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Errors as a Productive Context for Classroom Discussions: A Longitudinal Analysis of Norms in a Classroom Community

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Errors as a Productive Context for Classroom Discussions: A Longitudinal Analysis of Norms in a Classroom Community

By

Nicole Therese Leveille Buchanan

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Education in the Graduate Division of the University of California, Berkeley

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Professor Geoffrey Saxe, Chair
Professor Aki Murata
Professor Darlene Francis

Spring 2016
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Nicole Therese Leveille Buchanan
Abstract

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How do teachers and students create classroom environments in which mathematical errors are regarded as important opportunities for learning? What norms support students in learning from their errors, and how do these norms develop in a classroom community? This dissertation addresses these questions through a longitudinal case study investigating the emergence of classroom norms related to the treatment of errors. Classroom norms are here understood to be taken-as-shared expectations for behavior in a classroom. The fifth-grade case study classroom was selected because in prior research studies the experienced and highly-regarded teacher had engaged students in rich discussions of their mathematical errors and the opportunities presented by these errors for learning.

To answer the questions of (a) what norms related to mathematical errors were taken up, and (b) how these norms developed over the course of the school year, Saxe’s (2012) framework describing the relation of micro-, onto-, and sociogenetic processes was used as a guide for determining methods. Sociogenetic processes were the focus of this investigation, and Saxe’s framework points to microgenetic constructions, when studied collectively over time, as likely to illuminate otherwise difficult to observe sociogenetic processes, such as norm development. To provide information about microgenetic constructions related to errors, several types of evidence were collected throughout the school year during three data collection periods lasting two weeks each: at the beginning (September), middle (January), and end (April) of the 2014 to 2015 school year. During all three time periods, the teacher was interviewed, five “focal” students were interviewed, classroom mathematics lessons were video-recorded daily, and all students in the classroom answered a paper-and-pencil multiple choice survey about their expectations related to errors. Interviews were analyzed using grounded analysis methods, and video-recordings were analyzed using a focused coding procedure and StudioCode software. Sources of evidence were used in a triangulating fashion to identify norms in the classroom.

Through this analysis, seven norms were identified as having been taken up by the majority of members of the class by the end-of-year data collection period. Two of these norms were selected for in-depth description. The norm everyone has some mathematical understandings to which you should pay attention provides a good example of a norm that was closely tied to a specific collective practice, the “coaching” practice that was used frequently in the case-study
classroom. The norm *there are different types of errors, only some of which are acceptable* provides an example of a norm that emerged part-way through the school year in response to a problem with the way errors were being treated. Classroom interactions and teacher and student interview statements exemplifying these norms are described. The process through which these two norms emerged in the case study classroom over the course of the school year is detailed, using evidence collected throughout the school year. In general, the teacher strongly promoted these norms by frequently and persistently modeling, describing, and praising behaviors consistent with these norms and by correcting inconsistent behaviors.

Implications for how Saxe’s framework may be productively applied in future investigations of classrooms norms are discussed. In particular, attention to ontogenetic processes – that is, individuals’ shifting expectations over time – was found to be useful as an access point for identifying the norms of a classroom community, and the teacher’s actions and expectations were found to be especially important indicators of classroom norms. Examination of collective practices related to errors was also useful for the identification of norms because some norms were strongly associated with collective practices, such as the “coaching” practice. The results of this study also have implications for teaching practice. The findings indicate that children are capable of taking up challenging practices related to the study of errors, and teachers who promote these practices in their classrooms may be successful if they are persistent in modeling, explaining, and praising the desired practices.
Table of Contents

Abstract ........................................................................................................................................... 1

Acknowledgements ......................................................................................................................... ii

Chapter 1: Introduction ..................................................................................................................... 1

Chapter 2: Current Literature and Frameworks ............................................................................... 2
  Background and Review of the Literature ....................................................................................... 2
  Research Framework and Purpose .................................................................................................. 8

Chapter 3: General Study Design and Methods ............................................................................. 13
  Case Study Design .......................................................................................................................... 13
  Description of Case Study Classroom ............................................................................................ 15
  Data Sources and Data Collection Procedures ............................................................................. 18

Chapter 4: Analyses of frequency of errors and teacher and student talk related to norms in mathematics lessons ........................................................................................................ 27
  Methods: Video Coding Procedures ............................................................................................... 27
  Results: Frequency of Interactions Related to Errors and Classroom Behavioral Expectations ........................................................................................................................................ 34

Chapter 5: What Norms Emerged in the Case Study Classroom .................................................. 38
  Procedures for Identifying Norms ................................................................................................... 38
  Norms Identified in the Case Study Classroom .............................................................................. 41

Chapter 6: How Norms Emerged in the Case Study Classroom – Shifts in Interactions over Time ............................................................................................................................... 61
  Classroom Discussions of Values .................................................................................................... 61
  Emergence of the Coaching Practice ............................................................................................... 67
  Emergence of the Everyone Has Some Mathematical Ideas to which You Should Pay Attention Norm ........................................................................................................................................ 72
  Emergence of the There are Different Types of Errors, Only Some of Which are Acceptable Norm ........................................................................................................................................ 80
  Summary ......................................................................................................................................... 93

Chapter 7: Conclusions and Implications ...................................................................................... 94
  Summary of Results ......................................................................................................................... 94
  Implications for Future Research Methods ................................................................................... 95
  Implications For Teaching Practice and Teacher Professional Development ............................. 99

References .......................................................................................................................................... 101

Appendices ......................................................................................................................................... 104
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Chapter 1: Introduction

Researchers have shown that when classroom discussions challenge students to justify, compare, and explain their developing mathematical ideas, many students extend their prior understandings to new kinds of mathematical problems in ways that lead to errors. For example, students may use their knowledge of addition with whole numbers to interpret addition with fractions (Smith, di Sessa, & Roschelle, 1994). Research and theory have indicated that open classroom discussion of these errors and the partial understandings underlying them may support students’ conceptual learning and help them to further coordinate their developing ideas (Bray, 2013; Borasi, 1994; Carpenter et al., 1996; Chapin et al., 2009; Kazemi & Stipek, 2001; Santagata, 2005). To date, we have limited knowledge about how teachers support students in engaging in productive discussions about errors. In this dissertation, I examine such discussions through a case study of one upper-elementary classroom with a particular focus on emerging norms for the treatment of errors.

In the following chapters, I describe this case study in which I investigated what norms related to the treatment of mathematical errors were developed over the course of a school year, as well as how these norms developed. In Chapter 2, I describe the motivation for this study. I begin by situating this study in prior scholarship, reviewing current research on the utility of discussions of students’ mathematical errors and difficulties with implementing such discussions in classrooms. This review of current scholarship indicates that classroom norms related to discussions of errors are an appropriate focus for study. I then review the theoretical and analytic frameworks I used to approach the study of norms related to the discussion of errors, and I describe specific research questions for this study. In Chapter 3, I describe data collection methods, data sources, and analysis methods used for this case study, and then in Chapters 4 through 6, I detail specific methods used to analyze particular data sources used to address my focal research questions. Thus, in Chapter 4, I ask whether or not norms and mathematical errors came up during whole class discussions in the case-study class, and I describe methods and results for the analysis of video recordings of case-study classroom mathematics lessons. In Chapter 5, I ask what norms supported the discussion of students’ mathematical errors in the case-study class, and I describe specific methods and results related to this question. In Chapter 6, I ask how these norms became norms in the case-study class, and I describe specific results related to how these norms emerged over the school year in the class. Finally, in Chapter 7, I present some conclusions based on these results as well as suggestions for future research in this area.
Chapter 2: Current Literature and Frameworks

In this chapter, I first review current literature related to discussions of students’ mathematical errors and partial understandings in order to provide background on the relevant issues to be addressed by this study. Then, I review the theoretical and methodological frameworks used in this investigation. Finally, I summarize the specific research goals of the current study.

Background and Review of the Literature

Discussions in which teachers and/or students explain and build on students’ mathematical errors and partial understandings of mathematics are important for students’ learning and development (Bray, 2013; Carpenter, Fennema, & Franke, 1996; CCSSI, 2010). For this study, I define students’ mathematical partial understandings as ideas based in prior knowledge but incompletely coordinated with one another or with relevant mathematical principles for the task at hand. When students apply their prior knowledge and ideas to new types of mathematical tasks, their understandings may be inadequate for considering all relevant aspects of the task. Consequently, the student may arrive at an incorrect answer, here referred to as an error.\(^1\)

The benefit of discussions of students’ partial understandings. A constructivist epistemological framework identifies the production of a mathematical error as a potential opportunity for cognitive development through resolution of disequilibrium, a state in which current cognitive structures are found to be inadequate for dealing with new situations or information (Piaget, 1967, 1970). A student who produces an error in mathematics may realize that her solution is inadequate may experience disequilibrium as a result, recognizing that her current mathematical understandings do not produce the expected result when applied to the task at hand. However, in order for development of more coordinated understandings to follow, the student must not only realize the inadequacy of her solution and but also have the chance to grapple with her current partial understandings in light of the challenges of the new mathematical situation with which she is faced. This grappling—an attempt at sense-making—may lead to new, more coordinated understandings through equilibration processes as described by Piaget (1970).

Though this cognitive work may seem highly individual, classroom discussions can contribute to these processes as they afford the opportunity for errors to be identified and partial understandings to be explained, justified, and compared with other ideas through the collective activity (Smith, di Sessa, & Roschelle, 1993; see Confrey, 1990 for a review). Additionally, in the course of discussions other students may be invited to be part of the process of exploring, explaining, and building on partial understandings, and through this process these students may deepen or extend their own understandings of the topic. Discussions of errors and partial understandings thus afford the entire class the opportunity to reframe a problem, to explore mathematical contradictions highlighted by an error, and to consider alternative strategies and

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\(^1\) This type of error is here understood to result from incomplete coordination of relevant mathematical ideas. Errors can be simple or complex and conceptual or procedural. Additionally, a somewhat different type of error can occur when a student understands the solution strategy for a task and draws on the relevant mathematical ideas but arrives at an incorrect answer due to some other process (e.g., distraction by another student, carelessness).
ideas (Kazemi & Stipek, 2001). For these reasons, several authors advocate the use of discussions of students’ errors and underlying partial understandings in mathematics lessons (Bray, 2013; Chapin et al., 2009), and such discussions are highly aligned with current education policy recommendations (CCSSI, 2010).

**Differences from past conceptions of and recommendations about errors.** Notably, research and recommendations regarding the treatment of students’ errors and “misconceptions” have varied historically depending on the popularity of various epistemological frameworks and instructional techniques. For example, research from the information processing perspective (summarized in Confrey, 1990) conceptualized errors as analogous to “bugs” in the procedural “code” of students’ mathematical understandings (Brown & Burton, 1978; Brown & VanLehn, 1980) and indicated that the “bugs” should simply be replaced by correct procedures. Additionally, researchers have approached errors from the perspective of conceptual change, suggesting that students need to replace their misconceptions with correct conceptions through conceptual shifts analogous to Kuhn’s paradigm shifts. From a radical constructivist perspective, Borasi (1994) has advocated for the use of errors as “springboards for inquiry,” suggesting that students explore their errors and partial understandings through inquiry experiments. These various recommendations each recognize the need to address students’ errors and understanding in mathematics lessons. Current recommendations and research build on these historical ideas by incorporating current research on the benefits of classroom discussions for facilitating student learning and development (Carpenter et al., 1996; Chapin et al., 2009; Gearhart et al., 2013).

**Current recommendations and support for discussions of partial understandings.** Researchers currently recommend that teachers facilitate classroom discussions in which students explain, justify, and compare their mathematical ideas in respectful ways. To support enactment of these recommendations, some authors have described specific pedagogical strategies that teachers can use to facilitate these discussions (Bray, 2013; Chapin et al., 2009). Additionally, professional development programs that support teachers in holding class discussions based on students’ mathematical ideas have existed for years, including well-researched programs such as *Cognitively Guided Instruction* (Carpenter et al., 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003; Franke, Carpenter, Levi, & Fennema, 2001), *Developing Mathematical Ideas* (Bell, Wilson, Higgins, & McCoach, 2010; Schifter, Bastable, & Russell, 1999a, 1999b, 2003), and *Integrated Mathematics Assessment* (Saxe, Gearhart, & Nasir, 2001), as well as Fosnot and Dolk’s (2001a, 2001b, 2001c, 2002) *Young Mathematicians at Work* series. These programs have been shown to build teachers’ content and pedagogical content knowledge (PCK; Hill, Rowan, & Ball, 2005) related to teaching based on their students’ actual mathematical understandings (Franke et al., 2001; Bell et al., 2010).

While this knowledge is certainly an important component of what teachers need in order to facilitate classroom mathematical discussions of partial understandings, Bray (2011) found that teachers also need skills, knowledge, and beliefs that support them in developing classroom norms to create a “community of learners” who engage in thoughtful discussion, justification, and explanation of mathematical ideas, including partial understandings. To address this issue, Santagata and Bray (2015) developed and piloted a video-based professional development program with the intention of supporting teachers in helping students learn from their errors. Their results indicate some positive changes in teachers’ practice, as well as continued challenges, after engaging with the professional development program. The current study aims
to support efforts like Santagata and Bray’s by providing in-depth illustration of how a teacher might support classroom norms that promote thoughtful discussion of students’ errors.

**Difficulties with discussions of partial understandings.** Despite current research and policy that strongly indicate the benefits of discussions of students’ errors and partial understandings, many teachers in the U.S. are not implementing these recommendations. Quite the opposite, research has shown that teachers have difficulty adopting new discussion-based methods for teaching mathematics (Bray, 2011; Nathan & Knuth, 2003), and even very experienced mathematics teachers face frequent dilemmas when basing their teaching on students’ mathematical ideas, including partial understandings (Ball, 1993). Teachers have been shown to have difficulty providing feedback to students who make errors and connecting students’ mathematical ideas to canonical problem-solving methods (Son, 2016). Moreover, studies have found that teachers – particularly teachers in the United States – typically have difficulty with or resistance to holding discussions of students’ errors (Santagata, 2005; Schleppenbach, Flevares, Sims, & Perry, 2007) and may treat errors as something to avoid instead of opportunities for learning (Ingram, Pitt, & Baldry, 2015). Bray (2011) suggests that U.S. teachers’ beliefs and knowledge of mathematics and of mathematics learning and teaching may be contributing to their reluctance to facilitate discussions of students’ errors and partial understandings. Specifically, U.S. teachers may lack skills and knowledge needed to establish norms that support respectful discussions focused on relevant mathematical concepts.

Above, I have described current research, theory, and policy indicating that classroom discussions of students’ mathematical errors and partial understandings are a useful and desirable feature of mathematics lessons. I have also noted the current difficulty U.S. teachers face in implementing these recommendations, and I have identified research suggesting that a lack of knowledge and skill for establishment of appropriate classroom norms is a key aspect of this difficulty. I now turn to a more in-depth discussion of what classroom norms are, how they have been studied, and specifically what research has shown regarding classroom norms related to discussions of students’ mathematical partial understandings.

**Classroom norms for discussions of partial understandings.** Classroom norms are a part of the collective activity of the class (Saxe et al., 2014) and can be defined as taken-as-shared expectations that govern how the activity is carried out in the group. Here, the term collective indicates that norms belong to the group, not to individuals within the class. Norms can be specific to a single practice or subject matter, or they can be general expectations for behavior in the group regardless of the specific activity in which the group is engaged (Cobb & Yackel, 1996).

Practice-specific norms may be tied to a particular collective practice in which the group engages regularly. Collective practices are “recurring forms of activity in which norms, values, and social positions are constituted and re-constituted over time” (Saxe et al, 2015, p. 7; see also Saxe, 2012). For example, as is described in more detail in Chapters 5 and 6, the case-study class that was the focus of this dissertation engaged in a practice they called coaching, in which a student who had made an error or was struggling to solve a problem was supported in correctly solving the problem by another class member. Individuals engaged in a collective practice have expectations for typical behavior associated with that practice. These norms may be specific to the practice; for example, during the coaching practice in the case-study class, the “coach” was

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2 I use the term “taken-as-shared” rather than “shared” to indicate that in interactions, individuals cannot directly know what occurs in others’ minds (von Glasersfeld, 1989).
expected to ask the “person-being-coached” questions rather than to tell him or her the answer to the problem. This expectation was specific to the coaching practice and did not apply to other interactions in the classroom. However, norms that regulate behavior during collective practices need not be exclusive to that practice. For example, as described in Chapter 5, the case-study class developed a norm that “everyone has mathematical ideas to which you should pay attention.” This norm regulated behavior during the coaching practice as the coach was expected to attend to the mathematical ideas of the person-being coached. This norm applied to other situations it the case study class, as well.

Subject-specific norms are not specific to only one practice but are specific to certain types of academic content. For example, in mathematics sociomathematical norms may include "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (Cobb & Yackel, 1996, p. 178). These norms are specific to mathematics simply because they do not apply to other academic subjects.

General social norms, in contrast, apply broadly to interactions in a group. An example of a general social norm could be how one might legitimately get a turn to talk during a discussion (e.g., raising a hand and waiting to be called on by the teacher). All norms – general, subject-specific, and practice-specific – serve to regulate the interactions that occur in the classroom, so it is important for classrooms to develop norms that facilitate learning. The process of establishing these norms highlights the relationship between individual and collective activity in the classroom.

The individual, the collective, and the negotiation of norms. Norms are a property of the classroom as a collective; individual expectations are the analogous construct at the individual level (Cobb & Yackel, 1996). However, norms are established through interactions in the classroom, and these interactions clearly involve the actions and understandings of individuals: “Individual children's mathematical interpretations of the task at hand and of others' activity profoundly influence the nature of the social interactions in which they participate" (Cobb & Yackel, 1996, p. 599). When a new group of students and a teacher begin interacting, initial interactions of the group may be based on their individual expectations about math classrooms based on previous experiences. While they interact, however, the explicit (e.g., verbal directions of how to act) and implicit (e.g., negative reactions to a behavior, such as ignoring it) social cues put forth by group members start to establish some taken-as-shared expectations for further interaction in that classroom. Norms specific to that classroom begin to form and to govern individuals’ behaviors as they interact with each other in the shared space over time. In other words, individual constructive activity and social collective activity in the classroom are mutually constitutive; each informs the other (Saxe et al., 2014).

The establishment of norms in a classroom may be most apparent at the beginning of a school year, but certainly the process of continual negotiation and renegotiation of norms occurs throughout the school year. As individuals interact, they may shift their expectations for interactions to meet the needs of the group over time. For example, as different mathematical content becomes the focus of discussions, new expectations may be needed to fit the particular challenges of the content. Or, as students develop their abilities to explain their thinking, the teacher may push for more rigorous standards for justification of explanations based in mathematical principles. Therefore, investigations of the establishment of classroom norms must take a longitudinal (i.e., over the entire school year) perspective in order to capture both the
initial development of norms in the classroom and the renegotiation of these norms among members of the group over time.

Additionally, norms may develop related to specific collective practices that arise in a classroom to meet certain needs of the community. Collective practices have normative structures – that is, specific patterns of actions and interactions that are expected. These structures are reproduced as the practice is repeated, or they are altered over time as individuals engaged in the practice modify it so as to meet the requirements of individual goals during particular instances (Saxe et al., 2015). Consequently, norms for behavior are reproduced in interactions as the normative structure is reproduced, or they are altered as individuals modify behavior to meet the requirements of particular circumstances. Alterations to normative structures or to norms may result in long-lasting shifts if members of the class take up the altered forms over time. This relationship of reproduced and altered practices and norms is most clearly visible through the analysis of particular patterns of interaction over time.

While I have emphasized that all individuals in the classroom participate in the establishment of these norms, it is important to note that the teacher in the classroom typically holds more power than the students in setting up expectations. Therefore, it has been argued that teachers need to understand and have a plan for the types of norms that will most productively facilitate student learning in their classrooms (Kazemi & Stipek, 2001). Descriptions of how such productive norms are established through teacher and student interactions may therefore support teachers in understanding and establishing similarly productive norms in their own classrooms:

The notion of sociomathematical norms is very different from specific prescriptions for educational practice. Sociomathematical norms concern a set of expectations about what constitutes mathematical thinking. Supporting teachers to create sociomathematical norms in their classrooms requires more than describing discrete behaviors. Rather teachers need to understand what a sociomathematical norm is and construct pedagogical strategies that can be applied in a variety of contexts. (Kazemi & Stipek, 2001, p. 79)

The difficulty of establishing norms for discussions of partial understandings. Norms that support discussions of partial understandings must be in place in order for these discussions to be maximally useful (Kazemi & Stipek, 2001). However, research has shown that teachers have difficulty supporting the establishment of productive norms for discussions of partial understandings and errors. Through a multiple case studies, Kazemi and Stipek (2001) found that the general social norm that “students can make mistakes, which are a normal part of learning” (p. 62) was present in all four classrooms they examined. However, only the classrooms with a high press for student learning had developed the analogous specific norm that “errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies” (p. 64). Similarly, in her case study of a single classroom Cheval (2010) found that general social norms for accepting mistakes as part of the learning process were established, but the teacher failed to support the establishment of specific norms for the discussion of students’ partial understandings, despite his beliefs that such discussions were valuable for student learning. Thus, research has shown that establishing sociomathematical norms for productive discussion of students’ partial understandings is a useful and desirable goal, but teachers have difficulty doing so. Specifically, teachers in the studies described above were able to establish social norms in which mistakes were expected as part of the learning process. However, these same teachers declined to or were unable to establish specific norms that discussion of errors is useful for helping the class to reframe a problem or to explore
contradictions and alternative strategies. Researchers have recommended that careful description
of what such specific norms might be, as well as how they can be established in classrooms,
might help teachers understand these norms and gain ideas of how to establish them in their own
classrooms (Kazemi & Stipek, 2001).

In this dissertation, I therefore aim to build on current literature through an in-depth case
study of how such specific norms are established and continually negotiated in a classroom
throughout the school year. In particular, this dissertation will add to prior literature describing
norms related to errors by investigating the patterns of classroom interaction that over time
support the establishment of norms related to discussions of errors.

Current research on norms for discussions of partial understandings. As described
above, current literature contains little description of the successful establishment of specific
norms that support productive discussions of partial understandings. However, some authors do
allude to such norms while describing practices teachers may find useful in facilitating
discussions of students’ errors or partial understandings. For example, Bray (2013) recommends
establishing a “community of learners [in which] students actively engage in working
collaboratively to grapple with mathematical ideas” (p. 429). She recommends establishing
norms that support students in “uncover[ing] and unpack[ing] errors” themselves, perhaps by
assigning students specific tasks such as explaining, comparing, evaluating, or developing
questions based on each other’s ideas. However, Bray does not give a comprehensive or detailed
account of these norms or how they might be established in classrooms. Chapin and colleagues
(2009) also describe a variety of teaching practices to support classroom mathematical
discussions, including discussions of errors and partial understandings. As Kazemi and Stipek
(2001) point out, however, such practices may be helpful for teachers, but a deep understanding
of the types of specific norms that support classroom discussions of errors is essential for
supporting flexible selection of teaching practices that are likely to succeed in a variety of
contexts. In my dissertation study, I will build on these authors’ brief and general descriptions of
norms for discussion of errors by studying a single class intensively to produce in-depth
descriptions of norms supporting productive discussion of students’ errors in that classroom.

Furthermore, in these studies that provide some brief description of norms for discussions
of errors, the authors are for various reasons unable to provide description of how these norms
develop in classrooms. For example, in the multiple case studies described above, Kazemi and
Stipek (2001) relate the norm that “errors provideopportunities to reconceptualize a problem,
explore contradictions in solutions, or pursue alternative strategies” (p. 64) to teachers’
encouragement of all students in the class (a) to compare mathematical strategies, (b) to justify
their reasons using mathematical concepts, and (c) to persist in exploring alternative strategies if
they find that a strategy is inadequate. These findings are useful in naming a sociomathematical
norm that supports classroom discussions of partial understandings and in linking this norm to
increased opportunities for student learning. However, the data for this study was drawn from
only one lesson, so the authors were not able to investigate how this norm was developed or
negotiated between members of the classroom over time. Kazemi and Stipek themselves point
out that future research should “investigate, with longitudinal data, how… norms are created and
sustained, and how they influence students' mathematical understanding” (p. 79). In this
dissertation I follow this recommendation by studying the development, reproduction, and
alteration of specific norms supporting productive discussion of students’ errors in a single case-
study classroom over the course of an entire school year.
Cheval (2010) takes a longitudinal view of the development of norms in response to this recommendation by Kazemi and Stipek, but she is unable to describe the development of specific norms for the discussion of students’ errors because the classroom in her case study fails to fully develop these norms. Cheval’s study was limited, however, by a data collection period of only three weeks at the beginning of the school year, so she had no evidence of whether or not these norms ever fully developed after her case study ended. Furthermore, the author was interested in several other types of norms in the classroom aside from those related to discussions of errors, so her analysis of this particular type of norm was necessarily less detailed than would be expected in a study focusing exclusively on students’ errors and partial understandings.

The current literature provides some glimpses of specific norms that support discussions of student errors, but none of these descriptions are in depth or detailed and none contain illustrations of how these norms develop over time in classrooms. I aim to add to the current literature by providing an in depth illustration of specific norms supporting discussions of students’ partial understandings and errors and also by taking a longitudinal perspective that allows for the examination of how these norms develop over time. I draw from multiple sources of data in order to produce this in-depth and longitudinal description. I identify patterns of interaction through videotapes of classroom lessons, and I interviewed both the teacher and a sample of “focal” students in the classroom in order to better understand their expectations for the handling of and discussion of students’ errors. These observations and interviews occurred throughout the school year in order to provide a longitudinal perspective on the development of and any shifts in these expectations.

I have thus far reviewed current theory and findings related to norms for classroom discussions of students’ partial understandings. I have highlighted the importance of understanding what such norms are and how they might develop in classrooms, and I have also pointed to the lack of current descriptions supporting such an understanding. In the next section, I first review the framework from which I investigated these norms. Next, I summarize the research goals for this dissertation, and I then provide the rationale for my choice of a case study to address these questions.

Research Framework and Purpose

Framework for this study. To support analysis of the establishment and development of norms in classroom discussions over time, I turn to Saxe’s (2012; Saxe et al., 2014) framework outlining the relationship between microgenetic analyses and sociogenetic analyses. Within Saxe’s framework, microgenetic analyses involve attention to instances of individuals “turning cultural forms… into mathematical means for accomplishing emerging goals in activities” (Saxe, 2004, p. 248). In this study, my analysis at the microgenetic level includes attention to how individuals use particular mathematical ideas, representations, and language in public displays to show their thinking about particular mathematical problems during whole class discussions. These individual sense-making activities are understood to contribute to development of taken-as-shared understandings over time through what Saxe terms as sociogenetic processes: sociogenetic analyses require attention to the way microgenetic constructions are distributed over individuals. That is, a sociogenetic analysis considers how individual displays are part of a wider distribution…, as well as how such distributions shift over time. Thus, a sociogenetic analysis illuminates the reproduction and alteration of a common ground, taking multiple individuals into account in a classroom community. (Saxe et al., 2014, p. 17)
In this study, my analysis at the microgenetic level serves to contribute to an understanding of patterns in these individual constructions in a sociogenetic analysis. In other words, patterns in how individuals act and speak when solving mathematical problems illuminate the taken-as-shared expectations (or what Saxe and colleagues refer to as “common ground”) for public displays of mathematical thinking in the classroom, that is, the sociomathematical norms of the class. Additionally, responses of the group to an individual’s microgenetic construction (e.g., accepting or rejecting the individual’s actions) also point to taken-as-shared expectations in the classroom. Students’ individual actions that diverge from typical patterns of interaction during discussions may thus also inform a sociogenetic analysis through examination of the general collective response to the student’s atypical action.

These individual teacher and student actions and statements are shown in Figure 1 (adapted from Saxe et al., 2014) as ovals, each representing the activity of one individual’s microgenetic constructions at a single time point. For example, imagine that at Time Point 1, the teacher and students A, B, and C – along with the other students in the classroom, who are not represented in the figure simply for space considerations – are all engaged in a classroom mathematical discussion. Perhaps Student A is explaining her solution to a mathematical task, and the teacher praises Student A’s use of a particular mathematical strategy. Student B then voices his agreement with Student A’s solution strategy up to a point, but disagrees with Student A’s final answer, citing a problem with one step in Student A’s solution. Student C says nothing, but raises her hand when the teacher asks if other students agree with Student B’s response to Student A’s solution. All of these individual actions and statements result from the students’ and teacher’s own sense-making activity at that particular time regarding the particular problem and Student A’s explanation of her solution to that problem. These individual microgenetic constructions contribute to the classroom norms, as shown in the figure by the arrows from the ovals depicting the individuals’ actions to the diamond representing the classroom norms. These individual actions, when publically displayed in whole class discussions, contribute to the collective experience of discussions and the expectations that become taken-as-shared through sociogenetic processes over time, shown by the dashed horizontal line indicating the passage of time from left to right in the figure.
The framework shown in Figure 1 is necessarily a simplification of complicated processes involving many more students in repeated interactions over a much longer time period than can be clearly depicted here. However, the framework is useful for depicting how microgenetic processes at the level of the individual (teacher or student) and sociogenetic processes at the collective level are theoretically related in classroom discussions over time. Furthermore, this framework implies some types of data and analysis that will be necessary to meet the goals of this study seeking to describe in a detailed way norms are established and continue to develop in a classroom over time. That is, this framework suggests the need to investigate patterns in individuals’ public displays of thinking and how these patterns shift over time.

**Research goals and questions.** As described above, this study extends prior research by illustrating in depth norms related to classroom discussions of students’ partial understandings and how these norms develop over time. Researchers have shown that classrooms with a high press for student learning can include norms supporting classroom discussions of student errors (Kazemi & Stipek, 2001), suggesting that such norms support students’ learning of mathematics. However, to my knowledge research has not yet provided a detailed, in depth illustration of specific norms that support generative discussion of students’ errors and the partial understandings underlying those errors. Such an in-depth illustration is likely to be useful to support teachers in flexibly adapting their teaching to meet the challenges of their students’ needs (Ball, 1993) while also meeting the recommendations of the CCSS (2010) for students to engage in mathematical discussion in which they explain, justify, and compare their ideas.
Additionally, this study builds on prior research by examining how norms related to discussions of students’ errors are reproduced and altered during the school year. Previous studies have demonstrated that it is difficult for teachers to support the establishment of such norms (Cheval, 2010). However, researchers have not yet illustrated in detail the successful negotiation and maintenance of such norms in a classroom nor explained the mechanisms through which such negotiation occurs.

This study aims to contribute to the literature in two ways: (a) conceptually, and (b) for the purposes of teacher professional development. As a conceptual contribution, I plan for this study to serve as an explanatory case study revealing the mechanisms of the process of establishing and continually renegotiating norms supporting the discussion of students’ partial understandings and errors. The mechanisms uncovered in this study may support or suggest the need to modify the framework above as an explanatory model for the processes through which such norms may be negotiated in classrooms. As a contribution to professional development literature, this study aims to describe in depth some specific norms that support productive discussion of students’ errors and partial understandings in a case-study classroom. This description is intended to allow for teachers to more completely understand examples of such norms so that teachers might be able to support similar norms in their own classrooms, as suggested by Kazemi and Stipek (2001). Furthermore, the description of how these norms are negotiated in the case-study classroom may suggest how teachers might attempt to go about supporting the establishment of these norms with their own students. The purposes of this study lead to two particular research questions:

1. What norms support the discussion of students’ mathematical errors?
2. Over the course of the school year, how are norms that support discussions of errors established and negotiated by the teacher and students?

Rationale for a case study design. An embedded single-case study was selected as the most appropriate method to address these research questions. Many studies of norms use a case study inquiry method (e.g., Bowers, Cobb, & McClain, 1999; Cheval, 2010; Cobb et al., 1992; Kazemi & Stipek, 2001). Yin (2014) defines a case study using a two-part definition:

A case study is an empirical inquiry that investigates a contemporary phenomenon (the “case”) in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident. (p. 16)

A case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence, with data needed to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis. (p. 17)

This definition is helpful for identifying the utility of a case study as well as some specific characteristics of good case studies.

Based on Yin’s definition, a case study was an appropriate way to address the research questions of this study because the phenomenon of study – namely, norms supporting discussion of students’ mathematical errors as evident through teacher and student actions during collective activity – is not possible to realistically separate from the context (i.e., the social interaction of the classroom) in which it occurs. Furthermore, in order to provide a detailed description of these norms and their negotiation in the classroom, it was necessary to closely observe teacher and student actions and dialogue and to investigate the meaning of these actions for individuals
and within the social space of the classroom. A case study was the most appropriate way to conduct such a thorough investigation with multiple sources of data.

