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ASSET VALUES AND ECONOMIC FUNDAMENTALS

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Abstract

This paper examines the theoretical and observed co-movements of financial variables and the economic fundamentals. I use a real business cycle model to define the economic fundamentals, but most of the results apply to less precisely specified macroeconomic models. The data are not consistent with the theoretical prediction of a tight linkage between asset values and the economic fundamentals. The data easily reject the most stringent, and least reasonable, hypothesis that Tobin’s q always equals one. Vector autoregressions show none of the strong short-run feedback from real to financial variables predicted by the theory. And co-integration tests cannot reject the least stringent null hypothesis of no long-run equilibrium relationship.

*I would like to thank Greg Connor for some stimulating and valuable lessons in finance and Mark Carey for invaluable research assistance.

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Introduction

In any macroeconomic model the financial market explicitly or implicitly links household and firm decisions. Households use securities markets for intertemporal substitution and risk sharing. Firms receive signals for real investment decisions from securities markets and firms' real earnings determine the distribution of asset payoffs. In theory the value of financial assets is closely tied to the economic fundamentals, so in practice it should be easy to find evidence of the linkages in the data. But, six months after the US stock markets lost nearly one-third of their value in a week there is neither evidence that a dramatic change in the fundamentals preceded the crash, nor a consensus belief that the enormous October meltdown accurately predicts a decline in future real activity.

This paper presents the results from three sets of more formal tests to check the consistency of theoretical predictions of the co-movements of economic variables with the observed co-movements. The observed co-movements of financial and real variables do not conform closely to the co-movements predicted by economic theory. I use a general equilibrium real business cycle model to specify the theoretical co-movements, but most of the results in this paper have implications for any macroeconomic model.

The seminal papers by Kyland and Prescott (1982) and Long and
Plosser (1983) showed that the Pareto optimal allocation in stochastic intertemporal general equilibrium models display "business cycle" phenomena. Their work inspired a class of models known generically as real business cycle theory. Real business cycle models define the economic environment precisely in terms of agents' preferences, technology, and the probability distribution of an exogenous random shock(s). Maximization by individual agents defines economic behavior.

Most real business cycle modellers solve for the Pareto optimal allocation as a central planning solution. The central planning solution naturally leads to an examination of the co-movements or real variables since there are no explicit market prices. Kydland and Prescott, Long and Plosser, and Prescott showed that the sample statistics from artificial data generated by fairly simple real business cycle models were remarkably similar to the sample statistics for some key US time series on real variables.

In Section 1 I show the recursive competitive equilibrium that supports the Pareto optimal allocation in the central planning problem. The competitive equilibrium solution makes the financial sector and the role of markets explicit. The financial-real sector linkages are direct and very strong in this stylized model, but any description of a well functioning market economy implies fairly strong linkages between the financial and real sectors.
Section 2 presents testable restrictions imposed by the theory. Section 3 gives the empirical results. A literal interpretation of the specific real business cycle model in this paper imposes the unreasonably stringent restriction that Tobin's q always equals one. The data easily reject this very strong null hypothesis.

A weaker and more reasonable hypothesis that applies to a broad class of models only imposes strong linkages between the variables. In a dynamic general equilibrium model agents respond to shocks with intra- and inter-temporal substitutions. The substitution implies temporal linkages across variables; in general the variables should display non-zero cross-autocovariances. Vector autoregressions show no feedback from the real variables to the financial variable. The data display only weak feedback between the real variables, and the results are sensitive to the detrending technique.

The weakest hypothesis which applies to almost any model is that the fundamentals matter in the long run. I test this hypothesis with co-integration tests. The test requires no detrending, and allows for long periods of "disequilibrium". The data fail to reject the null hypothesis that capital and the financial value of the firm are not co-integrated, ie, the errors from Tobin's q are unbounded.
Section 1: A Simple Real Business Cycle Model

This section presents a very simple model to illustrate the financial-real sector linkages implicit in real business cycle models.

1.1 The Model

Household Preferences

The representative household is a stand-in for all households. The utility of the (infinitely lived) household depends on the expected value of the time-separable discounted utility function,

$$\sum_{\tau=0}^{\infty} \beta^\tau E_t U(c_{t+\tau}, 1-z_{t+\tau}).$$

Instantaneous utility is strictly concave in consumption, $c$, and leisure, $1-z$. $\beta$, the household time discount factor, is between zero and one.

Technology

The stochastic production function is homogeneous of degree one.

$$y_t = f(k_t, z_t, e_t).$$

Capital, $k_t$, is predetermined. Labor, $z_t$, is a current choice variable. The exogenous productivity shock, $e_t$, is an independently and identically distributed strictly positive random variable with a mean of one. I assume a discrete probability distribution where $\pi(e')$ denotes the probability of the realization $e'$, which is an element of the finite set of
potential realizations, $e_{t+1}^t$ is an Arrow-Debreu "state of nature".

1.2 The Central Planning Problem

Most real business cycle models are complicated stochastic growth models. In these models it is frequently easier to find the Pareto optimal resource allocation as the solution to a central planning problem, see Sargent's (1987) explanation in Chapter 3. A fundamental theorem in welfare economics shows a competitive equilibrium supports the Pareto optimal allocation (when the constraint set is convex), so the allocation applies to a decentralized market environment. The central planning solution gives the Pareto optimal allocation of real resources and the relationships between real variables used to test (or calibrate) the accuracy of real business cycle specifications.

