – 7.

As better described in next sections, the change in form is due to the appearance of high-order harmonics in the response such as \( \sin(3\omega t) \), \( \sin(5\omega t) \) etc. Distortion can be easily detected on an oscilloscope by observing the input and output time response signals. Figure 5.11 shows an example of harmonic waveform distortion where a sinusoidal response signal is warped due to nonlinearity.
Figure 5.11 – Response signals from a nonlinear system showing clear distortion only on the acceleration signal.

In the last Figure 5.11 the output response from a nonlinear system is shown in terms of the displacement, velocity and acceleration. The reason that the acceleration is more distorted compared with the corresponding velocity and displacement is easily explained. Let \( x(t) = \sin(\omega t) \) be the input to the nonlinear system. As previously stated, the output will generally be represented as a Fourier series composed of harmonics written as

\[
y(t) = A_1 \sin(\omega t - \phi_1) + A_2 \sin(2\omega t - \phi_2) + A_3 \sin(3\omega t - \phi_3) + \ldots
\]  

(5.8)

and the corresponding acceleration is

\[
y(t) = -\omega^2 B_1 \sin(\omega t - \phi_1) - 4\omega^2 B_2 \sin(2\omega t - \phi_2) - 9\omega^2 B_3 \sin(3\omega t - \phi_3) - \ldots
\]  

(5.9)

Thus the \( n \)th output acceleration term is weighted by the factor \( n^2 \) compared to the fundamental.
If non-sinusoidal waveforms are used, such as band-limited random signals, waveform distortion is generally impossible to detect and additional procedures are required such as the coherence function described in [62].

5.3.2 Sources of Nonlinearities

Nonlinearities arise from virtually all forces that govern the motion of mechanical systems. They may manifest themselves in the forces contributing to system stiffness, damping, and inertia. To appreciate the many ways in which nonlinearities arise, we can consider a general continuum illustrated in Figure 5.12.

![Figure 5.12 - General continuum subject to applied body forces b(x,t) and surface tractions T(x,t).](image)

Let \((x_1, x_2, x_3)\) be the components of a Cartesian triad and \((u_1, u_2, u_3)\) be the associated components of the three-dimensional displacement field of the continuum from some reference configuration. The body occupies the three-dimensional domain \(D\) bounded by the surface \(B\). The motion of the body, as described by the displacement components, is determined by the three momentum equations:
\[ \tau_{ij,\nu} + f_i = \rho \ddot{u}_i \quad \text{in } D \]  
(5.10)
and the boundary conditions:
\[ \tau_{ij} n_j = T_i \quad \text{on } B_1 \]  
(5.11)
\[ u_i = U_i \quad \text{on } B_2 \]  
(5.12)
where the index \( i=1, 2, 3, \) \( j \) denotes partial differentiation with respect to the independent spatial variable \( x_j \), \( \ddot{u}_i \) denotes total differentiation with respect to time (twice), and summation of repeated indices is implied.

Here, \( \tau_{ij} \) is the (Cauchy) stress tensor, \( f_i \) are components of applied body forces, \( \rho \) is the material density, \( n_j \) are components of the unit normal vector (outward) to the boundary \( B_1 \) on which tractions with components \( T_i \) are applied, and \( U_i \) denotes prescribed displacements on the remainder of the boundary \( B_2 \). This general formulation, which applies to any continuum (e.g., solids and fluids), includes all common structural elements (shells, plates, beams, etc.) as special cases and serves as a means to introduce major classes of nonlinearities.

Consider first nonlinearities introduced through the stress tensor \( \tau_{ij} \). These nonlinearities are referred to either as material nonlinearities or as geometric nonlinearities, depending upon how they arise in \( \tau_{ij} \). Material nonlinearities arise from nonlinear material laws (constitutive equations) that relate the stress tensor to the strain tensor (or derivatives thereof). By contrast, geometric nonlinearities arise from nonlinear strain-displacement relations. Thus, a linearly elastic material may exhibit nonlinear response when (finite) strain measures are used that capture the effect of geometrically large deformations. Several examples of material and geometric nonlinearities follow below. The body forces \( f_i \) become a source of nonlinearity whenever they are a nonlinear function of the displacement field (or derivatives thereof).
Examples may include magnetic and electrostatic forces and fluid/structure interaction forces (when treated as body forces). The inertia term on the right-hand side of Equation (5.1) may also introduce nonlinearities as seen in fluid mechanics (Navier-Stokes equation) where an Eulerian description of fluid motion is employed. Continuing to Equation (5.2), note that the tractions $T_i$ applied on the traction boundary may also be nonlinearly related to the displacement field (and derivatives thereof), with common examples being fluid/structure interaction forces (when treated as surface tractions), surface friction, and impact. The displacement boundary Equation (5.3) may also introduce nonlinearities as, for instance, in the case of clearance problems where traction boundary conditions are suddenly replaced by displacement boundary conditions.

Finally, the domain $D$ may depend on the displacement field and thereby capture the nonlinearities that arise, for example, in fluid/structure interaction problems with free surfaces or some contact problems with deformable bodies.

### 5.3.3 Classes of Nonlinearities

Many classifications of nonlinearities are possible, with one being based upon how they enter the equations of motion, as described above. An alternative classification may be based on the mathematical forms these nonlinearities take, including analytic forms (e.g., most geometric and material nonlinearities) versus nonanalytic (nondifferentiable) forms (e.g., dry friction and impact nonlinearities). In this section nonlinearities associated with four major sources for mechanical systems will be described including: (1) geometrically large motion, (2) nonlinear material behavior, (3) fluid/structure interaction, and (4) friction and impact. Each of the four major categories of nonlinearities is reviewed below.
The attention should be focused essentially on the last form of nonlinearity since, as better described in next Chapters 6 – 7, it constitutes the most plausible cause from where nonlinear effects arise.

5.3.3.1 Geometrically Large Motion

Geometrically large motions may introduce nonlinearities through inertia, damping, and stiffness terms. For the general continuum described above, geometric nonlinearities enter through the nonlinear strain-displacement relations associated with finite strain measures. For instance, consider the prototypical example of a flexible beam undergoing deformation in the plane, as depicted in Figure 5.13. Let \( x_i \) denote the independent spatial variable along the undeformed neutral axis of the beam and let \( u_1 \) and \( u_2 \) denote the longitudinal and transverse components of the displacement field of the neutral axis.

Using Kirchhoff assumptions for beam deformation, the strain of a beam element becomes:

\[
\varepsilon(x_1, x_2, t) = \varepsilon_0(x_1, t) - x_2 k(x_1, t)
\]  

(5.13)

where:

\[
\varepsilon_0(x_1, t) = u_{1,1} + \frac{1}{2}(u_{1,1}^2 + u_{2,1}^2)
\]  

(5.14)

represents the strain of the neutral axis and \(-x_2 k(x_1, t)\) represents the additional strain at the level \( x_2 \) from the neutral axis due to the curvature \( k(x_1, t) \) of the neutral axis. The curvature of the neutral axis is given by:

\[
k(x_1, t) = \frac{u_{2,1}}{(1 + u_{2,1}^2)^{3/2}}
\]  

(5.15)
Figure 5.13 - (A) Geometrically large planar motion of a beam as measured by the displacements $u_1(x_1,t)$ and $u_2(x_2,t)$ of the neutral axis. (B) Simply supported beam for which the dominant nonlinearity derives from the stretching of the neutral axis. (C) Cantilevered beam for which the dominant nonlinearity derives from large rotations of the beam cross-sections.

Inspection of the strain-displacement relation, Equation (5.13) with Equation (5.14) and Equation (5.15), reveals two sources of geometric nonlinearity for the beam. The first of these is the nonlinear strain-displacement relationship (5.14) that describes the stretching of the beam neutral axis, and the second is the nonlinear curvature-displacement relationship (5.15) describing large rotation of the beam cross-section. Either or both of these nonlinear mechanisms may arise in a particular application. For instance, in the case of the simply supported beam shown in Figure 5.13(B), the neutral axis must stretch as the beam deflects and the dominant nonlinear mechanism in this case is associated with neutral axis stretching (Equation (5.14)).
By contrast, large rotation (Equation (5.15)) represents the dominant nonlinear mechanism for the cantilevered beam of Figure 5.13(C). Here, the beam centerline may not stretch appreciably, but the cross-sections may rotate through moderate to large angles. This effect is captured in the classical problem of the elastica.

The discussion above for beams forms a special case of the general topic of the theory of rods. Qualitatively similar geometrical nonlinearities arise in other structural elements, including cables, membranes, plates, and shells. For instance, von Karman plate theory captures the nonlinearities produced by the stretching of the neutral plane of the plate in analogy to the effect modeled for the beam by Equation (5.14) above. It should also be noted that the geometric nonlinearities of structural elements can exhibit hardening or softening effects or both. Consider for instance the geometric nonlinearities produced by the stretching of the centerline of a sagged flexible cable, as shown in Figure 5.14. We can note that the strain of the centerline will initially decrease (i.e., tension will decrease) if a load $F$ is directed along the normal (i.e., towards the center of curvature). Thus the cable can exhibit softening behavior.

![Figure 5.14 – Sagged cable subjected to a central load.](image)

By contrast, hardening behavior is immediately observed if the direction of the load is reversed (applied away from the center of curvature). Thus, the overall force-deflection behavior of the cable is asymmetric (describable by both even and odd functions of deflection), as illustrated in Figure 5.15. This structural asymmetry is generic to all structural elements that are precurved (e.g., arches and shells).
5.3.3.2 Nonlinear Material Behavior

As discussed above, material nonlinearities may arise in Equation (5.10) and Equation (5.11) through the nonlinear dependence of the stress on the strain (and/or strain rate) measure. Common examples of materials that obey nonlinear material (constitutive) laws include ductile metals that yield, elastomeric materials (e.g., most rubbers), viscoelastic materials (e.g., rubbery compounds and polymers), biomaterials (membranes, skin, collagen fibers, muscle tissue), and various 'new' materials, including rheological fluids, shape memory alloys, etc. For the purpose of this overview, we shall consider two broad classes of nonlinear material behavior by distinguishing materials that are dissipative from those that are not.

The nondissipative materials are nonlinearly elastic and may exhibit hardening characteristics, softening characteristics or a combination of both. As a simple example, consider the scalar constitutive law:

\[ \tau = f(\varepsilon) \]  

(5.16)

which might be expanded in a Taylor series in the strain measure \( \varepsilon \) (about a stress-free state):

\[ \tau = f'(0)\varepsilon + \frac{1}{2} f''(0)\varepsilon^2 + \frac{1}{6} f'''(0)\varepsilon^3 + \ldots \]  

(5.17)

in which \( ()' \) denotes differentiation with respect to \( \varepsilon \).
In the expansion (Equation (5.17)), the quantity $f'(0) = E$ would be identified as Young's modulus. Many nonlinearly elastic materials are 'symmetric' in their material response in that the stress changes sign when the strain changes sign (but remains the same magnitude). For these materials, Equation (5.16) and Equation (5.17) are odd functions of the strain (hence the coefficient $f''(0)$ would vanish in Equation (5.17). The constitutive law (Equation (5.16) or Equation (5.17)) for a typical elastomeric material might appear as shown in Figure 5.16 and exhibit an initial softening behavior for small to moderate strains followed by hardening behavior for larger strains.

![Figure 5.16 – Nonlinear stress-strain relationship for an elastomeric material.](image)

Nonlinear material laws for dissipative materials include those materials exhibiting viscoelastic effects (dissipation produced by strain-rate dependency) as well as plastic effects (dissipation produced by material hysteresis). For instance, a general (scalar) model for a nonlinear viscoelastic material could be written:

$$\tau(\varepsilon, \dot{\varepsilon}) = f(\varepsilon, \dot{\varepsilon}) \varepsilon + g(\varepsilon, \dot{\varepsilon}) \dot{\varepsilon}$$  \hspace{1cm} (5.18)

in which the 'elastic' contribution $f$ and the 'visco' contribution $g$ could depend explicitly on the strain, strain rate, or both. Alternative (integral) material laws for viscoelastic materials have also been proposed that capture material memory effects. Materials exhibiting elastic-plastic behavior develop dissipation through the hysteresis loop associated with cyclical loading and unloading.
For instance, consider the material law for an ideal elastic/plastic solid, as depicted in Figure 5.17. Starting from the undeformed state, the material exhibits linearly elastic behavior up to the yield point, beyond which it exhibits ideal plastic behavior. Upon unloading, the behavior is first elastic then plastic, leading to the illustrated hysteresis loop.