Though case studies have been criticized as being ungeneralizable and therefore, not useful, Yin (2014) argues that “case studies, like experiments, are generalizable to theoretical propositions” (p. 21). The current case study is generalizable in the sense that it may support the theory described in the framework (see Figure 1) and provides some evidence for mechanisms of how this process might occur in a real-world classroom setting. This type of generalization is called *analytical generalization* and can be contrasted with *statistical generalization*, which is often used in quantitative studies (Yin, 2014) to generalize results from a representative sample to an entire population. This case study is intended to be explanatory – rather than solely descriptive – in that it aims to identify mechanisms through which norms are established in the case-study classroom. Through analytical generalization, these mechanisms may add to the framework described above to further refine an explanatory model for the negotiation and renegotiation of norms in classrooms. A case study is therefore the most appropriate way to address the specific goals of this investigation.

In the next chapter, Chapter 3, I describe the data sources, data collection methods, and general data analysis methods used in this case study. Further specific methods used to analyze particular types of data or for analysis related to each specific research question are described where appropriate in Chapters 4 through 6, in which I also present the results of these analyses. In Chapter 4, I describe general results related to the analysis of video recordings of classroom interactions, focusing on general characteristics of these interactions over time. In Chapter 5, I describe results related to the first research question of this study, the question of what norms related to the treatment of errors developed in the case study class. I describe results related to the second research question of how these norms developed over time in Chapter 6. Finally, in Chapter 7 I suggest some conclusions drawn from these results, indicating how the results of this study may contribute to the theoretical framework described in the current chapter.
Chapter 3: General Study Design and Methods

The research questions described in Chapter 3 necessitated certain features of the case study design. The case study needed to allow for analysis both of the classroom as a community and of individuals’ perspectives within the larger community, supporting multiple ways of uncovering what norms emerged in the community as they were interpreted by individuals and enacted in community practices. The case study design also needed to allow for investigation over time, which was important for investigation of the question of how these norms develop and are negotiated over the school year. In this chapter, I describe more fully the design of this case study, the particular class selected as the case, and the methods used to collect the various types of data for this study.

Case Study Design

An embedded single case study design was used for this study. A single case was appropriate for this investigation for several reasons, which I describe first, followed by a description of the particular classroom selected for this study. I then describe the embedded design of this case study and explain the rationale for this design.

Single case design. A single case design is appropriate when a unique or unusual case is under investigation. The particular classroom was identified as exceptional in that the teacher, Mr. Anderson (pseudonym), was previously observed to successfully facilitate meaningful discussion of students’ mathematical errors and partial understandings in whole class discussions. Based on observations of this teacher’s mathematics teaching in previous school years and on interviews with this teacher prior to this study, mathematical whole-class discussions are a part of his typical pedagogical practices. Because discussions of students’ errors and partial understandings are not typical in most U.S. classrooms (Bray, 2011; Santagata, 2005), this teacher’s classroom represents an unusual case worthy of study and is therefore appropriate for a single case study.

In any case study, it is important to define the boundaries of the case, that is, what (or who) will be included as a subject of study and what (or who) will not. For this study, the case was defined by the single 5th grade classroom in which Mr. Anderson was the primary teacher. Moreover, the students considered to be part of the classroom were those students who were typically present for mathematics instruction within this classroom. In other words, any students who were in the classroom for instruction in other subjects but who went elsewhere (e.g., a resource room) for all of mathematics instruction would not have been included. Furthermore, students who were present in the classroom for a minority of the school year (e.g., students who transferred in or out of the class) were not included except as their actions while present during math lessons related to general patterns in classroom practices. These restrictions were developed before data collection began and were intended to exclude students who were unlikely to participate in the typical practices of the class during mathematics instruction and/or who were unlikely to take up the expectations for mathematical discourse that became shared by members of the class over time.

For the current case study, one student was excluded from the analysis because this student transferred out of the class after only a few weeks of school. No students were excluded
because they went to another classroom for mathematics instruction; all students in the class were typically present for at least some part of mathematics instruction each day.

The case was further be bounded by time limitations; the case includes a single school-year. This time limitation is practical because students typically are rearranged into different classroom groupings with a different teacher each school year. However, U.S. elementary students typically have a single teacher for the entire school year (approximately August through June).

Embedded design. Though the case for the case study was the classroom community as a whole, thorough examination of classroom norms necessitated understanding the particular attitudes of individual members of the classroom community (Cobb & Yackel, 1996). Specifically, attention to individuals within the class was required to understand how the teacher and students made meaning of events in the classroom (e.g., someone making an error) and how the teacher and students came to understand (or not) the norms negotiated in the classroom. Therefore, I included several individuals as embedded units of analysis (Yin, 2014, p. 50) within the larger case of the classroom. I collected data about these individuals’ actions, thoughts, and feelings as related to mathematical errors in the classroom. The teacher and five focal students were treated as embedded units within the larger case. This design is illustrated in Figure 2, which is drawn from Yin’s illustration of embedded case study designs.

Figure 2. Embedded design for this single case study.

Selection of focal students. To select focal students for this analysis, I relied on initial survey data from the beginning-of-year survey administration (see below for further description of this survey) and the student roster for the class. I selected six focal students based on their having a range of responses to the survey items (e.g., very different expectations for responses to mathematical errors). I also attempted to choose equal numbers of apparently female names and apparently male names when selecting focal students. However, because I did not yet know all students at the point when I needed to select focal students, I inadvertently selected four students who presented as boys and two students who presented as girls. I also attempted to select students from a variety of apparent racial/ethnic backgrounds. The final group of six selected focal students included three students who appeared to be of Caucasian or white background, one student who appeared to be of Latino descent, one student who appeared to be of Pacific Islander descent, and one student who appeared to be of African-American or Caribbean Islander descent.
This group of students, at least in their appearances, was representative of the racial/ethnic diversity of the school at large, which is described below. In these ways, the focal students were selected to represent as diverse perspectives as possible.

In summary, the overall embedded single case study design was intended to allow for in-depth investigation of norms that were negotiated between members of this single fifth grade classroom over the course of the year. The teacher and focal students were treated as embedded units within the overall case in order to learn more about how these members of the class were taking up and making sense of the norms that developed in the class over time. The focal students were selected to be as representative as possible of the diverse perspectives of students in the classroom, and Mr. Anderson, the teacher, was selected as an embedded unit because of his unique social position as the teacher in the classroom.

Description of Case Study Classroom

In this section, I further describe the context of the case study by providing available information about the school in which the case-study class was situated and by describing my observations of the demographic make-up of the case-study class as compared to the school as a whole. I also describe the classroom teacher’s experience in education and research to provide some context that may support understanding his perspectives and practices. Finally, I describe the physical layout of the case study classroom, which of course resulted in both affordances and limitations for the purposes of data collection as well as for classroom practices described in the results of this study. Taken together, this information is intended to provide a sense of the physical and social space in which the case study class existed because, of course, these environmental factors shaped the way data was collected as well as the practices and norms that actually developed in the case study class.

School demographics. The case-study class was located in the larger community of an elementary school (Kindergarten through fifth grade) located in an urban area on the West Coast of the United States of America. The school district’s website reports demographic data for this school from the 2014 to 2015 school year, when data for this study was collected (Berkeley Unified School District, 2015). The report indicates that during the 2014 to 2015 school year the school had 392 students enrolled with the following racial/ethnic distribution: 12.5% African-American/Black, 8.2% Asian, 18.1% Latina/Latino, 42.9% White/Caucasian, and 17.1% identifying as belonging to two or more racial/ethnic groups. Of the students in the school, 32.7% were identified as socioeconomically disadvantaged, 8.4% were identified as English Language Learners, and 4.8% were identified as students with disabilities.

State test results for this school from the case-study data collection period are also available. Results of the California Assessment of Student Performance and Progress (CASPP) state testing administered in the spring of 2015 for this school are described in Table 1 below.
Case Study School State Testing Results from Spring 2015 for Grades 3 to 5

<table>
<thead>
<tr>
<th></th>
<th>English Language Arts</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Students “Not Meeting Grade Level Academic Standards”</td>
<td>18%</td>
<td>12%</td>
</tr>
<tr>
<td>Percent Students “Nearly Meeting Grade Level Academic Standards”</td>
<td>12%</td>
<td>19%</td>
</tr>
<tr>
<td>Percent Students “Meeting Grade Level Academic Standards”</td>
<td>34%</td>
<td>25%</td>
</tr>
<tr>
<td>Percent Students “Exceeding Grade Level Academic Standards”</td>
<td>36%</td>
<td>43%</td>
</tr>
</tbody>
</table>

**Case study class demographics.** Twenty-five students were enrolled in the case study class at the beginning of the school year. Specific racial and gender statistics are not available for the classroom. However, during my observations I estimated that about half of the students in the class appeared to be white/Caucasian, and the remaining half of students appeared to belong in fairly equal numbers to the following racial/ethnic groups: African-American or black, Latina/Latino, and Asian/Pacific Islander. The class appeared to include roughly half of students who presented as male and half who presented as female.

After a few weeks of school, one student was transferred out of the class and two students were transferred into the class. In the spring, one student – who was also one of the six focal students for this study – moved out of the area. Therefore, during the first data collection period the class had 25 students, one of whom is excluded from survey data analysis because this student left the class after only a few weeks. During the second data collection period, the class had 26 students, and during the last data collection period the class had 25 students.

**Case study classroom teacher.** The teacher of this class had worked with me before on previous research projects, so I had the opportunity to observe his class and to speak with him about his teaching before the start of the current study. This teacher has extensive experience as an elementary teacher in the same district and at the same school. He had been teaching for over 20 years when the case study began, and he had received a Master’s degree in education at a university program focused on child development.

Prior to this study, the teacher, Mark Anderson, had expressed an interest in supporting students in learning from their errors in mathematics. I had in prior school years observed his class to include whole-class discussions of mathematics topics, including in-depth discussions of students’ mathematical errors. Moreover, students in Mr. Anderson’s prior-year classes seemed comfortable discussing their mathematical errors in front of the entire class. Mr. Anderson’s class therefore seemed a suitable selection for the current case study because it seemed likely that it would develop norms that supported this kind of productive and open discussion of students’ errors in mathematics.

**Physical space of the case study classroom.** Mr. Anderson’s classroom was a large space easily accommodating individual desks for each student (see Figure 3). The desks were arranged in groups of four or five, with pairs of desks next to each other and facing another pair of desks. In this arrangement, four to five students were in close proximity to and facing each
other when seated at their desks. The groups of student desks were arranged on one side of the classroom around an overhead projector that projected on a screen above the whiteboard in the “front” of the room. During class, students sat at their desks or stood around the overhead projector to see images projected on the screen.

On the other side of the classroom from the students’ desks, a large area rug covered the floor and created a space where students often sat in a large group “on the rug” for various class activities, including class meetings and some academic lessons. The whiteboard extended across the entire “front” wall of the classroom so that it was accessible in this area by the rug as well. In the back of the classroom, two large tables held papers and various supplies that the teacher and students used for lessons. This table was also sometimes used for group work by the students. Near this table against the wall was a counter with a sink and water fountain. Also on in back of the classroom was a table with several computers on it. These computers were sometimes used by students for assignments, though rarely did this occur during mathematics lessons. The teacher had a desk located on the side of the room, but he rarely sat at it during class time.

Figure 3. Approximate layout of the case study classroom.

The walls of the classroom were fairly bare at the beginning of the school year, but over the course of the year many posters, art projects, and photographs were posted on the walls. Some of these were pre-made posters, such as the teacher’s posters promoting certain actions or values that he hoped his students would take up (See Figure 4). Other posters were artifacts created during lessons (e.g., a chart showing different measurement conversions, a poster of homonyms that students are not to misspell in writing assignments) or by students (i.e., projects of various kinds). One area of a wall was filled by the end of the year with photographs and short biographies of social-justice leaders in the local community and from the world at large.
Data collection for this study took place primarily within the space of this classroom, though student interviews were conducted in another area of the school for privacy reasons. Data sources and data collection procedures are discussed further in the section below.

**Data Sources and Data Collection Procedures**

In this section, I describe general rationale and procedures for data collection, including the timing of data collection, the types of data collected, and the procedure for obtaining parental permission and student assent. I then describe instruments and data collection procedures for each type of data collected. These data types are (a) student surveys, (b) classroom lesson video recordings, (c) focal student interviews, and (d) teacher interviews.

**Timing of data collection.** Data for this case study was collected in three “periods” lasting two weeks each: the beginning of the school year (September 2014), the middle of the school year (January 2015), and toward the end of the year (April 2015). The school year for the case study school began at the very end of August 2014 and lasted through the beginning of June 2015. See Figure 5 for timing of data collection in relation to the school year as a whole.
Data was collected at these three time points so that changes in classroom practices and expectations over the course of the school year could be observed. Of course, it is possible that some changes were not observed because they occurred during times when data was not being collected. However, it was decided that the beginning-, middle-, and end-of-year design was likely to provide enough information about common and persistent patterns of interaction and expectations in the classroom to allow for analysis of major norms in the class.

Types of data collected. During each data collection period, several types of data were collected: student surveys, video-recordings of mathematics lessons, focal student interviews, and teacher interviews. These types of data were selected in order to provide information at the level of the case, meaning the entire class, and at the level of the embedded units within the case, meaning the teacher and focal students as individuals. Table 2 shows various data sources that were used for information about the class, the teacher, and the focal students. Notably, some sources of data provide information about more than one unit within the embedded case study. For example, videotapes of mathematics lessons provide information about practices and expectations within the class as a whole (the case) as well as about the actions of individuals within the class, such as the teacher and focal students (embedded units). These sources of data were used in a triangulating fashion in analysis to arrive at results and conclusions from this study.

Table 2

Units of Analysis and Data Sources

<table>
<thead>
<tr>
<th>Level (Unit) of Analysis</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class (case)</td>
<td>videotapes of mathematics lessons</td>
</tr>
<tr>
<td></td>
<td>field notes of mathematics lessons</td>
</tr>
<tr>
<td></td>
<td>student surveys (collectively)</td>
</tr>
<tr>
<td>Teacher (embedded unit)</td>
<td>teacher interviews</td>
</tr>
<tr>
<td></td>
<td>videotapes of mathematics lessons</td>
</tr>
<tr>
<td>Focal students (embedded units)</td>
<td>focal student interviews</td>
</tr>
<tr>
<td></td>
<td>focal student surveys</td>
</tr>
<tr>
<td></td>
<td>videotapes of mathematics lessons</td>
</tr>
</tbody>
</table>
As a whole, the data sources afford an analysis of the reproduction and alteration of classroom norms over time. As described in Chapter 2, norms are defined as taken-as-shared expectations held within a group, which in this case is the case-study class. This theoretical definition indicates that norms might be revealed in patterns of interaction within the class, particularly when an expectation is violated during an interaction and a correction is made in some way. Videos and field notes of classroom mathematics lessons were collected to provide information about classroom interactions. Norms may also be revealed through individuals’ statements about their own expectations for interaction in the classroom. Though norms are a property of the collective, it is likely that individuals form beliefs expectations for behavior in the class aligned with the classroom norms that are part of the group. Therefore, student paper-and-pencil surveys and teacher and focal student interviews were collected to provide information about individuals’ expectations for classroom behavior related to mathematical errors. Together, these various data sources allowed for identification of norms in this study based on an operational definition that is intended to honor the collective nature of norms as well as individuals’ perspectives and interpretations. In this study, I identified norms as expectations that were endorsed by multiple students and the teacher in interviews and surveys and that appear to be consistent with patterns of behavior observed in classroom lessons. In Chapter 5, I further describe specific procedures for how I used these various data sources to identify norms in the case study classroom.

**Parental permission and student assent.** Before collection of any of these types of data began, I visited Mr. Anderson’s class at the very beginning of the school year during the school day to describe my purposes for visiting their class. Using a pre-prepared study description, I told them generally that I was interested in learning about how their class learned mathematics. I did not describe specific research questions, but I did describe the kinds of data I would collect and the kinds of questions I would ask in the survey and interviews. Students had the opportunity to ask me questions at that time.

I sent home parental permission forms and student assent forms with each student in the class, and within a few days all students had returned both forms. All parents and guardians granted permission for students to participate in the study and all students assented to participate in the study. Some parents asked that their students not be interviewed individually as focal students, and these students were therefore excluded from the pool of students from whom I selected focal students. Additionally, the teacher signed a consent form for participation in this study before the first data collection period began.

When two students joined the class after the first data-collection period, they were also provided with a description of the study and with parental permission forms and student assent forms, which they returned signed. All consent, permission, and assent forms are kept in a locked file cabinet separate from other data for this study for purposes of confidentiality.

**Student paper and pencil surveys.** During each data collection period, I administered a paper survey to all students in the class. The survey had been developed prior to data collection. It was piloted with students of similar age (4th through 6th grade) during the summer of 2014, and minor adjustments were made in the wording to support students’ understanding of the intent of the questions.

**Survey instrument.** The survey included 6 items (see Appendix A). The first five items were “scenario” questions; a classroom scenario was described and students were asked how they would respond or how they would expect others in the classroom to respond in the given situation. Students were to respond to the first four scenario items by selecting one of four
provided responses that described possible reactions to the scenario or by writing in their own response in space provided for that purpose.

The fifth item required students to respond by choosing the degree of likelihood that they would admit to getting a wrong answer in the provided scenario. The response options for this item were in the form of a four-point Likert scale: I definitely won’t, I probably won’t, I probably will, and I definitely will. During piloting of this survey item, students had no apparent difficulty understanding this item, but during the first data collection period of this case study students described having difficulty understanding the scenario and the meaning of this question. Therefore, during the second and third data collection periods, I paraphrased this item after reading it aloud, prompting students by saying “How likely is it that you would raise your hand to admit that you got the wrong answer? You can choose, I definitely won’t raise my hand to admit I got it wrong, I probably won’t raise my hand to admit I got it wrong, I probably will raise my hand to admit I got it wrong, or I definitely will raise my hand to admit I got it wrong.”

The final survey item consisted of a list of 12 activities that could occur during a mathematics class. Students were instructed to put a check mark in the box next to each of these activities that they expected to do in their mathematics class. Responses to this item varied greatly, and in many cases the responses did not make a lot of sense. For example, some students did not check the box next to “solve math problems,” indicating that they did not expect to solve math problems in their math classes. In final analyses, the responses to this question did not appear to provide any useful data about expectations in this class, and it appeared as though many students did not fully understand the purpose of this item.

The same version of the survey including these six items was administered at all time points. No changes were made except slight revisions to correct a formatting error that students pointed out during survey administration during the first data collection period.

Survey administration procedures. During the first data collection period in September, I administered this survey on the first day of data collection so that I would be able to use the results of this survey to select focal students for interviews. Students took the survey in their classroom during class time. I read the survey directions and questions aloud to the entire class, and students responded by writing on individual survey forms. At each time period, the survey administration took about 15 minutes. Students appeared attentive and to put forth reasonable effort on the survey. For example, several students asked clarifying questions and some students chose to write in answers rather than circle provided answer options. Therefore, it seems that the survey is likely to generally provide a reasonable representation of what students wanted to communicate to me about their expectations in their mathematics class.

I collected all surveys from students immediately after administration. The same day, I rendered anonymous all surveys by covering each student’s name on the survey paper with a label sticker on which I wrote an identification number that I had randomly assigned to each student. The “key” listing each student’s name and corresponding identification number for this study was kept in a locked filing cabinet in a locked lab room apart from other data for this study.

Survey data organization procedures. After survey data was collected at each data collection period, I recorded student responses to the survey in an electronic database file. In a separate spreadsheet for each data collection period, I recorded each student’s survey responses. Students were identified in this database file by their study identification number only. Responses for each item were recorded as either the letter of the response selected for multiple-choice responses or as a verbatim copy of what students had written in as their responses.
Responses were transformed into numerical data for the Likert-type item responses and for binary (yes or no) responses to each classroom activity listed for the sixth survey item. Responses for the Likert-type item were recorded as numbers 1 (*I definitely won’t*) to 4 (*I definitely will*) and responses to each activity for the sixth item were recorded as 1 (checked) or 0 (not checked). A key was created to map these numbers recorded in the database to the responses to which they correspond for these survey items. This organization of data supported quick and flexible analysis of survey results at all data collection periods, including during the first data collection period when these results were used to select focal students with varied responses to the survey questions, which were used as indicators of different initial expectations for classroom responses to students’ mathematical errors. The survey data were also intended to be part of a later analysis in which multiple data sources were triangulated to draw conclusions about norms that developed in this class. The survey data was unique among data sources in that it provided some information about the individual perspectives of all students in the case study class.

**Classroom mathematics lesson videos and field notes.** Recordings of each mathematics lesson were intended to provide evidence of the types of interactions that were actually occurring during classroom lessons. During each two-week data collection period, each mathematics lesson was video-recorded, meaning that roughly one to 1.5 hours of classroom video was recorded roughly each day of the data collection period. Some days, Mr. Anderson did not teach a mathematics lesson for various reasons, and no video recordings of lessons were made on those days. In all, 26 mathematics lessons were video recorded as part of this study. Ten of these lessons occurred during the first data collection period in September of 2014, eight occurred during the second data collection period in January of 2015, and eight occurred during the third data collection period in April of 2015.

**Lesson video recording procedures.** I collected roughly half of video recordings and the remaining video recordings were made by other graduate students who were trained in classroom video recording procedures. A high-quality video camera and tripod were used, along with a “shotgun” microphone to best capture voices of various members of the classroom community. The tripod was set up each day in a pre-specified location in the back and on the side of the classroom near the classroom sink (see Figure 3). Before data collection began, Mr. Anderson and I agreed on this location both as being out of the way of his and students’ general movement in the classroom and also as convenient for capturing classroom activities in most parts of the room (e.g., on the rug, at student desks, and around the overhead projector).

During lesson video recording, the camera was turned on and recording continuously. The lens was positioned to capture the portion of the classroom where most students and the teacher were located. If capturing all students on camera was not possible, the area of the classroom where most students were located was captured. If no clear location included most students, the teacher’s movements were followed with the camera. On days when I was recording lessons myself, I also took field notes that I used to support later coding of the lesson videos.

**Video uploading and storage procedures.** Periodically during and after data collection periods, I removed the memory “chip” from the camera and uploaded videos to an encrypted portable hard drive located in a locked lab room. I used iMovie to import videos and then exported them to the hard drive as Quicktime movie files. All videos were stored in encrypted form on two hard drives. Extensive analysis of classroom videos occurred after the conclusion of data collection and is described in more detail in Chapters 4, 5, and 6.
**Student interviews.** Interviews with focal students also occurred during each data collection period. Once focal students were selected based on survey data at the beginning of the first data collection period in September of 2014, these same students were interviewed individually at each time point. These focal student interviews were intended to provide insight into the students’ understandings of survey items and their individual perspectives on classroom routines and expectations over time. The final group of focal students includes five students (20% of the class of 25 students) because of the six students originally selected, only five remained enrolled in the class by the final data collection period in April of 2015. One of the original six focal students moved out of the area in between the second and third data collection periods of this study.

**Structure and development of the interview protocol.** Interviews were semi-structured and followed a protocol of seven questions (see Appendix B), some of which include more than one part. The same questions were asked at each time period of all focal students. The protocol was developed with research questions in mind and based on my experience with interviewing similarly-aged students both for past research projects and through practice in school as a teacher and as a psychologist intern.

The first question of the interview asked students to reflect on five different activities that they might do in their mathematics classes: (a) check answers to math problems, (b) ask the teacher for help, (c) ask other students for help, (d) watch the teacher help or coach other students in math, and (e) coach other students in math. These activities were selected because the teacher had identified them as activities that he found important for students’ learning from their errors in mathematics and learning mathematics generally. For each of these activities, I asked each focal student to rate how often they did the activity in their math class using a rating scale, which I provided written on a sheet of paper, with the following Likert-type options: never, about once a month, about once a week, about once a day, and more than once a day. For each activity, I also asked each student to describe how useful they found that activity for their own mathematics learning using another provided rating scale with the following Likert-type options: not at all, not much, a little, and a lot. I then asked students to tell me about their answers, meaning to describe why they thought each activity was useful or not and why they did each activity frequently or not. These questions were intended to provide information about students’ perspectives on how often they engaged in these activities identified by the teacher as important and to get a sense of what expectations students had for these activities.

The next three interview questions prompted students to explain their answers to three items from the paper-and-pencil survey, which all students had taken prior to the interviews at each data collection time period. All three of the survey questions that I queried were scenario questions. I provided each student with his or her actual survey during the interview, so students were not expected to remember how they responded to the survey and they were able to reread the questions and answer options.

The next three interview questions asked students general questions about how they thought about making errors in mathematics (*What does it mean when someone gets the answer to a math problem wrong?*) and about a collective practice called coaching that became common in this class. Before the final interview question, I showed students a very short video clip of an interaction that occurred in their classroom at the beginning of the first data collection period. During this interaction, the teacher and students had been doing a whole-class fluency exercise in which students practiced their times tables aloud using hand movements. One student said an incorrect response, and the teacher praised her effort, saying, “Make all the mistakes you want.”
I asked focal students to reflect on this instance and to rate how serious they thought the teacher was in this statement using the following Likert-type response scale: *not at all serious – he’s joking*, *not very serious, a little serious*, and *very serious – he means what he’s saying.* I showed students the same video clip at all three time periods.

**Interview procedures.** Interviews were conducted during the school day at times pre-arranged with the classroom teacher to cause as little disruption as possible in students’ learning. Interviews were conducted in a small, quiet room in the school. All interviews were video recorded, and the video camera was positioned so as to capture the student’s hands and the printed rating scales they used to answer some interview questions. Students’ faces were not captured on video. Because students’ faces were not captured, I held up a piece of paper with the students’ study identification number and the time period (*beginning of year, middle of year, or end of year*) at the beginning of each video recording so that I could easily identify the student being interviewed and the data collection period.

Students were interviewed individually at each data collection period. I conducted all interviews myself. I attempted to interview as many students as possible on the same day, but it was never possible to interview all focal students in one day. Therefore, interviews took place on more than one day at each data collection time period.

At each time period, I asked each student to assent again to being interviewed and reminded them that it was optional and not part of their classroom academic requirements. I also explained a bit about the purpose of my study. During the interviews, I asked clarifying questions as needed or rephrased questions if students asked for me to do so. I also gave students the opportunity to ask me questions about my study, the interviews, or myself generally. Each interview lasted approximately 15 minutes, and students returned to class after being interviewed.

All videos of interviews were transferred from the video camera memory chip to an external hard drive using the same procedure as used for classroom video recordings. All interviews were also stored on two external hard drives in encrypted format. I roughly transcribed all interviews into word processing documents for analysis. These transcripts are “rough” because although they generally include students’ statements verbatim, some words (e.g., “uh,” “um,” “ya know”) were often left out. I included timestamps periodically within the transcript documents so that I could refer back to the interview videos to make exact transcriptions of students’ interview responses for the purposes of data reporting in research or professional reports. These rough transcripts were used primarily in the analysis of focal student interview data, and the original videos were used to confirm students’ exact statements for reporting. Analysis of focal student interviews and results of this analysis are described further in Chapters 5 and 6.

**Teacher interviews.** Interviews with the classroom teacher, Mr. Anderson, also occurred during each data collection period. These interviews were intended to provide information about the types of practices Mr. Anderson intended students to engage in during mathematics classes, specifically practices related to errors or incorrect or incomplete understandings of mathematics. Additionally, these interviews were intended to provide information about Mr. Anderson’s own expectations for behavior in the class, that is, his understanding of the norms that developed over the course of the year and whether or not these norms were taken up in the ways and to the degree that he wanted students to take them up.

**Structure and development of the interview protocols.** Unlike the focal student interviews, for which the questions were identical at all three data collection time periods, the
teacher interview protocol varied somewhat at each time period (see Appendix C). For the first interview, which occurred at the beginning of the school year, I asked Mr. Anderson questions about his plans for the kinds of mathematical practices he wanted students to engage in that year. I asked both about general practices in mathematics and also specifically about practices related to mathematical errors and partial understandings. Additionally, I asked Mr. Anderson which of these practices he expected to be harder or easier for students and why. I intended this question to get information on Mr. Anderson’s expectations for how students might engage in or take up these practices and to what degree. The last two questions of the beginning-of-year interview protocol focused on how Mr. Anderson thought his students currently interpreted mathematical errors and also how he himself interpreted mathematical errors.

The middle-of-year teacher interview protocol and end-of-year teacher interview protocol were similar in that the first few questions focused on getting updates on how Mr. Anderson thought his students were doing with regards to the mathematical practices he identified as important at the beginning of the year. I asked questions about how often his students were engaging in these practices and how satisfied Mr. Anderson was with the way students were performing these practices at this point in the year. These questions were intended to support understanding of Mr. Anderson’s perception of how well these practices were being taken up by students and what kinds of norms these practices were creating or sustaining in the classroom. I also asked Mr. Anderson how he thought students were interpreting mathematical errors at this point in the school year. This question was intended to get a sense of how Mr. Anderson understood his students’ ideas about what errors imply about the person who makes them and how this understanding may have shifted over time. Finally, I asked Mr. Anderson to view the same short video clip of an interaction in his classroom that I showed during focal student interviews. I asked him to describe what was going on in that interaction.

The end-of-year interview expanded on these questions from the middle-of-year interview through questions about the values and habits of mind Mr. Anderson specifically teaches in his class. I had noticed throughout my time in the classroom that there were many practices that took place outside of the mathematics instruction time that seemed to support norms that affected mathematics lessons and practices as well. For example, students had referenced the “quote of the week” in focal student interviews to describe their practice of coaching in mathematics. I asked Mr. Anderson to describe these practices for teaching habits of mind and values and to list some of these important values. These questions were intended to get information about the context of the class and classroom expectations that apply both during mathematics lessons and also during other times in the class.

**Interview procedures.** Interviews were conducted after school hours in the case-study classroom at times prearranged with the teacher. All interviews were video recorded, and the video camera was positioned so that the video captured the teacher’s hands and the printed rating scales he used to answer some interview questions. The teacher’s face was not captured on video. I conducted all interviews myself, and during the interviews, I asked clarifying questions as needed or rephrased questions if Mr. Anderson asked me to do so. Each interview lasted between thirty minutes and one hour.

As with the student interviews, all videos of teacher interviews were transferred from the video camera memory chip to an external hard drive using the same procedure as used for classroom video recordings. All interviews were also stored on two external hard drives in encrypted format. I roughly transcribed all teacher interviews into word processing documents using the same procedure that I used for student interview transcription. These rough transcripts
were used primarily in the analysis of teacher interview data, and the original videos were used to confirm Mr. Anderson’s exact statements for reporting. Analysis of and results from teacher interviews are described further in Chapters 5 and 6.

The procedures described here were used to collect the four types of data for this embedded-design case study: (a) student surveys, (b) video-recordings of mathematics lessons, (c) interviews with focal students, and (d) interviews with the classroom teacher. This chapter has detailed these data collection procedures as well as some of the first steps of data analysis. Next, I present three results chapters. In the first results chapter, I describe in greater detail the methods used to analyze video recordings of classroom lessons, and I present some descriptive results of this analysis. In the following chapter, I describe results related to the research question of what norms related to mathematical errors developed in the case study classroom over the course of the year, and I describe specific methods used to analyze data for the purpose of identifying these norms. In the final results chapter, I describe results related to the research question of how these norms emerged and were reproduced and altered over the course of the school year, and I describe further methods used to analyze the data for this purpose.
Chapter 4: Analyses of frequency of errors and teacher and student talk related to norms in mathematics lessons

In this chapter, I describe initial analyses of the video recordings of classroom lessons, which were one type of data that was collected for this study, in order to provide context for more specific results described in subsequent chapters. Chapters 5 and 6 present results addressing the research questions of what norms (i.e., shared behavioral expectations) related to mathematical errors emerged and how those norms emerged over time in the case study classroom. I devote the current entire chapter to description of the analysis of video recordings of lessons because the results of this analysis indicate relevant features of discussions in the case-study class. In particular, these results provide evidence of whether or not mathematical errors occurred in a public way in the classroom and whether or not public references to norms were observable in the classroom. The results presented in this chapter also indicate how errors and references to norms came up during lessons.

I first describe procedures used for analysis of the video recordings of lessons, including (a) data reduction, (b) identification of relevant instances in the video, and (c) subcoding of “errors” and “norms” instances. I then describe findings indicating some characteristics of classroom interactions identified as related to errors and norms.

Methods: Video Coding Procedures

Analysis of the video recordings of classroom lessons followed several steps. First, it was necessary to identify relevant sections of the video recordings. This was accomplished first by limiting the video to be analyzed to whole-class mathematics discussions. Then, specific interactions that were determined to be related to either errors or norms were identified and coded. Finally, specific characteristics of identified interactions related to errors and norms were described in “subcodes.” For this analysis, I recruited two undergraduate research assistants, and we worked as a research team to create coding procedures and to implement these procedures. StudioCode software was used in all stages of video coding and for basic quantification of the results. In this section, I describe the stages of this coding process and the tools used in this process.

Data reduction. In total, 26 mathematics lessons were videotaped for this study, resulting in over 35 hours of video recording of classroom lessons. See Table 3 for the distribution of this data over the three time periods. This amount of video was not possible to analyze in detail, so it was necessary to select the most relevant sections of video for in-depth analysis through a process of data reduction.

