Title
What Covered Interest Parity Implies about the Theory of Uncovered Interest Parity.

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The literature assumes that the *theory* of uncovered interest parity fails because investing without cover is risky and investors are risk adverse. But covered interest parity implies that the theory can fail even when investors are risk neutral and hold when investors are risk adverse and there is a risk premium. The failure to fully appreciate the relation between uncovered interest parity and risk premiums has probably contributed to our failure to understand why UIP fails empirically.

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1.0 Introduction

Although this paper has implications about testing uncovered interest parity (UIP) it is not primarily about testing. It is primarily about the theory of uncovered interest parity. The literature assumes that the theory of uncovered interest parity fails because investing without cover is risky and investors are risk adverse. But covered interest parity (CIP) implies that the theory can fail even when investors are risk neutral and hold when investors are risk adverse and there is a risk premium. The failure to fully appreciate the relation between uncovered interest parity and risk premiums almost certainly has contributed to our failure to understand why UIP fails empirically.

Section 2 briefly reviews the theory of uncovered interest parity. Section 3 describes what covered CIP implies about UIP. Section 4 describes the role of risk premiums when CIP holds. Section 5 uses Sarno et al (forthcoming) to illustrate how ignoring the relation between the theory of UIP and risk developed in Sections 3 and 4 can affect empirical research.

2. Uncovered Interest Parity


1 Testing the theory requires additional assumptions such as rational expectations, capital mobility and the absence of taxes.
As this literature shows, empirically UIP routinely fails, particularly in the short run, at short maturities and between developed countries.

For reasons that will become clear shortly, I express the theory of uncovered interest parity as follows:

\[ E(s_{t+1}/\Phi_t) - s_t - (i_t - i_t^*) = E(R_{t+1}/\Phi_t) = 0.0 \]  (1)

Following the usual notation, \( s_t \) is the log of the spot domestic price of foreign exchange, \( i_t \) is a risk free domestic interest rate and \( i_t^* \) is a risk free foreign interest rate. The maturity of the interest rates should match the maturity of the expected change in spot rates. \( E(x_{t+1}/\Phi_t) \) is the expectation of \( x_{t+1} \) taken at time \( t \) based on the information \( \Phi_t \) available at time \( t \). So \( E(s_{t+1}/\Phi_t) \) is the expected future spot rate.

Ignoring any relevant transaction costs, the theory of uncovered interest parity holds when \( E(R_{t+1}/\Phi_t) \) is zero. The UIP literature usually interprets \( E(R_{t+1}/\Phi_t) \) as a risk premium. See for example Chinn (2006). If investors were risk neutral, presumably the theory of UIP would hold.

3.0 What CIP Implies about the Theory of UIP

Covered interest parity suggests that the theory of uncovered interest parity fails, not because investors are risk adverse, but because there is an expected excess return. That expected excess return may or may not equal some risk premium. The next section discusses the relationship between expected excess returns and risk premiums. This section concentrates on expected excess returns.

As that literature points out, covered interest parity is the equilibrium, or what is often called the no-arbitrage, condition associated with covered interest arbitrage. Effective arbitrage appears to quickly eliminate any potential profit from covered interest rate arbitrage. As Akram et al (2008) points out, “It seems generally accepted that financial markets do not offer risk-free arbitrage opportunities, at least when allowance is made for transaction costs.” Akram et al (2008) goes on to say that one “can safely assume arbitrage-free prices in FX markets when working with daily or lower frequency data”. From this point on I assume that covered interest parity holds empirically.

Ignoring the relevant transaction costs, eq. (2) describes the theory of covered interest parity where \( f_t \) is the log of the forward exchange rate with the same maturity as the interest rates.

\[
(f_t - s_t) - (i_t - i^*_t) = 0.0
\]  

(2)

When CIP holds for all \( t \), eq. (2) implies eq. (3).

\[
s_{t+1} = f_{t+1} - (i_{t+1} - i^*_{t+1})
\]  

(3)

The actual excess return from investing without cover is \( s_{t+1} \) minus \( f_t \), which I denote as \( R_{t+1} \). When \( s_{t+1} \) equals \( f_t \), the actual return from an uncovered investment is the same as the return from a covered investment. When \( s_{t+1} \) is greater than \( f_t \), borrowing dollars and investing in sterling without cover produces a higher return than the same investment with cover. When \( s_{t+1} \) is less than \( f_t \), borrowing sterling and investing in dollars produces a higher return than the same investment with cover.