The planner selects a contingent plan for capital and labor that maximizes the household utility function subject to the constraint that,

$$c_t + k_{t+1} = f(k_t, z_t, e_t) + k_t$$

consumption plus capital accumulation not exceed production.

Dynamic Programming

There are many ways to solve for the sequence of the optimal

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1 See Prescott and Merha (1980) for a more general specification.
contingent allocation actions. Prescott and Merha (1980) had the keen insight to recognize that dynamic programming provides an extremely useful representation for testing a wide class of general equilibrium theories with time series data. In contrast, the testable implications of the elegant and famous Arrow-Debreu representation are few. The Arrow-Debreu general equilibrium solution yields a vector of optimal actions contingent on the sequence of past and potential realizations of the Arrow-Debreu "state of nature". Dynamic programming exploits the recursive structure of the problem\(^2\) representing the effect of all past decisions and current information in a minimum dimensional dynamic programming state vector. The dynamic programming solution is a set of time invariant functions in the state variable.\(^3\)

The dynamic programming value function is the recursive form of the objective function,

\[
W(S_t) = \max_{z_t, k_{t+1}} \left[ U(c_t, 1-z_t) + \beta E_t W(S_{t+1}) \right],
\]

equation 1.1.1, where the resource constraint, equation 1.2.1, defines consumption. Here \(S\) denotes the dynamic programming state vector. In general, the state vector is not unique; but, the

\(^2\) The specification of a time-separable structure imposes a restriction on the Arrow-Debreu specification; nevertheless this is a natural specification for a wide class of economic models.

\(^3\) See Prescott and Merha, or Sargent, Chapter 1, or Harris, Chapter 2, and their references for a comprehensive discussion of dynamic programming.
dimension of the state vector is unique. In this example define the state vector, \( S_t \), as \( k_t \), which summarizes the effect of past decisions, and \( e'_t \), which summarizes current information.

The partial derivatives of the value function with respect to the controls give the first order conditions for a maximum. At a maximum labor satisfies the condition,

\[
1.2.3 \quad U_{1-2t}/U_{ct} = f_{zt},
\]

that the marginal utility of leisure relative to the marginal utility of consumption equals the marginal product of labor at each date for any realization \( e' \).\(^4\) Define \( w_{st} = U_{1-2t}/U_{ct} \) as the current (relative) shadow price of labor.

At a maximum capital satisfies the Euler equation,

\[
1.2.4 \quad U_{ct} = \beta \sum_{e'} \pi(e')U_{ct+1}(f_{kt+1} + 1) = \beta E_t U_{ct+1}(f_{kt+1} + 1),
\]

which says the decrease in current utility from sacrificing a unit of consumption must equal the present value of the increase in expected utility from having another unit of capital. Define \( p(e')_{t+1} = \beta \pi(e')U_{ct+1}/U_{ct} \) as the current (relative) shadow price of consumption one period in the future contingent on the error realization \( e' \). The shadow price is the current (period t) Arrow-Debreu contingent claim price of consumption in the "Arrow-Debreu state" \( e'_{t+1} \).

\(^4\) The notation, eg, \( U_{ct} \), is shorthand for the partial derivative evaluated at the value of the arguments, ie, \( U_c(C_t, 1-z_t) \).
The dynamic programming solution is the decision function,
1.2.5 \[ u_t = u(S_t), \]
where, \( u'_t = (k_{t+1}, z_t) \),
and the recursive vector transition equation for the state,
1.2.6 \[ S_{t+1} = g(S_t, u_t, e_{t+1}) = g(S_t, u(S_t), e_{t+1}). \]
The solution is the set of functions that solve the problem,
1.2.2 \[ W(S_t) = \max_{z_t, k_{t+1}} \left[ U(c_t, 1-z_t) + \beta E_t W(S_{t+1}) \right]. \]
\[ = U(u(S_t), S_t) + \beta E_t [W(g(S_t, u(S_t), e_{t+1})], \]
where the resource constraint, 1.2.1, defines consumption.

The technically direct central planning solution masks the relationship between quantities and prices that is explicit in market equilibrium. The central planning allocation values resources in terms of their shadow prices. The shadow value of labor is the marginal rate of substitution between leisure and consumption. The shadow value of the marginal unit of capital equals,
1.2.8 \[ l = \sum p(e')_{t+1} (f_{kt+1} + 1), \]
the gross payoffs, \((f_{kt+1} + 1)\), to an additional unit of capital in each Arrow-Debreu state, \( e'_{t+1} \), weighted by the current shadow price of consumption in that state. The shadow value of the marginal unit of investment must equal one since consumption and investment are perfect substitutes. Multiplying equation 1.2.8 by capital gives the shadow value of capital,
1.2.9 \[ k_{t+1} = \sum p(e')_{t+1} (f_{kt+1} + 1)k_{t+1}. \]
The current shadow value of capital is the sum of the gross
payoffs to capital in each Arrow-Debreu state weighted by the current shadow prices of consumption in that state. Equation 1.2.9 follows from the fact that the production function is homogeneous of degree one.

1.3 A Decentralized Rational Expectations Recursive Equilibrium
A competitive equilibrium also supports the Pareto optimal allocation. The competitive equilibrium decentralizes decisions. Labor, commodities, and equities trade in a competitive spot market. Agents treat prices as exogenous in their decision rules and form rational expectations about future economic variables. The market sends signals to agents which, in equilibrium, coordinate their decisions.