Because the subject of this study is classroom norms, it seemed reasonable to focus analysis on interactions that involved most or all of the members of the classroom. Furthermore, because of the limitations of using only one video camera, interactions between pairs or small groups of students were difficult or impossible to capture simultaneously during lessons. Therefore, I selected for analysis only sections of video that showed whole class discussions, meaning interactions between most or all of the class in which the teacher and/or one or more students addresses the larger group.
Table 3

<table>
<thead>
<tr>
<th>Amount of Video Data Collected at Each Time Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Lessons Videotaped</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Beginning of Year (September)</td>
</tr>
<tr>
<td>Middle of Year (January)</td>
</tr>
<tr>
<td>End of Year (April)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

Data reduction was accomplished by using StudioCode software to mark all sections of video that apparently included whole-class discussion for all 26 lessons that were video recorded. To identify these sections of video, all video was visually scanned (i.e., watched in “fast forward”), and sections of video in which students were apparently working individually or in small groups were omitted. Only the remaining whole-class discussions were considered for further analysis. After this procedure, a total of 1,192 minutes – or 19 hours and 52 minutes – of video of whole-class discussion during recorded lessons remained to be analyzed.

Because the whole-class lesson video was still too much video data to feasibly analyze in detail, it was decided that three lesson videos would be selected as a sample for analysis from each time period. To identify this sample, the three lesson videos with the most number of minutes of whole-class lesson time were selected from each data collection period. In Table 4, the rows indicate these lessons, which are noted by data collection period (Time 1, Time 2, or Time 3) and number of the day within that data collection period (e.g., Day 1, Day 2, etc.). The following lesson videos were selected as part of this sample: Time 1 Day 2, Time 1 Day 3, Time 1 Day 5, Time 2 Day 2, Time 2 Day 3, Time 2 Day 6, Time 3 Day 4, Time 3 Day 6, and Time 3 Day 8. This sample was intended to provide a reasonable estimate of the number of instances in which mathematical errors were made public or behavioral expectations were made publicly salient during a typical mathematics lessons at each time period during the year in this classroom.

Furthermore, because the research questions specifically relate to norms for the treatment of mathematical errors, segments of video in which the class was discussing another academic subject were eliminated from the analysis. During videotaping, occasionally discussions of English Language Arts or science topics were captured. These segments of video were not directly related to the treatment of mathematical errors, so they were not included for further analysis.

This process of selecting a sample of videos and limiting video to be analyzed to mathematics whole-class discussions only resulted in a sample of nine videos ranging from 36 minutes to 71 minutes in length (mean = 52.3 minutes; sd=12.2 minutes). In total, 471 minutes, or 7 hours and 51 minutes, of video recordings of class mathematics discussions was analyzed.
### Table 4

Sample of Mathematics Lesson Videos Selected for Analysis Length in Minutes

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beginning of Year (September)</strong></td>
<td>179</td>
</tr>
<tr>
<td>Time 1 Day 2</td>
<td>71</td>
</tr>
<tr>
<td>Time 1 Day 3</td>
<td>65</td>
</tr>
<tr>
<td>Time 1 Day 5</td>
<td>43</td>
</tr>
<tr>
<td><strong>Middle of Year (January)</strong></td>
<td>137</td>
</tr>
<tr>
<td>Time 2 Day 2</td>
<td>62</td>
</tr>
<tr>
<td>Time 2 Day 3</td>
<td>39</td>
</tr>
<tr>
<td>Time 2 Day 6</td>
<td>36</td>
</tr>
<tr>
<td><strong>End of Year (April)</strong></td>
<td>155</td>
</tr>
<tr>
<td>Time 3 Day 4</td>
<td>46</td>
</tr>
<tr>
<td>Time 3 Day 6</td>
<td>53</td>
</tr>
<tr>
<td>Time 3 Day 8</td>
<td>56</td>
</tr>
</tbody>
</table>

**Identification of relevant instances in the video.** Once the sample of whole-class mathematics discussions was identified, it was necessary to identify instances that were relevant to the research questions. All coding was conducted using StudioCode software. See Appendix D for the complete codebook used for identifying these instances and coding video data.

First-cycle coding was conducted using descriptive codes to identify classroom interactions involving errors or reference to norms (Miles & Huberman, 2014). Coders watched video of whole-class discussions from entire lessons and identified relevant instances. An instance was defined as a segment of the video long enough so that it was clear what was going on from just that segment (i.e., including enough context so that a person only seeing that video clip could make sense of the important parts of the clip). The *norms* code was applied to instances in which the class explicitly discussed behavioral or other expectations or in which a person’s behavior in the class was corrected or praised. The *errors* code was applied to instances that showed evidence of public discussion of canonically incorrect assertions (errors), making errors, or having *partial understandings*, meaning incomplete or at least partially incorrect ways of thinking about or solving a problem or mathematical situation. These codes used to identify relevant instances are described further in Table 5 below.

The research team decided to identify instances as related to *either* errors or public references to norms – rather than to instances related to *both* errors and norms – because identification of instances related to either norms or errors seemed feasible during initial coding. It was determined that identification of instances related specifically to norms about the treatment of errors was problematic for first-cycle coding because coders would be forced to make assumptions about which norms were related to errors. Therefore, instances related to any public reference to classroom norms were identified initially; in later analyses, subcodes were used to identify which of these instances were specifically related to norms about the treatment of errors (see Chapter 5).
Table 5

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Errors</td>
<td>Any instance in which a student or the teacher makes an error in mathematics. This error will be in their academic work, not in their classroom behavior. Or, any instance in which errors are explicitly discussed. Also, include any instance in which students or the teacher bring up partial understandings, meaning their incomplete (and at least partially incorrect) ways of thinking about a problem or mathematical situation or at least partially incorrect way(s) of solving a math problem (in this case, a wrong answer may not necessarily be identified, but a student or the teacher may indicate that they do not know how to solve the problem, for example, or may indicate that their process of solving the problem was confused or incorrect).</td>
</tr>
<tr>
<td>Norms</td>
<td>Any instance in which the class explicitly talks about the kind of behavior that is expected in the classroom. Or, any instance in which someone corrects another person’s behavior in a way that reveals something about the general behavioral expectations in the classroom. Additionally, because we are specifically interested in behavior related to “coaching”, code any instance in which the teacher or students give instructions related to coaching as “norms,” even if the instruction is somewhat specific to the situation.</td>
</tr>
</tbody>
</table>

The definitions of these categories were refined through cycles of coding and discussion. While coding, each rater watched each video at least twice. However, despite this codebook refinement and review of the video, rater agreement remained low, seemingly because of the ease of missing references to errors or norms in such a large amount of video. For example, a reference to an error could be very quick, such as a student calling out an incorrect answer and the teacher or another student correcting them. A public reference to a norm could also be quick and easy to miss, as in the common case of the teacher correcting a student’s behavior by telling the student to “stop.” It was rare for raters to miss more prolonged instances of public references to errors or norms. In general, the two raters agreed on all identified instances during discussions, but often one of the raters had originally failed to identify instances while coding independently. Therefore, it was decided that both raters would engage in First-Cycle coding for all nine lesson videos in the sample, and the union of their coding would be considered for further analysis. In other words, both raters identified instances pertaining to norms and errors in all nine videos, and all instances identified by either rater (or both) were considered for second-cycle coding. I reviewed all first-cycle coding as a third rater. Using this procedure, 267 instances pertaining to errors were identified across the nine lessons, and 414 instances pertaining to classroom norms were identified.

**Subcoding of errors and norms instances.** Specific aspects of the interactions identified as pertaining to errors and norms using first-cycle coding procedures were further distinguished using subcodes. For each interaction involving an error, research assistants and I identified to whom the error was attributed, who labeled it as an error, and whether or not the error was corrected. For each interaction involving norms, we identified how or why the norm came up.
during the class discussion. A codebook was also created for this subcoding coding procedure. Subcodes were applied to instances using the “label” feature in the StudioCode software. See Table 6 for selected subcodes applied to instances pertaining to errors and Table 7 for selected subcodes applied to instances pertaining to norms. The subcodes listed here are highlighted because they are relevant to findings presented in Chapters 5 and 6. A full list of subcodes applied during coding is provided in the codebook in Appendix D.

For most subcodes, each rater chose one option from each category. For example, the raters selected who identified an error that came up during a discussion as being incorrect (e.g., the teacher, a student, etc.), and raters could not select more than one option for this code category for each instance. However, for some subcode categories, a rater could select more than one option. The Resolution (how each interaction involving errors was resolved) and Why (why or how a discussion of classroom norms came up) categories required raters to select all options that applied. For example, an error could be corrected during a discussion and also lead to an in-depth discussion of mathematics, in which case the rater was expected to apply both subcodes to that instance.

Table 6

<table>
<thead>
<tr>
<th>Subcode Category Meaning</th>
<th>Subcode Category</th>
<th>Subcode Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who identified the error as incorrect (if applicable)?</td>
<td>Identify</td>
<td>Teacher</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student who made error: the student who made the error identified it as incorrect</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Other student: a student who did not make the error identified it as incorrect</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N/A: this is not applicable (e.g., because the instance is a general discussion of errors and no specific error was made)</td>
</tr>
<tr>
<td>What was the resolution to the interaction, if any?</td>
<td>Resolution</td>
<td>Error corrected: the error was corrected during the same lesson</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Leads to in-depth discussion: the error discussion leads to a more in-depth discussion of a mathematical topic</td>
</tr>
</tbody>
</table>
Table 7

Subcodes Used to Characterize Interactions in Norms Instances in Classroom Video

<table>
<thead>
<tr>
<th>Subcode Category</th>
<th>Subcode Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>How was a norm (or were norms) made public during this instance?</td>
<td>Why</td>
</tr>
<tr>
<td>Behavior correction: a member of the class corrected another member’s non-normative behavior</td>
<td></td>
</tr>
<tr>
<td>Behavior praise: a member of the class praised another member’s behavior, suggesting that the behavior is normative</td>
<td></td>
</tr>
<tr>
<td>Discussion of norms: the class explicitly discussed norms (expectations for behavior) in the class</td>
<td></td>
</tr>
</tbody>
</table>

For this subcoding process, the two raters and I met frequently and refined the codebook through a process of subcoding, comparing subcodes, and discussion of subcodes and procedures. Following this refinement of the codebook and subcoding procedures, rater agreement was sufficient to allow for only one rater to subcode each lesson video. That is, raters agreed on the application of subcodes for at least 70% of instances in a video used for practice of subcode application. So, it was decided that raters were generally applying subcodes in the same way, and it was unnecessary for both raters to subcode all lesson videos. Then, to determine interrater reliability, both raters coded a subset of three lessons (one from each data collection period). Interrater reliability is presented in Table 8 below in the form of percentages of instances on which raters agreed (i.e., percent agreement) on the application of subcodes. These measures of interrater reliability were considered sufficient for the purposes of this study.

Rater reliability varied somewhat by subcode, as shown in Table 8. Agreement on the application of the “error corrected” option of the Resolution subcode and the “behavior correction” and “behavior praise” options of the Why subcode was achieved on 80% or more of instances, indicating high levels of agreement. Agreement was only slightly lower than 80% for application of the options of the Identify subcode. Agreement was significantly lower for the application of the “leads to in-depth discussion” option of the Resolution subcode (57%) and the “discussion of norms” option of the Why subcode (43%). Discussion with raters indicated that these options were more difficult to apply because of the difficulty of judging whether a discussion was sufficiently “in depth” to warrant application of these subcodes. Because agreement was far below 80% for these two options, they were not considered as reliable sources of evidence in later analyses. For the subcodes with agreement above or close to 80%, it was considered that raters generally coded in a similar manner, so these subcodes could reasonably be used as indicators of the general characteristics of the relevant classroom interaction instances identified in the nine sample whole-class lessons.
Table 8

<table>
<thead>
<tr>
<th>Subcode Category</th>
<th>Subcodes</th>
<th>Percent Instances for which Raters Agreed on Subcode Application</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Errors Instances</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Who identified the error as incorrect (if applicable)? (Identify)</td>
<td>Teacher Student who made error Other student N/A</td>
<td>78% &lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>What was the resolution to the interaction, if any? (Resolution)</td>
<td>Error corrected (yes/no) Leads to in-depth discussion (yes/no)</td>
<td>80% 57%</td>
</tr>
<tr>
<td><strong>Norms Instances</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>How or why did the rule come up during the class discussion? (Why)</td>
<td>Behavior correction (yes/no) Behavior praise (yes/no) Discussion of norms (yes/no)</td>
<td>95% 81% 42%</td>
</tr>
</tbody>
</table>

In general, the coding and subcoding process was intended to provide further information about the general type of scenario in which errors and classroom expectations were coming up and being discussed in the case-study classroom. The entire video coding and subcoding process is outlined in Figure 6. The results of this analysis are discussed in the next section.

<sup>3</sup> This percentage indicates that coders agreed for 78% of errors instances whether the error was identified as incorrect by the teacher, the student who made the error, or another student, or they agreed that this code was not applicable (e.g., because the instance did not include a specific error but rather was a general discussion of making errors).
Results: Frequency of Interactions Related to Errors and Classroom Behavioral Expectations

The results of the descriptive coding and subcoding process indicate that errors and behavioral expectations came up in whole class discussions frequently, and subcodes provide further information about the circumstances in which they came up. In this section, I describe relevant results of descriptive coding and subcoding of the nine sample lessons.

**Frequency and characteristics of interactions coded as related to errors.** In general, errors came up frequently in each lesson. Across the nine sample mathematics lessons, 223 instances were coded as relating to errors using the coding definitions and procedure described above and in the codebook in Appendix D. The frequency of instances coded as relating to errors was higher in video of lessons recorded at the end of the school year than video recorded at the beginning of the school year. Figure 7 shows the frequency of instances coded as related to errors from each data collection period. Totals are summed from the three sample lessons from each data collection period. Looking at the frequency of coded errors instances identified in video from the different time periods, it is notable that roughly 50% more coded errors instances were identified in class discussion video from the end of the school year than from the beginning or middle of the school year. This difference does not appear to be solely due to differences in the length (i.e., number of minutes) of the sample video from each data collection period. In fact, the sample of video coded for the beginning-of-year data collection period was longer (179 minutes) than the sample of video coded for the end-of-year data collection period (155 minutes). See Table 4 for more details of the length of each video. Therefore, it is
possible that the increase in total number of instances related to errors in whole class discussions may be related to changes in classroom practices and norms related to errors over time. These changes could be related to any number of factors, including changes in students’ comfort bringing up and discussing errors and changes in the specific content (e.g., geometry, fractions) being discussed.

Figure 7. Number of instances identified as relevant to errors and labeled with the most common relevant subcodes.

Though the frequency with which errors occurred during discussions shifted over time, some characteristics of the way errors were treated remained consistent. Most errors were identified as incorrect by the teacher (82%), and most errors that came up during whole-class discussions were also corrected during that discussion (79%). In other words, usually the correct answer was stated and/or the class worked together to identify how to correct the solution strategy in order to arrive at a correct answer. The graph in Figure 7 also shows the number of these error instances in which subcodes were applied indicating that (a) the error was identified as incorrect by the teacher, and (b) that the error was corrected during the discussion. These were the most commonly applied subcodes for errors instances across all three data collection periods, indicating that at the beginning, middle, and end of the school year, most errors that came up during discussions tended to be identified as such by the teacher and tended to be corrected during the discussion.

The relative consistency across the school year of the characteristics of interactions related to errors is striking. Though the percentages of instances from the beginning, middle, and end of the year fluctuate somewhat for the subcoded characteristics of these interactions, the general patterns remain largely the same across the three data collection periods. Throughout the school year, most errors that came up were identified as errors by the teacher. This finding is consistent with the general tone of whole-class discussions in the case-study class; discussions were teacher-led with the teacher acting as the organizer and authority during discussions. Additionally, these results show that most errors that came up during discussions were corrected during that discussion across all three data collection time periods. That is, either the correct answer was stated, or a more elaborated discussion of how to correct the error was held. It is likely that these characteristics of most interactions involving errors were linked to class norms for the treatment of errors; the consistency across the school year of these characteristics

35
indicates that the class likely held shared expectations related to the identification of errors and related to errors being corrected.

**Frequency and characteristics of instances coded as related to norms.** In general, classroom behavior expectations came up frequently in the case study class during whole class discussions. In total, 359 instances of interactions coded as related to norms were identified in the sample mathematics whole-class discussion videos. The frequency of these instances identified at each time period is relatively high (see Figure 8), though the general trend appears to be a slight decrease over time in the frequency of instances of norms coming up in whole-class discussions. This trend does not seem to be solely due to the relative lengths of the video coded for each time period; in fact, the middle-of-year video sample was the shortest in length of the three. It is possible that the slight decrease in public references to norms is related to a shift in classroom interactions over time. However, it is notable that norms came up frequently in whole-class discussions across all three time-periods.

Most of interactions identified as related to norms took the form of behavior corrections or behavior reinforcements, meaning that someone (usually the teacher) corrected or praised a person’s behavior in a public way. In fact, by far most of the instances during which norms came up were instances of the teacher correcting a student’s behavior. See Figure 8 for the frequency of instances identified as related to norms for which each of these subcodes were applied from each data collection period.

As with the errors instances, these initial results indicate that the characteristics of interactions related to classroom norms in the case study class were fairly consistent over the course of the school year. Though there are fluctuations, the general patterns remain fairly consistent, in that most of these interactions involved a class member – usually the teacher – correcting another class member’s behavior. These interactions likely indicated to students what types of behavior were acceptable in the classroom and likely supported the taking up of classroom norms for behavior.

**Summary and implications for further analysis.** The results presented in this chapter show that errors came up frequently in whole-class discussions of mathematics, and behavioral expectations were frequently made salient during discussions as well. Some patterns related to how errors came up were consistent across the school year; the teacher generally identified errors...
as incorrect, and errors that came up were usually corrected during discussions. Additionally, some patterns related to behavioral expectations were also consistent across the school year. Most of the time the action that made behavioral expectations public was a class member correcting another class member’s behavior because he or she had done something that violated behavioral expectations (i.e., norms) in the classroom. Sometimes behavioral expectations became salient when a class member – usually the teacher – praised another class member’s behavior, indicating that this behavior was an example of expected behavior in the classroom. The teacher seemed to have used these two strategies of correcting students’ non-normative behavior and praising students’ normative behavior in order to support students in taking up particular norms in the classroom.

The results presented here also indicate the errors and norms did come up frequently in whole class discussions, suggesting that further analysis of the data collected for this study could provide some results related to the research questions of what norms related specifically to students’ mathematical errors developed in this class and how they developed over time. If errors had rarely come up during whole-class discussions or if behavioral expectations had been unclear, it may not have been possible to find evidence of classroom norms related to the treatment of mathematical errors using the data collected for this study.

In the next two chapters, I describe in more detail results related to the two research questions of this study. The next chapter, Chapter 5, details results related to the research question of what norms emerged related to discussions of students’ mathematical errors. In that chapter, I describe specific methods used to identify these norms, and I describe in detail two of the seven norms identified using these procedures. I also present more briefly the other five norms identified. In the following chapter, Chapter 6, I describe methods and results related to the question of how the norms identified in Chapter 5 emerged over time in the case study classroom. I focus on the development of the two norms detailed in Chapter 5. Following these two results chapters is a chapter of conclusions and implications of the results.
Chapter 5: What Norms Emerged in the Case Study Classroom

In this chapter I describe the methods I used to identify what norms related to mathematical errors had developed in the case-study class by the end of the school year, and I briefly describe each of the norms identified using this process. As described in Chapter 2, norms are here understood to be taken-as-shared expectations for behavior in the classroom community. I describe two of these norms in more detail, providing examples of teacher and student statements from the end-of-year interviews as examples of individuals’ expectations that seem aligned with these norms. I also provide examples of classroom interactions that seem to corroborate the presence of these norms in the classroom, and when possible I provide examples from student survey data of survey responses that provide further evidence. The descriptions provided in this chapter focus on the norms evident in the class at the end of the school year in order to illustrate what norms were actually taken up by the classroom community over the course of the school year. The question of how these norms came to be established in the classroom community – that is, by what processes these norms came to be taken-as-shared – is discussed in the next chapter, in which I present evidence from the beginning, middle, and end of the school year to illustrate the development of these norms over time.

Procedures for Identifying Norms

Because norms are intangible properties of a collective, they are not easily discerned by an outsider. The multiple sources of data collected as part of this case study were necessary in order to develop a triangulating method to identify and illustrate the norms that developed in the community of interest, namely the case study classroom.

A useful first step in this process was to analyze teacher and student interviews for common themes in these individuals’ expectations, because these common themes may indicate regularities in expectations of the classroom community as a whole. A second step was to analyze classroom video to identify confirming or disconfirming evidence that these common expectations were embodied in classroom interactions. A final step was to examine students’ responses to survey data for confirming or disconfirming evidence that these expectations seem to be consistent among student members of the class.

I focused this analysis on the interview, lesson video, and survey data from the third data collection period, which occurred in April, towards the end of the school year. This focus allowed for identification of norms that had developed and persisted in the class over the course of many months and were therefore likely to be integral to the classroom culture. In this section, I describe the procedures I used to develop theories about what norms existed in the case study classroom and then to provide support for or to refute these theories through triangulation of various types of data.

Open coding of student interview data. I began the process of identifying norms by using grounded theory techniques to analyze student interview data from the end of year interview. Beginning with the student interviews seemed reasonable because any classroom expectations held by the group would likely also be expressed by more than one student. Because the interview questions were the same for all students and because interview questions explicitly and implicitly probed for expectations students held for interactions related to errors in their classroom, the student interviews were a useful starting point for identification of expectations common to multiple members of the classroom.
Grounded theory techniques were an appropriate choice for this analysis because “…grounded theory methods consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories from the data themselves. Thus researchers construct a theory ‘grounded’ in their data” (Charmaz, 2014, p. 1). In this analysis, it was important for theories of what norms existed in the classroom to be “grounded” in the data in order to allow for identification of norms I did not anticipate. If I had instead used a coding scheme based on previous research or on my previous observations of the classroom, this method might have obfuscated norms that existed in the classroom but were less clear to me as an outside observer or that had not been previously described in research literature.

Charmaz (2014) describes the process of data analysis using grounded theory as follows: Conducting grounded theory coding involves you in at least two main phases: 1) an initial phase involving naming each word, line, or segment of data followed by 2) a focused, selective phase that uses the most significant or frequent initial codes to sort, synthesize, integrate, and organize large amounts of data. (p. 114)

I followed this basic process using the rough transcripts I had produced for each end-of-year student interview. I engaged in the first step described by Charmaz by paraphrasing or describing the content of each student turn of talk in as few words as possible. When turns-of-talk were quite short or did not make sense individually, I combined multiple turns into one unit for paraphrasing. This type of paraphrasing or description is often referred to as open coding or descriptive coding.

Once I finished the descriptive coding phase for the five focal student interviews, I began the second phase described by Charmaz by reviewing the descriptions I had written and identifying common types of descriptions that came up frequently. I will refer to these groups of similar descriptive codes as themes. I then reviewed these themes for consistency and coherence, and I reviewed the original text of the student interview transcripts to be sure that the content of the interview was consistent with my descriptive codes. Seven themes were identified using this process, and all were present in at least three students’ statements during interviews.

**Focused coding of the teacher interview, classroom video, and survey data.** I treated these seven themes as candidate norms, meaning that these were statements about expectations related to behavior in the classroom that were endorsed by at least three students and that might be common to the classroom community as a whole. In order to determine if these expectations did, in fact, seem to be norms in this community, I used a focused coding procedure to identify all instances of these candidate norms emerging in teacher interview, classroom video, and student survey data from the end of year data collection period. This procedure was intended to allow for examination of evidence from multiple sources that either confirmed or disconfirmed that the expectations expressed in these themes were in fact norms in this community.

**Analysis of the teacher interview for confirming or disconfirming evidence of norms.** I then engaged in a similar focused coding procedure for the end-of-year teacher interview transcript. In so doing, I identified all statements made by the teacher during the end of year interview that were related to each of the seven identified candidate norms. I then examined each identified statement to determine if it confirmed or contradicted the expectation expressed in the candidate norm. In fact, all teacher statements related to candidate norms confirmed these candidate norms. No statements made by the teacher contradicted the candidate norms identified through themes in students’ interviews.

**Analysis of video data for confirming or disconfirming evidence of norms.** I then reviewed all instances from the end-of-year classroom video previously identified as related to
mathematical errors or classroom behavioral expectations (see Chapter 5 for a description of the identification process) using a focused coding procedure to determine if these instances were related to any of the seven candidate norms (themes). I created a coding scheme (see Appendix D) to guide this focused coding procedure, and I used StudioCode software to label (i.e., apply subcodes) each previously identified instance as being related to one or more of the seven candidate norms if it fit the criteria described in the coding scheme. In general, this coding scheme describes each candidate norm and instructs the coder to apply labels to instances in which the teacher or students either (a) act or talk in ways clearly consistent with the expectations of the candidate norm, or (b) act or talk in ways clearly inconsistent with the candidate norm.

Using StudioCode software, it is possible to create a compilation video of all instances labeled with a given label. I used this function to create compilation videos of all instances labeled as related to each candidate norm. I was therefore easily able to review all instances of classroom interactions related to each candidate norm to investigate whether these instances seemed to offer evidence that supported or contradicted the norm. In general, all identified instances from the end-of-year classroom video supported the existence of the seven candidate norms in the case study class.

**Analysis of student survey data for confirming or disconfirming evidence of norms.**

Focused coding of student survey data was more challenging because the provided multiple-choice options on this survey were not written with the candidate norms in mind; they were written prior to data collection. The focus of survey questions was to elicit students’ expectations related to scenarios in which a student made an error in front of the class, but the types of scenarios included in the survey and the answer options for multiple choice questions did not specifically target the candidate norms (see Appendix A for student survey questions). The response options provided on the survey instrument therefore did not fit neatly with each of the candidate norms. However, some response options did align with candidate norms and some did contradict candidate norms.

The survey was administered at each data collection period – at the beginning, middle, and end of the school year – as described further in Chapter 3. Further information about shifts in students’ responses to the survey items over the course of the year is discussed in Chapter 6 in relation to how norms were taken up in the class over time.

I reviewed students’ responses to the end-of-year survey to determine if students tended to select responses that were aligned with the candidate norms or those that contradicted the candidate norms. In fact, most students selected responses that aligned with candidate norms at the end-of-year. Only two students’ responses to one survey question could be interpreted as contradicting a candidate norm, specifically the norm *the teacher and students should not give up on themselves or others who are struggling with math*. However, this norm aligned with the majority of students’ survey responses to two survey items, suggesting that most students appeared to hold expectations consistent with this norm. Other norms that were supported by students’ survey responses were *everyone has some mathematical understandings to which you should pay attention* and *you should admit when you get an answer wrong*. To a lesser degree, the norm *there are different types of mistakes/errors* was also supported by some students’ survey responses.

In summary, seven themes or “candidate norms” were identified through students’ interview statements, and these candidate norms were substantiated using triangulation of various sources of data, including teacher and student interviews, student surveys, and
classroom-lesson video recordings. Analysis of this data followed a grounded procedure of open-coding followed by focused-coding. Below, these candidate norms are referred to simply as norms. In this analysis, a norm is defined as a behavioral expectation expressed by at least three of the five focal students during end-of-year interviews and consistent with the teacher’s statements during the end-of-year interview as well as observed interactions found in classroom video data and student survey data from the end-of-year data collection period.

Norms Identified in the Case Study Classroom

The seven norms identified through the process described above are discussed in this chapter in order to answer the first research question of this study: What norms support the discussion of students’ mathematical errors? Though other norms may have existed in the case-study class, these are not discussed here because they did not emerge as themes during focal student interviews about the treatment of errors and were therefore not likely to be central classroom expectations related to the treatment of errors in the class.

Each of the seven identified norms is described briefly below, and two of these norms are described in more detail. These two norms were selected for in-depth discussion because the process of how each of these norms emerged in the classroom over the school year was of interest. The first norm is an example of an expectation that was originally conceived of by the teacher, promoted by him, and gradually taken up by students over the course of the year. This norm is also closely related to an interesting collective practice in this classroom, coaching, which is described in more detail below. In contrast, the second norm provides an example of an expectation that developed in response to a need in the class that had not been anticipated at the beginning of the year; it arose as a solution to the problem of students being overly comfortable making mathematical errors to the point of making many careless errors. The development of these norms over the school year is described in detail in Chapter 6.

In this chapter, I describe these two norms as they existed at the end of the school year in order to illustrate what these expectations were like after they had been taken up. I also more briefly describe the other five norms identified in this study. Though these other five norms are also interesting and were important in the case study classroom, constraints of space and time prevent me from fully describing all seven norms in detail.

To illustrate norms, I provide examples of statements from the teacher and student interviews, examples from transcripts of lesson videos, and where possible, evidence from student survey data. The evidence provided in this chapter and the next chapter is intended to be considered collectively. Each student or teacher statement quoted below expresses that person’s individual expectation with regards to behavior in the classroom. Each statement made by the teacher or a student, each example of interactions from classroom video, and each reference to the student survey is just one piece of evidence intended to be considered along with all other evidence in a triangulating fashion in order to indicate the existence of a norm in the case study class. No single quotation or example is intended to provide such evidence on its own. Below, I describe the evidence for the first two norms in detail, followed by a more brief description of the other five norms identified in this study. First, however, I describe the collective practice called coaching that was closely associated with the first norm and also associated to varying degrees with other norms described in this chapter.

The coaching practice. Over the course of the year a practice called coaching became common in the case-study classroom. This practice typically involved two class members engaging with each other at a time: the teacher or a student helping another student who was
struggling with a mathematics problem. During whole-class discussions observed for this study, the teacher often publicly coached students who had difficulty with math problems. Most focal students described coaching other students during independent practice time at least occasionally. In general, this practice seems to have been promoted and modeled by the teacher and taken up by students over the course of the year as a way to help students succeed in solving math problems and to help them understand mathematics concepts. The process through which this practice developed in the class over time is described in more detail in Chapter 6. Here, I provide a brief overview of the practice and how it was used in the class.

Several features of the coaching practice distinguish it from simply “helping” students with mathematics problems. These features were discussed explicitly by the teacher and students in the class and were also described to me by focal students during interviews. Specifically, focal students identified four key features of the coaching practice:

- The coach helps the person being coached to find the answer to a problem on their own rather than telling them the answer.
- The coach checks that the person being coached understands of the problem.
- The coach helps the person being coached to go through the steps of the problem, step by step.
- The coach asks the person being coached questions to help them think through the problem.

These defined features indicate that coaching was a distinct and shared practice in the classroom. The class members generally understood coaching to include these specific features, as evidenced by focal students’ descriptions during interviews at the end of the school year, and many times the class explicitly discussed these features of the coaching practice.

The coaching practice is relevant to the descriptions of classroom norms related to errors because many of these norms were related to how class members engaged in the coaching practice. Because this practice specifically involves one person helping another who has made an error or is struggling to understand a mathematics problem, norms related to the treatment of mathematical errors clearly relate to situations in which the coaching practice would be appropriate. As described in Chapter 2 community norms can sometimes be closely tied to collective practices in the community. Some of the norms identified during this study were observed primarily in the context of the coaching practice, and others were observed primarily in other contexts. The first norm described below frequently applied to coaching situations and was most clearly visible in those situations. Other norms described below also related to the coaching practice, and I describe these relationships as well.

Norm 1: Everyone has some mathematical understandings to which you should pay attention. This first norm is an example of a norm that was closely related to the collective practice called coaching that was common in the case-study class. This norm helped class members engage productively with each other in the coaching practice by assuming that all class members – even those who were struggling or who got an answer wrong – had valid mathematical knowledge that could be leveraged to facilitate learning of new ideas and problem solving strategies. Below I describe data indicating the existence of this norm in the case study class, and I use this data to illustrate how the norm regulated behavior in the classroom, particularly as related to the coaching practice, and how various members of the class made sense of each norm. In particular, the teacher and focal students seem to have understood this norm somewhat differently from each other.
Evidence from the teacher’s statements during interview. This norm seems to have been the foundation of how Mr. Anderson, the teacher, understood mathematical errors and how he wanted his students to think about mathematical errors. In interviews, Mr. Anderson consistently took the position that all people have some mathematical understandings with which they are operating. According to Mr. Anderson, when people make mistakes, their understandings are incomplete or not adapted to the type of problem that they are trying to solve. However, he focused on the fact that there are some mathematical understandings on which to build more complete understandings. Mr. Anderson said that he encouraged his students to think this way, too, and he explicitly taught students a process for supporting each other in building on their valuable mathematical understandings. He called this practice “coaching” and the features of this practice as it came to be used are described above. The norm that one should attend to the mathematical understandings that a person does have – rather than, for example, focusing on the understandings that are lacking – was one part of the coaching practice that Mr. Anderson attempted to promote, as described in the interview excerpt below.

**Interviewer:** What practices do you want students to engage in related to making errors or having partial understandings?