![Figure 5.17 – Nonlinear stress-strain relationship for an ideal elastic-plastic material.](image)

### 5.3.3.3 Fluid/Structure Interaction

Fluids surrounding a structure exert pressure (modeled either as body forces or tractions, following the discussion above) that ultimately influence the inertia and/or damping of the structure. In the present section the attention is focused on two common types of fluid/structure interactions that are governed by nonlinearities (recognizing that other nonlinear interactions also exist).

The first example will focus on the Morison relation that accounts for the added mass and nonlinear drag imparted to a structure in a surrounding fluid (e.g., water). The second example will focus on a nonlinear wake oscillator model proposed to describe the phenomenon of vortex-induced vibrations of structures.

Consider first a body moving with some speed $\dot{x}$ in an otherwise quiescent fluid. The surrounding fluid provides both a source of added mass to the body as well as drag. These features are captured in the classical Morison relation (scalar form):
\[ F_f = C_a \rho_f V \ddot{x} + \frac{1}{2} C_d \rho_f A |\dot{x}| \dot{x} \]  

(5.19)

in which the force due to the fluid \( F_f \) captures the effects of added fluid mass (first term) and fluid drag (second term). Here, \( \rho_f \) is the fluid density, \( V \) is the volume of the body, \( A \) is the projected area of the body in the plane orthogonal to the relative motion, \( C_a \) is the added mass coefficient, and \( C_d \) is the drag coefficient. These coefficients are often determined by experiments. In employing Equation (5.19), it is tacitly assumed that the Reynolds number is high enough such that the drag force is proportional to the square of the relative velocity. For low Reynolds numbers in the Stokes flow regime, the drag is adequately represented as being proportional to the relative speed, rendering the drag force linear.

The drag force modeled by the Morison relation above is strictly dissipative. However, under certain flow conditions, a slender bluff body can actually extract energy from the flow and achieve sustained vibrations. The mechanisms responsible for this behavior trace to the fluid vortices shed from the surface of the bluff body and the resulting motion is often referred to as vortex-induced vibration.

Consider, for instance, the vortex-induced vibrations of the subsurface instrumentation array depicted in Figure 5.18. While no one model for vortex-induced vibration is used exclusively, a class of nonlinear models described as wake oscillator models has received considerable attention. These phenomenological models introduce a single oscillator equation to describe the fluctuating lift forces (lift coefficient) caused by vortex shedding and invariably include several empirical coefficients that must be fit to experimental data to produce estimates of vortex-induced vibration. The wake oscillator is then coupled to a model of the structure (typically, a single-degree-of-freedom model) in forming the two equations of coupled fluid/structure interaction.
Figure 5.18 - Fluid vortices shed may resonantly drive sustained structural vibrations as, for example, in this subsurface instrumentation array.

For instance, let $x(t)$ denote the modal coordinate for a vibration mode of a slender structure having natural frequency $\omega_n$ and damping ratio $\zeta$. Let $q$ denote the excitation to this mode produced by vortices shed with Strouhal frequency $\omega_s$ (a frequency that depends on the flow velocity, the diameter of the structure, and the Strouhal number). A typical model proposed for the coupling of the vibration response $x(t)$ and the fluid lift coefficient $q(t)$ is:

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \mu \omega_s^2 q$$  \hspace{1cm} (5.20)$$

$$\ddot{q} - \omega_s G \left( C_{Lo}^2 - 4q^2 \right) \dot{q} + \omega_s^2 q = \omega_s F \dot{x}$$  \hspace{1cm} (5.21)$$

in which $\mu$ is a mass ratio parameter and $G$, $C_{Lo}^2$, and $F$ are three empirically determined coefficients. While the 'structure equation' (Equation (5.20)) is assumed linear, the 'fluid equation' (Equation (5.21)) is of the form of a van der Pol equation with both the sign and magnitude of the damping determined by the magnitude of $q$.

Inspection of Equation (5.21) reveals that the damping is negative (energy increases) when $q$ is small and becomes positive (energy decreases) when $q$ is large. Thus, there is an intermediate value of $q$ that leads to a self-sustained oscillation (a limit cycle) that drives a sustained vortex-induced vibration, as measured by $x$. This self-sustained vibration is governing
by the coupling of Equations (5.20) and (5.21) and the strength of this coupling is determined by
the resonance condition \( \omega_s \approx \omega_n \).

5.3.3.4 Friction and Impact

Most of the nonlinear models discussed above are analytic (differentiable) and thus are
amenable for analysis using asymptotic methods. Exceptions to this are the ideal elastic-plastic
material law (see above) and the nonlinear drag model (see above) that are described by
nonsmooth functions. The same is true of commonly used nonlinear models for friction and
impact, as will be discussed here. In addition, the nonlinearities associated with friction and
impact have little to do with large-amplitude motion as they are frequently exercised even for the
relatively small-scale motions of mechanical assemblies.

Friction and impact are also the underlying causes of squeak-and-rattle problems that
remain a concern in many applications.

The study of friction has a long history, with contributions from prominent scientists
including Coulomb, who proposed the classical laws for dry friction still commonly used in
models of mechanical systems. As an example, we can consider the Coulomb friction law
illustrated in Figure 5.19 that describes the friction force developed between two surfaces as a
function of the relative (sliding) velocity.

![Coulomb friction law](image)

**Figure 5.19** - Classical Coulomb friction law. F and N are friction and normal forces acting between two
surfaces, with n being the relative (sliding) velocity.
For vanishing sliding velocity, the friction force that develops is that required to maintain equilibrium and is limited by a maximum value as determined by a coefficient of static friction $\mu_s$. Upon sliding, the friction force has a constant magnitude, as determined by a coefficient of dynamic friction $\mu_d$. While this description is attractive because it is simple, it brings with it difficulties in analysis. This friction model renders the acceleration discontinuous in time as the friction force is a discontinuous function of the sliding velocity. Numerical and analytical methods frequently have difficulties in resolving this discontinuity.

An important nonlinear effect produced by dry friction is friction-induced vibration. These vibrations are often small-amplitude, high-frequency oscillations that produce audible tones. Friction-induced vibrations have been observed in many systems, with classic examples being the sustained vibration of the violin string due to friction from the bow and the sustained tone produced by circulating a wet finger around the perimeter of a wine glass. Other examples include the squeal of disk brakes, the screeching of railway wheels guided by rails, and the squeak of a wiper blade on a drying windshield.

Most assemblies containing moving mechanical parts have unavoidable clearances and/or backlash. Like the ideal model for friction above, ideal models representing systems with clearances and backlash are nonsmooth and may also involve impacts. Examples of mechanical systems modeled in this manner include mating gears, seating valves, seating chains, and impacting heat exchanger tubes, print hammers, pile-drivers, impulsive drills, jack-hammers, and the like.

For instance, we can consider a classical model for backlash represented by the system of Figure 5.20. Here, a body (e.g., a particle) is free to move within a 'dead zone' of length $d$ prior to contacting elastic stops. The motion of the body could be induced in many ways, one of which is through prescribing the base motion $y(t)$, as in Figure 5.20. The simplest force/displacement model for this process would appear as a piecewise linear model, as illustrated in Figure 5.21.
Piecewise nonlinearities represent an important class of nonsmooth nonlinearities and are frequently used as approximations for systems with clearances and backlash. A variation on this theme is an impact oscillator which can be obtained from the above example as the limiting case of infinite (contact) stop stiffness, as depicted in Figure 5.22. The resulting motion-limiting rigid stops now form sites where impacts occur as the body rattles about in between.

The simplest model for this process would be to represent the contact forces as impulses that produce discontinuities in the particle's velocity. These discontinuous changes in velocity might be modeled by introducing a coefficient of restitution for the impact process.

![Figure 5.20](image1) – Simple model for backlash.

![Figure 5.21](image2) – Piecewise linear model for backlash (clearance).
5.3.4 Nonlinear Parameter $\beta$

The generation of high-order harmonics is the main nonlinear effect deeply analyzed in the present research in order to provide a valid technique to monitor the prestress level in strand. High-order harmonics are related to the so called nonlinearity parameter, $\beta$. This section gives a theoretical background in one dimensional nonlinear wave propagation and the derivation of this nonlinearity parameter. Contribution from lattice elasticity and dislocation dipoles are considered.

A difference between wave propagation in linear and nonlinear medium is depicted in Figure 5.23.

The fundamental wave will distort as it propagates, therefore the second and higher harmonics will be generated. Particularly the lattice anharmonicity and dislocation structures contribute to the nonlinearity parameter.
A longitudinal stress perturbation $\sigma$ associated with a propagating ultrasonic wave produces a longitudinal strain

$$\varepsilon = \varepsilon_e + \varepsilon_{pl} \quad (5.22)$$

in the material where $\varepsilon_{pl}$ is the plastic strain component associated with the motion of dislocation in the dipole configuration. The relation between the stress perturbation and elastic strain can be written in the nonlinear form of Hooke’s law (quadratic nonlinear approach)

$$\sigma = A_2^e \varepsilon_e + \frac{1}{2} A_3^e \varepsilon_e^2 + ... \quad (5.23)$$

or

$$\varepsilon_e = \frac{1}{A_2^e} \sigma - \frac{1}{2} \left( \frac{A_3^e}{A_2^e} \right)^2 \sigma^2 + ... \quad (5.24)$$

where $A_2^e$ and $A_3^e$ are the Huang coefficients.

According to [63], the relation between the stress perturbation and the plastic strain $\varepsilon_{pl}$ can be obtained from a consideration of dipolar forces. For edge dislocation pairs with opposite polarity, it is possible to write

$$F_x = -\frac{Gb^2}{2\pi(1-\nu)} \frac{x(x^2-y^2)}{(x^2+y^2)^2} \quad (5.25)$$

where $b$ is the Burgers vector, $\nu$ is Poisson’s ratio, $G$ is the shear modulus and $x, y$ are the Cartesian coordinates of one dislocation in the pair relative to the other. At equilibrium, $y=h$, where $h$ is the dipole height and the total shear force per unit length on the dipole is

$$F_x + bR\sigma = 0 \quad (5.26)$$

where $R$ is the Schmid factor along slip planes.

Also the relation between $\varepsilon_{pl}$ and the relative dislocation displacement $\xi = x - h$ is given by
\[ \varepsilon_{pl} = \Omega \Lambda_{dp} b \xi \]  

(5.27)

where \( \Omega \) is a conversion factor and \( \Lambda_{dp} \) is the dipole density.

Using these relationships among \( F_x, \sigma, \varepsilon_{pl}, \xi \) and an expansion of Equation (5.25) in a power series in \( x \) with respect to \( h \) leads to the following equation

\[ \sigma = A_{2}^{dp} \varepsilon_{pl} + \frac{1}{2} A_{3}^{dp} \varepsilon_{pl}^2 + ... \]  

(5.28)

where \( A_{2}^{dp} = -\frac{G}{4 \pi M \Lambda h^2 (1 - \nu)} \) and \( A_{3}^{dp} = \frac{G}{4 \pi \Omega^2 R \Lambda h^3 (1 - \nu) b} \). The inverse is

\[ \varepsilon_{pl} = \frac{1}{A_{2}^{dp}} \sigma - \frac{1}{2} \left( \frac{A_{3}^{dp}}{(A_{2}^{dp})^3} \right) \sigma^2 + ... \]  

(5.29)

Substituting Equations (5.29) and (5.23) in Equation (5.22) results in

\[ \varepsilon = \left( \frac{1}{A_{2}^c} - \frac{1}{A_{2}^{dp}} \right) \sigma - \frac{1}{2} \left( \frac{A_{3}^c}{(A_{2}^c)^3} + \frac{A_{3}^{dp}}{(A_{2}^{dp})^3} \right) \sigma^2 + ... \]  

(5.30)

or the inverse relation

\[ \sigma = A_{2}^c \left[ \varepsilon - \frac{1}{2} \left( \frac{A_{3}^c}{(A_{2}^c)^3} + \frac{A_{3}^{dp}}{(A_{2}^{dp})^3} \right) \varepsilon^2 + ... \right] \]  

(5.31)

The wave equation with respect to the Lagrangian coordinate \( X \) is given as

\[ \rho \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \sigma}{\partial X^2} \]  

(5.32)

Combining Equations (5.31) and (5.32) we obtain

\[ \frac{\partial^2 \varepsilon}{\partial t^2} - c^2 \frac{\partial^2 \varepsilon}{\partial X^2} = -c^2 \beta \left[ \varepsilon \frac{\partial^2 \varepsilon}{\partial X^2} + \left( \frac{\partial \varepsilon}{\partial X} \right)^2 \right] \]  

(5.33)
where \( c = \sqrt{A_e^2 / \rho} \) and \( \beta = \beta_e + \beta_{\phi} \) with \( \beta_e = -\frac{A_e}{A^2} \) and

\[
\beta_{\phi} = -\frac{16 \pi \Omega R^2 A_{\phi} h^3 (1-\nu)^2 (A_e^2)^2}{G^2 b}.
\]

In the literature, the Huang coefficients are often written in terms of higher elastic constants, that is \( A'_e = C_1 \) where \( C_1 \) is equal to the initial stress. Moreover, \( A'_e = C_1 + C_{11} \) and \( A'_t = 3C_{11} + C_{111} \). Assuming zero initial stress, the portion of \( \beta \) describing the nonlinearity contribution from lattice elasticity can be expressed as \( \beta_e = -\left(3 + \frac{C_{111}}{C_{11}}\right) \).