Households
The household wants to maximize expected lifetime utility, equation 1.1.1, subject to its budget constraint. The budget constraint limits household consumption plus end-of-period wealth, \( s_{t+1}V_t \), to,

\[
1.3.1 \quad c_t + s_{t+1}V_t = w_t z_t + s_t(V_t + d_t),
\]

labor income plus initial wealth and the current payoff from wealth.\(^5\) Here \( V_t \) denotes the current (spot market) price of the

\(^5\) The additional constraint that \( \beta'V_{t+1} \) goes to zero as \( r \) goes to infinity is required to rule out unbounded borrowing (short sales).
firm's equity and \( d_t \) the dividend. \( s_t \) the "number of shares"\(^6\) owned by the household at the beginning of the period and \( s_{t+1} \) is the number of shares owned by the household at the end of the period. \( w_t \) is the spot market wage. The spot market prices are relative to the price of consumption which I normalize at one. The household chooses contingent plans for labor and asset accumulation.

The household value function is,

\[
W(S_t) = \max_{z_t, S_{t+1}} \left[ U(c_t, 1-z_t) + \beta E_t W(S_{t+1}) \right].
\]

where the household budget constraint 1.3.1 defines consumption. Define the state vector, \( S_t = (s_t(V_t + d_t), w_t) \), as the current spot market wage and beginning-of-period gross wealth. The household control vector is the labor supply and end-of-period shareholding decision. Assume the household knows (has rational expectations about) the probability distribution of wages and gross asset values. That is, the household knows the conditional probability distribution \( \pi((V_{t+1} + d_{t+1})|S_t) \) and \( \pi(w_{t+1}|S_t) \). As shown below, knowledge of the conditional distribution of gross asset values and wages is equivalent to complete knowledge about the activities of the firm and the probability distribution of shocks, i.e., the household solves the planning problem and

\(^6\) I assume there is one share of infinitely divisible stock outstanding in the firm. So, \( 0 \leq s \leq 1 \), is the fraction of the firm owned by the household, and \( V \) is the equity value of the firm. The Modigliani-Miller theorem holds in this environment so \( V \) would represent the market value of the firm (equity plus debt) in a model with a richer set of financial contracts.
essentially requires the knowledge of the central planner.

At a maximum the household supplies labor until,

1.3.3 \( U_{l-zt}/U_{ct} = w_t, \)

the marginal utility of leisure relative to the marginal utility of consumption equals the wage. The household chooses to accumulate (or sell) shares of stock until,

1.3.4 \( U_{ct}V_t = \beta E_t[U_{ct+1}(V_{t+1}+d_{t+1})], \)

the decrease in current utility from purchasing a share of stock equals the present value of the increase in expected utility from owning another share of stock. This is the deservedly famous consumption-capital asset pricing equation.

The Firm

The representative firm is a stand-in for all firms. The firm produces the commodity with a stochastic technology (equation 1.1.2). It sells part of output to households and retains part for capital accumulation,

1.3.5 \( c_t + k_{t+1} = f(k_t,z_t,e_t) + k_t. \)

The owners of the firm (shareholders) instruct the firm manager to hire labor until its marginal product equals the wage,

1.3.6 \( f_{zt} = w_t. \)

And they instruct the manager to accumulate capital until the end-of-period's capital stock equals the equity price (market
value of the firm),\(^7\)

1.3.7 \(k_{t+1} = V_t\).

The capital accumulation rule sets Tobin's \(q (q = V_t/k_{t+1})\) to one. These rules give a Pareto optimal allocation, so any maximization problem the firm solves must yield equations 1.3.6 and 1.3.7 as decision rules.

The firm returns all net earnings to shareholders in dividend payments,

1.3.8 \(d_t = y_t - (k_{t+1} - k_t) - w_t z_t\)

Since the production function is homogeneous of degree one, dividends equal capital's share of output minus net capital accumulation,

1.3.8' \(d_t = f_k k_t - (k_{t+1} - k_t)\).

Equilibrium

An equilibrium is a vector of spot prices where the excess demand for consumption, labor, and equities equals zero. In equilibrium the (relative) spot wage, \(w_t\), clears the labor market. To clear the stock market the equity price must adjust to make the household content to hold the number of shares it began the period with, ie, \(s_{t+1} = s_t = s\) (this is a property of the representative individual assumption; in general the aggregate demand for shares must equal the aggregate supply). The spot

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\(^7\) The Modigliani-Miller theorem holds in this environment so \(V\) represents the financial value of the firm.
price of consumption and capital is normalized at one.

If the equilibrium allocation is Pareto optimal it must also satisfy the conditions for a maximum in the central planning problem given in equations 1.2.3 and 1.2.4. To verify that the allocation is Pareto optimal, notice that the equilibrium wage equates the marginal product of labor with the marginal utility of leisure relative to the marginal utility of consumption, satisfying equation 1.2.3.