**Mr. Anderson:** The most important thing that I want them to get is that “partial understandings” approach – that idea that every mistake they make, there is something about it they do understand. I’m trying to start from there myself and build on that. And get the kids to see that, too. To look at what they do know. Let’s show the good math, the strong math that they really do know about this, and what step do they need to take next. How would you coach them? Coaching is really the word I want to use this year. It’s something we started at the end of the year last year. My [colleague] and I are working on the language to help kids understand what good coaching would look like. And part of that is this idea of starting with what somebody knows and trying to work upward from that. Trying to find that place first. (Teacher Interview, Beginning of Year, September 2013)

In this statement, Mr. Anderson made clear that when mathematical errors occur, he wanted his focus to be on “the good math, the strong math that [the student] really did know” rather than on the understanding that the student was missing. Furthermore, he expressed that he wanted to “get the kids to see that, too,” meaning to see the correct mathematical ideas. He gave the example of a specific way of helping someone learn from his or her mistakes, a practice he called “coaching.” He suggested that he was developing a way to teach students this practice, helping students learn to start “with what somebody knows and to try to work upward from that,” in other words, to identify the useful mathematical ideas in a person’s mathematical solution, even if that person’s answer is incorrect.

Evidence from focal students’ statements during interviews. Focal students seem to have made sense of this norm somewhat differently from Mr. Anderson, though they also discussed the first norm in the context of the coaching practice. When I asked focal students to describe the process of coaching during the end-of-year interviews, they often brought up the idea that students have correct mathematical ideas even when they get the answer to math problems wrong. The focal students often indicated that they found value of some sort in other students’ mathematical ideas, saying that they expected that as a coach they would not only teach the person they were coaching but also learn from that person’s mathematical ideas. This expectation was expressed by all five focal students during their end-of-year interviews. This expectation is interesting because the coaching process is only supposed to occur when the
student being coached does not understand how to do a problem or has gotten the answer to a problem incorrect. Therefore, focal students were saying that even if a person doesn’t fully understand a problem or has an incorrect answer, they still expect to be able to learn from that person’s mathematical ideas about the problem. This way of making sense of the norm differs somewhat from Mr. Anderson’s focus on using the person-being-coached’s mathematical ideas as a starting place to build his or her understandings. Instead, focal students seem to have understood the importance of identifying another person’s correct mathematical ideas as potentially useful for their own learning.

For example, during our end-of-year interview, Rowan described using the coaching practice to “check in” with his tablemates during independent practice on math problems. He indicated that when he was coaching someone, that person might help him to identify his own mathematical errors. For Rowan, the coaching practice is really a process of the coach and the person-being-coached helping each other.

Interviewer: How often do you coach other students who are stuck or who get an answer wrong?
Rowan: More than once a day, because … as I go along in the problem set, I ask for help and people also ask for help from me. And the whole table group checks with each other and asks for help from each other more. When people ask, I help them, and it happens a lot every day.

[brief discussion between the interviewer and Rowan of why Rowan checks in with his tablemates instead of other students in the class]

Interviewer: Sure. And how important is that for you learning math, to coach other kids?
Rowan: I think it’s a lot because like I said earlier it strengthens my understanding, too. Sometimes when I’m coaching someone through something, maybe I got it wrong, too. That person might point it out that it doesn’t make sense. And usually when it’s coaching, like, we’re coaching each other. (Focal Student Interview, End of Year, April 2014)

Rowan’s comments suggest that he expects other students in the class to have mathematical understandings and strategies from which he can learn, even if they are struggling to solve math problems. He expresses his experience that “when it’s coaching, like, we’re coaching each other,” indicating that he expects that coaching the other person might “strengthen” his own understanding. Furthermore, Rowan expects that when he is coaching a student who is struggling with a math problem, that student might be able to analyze Rowan’s own solution and “point out that it doesn’t make sense” if Rowan has solved the problem incorrectly. In other words, he expects other students in the class to have mathematical understandings that may allow them to understand his solution and also to point out his own errors, even if they are struggling with how to solve the problem correctly themselves.

Maria, another focal student, also expressed the idea that students can learn from each other’s ideas during the coaching practice. She also seemed to think that even when she gets an answer correct, she can learn from another student’s mathematical ideas.

Interviewer: … And how often do you coach other kids who are stuck or who get an answer wrong in math?
Maria: Um, once a week because when I’m doing math, you know, when I’m half way through there are a few people who are close to being done or done, and by the time I’m done with my problem set, that’s when, you know, the math session is done.
Interviewer: Okay, so you usually don’t have time.
Maria: About once a week. But you know, when I really understand every problem, then I get it done fast and I can go help coach other kids.

Interviewer: Okay, so sometimes you have time. And how important is that for your math learning?

Maria: A lot, because then I can see their point of view of how to solve the math problem. If they have a different strategy, then I can fix their strategy to see how they got it wrong. Or right. And maybe use that strategy. (Focal Student Interview, End of Year, April 2014)

Maria focuses on the idea of students’ different strategies, saying that she might learn from another student’s strategy and use it herself. Her statement suggests that she understands that strategies can be partially correct and may be useful, even if the student using that strategy gets the final answer to the problem wrong. She finds value in understanding other students’ “point[s] of view of how to solve the math.”

Another focal student, Domingo, generally expressed low confidence in his own mathematics understandings but nonetheless voiced a similar idea about the coaching practice during our end-of-year interview. He described often not feeling confident to coach other students, but he said that when he did coach another student, he often felt that they were both learning from each other:

Interviewer: How often do you coach other students in math?

Domingo: Well, it’s these two [pointing back and forth at “about once a week” and “about once a day” on the printed rating scale] because sometimes there’s not really that many people that go to me for help. Um, only when I really get the problems, and some of the people don’t, that’s really when people come to me and ask for help. Otherwise, if I don’t get the problem really well, I don’t trust myself to talk to other people because I might give them the wrong answer, so I just don’t do it. …

Interviewer: … Sometimes that might be in between once a week and once a day that you might feel confident to help. When you do coach other people, how important is that for you learning math?

Domingo: A lot because I can listen to them. It’s basically like we’re both helping each other because I might get something wrong and they got it right, and they thought they got it wrong. So we’re both helping each other at the same time. (Focal Student Interview, End of Year, April 2014)

In this segment, Domingo expressed the idea that both students can learn from each other when one is coaching the other. He described coaching other students only when he was confident about his own understanding of a problem, but he said that even at these times, “it’s basically like we’re both helping each other.” This statement indicates that, like Rowan and Maria, Domingo seemed to expect other students who were struggling with a math problem to have mathematical ideas from which he could learn.

The other two focal students, Jordan and Kai, expressed similar ideas about learning from each other’s mathematical ideas when they coach other students in the mathematics class. All five focal students brought up this idea when asked whether or not they thought that coaching other students was valuable for their own mathematics learning. This theme of learning from each other is not exactly the same as the idea that the teacher Mr. Anderson expressed in his description of looking for the “the good math, the strong math that [the student] really do[es] know” in order to build on that understanding while coaching another student. However, like Mr. Anderson, the focal students do seem to focus on other students’ mathematical knowledge,
even when those students are being coached and therefore must be struggling with a math problem.

Furthermore, coaching was not the sole context in which focal students focused on other students’ mathematical ideas as something to be attended to during class. During the end-of-year interview, I asked focal students why they chose particular survey response options to a question about what they expected to happen if another student solved a problem incorrectly in front of the class. Rowan said that he chose the survey response indicating that he would “try to figure out what [his] classmate was thinking about when he got the answer wrong” because he expected that he might learn from that student’s strategy, even if their final answer was incorrect.

Interviewer: Now I’m going to ask you some questions about your survey. Let’s take a look at question number two.

Interviewer reads question #2 on the survey and identifies that Rowan chose option D, that he’d try to figure out what his classmate was thinking when he got the answer wrong.

Interviewer: Can you tell me more about why you chose that?
Rowan: Well, sometimes they might have, this actually just came to me, it might give an idea of a faster way to think it through. And when you do this, it’s more comparing your [solution] to that person’s [solution]. Sometimes when you go through your [solution] and that other person’s [solution] – how do I put this – you can find maybe you made a mistake or something you did some way that you could have done much faster.

Interviewer: So even though they got the answer wrong, they might have had some good thinking.
Rowan: Yeah, something might pop into your head like, oh, maybe that person was trying to do this. And then you have better ways to solve problems. (Focal Student Interview, End of Year, April 2014)

Rowan’s response suggests further that he understands that he can learn from other students’ mathematical ideas even if the student gets an answer wrong. Rowan discusses these ideas as possibly helping him find “faster” or “better” strategies for solving problems. As before, Rowan here indicates the importance of attention to other students’ canonical or correct mathematical understandings and strategies, even in the context of an incorrect answer. Unlike Rowan’s first quoted statement above, here Rowan was not necessarily linking this norm to the coaching practice, suggesting that he understood the expectation of focusing on correct mathematical ideas to extend other contexts as well.

In general, all focal students seemed to expect to attend to other students’ ideas and to learn from other students’ ideas, whether those students got the answers to math problems correct or not. This expectation is generally consistent with Mr. Anderson’s expectation that students all have valuable mathematical knowledge on which to build deeper understandings. Though the focal students expressed less nuanced ideas than Mr. Anderson, all of the focal students and the teacher at least seemed to expect that everyone has some understandings in math to which you should pay attention. Often this expectation was linked to the coaching practice, but evidence suggests that class members may have also understood this norm to apply to other situations.

Evidence from student surveys. Student survey data also suggests that the majority of students – not only the focal students – took up the expectation that one should focus on correct mathematical understandings. The survey data from the survey question described in Rowan’s interview excerpt above is suggestive of the joint class expectation that members should attend to
others’ mathematical ideas. The majority of the class responded to this survey item in the way that Rowan did, and the answer option that Rowan and most of his classmates selected in response to this survey question was consistent with this norm. The survey question was, “Imagine that a classmate is talking in front of the whole class about how they solved a math problem, and they get the final answer wrong. What would you do?” Students were provided with four options from which to select an answer, or students could write in their own answer. Like Rowan, most students (76%) in the case study class selected this option, suggesting that the majority of the class also expected to analyze other students’ mathematical thinking.

**Classroom video-recording data related to this expectation.** Focused coding of the classroom video recordings from the sample taken at the end of the school year did not reveal many instances in which this norm was obvious in classroom interactions during whole-class discussions, possibly because it is difficult to observe where class members’ attention lies during problem solving. However, at times this norm was suggested by statements made by a class member (often the teacher). For example, during one whole-class discussion of a problem, the teacher focused on the effort and correct mathematical strategies a student was using, though that student had and incorrect answer to the problem.

*Teacher:* [Student] got forty-one and seven-eighths.
*Several students:* No.
*Teacher:* No? You don’t like that? Let’s try it. Let’s see where he made his mistake.

The teacher begins writing the problem on a piece of paper taped to the whiteboard. He articulates each step of the problem and asks the student who originally had the answer of 41 7/8 to do the calculation for each step. Eventually, it is apparent that the student was following the correct steps to solve the problem but had multiplied incorrectly when performing a long-division step.

*Teacher:* Excellent job, [Student]. Super proud. You’re doing it all on your own. Make all the mistakes you want. Uh, and then we bring down that five. How many eights are in 25?
*Student:* [difficult to hear] … I mean three.
*Teacher:* Three, perfect. What do we do with that one? What fraction?
*Student:* [inaudible]
*Teacher:* [nods, writing solution on a paper taped to the whiteboard and visible to the class] How many of you got fifty-three and one eighth?

(Classroom Video Recording, End-of-Year Data Collection Period, Day 4)

In this example, the teacher worked through a problem with a student who had gotten an apparently incorrect answer, and the teacher specifically pointed out the correct mathematical thinking that the student showed despite an arithmetic mistake. The teacher praised the student’s thinking and independence in his work. This example illustrates the focus on students’ mathematical understandings in this classroom. The student whom the teacher was talking to in this example often asked for help in this class and might have been regarded by some observers as a “struggling” student. However, the teacher positioned his solution strategy as valuable and praised his independent work, further emphasizing that all students – even those who struggle – have mathematical understandings.

During the same lesson, the teacher also reminded students to attend to each other’s mathematical ideas by praising students for showing that they were listening to a student who was sharing his mathematical idea.

*Teacher:* Anybody have a different method?
Domingo raises his hand
Teacher: Yeah?
Domingo: Well, I was thinking of doing the same thing [as the strategy Rowan had just shared] but with play-doh.
Teacher: [looking at Domingo] Explain.
Domingo: Well –
Teacher: [interrupting, and looking up at other students] Nice, guys. Well done.
Domingo: Uh –
Teacher: [looking at Domingo] Wait, stop. [looking at the whole class] You know how I’ve talked to you before about how whether you give the same attention to everyone or it’s just some people who get it? There was a great example of you giving the same attention no matter who’s talking. On your own. The second somebody else talked, you did the same thing for him that you did for [Rowan]. Perfect. Integrity. [looking at Domingo] Carry on.
Domingo explains his strategy for trying to figure out the area of an irregular shape.
(Classroom Video Recording, End-of-Year Data Collection Period, Day 4)
In this example, Mr. Anderson publicly praised students for showing that they were listening to Domingo when he began speaking after Rowan. Mr. Anderson referenced discussions the class had had before about “giv[ing] the same attention to everyone,” presumably because it is important to attend to everyone’s mathematical ideas. In this case, Mr. Anderson noticed that many students in the class physically turned their bodies to face Domingo when he began speaking, indicating that they were likely listening as he shared his mathematical idea. This kind of attention is aligned with the norm that the class expects everyone to have mathematical ideas to which they should attend.

Summary. The norm that everyone has mathematical ideas to which you should attend seems to have been understood in different ways in the case-study class and to have been salient primarily – but not solely – in the context of the coaching practice. It appears that class members were expected to attend to each other’s mathematical ideas and to focus on what others do know and understand, even if the other person has an incorrect answer. This expectation seems to be salient for students when they are coaching other students on how to solve math problems; they expect to attend to the other student’s mathematical thinking and perhaps to learn from it. Additionally, Mr. Anderson expressed that he expected to attend to students’ “good” mathematical thinking and to help them build on that thinking when supporting them in learning from errors during coaching. This attention to students’ mathematical thinking was sometimes made clear during whole-class discussions, as in the examples above. Furthermore, student survey data suggests that most students in the class expected to try to understand another student’s mathematical thinking, even when the student got an answer to a problem incorrect. Therefore, all of these pieces of evidence together suggest that this norm indeed regulated behavior in the classroom; class members seem to have held the shared expectation that everyone had mathematical ideas to which they should attend.

Norm 2: There are different types of errors, only some of which are acceptable. This norm emerged in the classroom because the teacher identified a need to distinguish errors while learning, which were positioned as acceptable, and careless errors, which he wanted students to avoid. By the end of the school year, this distinction between types of errors had become a norm that helped class members regulate their reactions to different types of errors. Errors while learning were expected to be good for learning and to arise from productive struggle while
learning tough new concepts; these were treated as acceptable and often were responded to with coaching. *Careless errors*, which occurred when students were not careful in their responses and made errors that could have been avoided, were considered unacceptable if they occurred frequently and were treated as something to be avoided as much as possible. Typically careless errors were not addressed through coaching, or if they were, the coaching practice was abbreviated because the person being coached quickly identified their error and already understood how to solve the problem correctly. In this way, the distinction between careless errors and errors while learning supported class members in identifying how to react to mathematical errors depending on the type of error.

This particular norm was not apparent to me during my observations of the class, perhaps because during class discussions, only one of these types of errors seemed salient in any particular interaction. That is, I observed the teacher talking to the class about errors as acceptable, or I observed the teacher talking about avoiding making errors. These messages appeared contradictory to me until the teacher and students articulated during interviews the difference in acceptability between careless errors and errors while learning. Below, I present data from end-of-year interviews in which the teacher and students describe classroom expectations related to these different types of errors.

**Evidence from the teacher interview.** During our end-of-year interview, Mr. Anderson clarified the distinction between careless errors and errors that students make when they are legitimately confused while learning something new. He said that he was explicitly trying to teach students to understand the difference between these types of errors, though he understood that they could be difficult to distinguish at times. In the long excerpt from this interview transcribed below, Mr. Anderson explains why this distinction is important and how he tries to help students understand it.

*Interviewer:* How do you think your students are interpreting making an error on a math problem at this point in the school year?

*Mr. Anderson:* I think they could see it two ways. Sometimes it’s carelessness, and sometimes it’s a legitimate way of thinking about it that they got confused about. And I think they think it’s one of those two things.

*Interviewer:* Based on my interviews, that might be correct. Um, do you want them to think that way, or do you want them to think a different way?

*Mr. Anderson:* I do, because I think a lot of the work they do, they rush through, or they, you know, there’s a quality aspect to it. I was doing it with the exit tickets, you know talking to them about how it’s a form of a test. You know, getting them to refocus on how they think about that. Because they were kind of just dashing it off as fast as they could and they didn’t care if they got it right or wrong. And they were, you know, able to go back and fix it. They were losing the sense of, “I’ve got to do it right the first time.” And it’s a tightrope walk, you know, to think about there are times when you really need to get things right the first time. Um, and there are other times, and it’s also okay at the same time as you work on real difficult problems you’re going to need to make some mistakes. And try again. But knowing kind of when is when…

*Interviewer:* Is that something that the class has been working on?

*Mr. Anderson:* Yeah, it’s a time-and-place kind of issue. When do I really need to make sure it’s right, and when do I still need to make sure it’s right but it’s okay if I make a mistake and I can fix it.
Interviewer: So you’ve been talking about how it’s okay on really difficult problems to make mistakes…

Mr. Anderson: Yeah, it’s hard to talk about because it’s, it’s, it doesn’t even have to be real difficult problems sometimes because once you say it’s difficult, then it defines difficult for whom. So that’s the trick part of it. It’s messy. It’s nuanced. And there’s not a nice line to separate the two sides. Um, it’s more an attitude I’m trying to instill in them, in everything they do, they need to be thinking about quality. Doing your best. And then I can live with it. If you take an exit ticket as a test and you do your best, I’m okay. If you’ve checked your work, you do all the things you do when you do your best, fine. It’ll be right or wrong, and I can live with that. And then if it’s wrong I can help you and figure out why. If you’re, you know, but then conversely you’re not giving your best and doing sloppy work, it won’t matter how hard the problem is. You’ll get it wrong when it’s easy, hard, who knows what you’re doing. (Teacher Interview, End of Year, April 2015)

Mr. Anderson described himself as expecting students to “be thinking about quality” and “doing [their] best” in class. He said he “can live with” whatever the results of students trying their best was, so long as they were indeed trying. The implications of this expectation are what he describes earlier in the interview about how he expects students think about errors: that careless errors are to be avoided, but other errors come from grappling with difficult ideas while learning and are acceptable. For the sake of simplicity, I refer to the former as careless errors and the latter as errors while learning.

Evidence from focal student interviews. My interviews with focal students suggest that by the end of the school year students did think about errors as either careless errors or errors while learning. When I interviewed students towards the end of the school year, all five focal students brought up this idea of different types of errors, and they identified some errors as good for learning and the others as careless and to be avoided. Though they did not use the terms careless errors and errors while learning, their descriptions align with these terms as defined above. For example, Rowan and Maria both brought up this distinction in response to a question about what errors indicate about a person.

Interviewer: What do you think it means when someone gets the wrong answer on a math problem? Like what does it tell you about that person?

Rowan: Well, uh, it tells me that that person is like having, usually their thinking is a little off. And they just need to understand something more, or they need a better way to solve it. Or sometimes people just make silly mistakes, and yeah, uh, they just make silly mistakes and they realize it fast.

Interviewer: So let me make sure I’m understanding you right. It sounds like there are two different kinds of mistakes you’re talking about: one is when somebody doesn’t quite understand something yet, and that’s kinda different from when you make a silly mistake, like you added 2 plus 3 and got 6 or something like that.

Rowan: Yeah.

Interviewer: So those are different things.

Rowan: Yeah. And, uh, you’ll be able to tell which one is which because if it’s a silly mistake, they’ll realize it really fast. And if it’s something more technical like the strategy they used, then it’s more like they need help. They need a little help, they need more understanding.
Interviewer: Okay. And when somebody makes one of those silly mistakes, what does that tell you about them?
Rowan: Um, it tells me that well, one silly mistake can’t really tell you about a person, because if it’s someone that you know is like one of the smartest kids in the class and they make a silly mistake, like everyone makes silly mistakes. Like, it can’t really affect that.
Interviewer: So everyone makes silly mistakes.
Rowan: Silly mistakes, like if you make a lot of them, that could tell you that it might lead to something more technical and you might need more help.
Interviewer: So making a lot of silly mistakes might mean there’s a bigger problem with their understanding?
Rowan: Yeah. (Focal Student Interview, End of Year, April 2014)
Rowan described both errors while learning, which he said indicate that a person “need[s] more understanding” and also careless errors, which he called “silly mistakes.” Rowan insisted that careless errors “can’t really tell you about a person, because… like everyone makes silly mistakes.” However, he went on to say that if you make a lot of silly mistakes, “it might lead to something more technical and you might need help,” meaning that consistently making careless errors might lead to serious problems with understanding mathematics topics.

During my interview with her, Maria also called careless errors “silly mistakes,” and she also suggested that everyone makes such errors. She also distinguished these from errors that indicate that someone really doesn’t understand a problem.

Interviewer: What do you think it means when someone gets the wrong answer on a math problem? Like what does that tell you about the person?
Maria: Well, it tells me that they either don’t understand the problem, or they made an adding problem or a multiplying problem and they wrote the wrong numbers, so. Either they’re just like quickly doing it and they don’t pay attention to what they’re doing, or they don’t know how to solve those types of problems.
Interviewer: So it sounds like there are kind of two possibilities, they’re either…
Maria: Yeah. Just rushing through…
Interviewer: And making silly mistakes, or they really don’t understand how to solve the problem. So what does that mean when they really don’t know how to solve the problem?
Maria: When they really don’t know how to solve the problem it means that either they missed something that [Mark4] said about how to solve those types of problems.
Interviewer: So they might have missed something. What does it mean when they’re rushing and they…
Maria: When they’re rushing through and they make a mistake. It’s not that they don’t care, it’s just that they think they know what to do, and they you know I guess they’re writing, I don’t know, like 18 times 9 and they write 18 times 8. Some silly mistake they make that everybody has done at least once, I think. (Focal Student Interview, End of Year, April 2014)

Maria explained that she thought errors while learning indicated that a student may have “missed something” that the teacher explained about that type of problem, indicating that she expected these errors to occur when someone had incomplete understanding of a mathematical topic that

4 Students in the class referred to Mr. Anderson by his first name, Mark.
they were learning. She distinguished these errors while learning from careless errors, which she said come from someone “just like quickly doing it and they don’t pay attention to what they’re doing.” Later, she went on to say that “everybody has [made careless errors] at least once, I think.” Like Rowan, Maria drew a distinction between careless errors and errors while learning and described situations in which she expected each type of error to occur. Also, both Rowan and Maria were careful to say that everyone makes careless errors sometimes, so just making one does not necessarily indicate that a person is always careless.

Domingo expressed a similar distinction during his end-of-year interview, though he did so in the context of discussing what he would expect the teacher, Mr. Anderson, to do if he said he didn’t know how to solve a mathematics problem during a class discussion. On the end-of-year student survey, Domingo had selected a response indicating that if he realized that he didn’t know how to solve a problem that the teacher asked him to solve in front of the class, he would say he didn’t know how to solve the problem and ask for help. In the interview, I asked Domingo to explain why he chose this response option, and he explained that he expected that Mr. Anderson would help him learn how to solve the problem, unless it was a problem that he should have already known how to solve.

Interviewer: Now let’s take a look at number 3. [Interviewer reads question #3 from the survey and identifies that Domingo circled option D, which is that he would say he doesn’t know how to solve the problem and she would ask for help.] Tell me more about why you chose that.

Domingo: I picked that because, well, I wouldn’t want to guess because [Mark] would know I’m guessing. So I would say that I don’t understand it and I would ask [Mark] for help. And after that he usually either says “I’ll help you,” and he’ll do a different problem with me that’s similar, and I would try to do the same problem. And the other way, if it’s something really simple that he kept on teaching to the class, he would just ask me why I wasn’t paying attention or anything. (Focal Student Interview, End of Year, April 2014)

Here, Domingo expressed that he expected the teacher to help him learn how to solve a problem if he really did not understand it. He implied that in this situation, he expected that his lack of understanding of the problem would be seen as acceptable because he was in the process of learning that mathematics concept or procedure. He distinguished this type of error (or lack of understanding) while learning from the situation in which he did not know how to do “something really simple that [the teacher] kept on teaching to the class,” which he would be expected to understand already. In that case, he expected that his confusion would be seen as unacceptable, and it would be assumed that he hadn’t been “paying attention” when the concept was taught repeatedly.

Kai also made a distinction between careless errors and errors while learning during his end-of-year interview, though he did so more subtly than Rowan, Maria, and Domingo. When asked what the teacher meant during a classroom interaction shown on a video clip, Kai described the teacher as being okay with students making errors, but also expecting students to learn from their errors so that eventually they get correct answers:

Interviewer: Okay. Now we’re going to watch the same short video clip of your class that we did last time. [plays video clip] How serious do you think [Mark] was when he said that [referring to teacher saying “That’s okay honey, I’m glad your voice is out there, make all the mistakes you want!”]? Was he not at all serious, not very serious, a little serious, or definitely serious, he means what he’s saying? [puts rating scale paper in front of Kai]
Kai: He’s both of these [pointing back and forth to “A little serious” and “Definitely serious” on the rating scale paper]

Interviewer: A little serious and definitely serious? Tell me why that is.
Kai: Mmm hmm. Like in the middle of both of those, because he wants you to understand in a good way how to get the right answer. Like how to spell that word or do the math, um, multiples.

Interviewer: So when he says “make all the mistakes you want,” he does want you to be able do it right?
Kai: Yeah, he means like don’t make the same mistake over. You can make all the mistakes you want, but you have to learn from them.

Interviewer: Okay, so he wants you to try, and it’s okay to make mistakes while you’re learning, but then you need to be able to learn from them.
Kai: He wants you to, it’s okay if you make the mistake a couple of times, but you should learn from it. (Focal Student Interview, End of Year, April 2014)

Here, Kai indicated that he expected the teacher to find errors that students make while learning acceptable. He also expected, however, that the teacher did want students to “learn from them” and eventually “to understand in a good way how to get the right answer.” Here, Kai implied that after one has made “the mistake a couple of times,” one is expected to no longer make such mistakes. He did not explicitly say that once someone has learned a concept, errors of that type of problem would be viewed as “careless” and to be avoided, but he implies that this is the case.

The fifth focal student, Jordan, also made this kind of distinction during his interview, though not as concisely as the other students. At various times in the interview, he explained that he expected the teacher to help him with math topics that he was “struggling with,” but he also expected that sometimes students made careless errors because they “weren’t paying attention” or “fired before they aimed,” meaning that they called out an answer before thinking carefully (referring to the saying “aim before you fire,” which Jordan had discussed as a saying that the teacher uses often).

In general, all focal students expressed expectations that the two types of errors – careless errors and errors while learning – would be treated differently in the classroom. They expected that the teacher or other students would accept errors while learning as part of the learning process and would help the person who made the error gain more understanding of the topic. They also expected that though everyone makes careless errors, these were to be avoided generally and often occurred because people were not paying attention to what they were doing.

Evidence from student surveys. The student survey did not include any items or answer options that aligned well with this particular norm because I had not anticipated it at the time I was writing survey items. However, one answer option to the fourth survey question can be interpreted as aligning with this norm. The question asks what students expect the teacher to do if they are solving a problem in front of the class and they get stuck, meaning that they do not know how to continue the solution process. At the end of year, some students (48%) selected the response option indicating that they expected the teacher would ask them questions about their mathematical thinking. This response suggests that they expected the teacher to treat their confusion as acceptable and to help them work through it. However, some students (24%) selected the option indicating that they expected the teacher to “ask you why you weren’t paying attention earlier when he taught you how to do this kind of problem,” suggesting that they expected Mr. Anderson to view their confusion as unacceptable because they should have already learned how to solve that type of problem. Further, some students (12%) selected both
of these answer options, though I did not explicitly tell them that they could select more than one answer. This variation in how students responded to this item is consistent with the second norm in that students recognized that more than one type of response to errors may occur in the class. When interpreted using the more elaborated descriptions from focal students’ interviews, these survey responses are consistent with the norm that there are different types of errors, only some of which are acceptable.

**Evidence from classroom lesson video recording data.** This norm was not frequently apparent in whole-class interactions related to errors, though class members typically acted in ways consistent with the norm. More often, one or the other type of error was discussed; rarely was the contrast between careless errors and errors while learning made salient during the lessons recorded for this study. However, errors while learning were consistently treated as acceptable, and by the end of the school year careless errors were consistently treated as something to be avoided.

In one instance from the end-of-year video recordings of whole-class discussions, the teacher did make clear that mistakes while learning are acceptable, so long as they are not careless.

*Teacher:* Two and eleven twelfths is correct.
*Several students:* What? [sounding confused]
*Teacher:* Who’s got a question.
*Some students raise hands, and teacher calls on one of them.*
*Student:* How’d you get it?
*Teacher:* What did you do with this [pointing to a mixed number written on a paper on the board, visible to the whole class]? What fraction did you make it in to?
*Student:* Seven halves.
*Teacher:* How many of you did seven halves for that? [many students raise their hands] Okay, we’re good so far. [writes “7/2” on the paper] And then what’d you do?
*Student:* I did five sixths.
*Teacher:* Multiplied it by five sixths. [writes “5/6” on the paper] That’s what I did, too. What is five times seven? [pointing to the five in “5/6” and the seven in “7/2”]
*Some students:* Thirty-five.
*Teacher:* And what’d you get for the denominator? [writing 35 on the paper]
*Student:* Twelve.
*Teacher:* Good. [writing 12 on the paper] We’re all good so far. Then what?
*Student:* I doubled twelve, which is twenty-four.
*Teacher:* [pointing] Good, there’s our two, double.
*Student:* Ummm…
*Teacher:* And thirty-five minus twenty-four is?
*Student:* Oh, eleven.
*Teacher:* [nodding and smiling] Eleven. You did everything right, you just got stuck on this part. Nice job. Well done. So you know what you’re doing, that’s it’s important to know that you know what you’re doing, you just make a little subtraction mistake. But you gotta be a little more careful about that.

(Classroom Video Recording, End-of-Year Data Collection Period, Day 4)

In this instance, it is apparent that the expectation expressed by some students that the teacher would treat confusion about how to solve a problem as valid was correct; the teacher helped the student make sense of his solution by asking questions about how he solved the problem until the
student realized his own careless error. The teacher highlighted the “right” thinking and said that the student “know[s] what [he’s] doing.” However, the teacher also indicated that subtraction mistakes like the one this student made were careless errors that were to be avoided by being “careful.”

In another instance, the teacher indicated that tests were particular situations in which students should not make careless errors, such as leaving the units off an answer that should be expressed with units. The teacher suggested that students had been practicing including units, and by the time they took a test on this type of problem, it would not be acceptable to forget the units.

*Teacher:* [writing “6 ¼” on a paper taped to the whiteboard] Did you label the numbers? Did you put “inches” after it?

*Many students:* Yes.

*Teacher:* If you didn’t, it’s wrong. On your diagram, if you didn’t put “inches” there, I would mark it wrong on a test. Listen carefully. I’m looking around, and I can see ten people who don’t have it there. All you wrote was six and a quarter; you didn’t write inches. I would mark that wrong on a test. If you’re not labeling your diagrams correctly, you’re going to get them wrong.

(Classroom Video Recording, End-of-Year Data Collection Period, Day 4)

Here, we see that the teacher repeated that on tests, students were expected to know how to perform this type of mathematical task and he would not accept errors. Instead, he said he would “mark that wrong,” meaning that he would not assume that students meant “6 ¼ inches” when they wrote “6 ¼.” He made explicit that he expected students to know that they must include units and to do so consistently on tests. In this example, the teacher indicated that once students have had sufficient time to practice and learn a skill, errors would no longer be acceptable and were instead considered to be careless errors.

In addition to making it clear that careless errors were to be avoided, the teacher also made many statements indicating that mistakes while learning were acceptable. For example, in the instance quoted at length in the section on the *everyone has some mathematical understandings to which you should pay attention* norm above, the teacher explicitly praised a student for trying to work through a problem that he was learning to solve, even though he made an error in the process, saying “excellent job, [Student]. Super proud. You’re doing it all on your own. Make all the mistakes you want” (Classroom Video Recording, End-of-Year Data Collection Period, Day 4). This type of statement indicates that the teacher viewed errors as acceptable as long as students were really trying their best. Taken together, these classroom interactions suggest that the teacher, at least, discussed both careless errors and errors while learning as distinct types of errors during whole-class discussions. Though instances of students voicing this distinction during whole-class discussions were not apparent in the video sample analyzed, focal student interviews suggest that students in the class also understood these two types of errors as being different.