Assuming a purely sinusoidal input wave of the form \( \varepsilon_0 \sin(\omega t - kX) \), a solution to Equation (5.33) is

\[
\varepsilon = \varepsilon_0 \sin(\omega t - kX) - \frac{1}{4} \beta k \varepsilon_0^2 X \sin[2(\omega t - kX)]
\]  \hspace{1cm} (5.34)

Therefore \( \beta \) can be described by the amplitudes \( A_1 \) and \( A_2 \) of the fundamental frequency and the second harmonic respectively as

\[
\beta = \frac{4k A_2}{X A_1^2}
\]  \hspace{1cm} (5.35)

which permits the experimental determination of the nonlinearity parameter.

Similar expression can be obtained for displacements instead of strains. Starting with Newton’s law we can write

\[
\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 \sigma}{\partial X^2}
\]  \hspace{1cm} (5.36)

Substituting Equation (5.31) in Equation (5.36) we obtain the displacement based nonlinear wave equation.
\[
\frac{\partial^2 u}{\partial t^2} = c^2 \left[ 1 - \beta \frac{\partial u}{\partial X} \right] \frac{\partial^2 u}{\partial X^2}
\]

Assuming now an input wave of the form \( u_0 \cos(kX - \omega t) \), a solution to Equation (5.37) is

\[
u = \frac{1}{8} \beta k^2 u_0^2 X + u_0 \cos(kX - \omega t) - \frac{1}{8} \beta k^2 u_0^2 X \cos[2(kX - \omega t)] + ... \tag{5.38}
\]

Again, \( \beta \) can be expressed by the amplitudes \( A_1 \) and \( A_2 \) of the fundamental frequency and the second harmonic respectively which leads to the following expression:

\[
\beta = \frac{8 A_2}{A_1^2 \omega^2 X} \quad \text{or} \quad \beta = \left( \frac{A_2}{A_1^2} \right) \frac{2c^2}{Xf^2 \pi^2}
\]

The nonlinearity parameter \( \beta \) depends on the amplitudes of the fundamental wave as well as the second harmonic, the wave speed, the propagation distance and frequency. If the amplitude of the second harmonic is determined experimentally using a certain frequency and propagation distance, \( \beta \) can be determined.

Starting from the definitions described above, the nonlinearity analyzed in the experimental tests (Chapter 6) was characterized in terms of a simplified version of \( \beta \) that is

\[
\beta_2' = \frac{A_2}{A_1^2}
\]

Since also third order harmonics were found, the nonlinearity related to them was characterized in a similar way defining

\[
\beta_3' = \frac{A_3}{A_1^3}
\]
Chapter 6

Experimental Study of Prestress Level Monitoring in Free and Embedded Strands

In this chapter a promising technique based on ultrasonic guided waves to monitor the structural condition (essentially the applied load but the presence of defects as well) of multi-wire strands (embedded and free) widely used in prestressed concrete structures and cable-stayed or suspension bridges is presented. The experimental tests developed during the present research, in particular, involved free and embedded strands and were aimed to analyze nonlinearities arising in outputs signals and their relationship with the load level of the strand in order to improve a valid prestress load level monitoring method based on ultrasonic tests.

The first section of the chapter is focused on the description of the measurement equipment used to develop the tests. Later the second section describes the experimental tests on embedded strands. Finally, in the last section, a synthetic description of the results achieved on a free strand is reported.

6.1 Measurement Technique

In this section a brief description of electronic devices and instrumentations involved in all the experiments on both free and embedded tendons is presented.
6.1.1 National Instruments PXI®

This device constitutes the core of the measurement equipment and easily allowed to control the sensors, acquire and process the data using a LabView® software developed at UCSD.

PCI eXtensions for Instrumentation (PXI®) is a modular instrumentation platform originally introduced in 1997 by National Instruments and designed for measurement and automation applications that require high-performance and a rugged industrial form-factor.

Based on industry-standard computer buses and loaded up with extra features to facilitate electronic test, it permits a great deal of flexibility in building the exact test equipment or automation system required. Often it is fitted with custom software to manage the system. PXI uses PCI-based technology and an industry standard governed by the PXI Systems Alliance (PXISA) to ensure standards compliance and system interoperability.

The model used in experimental tests on embedded strands is PXI-1010 depicted in Figures 6.1 - 6.3.

![Image of PXI-1010](image-url)

**Figure 6.1** – National Instruments PXI-1010 used in experimental tests of embedded strands.
The acquisition module installed on this chassis was the PXI-5122 (Figure 6.4). This high-speed digitizer features two 100 MS/s simultaneously sampled input channels with 14-bit resolution, 100 MHz bandwidth, and up to 512 MB of memory per channel in a compact, 3U PXI Express, PXI, or PCI device. With its high sampling rate and low-distortion front end, this device
is ideal for a wide range of applications in automotive, communications, scientific research, military/aerospace, and consumer electronics.

In the used chassis two of these digitizers were installed so that up to four channels could be recorded simultaneously.

Figure 6.4 – NI PXI-5122 Acquisition Module.

The chassis used in experimental tests on a free strand, instead, was the PXI-1042 represented in Figures 6.5 - 6.7. The installed digitizer here was the more powerful PXI-5105 illustrated in Figure 6.8. This high-resolution digitizer features eight 60 MS/s simultaneously sampled input channels with 12-bit resolution, 60 MHz bandwidth, and up to 512 MB of memory in a compact, 3U PXI/PXI Express or PCI device. This device uses the National Instruments Synchronization and Memory Core (SMC) architecture, so it’s possible to combine multiple devices to build up to 136 phase-coherent channels in a single PXI chassis. It is also possible to synchronize an NI 5105 with other analogic and digital instruments to create mixed-signal test systems.
Figure 6.5 - National Instruments PXI-1042 used in experimental tests of a free strand.

The NI 5105 is ideal for a wide range of applications including ultrasonic nondestructive test (NDT), medical imaging, scientific research, military/aerospace, and consumer electronics.

In the used chassis two of these digitizers were installed so that up to sixteen channels could be recorded simultaneously.

Figure 6.6 - Front view of the PXI-1042.
Both devices described above used an Arbitrary Waveform Generator NI PXI-5411 depicted in next Figure 6.9. This device is a full-featured 40 MS/s arbitrary waveform generators (AWGs) for PCI and PXI capable of combining the power and capability of a stand-alone AWG with the flexibility and benefits of a computer to create highly capable virtual instrument solutions that take advantage of the Internet, Pentium processing power, and deep high-speed data storage.
With the 40 MS/s waveform update rate, up to 8 million samples of standard waveform memory per channel, and linking and looping capabilities, PXI-5411 allows to build complex waveforms. It includes all the features of arbitrary waveform generators, sweep generators, and function generators.

![Figure 6.9 – NI PXI-5411 Arbitrary Waveform Generator.](image)

### 6.1.2 Ultrasonic Transducers

Ultrasonic waves are created (or transduced) from electrical or optical signals by ultrasonic transducers. These devices also detect ultrasonic waves by transducing the ultrasonic waves back into electrical or optical signals. Because a transducer functions as both the source and the detector of ultrasound in a medium, the limitations of a given transducer dictate our ability to use ultrasound in general. Various transducers offer different methods for creating ultrasonic waves. The transducers used in the present work are piezoelectric ones.

Piezoelectric transducers are the most common method used today to create and detect ultrasonic waves. In the direct piezoelectric effect, discovered in 1880 by Jacques and Pierre Curie, a piezoelectric material responds to a mechanical deformation by developing an electrical charge on its surface. The direct piezoelectric effect relates the applied mechanical stress to the
output charge and is a measure of the quality of the material to act as a receiver. The reverse phenomenon, the *indirect piezoelectric effect* (discovered by Lippmann in 1881), produces a mechanical deformation when the piezoelectric material is subjected to an electric field. The indirect piezoelectric effect relates the input charge on the material surface to the output strain, and is a measure of the quality of the material to generate ultrasound.

The quality of a particular piezoelectric material to act as a good receiver or as a good generator of ultrasound depends on its specific material properties. Let α and β represent the direct and indirect piezoelectric coefficients such that \( V_{\text{output}} = \alpha \Delta d_{\text{applied}} \) and \( \Delta d_{\text{output}} = \beta V_{\text{applied}} \) where \( \Delta d \) = thickness and \( V \) is a voltage. Ideally, a transducer material would be both a good receiver and a good generator of ultrasound. But α and β are competing effects, so some compromise is required. Typically, materials are chosen with higher α to improve reception, and simply pulsed at a higher voltage to generate the ultrasound.

If separate transducers are used for generation and reception, the piezoelectric crystals can be chosen to match their function as in a pitch–catch element transducer. For example, PVDF, a polymer film used in the packing industry, has an exceedingly high α value, and acts as excellent receiver material. However, its low mechanical durability limits its use.

In the developed tests three different forms of ultrasonic piezoelectric transducers were used: PZT, PICO and PINDUCER.

### 6.1.2.1 PZT®

This particular type of ultrasonic transducer (6.35 mm x 3.2 mm) is produced by *APC International Ltd*. The experiments highlighted its sensibility, efficiency and suitability for the measurement of in-plane displacements due to its particular flat shape. The sensor was essentially
glued to the specimen and connected to the acquisition system. Figures 6.10 – 6.12 illustrate this device and provide an overall view of its dimensions.

**Figure 6.10** – PZT Transducer attached to the free strand.

**Figure 6.11** - PZT Transducer.

**Figure 6.12** – Overall view of PZT dimensions.
6.1.2.2 PICO®

The PICO transducer is a micro-miniature sensor with a wideband and relatively fat frequency response over the range of 200 – 750 kHz produced by Physical Acoustic Company. Due to its small size (5 x 4 mm) and its high sensibility, the sensor was ideal in recording out-of-plane displacements in both free strand and embedded strand experimental tests. Figures 6.13 and 6.14 depict this particular sensor and its dimensions.

Figure 6.13 – PICO transducer.

Figure 6.14 – PICO transducer – dimensions.
6.1.2.3 PINDUCER®

This device was used just in the embedded strands tests as better described later in section 6.2. It is developed and produced by Valpey Fisher Company and essentially consists of a 2 mm diameter piezoelectric disc element mounted on the end of a 33 mm long brass stern. Figures 6.15 - 6.17 provide a schematic view and an overall view of the present sensor and its dimensions. Also here, the small element dimensions give the Pinducer a wide operating range, which is quoted by the manufacturer to be from DC up to 10 MHz (40 dB down points). Moreover, its particular shape, likewise the PICO, makes it very suitable in recoding out-of-plane displacements.

![PINDUCER Diagram](image)

**Figure 6.15** – Schematic view of PINDUCER sensor.

![PINDUCER Image](image)

**Figure 6.16** – PINDUCER sensor.
6.1.3 Couplant

A couplant is a material (usually liquid) that facilitates the transmission of ultrasonic energy from the transducer into the test specimen (Figure 6.18)

Couplant is generally necessary because the acoustic impedance mismatch between air and solids (i.e. such as the test specimen) is large. Therefore, nearly all of the energy is reflected and very little is transmitted into the test material. The couplant displaces the air and makes it possible to get more sound energy into the test specimen so that a usable ultrasonic signal can be obtained. In contact ultrasonic testing a thin film of oil, glycerin or water is generally used between the transducer and the test surface.
When scanning over the part or making precise measurements, an immersion technique is often used as illustrated in Figure 6.19. In immersion ultrasonic testing both the transducer and the part are immersed in the couplant, which is typically water. This method of coupling makes it easier to maintain consistent coupling while moving and manipulating the transducer and/or the part.

![Figure 6.19 – Schematic view of an immersion ultrasonic test.](image)

In the tests object of this chapter, glycerin was used as a couplant (Figure 6.20). Next Figure 6.21 shows the application of this couplant on an embedded strand specimen.