To verify the optimality of the capital allocation, substitute the firm's capital accumulation rule, \( k_{t+1} = V_t \), and the definition of dividends, \( d_t = f_k k_t - (k_{t+1} - k_t) \), in the household Euler equation, 1.3.4, giving,

\[
U_{ct} k_{t+1} = \beta E_t [U_{ct+1} (f_{kt+1} k_{t+1} - (k_{t+2} - k_{t+1}) + k_{t+2})],
\]

\[ k_{t+1} = \beta E_t [(U_{ct+1} / U_{ct}) (f_{kt+1} + 1) k_{t+1}]. \]

Equation 1.3.9 is \( k_{t+1} \) times the Euler equation, 1.2.4, in the central planning problem. The shadow value of capital in the central planning solution (equation 1.2.9) equals the equity value of the firm in market equilibrium. Thus the market allocation is Pareto optimal.

Information and Decision Structure

In the recursive competitive equilibrium the household solves the planning problem. The firm's manager follows simple instructions
using observable market prices as signals. Management has no economic role and is not specified as an input in the production process. Households make all the essential decisions. The household needs more information than it can obtain from observable market signals. The rational expectations equilibrium assumption closes the model.

The household chooses the number of shares of equity to hold. The number of shares determines the household's end-of-period wealth, $s_{t+1}V_t$. The equilibrium end-of-period aggregate financial wealth ($s=1$) equals the aggregate capital stock, $k_{t+1}$. The central planner chooses society's end-of-period capital, $k_{t+1}$, directly. The household chooses end-of-period wealth indirectly through financial transactions. Either decision depends on the probability distribution of future values of gross wealth.

The gross value of equity next period in the Arrow-Debreu state $e_{t+1}'$ equals,

$$1.3.10 \quad (d(e') + V(e'))_{t+1} = (f_k(e_{t+1}') + 1)k_{t+1},$$

the gross payoff to capital. Substituting the capital transition equation, $k_{t+1} = k(S_t) = k(k_t, e_t')$, and the decision rule for labor, $z_{t+1} = z(S_{t+1}) = z(g(S_t, e_t'))$, from the central planning problem, in the right-hand side of 1.3.10 gives,

$$1.3.11 \quad \pi((f_k(k(S_t)), z(g(S_t, e_{t+1}'), e_{t+1}')) + 1)k(S_t) | S_t; \ e_{t+1}' \in \mathcal{E},$$

the probability distribution for gross payoffs to capital conditional on the current dynamic programming state,
$S_t = (k_t, e_t')$. By the assumption of rational expectations the household knows,

$$1.3.12 \pi((d(e') + V(e'))_{t+1} | (S_t(d+V)_t, w_t)); \; e'_{t+1} \in e,$$

the probability distribution of gross equity payoffs conditional on the current state, which is identical to the conditional distribution of gross capital payoffs.

In a stationary state knowledge of the conditional distribution of gross asset values, equation 1.3.12, (and wages) is sufficient for optimal household decisions. The household might infer the conditional distributions from historical data with no direct knowledge of the activities of the firm. But the evaluation of any intervention, eg, a new production process or a new firm, requires knowledge of the structure, not just historical correlations.\(^8\)

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\(^8\) This is simply Lucas'(1976) famous econometric critique that proper evaluation of an intervention requires knowledge of the structure.
Section 2: Testable Restrictions

Section 1 presented a simple stylized real business cycle model to illustrate the strong linkages between the financial and real sectors. More "realistic" models have more complicated structures, but they still imply strong financial real sector linkages. This Section gives two testable restrictions. The first is not robust with respect to relatively minor changes in the model specification; the second is robust with respect to fairly major changes in the specification.

A Stringent Restriction

In the model in Section 1 the gross payoffs to physical and financial capital have the same conditional probability distribution and follow the same stochastic process. The equity (market) value of the firm equals the value of the firm's capital. As a result, the transition equation that describes the evolution of capital,

$$k_{t+1} = k(k_t, e'_t) = V_t = k(V_{t-1}, e'_t),$$

also exactly describes the evolution of equity value. Since the dynamic programming state vector is not unique there are other representations of the transition equation, eg,

$$V_t = V(V_{t-1} + d_{t-1}, w_t),$$

but they all contain the same number of state variables—-in this example, one to summarize past decisions and one to represent the current shock realization. A single stochastic process describes the evolution of equity and physical capital. Equity represents a
claim to the firm's future net earnings. In this model, since the production function is homogeneous of degree one and there are no restrictions on trading capital, net earnings equal the marginal product of capital times capital minus investment. Capital and equity promise the same payoff stream and have the same value.

Most real business cycle models, which focus on the co-movements of real variables, specify a more complicated technology, and/or utility function, and/or error process (eg, see Kydland and Prescott) to replicate more complex dynamic patterns observed in most real variables. The Appendix shows these changes make the dynamics richer but do not break the stringent restriction that Tobin's q always equals one. In addition if agents are heterogeneous and markets are complete (so the allocation is Pareto optimal) Tobin's q still equals one. These changes in the specification make the dynamics richer so the model approximates the observed co-movements of real variables. But, the observed change in the value of many financial assets has a very simple dynamic pattern which is closely approximated by a first-order autoregression. Adjustment costs, decreasing returns, or restrictions on trading capital cause physical capital and equity to have different dynamic patterns breaking the stringent restriction that Tobin's q always equals one.