**Summary.** Evidence from the teacher and student interviews, student survey, and video-recordings of whole-class discussions suggests that the teacher and students in the class expected careless errors and errors while learning to be treated differently in the class. It appears that careless errors were expected to be avoided and treated as unacceptable, particularly on tests. However, students also indicated during interviews that they expected that everyone would make careless errors occasionally, even if they tried to avoid doing so. In contrast, class members seem to have shared the expectation that errors while learning were acceptable and are normal
during the learning process. Furthermore, they seemed to expect that the teacher or other class members would help them learn from these errors that they made while learning mathematics. This distinction between careless errors and errors while learning allowed class members to hold simultaneously the apparently contradictory values for accuracy and for learning from mistakes. This distinction also supported class members in knowing what responses would be expected to the different types of errors. Errors while learning were often treated with coaching, and careless errors were often quickly corrected.

The norms everyone has some mathematical understandings to which you should pay attention and there are different types of errors, only some of which are acceptable are described in detail here for three reasons. First, these norms exemplify some different ways that norms may be related to collective practices: the first norm provides an example of how a norm may be closely tied to a collective practice (in this case, the coaching practice), and the second norm illustrates how a norm may define situations in which a collective practice is used. Second, the elaborated descriptions of these two norms provide a sense of the type of evidence that exists for all seven norms identified in this analysis. Finally, the development of these two norms over the course of the school year is analyzed in the next chapter (see Chapter 6), so an in-depth description of these norms is particularly useful as foregrounding for that analysis. Below, I conclude this chapter with a brief description of the other five norms identified as related to the case-study class’s treatment of errors during mathematical discussions.

Norm 3: Multiple methods may be used to solve a problem. The norm that multiple methods may be used to solve a problem was expressed in the teacher’s end-of-year interview as well as in interviews with some of the focal students. This norm supported class members in expecting to analyze solution methods and to accept a variety of methods so long as they were mathematically correct. In general, students talked about there being more than one “strategy” or “way” to solve problems, and they mentioned looking for “better” or “faster” strategies for finding solutions. During lessons in the case-study class, this norm appeared evident when the teacher and students described more than one way to solve the same problem, often writing two or more solution strategies on the board. The teacher often asked “did anyone do it a different way?” and prompted students to share different solution strategies for the same problem.

This norm is less directly related to how errors were treated in the case-study class than some of the other norms described here. However, this norm came up repeatedly in student interviews as students described the value of attending to other students’ thinking, even when they got the answer to problems wrong, as described in the section on first norm above. It appears that the expectation that there were multiple ways to solve a problem was a basis for class members to attend to and look for value in each other’s mathematical ideas and solutions, even if those solutions were only partially correct or included one or more errors. Therefore, this norm can be viewed as a precursor or foundation for the norm everyone has some mathematical understandings to which you should pay attention.

Norm 4: You can and should learn from your errors to increase your understanding and also so that you don’t keep making the same errors. The expectation that one should learn from errors supported class members in responding to errors by correcting them and also striving to better understand the mathematical ideas so as to not make the same error again. This norm was salient in the coaching practice as coaches supported persons-being-coached in understanding the problem, and this expectation also came up in other contexts in the class. For example, this norm was often referenced in focal student interviews when students explained why they thought it was important to check their own answers to math problems. Many students
indicated that it was important to make sure that they were solving math problems correctly so that they would be able to figure out any errors they were making and correct those errors. In other words, they needed to know that they were making errors before they could learn from them. For example, Rowan, who is quoted above as thinking that checking answers is valuable for learning math, described a reason he thought that it was important to check his answers to math problems:

… if you, uh, get the answer wrong and you don’t fix it or anything, you don’t really learn from your mistakes. You just – you might be thinking, like maybe you’re using a certain strategy or something and you might be doing it the wrong way, and maybe you keep using that strategy and you keep getting answers wrong. (Focal Student Interview, End of Year, April 2014)

Here, Rowan indicated that learning from errors could mean that you do not make the same errors over and over again.

The norm that one should learn from errors is related to several other norms that seemed to regulate the case study class. The norm there are different types of errors, only some of which are acceptable, discussed in-depth above, is highly related to this norm because the errors while learning type of errors were seen as acceptable in the class only in the context of learning from errors. Furthermore, careless errors were seen as unacceptable in part because the class expected that you can and should learn from your errors.

Additionally, this norm is related to the norm described below that you should take risks while learning because in this class “risks” often included trying new things while learning, sometimes resulting in errors. These errors were seen as acceptable errors while learning, but students were expected to learn from them and to use them to increase their mathematical understanding. Relatedly, this norm is tied to the norm you should admit when you get an answer wrong because that norm suggests that getting help is an important part of learning from errors.

These norms are highly related and often interdependent. Together, they suggest that students were expected to be active in the learning process; students in the case-study class were expected to correct their mistakes, to seek knowledge, and to increase their understandings. That you can and should learn from your errors was expected to be part of this process.

**Norm 5: You should admit when you get an answer wrong** or don’t know how to solve a math problem so that you can get help and learn how to solve it, even if it’s scary or embarrassing. The idea that you should admit when you get an answer wrong supported class members in regulating behavior related to making errors public so that they could learn from these errors. This norm was apparent in the case-study class even from the beginning of the school year, and many students indicated that they would in fact publicly say when they got a math problem incorrect. At the beginning of the year, some students said that they did not like admitting when they get an answer wrong because it was embarrassing to them. However, by the end of the year students rarely mentioned feelings of embarrassment during interviews, instead discussing how important it was to acknowledge when they made an error or did not understand how to solve a problem so that they could get help from the teacher or another student. For example, in response to a question about why he had indicated on his survey that he would “definitely” admit that he got an answer wrong, Kai said,

Well, everybody will know that you have the answer wrong by seeing what you have on your whiteboard, and plus, why would you say that when you got the answer wrong? Just to say I got the answer right. I mean, nobody’s perfect. So, they might get answers
wrong every once in a while. I definitely will raise my hand if I got it wrong so that [Mark] can help me. (Focal Student Interview, End of Year, April 2014)

Kai clearly indicated that he thought it was important to acknowledge that he got an answer wrong so that he could ask for help. Other students expressed similar ideas during interviews.

The student survey also provides some information related to this norm. Two survey questions required students to indicate if they would admit in front of the class to getting an incorrect answer to a math problem or that they did not know how to solve a math problem. To the first of these (survey question #3), 96% of students in the class indicated that they would say they didn’t know how to solve a problem. Similarly, in response to a question of how likely students thought it was that they would admit that they got an incorrect answer, 96% of students said that they “probably” or “definitely” would do so. In general, there seems to be some consensus among the class that it is important to acknowledge errors and partial understandings in order to learn from them. This norm is clearly related to several others, including that you can and should learn from errors and there are different types of errors.

Norm 6: You should take risks while learning. The idea that you should take risks while learning supported class members in engaging in productive struggle while learning such that they were expected to try ideas that made sense to them even if they were not completely sure those ideas would prove useful or correct in problem solving. This idea was present in interviews and classroom video recordings even at the beginning of the year, and the teacher explicitly said that he valued students taking risks with their learning by trying things that are difficult. For example, in the video clip I presented during focal student interviews, the teacher said “Glad your voice is out there, honey. Make all the mistakes you want” after a student incorrectly called out an incorrect number during a times-tables practice activity. He praised her taking the emotional risk to put her “voice… out there,” even though she may not have been sure of her answer and indeed her answer was incorrect.

This idea remained fairly consistent throughout the school year. Students expected to try new things, to be challenged, and to make acceptable mistakes while learning when doing so. This type of intellectual risk-taking was seen as part of the active learning process that became part of the shared expectations of this classroom.

Norm 7: You should support others who are struggling. The norm that you should support others who are struggling regulated behavior related to errors and partial understandings by requiring that class members not “give up” on anyone who was having difficulty understanding a topic or solving a problem. Instead, class members were expected to support each other by using the coaching practice or other methods. This norm was expressed in interviews and classroom video recordings even at the beginning of the year. The teacher consistently coached students who struggled with mathematics ideas and encouraged students not to give up on themselves or each other. This expectation was closely related to the idea of learning from errors and seeking help when you do not understand a concept or procedure.

Students in the class clearly took up this idea, indicating in interviews and through the student survey that they expected the teacher not to give up on them but instead to help them when they struggled with math. For example, Jordan described “trusting” the teacher to help him if he admitted he got an answer incorrect:

44% of students responded that they would “probably” raise their hands and 52% of students said that they would “definitely” raise their hands to admit that they got an answer wrong. Only one student responded that he was unsure of what he would do.

5
Interviewer: Why would you admit that you got the answer wrong?

Jordan: Mmm, cuz it will help me.

Interviewer: How will it help you?

Jordan: Cuz [Mark] will prolly coach me. And he’ll ask me questions about why I did that, and then he’ll teach me about why that answer’s wrong, and I trust [Mark] to do that. (Focal Student Interview, End of Year, April 2014)

Jordan made it clear that he expected Mr. Anderson to help him if he got a problem incorrect. Other students in the class expressed a similar expectation in response to the first survey question, which asked what students expected the teacher to do if a classmate got the answer to a math problem incorrect. The majority of students (88%) selected the option that the teacher would “ask the student who made the mistake questions about how he was thinking about the problem,” indicating that they expected the teacher to work with students who make errors to help them realize their errors and build on their mathematical thinking.

The norm you should support others who are struggling can be seen as complementary to the norm you should admit when you get an answer wrong, which regulated what class members should do when they get math problems incorrect. The norm you should support others who are struggling indicates what other students and the teacher should do if another class member gets a problem incorrect. In other words, they should help that person learn. Furthermore, the idea of not giving up on oneself, which is also implied by this norm, is closely tied to the ideas of actively learning from errors and seeking help when needed by admitting to making errors. These norms therefore functioned together to regulate class members’ behavior related to making and learning from errors as a community.

Summary. These seven taken-as-shared expectations – norms – were identified through a grounded theory coding process. Open coding of focal student interviews from the end-of-year data collection period suggested some themes in students’ statements about their expectations related to mathematical errors in their classroom. These themes were taken as “candidate norms” and confirmed as appearing to indeed be norms in the case-study class through focused-coding of the teacher interview, classroom video recordings, and student survey data from the end-of-year data collection.

These seven norms appear to have regulated classroom interactions related to the treatment of students’ mathematical errors. Of course, many other norms also regulated behavior in the case study classroom, but these seven seemed specifically related to the treatment of errors. Furthermore, these seven were brought up by focal students during interviews, suggesting that they were salient to these students and therefore likely important to the class as a whole. These seven, highly interrelated norms are:

1. Norm 1: Everyone has some mathematical understandings to which you should pay attention.
2. Norm 2: There are different types of errors, only some of which are acceptable.
3. Norm 3: Multiple methods may be used to solve a problem.
4. Norm 4: You can and should learn from your errors to increase your understanding and also so that you don’t keep making the same errors.
5. Norm 5: You should admit when you get an answer wrong or don’t know how to solve a math problem so that you can get help and learn how to solve it, even if it’s scary or embarrassing.
7. Norm 7: You should support others who are struggling.
Many of these norms were closely linked, and some were supported by or dependent on others in order to function effectively in the case-study class. These norms are described in more detail above, and the first two are described extensively with examples of individual interview statements that suggest that the teacher and focal students held expectations aligned with these community norms. Examples from the classroom video-recording data are also provided, and where possible, evidence from student survey data is described as well.

In the next chapter, I describe evidence of how the first two of these norms, *everyone has some mathematical understandings to which you should pay attention and there are different types of errors, only some of which are acceptable* emerged over time in the case study class. Students and the teacher began the school year with certain expectations, and over time their interactions as a class shaped, shifted, and/or reinforced the expectations of individuals. This process seems to have created a set of common expectations for the class as a whole, namely the seven norms described in the current chapter. In the next chapter, I describe evidence of the kinds of initial expectations that individuals in the class seemed to hold and the interactions through which their expectations seem to have shifted and/or been reinforced over time in order to arrive at the shared expectations described in the current chapter.
Chapter 6: How Norms Emerged in the Case Study Classroom – Shifts in Interactions over Time

In this chapter, I present results addressing the second research question of this study: Over the course of the school year, how are norms that support discussions of errors established and negotiated by the teacher and students? I focus on elaborating how two of the norms described in Chapter 5 developed in the case-study class. The everyone has mathematical ideas to which you should attend norm is a good example of an expectation that was originally promoted by the teacher at the beginning of the school year, and that was then taken up slowly by the rest of the classroom community throughout the year. The norm there are different types of errors, only some of which are acceptable is an example of a norm that was not part of the teacher’s initial plan. Instead, the teacher began promoting the norm mid-year in response to what the teacher understood as an emergent problem: Students actions indicated that they overgeneralized the acceptability of errors in ways that led them to poor performances on assessments.

To provide some context for the development of these norms, I describe two related features of the classroom: routines for discussing values, and the development of the coaching practice. Common understandings of values, like honesty and integrity, are inherently related to students’ understandings of norms, including norms related to the treatment of errors. Therefore, understanding the character of classroom discussions of values is important for an analysis of emerging norms related to errors. Similarly, the coaching practice, which is described in Chapter 5 as well, was related to norms for the treatment of mathematical errors, so its development over time is presented here as well. I begin by describing these two classroom features – discussions of values and the coaching practice – and then I describe in detail the development of the two selected norms.

Classroom Discussions of Values

In this section, I present the classroom routines and techniques through which the case-study class discussed various values, such as honesty and integrity, because the promotion of these values seems to have influenced class norms related to mathematical errors (described in Chapter 5). Throughout the school year, the classroom teacher consciously promoted certain values, or aspirational qualities of character, that he framed as important for success and well-being. Students sometimes referenced various values, such as honesty, during mathematics instruction and during interviews for this study when discussing expectations for the treatment of mathematical errors.

To learn about classroom interactions related to values, I asked the teacher, Mr. Anderson, to describe how values came up and in what context they were discussed. Mr. Anderson described four main pedagogical techniques he had created as contexts to teach values in his class, and he confirmed that this instruction occurred generally at times other than mathematics instruction. He reported that the class held a “class meeting” once a week on Mondays, and during this time two techniques were used: a) quotations of the week, and b) people of the week. Two other techniques were used more sporadically as the teacher saw fit: c) posters, and d) “Monkey’s Fists.” Each of these techniques will be described in more detail below, based on Mr. Anderson’s report and on observations from classroom video data.
**Quotations of the week.** Mr. Anderson described holding a weekly discussion about a quotation he selected to share with students and to promote a particular value or character trait. These quotations were printed out and posted on a particular wall in the classroom, so they remained visible over the course of the school year. Some of these quotations clearly related to expectations for the treatment of mathematical errors; I noticed several times during student interviews that various focal students referenced such quotations. These students referenced these quotations in response to questions about their expectations for the treatment of mathematical errors, suggesting that at least some students linked these values to expectations for reactions to errors. For example, during our middle-of-year interview, Jordan linked one quotation to the practice of coaching other students who are struggling with a mathematics problem:

*Interviewer:* What does it mean to coach or help someone when they get the answer to a math problem wrong? What do you do?

*Jordan:* We have this quote, we have like a quote of the week, and one of the quotes was “Teach a man how to fish and he will have fish for the rest of his life. Give him a fish and he will not have fish for the rest of his life.” And what it’s saying is, if you just give them the answer – like I’m pretty sure everyone would know this, but like two plus two, well that’s four, that one’s done – then they wouldn’t know how to do it. And so – but if you teach them how to fish, then you would basically be teaching them how to do the problem, and then they could do problems like that again. And so you won’t have to keep giving them a fish.

*Interviewer:* So coaching them is more like teaching them how to fish.

*Jordan:* Yeah.

*Interviewer:* So showing them how to do the problem, helping them understand how to do the problem.

*Jordan:* Yeah, I usually try not to give them a fish, cuz, um – usually I have a bit of trouble not giving them a fish. For a while I didn’t understand what the quote meant. I didn’t understand what giving them a fish meant for a while. (Focal Student Interview, Middle of Year, January 2015)

Though Jordan misquotes the specific quotation he is referencing, he focuses on a correct meaning of the quotation, which he links to helping another student understand a mathematics problem rather than simply giving the person the answer to the problem. He uses this quotation to explain why he expects to react to another student’s error or partial understanding by helping them understand the problem. He references a “quote of the week” in the context of explaining how to engage in the coaching practice with students who are struggling with math. This type of reference suggests that students may have used these quotations and the values they exemplify to understand expectations for handling errors in mathematics as well as for understanding the coaching collective practice.

The teacher described the quotations of the week as part of the way he and the students in the class discuss values and life skills on a regular basis.

We start with this list at the beginning of the year of what makes a successful fifth grader. And it comes out of what kids are doing, like a kid is being flexible and I say that’s the kind of thing that makes you [successful]. Somebody who’s a flexible person, who’s willing to – How many of you like somebody like that? Like to be around somebody like that? That’s how we identify all of this stuff. All of this stuff. From the people to the quotes, to everything, it’s all about these are the examples we want to learn from.
Whether they’re famous people or not, people who have said important things, those are the bars that we want to reach for. (Teacher Interview, End of Year, April 2015)

Here, the teacher indicated that he had been trying to teach students how to be “successful fifth grader(s)” since the beginning of the year using various techniques, including quotations of the week. He gave the example of “being flexible” as a quality that helps fifth graders be successful, and he talked about how he and the students identified these qualities and discussed them through various examples, including the quotations of the week.

Further, the teacher described how quotations of the week were introduced during class meetings and the types of discussions the class had about these quotations.

**Interviewer:** Um, are the quotations every week?

**Mr. Anderson:** Yeah, every week. We have a class meeting on Mondays, and there’s a quote of the week.

**Interviewer:** And you talk about it?

**Mr. Anderson:** Yeah. The kids first try to figure out what it means, and we have kids explain it in three different ways, and then we talk about how it applies to them, the world. (Teacher Interview, End of Year, April 2015)

Mr. Anderson’s description makes it clear that students were expected to think deeply about the quotations and their implications. These quotations of the week became shared knowledge among members of the class. These quotations were sometimes referenced by students and by the teacher during lessons and interviews for this study. They appear to have been one way that members of the class referenced the values that were discussed during class meetings and that seem to have influenced classroom norms related to errors.

**People of the week.** In addition to a quotation of the week, the teacher shared with students a “person of the week” during class meetings, and like the quotations, these people served as examples of the values that were discussed in the class. In the interview at the end of the school year, the teacher described the reasons for discussing these selected people and how he chose people to discuss.

**Interviewer:** And, let’s see, what else. The various role models you have up here.

**Mr. Anderson:** Yeah, we do a person of the week, too.

**Interviewer:** Oh, you do a person of the week. So, those are people from all sorts of different backgrounds.

**Mr. Anderson:** The idea that started this was, we’d been talking about equity, and I was thinking about how for a lot of kids, the only time they see somebody who looks like them is when somebody’s done something terrible. And they don’t get to see people who look like them doing regular jobs, or great important things, other than the heroes they study once a year and all that kind of stuff. So I started combing the news and looking for things that are happening here and now that were women, that were people of color, or you know white people every once in a while, but just you know sometimes they’re famous, sometimes there’s just people, you know [brief discussion of some specific people he has selected to be the person of the week]… People from all different walks of life, different genders, different spaces on the gender spectrum, just so they can, you know, what I wanted was for them to see people and go, oh, that looks like me, and they’re a lawyer, they’re a doctor, they’re a bus driver, they’re whatever. (Teacher Interview, End of Year, April 2015)

Here the teacher indicated that he intended for the person of the week to be a person that students could relate to as a role model, but not a person whose accomplishments seemed so extraordinary
that students would feel intimidated. He described looking through local and national news for people who had done interesting things and to whom the students might relate. As each person was discussed in a class meeting, his or her photo and brief biography were posted on a wall in the back of the classroom. These people of the week included famous people and also many less well-known or “regular” people, including a local city councilwoman and several local social-justice advocates.

Students less frequently mentioned these “people of the week” during interviews or during mathematics lessons, but it is likely that the discussions of these role models included talk about values, such as honesty and integrity, that may have influenced class members’ expectations for their own behavior and that of others in the class during mathematics discussions.

**Posters.** The teacher also used posters about norms and values as topics of discussion during lessons. As is noted in Chapter 3, posters appeared on the walls of the classroom over the course of the year. Some of these posters were designed to communicate messages about values and expectations in the class. See Figure 9 below for a picture of some of these posters. In particular, several caught my eye as likely being related to class norms for the treatment of errors. They read: “Practice makes progress,” “own your mistakes,” and “mistakes = information.”

![Figure 9. Posters on and around the classroom whiteboard. This figure is included in Chapter 3 and reproduced here for reference.](image)

In our end-of-year interview, the teacher described having a “stack” of these posters, which he used in a spontaneous way. He described the posters as “com[ing] up when they come up,” (Teacher Interview, End of Year, April 2015), indicating that he would pull out a poster and talk about it with the class when a situation occurred for which the message of the poster was appropriate. He then put the poster up somewhere visible in the classroom as a reminder of this
message. It appears as though the posters were used as visual reminders of various expectations in the classroom, including several related to making errors, as in the case of those listed above.

**Monkey’s Fist.** The fourth technique that was used in the case-study class to teach or reinforce values was referred to as the “Monkey’s Fist.” This technique may be somewhat less related to norms for the treatment of mathematical errors than the other three techniques described above, and – perhaps for that reason – I had been completely unaware of this technique until the teacher described it in our end-of-year interview. The Monkey’s Fist is actually a knotted rope that is worn around the neck; I had noticed the teacher wearing one throughout the year and several students wearing them later in the school year. However, the significance of these ropes was not apparent until the teacher discussed them during the interview.

Mr. Anderson described these ropes as signifying that the people wearing them were working on some aspect of their character that was difficult for them. For example, the white rope indicated that the person wearing it was working on being more independent, whereas a blue rope indicated that the person had chosen some other challenge to work on, such as being honest.

Teacher: … So there’s that, and then there’s our Monkey’s Fist stuff -
Mr. Anderson holds up the knotted rope he wears around his neck.
Teacher: where we talk a lot about, like our white monkey’s fist these.
Mr. Anderson holds up the knotted rope again.
Interviewer: Oh. What are those?
Teacher: These are symbols of um kind of working on something that’s challenging for you. The idea is wearing one means that you have committed to working on something that’s difficult for you. It’s not an award, it’s not meant to be something like that. I tell them it’s more like an advertisement that there’s something wrong with you. It’s like, “I have this huge struggle. I want everybody to know I’m struggling with something.” So the first one, there’s a white one, that’s all about independence. So they work on being more independent, when they’re showing that they’re making it a habit they earn one of those. Then there’s a blue one they’re working on right now, that they’ve chosen the challenge. It’s another level of it. They choose something, open it up for the class to know, “I struggle with this.” They make it public. And for the purpose of everybody supporting, for everybody to know we’ve all got our struggles, every one of us is working on something.

Interviewer: They choose to or not?
Teacher: They choose to or not, yeah. It’s completely optional. Um, some of them are working on it more than others. (Teacher Interview, End of Year, April 2015)

In this segment of the interview, Mr. Anderson described the purpose of the Monkey’s Fist as being “an advertisement” that a person is “struggling with something.” He indicated that the “something” with which someone is struggling is a character trait or value that they want to take up but find difficult to enact, such as independence or honesty. He wore a Monkey’s Fist rope throughout the year, apparently as an example to the students that he is constantly working on improving his own character. Mr. Anderson also indicated that students could choose to try to earn a Monkey’s Fist by showing that they were working on being more independent or some other character trait, and some students seemed highly invested in the Monkey’s Fist.

One instance recorded during data collection for this study showed the teacher teaching students about the Monkey’s Fist by praising one student’s behavior as an example of the type of behavior that, if it becomes a habit, might earn a student a Monkey’s Fist rope. This instance
occurred at the beginning of a whole-class discussion, after the teacher had just called students over to the rug area of the classroom. Most students sat down on the rug, but one girl remained standing, as did Mr. Anderson.

*Teacher: [tapping a girl on the shoulder]* Why weren’t you sitting like the rest of them?
*Girl: Because you were standing.*

*Teacher: That’s a Monkey’s Fist decision. It must have felt very uncomfortable standing there when everyone else was sitting, huh?*
*Girl nods.*

*Teacher: That’s what a Monkey’s Fist decision feels like. It’s not easy. Are you getting it?*
*Some students: Yes. Yeah.*

*Teacher: Cuz I haven’t seen it a habit yet. Which is why nobody’s wearing one yet. I see it every once in a while, but I’m not seeing anybody making it a habit. Okay?*
*Some students: Okay.*

*Teacher: [holding up a Monkey’s Fist rope] It’s not a present. It’s not an award. One thing that I didn’t tell you about this that I want to make sure I say is that when somebody earns one of these, never think it means they’re done. The fact that I’m wearing one [teacher motions to the Monkey’s Fist rope he is wearing] doesn’t mean I’m like an expert at something and I don’t have to work on it anymore. It’s the opposite for me. Wearing this [gestures to Monkey’s Fist rope again] means I have something I struggle with. It’s almost like an advertisement, “Hey, everybody! I’m struggling with something.” And don’t think of it like a trophy, like I won something. Think of it as, “I’m serious about working on something, and this is my reminder.” (Classroom Video Recording, Middle-of-Year Data Collection Period, Day 3)

During this interaction, Mr. Anderson used a student’s decision to stand up rather than sitting down as her classmates were as an example of taking a risk, being independent, and figuring out what to do based on logic rather than what everyone else was doing. Mr. Anderson praised the girl’s decision and pointed out that it felt uncomfortable for her to do something different from the rest of students, even though she thought it was the right thing to do. It is apparent that the value for independence was something that Mr. Anderson was trying to encourage students to take up, and the Monkey’s Fist was a way to reinforce independence. Many students in the class were wearing Monkey’s Fist ropes by the end of the school year, suggesting that they were invested in the technique by that point.

Along with the quotations of the week, people of the week, and posters hung on the classroom walls, the Monkey’s Fist technique reinforced to students various values that seem to have been taken up in the class as characteristics that students and the teacher were working towards. The Monkey’s Fist technique also indicated that the teacher and students did not necessarily expect that living these values would be easy or free from mistakes; rather, the Monkey’s Fist indicated that living according to these values was a constant challenge that class members accepted. In this way, the Monkey’s Fist technique seems thematically linked to the class’s norms related to errors. Furthermore, the values that students chose to work on, such as independence and honesty, had implications for how they might expect to react to mathematical errors. *Honesty*, for example, would imply that students expect to try to be open and honest about their own errors and partial understandings.

**Relation of classroom discussions of values to norms for the treatment of mathematical errors.** Together, these class techniques for teaching and reinforcing values seem
to have supported common understandings of values that class members were expected to work towards, having implications for norms for classroom behavior. Focal students’ statements during interviews suggest that they associated these values with the treatment of errors in the class, and certainly values like honesty and integrity are related to the recognition and treatment of errors. Some of these values or character traits were listed on the board at one point during the year as shown in Figure 10. The values shown were: flexibility, trust, problem-solving, courage, perseverance, responsibility, initiative, integrity, and patience. These appear to be just some of the values or character traits that were discussed in the classroom over the course of the year.

![Figure 10. Values listed on the board in the case-study class.](image)

Many of these values appear logically related to classroom norms related to mathematical errors in that recognizing and learning from errors requires many of these traits. For example, recognizing one’s own errors can require integrity and courage, and learning from errors can require patience and perseverance. Furthermore, the teacher and students seemed to link these values to expectations for the treatment of mathematical errors; they sometimes brought up these values during interviews focused on expectations related to mathematical errors.

This description of the classroom techniques that were used to teach and reinforce these values is intended to provide some context related to how specific norms developed over time in the case study class. Below, I also describe the emergence of the coaching practice. This description is provided as further context related to the development of classroom norms. I then discuss the emergence of two norms in detail. These two norms, everyone has some mathematical ideas to which you should pay attention and there are different types of errors, were taken up by the class over time through interactions that often seemed related to the values the class was focused on. Evidence for how the two example norms emerged is discussed in detail below.

**Emergence of the Coaching Practice**

As described in Chapter 5, the case-study class members often engaged in a specific practice called coaching that was used to support people who were struggling, and this practice seems closely tied to norms for the treatment of students’ errors. The classroom teacher
originally designed the concept of the coaching practice before the school year began, but the actual practice developed over the course of the year as the teacher and students engaged in the practice and discussed and refined specific aspects of it. An example of one such discussion is described in Chapter 5. In general, it appears as though the students began the school year with a vague notion of coaching as simply helping another student who was struggling with a mathematics problem, but over the course of the year students refined this understanding to include a few key features. Specifically, they came to identify the practice of coaching as meaning:

• The coach helps the person being coached to find the answer to a problem on their own rather than telling them the answer.
• The coach checks that the person being coached understands of the problem.
• The coach helps the person being coached to go through the steps of the problem, step by step.
• The coach asks the person being coached questions to help them think through the problem.

These particular expectations for enactment of the coaching practice differentiate it from simply helping another student who is struggling.

The coaching practice also seems to have been regulated somewhat by behavioral systems in the classroom. For example, students were expected to finish their own independent practice work, to have that work checked for accuracy by the teacher, and then to receive permission from the teacher before coaching other students who were struggling with the independent practice problems.

In this section, I describe the emergence of the coaching practice over the course of the year. I describe students’ interview statements indicating their understandings of the coaching practice at the beginning, the middle, and the end of the school year to illustrate how the focal students’ understandings of and expectations for coaching developed over the school year. The focal students’ understandings of coaching cannot be taken as representative of those of the entire class, but they provide some indication of the gradual development of the coaching practice over the year.

**Students’ descriptions of the coaching practice at the beginning of the year.** Evidence from interviews with focal students suggests that they began the year either with vague notions of the coaching practice as simply helping or with and understanding of coaching as a somewhat more specific practice involving asking questions. Their descriptions of the practice at the beginning of the year were generally far less detailed than their descriptions at the end of the year. For example, at the beginning of the school year Rowan seemed unsure of how to answer a question about what he would expect to do when coaching another student:

*Interviewer*: What does it mean to coach or help someone when they get the answer to a math problem wrong?
*Rowan*: It means to give them support, and, uh.

*Interviewer*: So what’s that like? What would [Mark] do if he was coaching somebody?
*Rowan*: He’d get them part of the problem done. He’d tell them, like, “You have to do this. So what do you do next?” (Focal Student Interview, Beginning of Year, September 2014)

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6 Students called the teacher by his first name, Mark.
In this excerpt from the beginning-of-year interview, Rowan was unable to describe specific aspects of the coaching practice. He indicated that he had seen the classroom teacher coach other students, and he seemed to have a sense of the teacher asking those students questions. He did not, however, verbalize specific features of the coaching practice.

On the other hand, a short time after the school year began, Maria had already begun to identify key aspects of the coaching practice, specifically that it involved asking a person questions to help them figure out how to solve a math problem. In her beginning-of-year interview, she voiced this feature clearly:

_Interviewer:_ What does it mean to coach or help someone when they get the answer to a math problem wrong?

_Maria:_ It means that you don’t directly tell them the answer, like “Oh you have to do this, and this, and this.” It means you like ask them questions about how to solve it so that they get the idea of “Oh yeah, you have to do this and this.” And then I might say, “Yeah that’s correct.” And they just do what they thought about it.

_Interviewer:_ So you ask them questions to help them figure it out.

_Maria:_ Yeah. Instead of just directly giving them the answers. (Focal Student Interview, Beginning of Year, September 2014)

Here, Maria clearly describes asking a student questions while coaching them to help them figure out the correct way to solve a problem. She has already identified this feature of the coaching practice, and she also understands that she should not “directly tell them the answer.” Maria’s description of the coaching practice at the beginning of the school year was by far the most detailed of the focal students’ descriptions. In general, most focal students had few ideas about specific features of the coaching practice at the beginning of the school year.

**Students’ descriptions of the coaching practice at the middle of the year.** By the middle of the year, focal students seemed to be developing more ideas about the coaching practice, though they did not yet express detailed understandings of the features of the practice. At this point, all focal students indicated that a coach does not tell the person being coached the answer to a problem. Additionally, Maria continued to identify asking questions as a central feature of the coaching practice. Another feature of the coaching practice also came up in middle-of-year interviews with two other focal students; both Rowan and Domingo reported that coaching involves taking the person through the math problem step by step. The fourth feature – checking for understanding – was discussed by Kai during his interview. It seems as though focal students were continuing to develop their understandings of the features of this practice by the middle of the school year.

**Students’ descriptions of the coaching practice at the end of the year.** At the end of the school year, focal students’ statements during interviews indicated that they had developed their ideas about coaching even further. Not only did all four features of the coaching practice described above emerge during these interviews, but also multiple students brought up each of these features, suggesting that these features were becoming common expectations for this practice. The focal students’ descriptions of the coaching practice were also much more detailed at the end of the year, often including specific examples of what they might say or do while coaching a peer. For example, Jordan described specifics of what he would do when coaching a classmate:

_Interviewer:_ What does it mean to coach or help someone when they get the answer to a math problem wrong? What do you do?
Jordan: I usually, when they’re stuck on a problem, I usually—not I help them figure out the problem. Like I ask them questions about it. Like usually I’ll say, “Okay, so you did this, now what do you think is the next step?” And I’ll see what they think and then if they get it wrong, I’ll correct them. And once we’re done with that problem, I’ll usually like give them a different problem, just like that problem except with different numbers. And see if they can figure it out, and then I’ll know if they get it. (Focal Student Interview, End of Year, April 2015)

Here, Jordan’s description includes specific phrases he might use while coaching other students. Similarly, Rowan described how he would coach another student in much more detail than in previous interviews:

Interviewer: Alright. What does it mean to coach or help someone when they get the answer to a math problem wrong?
Rowan: First I like look through what they did, and I try to find the mistake. And then I go through it with them and help them realize the mistake.