![Figure 6.20 – Glycerin.](image)
6.1.4 Ultrasonic Preamplifier

This particular device is widely used in ultrasonic applications in order to provide an additional gain and signal-to-noise enhancement. Fundamentally it is an electronic amplifier which precedes another amplifier to prepare an electronic signal for further amplification or processing. In general, the function of a preamp is to amplify a low-level signal to line-level one.

The model used in the present tests is Panametrics Preamp 5665 (Figure 6.22) characterized by a voltage gain (invertible) of 34 and 54 dB (Switchable), a bandwidth ranging from 50 kHz to 5 MHz and an input resistance of 100k ohm.
6.2 Experimental Study in Embedded Strands

These experiments were conducted on three strands, each embedded in a 152 mm (6 in) × 152 mm (6 in) × 1016 mm (40 in) concrete block (Figures 6.23 – 6.25). A layer of grout was also present in the strand ducts. After the concrete cured, two of the strands were post-tensioned at two different stress levels, namely 70\% and 45\% of U.T.S. The third strand was left unstressed to provide a total of three post-tensioned blocks, each with a different level of prestress.

A through-transmission ultrasonic setup, represented in Figure 6.26, was adopted for this investigation and two different layouts were used as described below.

Figure 6.23 – Embedded strands specimens used for the tests.

Figure 6.24 – Embedded strand specimen before the application of the transducers.
In the first layout a broadband PICO transducer (see Section 6.1.2.2) was used to excite waves in the central wire at one of the strands’ ends (Figure 6.27 – 6.29) and the waves were detected by a PINDUCER sensor (see Section 6.1.2.3) located on the central wire at the strands’ opposite ends (Figure 6.30 – 6.31).
Figure 6.27 – PICO transducer applied to the specimen.

Figure 6.28 – Application of the PICO transducer to the central wire of the strand.

Figure 6.29 – Zoomed view on the PICO sensor applied to the central wire.
The second layout essentially maintained the same setup except for the PINDUCER sensor that in this case was applied to a peripheral wire (Figure 6.31). Since no significant differences in results were found with this second configuration, the results presented below are referred to the first layout.

Figure 6.31 – PINDUCER applied to the central wire.
Once the experimental setup (Figure 6.33) was configured and optimized, the test started generating toneburst signals by sweeping the generation frequency from 50 kHz to 1 MHz through the PXI and LabVIEW software (Figure 6.34). Using a purposely compiled program, it was possible to easily monitor and module all the characteristics of the input signals and to control the evolution of the test. The number of cycles of toneburst signals was varied from 5 to 60 with a step of 5. However the best results were found with a toneburst of 30 cycles; for this reason all the results presented later on this section are related to this particular configuration. The generated waveforms were conveyed into the specimen through the PICO transducer.
The output signals were acquired from the PINDUCER (on the other strand’s end) and carried into the ultrasonic preamplifier in order to gain the signal’s amplitude and increase the Signal-to-Noise Ratio. After this step, output signals were conveyed into the PXI Acquisition Card and were recorded and stored through LabVIEW software.

![Figure 6.34 - Screen of LabVIEW strand monitoring software.](image1)

![Figure 6.35 – Screen of Matlab software.](image2)
The recorded data were later imported in Matlab (Figure 6.35) in order to be analyzed using the FFT algorithm. Once the response in the Frequency Domain was determined, as mentioned before at the beginning of the chapter, the attention was focused on nonlinear aspects and, in particular, on higher-order harmonics generation. Repeating the test for each specimen and analyzing corresponding results then it was possible to relate these data and focus the attention on the variability of the nonlinear parameter (referred to the 2\textsuperscript{nd} harmonic and to the 3\textsuperscript{rd} harmonic as well) with the load level acting on the embedded strand.

The obtained results were found particularly satisfactory inside the frequency range (300 kHz – 500 kHz). The illustrations in next pages show these results for a given toneburst input signal of 30 cycles and for some specific frequencies in terms of Time Domain Response, Frequency Domain Response and variability of the nonlinear parameters with load level.
Toneburst (30 cycles) at 390 kHz

Figure 6.36 – Time Domain Response for each load level with a toneburst at 390 kHz.

Figure 6.37 - Frequency Domain Response for each load level with a toneburst at 390 kHz.

Figure 6.38 – Nonlinear Parameter vs. Load level with a toneburst at 390 kHz. (a) related to the 2nd harmonic; (b) related to the 3rd harmonic.
Toneburst (30 cycles) at 420 kHz

Figure 6.39 – Time Domain Response for each load level with a toneburst at 420 kHz.

Figure 6.40 - Frequency Domain Response for each load level with a toneburst at 420 kHz.

Figure 6.41 – Nonlinear Parameter vs. Load level with a toneburst at 420 kHz. (a) related to the 2nd harmonic; (b) related to the 3rd harmonic.
**Toneburst (30 cycles) at 430 kHz**

Figure 6.42 - Time Domain Response for each load level with a toneburst at 430 kHz.

Figure 6.43 - Frequency Domain Response for each load level with a toneburst at 430 kHz.

Figure 6.44 – Nonlinear Parameter vs. Load level with a toneburst at 430 kHz. (a) related to the 2nd harmonic; (b) related to the 3rd harmonic.
In these plots, the frequency domain responses are normalized to the fundamental harmonic amplitude so that the representation becomes clearer and the definition of $\beta'$ (see Section 5.3.4), being the amplitude of the fundamental harmonic equals to the unity, brings to the value of the second (or third) harmonic amplitude.

Analyzing obtained results it is possible to notice a clear appearance of higher harmonics, namely the 2nd harmonic and 3rd harmonic. This fact, as expected, denotes nonlinear effects. Since both the load level acting in the strand and the displacement field during the wave propagation are not so high to produce material or geometrical nonlinearities, contact and friction phenomena are the most probable and plausible causes of this behavior (see Section 5.3.3.4).

Figures 6.38, 6.41 and 6.44 (and the results presented in next section 6.3) confirm this scenario. In fact, the value of nonlinear parameter $\beta'$ related to the second and third higher-order harmonics decreases with increasing prestress level from 0% to 45% U.T.S. and 70% U.T.S.. In other words, as the load level increases, the interwire stress increases as well and the result is a damping of nonlinear effects and then of nonlinear parameter amplitudes.

This fact is extremely important because it outlines a concrete possibility to monitor the applied prestress in strands by the use of nonlinear ultrasonic waves, being the found trends reasonably linear.

In conclusion, this proof-of-principle study demonstrated that nonlinear waves and their nonlinear characteristics are promising features for monitoring prestress levels in embedded multi-wire strands. However, these results were obtained by installing sensors at both ends of a short strand. This option is not achievable in real post-tensioned structures given the substantial lengths. Hence the option of installing sensors in the embedded portion of the strand has to be considered. The next Section 6.3 presents the first attempt at monitoring prestress levels in strands using a sensor lay-out that is applicable to real post-tensioned structures.
6.3 Experimental Study in a Free Strand

Tests were performed at UCSD’s Powell Labs on the SATEC M600XWHVL (Figure 6.45a), 6000 kip capacity, pneumatic test apparatus configured for tensile loading and controlled by a specific computer (Figure 6.45b). A single seven-wire strand specimen was used for all the tests performed in this phase. The specimen was a Grade 270, 15.2 mm (0.6”) diameter, seven-wire strand having an U.T.S. of 1.86 GPa (270 ksi), an yield stress of 1.67 GPa (243 ksi), 1.82 m (72”) in length and a cross-sectional area of 140 mm$^2$.

Next Table 6.1 summarizes the properties of the strand. A 10° serrated wedge was placed on the tendon and inserted into a corresponding collar that was clamped pneumatically into the machine to transfer loads (see next Figures 6.48 – 6.49). The tested length was 1.4 m (56”) with 0.33 m (13”) extending on one end and 76 mm (3”) at the other end to allow for the wedge, collar and sensor placement. Figure 6.46 shows the strand installed on the testing machine.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength $f_y$</td>
<td>ksi</td>
<td>243</td>
<td>1.670</td>
</tr>
<tr>
<td>Ultimate strength $f_u$</td>
<td>ksi</td>
<td>270</td>
<td>1.860</td>
</tr>
<tr>
<td>Nom. diameter</td>
<td>in</td>
<td>0.6</td>
<td>15.24</td>
</tr>
<tr>
<td>Cross sectional area</td>
<td>in$^2$</td>
<td>0.217</td>
<td>140</td>
</tr>
<tr>
<td>Weight</td>
<td>lbs/ft</td>
<td>0.74</td>
<td>1.102</td>
</tr>
<tr>
<td>Ultimate load</td>
<td>kips</td>
<td>58.6</td>
<td>260.7</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>ksi</td>
<td>28,000</td>
<td>195,000</td>
</tr>
</tbody>
</table>

Table 6.1 - Properties of the seven-wire strand tested on the SATEC machine (http://www.dsiamerica.com/products/MultistrandSystem.html)
Figure 6.45 – (a) SATEC testing machine for stress monitoring tests; (b) Computer to control the SATEC machine.

Figure 6.46 – The 1.82-m strand installed in the SATEC testing machine for stress monitoring tests.
Next Figures 6.47 – 6.49 illustrate in detail the specimen and the anchorage system.

**Figure 6.47** – Seven-wire steel strand. (a) Overall view; (b) Section view; (c) Section view with dimensions (in cm).

**Figure 6.48** – External view of the anchorage system. (a) Overall view; (b) Dimensions (in cm).

Load-unload cycles were performed with 11 load steps in each cycle. The steps were based on a percentage of 70% of ultimate load 182.4 kN (41 kip), consisting of a 0%, 20% (8.2
kip), 40% (16.4 kip), 60% (24.6 kip), 80% (32.8 kip), 100% (41.0 kip), and down to 80% (32.8 kip), 60% (24.6 kip), 40% (16.4 kip), 20% (8.2 kip), and 0%.

![Figure 6.49 – Internal view of the anchorage system. (a) Overall view; (b) Dimensions (in cm).](image)

A variety of sensors were installed on the specimen. A schematic representation of the sensor layout is shown in Figure 6.50. Two strain gages were placed at the strand’s midpoint to measure the axial strain on two opposite peripheral wires. The following two different sensors were used to transmit and receive ultrasonic waves in the strand:

- Three ultra-mini broadband PICO sensors (see Section 6.1.2.2) placed on the bottom end of the strand, one on the central wire and the other two on two different peripheral wires (one of these two peripheral wires is the one where the excitation was applied).
- Five, 6.35 mm x 3.2 mm piezoelectric PZT sensors (see Section 6.1.2.1) placed on the same peripheral wire as the previously mentioned PICO sensor, all of them with an interval of 21 cm starting from the bottom end of the strand were PICO sensors were installed.

The installation of PZT sensors was developed using *Araldite 2014* (AW 139/XB 5323) epoxy paste adhesive (Figure 6.51). This product is a two component, room temperature curing, thixotropic paste adhesive of high strength with good environmental and excellent chemical resistance, used for bonding of metals, electronic components, GRP structures and many other
items where a higher than normal temperature or more aggressive environment is to be encountered in service. One of the five installed PZT sensors is illustrated in Figure 6.52.

**Figure 6.50** - Schematic representation of the sensor layout.

**Figure 6.51** – *Araldite 2014* epoxy paste adhesive used to install PZT sensors.

**Figure 6.52** – PZT transducer installed to the specimen using Araldite epoxy paste adhesive.
The installation of PICO sensors, instead, required a particular device purposely designed in order to firmly hold these sensors to each wire. Next figures show the Solidworks model (normal and exploded 3D views) and the technical drawings of this component.

Figure 6.53 – Solidworks model of the PICO holder. (a) Normal 3D view; (b) Exploded 3D view.

Figure 6.54 – Technical drawing of the PICO holder – Sheet 1.
Figure 6.55 - Technical drawing of the PICO holder – Sheet 2.

Figure 6.56 - Technical drawing of the PICO holder cover.

Next Figure 6.57 shows in detail this particular device installed to the strand.
The measurement equipment used for signal generation and data acquisition was mainly
constituted by the National Instruments (NI), modular PXI 1042 unit previously described in
Section 6.1.1. Once the experimental setup (Figure 6.58) was configured and optimized, a
frequency sweep of the transmitter of 250 kHz – 700 kHz was performed at each load step in the
cycles. LabVIEW software was used to control the sensors, acquire and process the data. Likewise
the previous test, the number of cycles of toneburst signals was varied, but this time from 10 to 60
with a step of 10.