Subtracting \( f_t \) from both sides of eq.(3) produces eq. (4) .
\[ R_{t+1} = s_{t+1} - f_t = \Delta f_{t+1} - (i_{t+1} - i^*_{t+1}) \quad (4) \]

Note that \( R_{t+1} \) equals \( s_{t+1} - f_t \) by definition while the equality between \( R_{t+1} \) and \( \Delta f_{t+1} \) minus \( (i_{t+1} - i^*_{t+1}) \) is an implication of the theory of covered interest parity.

Taking the first difference of eq. (3) and doing a little rearranging produces eq. (5).

\[ \Delta s_{t+1} = (i_t - i^*_t) + \Delta f_{t+1} - (i_{t+1} - i^*_{t+1}) \quad (5) \]

Using eq. 4, eq. (5) can be rewritten as eq. (6).

\[ \Delta s_{t+1} = (i_t - i^*_t) + \Delta f_{t+1} - (i_{t+1} - i^*_{t+1}) = (i_t - i^*_t) + R_{t+1} \quad (6) \]

Since \( R_{t+1} \) is the actual excess return to investing without cover, it follows that the expected excess return is \( E(R_{t+1}/\Phi_t) \). Rearranging eq. (6) and taking expectations produces eq. (7).

\[ [E(s_{t+1}/\Phi_t) - s_t] - (i_t - i^*_t) = E(\Delta f_{t+1}/\Phi_t) - E(i_{t+1} - i^*_{t+1}/\Phi_t) = E(R_{t+1}/\Phi_t) \quad (7) \]

Eq. (7) says that uncovered interest parity holds if and only if the expected excess return is zero. This interpretation of the theory of uncovered interest parity does not assume rational expectations, risk neutrality or anything about risk premiums. All it assumes is that covered interest parity holds.

### 4.0 Expected Excess Returns and Risk premiums

When the UIP literature interprets \( E(R_{t+1}/\Phi_t) \) in eq. (1) as a risk premium it implicitly assumes stock equilibrium. That assumption may be reasonable for the long run, but it is unlikely to hold in the very short run.\(^2\) In either case, the assumption needs to be explicit because it has important implications.

To make my point as simply as possible, let all uncovered investment involve forward contracts. \( A_t \) represents the actual stock of uncovered forward claims on sterling

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\(^2\) The theory of covered interest parity also assumes stock equilibrium. But that stock equilibrium does not imply stock equilibrium for uncovered interest parity. Investors may be willing to move quickly to earn riskless returns and at the same time be slow to exploit risky profits even when they exceed risk premiums.
and $A^D_t$ the desired stock of $A_t$. When $A_t$ is positive investors hold contracts to buy sterling forward that are not covered. When $A_t$ is negative, investors hold contracts to sell sterling forward that are not covered.

Buying sterling forward without cover is equivalent to borrowing in the U.S. and lending in the U.K. without cover. In both cases the actual excess return is $R_{t+1}$.

When investors are risk neutral, desired changes in $A_t$ presumably depend on only the expected excess return $E(R_{t+1}/\Phi_t)$. When investors are risk adverse, desired changes in $A_t$ depend on risk adjusted expected excess returns where $\nu_t$ is the risk premium.\footnote{The relevant risk premium is positive when the expected excess return is positive and negative when the expected excess return is negative.}

\[ A^D_t - A_t = a(E(R_{t+1}/\Phi_t) - \nu_t) \]  

(8)

When $E(R_{t+1}/\Phi_t)$ is positive and greater than $\nu_t$, risk adverse investors buy sterling forward without cover. When $E(R_{t+1}/\Phi_t)$ is negative and less than $\nu_t$, they sell sterling forward without cover. When $E(R_{t+1}/\Phi_t) - \nu_t$ is zero they neither buy nor sell.