A More Robust Restriction

Intra- and intertemporal substitution link all the variables in
dynamic general equilibrium models. Any shock eventually affects all the variables in the system. The dynamic programming solution,

2.3 \( u_t = u(S_t) \),

2.4 \( S_{t+1} = g(S_t, u(S_t), e_{t+1}) \),

shows the linkage. The state transition equation is a minimal dimensional (possibly unobservable) factor model describing the fundamental dynamics of the system. The observable decision variables are functions of the state vector and reflect current shocks. The decision variables are temporally cross-related by their linkage to the state vector. For example, the cross-autocovariance,

2.5 \( E[u^i_t, u^j_{t-1}] = E[u^i(S_t), u^j(S_{t-1})] 
\quad = E[u^i(g(S_{t-1}, e_t)), u^j(S_{t-1})], \)

measures the linear relationship between variables across time (where the superscripts denote elements of the (demeaned) control vector). Notice that the correlation is bi-directional (correlation at leads and lags). Furthermore the other variables in the system are also temporally related since they are indirectly linked to the state vector through the constraints.

The complete set of temporal cross-correlation linkages follows from the general equilibrium specification. In any model, however, where households use financial assets to transfer consumption between periods and the distribution of asset payoffs depends on firms' real earnings (real allocation decisions) one
would expect strong temporal cross-correlations between asset values and the economic fundamentals. Shocks which may appear in the financial market should affect real allocation decisions which affect future earnings and asset values. Bi-directional temporal correlation does not necessarily imply any risk adjusted profit opportunities. All the variables in the theoretical real business cycle model are temporally related and the allocation is Pareto optimal.
Section 3: Empirical Evidence

The explicit specification of real business cycle models coupled with the dynamic programming representation yields a rich set of testable hypotheses. Any model, however, is intended only as a useful approximation, not as an exact replica of the economy. The relevant empirical question is: how good is the approximation, and how many characteristics of the actual economy will the model replicate? The outstanding seminal papers by Kydland and Prescott, and Long and Plosser, demonstrated that sample statistics on data generated by fairly simple real business cycle models match the sample statistics of some important US aggregate time series on real variables remarkably well.

This section presents the results from a sequence of tests focusing on the relationship between the values of financial and real capital. The most stringent null hypothesis is that Tobin's q always equals one. It is not a surprise that the data soundly reject this hypothesis. It is a surprise, however, that the data also fail to reject the least stringent null hypothesis that the market value of the firm and the value of its physical capital are not co-integrated series. I also test the degree of temporal interdependence with cross-autocorrelations and vector autoregressions. The tests show little significant feedback from the real variables to the financial variable.
Data

The data on the financial value of the "firm" is the equity plus debt of nonfinancial corporations. This is the series used by Abel and Blanchard (1986). The capital stock series is capital in all manufacturing from DRI.\(^1\) The other series come from the CITIBASE data bank. The data appendix gives detailed definitions. The series consist of quarterly observations from 1958-4 through 1985-4.

Tests

3.1 q Always Equals One

Figure 1 shows a plot of the financial value of the firm "V" and the value of real capital, k. The plot shows deviations from the sample means. It is fairly obvious that the two series are not identical. The financial value exceeds the physical value from the secular stock market boom in the early '60s until the oil shock and severe recession in '75. After that the financial series lies below the physical capital series until almost the end of the sample. The plot coincides with the folk wisdom that the stock market was "overvalued" in the '60s and "undervalued" in the '80s. Of course, the crossover points and the magnitudes depend on the normalization. The plot also shows the renowned volatility of financial values relative to real values. Capital

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\(^1\) I couldn't find the series Abel and Blanchard used. DRI updated their model and revised the data series. Abel (conversation) believes their series came from the model database. This series appears to be the nominal equivalent of the series they used. I deflated the series to obtain real capital.
follows a fairly smooth secular trend, while the financial series exhibits much more irregularity. More formal statistical tests also reject the null hypothesis that a single stochastic process describes both series at any reasonable significance level.

3.2 Tests for Feedback
The dynamic programming representation has testable implications even though the variables in the dynamic programming state vector are not necessarily observable or unique. The state transition equation summarizes the fundamental driving dynamics in the system. The state vector affects all current decisions, and the decision variables affect all the other variables through the constraints. As a consequence, there is an intertemporal relationship between all variables in the system. Cross-autocovariances,

\[ E[u_i^t, u_j^{t-1}] = E[u_i^i(S_t), u_j^j(S_{t-1})] \]

\[ = E[u_i^i(g(S_{t-1}, e_t)), u_j^j(S_{t-1})], \]

measure the linear intertemporal relationship between variables. (Where the u are deviations from means.)

The coefficients in vector autoregressions also give a measure of the feedback in the system. The coefficients are proportional to the cross-autocovariances,

\[ b_{ij} \propto E[(u_i^i, u_j^{j_{t-1}} | u_{n,t-1}, n \neq j)] \]

conditional on the linear information contained in the other variables in the regression.
The cross-autocovariances and the vector auto-regressions should give approximately the same results unless a linear combination of observables is a good proxy for the state vector. Straightforward interpretation of the test statistics requires stationary stochastic processes, which means detrending the raw data.

Table 1 shows the cross-autocorrelations. I detrended the data by first-differencing and with the Hodrick and Prescott procedure (see Prescott p10). A * indicates significance at the 5% level.

Table 1: Cross-Autocorrelations

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Hodrick-Prescott Detrending

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<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>*</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

The results are sensitive to the detrending procedure. The cross-autocovariances from the first-differenced data show no feedback from the real to the financial sector. The financial variable is correlated with led values of consumption and labor, but the lags
of real variables are not correlated with the financial variable. Detrending with the Hodrick-Prescott procedure shows significant cross-autocorrelations between most of the variables.