After several preliminary tests in order to identify the best testing configuration, it was
adopted a test protocol where the generated waveforms were conveyed into the specimen through
the PZT1 transducer (see Figure 6.50) while the output signals were acquired from the other four
PZT transducers (PZT2, PZT3, PZT4 and PZT5 in Figure 6.50) and from the three PICO sensors
installed at the strand’s bottom end (PICO_CENT, PICO_PER and PICO_PER_EXTRA in Figure 6.50). According to Figure 6.50, the test protocol is summarized in next table 6.2.

![Experimental setup](image)

**Figure 6.58** – Experimental setup.

**Table 6.2** - Test protocol for prestress level monitoring in seven-wire strand (Trans = ultrasound transmitter; Receiv = ultrasound receiver).

<table>
<thead>
<tr>
<th>PZT1</th>
<th>PZT2</th>
<th>PZT3</th>
<th>PZT4</th>
<th>PZT5</th>
<th>PICO_CENT</th>
<th>PICO_PER</th>
<th>PICO_PER_EXTRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans</td>
<td>Receiv</td>
<td>Receiv</td>
<td>Receiv</td>
<td>Receiv</td>
<td>Receiv</td>
<td>Receiv</td>
<td>Receiv</td>
</tr>
</tbody>
</table>

In this test the acquired signals were carried into the ultrasonic preamplifier in order to gain the signal’s amplitude and increase the Signal-to-Noise Ratio. After this step, the amplified output signals were conveyed into the PXI Acquisition Card and were recorded and stored through LabVIEW software. Later the acquired signals were imported in Matlab and analyzed in the frequency domain, likewise in the embedded strands test described before.

The obtained results are really promising for prestress load monitoring and, furthermore, they are in accord with the ones obtained from the previous test showing a reduction of the
nonlinear parameter’s amplitude as the load level increases (loading phase of the test) and an opposite trend as the load level decreases (unloading phase of the test). Clear nonlinear effects appeared in form of high-order harmonics generation and they were particularly satisfactory inside the frequency range (500 kHz – 600 kHz). In contrast to the results of the previous test, here just the second harmonic was well defined and this fact is most likely related to the absence of the grout surrounding the strand in this configuration.

After the analysis of all acquired results, it was possible to focus the attention on the variability of the nonlinear parameter (referred to the 2nd harmonic and to the 3rd harmonic as well) with the load level acting on the embedded strand. The illustrations in next pages show the best results for a given transducer (used as a receiver) and a given toneburst input signal of a specific frequency and number of cycles in terms of Time Domain Response, Frequency Domain Response and variability of the nonlinear parameter (related to the second harmonic) with load level.
Figure 6.59 – Time Domain Response acquired from PZT2 for each load step during loading phase (first six pictures) and the unloading phase (last five pictures). Toneburst with 40 cycles at 500 kHz.
Figure 6.60 - Frequency Domain Response acquired from PZT2 for each load step during loading phase. Toneburst with 40 cycles at 500 kHz.

Figure 6.61 – Nonlinear parameter (related to the second harmonic) vs. load level during loading phase. PZT2 receiving and toneburst with 40 cycles at 500 kHz.
Figure 6.62 - Frequency Domain Response acquired from PZT2 for each load step during unloading phase. Toneburst with 40 cycles at 500 kHz.

Figure 6.63 - Nonlinear parameter (related to the second harmonic) vs. load level during unloading phase. PZT2 receiving and toneburst with 40 cycles at 500 kHz.
Figure 6.64 - Time Domain Response acquired from PICO_CENT for each load step during loading phase (first six pictures) and the unloading phase (last five pictures). Toneburst with 30 cycles at 350 kHz.
Figure 6.65 - Frequency Domain Response acquired from PICO_CENT for each load step during loading phase. Toneburst with 30 cycles at 350 kHz.

Figure 6.66 - Nonlinear parameter (related to the second harmonic) vs. load level during loading phase. PICO_CENT receiving and toneburst with 30 cycles at 350 kHz.
Figure 6.67 - Frequency Domain Response acquired from PICO_CENT for each load step during unloading phase. Toneburst with 30 cycles at 350 kHz.

Figure 6.68 - Nonlinear parameter (related to the second harmonic) vs. load level during unloading phase. PICO_CENT receiving and toneburst with 30 cycles at 350 kHz.
PZT5 receiving – Toneburst with 30 CYCLES at 550 KHZ

Figure 6.69 - Time Domain Response acquired from PZT5 for each load step during loading phase (first six pictures) and the unloading phase (last five pictures). Toneburst with 30 cycles at 550 kHz.
Figure 6.70 - Frequency Domain Response acquired from PZT5 for each load step during loading phase. Toneburst with 30 cycles at 550 kHz.

Figure 6.71 - Nonlinear parameter (related to the second harmonic) vs. load level during loading phase. PZT5 receiving and toneburst with 30 cycles at 550 kHz.
Figure 6.72 - Frequency Domain Response acquired from PZT5 for each load step during unloading phase. Toneburst with 30 cycles at 550 kHz.

Figure 6.73 - Nonlinear parameter (related to the second harmonic) vs. load level during unloading phase. PZT5 receiving and toneburst with 30 cycles at 550 kHz.
Figure 6.74 - Time Domain Response acquired from PICO_PER for each load step during loading phase (first six pictures) and the unloading phase (last five pictures). Toneburst with 30 cycles at 450 kHz.
Figure 6.75 - Frequency Domain Response acquired from PICO_PER for each load step during loading phase. Toneburst with 30 cycles at 450 kHz.

Figure 6.76 - Nonlinear parameter (related to the second harmonic) vs. load level during loading phase. PICO_PER receiving and toneburst with 30 cycles at 450 kHz.
Figure 6.77 - Frequency Domain Response acquired from PICO_PER for each load step during unloading phase. Toneburst with 30 cycles at 450 kHz.

Figure 6.78 - Nonlinear parameter (related to the second harmonic) vs. load level during unloading phase. PICO_PER receiving and toneburst with 30 cycles at 450 kHz.
Here again the frequency domain responses are normalized to the fundamental harmonic amplitude.

It is apparent that the FFT spectra reveal the generation of higher-order harmonics, beyond the fundamental frequency, indicating nonlinear behavior. For the same reasons previously discussed, contact and friction phenomena are the most probable and plausible causes for this behavior (see Section 5.3.3.4) since $\beta'$ related to the second higher-order harmonic decreases as the load level increase from 0% to 100% and the opposite situation during the unloading phase.

It can also be seen that the trend of $\beta'$ vs. Load is reasonably linear, indicating the suitability for this parameter to provide a direct indication of the level of prestress in the strand once the slope of the $\beta'$ vs. Load line is known. Clearly, the slope of $\beta'$ vs. Load for a given excitation frequency is expected to be different between a free strand and an embedded strand because of the different wave propagation behavior in the two cases. However, the net result is consistent, indicating the suitability of the nonlinear parameter $\beta'$ to monitoring load levels in both free and embedded strands. The field application of this technique will require knowledge of the calibration factor (i.e. the slope of the $\beta'$ vs. Load line), which will be different between free strand and embedded strand.

In conclusion, the present experimental study strengthened the idea that nonlinear waves and their nonlinear characteristics are promising features for monitoring prestress levels in multi-wire strands. The developed measurements, in fact, confirmed that the nonlinear parameter $\beta'$ can indeed be used for the measurement of load levels in free strands because of (a) its linear dependence on applied load and (b) its increased sensitivity compared to any linear ultrasonic feature examined previously.
Chapter 7

Numerical Analysis of Higher Harmonics Generation in Seven-Wire Strand Ultrasonic Test

7.1 Introduction

The experimental studies discussed in previous chapters suggested that the observed nonlinear ultrasonic behavior was caused by the interwire stresses between adjacent wires comprising the seven-wire strand, which are in turn proportional to the level of prestress applied to the strand. Finite Element Analyses (FEA) were carried out to corroborate this suggestion.

In this chapter, a series Numerical Simulations developed using ABAQUS commercial finite element software will be presented. The purpose of these numerical analyses essentially was to explore in detail contact and friction phenomena in seven-wire strand ultrasonic tests (that are supposed to be responsible of nonlinear effects noticed in the previous tests) and outline an analytical confirmation to the obtained experimental results described in previous two chapters.

The first part of the chapter focuses the attention on a brief review of the basics of contact mechanics and the formulation of the contact problem using the finite element formulation; the second part gives a detailed description of the developed ABAQUS 2D and 3D analyses with a special emphasis to nonlinear effects.
7.2 Basics of Contact Mechanics

Contact phenomena are abundant in everyday life and play a very important role in engineering structures and systems. They include friction, wear, adhesion and lubrication, among other things; are inherently complex and time dependent; take place on the outer surfaces of parts and components, and involve thermal, physical and chemical processes. Contact Mechanics is the study of relative motion, interactive forces and tribological behavior of two rigid or deformable solid bodies which touch or rub on each other over parts of their boundaries during lapses of time. However, the contact between deformable bodies is very complicated and it is not yet well understood. This is mainly due to the fact that it is very difficult to measure directly the evolution of surface quantities during contact processes [64]. In [65] the authors have made a brief survey on the recent developments in the field of contact mechanics.

The contact theory was originally developed by Hertz (1882) (see Section 4.8) and it remains the foundation for most contact problems encountered in engineering. However, the Hertz theory is restricted to frictionless surfaces and perfectly elastic solids. Later Johnson (1985) [66] gave an elaborate treatment to mechanics of contacts between non-conforming surfaces, contacts involving inelastic solids, sliding contacts, rolling contacts and also contact between rough surfaces.

The basic assumptions governing contact mechanics for application to any arbitrary geometry or with materials that deviate significantly from the elastic properties will be discussed in the following, however, some of these assumptions are included in the Hertz theory discussed in Section 4.8. With these assumptions, the solution of contact problems can be more easily implemented within a finite element program. Geometrically two bodies are said to be in unilateral contact if they are contiguous, impenetrable but separable [67]. The interaction between the contacting surfaces consists of two components (i) normal to the surface and (ii) tangential to
the surface. The normal interaction in a mechanical contact between two surfaces is modeled by three conditions:

- A geometric in-equality condition of impenetrability on the contact gap,
  \[ g \geq 0 \]  
  \[ (7.1) \]

- A static inequality condition of intensility on the contact pressure (contact pressure has a negative sign as it is compressive in nature),
  \[ p \leq 0 \]  
  \[ (7.2) \]

- An energetic equality condition of complimentarity on the gap-contact pressure pair
  \[ g \cdot p = 0 \]  
  \[ (7.3) \]

All together, these three conditions (Equations (7.1), (7.2), (7.3)) form the Hertz-Signorini-Moreau law of unilateral normal contact. As seen above, unilateral contacts are governed by a non-smooth multi-valued contact law relating the normal pressure to the normal gap between the two bodies. Being non-smooth, contact problems are inherently nonlinear. In the commercial finite element software, ABAQUS, used in this work, the contact constraint is applied when the gap between the surfaces becomes zero. In the present work, a “hard” contact formulation is employed where there is no limit on the magnitude of the contact pressure that can be transmitted between the surfaces (see Figure 7.1 (a)).

When surfaces are in contact, they usually transmit shear as well as normal forces across their interface. Thus, the analysis may need to take frictional forces, which resist the relative sliding of the surfaces, into account. The laws of friction were introduced by Amonton in 1699 [68] which were further developed by Coulomb in 1785 [69]. Leonardo da Vinci also had observed the above phenomena (see [70] for more details). Coulomb friction (as it is commonly termed) is a popular friction model used to describe the interaction of contacting surfaces.
Figure 7.1 - (a) Contact pressure-clearance relationship for "hard" contact; (b) Frictional behavior at the contact.

The Coulomb friction model characterizes the frictional behavior between the surfaces using a coefficient of friction, $\mu$. The tangential motion is zero until the surface traction reaches a critical shear stress value, $\tau_{\text{crit}}$, which depends on the normal contact pressure, according to the following equation (see Figure 7.1 (b)):

$$\tau_{\text{crit}} = \mu \cdot p$$  

(7.4)

7.2.1 Formulation of the contact problem using the Finite Element Method

The simulation of ultrasonic tests in a seven-wire strand to be discussed in this chapter involves the solution of the contact problem using a commercial finite element code, ABAQUS. In the present section, the mathematical formulation for the solution of a general contact problem using the finite element method will be presented.

In finite element analysis, the contact conditions are a special class of discontinuous constraint, allowing the forces to be transmitted from one part of the model to another. The
constraint is discontinuous because it is applied when the two surfaces come in contact. When the two surfaces separate no constraint is applied thus making the problem nonlinear.