Interviewer: How do you help them realize the mistake?
Rowan: I kinda go through, “Oh, so now you divided [hand motion like turning the page], and do this [hand motion like turning the page], and this” and I kinda ask them questions so like, or more like I kinda—how do I put this—I kinda like ask them, “What’s next? What do you do next?” And if it’s something that they should have subtracted but they added, I kinda help them with that, and tell them why. (Focal Student Interview, End of Year, April 2015)

Here, Rowan also mentioned specific things he might say when coaching another student. He also identifies multiple features of the coaching practice including asking questions, focusing on understanding (“tell them why”), and going through the steps of the problem (“What’s next? What do you do next?”). Other focal students also gave specific descriptions of what they would do to coach other students, suggesting that their experience with and understanding of this practice had developed over the school year.

Types of interactions related to development of the coaching practice. It seems that several types of interactions in the classroom contributed to the students’ developing understandings and use of the coaching practice. The teacher described how he attempted to teach the students the coaching practice in our end-of-year interview. He described explicitly talking to students about coaching, modeling coaching for students, having students role-play what coaching is like, and giving students specific language to use while coaching or being coached.

Interviewer: And how satisfied are you with how [students are] asking each other for help?
Teacher: I’m mostly very satisfied with that. I think they do a pretty good job of choosing the right people to ask. And it’s not always the same person. We’ve done a lot of work on that. About who you get to coach you. And I think the coaches do a pretty good job. We make kind of a big deal over what it feels like to be the coach, what it feels like to be coached.

Interviewer: So you’ve given them some really explicit instruction on that.
Teacher: And how important it is to—that if someone asks you to coach, that’s an expression of trust. That they believe that you’re really going to help them, and that’s a compliment. And same way the other way, that if you’re being coached, this is a person who’s willing to help you and that’s a compliment. So I think they generally do a really
good job of that. Again, there’s a few kids – about three – who go to other people who don’t know anything. They’re really afraid to branch out of that, and it’s partly that they feel insecure. And they feel like they don’t want to show that they can’t do things – it seems to be all the time – and that must be really frustrating for them. And they don’t want to always be the one to have to ask. But other than that, I think I’m very satisfied.

_Interviewer:_ So when you were teaching kids how to ask for coaching, how to coach each other, obviously you talked to them explicitly about it, how else did you teach them to do this?

_Teacher:_ We did a little role playing, a little fish-bowling. You know. Bring everybody around somebody who’s done it, so you know, how’d this happen. They talk through it. Carry on. That’s how you do it.

_Interviewer:_ I’ve also seen you kind of model it, I think. Like what it would look like.

So you coaching.

_Teacher:_ With myself? Yeah. Oh yeah.

_Interviewer:_ So modeling, and role playing.

_Teacher:_ I give them a lot of language around that kind of stuff, just give them explicitly what to say. Go try it, do it with me. (Teacher Interview, End of Year, April 2015)

In this excerpt, the teacher clearly describes using several methods to support students in understanding and developing their use of the coaching practice. He talks about directly discussing with students how to ask for coaching and what language to use while coaching, and he talks about modeling what coaching looks like and having students role-play what coaching is like in order to practice the skills and procedures involved. These methods seem to have supported students in developing the understandings of the coaching practice that focal students expressed in their interviews.

Importantly, the coaching practice was used during mathematics instruction and for supporting students with struggles with mathematics, but it was also a part of the general classroom practices applied across subject areas. The students therefore had opportunities to practice coaching throughout the school day, and the teacher talked to them about coaching frequently. In our end-of-year interview, Mr. Anderson made clear that he viewed coaching as a central practice in the classroom:

_Interviewer:_ And you said that they are very good at coaching each other. Do you have ideas about why?

_Teacher:_ It’s something I teach in everything. Coaching comes across everything.

_Interviewer:_ In other subjects you mean.

_Teacher:_ In every subject, in life, everything. When kids are struggling with personal problems, you need a coach. We call it coaching, we talk about coaching, we talk about how to do it.

_Interviewer:_ What it means.

_Teacher:_ Yeah, we talk about everything. You know, and when it happens, I make an effort to say, you know, did you thank the person that coached you? Did you thank the person that let you coach them? Do you remember that that’s a risk for people? All that. It’s part of our culture. (Teacher Interview, End of Year, April 2015)

In this excerpt, the teacher described coaching as a practice that class members talked about and used frequently, in mathematics as well as in other subjects and “in life, everything.”

Students seem to have had many opportunities to observe and practice coaching. In fact, during interviews all focal students reported observing the teacher coach other students in
mathematics class at least daily. Focal students reported varying frequencies of actually coaching other students themselves. By the end of the year, all focal students reported having some experience with coaching other students, though some students—particularly Jordan and Kai—reported rarely having the time to coach other students during mathematics class because they were often too busy doing their own work. Other students reported coaching classmates far more often. In general, this practice seemed common in the case study class, and students seemed to at least have familiarity with it by the end of the school year.

The development of the coaching practice over the course of the school year doubtlessly impacted class norms for the treatment of errors. Because this practice is intended to help students who make errors or struggle with understanding, the specific features of the practice have implications for classroom expectations for how to handle errors. Below, I describe in detail the development over time of two of these norms, the everyone has some mathematical ideas to which you should pay attention norm and there are different types of errors, only some of which are acceptable norm. Both of these are closely related to the coaching practice. In fact, the everyone has some mathematical ideas norm was often apparent during coaching interactions in the classroom. Throughout the discussion of the development of these norms over time, I reference relations of these developing norms to the coaching practice, as well as to the classroom values described above. The elaboration provided here of the coaching practice and techniques for promoting classroom values is intended to support understanding of the community features that influenced the development of these particular norms.

**Emergence of the Everyone Has Some Mathematical Ideas to which You Should Pay Attention Norm**

The norm everyone has some mathematical ideas to which you should pay attention, which is described in Chapter 5, is a good example of a norm that was taken up gradually over time in the case study class. Although the teacher described his intention to focus on students’ “good math, the strong math that they really do know,” (Teacher Interview, Beginning of Year, September 2013) at the beginning of the school year, many students did not seem to take up this idea until later in the school year. References to this norm were not apparent in students’ statements during interviews until the middle of the school year, and most students did not indicate that they took up this idea until the end of the school year interviews. It seems that this expectation originated with the teacher’s intention and slowly become a norm in the classroom over time.

In this section, I describe evidence for the development of this norm in the classroom over time. First, I describe evidence from classroom interactions, which generally show that the teacher models this norm throughout the school year, particularly when publicly coaching a student through a math problem. Students’ own actions related to this norm are less apparent in classroom video data, possibly because students were rarely observed to coach each other during recorded whole-class discussions and this norm seems linked to the coaching practice. I then describe student interview and survey data, which suggests that students slowly took up this idea of focusing on correct mathematical ideas. Taken together, this evidence suggests that the teacher’s repeated modeling of this norm and explicit statements related to it seem to have lead to students taking up the idea over time until it became taken-as-shared by the class.

**Evidence from video recordings of classroom interactions throughout the year.** The everyone has mathematical ideas to which you should pay attention norm was apparent in the teacher’s statements and actions during whole-class discussions throughout the school year. Few instances of students’ actions related to this norm were evident, possibly because the whole-class
discussions, which are the focus of this analysis, were very much teacher-led in this particular class. Discussions generally involved back-and-forth talk between the teacher and students; rarely were students seen talking to each other publicly in this class’s whole-class discussions except during brief periods of partner-talk, which is difficult to make sense of on video recordings because many students are talking at once. It is possible that students engaged in behavior related to this norm when working in small groups or partners, but during interpretable parts of whole-class discussions in the data collected for this study, only the teacher’s actions clearly showed any relation to this norm.

The finding that the teacher was most often observed to publicly attend to and draw others’ attention to students’ mathematical ideas is consistent with the finding that most errors that came up during classroom interactions in the classroom video recordings were identified as errors by the teacher (see Chapter 4). The teacher was typically acting as the authority in class discussions, and any recorded instances of coaching typically involved the teacher publicly coaching a student. The video evidence therefore includes many examples of the teacher modeling the norm *everyone has some mathematical ideas to which you should pay attention*, and the degree to which students had taken up this norm is unclear from recorded classroom interactions.

In fact, the teacher’s actions related to this norm were fairly similar in instances identified from all three data collection time-points. At all time-points, the teacher can be observed (a) recognizing students’ correct mathematical thinking when they were struggling or while he was coaching them, (b) publicly praising and sharing students’ correct mathematical strategies, and (c) explicitly talking about focusing on what was correct in students’ solutions or strategies.

The teacher was often observed to coach students who were struggling or who solved mathematics problems incorrectly during whole-class discussions, and during these episodes he often pointed out these students’ correct mathematical ideas. For example, during a whole-class discussion of a problem in which students were to compare the number 15.202 and 15.21 to determine which is bigger, a student raised her hand and said she was confused.

Mr. Anderson: Who’s confused?
A few students raise their hands.
Mr. Anderson: Michelle, what’s going on?
Michelle: I don’t get it because, I got the last one right but I don’t get it because [some discussion of her reasoning on the last problem the class solved, which involved lining up place values of 0.7 and 0.705 and determining that 0.705 was bigger], because you put fifteen point twenty-one as bigger…
Mr. Anderson: Oh, because you’re thinking it’s about how many digits they have.
Michelle: Yeah.
Mr. Anderson: It’s not. It’s about which number has a bigger value in each place. So you’re thinking this one [gesturing to “.705” written on the board] has three numbers, this one [gesturing to “.7” written on the board] only has one, so it’s [gesturing to “.705” again] bigger. This one has five numbers [gesturing to “15.202” written on the board], this one only has four [gesturing to “15.21” written on the board], so it [gesturing to “15.202” again] should be bigger.
Michelle: Oh, cuz the one is bigger than the zero.
Mr. Anderson: That’s a common thing that people think. And you know something about how when numbers have more digits, they are usually bigger. And that’s true on the whole number side [gesturing to the whole number side of a place value chart]...

[some discussion of whole numbers and place value followed by a guided comparison of 15.202 and 15.21 comparing the digits in each place value]

Mr. Anderson: It’s not about how many digits it has; you’ve got to compare the places. You just cleared that up for a lot of people. Thank you for being so brave and asking.

(Classroom Video Recording, Beginning-of-Year Data Collection Period, Day 2)

In this beginning-of-year interaction, the teacher responds to Michelle’s question by coaching her through how to solve the problem. While coaching he points out her correct mathematical understandings about whole numbers and place value, saying “you know something about how when numbers have more digits, they’re usually bigger. And that’s true on the whole number side.” He demonstrates how this understanding does not work the same way for decimal numbers, but he places emphasis on why the student’s reasoning made sense based on correct mathematical knowledge. Interactions such as this one were common in the case study classroom throughout the year. The teacher often coached students in front of the whole class, and while doing so he often focused on their correct mathematical thinking.

The teacher also publicly praised students’ correct mathematical strategies throughout the school year, particularly drawing attention to the mathematical strategies of students in the class who often struggled with mathematics. For example, during a whole-class discussion in the middle-of-year data collection period, Mr. Anderson instructed students to begin solving the problem “What is five-sixths of twenty-four?” individually by drawing a diagram on their individual whiteboards. While walking around the room to look at students’ work, Mr. Anderson noticed a particular student was using a useful strategy to represent and solve the problem. She had created six columns on her board and was drawing twenty-four circles in a systematic fashion so that first she drew one circle in each column, then a second circle in each column, and so on. Mr. Anderson praised this strategy as “smart” and demonstrated its utility to the whole class:

Mr. Anderson: [walking around the carpet area, where students are seated on the floor working on a division problem on their individual whiteboards] That’s smart, young lady. [stopping in front of a student named Daysha] That’s smart. That’s super cool.

Daysha started with this. [draws on the board a diagram with six columns similar to the one Daysha had been drawing on her board] Then she started putting in her circles [begins drawing one circle in each column] One, two, three, four, five, six. [begins drawing a second circle in each column] Eight, nine, ten, eleven, twelve. [continues drawing circles] … eighteen. [mumbles] twenty-three, twenty-four. Did she make six sets?

Some students: Yeah. Yes.

Mr. Anderson: Yes. What does this set represent? [gestures to the first column of circles on his drawing]

Some students: One sixth.

[more teacher-lead discussion of the use of this representation to solve the problem]

(Classroom Video Recording, Middle-of-Year Data Collection Period, Day 6)

In this instance, Mr. Anderson’s public praise of Daysha’s mathematical strategy shows a clear focus on students’ correct and useful mathematical strategies. Furthermore, because Daysha was
a student who typically struggled with mathematics, Mr. Anderson’s focus on her “smart” strategy positioned her as having useful mathematical ideas, even though at other times she and other students in the class may have focused on her difficulties with mathematics, which were often more salient than her correct ideas. In this particular instance, Mr. Anderson focused instead on her correct ideas, effectively highlighting that all students – even those who struggle – have correct mathematical ideas and can contribute useful strategies. The teacher often used this technique to draw students’ attention to others’ correct mathematical ideas.

Throughout the year, the teacher also directly talked about focusing on the correct aspects of a student’s strategy, even if the student made a mistake while using the strategy. For example, during one beginning-of-year whole-class discussion that occurred during students’ independent practice time, the teacher focused on students’ correct strategies and explicitly talked about ignoring their errors at that time. The teacher had collected several students’ notebooks and projected the students’ work onto the whiteboard using a document camera so that all students could see these students’ strategies:

Mr. Anderson: And pause [working on the problems]. Look at these strategies for organizing the work.

Mr. Anderson puts a student’s notebook under a projector so that the page is projected for all students to see.

Mr. Anderson: Quickly made a place-value chart so it’s easy to tell, and then numbered them, put them in order like that. Mathematicians love that, when you put things in order with numbers – one, two, three, four. Here’s another way to show the same thing.

Mr. Anderson puts Andy’s notebook under the projector.

Andy: [Mark], I did something wrong.

Mr. Anderson: So look how he wrote them out first just using initials with the numbers right next door.

Mr. Anderson pauses to praise another student for paying attention.

Mr. Anderson: And then he checked off which one, and as he figured out which one was the best, he put that there.

Mr. Anderson gestures to Andy’s page, projected for the class to see.

Mr. Anderson: And then he wrote out the names and put the numbers right next door to them.

Some students make comments indicating that something in Andy’s solution is incorrect.

Mr. Anderson: I’m not worried about whether it’s right or not right now, I’m worried about your strategies. (Classroom Video Recording, Beginning-of-Year Data Collection Period, Day 3)

In this instance, Mr. Anderson made it clear that he was focused on these students’ correct and useful solution strategies rather than on any mistakes they may have made while solving the problem. He directly told students that he finds value in the correct and organized mathematical strategy Andy used, and he clearly stated that he was not concerned with the student’s errors at that time. This kind of interaction was observed throughout the school year during data collection periods. Mr. Anderson often highlighted valuable aspects of students’ solution strategies, ignoring or deemphasizing any mistakes had been made while solving problems using these strategies. He sometimes made this emphasis explicit, pointing out to students that he was focused on the correct mathematical thinking.

As the examples above show, Mr. Anderson’s actions and statements during whole-class discussions exemplified the norm everyone has some mathematical ideas to which you should
pay attention in several ways. He sometimes coached students who were struggling during whole-class discussions, and while doing so he often focused on their correct mathematical thinking. Mr. Anderson also often praised a student’s specific mathematical strategy, sometimes referring to that particular strategy using the student’s name (e.g., “Daysha’s strategy”) throughout the lesson or in later lessons. Additionally, Mr. Anderson sometimes explicitly talked about focusing on correct aspects of a student’s strategy or thinking, even when that student had made an error in part of their solution. Mr. Anderson engaged in these types of actions and statements throughout the school year; his actions during whole class discussions seemed to be consistent with this norm throughout the school year. Student’s actions and expectations related to this norm were often unclear from the classroom video, however, likely because the teacher-led whole-class discussions offered few opportunities for students to engage in coaching each other.

Evidence from focal students’ statements during interviews throughout the year. Interviews with the focal students illustrate students’ development over time of expectations that everyone has some mathematical ideas to which you should pay attention. The shift from the beginning of the year to the end of the year in students’ statements about expectations for responses to mathematical thinking is striking. At the beginning of the year, focal students either said almost nothing indicating that they were interested in others’ mathematical thinking and ideas, or they made comments indicating that they tended to look for a person’s incorrect thinking when they made an error. By the end of the year, students emphasized looking for the correct mathematics or useful strategies in others’ thinking.

It is possible that this shift in students’ talk about focusing on correct or incorrect parts of others’ mathematical thinking is tied to the emergence of the coaching practice over time. The teacher modeled the coaching practice throughout the school year, but students seem to have taken up and refined the practice slowly, as described above. The norm that everyone has some mathematical ideas to which you should pay attention seems to have been an integral part of the coaching practice by the end of the school year, and students’ statements reflect that relationship.

For example, I asked Maria the same question about her response to the paper-and-pencil survey in the beginning, middle, and end of year data collection time periods. At all three times, she had responded to a survey item by indicating that if a classmate got a math problem wrong in front of the class, she would try to figure out what the classmate was thinking. However, Maria’s explanation of why she would think about her classmate’s ideas was very different at the beginning of the year from her explanation later in the year. At the beginning of the year, Maria explained that she would be trying to figure out what mistake her classmate made while solving the problem:

Interviewer: Now let’s take a look at that survey I gave you.
Interviewer reads the survey question and Maria’s response aloud.
Interviewer: Can you tell me more about why you chose that?
Maria: Because sometimes they get the answer wrong, and I’m trying to see what mistake they made to get the answer wrong.
Interviewer: Why are you trying to figure that out?
Maria: Because sometimes the teacher asks kids to help the student that’s getting the answer wrong. (Focal Student Interview, Beginning of Year, August 2014)

Maria’s answer to this question at the beginning of the year is clear. She has the intention of helping the student who made the error, but her focus is on identifying exactly where in the solution process the student’s thinking went “wrong.”
In contrast, by the middle-of-year interview, Maria seemed to be taking up the idea that she should focus on the correct and useful math in other’s mathematical thinking, even when they get the answers to problems incorrect. She described trying to understand the student’s “point of view,” in order to understand their solution strategy:

*Interviewer:* Now let’s take a look at that survey I gave you. *Interviewer reads the survey question and Maria’s response aloud.*

*Maria:* Well, I circled that because I would try to see their point of view about how they solved the problem. So, maybe next time I could use the same strategy that they used. But just try to get the right answer.

*Interviewer:* Okay, so maybe taking some ideas from them?

*Maria:* Taking ideas from their answer. (Focal Student Interview, Middle of Year, January 2015)

By the middle of the school year, Maria already expressed the expectation that she would be looking for useful and correct ideas – particularly strategies – in a student’s mathematical thinking, even when that student made an error. She indicated that she focused on other students’ mathematical ideas in order to “take ideas from their answer,” suggesting that she expected not only to focus on the correct aspects of others’ mathematical thinking but also to find these ideas useful in her own problem solving.

Not all students took up this idea as quickly as Maria did. For example, at both the beginning-of-year and the middle-of-year interviews, Kai indicated that an important part of the coaching process was to identify the error that the student made and point that error out to the student:

*Interviewer:* What does it mean to coach or help someone when they get the answer to a math problem wrong?

*Kai:* Sometimes it’s pretty fun, when they’re like listening to me. But sometimes when they’re like yelling at me, saying, “How did you get that?” I don’t really want to help them any more.

*Interviewer:* Okay, and how is it that you’re helping them when you coach somebody. Like what do you do?

*Kai:* Well, at first I ask, “How did you get that answer?” And then usually it’s like me, I forget something or I put something on that’s extra. And I show him what he or she does wrong. (Focal Student Interview, Beginning of Year, August 2014)

In this excerpt from our interview at the beginning of the school year, Kai focused on identifying what the student he was coaching did incorrectly when solving a math problem. Kai seems to have understood this identification of “what he or she does wrong” as a central part of the coaching process. Similarly, during our middle-of-year interview, Kai continued to talk about identifying what another student “did wrong” as an important part of coaching:

*Interviewer:* What does it mean to coach or help someone when they get the answer to a math problem wrong?

*Kai:* Well, first I ask them to do what they did. First I ask them to show me what they did to answer the problem. And then I see what they did wrong and I try to help them see that and tell them the correct way to do it. And then I would make up a new problem of my own that was just like that one and ask them to do it to see if they understood it. (Focal Student Interview, Middle of Year, January 2015)
Unlike Maria, by the middle-of-year data collection period Kai did not seem to have fully taken up the idea that everyone has some mathematical ideas to which you should pay attention as meaning that he should focus on others’ correct mathematical ideas.

In fact, Kai’s statements at the end-of-year interview indicate that he continued to view identification of another student’s incorrect thinking as central to the coaching practice. However, he also indicated that he was beginning to take up the idea that the student being coached might also have useful mathematical ideas to which he should attend:

Interviewer: Okay, great. And how often do you coach other students in math?
Kai: Mmm, about once a month.
Interviewer: About once a month. So not very often.
Kai: Yeah.
Interviewer: When you do, how important is that for you learning math?
Kai: A lot.
Interviewer: A lot. Tell me about that.
Kai: Well, sometimes when I’m helping somebody they find out something and then they like actually do the right answer when I’m doing the wrong answer and everybody else is, too, so they find the actual answer that [Mark] is agreeing to, so it helps to know.
Interviewer: So you can see how they’re solving it, and that helps you.
Kai: Even though I’m the coach. I’m the one who’s helping. (Focal Student Interview, End of Year, April 2015)

In this excerpt, Kai expressed his idea that when he was coaching another student, that student might have an idea from which he could learn. Kai continued to express thinking that is very “black and white,” meaning that he seemed to view thinking as either right or wrong, without any indication that he understood that a person could make an error in one part of their solution while still having correct ideas in other parts. However, Kai did seem to be starting to develop the idea that focusing on others’ correct mathematical thinking was important.

All focal students’ statements related to this norm indicated some taking up of the idea that everyone has some mathematical ideas to which you should pay attention over time. No focal students expressed this idea at the beginning of the year, and all focal students at least began to express this idea — like Kai — by the end of the year.

In many cases, students’ statements related to this norm were tied to the coaching practice. Students talked about their expectations for focusing on incorrect parts of others’ solutions (mostly at the beginning of the year) or correct parts of others’ solutions (mostly at the end of the year) in terms of situations in which they were coaching other students or in which they were observing the teacher coach other students. While this norm certainly seems to have applied in other situations than coaching, it was also clearly a central part of these students’ thinking about coaching practice by the end of the school year.

Evidence from students’ survey responses throughout the school year. Student survey data suggests that most students in the class eventually took up the everyone has some mathematical ideas to which you should pay attention norm, and roughly half of students began the year with some expectation that they would attend to others’ mathematical ideas. As stated previously in Chapter 3, the student survey was designed prior to identification of the seven norms described in Chapter 5, so survey questions were not designed to elicit students’ expectations related to these norms in particular. Evidence from the student survey is therefore suggestive that students took up this norm over time, but it must be considered in relation to focal students’ explanations during interviews.
Only one survey question (survey question #2, see Appendix A) required students to choose what response they would expect to have if a classmate solved a problem in front of the class and had an incorrect final answer. One answer option provided for students indicated that they would “try to figure out what my classmate was thinking about when he got the answer wrong.” Selection of this answer option seems to indicate that a student expects that despite the incorrect answer, the classmate has mathematical ideas to which they should attend. Maria’s explanations of her reasoning for selecting this answer option, described above, suggest that students’ selection of this answer option should not be taken to mean that students necessarily expected to focus on others’ correct mathematical ideas. Rather, students may have interpreted differently what trying to “figure out what my classmate was thinking” meant. Nonetheless, students’ selection of this answer option does indicate a focus on others’ mathematical thinking and suggests at least the possibility that these students might be thinking about others’ correct mathematical ideas.

At the beginning of the school year when the survey was first administered to all students in the class, 52% of students (13 out of the 25 students in the class) responded to this survey item by choosing the response indicating that they would attend to the classmate’s mathematical thinking. At the middle-of-year survey administration in January, 63% of students (15 out of 25) in the class selected this answer option. Additionally, one student responded by writing in an answer that suggests that they expected to attend to their classmate’s thinking: “I would try to figure out what he/she was thinking, then I would raise my hand and question their reasoning.” Another student wrote in an answer linking their expectations to the coaching practice, which may indicate that he or she also expected to attend to his or her classmate’s thinking: “I would help them to correct their mistakes by coaching them and going through thoroughly.” It appears that more students in the class were starting to take up this norm by the middle of the school year, at least by attending to others’ mathematical ideas, though not necessarily others’ correct mathematical ideas.

By the end of the school year, as stated in Chapter 5, 76% of students (19 out of 25) responded to this survey item by selecting the option that indicated they would expect to attend to their classmate’s mathematical thinking. These results are consistent with the student interview data, suggesting that students in the class took up this norm over time, with some students taking up the idea more quickly than others. Of course, interview data also suggests that the way students interpreted this question and this particular response option may have shifted over time, as Maria’s interpretation did. Nonetheless, in combination with interview data, the survey results suggest that more students began to expect to attend to others’ mathematical ideas over time.

**Summary of evidence throughout the school year.** Recordings of whole-class discussions, student interviews, student surveys, and the teacher interviews suggest that this norm originated with the teacher’s conviction that it is important to focus on and build from students’ correct mathematical ideas, and over time students in the class took up this expectation until it became taken-as-shared by most class members. The teacher engaged in similar practices during whole-class discussions throughout the year, effectively modeling for students how to focus on others’ correct mathematical ideas while coaching or while discussing a mathematical solution. During whole-class discussions, the teacher often (a) highlighted students’ correct mathematical ideas while coaching them through a problem they got wrong or struggled with, (b) publicly praised students’ solution strategies, and (c) explicitly talked about focusing on the correct parts of others’ strategies and ideas even when they made errors.
Student interviews and survey data suggest that students slowly began to expect themselves and others to focus on others’ mathematical ideas, particularly on others’ correct mathematical ideas. Interviews suggest that students more quickly took up the idea that identifying other’s incorrect mathematical ideas – that is, their errors – was important, perhaps because this practice is common in many mathematics classrooms, and students may have experienced it before. However, students seem to have more slowly come to expect that they should focus on others’ correct mathematical ideas. This understanding seems to have developed in relation to the development of the coaching practice.

The understanding that part of a solution can be correct and useful though another part may be incorrect seems to have been new for many students in the class, and evidence suggests that students generally took up this idea slowly over the course of the year. In an interview, the teacher recognized that this concept is advanced and difficult to grasp even for some adults:

Well, what I’d like them to do is be able to think about it in terms of what they do know, instead of what they don’t know. But that would require a lot, that’s a lot of empathy. It’s not even empathy. They have a lot of empathy that they definitely can do that. I think it’s more an understanding of math that goes beyond just understanding the math. That understands how to critically analyze the pieces of it so that they can pull it apart and see what it is that they really do understand and what little piece they’re missing. That’s a tall order for a fifth grader. It’s what I’m asking them to do. It is what I’m asking them to do. But I would like it more I suppose if they would explicitly say to kids, “Oh, I see you understand…” and being able to verbalize that to kids would be great. But that’s a very advanced way to coach. There’s a lot of teachers I know who can’t do that.

(Teacher Interview, Middle of Year, January 2015)

Because of the difficulty of identifying parts of a mathematical solution that are correct as well as parts that are incorrect, it is understandable that students took up slowly the norm of focusing on the correct parts of mathematical solutions involving errors. As students developed the ability to identify correct and incorrect parts of solutions, particularly while engaged in the coaching practice, they appear to have taken up the expectation that one should focus on the correct mathematical ideas as useful for learning. The development of this norm in the case-study class is therefore a good example of how an expectation may be initially promoted by the teacher and slowly taken up over time by members of a class as collective practices related to that norm are also developed over time.

**Emergence of the There are Different Types of Errors, Only Some of Which are Acceptable Norm**

The norm *there are different types of errors, only some of which are acceptable* is an example of a norm that was co-constructed by members of the class over time. The classroom teacher did not seem to have had this specific expectation in mind at the beginning of the school year, as he did with the *everyone has some mathematical ideas to which you should pay attention* norm. Rather, this norm appears to have developed over the course of the school year in response to the class’s need to define under what circumstances errors were acceptable. From the beginning of the year, the teacher had promoted related expectations, which became the *you can and should learn from your errors* (Norm 4), *you should admit when you get an answer wrong* (Norm 5), and *you should take risks while learning* (Norm 6) norms; the teacher encouraged students to expect to make errors while learning, to “own” those errors, and to learn from them. However, this set of norms lead to the problem of students becoming so comfortable
making errors that they were not always careful in their responses on assessments. Through interactions over the course of the year, the class negotiated expectations related to when and how errors were acceptable, leading to the there are different types of errors, only some of which are acceptable norm being taken up.

This norm also appears to be related to the coaching practice in that mistakes while learning, which were considered “acceptable,” often resulted in coaching or were salient when the teacher coached a student through a difficulty with a mathematics problem during whole-class discussion. On the other hand, careless errors, which were considered “unacceptable” or to be avoided, were typically not treated as opportunities for coaching.

In this section, I describe evidence from interviews and video recordings of classroom interactions that suggest that this norm was developed slowly over time in the case-study class. To illustrate this process, I present evidence chronologically. First, I describe evidence from teacher and student interviews and recordings of classroom interactions from the beginning of the school year, then from the middle of the school year, and finally from the end of the school year.

Evidence from the beginning of the school year. Interviews and recordings of classroom interactions indicate that at the beginning of the year, the class had not defined when and what types of errors were acceptable, so this norm was not yet developed in the class. Rather, the focus of class members seems to have been on defining errors as a normal part of learning.

Evidence from teacher statements during the interview at beginning of the school year. At the beginning of the school year, the teacher focused on the need to help students accept errors rather than becoming upset by them. He expressed some ideas that errors were sometimes to be embraced and learned from and at other times to be avoided, but he seemed perplexed as to how to present these ideas to students. Instead, he problematized his message of accepting errors as a “double-edged sword:”

Interviewer: How do you think your students currently interpret making an error on a math problem? What does that mean to them right now?
Mr. Anderson: At this point, it’s hard to say. I think they see it as a disappointment. I know the teachers last year tried really hard. I mean, it’s a double-edged sword; we don’t want them to be proud of getting the wrong answer. We do want accuracy. I mean, it’s one of the math practices [in the Common Core]. We want them to feel like the mistakes they make are novel. We don’t want them to make the same mistakes over and over again. I have several students who get down on themselves. I just watched it happen today. I was like, “Whoa, easy, it’s your first mistake of the day, relax.” I can tell that it’s a thing for them. They’re predisposed to an intimidation when it comes to math—that it’s just not going to make sense to them. They think that already. (Teacher Interview, Beginning of Year, August 2014)

It is clear from this statement that at this point in the year, the teacher viewed his students as wanting to avoid errors, so he saw his challenge as helping students to accept errors as part of learning. He acknowledged that accuracy was also important, but it seemed to be less of a focus at that time.

Evidence from focal students’ statements during interviews at the beginning of the school year. At the beginning of the year, students did not express clear ideas about when or what kinds of errors were acceptable in mathematics. In general, focal students tended to talk about errors as careless errors, and they tended to describe these as common among students.
For example, when asked an open-ended question about errors, Maria interpreted it as a question about careless errors in math:

_Interviewer_: What do you think it means when someone gets the wrong answer on a math problem?
_Maria_: I means that maybe, like if we’re adding decimals or something, it means that they probably added one more or one less, and they have to. Cuz some kids, they just do the answer and they think that it’s alright, but then when they double check it they have a little mistake.

_Interviewer_: So it might mean that they didn’t check their work?
_Maria_: Yeah.

_Interviewer_: Does it tell you anything about the person who got the answer wrong?
_Maria_: Mmm, I don’t think so.

_Interviewer_: The reason I ask that is because some people say that people who get the answers wrong need to work harder to understand math, or maybe that they’re just not very good at math, or maybe some people say that everybody gets problems wrong and it’s part of learning.
_Maria_: Yeah, everybody might get a problem wrong once in a while. (Focal Student Interview, Beginning of Year, September 2014)

Maria focused on students making “a little mistake” while solving a problem, such as “add[ing] one more or one less.” She seemed to be referring to careless errors, and she also seemed to view these as fairly common because “everybody might get a problem wrong once in a while.”