Table 2 shows the results from VARs. Appendix 2 gives the estimated coefficients and more detail.

Table 2: Vector Autoregression Tests

First Differences

<table>
<thead>
<tr>
<th></th>
<th>V(-1)</th>
<th>K(-1)</th>
<th>C(-1)</th>
<th>Z(-1)</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
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<td></td>
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</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
</tr>
</tbody>
</table>

Hodrick-Prescott Detrending

<table>
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<tr>
<th></th>
<th>V(-1)</th>
<th>K(-1)</th>
<th>C(-1)</th>
<th>Z(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
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<td>*</td>
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<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
</tr>
</tbody>
</table>

The test results from the VARs do not indicate linear interdependence between the financial and real sectors for either detrending procedure. None of the real variables feed back on the financial value variable. Market value is a leading indicator.

---

2 As a crude test for nonlinearity I added squared lagged terms to the regressions. The squared terms were insignificant and did not change the pattern of significance for the other variables.
for some of the real variables, but the feedback loop from real to financial values is missing.

The real sector only displays weak and asymmetric feedback among variables and the results are sensitive to the detrending procedure. When the data are first differenced only the change in financial value feeds back on the change in consumption. Detrending with the Hodrick-Prescott procedure reverses this result. The detrended data show that all the real variables feed back to consumption, but the financial variable does not. Neither set of results conform closely to the theoretical prediction of strong interdependence.

3.3 Co-Integration Tests
The least stringent restriction implied by any interdependent economic structure is that financial and real asset values move together in the long run. In the short run the equilibrating forces pulling the values together might be very weak, so one can observe long periods of disequilibrium.

Engle and Granger (1987) developed formal tests for co-integration to detect the long run (low frequency) co-movement of series. A test is to run the regression,

\[ 3.3 \quad K_t = a + bV_t + e_t. \]

and check the time series properties of the residual, e. Least squares picks the linear combination that minimizes the residual
sum of squares. The serial correlation of the residual shows the tendency of the series to move together. If the residual is white noise the equilibrium condition is satisfied each period up to a noisy linear transform. At the other extreme, if the residual is nonstationary the system has no tendency to satisfy the hypothesized relationship even in the very long run.

The Durbin-Watson statistic in the estimated regression, and the Dickey-Fuller regression and the augmented Dickey-Fuller regression on the residuals all fail to reject the null of no co-integration at the 10% level using Engle and Granger's table on p269. This result is not sensitive to changing (i) the sample period, or (ii) the deflators or not deflating, or (iii) substituting the Commerce Department's capital stock series, or (iv) substituting the S&P 500 index for V. The Stock-Watson test on the bi-variate series, V and k, fails to reject the null of two units roots. Appendix 2 gives the detailed results.

Comments
The results of the three tests are roughly consistent with each other, but not with the hypothesis that asset values are closely linked to the economic fundamentals. The test that Tobin's q always equals one is unreasonable. Nevertheless the figure reveals extremely large deviations of long duration. The temporal interdependence tests are sensitive to detrending and to the particular data series. The data are poorly measured
aggregated time-average data. Usually one would suspect these errors would introduce spurious correlation, yet the data show a surprising lack of intertemporal correlation. The vector autoregression tests on the changes in the variables are fairly consistent with other empirical evidence from different data sets and sample periods. Hall (1978) found a random walk described consumption except for correlation with stock market variables. Geske and Roll (1983) found the stock market led real activity, but found no feedback from the real sector.\(^3\)

The co-integration tests also indicate a decoupled system where financial asset values live a life of their own.

---

\(^3\) The market value of the "firm" (V) variable in this study includes a proxy for debt. Replacing the market value series with the first difference of S&P 500 index gives essentially the same results.
Section 3: Summary

This paper examines the theoretical and observed co-movement of financial variables and the economic fundamentals. I use a real business cycle model to define the economic fundamentals, but most of the results apply to less precisely specified macroeconomic models.

The financial market plays a fundamental role in co-ordinating agents' decisions in theoretical decentralized economic systems. Households trade securities to transfer consumption between periods and achieve optimal risk sharing. Firms use equity values as a signal for real investment decisions and firms' real allocation decisions determine the distribution of asset payoffs.

The data are not consistent with the theoretical predictions of a strongly interdependent system. Vector autoregression tests of the short-run dynamics indicate only weak feedback between the real variables, and no feedback from real to financial values. Co-integration tests indicate the linkage is very weak—at best—even in the long run. These results are consistent with many economists' responses to the October '87 stock market crash. There is no evidence that a dramatic change in the fundamentals led to the crash, but there is some concern that the crash may signal a reduction in real economic activity.

The empirical results raise an important and troublesome
question. If financial values are not closely linked to the fundamentals, then what determines financial values and real allocation decisions? Or if they are closely linked, why is the empirical evidence so weak?

This paper has no answer. One answer might be bad data. The problems in measuring capital are legend\(^1\) and aggregate data probably contain more measurement error than firm or household data. Studies relating consumption to asset returns also fail to find a close linkage. If the measurement error in real aggregate data obscures very basic economic relationships, then we learn little from empirical studies on aggregate data.\(^2\) On the other hand if the data correctly reveal a weak relationship between financial values and the economic fundamentals, then we need models with a different focus to explain the dichotomy and understand its implications.