Contact problems range from frictionless contact in small sliding to contact with friction in finite sliding and involving interaction between rigid and deformable bodies or interaction between two or more deformable bodies. If the absolute and hence the relative displacement undergone by the contacting bodies are small in comparison with the body sizes, then the contact principles are simple to establish and most of the novelty and the complexity comes from the tribological laws. When on the contrary the two bodies undergo large relative motions, the general principles become more complicated whereas the tribological laws remain virtually the same from the computational standpoint. The formulation of contact conditions is the same in all these cases, the solution of the nonlinear problems can in some analysis be much more difficult than in other cases.

In the following, the solution of the contact problem using the finite element method will be presented (see [71]; for a summary also see [72]). The general form of the governing equilibrium equations is formulated. Then the terms in the equilibrium equations are manipulated to include the contact condition. The theoretical considerations in the following are:

1. Arbitrarily shaped body,
2. Small deformations,
3. Linear elastic material property,
4. Static problem,
5. External forces include volume forces, surface forces and concentrated forces (which includes the contact forces).

Assuming a linear elastic continuum, the total potential is given by:
where, $\varepsilon$ is the strain tensor, $D$ is the stress-strain matrix for the linear elastic material, $u$ is the displacement vector, $f^v$ is the body force vector, $f^s$ is the vector of surface forces, $F^C$ is the vector of concentrated forces, $\lambda^k$ is the vector of contact forces for contact node $k$ (see Subsection 7.2.2), $Q^k$ is the contact matrix for contact node $k$ (see Subsection 7.2.2), $\Delta^k$ is the penetration for contact node $k$ (see Subsection 7.2.2), $V$ is the volume, $S$ is the surface, ($)^C$ is the superscript for concentrated forces and the respective displacements and ($)^k$ is the superscript for contact node $k$.

The governing equilibrium equations can be formulated by invoking the stationarity of the total potential, $\Pi$ (principle of minimum potential energy), i.e., $\delta \Pi = 0$. The contact constraints are imposed using the Lagrange Multiplier Method and using that $D$ is symmetric, we obtain:

$$
\delta \Pi = \int_V \delta \varepsilon^T D \varepsilon dV - \int_V \delta u^T f^v dV - \int_S \delta u^T f^s dS - \sum_C \delta u^C F^C + \sum_k \left[ \delta \lambda^k \left( Q^k u^k - \Delta^k \right) + \lambda^k Q^k \delta u^k \right]
$$

(7.6)

To evaluate Equation (7.6), the displacements should satisfy the geometric (essential/Dirichlet/displacement) boundary conditions. Hence, any variations on the displacements that satisfy the geometric boundary conditions and their corresponding variations
in strains are considered. In finite element analysis, the continuous body is approximated as an assemblage of discrete finite elements with the elements being interconnected at nodal points on the element boundaries. Therefore the equilibrium equations that correspond to the nodal point displacements of the assemblage of finite elements is written as a sum of integrations over the volume and areas of all finite elements; i.e.,

\[
\delta \Pi = \sum_{M} \int_{V(M)} \delta \varepsilon^{(M)^T} D \varepsilon^{(M)} dV^{(M)} - \sum_{M} \int_{V^{(M)}} \delta u^{(M)^T} f^{V^{(M)}} dV^{(M)} - \sum_{S} \int_{S} \delta u^{(S)^T} f^{S^{(M)}} dS^{(M)}
\]

\[-\sum_{C} \delta u^{C^T} F^{C} + \sum_{k} \delta \lambda^{k^T} \left( Q^{k^T} u^{k} - \Delta^{k} \right) + \lambda^{k^T} Q^{k^T} \delta u^{k} \]

(7.7)

The displacements measured in a local coordinate system within each finite element are assumed to be a function of the displacements at the finite element nodal points. Therefore for element \( M \) we have

\[
u^{(M)}(x,y,z) = H^{(M)}(x,y,z) \hat{U}
\]

(7.8)

where \( H^{(M)} \) is the displacement interpolation matrix, \( \hat{U} \) is the vector of global displacements at all nodal points. The variation of \( u^{(M)} \) is given by:

\[
\delta u^{(M)}(x,y,z) = H^{(M)}(x,y,z) \delta \hat{U}
\]

(7.9)

With the assumptions on displacements in Equation (7.8), the corresponding element strains are given by:

\[
\varepsilon^{(M)}(x,y,z) = B^{(M)}(x,y,z) \hat{U}
\]

(7.10)

where \( B^{(M)} \) is the strain displacement matrix. The variation of \( \varepsilon^{(M)} \) is given by:

\[
\delta \varepsilon^{(M)}(x,y,z) = B^{(M)}(x,y,z) \delta \hat{U}
\]

(7.11)

Substituting for the element displacements and strains we obtain:
\[
\delta \Pi = \sum_M \int_{\Gamma^{(M)}} \delta \hat{U}^T B^{(M)^T} D^{(M)} B^{(M)} \hat{U} dV^{(M)} - \sum_M \int_{\Gamma^{(M)}} \delta \hat{U}^T H^{(M)^T} f^{(M)} dV^{(M)} \\
- \sum_M \int_{S^{(M)}} \delta \hat{U}^T H^{(S^{(M)^T}} f^{(S^{(M)}} dS^{(M)} - \sum_{\kappa^{(M)}} \delta \hat{U}^T F^{C} + \sum_{\kappa^{(M)}} \left[ \delta \hat{\lambda}^{T} \left( Q^{T} \hat{U} - \Delta^{k} \right) + \delta \hat{U}^T Q \delta \hat{\lambda} \right]
\]

(7.12)

Applying the principle of the stationarity of the total potential we get:

\[
\delta \hat{U}^T \left[ \sum_M \int_{\Gamma^{(M)}} B^{(M)^T} D^{(M)} B^{(M)} dV^{(M)} \right] \hat{U} = \delta \hat{U}^T \sum_M \int_{\Gamma^{(M)}} H^{(M)^T} f^{(M)} dV^{(M)} \\
+ \delta \hat{U}^T \sum_M \int_{S^{(M)}} H^{(S^{(M)^T}} f^{(S^{(M)}} dS^{(M)} - \delta \hat{U}^T \sum_{\kappa^{(M)}} Q \hat{\lambda} + \delta \hat{U}^T \sum_{\kappa^{(M)}} F^{C} - \delta \hat{\lambda}^{T} \left[ Q^{T} \hat{U} - \Delta^{k} \right]
\]

(7.13)

where \( F \) is the vector of all the concentrated forces, \( \hat{\lambda} \) is the vector of all the contact forces, \( Q \) is the complete contact matrix of the assemblage, \( \Delta \) is the vector of all the penetrations.

Since, for a linear elastic continuum, the principle of minimum potential energy is identical to the principle of virtual displacement, the unknown nodal point displacements can be obtained from Equation (7.13) by imposing unit virtual displacements in turn at all displacement components. In this way we have

\( \delta \hat{U} = I \)

The equilibrium equations of the element assemblage corresponding to the nodal point displacements are given by:

\[
\begin{bmatrix}
K & Q \\
Q^T & 0
\end{bmatrix}
\begin{bmatrix}
\hat{U} \\
\hat{\lambda}
\end{bmatrix}
= \begin{bmatrix} R_k \\
0 \end{bmatrix} + \begin{bmatrix}
\Delta
\end{bmatrix}
\]

(7.14)

where,

\[
K = \sum_M \int_{\Gamma^{(M)}} B^{(M)^T} D^{(M)} B^{(M)} dV^{(M)}
\]

\[
R = R_b + R_s = \sum_M \int_{\Gamma^{(M)}} H^{(b)^T} f^{(b)} dV^{(M)} + \sum_M \int_{S^{(M)}} H^{(s)^T} f^{(s)} dS^{(M)}
\]

\[
R_s = -Q \hat{\lambda}^T
\]
For nonlinear problems (e.g., contact problems), the system of equations is formulated in an appropriate way and is solved for equilibrium using the Newton-Raphson Method as shown in the following:

$$\begin{bmatrix} K^{(i-1)} & Q^{(i-1)} \\ t+\Delta t & 0 \end{bmatrix} \begin{bmatrix} \Delta U^{(i)} \\ \Delta \lambda^{(i)} \end{bmatrix} = \begin{bmatrix} R^{(i)} \\ 0 \end{bmatrix} - \begin{bmatrix} F^{(i-1)} \\ 0 \end{bmatrix} + \begin{bmatrix} R_k^{(i-1)} \\ \Delta^{(i-1)} \end{bmatrix}$$

(7.15)

where $\Delta U^{(i)}$ is the vector containing the nodal displacements in the $i^{th}$ iteration, $\Delta \lambda^{(i)}$ is the vector containing the nodal contact forces in the $i^{th}$ iteration, $t+\Delta t$ is the force vector at time $t + \Delta t$, $R_k^{(i-1)}$ is the vector of the contact force on the contact nodes after the $(i-1)^{th}$ iteration, $\Delta^{(i-1)}$ is the penetration of the contact node after the $(i-1)^{th}$ iteration, $K^{(i-1)}$ is the stiffness matrix of the element, $Q^{(i-1)}$ is the contact matrix and $F^{(i-1)}$ is the force from stress distribution after the $(i-1)^{th}$ iteration.

### 7.3 ABAQUS Analyses

The numerical analyses object of this section was developed using ABAQUS software with the main purpose of exploring in detail contact and friction phenomena in seven-wire strand ultrasonic tests and outlining an analytical confirmation to the obtained experimental results described in previous two chapters.

All the calculations were performed on a powerful Lenovo D10 Workstation (Figure 7.2) configured with two Intel Xeon® Quad Core Processors working in parallel (eight processors in total) and 8 Gb of dedicated DDR3 RAM with a 64 bit architecture Windows operating system.

The developed analyses were particularly complex since a high velocity dynamic phenomenon (wave excitation to the strand) was modeled and, furthermore, it involved multiple contact interactions, transient dynamic effects, a nonlinear behavior and, in 3D analysis (Section
7.3.2), a huge finite element model consisting in almost one million elements. In addition to this, a very fine mesh was required in order to correctly catch changing contact conditions.

For these reasons ABAQUS/Explicit module was used. It is a special-purpose analysis product that uses an explicit dynamic finite element formulation and that is suitable for modeling brief, transient dynamic events, such as impact and blast problems, and is also very efficient for highly nonlinear problems involving changing contact conditions, such as forming simulations.

Figure 7.2 - Lenovo D10 Workstation used to perform numerical analyses.

### 7.3.1 2D Analysis

Starting from a 3D model of the strand (Figure 7.3), in order to optimize both computational efficiency and calculation time, it was firstly decided to simplify the model taking advantage from its symmetry properties. In accord to this, 1/6 sector of a 2D cross section was assumed as finite element model as depicted in next Figure 7.4. This model is an assembly of three different parts created as 2D Planar Deformable bodies. Likewise the experimental specimen, the diameter of each strand of the model is 5.08 mm.
The geometry was conveniently partitioned in order to obtain a valid and adequate mesh.

7.3.1.1 Material

In accordance with the specimen used in previous tests, the defined material was Steel characterized by:

- Type = Isotropic;
• Mass Density = 7800 kg/m$^3$;
• Young’s Modulus = 209E09 N/m$^2$;
• Poisson’s Ratio = 0.3.

7.3.1.2 Mesh

Once the overall geometry was defined the next step consisted in mesh definition. Every single portion of the model was subdivided in triangular elements and, after several preliminary mesh analyses, a mesh density progressively growing approaching the contact zones was defined. The elements used to define the mesh are explicit, linear, triangular with plane strain condition.

Next Figures 7.5 and 7.6 show an overall view of the final mesh and a zoomed view on a contact zone, respectively. With this mesh the computational features of the finite element model were:
• 19993 nodes;
• 39112 3-node linear triangular elements of type CPE3 (see [68] for further details).

The created mesh was then verified and no error or warning was found confirming its validity.

![Finite Element Mesh – Overall view.](image)
7.3.1.3 Boundary Conditions

Using model’s symmetry properties, boundary conditions essentially consisted in two lines of mechanical restraints over the two symmetry planes traces such that displacements perpendicular to these plain were constrained. This boundary layout is represented in next Figure 7.7.
7.3.1.4 Interaction Properties

In this step contact and fiction properties were defined. The main features defined for this interaction are:

**Tangential Behavior**

- Friction Formulation = Penalty;
- Friction Coefficient = 0.6.

**Normal Behavior**

- Pressure-Overclosure = “Hard” Contact.