Kai expressed a similar idea during the beginning of year interview, saying that “it’s not a big deal if you get it wrong. Because eventually, like if you’re a person who gets everything right, eventually you’ll get something wrong. Because nobody’s perfect” (Focal Student Interview, Beginning of Year, September 2014). Like Maria, Kai seemed to view errors as common among students, implying that he viewed errors as acceptable or “not a big deal.” Also, neither Kai nor Maria differentiate different circumstances in which errors may or may not be acceptable or what kinds of errors may or may not be acceptable.

Jordan also described errors as generally acceptable, focusing on expectations for homework:

_Interviewer_: Okay. Now we’re going to watch a really short video clip of your class.
_Interviewer_ puts a printed rating scale with these options in front of the student.

_Jordan_: I think he would be a little serious because on our homework that he gives us he just cares that we actually try. He’s not really uptight about if we get the answers correct or not.

Like Maria and Kai, Jordan seemed to expect that errors were acceptable, but he focused his response on homework specifically. It is unclear if Jordan expected there to be other situations in which errors were unacceptable or to be avoided. The other focal students, Domingo and
Rowan, made ambiguous statements about errors indicating that they also did not have clear ideas about when or what kind of errors were acceptable.

It is interesting that even at the beginning of the school year, the focal students expressed the idea that errors are common and implied that they expected errors to be accepted. In our interview at the middle of the school year, the teacher shed some light on why students may have begun the school year already having these ideas about errors. He explained that the school culture was based on a growth mindset (Dweck, 2002), so teachers in the school generally supported students in thinking that everyone can learn:

*Interviewer:* Do you have any kids that have an idea that if you get things wrong in math, you’re just not good at math, it’s just something you’re not good at.
*Mr. Anderson:* There probably are kids that feel that way really, but they don’t express it that way.
*Interviewer:* Why not?
*Mr. Anderson:* Uh, it’s not the culture. It wasn’t the culture when they came in here. I mean, I think I’ve done things to encourage that kind of culture, but they came in like that. It’s sort of a school culture here.

*Interviewer:* It’s a school wide thing that you can keep learning and-
*Mr. Anderson:* Everybody can learn. Eventually. It may take longer, you may have to work harder, you may have to spend extra time on it, but you can learn. (Teacher Interview, Middle of Year, January 2015)

Here, Mr. Anderson indicated that the entire school had a culture that supported students and teachers in adopting a growth mindset, which means that they believe that everyone can learn by working hard. This mindset can be contrasted with the belief that people who make errors are simply not good at math and are not going to be able to get better at it. The growth-mindset school culture seems to have supported focal students in at least viewing errors as common and acceptable rather than as indicating an inability to learn mathematics.

Based on the teacher and students’ statements above, it is not surprising that video recordings of classroom interactions from the beginning of the school year show the teacher generally encouraging students to learn from their errors. Maria, Kai, and Jordan’s statements from the beginning of the year suggest that they had begun to take up the idea that errors were part of learning, and Mr. Anderson’s interview statements indicate that he intended to further support students in taking up the expectation that errors are part of the learning process.

**Evidence from video recordings of classroom interactions at the beginning of the school year.** Recordings of classroom interactions indicate that ideas about errors being acceptable or not are generally expressed through the teacher’s actions and statements during whole-class discussions. As mentioned previously, the whole class discussions in this classroom are teacher-led, and the teacher generally identified most errors as incorrect (see Chapter 4) and coached students who made errors during discussions. At the beginning of the year, the teacher’s actions and statements during discussions showed that he both implicitly and explicitly indicated to students that mistakes are an acceptable part of the learning process. Consistent with his statements during beginning-of-year interviews, the teacher seemed to have focused on helping students accept mistakes while learning.

For example, the teacher often encouraged students to share their errors and how they learned from their errors. In one instance, the teacher prompted a student Andy to share an error he had made and how another student in the class helped him to understand how to solve the problem correctly. This instance indicates that the teacher was supporting students in feeling
comfortable sharing their errors by praising students who made and learned from errors and by encouraging students to share their errors publicly:

Students are talking in pairs and small groups about the solution to a word problem in which students must compare decimal numbers to determine which of several students finished a race first.

Teacher (to Andy): Say it just like you said it to me. [inaudible] What you said was brilliant just now.

Teacher whistles to get all students’ attention.

Teacher: Andy’s got something to say to you.

Teacher (to Andy): At first I thought – wait, wait, it’s right here, I wrote it out for you. At first I thought –

Andy: At first I thought…

Teacher briefly interrupts to clarify some notes he wrote that he is projecting for students to see.

Andy: At first I thought, um, Charlie was first, I mean uh Craig was first. But [another student] helped me figure out that he’s not first, because she added this right here.

Andy writes a zero on the projected page, so that now the number “25.9” reads “25.90”

Teacher: And then when you, when I saw the –

Andy: I saw the

Teacher: That whole number there

Andy (referring to where “25.90” and “25.85” are written on the projection): That ninety and the eighty-five, I knew that Sam was first.

Teacher: Because eighty-five is?

Andy: Is, um, less than ninety.

Teacher: Isn’t that cool? Putting that zero on the end really matters, doesn’t it? And putting the zero on the end there makes them all so they have the same number of digits. And that can really help you figure things out that way. That was amazing, thank you.

(Classroom Video Recording, Beginning-of-Year Data Collection Period, Day 3)

In this instance, the teacher referred to Andy’s description of his error and how another student helped him learn from it as “brilliant.” He supported Andy in publicly describing his thinking and how it changed based on another student showing him how to add a zero to the end of a decimal number to make it clear which of the two numbers 25.90 and 25.85 was bigger. Further, the teacher praised Andy’s description as “amazing” and thanked him publicly for sharing with the class. These actions indicate that the teacher was encouraging students to share their errors and to view errors as an acceptable part of learning. The teacher frequently made this kind of statement during lessons at the beginning of the school year.

The teacher also sometimes directly stated to students that errors while learning were acceptable. These statements often occurred after one or more students made errors while solving problems during a whole-class discussion, and the teacher paused the lesson in order to talk about errors as part of learning. For example, during one lesson at the beginning of the school year, after many students had struggled to correctly read the number 6,030.42 aloud, the teacher said,

Good. I love all of you that are trying and making mistakes, but you’re still talking. It drives me crazy when you sit there and you don’t say anything cuz you’re worried about saying the wrong thing. Take a risk! Learn. When you say it wrong, fix it. It’s fine. I
don’t care. (Classroom Video Recording, Beginning-of-Year Data Collection Period, Day 3)

Here, the teacher indicated to students that making mistakes is acceptable, and he encouraged them to “take a risk,” rather than playing it safe by remaining quiet. He went on to indicate that errors should also be corrected and learned from in this class. This kind of explicit statement about the acceptability of errors while learning seems to have been another method the teacher used to encourage students to view errors as an acceptable part of the learning process.

The teacher’s actions during beginning-of-year lessons suggest that he mostly focused on supporting students in accepting errors while learning; however, at times some of the teacher’s statements implied that he viewed careless errors as unacceptable. This view of careless errors was apparently not the teacher’s focus at this time, but nonetheless it did come up. For example, during one instance the teacher asked the whole class several times to read a number aloud correctly, but several students continued to say the number incorrectly. The teacher indicated that he thought that these students were saying the number incorrectly either because they were not paying attention or because they were trying to be funny, and he made it clear that he viewed these reasons as unacceptable for making an error.

Teacher: Okay, so now, how much does one baseball weigh?

Some students: One hundred forty-one and seventy-four hundredths grams.

Other students (at the same time as above): One hundred forty-one point seventy-four grams.

Teacher: Okay, read it correctly please.

Most students: One hundred forty-one and seventy-four hundredths grams.

Some students apparently continue to say “one hundred forty-one point seventy-four grams,” though this is not audible on the recording.

Teacher: Okay, now let’s get everybody to read it correctly. We still have some people who think its funny to keep saying “point” even though I keep saying, “Say the number correctly.” All of you who are doing that, you’re not funny. You’re wasting our time. Stop wasting our time. One, two, three.

Most students: One hundred forty-one and seventy-four hundredths grams.

(Classroom Video Recording, Beginning-of-Year Data Collection Period, Day 5)

In this instance, the teacher clearly rebuked students who said “one hundred forty-one point seventy-four grams,” and he suggested that “say[ing] the number correctly” was a skill that the class has been learning for some time. He made it clear that at this point students were expected to know how to “say the number correctly,” so making an error in doing so was not acceptable. Though he did not clearly say so at this point in the year, the teacher did imply that careless errors – or errors in skills that students should know – were not acceptable. Again, this defining of careless errors as unacceptable did not seem to be the teacher’s focus at this point in the year.

The teacher’s and students’ statements from interviews and the excerpts of classroom interactions described above indicate that at the beginning of the year, the teacher and students were developing classroom norms related to learning from errors and taking risks. The teacher did not yet express any clear messages about different types of errors being acceptable or not or different circumstances in which errors are acceptable or not. At times, classroom interactions revealed that he did expect students to avoid making errors when they should know better, but he did not make it clear how students can differentiate these unacceptable errors from making errors while learning. Accordingly, focal students’ statements indicated that they had some ideas about errors being common, and they seemed to view errors as usually being careless errors.
They generally seemed to view errors as somewhat acceptable, and they did not differentiate different types of errors or different situations where errors may or may not be acceptable. At the beginning of the year, the norm *there are different types of errors, only some of which are acceptable* did not yet appear to be a shared expectation in the classroom.

**Evidence from the middle of the school year.** By the middle of the school year, the teacher’s statements indicate that he clearly identified types of errors and circumstances in which errors were or were not acceptable to him. Students seemed to have begun to pick up on these expectations, and some focal students clearly expressed a distinction between types of errors and which were and were not acceptable. Not all focal students expressed these ideas at this point in the year, however. Classroom interactions indicate that at this time in the school year, the teacher was continuing to send a clear message to students that errors while learning were acceptable. He had additionally begun emphasizing to students that it is important to learn from those errors so as not to continue making the same types of errors over and over again. Furthermore, the teacher was observed to make some direct statements to students that careless errors were not acceptable, distinguishing errors while learning from careless errors and making it clear which were acceptable. This distinction was much clearer in classroom interactions observed at the middle of the school year than it had been at the beginning of the school year.

**Evidence from the teacher’s statements during the interview at the middle of the school year.** During our middle of year interview, the teacher clearly stated that he wanted students both to accept errors while learning and also to avoid careless errors. His ideas about the kinds of expectations related to different kids of errors that he wanted to foster in the class were much clearer at this point in the school year than at the beginning of the year. He described thinking that students had not quite taken up these expectations yet, though.

Notably, Mr. Anderson continued to emphasize mistakes while learning as valuable opportunities for learning and as acceptable. Mr. Anderson remained consistent in emphasizing this point throughout the school year. For example, during our middle-of-year interview, Mr. Anderson discussed valuing students’ errors during whole-class mathematics practice because he was glad these students were attempting the problems, even if they were unsuccessful:

*Interviewer plays video clip in which teacher says “That’s okay honey, I’m glad your voice is out there, make all the mistakes you want!”*

*Interviewer: So, just generally, what’s going on?*  
*Mr. Anderson: The activity or, everything?*  
*Interviewer: Well…*  
*Mr. Anderson: [brief description of the activity shown in the video and its purpose] But the kids who were making the mistakes in there, they were serious, they just didn’t really know what came next and they made a mistake. And I want that to be valued, I want that to be something that, you know, that to me is way better than the kid who’s not saying anything. Or the kid who’s just mouthing with everybody else and just trying to copy what they’re doing. There’s no thinking going on there, there’s nothing happening there. The kid who’s making the mistake is thinking, and they had an idea of where they were going, and something didn’t work, and then they heard the right answer and realized it was wrong. There’s so much good thinking going on there that I want to value.* (Teacher Interview, Middle of Year, January 2015)

Here, Mr. Anderson’s statements are very consistent with his ideas from the beginning of the school year. He had not wavered in his commitment to supporting the expectation that errors are
part of learning and that it is acceptable to make errors while grappling with new mathematical ideas or procedures.

In this same part of the middle-of-year interview, Mr. Anderson went on to indicate that his emphasis on the acceptability of errors while learning may have lead students in the class to over-generalize the acceptability of errors. In particular, Mr. Anderson referenced his observation that students at that time of year were often rushing through “exit tickets,” which were short daily assessments, and making many careless errors. He suggested that students may have thought that these careless errors were acceptable, too, and may not have distinguished them from errors while learning:

Interviewer: So they’re able to come up with something, it might be wrong, but they can then compare it.
Mr. Anderson: And they’re evaluating it, and realizing it’s wrong. I think there’s a lot going on. And then they get to try again. I want that attitude, I want that kind of culture to be fostered, you know that idea of go ahead and make a mistake, let’s try again. And now what. Let’s revise and evaluate. [laughs] And maybe that’s why they have the problem with the exit ticket. They think, “Oh, I’ll just screw up, and I get to try again.” I don’t know. It’s a fine line to walk, because there is an accuracy piece that I do want, you want kids to be able to do it right the first time. But I don’t want to engrain that in them so much that they’re afraid to try.

Interviewer: Right, so it’s kind of a balance of you want them to try things and make mistakes if they don’t know, but when they do know how to do something, you want them to be careful and accurate.
Mr. Anderson: Then I want them to be careful and really sure of themselves when they do it. And those two things are at cross-purposes.
Interviewer: That’s tricky.

Teacher: The way I’ve tried to go about it is this sort of time and place idea. You know that that [points to computer screen where video clip was played, indicating the situation in the video clip] is a time to just go for it, but an assessment is a time to practice being careful, do it right.

Interviewer: And that’s something you’ve been trying to tell them about?
Mr. Anderson: Yeah, and they’re not all getting it yet. (Teacher Interview, Middle of Year, January 2015)

In this excerpt, Mr. Anderson clearly described wanting to create a “culture” in which students felt comfortable to “make mistakes if they don’t know,” but he distinguished these errors while learning from times “when they do know something” and students should be expected to “be careful and accurate.” He described students as having problems making careless errors on the “exit ticket[s],” and he suggested that perhaps this problem was because he had emphasized the acceptability of errors without making it clear to students that there were times when errors were not acceptable. Mr. Anderson also indicated that he had been trying to communicate to students the expectation that mistakes while learning are acceptable while careless errors are not, but his impression was “they’re not all getting it yet.”

Mr. Anderson’s expectations related to different types of errors as acceptable or not and situations in which errors are acceptable or not were much clearer at the middle of the school year than they had been at the beginning of the school year. He continued to emphasize mistakes while learning as acceptable, but he also clearly stated that careless errors – meaning errors in situations where students should know better – were not acceptable.
Students’ statements related to this norm at the middle of the school year. At the middle of the school year, focal students generally expressed some understanding that there are different types of errors and some understanding that errors are sometimes acceptable and sometimes not. That is, most focal students made statements at the middle of the year interview that indicated they had begun to take up the norm there are different types of errors, only some of which are acceptable.

For example, in his middle-of-year interview Rowan at one point indicated that he understood careless errors to be unacceptable. At another point in the interview, Rowan indicated that he expected errors while learning to be acceptable and to be addressed with coaching. These two statements were made at different points in the interview, so it is unclear how integrated these ideas were for Rowan, but he did seem to be holding both ideas by this point in the school year. To illustrate, in this excerpt, Rowan clearly described the teacher as not wanting students to continue making the same kinds of errors over and over again:

Interviewer: Okay. Now we’re going to watch a really short video clip of your class.

Interviewer plays video clip of classroom interaction in which teacher says, “That’s okay honey, I’m glad your voice is out there, make all the mistakes you want!”

Interviewer: How serious do you think [Mark] was when he said that? Was he not at all serious – was he joking, – not very serious, a little serious, or definitely serious – he means what he’s saying?

Interviewer puts rating scale in front of Rowan showing the options she just stated.

Rowan: Uh, I think he’s a little serious. Because as the year goes, it changes. At the beginning of the year he was a lot more flexible with making mistakes than he is now.

Interviewer: What do you mean by that?

Rowan: Like he’ll give you a second chance more easily. Because now he wants, in math, he wants you to get it. He doesn’t just want you to make mistakes.

Interviewer: So maybe it’s okay to make some mistakes, but-

Rowan: He wants you to get there eventually. (Focal Student Interview, Middle of Year, January 2015)

In this exchange, Rowan told the interviewer that he perceived the teacher’s position on errors to have changed from the beginning of the year to the middle of the school year. Rowan expected the teacher to now find errors less acceptable because “he wants you to get it,” and if students did make errors, the teacher would want them “to get there eventually,” meaning to learn from their errors so that they did not make them anymore. While Rowan’s perspective expressed here clearly indicates that he understood that the teacher expected accuracy and found careless errors unacceptable, it is not clear from these statements if Rowan still viewed mistakes while learning as acceptable. During another part of the same interview, Rowan did express the expectation that mistakes while learning were acceptable and should be addressed with coaching:

Interviewer: What do you think it means when someone gets the wrong answer on a math problem?

Rowan: Well, uh, it means that they don’t exactly get how to do it yet and they need help. And if you get it wrong, usually they’re on the right path sometimes if they get it wrong. But they always need a little coaching, that’s really what it means.

Interviewer: They need some coaching.

Rowan: For that problem.

Interviewer: But they might be on the right path, they just need some help.

Rowan: Yeah. (Focal Student Interview, Middle of Year, January 2015)
In this excerpt, Rowan indicates that he expects mistakes while learning to be addressed with coaching. Taken together, these two excerpts from the interview with Rowan at the middle of the year suggest that he is developing an understanding of the distinction between errors while learning and careless errors, though he did not yet clearly express a coordinated understanding of this distinction.

Maria, on the other hand, did express a more clear distinction between types of errors during her middle-of-year interview, but she did not describe one or the other as more acceptable. It was therefore unclear if she expected errors while learning and careless errors to be acceptable or not in the class. Her statements therefore also indicate that she may not have yet fully taken up this norm:

*Interviewer:* What do you think it means when someone gets the wrong answer on a math problem?

*Maria:* Well, sometimes I think it means that they just made a little mistake, like with an adding problem or a subtracting problem or multiplying or dividing problem. And sometimes they just didn’t really get how to do it, and they just kind of guessed.

*Interviewer:* So when you say “a little mistake” that’s like they just added wrong or something simple?

*Maria:* Yeah. Like five plus six, they would put like 13 or 12.

*Interviewer:* Yeah, like maybe they were just going too fast and just writing the answers quickly.

*Maria:* But it could also mean that they don’t understand the bigger problem.

*Interviewer:* Yeah, like they don’t understand what to add, what to multiply, what to divide, what to subtract. (Focal Student Interview, Middle of Year, January 2015)

In her statements, Maria distinguished between making “a little mistake,” by which she seemed to mean a careless error, and “just [not] really get[ting] how to do it,” meaning not yet understanding the problem and making an error while learning. She gave an example of each type of error, indicating that she clearly understood the difference. However, it is not clear from her statements during the interview that she yet understood which type of errors were acceptable and which were not.

Other focal students made similar kinds of statements indicating that they understood that there were some differences in types of errors or differences in the acceptability of errors in different circumstances. No focal students made any clear statements at this time in the school year explicitly defining this distinction as well as which errors were acceptable. These statements by the focal students are consistent with the teacher’s impression that “they’re not all getting it yet,” meaning that at this point in the school year, students did not seem to have fully taken up the norm that there are different types of errors, only some of which are acceptable. Students seem to have begun to make these distinctions, but their expectations related to different types of errors were not yet fully clarified nor coordinated.

**Classroom interactions related to this norm at the middle of the school year.** Classroom interactions observed during the middle-of-year data collection period indicate that the teacher communicated to students the difference between the types of errors and which were acceptable or not. He continued to express explicitly and implicitly to students that mistakes while learning were acceptable, and he additionally emphasized that students were expected to learn from errors so that they do not make the same errors again and again.
Many instances observed during the middle of the school year are similar to those from the beginning of the school year in which the teacher demonstrated to students or directly told students that mistakes while learning are acceptable and are opportunities for learning. Students continued to make errors during whole-class discussions, and the teacher continued to publicly coach them through their difficulties with problems. The teacher also continued to encourage students to learn from their errors. These sorts of interactions generally seem to be similar to those from the beginning of the year.

For example, after many students made errors on an independent practice problem, the teacher went over the problem and how to solve it correctly with the whole class. After they solved the problem, the teacher asked students how many of them had initially had the correct answer to the problem, and he praised students for honestly admitting that they had not. He then went on to ask them if they learned from their errors through the class discussion of the problem, and many students indicated that they did.

*Teacher:* How many of you got that right? Just curious.
*One student tentatively raises his hand.*
*Teacher laugh.*
*Teacher:* Cool, way to go. Thanks for being honest. I didn’t get it right the first time I did it either. I misread it completely.
*A student:* Only one person got it right.
*Teacher:* Yeah, that’s a hard question isn’t it. How many of you understand it better now?
*Some students raise their hands.*
*A student:* Definitely.
*Teacher:* Cool. (Classroom Video Recording, Middle-of-Year Data Collection Period, Day 3)

In this instance, the teacher praised students for admitting their errors as he did in many other instances. He also went on to ask students if they “understand it better now,” emphasizing that learning from mistakes was also expected. This statement implies that errors while learning are acceptable, but students are expected to learn from those errors, suggesting that it may not be acceptable to continue making the same errors repeatedly.

The interactions observed at the middle of the school year differ from those at the beginning of the school year in that the teacher began to make clear statements to students that at times they were expected not to make errors, and careless errors were not acceptable. For example, in one instance the teacher asked the class a question, and many students quickly responded with incorrect answers. The teacher referenced a phrase that he commonly used in the class to indicate that students should slow down and try to be accurate, “aim before you fire:”

*Teacher:* When I divide it into fifths, how many fifths do I have?
*Students (variously):* Four. Four-fifths. Wait.
*Teacher:* Aim before you fire.
*Students:* Twenty fifths!
*Teacher:* Thank you. Please aim before you fire. [shaking his head] It’s no good being first if you’re saying nonsense. Please aim before you fire. (Classroom Video Recording, Middle-of-Year Data Collection Period, Day 3)

Here, the teacher emphasized that students should take time to figure out the answer to a question before responding, indicating that accuracy was important and expected. He indicated that the careless errors students made when they did not think before responding were
“nonsense” and were not valued. The teacher used this “aim before you fire,” analogy frequently, suggesting the expectation of accuracy and – conversely – that careless errors were unacceptable.

Furthermore, the teacher at times explicitly discussed with students situations in which errors were not acceptable and made the distinction between errors while learning and careless errors. For example, the teacher addressed the problem of students rushing through the “exit tickets” (end of day assessments) by making this distinction:

Teacher: I think I’ve done something wrong that I want to correct. I think I’ve made the exit ticket into a race.
A student: How?
Another student: No, we do it! We like to!
Another student: It’s not a race.
Teacher: Well, the problem is that a lot of you are coming up here with it wrong. And then you go back and correct it and everything, which is good, but the exit ticket is not a place where you should be getting things wrong. [shaking his head] The exit ticket is supposed to show – it’s like a test. And a lot of you are rushing through it and making mistakes that you shouldn’t make. [shaking his head] And I want you to start thinking of the exit ticket as more of a test. That “When I come up here, I want to be right the first time.” Cuz when you take tests do you get to change it after you’ve been wrong?
Students: No.
Teacher: No. It’s got to be right the first time, and I want you to get practice with that. Is it okay to get things wrong on the problem sets and correct them?
Students: Yes.
Teacher: Yeah, but you’ve got to correct them. That’s why I always let you talk to somebody, and figure it out with them, and make sure you get correct answers. That’s the time to raise your hand and learn what it is that you’re missing so that you can get it correct on the exit ticket. I want you to think of the exit ticket as a test. [nodding] Okay?
Students (variously): Okay. Got it. (Classroom Video Recording, Middle-of-Year Data Collection Period, Day 6)

In this instance, the teacher had a direct discussion with students of various types of errors and which of them were acceptable. He described errors while learning as acceptable, situating these in the “problem sets,” which were independent practice problems that students did on their own and then corrected with partners or as a whole class. He does not specifically talk about careless errors, but he does make clear that after students had learned how to do something correctly during the problem sets, they were expected to be able to get the problems of the exit ticket correct. The implication is that any errors students make while rushing through the exit ticket were careless errors, and these were unacceptable.

These middle-of-year interactions show that the teacher continued to emphasize errors while learning as acceptable and as opportunities for growth, and he also began to support students in drawing distinctions between errors while learning and careless errors, framing careless errors as unacceptable. As before, the students’ expectations related to this norm are less clear from video recordings of whole-class discussions, likely because these discussions were primarily teacher-led. However, the focal students’ statements described above indicate that they had begun taking up the idea that there were different types of errors, only some of which were acceptable in their class. At the middle of the school year, the teacher seemed to
have been clearly attempting students to take up this norm, and students seem to have begun to take it up.

**Evidence from the end of the school year.** By the end of the school year, it appeared that the class had more fully taken up the norm that there are different kinds of errors, only some of which are acceptable. Because evidence from interviews, classroom interactions, and the student surveys from the end of the school year is described in detail in Chapter 5, it is summarized briefly here.

At the end of the school year, the teacher clearly distinguished between errors while learning and careless errors, and he made clear that he viewed careless errors as unacceptable but errors while learning as acceptable and as opportunities for further learning. The teacher also indicated that he thought that students were making this distinction as well, viewing errors either as “carelessness” or as “a legitimate way of thinking about it that they got confused about” (Teacher Interview, End of Year, April 2014).

Focal students’ interview statements at the end of the year indicate that the teacher’s ideas about their expectations were correct; students seemed to distinguish different types of errors and possibly to understand which were acceptable. Some students’ statements about which types of errors were acceptable or unacceptable were clearer than others, but all focal students at least indicated that they understood that there were different types of errors – errors while learning and careless errors. Furthermore, student survey data indicate that by the end of the school year some students in the class seemed to recognize that different responses to errors could be expected, though the survey data does not provide enough detail to understand exactly why. Taken together, focal students’ statements and the survey data suggest that some students had fully taken up this norm and it was at least partially shared by other students in the class by the end of the school year.

Classroom interactions recorded on video at the end of the school year were similar to those from the middle of the school year, indicating that the teacher continued to encourage students to accept errors while learning while avoiding careless errors. He emphasized that students should learn from their errors so as not to continue making the same errors through carelessness or lack of effort. This message continued to be strong by the end-of-year data collection period.

Altogether, the class seemed to have largely taken up this norm by the end of the school year. The teacher’s and focal students’ statements indicate that they distinguished between careless errors and errors while learning, and at least some students also explained that careless errors were unacceptable but errors while learning were accepted. As the teacher explained, this distinction was difficult for students to learn, and it appears to have taken the entire school year for the teacher to first clarify the expectation to himself and then to support students in taking up this expectation.

**Summary of evidence throughout the school year.** The norm there are different types of errors, only some of which are acceptable developed slowly over the course of the school year. Unlike the everyone has mathematical ideas to which you should pay attention norm, the teacher did not seem to have the different types of errors norm in mind yet at the beginning of the school year. Rather, he seemed to clarify his own expectations related to different types of errors and their acceptability over the first half of the year in response to students overgeneralization of the acceptability of errors. Students had begun to act as though all errors were acceptable, even errors on assessments. To counteract this overgeneralization, the teacher supported students in understanding the difference between careless errors and errors while
learning, encouraging students to share the expectation that careless errors were to be avoided but errors while learning were acceptable. Students seemed to have taken up this expectation slowly; at the middle of the year, they seemed to have only partially understood and accepted it. By the end of the year, students seem to have more fully taken up this norm such that it appears to have been shared by at least most of the class.

This norm is a good example of a norm that was truly co-constructed over the course of a school year. Though the teacher appears to have been the primary authority in promoting this norm, it arose out of students’ actions that made it clear that they expected all errors to be acceptable. The teacher disagreed with this expectation, and in response he promoted the different types of errors norm. The course of development of this norm exemplifies the possibility of shifting norms over the course of a school year.

Summary

In this chapter, I have described in detail how two norms related to the discussion of students’ mathematical errors developed over the course of the school year in the case study class. The first of these norms, everyone has some mathematical ideas to which you should pay attention, is a good example of a norm that originated with the teacher’s intention at the beginning of the year and was taken up by most students over the course of the school year. The norm there are different types of errors, only some of which are acceptable differs in that the teacher did not have a clear intention to promote this norm at the beginning of the year; rather, he developed this expectation and promoted it in response to students’ apparent expectations at the middle of the school year that all errors were acceptable. To provide context for how these two norms developed, I also described the development of the coaching practice over the course of the school year and classroom techniques for discussing values that were used throughout the school year. Both the coaching practice and certain values, such as honesty and integrity, were clearly linked to the development of norms related to students’ errors.

In general, I have described the norms everyone has some mathematical ideas to which you should pay attention and there are different types of errors, only some of which are acceptable as developing largely as a result of the teacher’s promotion of these expectations. The teacher directly talked to the class about these expectations, modeled these behaviors, and implied the value of these behaviors through his actions. I do not mean to suggest, however, that students passively received these expectations from the teacher. Rather, students’ statements during interviews suggest that they actively attempted to make sense of the teacher’s expectations. Students also reacted to the teacher’s expectations in ways he did not expect, as evidenced by students’ overgeneralization of the acceptability of errors. These actions of students were central in the development of norms over time; norms can only be taken-as-shared when most members of the class actively take them up and act in ways consistent with the norms.

These results are intended to provide insight into the types of norms that may support students in discussing and learning from their errors and the ways in which these norms may be developed in a classroom. In the next chapter, I describe conclusions that can be drawn from the results described in this chapter and in Chapters 4 and 5. I attend to conclusions related to how teachers may apply the results of this study to their own teaching practice, and I also present some conclusions related to theory of the development of norms. Further, I suggest implications of these results for future research.
Chapter 7: Conclusions and Implications

In this chapter, I summarize the results of this study (presented in Chapters 4, 5, and 6), and I propose some implications of these results both as related to the research methods drawing on the analytic framework presented in Chapter 2 and for teaching practice. Implications related to research methods are intended to be helpful for future investigations of classroom norms, providing some insights as to the aspects of the framework that were useful for this study and suggesting methodological implications for future studies. Implications related to teaching practice are intended to be useful for current teachers, teacher educators, and education professionals.

Summary of Results

I described in Chapters 4 through 6 the results of a case study examining the development of norms related to the treatment of mathematical errors over the course of the school year in a single fifth grade classroom. Throughout the school year, I interviewed the classroom teacher and five “focal” students, administered surveys to all students in the classroom, and video recorded class mathematics lessons. I drew on these several data sources to identify norms that appeared to be shared by most members of the class. The language I use to describe these norms is intended to capture the gist of the shared expectations, and it is likely that if queried individuals in the class might describe these norms somewhat differently. These seven expectations generally seemed to be shared by most individuals in the class, and were identified in this study as classroom norms related to errors:

1. Everyone has some mathematical understandings to which you should pay attention.
2. There are different types of errors, only some of which are acceptable.
3. Multiple methods may be used to solve a problem.
4. You can and should learn from your errors to increase your understanding and also so that you do not keep making the same errors.
5. You should admit when you get an answer wrong or don’t know how to solve a math problem so that you can get help and learn how to solve it, even if it is scary or embarrassing.
6. You should take risks while learning.
7. You should support others who are struggling.

I described the first two of these norms in detail in Chapters 5 and 6, drawing on evidence from focal student interviews, teacher interviews, episodes from classroom lessons, and student survey data. The first norm provides an example of a norm that is closely related to a specific collective practice, the coaching practice that was common in the case-study class (see Chapters 5 and 6 for a description of this practice). The second norm provides an example of a norm that the teacher began promoting part way through the year in response to students’ actions. Unlike Norm 1, Norm 2 was not associated with a particular practice but rather guided many types of interactions involving errors. Both norms appear to have been strongly promoted by the teacher through explicit teaching of these expectations and through his modeling of these norms by his own behavior. These results have implications both for theories of how classroom norms develop as well as for teaching practice.
Implications for Future Research Methods

Relation of Saxe’s genetic processes framework to the design of this study. As described in Chapter 2, I drew on Saxe’s (2012) framework because it provides a useful conceptualization of the relationship between individuals’ actions and community practices and expectations, which seemed to be important for a study of classroom norms. I particularly focused on microgenetic processes, which occur as individuals engaged in activities use cultural forms to accomplish particular goals. These processes are local, in the sense that they are located in a particular activity at a particular time. I also focused on sociogenetic processes, meaning the distribution of individuals’ microgenetic processes throughout a community over time. In addition, Saxe describes ontogenetic processes, which are qualitative shifts over time in an individual’s activity and cognitive constructions. Saxe describes micro-, onto-, and sociogenetic processes as related. Microgenetic processes throughout a community over time may lead to sociogenetic shifts in patterns of activity. Similarly, an individual’s microgenetic constructions over time may lead to ontogenetic shifts as the individual makes sense of new situations and emerging goals.