\(^1\) It is easy to imagine that a stationary measurement error in investment could lead to a nonstationary measurement error in capital. Co-integration tests for investment and the financial value of the firm also fail to reject the no co-integration null. This is consistent with the poor empirical results from marginal q models.

\(^2\) Volatility tests, initiated by LeRoy and Porter, and Shiller, that only use financial data also fail to find a tight relationship between asset values and dividends.
References


Econometrica, 49, 555-74.


Appendix 1: Time to Build

The time to build specification makes capital heterogeneous. Installed units of capital contribute to current and future output. Newly purchased units of capital don't contribute to output until they are installed, which takes more than one period. As Kydland and Prescott (p1345) say, "That wine is not made in a day has long been recognized by economists". As a consequence grape juice and vintage wine have different prices, and capital of different vintages has different values. In a time to build model the firm holds capital of different vintages which have different values. The total value of the capital the firm owns equals the market value of the firm.

Consider the central planning problem in a time to build environment with a representative agent. The central planner wants to choose contingent plans for investment and labor that maximize the household utility function,

\[ W(S_t) = \sum_{\tau=0}^{\infty} \sum_{e' \varepsilon e} \pi(e') U(C_{t+\tau}, 1-Z_{t+\tau}), \]

subject to the constraint,

\[ C_t = f(k_t, z_t, e'_t) - I_t \]
\[ k_{t+1} = k_t + I_{t-J}. \]

The setup is the same as in section 1.2 except J+1 periods pass before current investment, I_t, contributes to production.

At a maximum,
A.3 \( W(S_t)_{It} = -U_{ct} + \sum_{r=J+1}^{\infty} \sum_{\epsilon'\epsilon} \pi(\epsilon')U_{ct+r}f_{kt+r} = 0. \)

Rewriting A.3 in terms of the shadow prices of consumption gives,

A.4 \( 1 = \sum_{r=1}^{\infty} \sum_{\epsilon'\epsilon} \Sigma p(\epsilon')_{t+J+r}f_{kt+J+r}. \)

The value of the marginal unit of investment equals the present value of the payoff stream which starts \( J+1 \) periods in the future. The shadow prices discount for time and risk. Since current output can be consumed or invested, the shadow value of the marginal unit of investment equals one at a maximum. We can also calculate the current shadow value of a unit of investment made in period \( t-1 \), ie a unit of capital of vintage one,

A.4 \( PI(1)_{t} = \sum_{r=1}^{\infty} \sum_{\epsilon'\epsilon} \Sigma p(\epsilon')_{t+J-1+r}f_{kt+J-1+r} \) \( \gg 1. \)

The current shadow value of a unit of vintage one capital exceeds the shadow value of a unit of vintage zero investment because its payoff stream starts one period earlier. And, in general, we can compute the current shadow value of a unit of capital of any vintage \( i \) (planted \( i \) periods ago) by the shadow price weighted sum of its payoffs.\(^1\)

Since a vintage \( i \) unit of capital will become a vintage \( i+1 \) unit

\(^1\) In an important sequence of papers Ross (1976,1978,1987) and Harrison and Kreps (1979) show that any asset can be valued as the shadow price weighted stream of payoffs.
of capital next period, the shadow values can be summarized in a compact recursive form,

\[ PI(i)_t = \sum_{e' \in \mathcal{E}} p(e')_t f(i+1)_{kt+1} + PI(i+1)_{t+1} \]

where,

\[ f(i+1)_{kt+1} = \begin{cases} 0 & i+1 < J \\ f_{kt+1} & i+1 \geq J \end{cases} \]

That is the current shadow value of a unit of capital of vintage \( i \) equals the payoff to that unit of capital next period, \( f(i+1)_{kt+1} \), plus the value of a unit of vintage \( i+1 \) capital, all weighted by the shadow prices of consumption.

Equation A.5 has the familiar look of an asset pricing equation, which it is. Since the production function is homogeneous of degree one we can think of any vintage capital as a separate plant (maybe plantings in the vineyard). The total current value of a vintage \( i \) plant is the shadow price times the quantity of vintage \( i \) capital,

\[ PI(i)_t I(i)_t = \sum_{e' \in \mathcal{E}} p(e')_t f(i+1)_{kt+1} + PI(i+1)_{t+1} I(i)_t \]

If each plant is a firm, then the equity value of a vintage \( i \) firm equals the value of its capital,

\[ V(i)_t = \sum_{e' \in \mathcal{E}} p(e')_t d(i+1)_{t+1} + V(i+1)_{t+1} = PI(i)_t I(i)_t \]

since, \( d(i+1)_{t+1} = f(i+1)_{kt+1} I(i)_t \), (there is no capital accumulation) and \( V(i+1)_{t+1} = PI(i+1)_{t+1} I(i)_t \). When each vintage of capital is a firm, current investment is a new firm. The owners of the new firm (purchasers of a new issue) instruct the
manager to buy capital (the consumption good) until, \( I(0)_t = V(0)_t \), the capital stock equals the value of the new issue. \(^2\)

In a model with a representative firm the firm holds a portfolio of capital of different vintages. The value of the portfolio of capital, say \( K \), is,

\[ K_{t+1} = \sum_{i=0}^{A.9} PI(i)_t I(i)_t, \]

the sum of the components. The equity value of the firm, \( V_t \), equals the present value of the payoff stream to all of the firm's capital, which equals the market value of \( K \). Now if there are \( J \) competitive markets in vintage capital the prices of vintage capital are observable and equal \( PI(i)_t \) in equilibrium. So the owners still instruct the manager to invest until the market value of its capital equals the equity value of the firm. If markets don't exist in vintage capital, then the owners must also inform the manager of the shadow values.