This interaction property was then applied to the contact surfaces of the model using:

- Mechanical constraint formulation = Penalty contact method;
- Sliding formulation = Finite sliding.

Next Figure 7.8 shows interaction surfaces of the models after the definition of all the properties described above.

![Figure 7.8](image)

**Figure 7.8** – Contact surfaces characterized with an adequate interaction model.
7.3.1.5 Loads

The loads applied to the model involved two different analysis steps as better described in next Section 7.3.1.6. In the first “preload” step two radial interwire loads applied to the center of the two peripheral wires and directed to the center of the central wire were imposed corresponding to 70% U.T.S.. Their amplitude was varied in three steps 28950 N, 38600 N and 48250 N (being 48250 N half of the interwire force that two wires of the strand transmit each other at a load level of 70% of U.T.S. in accordance with Machida – Durelli formulation [60]; in the same manner, the other two interwire forces of 38600 N and 28950 N corresponded to 55 % and 40 % of U.T.S., respectively) in order to focus the attention on nonlinear effects and their relationship with the load level applied to the strand. The loads applied inside the second step, instead, modeled the waveform excited during the ultrasonic test and consisted in two sinusoidal functions applied again to the center of the two peripheral wires, directed to the center of the central wire and characterized from:

- Amplitude = 5000N;
- Frequency = 500 kHz;

Figure 7.9 below represents the applied loads described before.

![Figure 7.9 – Loads applied to the model.](image-url)
7.3.1.6 Analysis Steps

As mentioned before, the analysis was split into two explicit steps of different duration. The most delicate part consisted in the definition of the duration of each step in order to obtain a preload step sufficiently long to not create dynamic effects (that could disturb and influence the subsequent step’s response) and a second step efficiently optimized (see Section 7.3.2) so that the dynamic transient response following the ultrasonic wave propagation was correctly explored.

After some preliminary analyses, the best configuration found consisted in:

**STEP 1 – Preload Phase**
- Duration = 3E-004 seconds;

**STEP 2 – Ultrasonic Waves Propagation Phase**
- Duration = 3E-005 seconds;

Inside these time intervals, two different amplitude curves were defined in order to describe the evolution of the loads’ amplitude inside each analysis step.

![Preload Amplitude Curve](image)

**Figure 7.10** – Preload Amplitude Curve – Step 1.
For the preload phase, a ramp function (Figure 7.10) was defined so that preload’s amplitude varied linearly from 0 to 1 during the first step. For the wave propagation phase, instead, a sinusoidal amplitude curve (Figure 7.11) was defined so that waveforms’ amplitude varied periodically from -1 to 1 during the first $2E-005$ seconds (developing ten complete cycles). Other $1E-005$ seconds after the end of the waveforms were considered to include latent dynamic effects in the analysis.

![Wave Propagation Amplitude Curve](image)

*Figure 7.11 – Wave Propagation Amplitude Curve – Step 2.*

### 7.3.1.7 Analysis

The analysis was performed using ABAQUS/Explicit code because of its suitability for transient problems. The explicit dynamics procedure performs a large number of small time increments efficiently. An explicit central-difference time integration rule is used; each increment is relatively inexpensive (compared to the direct-integration dynamic analysis procedure available in ABAQUS/Standard) because there is no solution for a set of simultaneous equations. The explicit central-difference operator satisfies the dynamic equilibrium equations at the beginning.
of the increment, \( t \); the accelerations calculated at time \( t \) are used to advance the velocity solution to time \( t + \Delta t/2 \) and the displacement solution to time \( t + \Delta t \).

Clearly, the stability of the numerical solution is dependent upon the temporal and the spatial resolution of the analysis. To avoid numerical instability ABAQUS/Explicit recommends a limit for the integration time step \( \Delta t \) equal to \( \Delta t = L_{\text{min}} / c_L \), where \( L_{\text{min}} \) is the smallest dimension of the smallest finite element in the model, and \( c_L \) is the bulk longitudinal wave velocity in the material [73]. This limit thus represents the time of travel of a longitudinal wave across the element. In terms of element size, a good rule is keeping a minimum of 20 points per cycle at the highest frequency (or smallest wavelength), that is a maximum element size of \( L_e = \lambda_{\text{min}} / 20 \).

The time increment used in an analysis must be smaller than the stability limit of the central-difference operator. Failure to use a small enough time increment will result in an unstable solution. When the solution becomes unstable, the time history response of solution variables such as displacements will usually oscillate with increasing amplitudes. The total energy balance will also change significantly. If the model contains only one material type, the initial time increment is directly proportional to the size of the smallest element in the mesh. If the mesh contains uniform size elements but contains multiple material descriptions, the element with the highest wave speed will determine the initial time increment.

In nonlinear problems - those with large deformations and/or nonlinear material response – the highest frequency of the model will continually change, which consequently changes the stability limit. ABAQUS/Explicit has two strategies for time incrementation control: fully automatic time incrementation (where the code accounts for changes in the stability limit) and fixed time incrementation. The first one was used in the present analysis.

Moreover, this automatic time incrementation was based on a so called *global estimation* of the stability limit. The adaptive, global estimation algorithm determines the maximum frequency of the entire model using the current dilatational wave speed. This algorithm
continuously updates the estimate for the maximum frequency. ABAQUS/Explicit monitors the
effectiveness of the global estimation algorithm.

In terms of computational costs, the analysis explained so far needed around three hours
to be completed on Lenovo Workstation described at the beginning of Section 7.3. The same
analysis was then repeated for each preload value, as mentioned before.

7.3.1.8 Results

In order to analyze nonlinear effects (higher-order harmonics generation) and their
relationship with the preload force acting on the strand, firstly the results for each preload level
were examined singularly, concentrating on the dynamic response of the system and on the
higher-order harmonics generation; later these results were combined together, focusing on the
connection between nonlinear aspects and preload level.

To clarify this scenario, next pictures illustrate the most salient results obtained from the
analysis with an interwire force of 48250 N (70% of U.T.S). In particular, Figures 7.12 – 7.14
illustrate three overall contour plots of Von Mises Equivalent Stress at three different instants
during Step 1; Figures 7.15 – 7.17 illustrate the same plots but with a magnified view on one of
the contact zones and with an isolinear style; Figures 7.18 – 7.20, finally, depict again three
overall contour plots of Von Mises Equivalent Stress at three different instants but this time
during Step 2. From these illustrations it is possible to notice the diffusion of contact stresses
while the preload increases (following the ramp amplitude curve) and the complex stress
distribution and the reverberation of the waves between the boundaries in Step 2 due to nonlinear
dynamic effects as time elapses.

Now, considering a generic node inside the central wire (Figure 7.21), it is possible to
emphasize the dynamic response during the wave propagation phase plotting Von Mises
Equivalent Stress vs. Time during both Step 1 and Step 2. Next Figures 7.22 – 7.23 describe this result with an overall view and a particular view on Step 2.

In order to analyze higher-order harmonics generation, a node inside the central wire and close to a contact zone was considered (Figure 7.24) to plot Global Displacements vs. Time (U1 for X-Axis (horizontal direction in figures) and U2 for Y-Axis (vertical direction in figures)). Then, these output signals were imported in *Matlab* and processed removing the part of the output related to the preload phase (clearly not responsible of nonlinear effects) and adding two zero-value fields on the right and left sides of the original output in order to remove abrupt ending parts, generally responsible of a broadband frequency content. After this signal processing phase, the modified output signal was analyzed in the Frequency Domain, likewise in previous tests.

The obtained results were particularly satisfactory since nonlinearity was well present with a clear appearance of higher-order harmonics and a sub-harmonic as well in addition to the primary excitation at 500 kHz. Figures 7.25 – 7.26 depict the original output signals from *ABAQUS*, while Figures 7.27 – 7.28 show Time Domain and Frequency Domain representations of the processed output signals from *Matlab* corresponding to 70 % of U.T.S. as preload level.
Figure 7.12 – Von Mises Stress Contour Plot – Step 1 – 1E-004 seconds.

Figure 7.13 - Von Mises Stress Contour Plot – Step 1 – 2E-004 seconds.

Figure 7.14 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds.
Figure 7.15 - Von Mises Stress Isolinear Plot – Step 1 – 1E-004 seconds.

Figure 7.16 - Von Mises Stress Isolinear Plot – Step 1 – 2E-004 seconds.

Figure 7.17 - Von Mises Stress Isolinear Plot – Step 1 – 3E-004 seconds.
Figure 7.18 - Von Mises Stress Contour Plot – Step 2 – 4.4168E-006 seconds.

Figure 7.19 - Von Mises Stress Contour Plot – Step 2 – 1.101E-005 seconds.

Figure 7.20 - Von Mises Stress Contour Plot – Step 2 – 2.3667E-005 seconds.
Figure 7.21 – Node 2280 considered to plot Von Mises Stress vs. Time.

Figure 7.22 – Von Mises Stress vs. Time during both Step 1 and Step 2.

Figure 7.23 - Von Mises Stress vs. Time during Step 2.
Figure 7.24 – Node 1575 considered to analyze high order harmonics generation phenomenon.

Figure 7.25 – U1 (horizontal displacement) vs. Time from ABAQUS in node 1575.

Figure 7.26 – U2 (vertical displacement) vs. Time from ABAQUS in node 1575.
The most important result of this numerical simulation is surely the one depicted in Figure 7.28 where it is possible to see the principal peak corresponding to the fundamental harmonic (in this
case at 500 kHz) and the other peaks related to the sub-harmonic and to the higher-order harmonics. After repeating these calculations for the other two interwire force values (40% and 55% of U.T.S.), in the last step of the analysis all the obtained results were combined together and processed in Matlab in order to outline the influence of the prestress load acting on the strand (and consequently the interwire stress) to the nonlinear effects’ amplitude. Next Figures 7.29 – 7.30 show FFT spectra referred to displacements U1 and U2 at node 1575 (see Figure 7.21 or 7.24) for the three different levels of imposed prestress, namely 40% U.T.S., 55% U.T.S. and 70% U.T.S..

Figure 7.29 – Frequency Domain responses referred to node 1575 for the three considered interwire force values.

Figure 7.30 - Frequency Domain responses referred to node 1575 for the three considered interwire force values – Magnified view on U2 response.
In last picture it is possible to note that, in accordance to what obtained from previous experimental results (Chapters 5 – 6), the higher-order harmonic peak magnitude (i.e. the extent of the nonlinearity) increases with decreasing prestress level.

This fact is extremely important since it constitutes a numerical confirmation of obtained experimental results. The conclusion from both experimental and numerical standpoints is that the nonlinear parameter $\beta'$ increases with decreasing prestress level as a result of the strand “loosening up.” However, the discussed analysis was a simplified 2D model of the real strand that surely constitutes an important first step but was not exempt from approximations mostly related to the geometric boundaries. For this reason a detailed and onerous 3D finite element analysis was developed considering the whole strand. This analysis is described in next Section 7.3.2.

### 7.3.2 3D Analysis

In this analysis a complete 3D model of the strand was considered (Figure 7.31), removing almost all the approximations and simplifications introduced into the 2D model described before, in order to develop a comprehensive numerical model able to predict the full $\beta'$ vs.

![Figure 7.31 - 3D Finite Element Model assumed for the 3D analysis.](image)
Load curve required for calibration of the prestress monitoring technique. Likewise the previous case, the analysis proceeded defining the model and its characteristics, solving the model and illustrating the obtained results, emphasizing on nonlinear aspects of output responses.

The computational model is constituted by seven different parts (seven different wires) assembled together in ABAQUS/CAE environment. The diameter of each wire is 5.08 mm and, after some preliminary analyses, a length of 26 cm was assumed to correctly explore the dynamics of the problem without drastically increasing the computational cost.

### 7.3.2.1 Material

In accordance with the specimen used in previous tests, the defined material was Steel characterized by:

- Type = Isotropic;
- Mass Density = 7800 kg/m$^3$;
- Young’s Modulus = 209E09 N/m$^2$;
- Poisson’s Ratio = 0.3.

### 7.3.2.2 Mesh

After the definition of the geometrical properties, the attention was focused in mesh definition. This was a crucial phase where it was necessary to find a valid compromise between precision and computational cost. A mesh density progressively growing approaching the contact zones was adopted here too. The elements used to define the mesh are explicit, linear, hexagonal with 3D Stress condition. The adopted mesh layout was particularly onerous from a computational standpoint since involved more than 650000 nodes in order to effectively describe the ultrasonic wave propagation phenomenon on a real seven-wire strand.
Next Figures 7.32-7.34 show an overall view of the final mesh and two specific views representing central wire mesh and peripheral wires mesh. With this mesh the computational features of the finite element model were:

- 671113 nodes;
- 624130 8-node linear hexahedral elements of type C3D8R (see [68] for further details).