I drew on this framework to inform the design of the case study as well as methods of data analysis, particularly focusing on microgenetic and sociogenetic processes because I believed these would be most pertinent to a study of classroom norms. Because my study focused on classroom norms, I was intent on identifying sociogenetic processes through which these norms were established, reproduced, and altered in the case study class. Saxe’s framework indicates that such processes may be most clearly visible through examination of the distribution of local interactions, that is, through identification of patterns and shifts in the interaction of individuals in the community. In order collect information about these patterns of activity in the case study classroom, I video recorded mathematics lessons at three time points during the school year, hoping to identify patterns of interactions at the beginning, middle, and end of the school year and so to identify any shifts in these patterns over time.

In designing the study, I intended for analysis of the classroom lesson video recordings to primarily provide information about patterns of interaction in the classroom, and I intended interviews with the teacher and focal students to provide insight into how those patterns of activity were interpreted by the actors in those interactions. In other words, I hoped to identify classroom collective practices and the normative structure of these practices through analysis of the video recordings, and to confirm individuals’ understandings of these practices and the normative structure of these practices through their statements in interviews. This analysis plan followed from my understanding of Saxe’s description of the relation between microgenetic and sociogenetic processes and how this framework may be applied to a study of classroom norms (see Figure 11).
Adapted methods used to identify collective practices and classroom norms. In fact, the actual methods I used to make sense of this particular case differed somewhat from my original analysis plan, and these differences suggest implications for future studies of classroom norms. I found that it was almost impossible to make sense of patterns of interaction in the classroom as viewed from video recordings of lessons without teacher and students’ interview statements as a lens through which to identify the relevant features of these interactions. Analysis of individuals’ sense-making about particular interactions – such as the video clip shown during the focal student interviews and teacher interview – proved useful as a method to gain insight into individuals’ expectations that were at play during these interactions. Without information from interviews as a guide, I would likely not have identified important features of interactions related to norms. For example, I may not have noticed the teacher’s tendency to focus on the correct aspects of a student’s solution while coaching that student through correcting incorrect aspects of the solution.

The findings of this study suggest that use of this framework for the identification of norms would benefit from attention to ontogenetic shifts in addition to microgenetic and sociogenetic processes. In other words, it may be useful to consider the development of individuals’ thinking and expectations over time. In this study, I found that microgenetic processes occurring in classroom interactions were difficult to interpret and sociogenetic processes of norm reproduction and alteration were difficult to identify without the context of individuals’ interview statements regarding their own expectations over time. Individuals’ observations and explanations served as evidence of their own ontogenetic processes of
development and as a lens through which to identify key features of interactions in the classroom in order to identify norms. Figure 12 provides an illustration of the framework with the addition of ontogenetic processes (adapted from Saxe, 2012). I have represented ontogenesis in this figure using dotted lines to indicate that these processes are not the focus of an investigation of classroom norms. However, attention to these processes is important to include in such an investigation as a way to access the sociogenetic processes that may otherwise be invisible to the outside observer.

The findings of this investigation also suggest that to this framework might be added an emphasis on the teacher as promoting certain norms in the classroom. The particular classroom that was the focus of this case study was clearly teacher-led in many respects, and the teacher’s promotion of the seven norms identified in this study seems to have been a key factor in their being taken up by the class. The teacher was found to frequently model through his own behavior the expectations that he wanted the class to adopt, and he often publicly praised students who displayed behavior aligned with these expectations. Additionally, the teacher frequently corrected the students’ behavior that was not aligned with the expectations he was attempting to promote.

Modeling, praising, and correcting behavior are, of course, standard teaching practices. The teacher in the case-study classroom, Mr. Anderson, used these practices frequently and consistently throughout the year to promote certain expectations that eventually were taken up and became norms in the class. Additionally, when he determined that a norm needed to be
altered because of the way students had interpreted it— as was the case with students’ overgeneralization of the acceptability of errors, as described in Chapter 6—Mr. Anderson was able to explicitly discuss the need for different expectations with students and then use the practices of modeling, praising, and correcting behavior to successfully promote the altered norm, as in the case of Norm 2. It may be useful, then, to add to the framework an emphasis on the teacher’s promotion of certain expectations as likely to be influential in their being taken up by the majority of the class. Figure 12 shows the framework illustration with such an emphasis. The teacher’s actions are depicted in a large oval to place emphasis on his/her actions as related to the reproduction and alteration of norms. Furthermore, specific teacher actions such as modeling, praise, and correction of behavior are highlighted as useful features of interactions to analyze in a study of classroom norms.

The use of this framework was also found to be helpful in this study for the identification of collective practices (Saxe et al., 2015) and the classroom norms associated with these practices. I found it useful to identify key collective practices that were strongly associated with the class’s treatment of mathematical errors. By attending to the collective practice of coaching (described in Chapters 5 and 6), I was able to identify norms that were most clearly evident during this practice, such as Norm 1. These results indicate that attention to collective practices may also be useful in future studies of classroom norms.

Further recommendations for future studies of classroom norms. It is worth mentioning that future studies of classroom norms may benefit from investigation of students’ interactions during partner and group work. Though these interactions are difficult to capture due to logistical limitations of video recordings, it may be interesting and useful to identify patterns of interaction during small-group student interactions in order to determine if the same norms appear to govern these interactions as whole-class interactions. It is possible that when in small groups without the presence of the teacher, students have very different behavioral expectations. Anecdotally, I did not observe students in the class to behave differently with respect to the treatment of mathematical errors when working in small groups as compared with during whole-class interactions. While video-recording for this study, I sometimes walked around the classroom while students were engaged in small-group work, and I noticed students generally talking about errors in similar ways to what I observed during whole-class lessons. Nonetheless, it may be useful for future investigations to more systematically capture and analyze these student interactions.

Summary of implications for studies of classroom norms. The results of this study suggest that future investigations of classroom norms may benefit from the use of Saxe’s framework, including attention to micro-, onto-, and sociogenetic processes. It may be particularly useful to attend to the teacher’s actions, as the teacher in many classrooms has power in promoting norms that he or she would like to see taken up by the class. Attention to collective practices related to the type of norms being studied may also be useful; when class members engage in these collective practices, they are likely to apply norms that are the target of study. Finally, it may be useful for future investigations of classroom norms to attempt to analyze the interactions of students working in small groups or partners in order to determine whether students appear to use the same norms for behavior in these situations as in whole-class interactions. Though the current study focused on norms related to the treatment of mathematical errors, it is likely that these recommendations would be beneficial for any study of classroom norms.
Implications For Teaching Practice and Teacher Professional Development

The results of this study also suggest some implications for teaching practice. As recommended by Kazemi and Stipek (2001), it may be useful for classroom teachers to engage with the in-depth descriptions of classroom norms for the treatment of errors provided in this dissertation. These descriptions may provide insight into the types of norms that could be useful in teacher’s classrooms.

Additionally, the results of this study indicate that young students – in this case, upper-elementary students – are capable of taking up challenging practices and ideas related to learning from errors. Many of the norms taken up by students in the case-study class would be difficult for adults to apply, yet these young students were applying these norms by the end of the school year. For example, the expectation that one should focus on the correct and useful mathematical ideas of a person’s solution, even when that solution resulted in an incorrect answer, is complex and can be difficult to apply. This expectation requires not only that one recognize the correct and useful parts of a person’s solution but also emphasize those correct features. Because often the incorrect parts of a solution are most salient, this expectation can be difficult to implement, yet fifth grade students were able to do so in the case-study class. These results suggest that with strong modeling, encouragement, and teaching, elementary grade students can engage in these challenging practices related to learning from errors.

The results of this study also indicate that teachers who consistently promote certain norms can be successful in doing so. The case-study classroom teacher was clearly very committed to the norms he promoted – such as the norm *everyone has some mathematical understandings to which you should pay attention* (Norm 1), and he repeatedly and consistently promoted these norms over time. As a result of his consistent promotion, the students in the class eventually took up these norms, as evidenced by focal students’ interview statements. However, it took several months of consistent effort for some norms, such as Norm 1, to apparently be taken up by most students in the class. Like Mr. Anderson, other teachers may be successful in promoting norms if they consistently promote them over time, particularly by using behavior management practices such as modeling, explaining, correcting, and praising behavior.

Because of the great effort that may be required to successfully promote such norms, it seems likely that teachers will be most successful in promoting norms to which they have a strong commitment. For this reason, it seems likely that teacher beliefs are relevant to the norms they promote successfully, as suggested by Santagata and Bray (2015). It may be useful for creators of professional development programs and for teacher educators to attend to teachers’ beliefs in order to support teachers in promoting norms that align with those beliefs.

Finally, though this study was focused on norms related to mathematical errors specifically, these results have implications for norms related to learning from errors in any academic subject or, indeed, in other aspects of life. Many of the norms that emerged as related to the case-study class’s treatment of mathematical errors can be applied to other types of errors, and in fact, the case study class did apply these norms in situations outside of mathematics instruction. For example, while in the classroom I observed the norm *there are different types of errors, only some of which are acceptable* (Norm 2) to be applied to Language Arts instruction. Others of the seven norms identified in this study were also applied to English Language Arts, science, and history lessons, and the teacher indicated that class members applied these norms even to activities that were not strictly academic.
The classroom teacher indicated that he thought of norms for learning from errors as behaviors that generally support success in life, and he said he attempted to support students in thinking the same:

*Interviewer:* And do you feel like the [students] …do you think they’re generally taking on these values?

*Teacher:* I do. For the most part, I do. I think they’re really starting to understand the value of a good challenge. And understanding that challenges are – when they came in in the beginning they thought of challenges as things when I bring out a rope or I bring out some prop or something. I think they’re starting to get the idea that challenges are every minute of every day. That it’s in front of them all the time.

*Interviewer:* And valuing that as something to be accepted and conquered rather than…

*Teacher:* Taken on, yeah. It’s a… “gimme, let’s see what you got.” That kind of attitude. (Teacher Interview, End of Year, April 2015)

In this excerpt, the teacher indicated that he thought that the students in his class had begun to take on the “values” that had been discussed in the class, such as integrity. He said that at the beginning of the school year, students seemed to think of a “challenge” as a particular type of problem-solving situation presented by the teacher, such as “when [the teacher] bring[s] out a rope or [the teacher] bring[s] out some prop or something.” By the end of the school year, the teacher said that students understood “the idea that challenges are every minute of every day,” suggesting that they understood that there is a challenge in how one approaches situations – that there is always the possibility of approaching situations in ways that promote learning. The teacher clearly suggested that norms related to learning from errors could be applied to many situations. It may be useful for other teachers to conceptualize such norms in a similar way.

To summarize, the results of this study have several implications for teaching practice. First, these results indicate that students – even elementary grade children – are capable of engaging in complex practices and adopting challenging norms related to learning from errors. However, these norms can be difficult to promote, and a teacher may need to consistently promote norms over many months before students take them up. Teachers may be most successful in promoting norms to which they are strongly committed. Finally, this study focused exclusively on norms related to the treatment of mathematical errors, but these norms are related to the treatment of many types of errors. Teachers may find success in approaching the promotion of norms in a holistic way, talking to students about and promoting general behaviors that support success in mathematics, in academics, and in life generally.
References


Appendices

Appendix A
Student Survey

Name: _____________________________ Date: ___________________

1. Imagine that a classmate is talking in front of the whole class about how they solved a math problem, and they get the final answer wrong. What do you expect your teacher would do?
   a. tell the student the correct answer, and move on to talking about something else.
   b. call on another student to tell the correct answer.
   c. ask the student who made the mistake questions about how he was thinking about the problem.
   d. ignore the student’s mistake.
   e. none of these. Instead, he would do this:

   ________________________________________________________________
   __________________________________________________________________

2. Imagine that a classmate is talking in front of the whole class about how they solved a math problem, and they get the final answer wrong. What would you do?
   a. raise your hand to say the correct answer
   b. laugh
   c. wait for the teacher to correct my classmate
   d. try to figure out what my classmate was thinking about when he got the answer wrong
   e. none of these. Instead, I would do this:

   ________________________________________________________________
   __________________________________________________________________

3. Imagine that you don’t know the answer to a problem – or even how to start solving it – and the teacher asks you to come up to the whiteboard and show the class how you would solve it. What would you do?
   a. Say you don’t know and hope the teacher moves on to someone else
   b. Try to figure it out really fast, or make something up
   c. Look at another student’s answer and put that up on the board
   d. Say you don’t know how to solve it and ask for help
   e. None of these. Instead, I would do this:

   ________________________________________________________________
   __________________________________________________________________

4. Continuing the imaginary situation from the last question, the teacher makes you come up to the whiteboard anyway, and you start trying to solve the problem but quickly get stuck. What do you expect the teacher to do?
   a. Ask you questions about what you’re doing and thinking
b. Ask another student to come up and solve the problem instead of you

c. Ask you why you weren’t paying attention earlier when he taught you how to do this kind of problem

d. Solve the problem for you

e. None of these. Instead, he would do this:

_________________________________________

__________________________________________________________________
_______________________

5. A classmate gives the wrong answer to a math problem in front of the whole class. You got the same wrong answer! The teacher asks for everyone who got the same answer to raise their hands. What are the chances you’re going to raise your hand? (circle one):

I definitely won’t   I probably won’t   I probably will   I definitely will

6. What kinds of things do you expect to do during math lessons? (check all of the ones you expect)

☐ solve math problems

☐ explain how I solved problems to the teacher (out loud or in writing)

☐ explain how I solved math problems to other students

☐ listen to other students talk about how they solved math problems

☐ take paper-and-pencil tests and quizzes

☐ watch other students solve math problems on the board

☐ watch the teacher solve math problems on the board

☐ read a math textbook

☐ correct my own mistakes on math problems

☐ correct other students’ mistakes on math problems

☐ talk about why I or other students made a mistake on a math problem

☐ play games about math
Appendix B
Student Semi-Structured Interview Protocol

Beginning, Middle, and End of Year Interviews

Thank you for talking with me today. I’ll ask you a few questions about your math class and about learning math. I want you to know that I won’t tell your teacher, parents, or friends how you answer these questions. When I use your answers for my research, I’ll either use a number or a made-up name to identify you – I won’t use your real name.

If you don’t want to answer a question, you don’t have to, and if you want to stop this interview at any point you can let me know and we’ll stop. If you’re not sure what I mean by a question, just ask!

1. Here are some things that you might do in your math class. I’m going to ask you to use this chart to tell or show me by pointing to how often you do each of these in your class this year. Then, I’ll ask you how important you think doing this is for your math learning. [See attached page for frequency/importance Likert scales]
   • How often do you do [activity 1, etc.] in your math class this year? You can choose “never” “about once a month” “about once a week” “about every day” or “several times a day.”
   • How important do you think [activity 1, etc.] is for your math learning? By that I mean, how much does doing [activity 1, etc.] help you learn math? You can choose “not at all” “not much” “a little” or “a lot.”

   Activities:
   1) check my answer to a problem (if it makes sense, is reasonable, fits with the problem)
   2) ask the teacher for help (when I’m stuck or I’ve gotten a problem wrong and don’t know how to fix it)
   3) ask another student for help (when I’m stuck, etc.)
   4) watch the teacher help/coach other students who are stuck or who get an answer wrong
   5) coach other students who are stuck or who get an answer wrong

2. Let’s take a look at the survey that you filled out [earlier today / earlier this week]. I see you circled [answer] for #2. Tell me more about why you chose that answer. *Interviewer asks follow up questions as necessary (i.e., “tell me more,” “explain what you mean by that”)*

3. Continuing with the survey, I see that you circled [answer] for #3. Tell me more about why you chose that answer. *Interviewer asks follow up questions as necessary (i.e., “tell me more,” “explain what you mean by that”)*

4. I also see that you circled [answer] for question #5 on the survey. Tell me more about why you chose that answer.
5. What does it mean when someone gets the answer to a math problem wrong? *(If clarification is needed, can list some common ideas: “they need to work harder to understand math,” “they’re not very good at math,” “everyone gets problems wrong sometimes because it’s part of learning”)*

6. What does it mean to coach or help someone when they get the answer to a math problem wrong? *(If clarification is needed, “If your teacher said he would help or coach someone with a problem they got wrong, what would he do first? What would he do next?”)*

7. Now we’re going to watch a very short video clip of your teacher talking to a class. *Interviewer plays a short video clip of the teacher saying that “mistakes equal learning.”* How serious do you think [teacher’s name] is when he says that? You can choose “not at all serious – he’s kidding,” “not really serious,” “a little serious,” or “definitely serious – he means what he’s saying.” *[After student chooses answer.]* Tell me about why you chose that answer.

Thanks for your time!
### Likert Scales Used During Student Interviews

<table>
<thead>
<tr>
<th>never</th>
<th>about once a month</th>
<th>about once a week</th>
<th>about once a day</th>
<th>more than once a day</th>
</tr>
</thead>
<tbody>
<tr>
<td>not at all</td>
<td>not much</td>
<td>a little</td>
<td>a lot</td>
<td></td>
</tr>
<tr>
<td>serious – he’s joking</td>
<td>not very serious</td>
<td>a little serious</td>
<td>definitely serious – he means what he’s saying</td>
<td></td>
</tr>
</tbody>
</table>
Appendix C
Teacher Semi-Structured Interview Protocol: Beginning of Year Interview

Hi, and thanks for meeting with me today. This will be the first of three interviews for this research study this school year. We’ll meet again in December or January and for the third time in May or April.

I’m going to ask you some questions about your plans for your math class this year and your expectations for how students will talk about math in your classroom, specifically how they talk about errors or partial understandings of math topics.

1. First, let’s talk about your plans for your math class this year. If you have a plan for units you intend to teach or a sequence of topics you intend to cover, I’d appreciate the opportunity to make a copy.

2. During your math classes, what kinds of activities will you expect students to engage in? (If clarification is needed, list examples of whole-class discussions, talking with a partner about math, solving problems with a partner, explaining how they solved a problem, justifying why a solution is correct or incorrect, etc.)

3. Which of these activities [from #2] do you expect to be most challenging for students? Why? Easiest? Why?

4. Specifically, what practices do you want students to engage in related to making errors or having partial understandings in math. (if needed for clarification: For example, what do you expect a student to do if she realizes she got an incorrect answer to a problem? What do you expect other students to do if they realize that a student got an incorrect answer? What about when students are explaining how they solved a problem in from of the whole class and they make mistakes – what do you expect them to do?) Even more specifically, what practices do you want students to engage in when they make an error or a classmate makes an error during a whole-class discussion? (e.g., explaining what they/the other student was thinking, giving the correct answer, justifying why the incorrect answer is incorrect using mathematical principles)

5. Which of these practices [from #4] do you expect to be most challenging for your students this year? Why? Easiest? Why?

6. For the practices you identified as being hardest for students [list from #5 response], how do you plan to support students in developing their abilities to engage in these practices? You can talk about each one separately, or if you use the same strategy to support kids with more than one practice, you can talk about them together.

7. How do you think your students interpret making an error on a math problem – what do you think they think it means right now? Is that what you would like them to think? [If not] What would you like them to think instead?

8. How do you interpret making an error on a math problem?
Teacher Semi-Structured Interview Protocol: Middle/End of Year Interview

Hi, and thanks for meeting with me today. This will be the second/last of three interviews for this research study this school year.

I’m going to ask you some questions about how things are going in your math class this year and how students are doing with classroom discussions about math. Specifically, I’ll be asking about how students talk about errors or partial understandings of math topics.

1. At the beginning of the year, I asked you what kinds of activities you expected students to engage in during math class. You listed the following: speed-drill exercises (Parts A and B), application problems, lessons (in the form of a whole-class discussion), partner or group work, exit tickets or other checks for understanding.

I’ll now ask you to rate how often your class as a whole, in general, is doing each activity using the following scale [see attached]: “never does this or cannot do this”; “does this once a month”; “does this once a week”; “does this about once a day”; “does this multiple times a day.”

Then, I’ll ask you to rate how satisfied you are with the class’s performance, in general, of each activity, given the current point in the school year: “very dissatisfied,” “somewhat dissatisfied,” “somewhat satisfied,” or “very satisfied.”

2. Of the activities that are most challenging for your students, why do you think these are so challenging?

3. At the beginning of the year, I also asked you specifically about what practices you wanted students to engage in related to making errors or having partial understandings in math, particularly during whole class discussions. You listed the following: students should check the reasonableness of their answers, students should be able to ask another student or the teacher for coaching when they are stuck, students should be learning how to coach each other in solving math problems.

I’ll now also ask you to rate how often your class as a whole, in general – or maybe how often the average student in your class – is doing each activity in math using the same scale: “never does this or cannot do this”; “does this once a month”; “does this once a week”; “does this about once a day”; “does this multiple times a day.”

I’ll also ask you to rate how satisfied you are with the class as a whole’s performance of each activity, given the current point in the school year: “very dissatisfied,” “somewhat dissatisfied,” “somewhat satisfied,” or “very satisfied.”

4. Of these practices that are most challenging for your students, why are these so challenging?
5. Similarly, of the practices that your students seem to be doing well, why or how have your students taken these up so well at this point in the school year?

6. How do you think your students interpret making an error on a math problem – what do you think they think it means right now? Is that what you would like them to think? [If not] What would you like them to think instead?

8. Let’s watch a short video clip of a discussion that happened in your classroom. I’d like to hear your thoughts about what was going on during this discussion.

For end-of-year only:
9. I’ve noticed that values, or habits of mind, are really a very strong part of this class. Is there a list of them, or are there some you can name that are really important for you? And how are these usually discussed in the class?
Appendix D

Video Data Analysis Codebook

All coding and subcoding was done using StudioCode software. Coding (i.e., identification of relevant instances) was done using StudioCode “codes” and subcoding was done using StudioCode “labels.”

General coding agreements made by the research team:

- When the teacher makes a correction to multiple students’ behavior for the same behavior at roughly the same time, we decided to code for each of these corrections in a separate instance. Similarly, when multiple students make errors one right after another, we count those as separate instances. When the whole class is making an error (e.g., students calling out an answer), and then a single student also makes an error on the same problem, we count those as separate instances.
- When a student blurts out something, we still code it if appropriate, whether or not the teacher recognizes the statement, except when it is unclear what it means or if anyone heard it at all (i.e., the statement is incoherent or barely audible).
- When identifying relevant instances by coding, make the segment long enough so that it is clear what is going on from just that segment. In other words, include enough context so that a person only seeing that video clip can make sense of the important parts of the clip.
- It is okay if a single segment of video is coded for more than one of the codes below.

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Notes on this Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>Any instance in which a student or the teacher makes an error in mathematics. This error will be in their academic work, not in their classroom behavior. Or, any instance in which errors are explicitly discussed. Also, include any instance in which students or the teacher bring up <em>partial understandings</em>, meaning their incomplete (and at least partially incorrect) ways of thinking about a problem or mathematical situation or at least partially incorrect way(s) of solving a math problem (in this case, a wrong answer may not necessarily be identified, but a student or the teacher may indicate that they do not know how to solve the problem, for example, or may indicate that their process of solving the problem was confused or incorrect).</td>
<td>This code may overlap with the <em>norms</em> code when a person in the class makes an error in their academic work and then the class discusses values or norms in relation to that error. This code can include times when students and/or the teacher are talking about their previous incorrect thinking about something in math. For example, a student might describe how they used to think that the marks on a number line always have to be evenly spaced, but now they understand that they don’t. Note that simple disagreement among students doesn’t necessarily indicate an error unless someone (a student or the teacher) identifies someone’s answer as being wrong. If a bunch of students call out different answers but no one identifies that at least one of them is wrong, then do</td>
</tr>
</tbody>
</table>
not code as “error”. Note that the teacher continuing to ask questions about a problem may be enough to indicate that one or more answers is/are wrong, and in this case you should code as an “error.” Just saying the correct answer may not be enough to identify other answers as wrong. If a student is called on and “has the floor,” and gets an answer wrong, then we code it as an error instance whether or not it is clearly identified as wrong. When a student or multiple students say “oh” or otherwise seem to indicate that they just “got” something, but it is not explicitly clear that one or more of them actually got an answer or problem-solving process wrong, do not code as an error. In a situation where a student or students do not complete a problem or follow the math procedure that the teacher instructed, that counts as an error rather than a norms code.

| Norms | Any instance in which the class explicitly talks about the kind of behavior that is expected in the classroom. Or, any instance in which someone corrects another person’s behavior in a way that reveals something about the general behavioral expectations in the classroom. Additionally, because we are specifically interested in behavior related to “coaching” (helping other students understand a problem or procedure), code any instance in which the teacher or students give instructions related to coaching as “norms,” even if the instruction is somewhat specific to the situation. | May overlap with error code when the explicit talk about norms or expectations is connected to a student’s or the teacher’s error in academic work. Behavior can include both positive and negative things that people do. For example, sitting still, being quiet, and giving a “thumbs-up” are all behaviors. Instances in which the class talks about behavior can include when someone does something wrong and they are corrected, and can also include when someone does something right and they are praised/rewarded. This code should not be applied when someone is giving instructions that are specific to that situation (unless that situation is related to the practice of coaching or helping peers with math). When the teacher apparently corrects a student’s behavior by just saying his/her name but it’s not clear exactly what the behavior was, do not code this instance as |
### Subcodes Applied to Instances coded as *Errors*

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Who identified it as an error (not correct)?</td>
<td><em>You should only use one of the labels in this category for each instance. If more than one seems to apply, then consider who first identified the answer/idea as incorrect. If needed, use the “other” label and we will discuss as a team.</em></td>
<td></td>
</tr>
<tr>
<td>Identify: teacher</td>
<td>The teacher is the first to indicate somehow that the idea/process/answer is not correct. This may be explicit (e.g., saying “that’s wrong”) or he may make it obvious that the answer or idea is incorrect in some other way.</td>
<td>For this label, it doesn’t matter whether it is a student’s idea, the teacher’s own idea, or someone else’s idea.</td>
</tr>
<tr>
<td>Identify: student who made error</td>
<td>The student who made the error also is the first to identify it as an error. This may happen when a student volunteers to share their idea and makes it clear that he/she knows it is incorrect.</td>
<td>This label can only be used when the error is attributed to a student (i.e., “student error” label).</td>
</tr>
<tr>
<td>Identify: other student(s)</td>
<td>One or more students identify the error as incorrect, but these are not the students to whom the idea/answer belong or is attributed. This could mean that a student shares an idea, and then another student says that it is not correct.</td>
<td>Note that a student who simply shares a different idea or answer from one that is shared already is not necessarily identifying that answer as incorrect. In this case, there may be two or more answers or ideas that are proposed, and at some point either the teacher or one or more students will clarify which one(s) are correct or incorrect. Use the appropriate label for who eventually identifies the idea/answer as incorrect.</td>
</tr>
<tr>
<td>Identify: N/A</td>
<td>There is no specific error (e.g., “Tommy multiplied one by two and got three”) or general error (e.g., “you’re all multiplying wrong…”)) in this segment, so there is nothing to identify as wrong.</td>
<td>Note, this will usually be the case only for discussions of errors generally (e.g., discussions of how it’s good to learn from errors and other general discussions about making errors rather than about specific errors or types of errors).</td>
</tr>
</tbody>
</table>
**How does the situation end? (select all that apply)**

<table>
<thead>
<tr>
<th>Resolution: error corrected</th>
<th>The error is corrected by someone in the class (the teacher, one or more other students, and/or the person who made the error).</th>
<th>You must apply this label any time you use one of the following: ESReaction: tell correct answer/ process or ETReaction: tell correct answer/ process.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution: error leads to in-depth discussion</td>
<td>The error itself or something about the topic related to the error leads to an in-depth discussion about the subject matter. The key feature here is that the bringing up of the error leads to the discussion; if the class was already discussing the topic when the error comes up, this label does not apply.</td>
<td>The point of this label is to help us identify situations where errors lead to useful or interesting discussions of subject-matter content.</td>
</tr>
</tbody>
</table>

**Subcodes Applied to Instances coded as Norms**

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why: behavior correction</td>
<td>The teacher or one or more students corrects the behavior of a person in the class. This means that the person was doing something he/she/they should not have been doing, or he/she/they was/were not doing something they should have been.</td>
<td></td>
</tr>
<tr>
<td>Why: behavior reinforcement/praise</td>
<td>The teacher or one or more students thanks or praises a person for showing good/positive behavior.</td>
<td>Note that this may occur with a behavior correction or behavior reinforcement. You may apply both labels in that instance. For example, the teacher may praise a student’s behavior</td>
</tr>
<tr>
<td>Why: explicit discussion of classroom norms</td>
<td>The reason the (s) is/are brought up is because the teacher or one or more students ask or begin talking about the classroom</td>
<td></td>
</tr>
</tbody>
</table>

**Why is the norm brought up? (select all that apply)**

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why: behavior correction</td>
<td>The teacher or one or more students corrects the behavior of a person in the class. This means that the person was doing something he/she/they should not have been doing, or he/she/they was/were not doing something they should have been.</td>
</tr>
<tr>
<td>Why: behavior reinforcement/praise</td>
<td>The teacher or one or more students thanks or praises a person for showing good/positive behavior.</td>
</tr>
<tr>
<td>Why: explicit discussion of classroom norms</td>
<td>The reason the (s) is/are brought up is because the teacher or one or more students ask or begin talking about the classroom</td>
</tr>
</tbody>
</table>

115
norms in a direct way. and then start talking generally about the classroom norms related to that behavior. In this case, you would apply both the \textit{RWhySurfaced: explicit discussion of classroom norms} and \textit{RWhySurfaced: behavior reinforcement/praise} labels.

### Codebook for Candidate Norms-Specific Subcoding

Subcodes ("labels") to be applied to "errors" or "norms" coded instances identified using the codes used in first-cycle coding described above. More than one of the labels below may be applied to any instance.

<table>
<thead>
<tr>
<th>Name of Label</th>
<th>Description of when to apply label</th>
<th>Examples/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>everyone value</td>
<td>Apply to instances in which some reference is made to the expectation that everyone has some valuable or correct ideas in math, or that even though a person made a mistake/error, they still have some correct/valuable/useful mathematical ideas. Also apply to instances in which a statement is made that clearly contradicts these expectations. In general, this label is to be applied when students and/or the teacher talk about or make reference to the correctness or value in people’s mathematical ideas.</td>
<td>The teacher may reference a particular student’s mathematical strategy as valuable/useful.</td>
</tr>
<tr>
<td>multiple strategies</td>
<td>Apply to instances in which the teacher and/or student(s) make reference to the idea that there are multiple strategies that can be used to solve a math problem. For example, if the class elicits more than one idea for how to approach a math problem. Also apply to instances that expressly contradict this idea. Note that simply showing one strategy does not necessarily contradict the expectation that multiple strategies exist; rather, some explicit reference to only one strategy being possible must be made.</td>
<td></td>
</tr>
<tr>
<td>learn from mistakes</td>
<td>Apply to instances in which the teacher and/or student(s) make reference to the idea that one should learn from mistakes (or other evidence that they don’t yet know/understand something, such as not knowing how to answer a problem) either to increase one’s understanding of mathematics or to emphasize on \textit{learning} from mistakes</td>
<td></td>
</tr>
<tr>
<td>make sure that one doesn’t keep making the same errors. This may be combined with the idea that it’s okay to make mistakes while learning. Also apply to instances in which the teacher and/or students reference an idea that is expressly contradictory to the expectation that one should learn from mistakes.</td>
<td></td>
<td></td>
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<tr>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>mult types errors</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply to instances in which the teacher and/or student(s) make reference to one or another types of errors: (a) errors you make while learning, which are okay, or (b) careless errors, which should be avoided. The class may make reference to one or the other or both types. Or, the class may describe another type of error. Note that some reference to the type of error and/or whether or not it is okay (accepted) in the given context is necessary to apply this label. Also apply to any instances that expressly contradict the expectation that there are multiple types of errors. For example, if the teacher or students reference an idea that all errors are the acceptable, or that only one type of error exists.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>admit wrong get help</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply to instances in which the teacher and/or student(s) make reference to the idea that it is important to admit (to yourself, to others) when you are wrong or have made and error or don’t understand something. This may also be combined with an explanation of why it is important to admit that you are wrong (to get help, to understand why you are wrong, etc.). Also apply to instances in which the teacher and/or student(s) state that they do not understand something, need help, or got an answer wrong. Also apply to instances in which the teacher and/or students make reference to an idea contradictory to the expectation that you should admit when you are wrong.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>take risks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply to instances in which the teacher and/or student(s) make reference to the idea that taking risks is valued. This may be coupled with an explanation of why risks are valued (e.g., to support learning). Also apply to instances in which the teacher and/or students make reference to a contradictory expectations, such as an expectation that risks should be avoided or are unacceptable.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>don’t give up</strong></td>
<td>Apply to instances in which the teacher and/or student(s) make reference to the idea that the teacher and/or student(s) should not give up on a student who is struggling or who doesn’t understand something or who makes a mistake. This label may also be applied to instances in which a student makes a mistake and the teacher or another student helps them figure out their mistake and how to correct it (i.e., not simply telling them the answer) AND checks for their learning/understanding of the correct solution/strategy. It may also be applied to instances in which there is reference made to not giving up on oneself while learning. Also, apply this label to instances in which the teacher and/or students express a contradictory expectation, such as giving up on themselves or others who are struggling.</td>
<td></td>
</tr>
</tbody>
</table>