Vintage capital does not change the basic relationship between financial asset values and real asset values, but it makes the dynamics richer for both. Vintage capital requires a higher dimensional state vector to summarize the effect of past decisions. Now the state vector has dimension \( J+2 \), and

\(^2\) This market set up where each plant is a firm is closer to Long and Plosser's multistage production process where a firm produces an input for the next stage of production which occurs at a different firm. In their model the equity value of each firm equals the value of its "capital".
A.10 $K_{t+1} = K(S_t) = K(k_t, I_{t-1}, \ldots I_{t-J}, e'_t)$.

The state vector for equity value also has dimension $J+2$, eg,

A.11 $V_t = K(S_t) = K(V_{t-1}, I_{t-1}, \ldots I_{t-J}, e'_t)$.

The dynamics in the state equation show up in the dynamics for physical and financial capital.
Appendix 2: Detailed Empirical Results

Vector Autoregression Tests:

Table 1: Detrending by Differencing

<table>
<thead>
<tr>
<th>Dep</th>
<th>Constant</th>
<th>DV(-1)</th>
<th>DK(-1)</th>
<th>DC(-1)</th>
<th>DZ(-1)</th>
<th>F Stat</th>
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<td>-0.00</td>
<td>0.90</td>
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<td>1.22</td>
<td>4.04</td>
<td>-0.81</td>
<td>-0.24</td>
<td>-0.91</td>
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<td></td>
<td>0.71</td>
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<td>54.82</td>
<td>3.89</td>
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<td>DC</td>
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<td></td>
<td>1.69</td>
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<td>0.77</td>
<td>0.94</td>
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<tr>
<td>DZ</td>
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<td>0.02</td>
<td>1.00</td>
<td>0.00</td>
<td>0.35</td>
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<tr>
<td></td>
<td>-0.70</td>
<td>0.50</td>
<td>0.93</td>
<td>3.94</td>
<td>0.28</td>
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</tbody>
</table>

The t-ratios are below the coefficient estimates. The F statistic in column seven measures total contribution of all the variables with insignificant t-ratios in each equation. The critical F value at the 5% level for F(3, 60) is 2.76. The t-ratios properly summarize the significant contributions for variables either singly or jointly in these regressions.

Table 2: Detrended by Hodrick-Prescott Procedure

<table>
<thead>
<tr>
<th>Dep</th>
<th>Constant</th>
<th>DTV</th>
<th>DTK</th>
<th>DTC</th>
<th>DTZ</th>
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</thead>
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<td>30.97</td>
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<td>0.02</td>
<td>-0.01</td>
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<td>-1.52</td>
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<td>0.58</td>
<td>0.74</td>
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<tr>
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<td>1.72</td>
<td>0.70</td>
<td>1.87</td>
<td>5.36</td>
<td>15.79</td>
</tr>
</tbody>
</table>

Some of the detrended variables fail to reject the Dickey-Fuller test for no unit root.
Co-integration Tests

Co-integrating Regression Durbin-Watson,

\[ k_t = 101.6 + 0.45 V_t + u_t \quad \text{DW} = 0.085 \]

1.90 6.29

The critical value for the Durbin-Watson statistic at 10% is .322 in Granger and Engle's Table II.

Dickey-Fuller Regression,

\[ D_{ut} = -(-0.00082) u_{t-1} + e_t \quad -(-0.0721) \]

The critical value for the t statistic on the coefficient of \( u_{t-1} \) at 10% is 3.03 in Granger and Engle's table.

Augmented Dickey-Fuller Regression,

\[ D_{ut} = -(-0.0067) u_{t-1} + \]

\[ -(-0.6093) \]

\[ \{ 0.47 D_{ut-1} - 0.14 D_{ut-2} + 0.02 D_{ut-3} + 0.04 D_{ut-4} + 3.25 \} \]

The critical value for the t statistic on the coefficient of \( u_{t-1} \) at 10% is 2.84.
Data Appendix:
Definitions

NV = MVD + MVE

MVD = INT/YA, the market value of debt
MVE = DIV/YSP, the market value of equity

This follows Abel and Blanchard's construction of the financial value of the firm, see their appendix. The data come from DRI's data bank with the DRI mnemonic in parenthesis.

INT is net interest payments by nonfinancial business corporations (INTBUSCORPNF)

YA is the yield on Moody's A corporate bonds (RMMBCANS)

DIV is dividends paid by nonfinancial business corporations (NFCDIV)

YSP is the quarterly average of the monthly yield on the S&P 500.

NK is nonresidential manufacturing capital (KGFIXNRM) interpolated to follow the quarterly pattern of investment in plant and equipment (IP&EM)

More Definitions

V = NV/PUNEW, financial value of the firm in consumption units
k = NK/GDIF, real value of capital

The remaining data series come from CITIBASE. All capital letters indicate the CITIBASE mnemonic.

PUNEW is the consumer price index for all urban consumers

GDIF is the implicit price deflator for gross private domestic investment.

z = LHOURS manhours employed per week. I use a quarterly average of the monthly series.

C = GC82 personal consumption expenditures in 1982 dollars