The created mesh was then verified and no error or warning was found confirming its validity.

**Figure 7.32** - 3D Finite Element Mesh – Overall view.

**Figure 7.33** - 3D Finite Element Mesh – Central wire.
7.3.2.3 Boundary Conditions

These restraints were aimed to model the used anchorage system during the experimental tests. They essentially consisted in two surfaces of mechanical restraints applied to both strand’s ends such that displacements perpendicular to the strand’s longitudinal axis and twisting rotation were constrained. Furthermore, in one strand’s end the longitudinal displacement was also constrained. This boundary layout is represented in next Figure 7.34.

Figure 7.34 - 3D Finite Element Mesh – Peripheral wire.

Figure 7.35 - Boundary Conditions (1 = x-axis; 2 = y-axis; 3 = z-axis).
7.3.2.4 Interaction Properties

In this phase contact and fiction properties were defined according to the Explicit Code.

The main features defined for this interaction are:

* **Tangential Behavior**
  - Friction Formulation = Penalty;
  - Friction Coefficient = 0.6.

* **Normal Behavior**
  - Pressure-Overclosure = “Hard” Contact.

In contrast to the previous 2D model, here this property was used to model the contact interaction between the seven wires of the strand using a General Contact (Explicit) formulation where ABAQUS/Explicit enforces contact constraints using a penalty contact method, which searches for node-into-face and edge-into-edge penetrations in the current configuration. In this context, the penalty stiffness that relates the contact force to the penetration distance is chosen automatically by ABAQUS/Explicit so that the effect on the time increment is minimal yet the penetration is not significant (see [73] for further details).

7.3.2.5 Loads

The loads applied to the model involved two different analysis steps as better described in next Section 7.3.2.6. The loads applied inside the first step concern the preload phase and consist in a uniform pressure (negative in order to model a tension) applied to one of the strand’s ends. Its amplitude was varied in three steps, namely 1.302E09 N/m², 1.116E09 N/m² and 0.930E09 N/m² corresponding to an applied prestress on the strand of 70% U.T.S., 60% U.T.S. and 50% U.T.S., respectively. This was done in order to focus the attention on nonlinear effects and their relationship with the load level applied to the strand. The loads applied inside the second step, instead, modeled the waveform excited during the ultrasonic test and consisted in a uniform
pressure applied to the end of one peripheral wire (following a purposely created sinusoidal amplitude function) with a frequency of 500 kHz and an amplitude of 0.930 N/m². Figures 7.36 - 7.37 represent the applied loads described before.

![Image](image1.png)

**Figure 7.36** – Preload applied to the strand.

![Image](image2.png)

**Figure 7.37** – Pressure used to model the ultrasonic wave.

### 7.3.2.6 Analysis Steps

The analysis was split into two explicit steps of different duration modeling the preload phase and the wave propagation phase, respectively. The definition of the duration resulted from similar considerations like the ones discussed in Section 7.3.1.6.
After some preliminary analyses, here the best configuration found consisted in:

**STEP 1 – Preload Phase**
- Duration = 3E-004 seconds;

**STEP 2 – Ultrasonic Waves Propagation Phase**
- Duration = 5E-005 seconds;

Inside these time intervals, two different amplitude curves were defined in order to describe the evolution of the loads’ amplitude inside each analysis step.

For the preload phase, the same ramp function of the 2D analysis (Figure 7.10) was used. For the wave propagation phase, instead, a different sinusoidal amplitude curve (Figure 7.38) was defined so that waveforms’ amplitude varied periodically from -1 to 1 during the first 3E-005 seconds (developing fifteen complete cycles). Other 2E-005 seconds after the end of the waveforms were considered to include latent dynamic effects in the analysis.

![Wave Propagation Amplitude Curve](image)

**Figure 7.38 - Wave Propagation Amplitude Curve – Step 2.**

### 7.3.2.7 Analysis

The analysis was performed with the same modality described in Section 7.3.1.7 and that part is recalled for any further details. Here in terms of computational costs, the analysis was more complex and onerous than the previous one and needed around seven hours to be performed.
on Lenovo Workstation described at the beginning of Section 7.3. The same analysis was then repeated for each preload value, as mentioned before.

### 7.3.2.8 Results

Following the same examination procedure described in Section 7.3.1.8, in order to analyze nonlinear effects (higher-order harmonics generation) and their relationship with the preload force acting on the strand, firstly the results for each preload level were examined singularly, concentrating on the dynamic response of the system and on the higher-order harmonics generation; later these results were combined together, focusing on the connection between nonlinear aspects and preload level.

The results related to the preload phase were of particular interest since they allowed a better understanding of the interaction between the seven wires in static conditions and the peculiar distribution of contact stresses along helicoidal lines, in accordance with the strand’s geometric properties. This result is shown in Figures 7.39 – 7.43 in terms of overall view and section views of Von Mises Equivalent Stresses contour plots at the end of the preload phase (at 3E-004 seconds) with a preload level corresponding to 70 % of U.T.S..

In Figure 7.43, in particular, helicoidal contact zones are clearly apparent and well defined in accordance to what expected. Considering the ultrasonic wave propagation phase (Step 2), next Figures 7.44 – 7.46 illustrate three contour plots of Von Mises Equivalent Stress related to the central wire at three different instants.
Figure 7.39 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds – Overall View.

Figure 7.40 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds – X-Plane Section View.
Figure 7.41 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds – Y-Plane Section View.

Figure 7.42 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds – Z-Plane Section View.
Figure 7.43 - Von Mises Stress Contour Plot – Step 1 – 3E-004 seconds – Central Wire.

Figure 7.44 - Von Mises Stress Contour Plot in Central Wire – Step 2 – 2.401E-005 seconds.
Figure 7.45 - Von Mises Stress Contour Plot in Central Wire – Step 2 – 2.501E-005 seconds.

Figure 7.46 - Von Mises Stress Contour Plot in Central Wire – Step 2 – 2.601E-005 seconds.
From latter illustrations it is possible to notice the complex stress distribution in Step 2 due to nonlinear dynamic effects arising during the wave propagation.

Wave propagation phenomenon is efficiently outlined also from Figure 7.47 where it is possible to see the wave propagation in the peripheral wire (where the wave was conveyed) and in the others wires as well.

**Figure 7.47** – Contour plot of Von Mises Stress at 4.871E-005 sec during Step 2 showing the propagation of the ultrasonic wave.

Likewise the previous finite element analysis (Section 7.3.1), in order to analyze higher-order harmonics generation, a generic node inside the central wire and in proximity of an helicoidal contact zone (*detection node*) was considered to plot Global Displacements vs. Time.
(U1 for X-Axis (horizontal direction in figures) and U2 for Y-Axis (vertical direction in figures)), emphasizing the dynamic response during the wave propagation phase. Again these output signals were imported and processed in Matlab. After this signal processing phase, the modified output signal was finally analyzed in the Frequency Domain.

Also in this case the obtained results were particularly satisfactory since nonlinearity was well present with clear higher-order harmonics and sub-harmonics as well. Moreover, the developed 3D finite element analysis provided very precise results in tangible accordance with experimental results since, as expected from the theory (Chapter 5), the higher-order harmonics were found perfectly at frequency values (2f, 3f,...) that are integer multiples of the fundamental frequency (f; in this analyses f = 500 kHz).

Next Figure 7.48 shows the Time Domain Responses of the processed output signals from Matlab corresponding to 70 % of U.T.S. as preload level.

Figure 7.48 - Time Domain representation of the processed output signals – 3D Finite Element Analysis.
Also in this case the FFT analysis of the aforementioned signals showed a nonlinear character with a peak corresponding to the fundamental harmonic (in this case at \( f = 500 \text{ kHz} \)) and other peaks related to the higher-order harmonics at 1000 kHz and 1500 kHz \((2f \text{ and } 3f)\).

Last analysis step consisted in combining all the obtained results and processing them in Matlab in order to outline the influence of the prestress load acting in the strand to the nonlinear effects’ amplitude. The final result is depicted in next Figure 7.49 where the influence of the preload level on the nonlinearity of the response (namely the amplitude of the second and third harmonics) is evident. The following plot is referred to the component U2 of the displacement.

Figure 7.49 – FFT Spectrum of U2 displacement time history at detection node for the two different preload levels.

Obtained results are in perfect accordance to the ones from 2D Finite Element Analysis showing the second harmonic peak (at 1000 kHz) decreasing with increasing of the prestress load acting in the strand. This fact constitutes a very important confirmation and consolidates the idea of using nonlinear ultrasonic parameter \( \beta' \) as a suitable feature for monitoring prestress level in free and embedded strands.
Chapter 8

Conclusions and Recommendations for Future Studies

8.1 Conclusions

In the present thesis, a technique based on ultrasonic guided waves to monitor the structural condition of multi-wire strands used in prestressed concrete structures and cable-stayed or suspension bridges was presented.

Firstly a brief review of the basics aspects of structural health monitoring was presented in order to outline the present state-of-art in this field. This introductory part was followed by a description of the basic theory of the wave propagation in rods and of the mechanics of seven-wire cables, focusing the attention on the interwire stresses and their connection with the prestress force applied to the strand since, as better described later in the research, contact and friction phenomena (strictly related to interwire stress) are the most plausible causes of nonlinearities.

Later the attention was moved on nonlinear effects, with particular attention to the ones related to the strand.

On this background, two experimental studies were developed and described, providing a very important results connecting nonlinear parameter’s amplitude to the prestress load applied to the strand. This represents the core of the present thesis since it is really a promising feature to monitor the structural condition of multi-wire strands through the installation of purposely designed sensors on new or existent structures and the execution of ultrasonic tests, as described in Chapters 5 and 6.
In order to confirm these experimental results and support them with an theoretical-analytical basis, a series of numerical simulations using a the commercial Finite Element code ABAQUS was performed. The nature of the examined phenomenon was particularly onerous from a computational standpoint since it involved nonlinear dynamic transient processes. The use of the powerful workstation surely helped a lot in containing calculation time to reasonable limits.

This analytical study was crucial to acquire a deep understanding of the developed experimental tests, constituting a solid and reliable tool to validate the experimental evidence. Numerical results, in fact, confirmed experimental trends efficiently, consolidating the concrete possibility to use this feature to monitor prestress load in multi-wire strands.

Both experimental and numerical results presented in the present research have indicated that the nonlinear ultrasonic parameter $\beta'$ is a suitable feature for monitoring prestress levels in free and embedded strands. In the field, the measurement of $\beta'$ would require placing one transmitting transducer at the strand’s free end (or at a location embedded in concrete), and placing the receiving transducer at a distance of ~1’ or more from the transmitter. Hence the method could be applicable to both new structures and existing structures, once access is provided to the strand’s anchored end and to at least one point in the embedded length close to the anchorage.

The nonlinear method does not require a baseline for each monitored strand (as long as it is of the same dimension and material, e.g. 0.6-in Grade 270 steel strand). The method is also insensitive to transducer-structure bonding conditions because the calculation of $\beta'$ is normalized by the primary excitation amplitude. What remains to be determined is the calibration factor that allows to calculate the exact level of applied prestress corresponding to a measured value of the nonlinear parameter $\beta'$, i.e. the slope of the $\beta'$ vs. Load curve.
8.2 Recommendations for Future Studies

The topic faced in the present research is surely in the forefront and far away to be completely understood. For this reason a lot of both experimental and theoretical research is needed.

Results presented in the second part of Chapter 6 were obtained on a single, free strand and a possible improvement is to test two additional strands to better assess the repeatability of the results among different specimens.

Furthermore, as a guideline for future studies and improvements, it should be noticed that the results presented in the first part of Chapter 6 were obtained on short, post-tensioned concrete beams and then a valid next step consists in developing more extensive and realistic tests, constructing and instrumenting larger post-tensioned concrete beams.

Considering the numerical simulations, here the possibility for future studies is really consistent. Firstly a broader range of ultrasonic frequency should be analyzed to outline the influence of this feature on nonlinear aspects of the output. A valid proposal consists in furthering the FEA ABAQUS simulation of the 3-D wave propagation in loaded strands. The analyzed model needs to be expanded to the case of strand embedded in grout and concrete, so as to provide a numerical framework able to extract the $\beta$’ vs. Load calibration factor. The development of such a model will enable predicting calibration factors for several types of strands (e.g. different numbers of wires, different materials) without the need to perform costly experimental calibration tests in each case.
BIBLIOGRAPHY