For simplicity, I assume that risk premiums depend on $A_t$ and a vector of exogenous variables $Z_t$.

\[ \nu_t = \Psi(A_t, Z_t) \]  

(9)

The larger $A_t$, the larger $\nu_t$.

$E(R_{t+1}/\Phi_t)$ also depends on $A_t$ and a different vector of exogenous variables $z_t$.

\[ E(R_{t+1}/\Phi_t) = \Omega(A_t, z_t) \]  

(10)

As $A_t$ increases $E(R_{t+1}/\Phi_t)$ tends to decrease.

With only flow equilibrium, expected excess returns and risk premiums can diverge. With only flow equilibrium, there is nothing preventing a risk neutral investor from expecting an excess return.
With stock equilibrium \((A_{t}^{D} - A_{t})\) is zero. The standard assumption that risk neutrality produces UIP depends on the implicit assumption of stock equilibrium. Stock equilibrium and risk neutrality require that \(E(R_{t+1}/\Phi_{t})\) is zero. On the other hand, stock equilibrium and risk aversion requires that, in general, risk premiums just offset expected excess returns.

But when \(A_{t}\) is zero, risk adverse investors also do not want to buy sterling forward without cover as long as \(\upsilon_{t}\) is greater than \(E(R_{t+1}/\Phi_{t})\). Investors also do not want to sell sterling forward without cover as long as \(\upsilon_{t}\) is less than \(E(R_{t+1}/\Phi_{t})\). Therefore stock equilibrium with risk aversion implies eq. (11).

\[
\begin{align*}
E(R_{t+1}/\Phi_{t}) &= \upsilon_{t} & A_{t} \neq 0 \\
E(R_{t+1}/\Phi_{t}) &\leq \upsilon_{t} & A_{t} = 0 \text{ and } E(R_{t+1}/\Phi_{t}) \geq 0 \\
E(R_{t+1}/\Phi_{t}) &\geq \upsilon_{t} & A_{t} = 0 \text{ and } E(R_{t+1}/\Phi_{t}) \leq 0
\end{align*}
\]

Equilibria like the one described by eq. (11) are not the result of my simplifications. When risk adverse investors do not hold uncovered assets or liabilities, they have no incentive to acquire them as long as the risk of doing so equals or exceeds the expected excess return. As a result, if both \(A_{t}\) and \(E(R_{t+1}/\Phi_{t})\) are zero, uncovered interest parity holds even though there may be a risk premium.

With the addition of the equilibrium condition that \(A_{t}^{D}\) equals \(A_{t}\), eqs. (9) to (11) describe a system of four equations with four endogenous variables: \(A_{t}\), \(A_{t}^{D}\), \(E(R_{t+1}/\Phi_{t})\) and \(\upsilon_{t}\). The UIP literature consistently ignores what a system of equations like this implies about the role of risk premiums in the theory of UIP. Sarno et al (forthcoming) is an example of what can happen.
5. Sarno, Schneider and Wagner

Sarno et al (forthcoming) tries to estimate risk premiums using a model based on covered interest rate parity. Their model includes the assumption that \( E(s_{t+1}/\Phi_t) \) equals \( f_t \) plus a risk premium \( \nu_t \). By including that assumption they implicitly assume that the expected excess return equals the risk premium. From there they go on to ignore the expected excess return and estimate a model of the risk premium.

What they do is equivalent to assuming that demand equals supply and then going on to estimate a demand schedule while ignoring the supply schedule. As a result, their estimates of the risk premium suffer from simultaneous equations bias.

6.0 Summary

CIP implies that the theory UIP fails if and only if expected excess returns are zero. The UIP literature routinely attributes that failure to risk premiums. But that attribution implicitly assumes a stock equilibrium in which actual and desired investments without cover are equal. Without that implicit assumption there is no theoretical reason to equate expected excess returns and risk premiums.

Failing to recognize the crucial role of stock equilibrium can lead to serious problems. Sarno et al (forthcoming) is an example.
References


