CONSTITUTIVE MODEL FOR THE DRAINED COMPRESSION
OF UNSATURATED CLAY TO HIGH STRESSES

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Abstract: A constitutive model is presented in this paper to describe the isotropic compression response of unsaturated, compacted clay under drained conditions over a wide range of mean effective stresses. The model captures the key transition points of the compression curves at different stress levels, ranging from the preconsolidation stress, to pressurized saturation, to the initiation of void closure. The results from drained, isotropic compression tests on compacted clay specimens having different initial degrees of saturation up to a mean total stress of 160 MPa were used for model calibration. The suction hardening effect on the preconsolidation stress and the nonlinear compression curve of unsaturated clay up to the point of pressurized saturation were captured using an extended form of an existing effective stress-based constitutive model. For higher mean stresses, an empirical relationship to consider the transition to void closure was incorporated to fit the observed compression curves of the compacted clays specimens. The transition to void closure was found to be affected by the initial compaction conditions despite the fact that all of the specimens were pressure-saturated in this mean stress range.

INTRODUCTION

An understanding of the compression of soils to high stresses is required in the simulation of applications such as buried explosives, penetration testing, and sub-bases for high-speed rail. Although many of these applications involve high loading rates, the equilibrium compression curve defined by using the quasi-static compression testing is commonly used in the analyses of

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these problems, such as in the Hybrid-Elastic-Plastic (HEP) constitutive model developed by Zimmerman et al. (1987) and Akers et al. (1995). Although the HEP model has been calibrated using 1-D compression tests on dry soils to mean stresses up to 1000 MPa, it is not capable of considering the behavior of unsaturated soils. Several efforts have been made to simulate the volume change of unsaturated soils considering changes in mean net stress and suction (Alonso et al. 1990; Josa et al. 1992; Wheeler and Sivakumar 1995) or changes in effective stress (Bolzon et al. 1996; Loret and Khalili 2002; Wheeler et al. 2003; Khalili et al. 2008), but in all of these studies the soil behavior has only been evaluated up to mean stresses of 10 MPa. The current hydro-mechanical models are capable of reproducing essential features of the behavior of unsaturated soils such as the change in preconsolidation stress and the change in slope of the virgin compression line (VCL), but they usually do not consider phenomena encountered at high stresses such as pressurized saturation and the transition to void closure. Further, it is difficult to extend these models to high stresses using the available data in the literature, as only a few studies have considered changes in the degree of saturation during drained compression under constant suction conditions (Lloret et al. 2003).

This study builds upon a testing approach developed by Mun and McCartney (2015) to consider the important transition points in the isotropic compression response of unsaturated, compacted clay to high stresses under drained conditions, including the mean apparent preconsolidation stress, the stress at pressurized saturation, and the transition to void closure. All of these points may depend on the initial degree of saturation and compaction conditions. Mun and McCartney (2015) found that the drainage conditions play a major role in the compression response of unsaturated soils to high stresses, due to the compressibility of air, interactions between the air and water phases, and the lower hydraulic conductivity of unsaturated soils.
compared to the saturated soils. Using data collected from this testing approach, this study seeks to establish a constitutive model for the compression of unsaturated, compacted soils to high stresses under drained conditions. The approach followed to define the compression curve was to extend established models up to high stresses for each of the key transition points. Specifically, a generalized effective stress-based relationship between the mean preconsolidation (yield) stress and the point of pressurized saturation developed by Zhou et al. (2012a, 2012b) was used to represent the drained compression behavior of unsaturated soil. Their model, which uses the effective saturation ($S_e$) instead of the matric suction ($\psi$) as the primary variable to describe the nonlinear evolution in the compression curve, is extended in this study to consider the phenomena of pressurized saturation by modifying the equation for the compression index of unsaturated soil after the effective saturation approaches 1.0. Further, the model of Zhou et al. (2012a, 2012b) is combined with a new empirical relationship to consider the transition to void closure in the compression curve at extremely high stresses beyond the point of pressurized saturation. The extended model was calibrated using the results from a series of drained, isotropic compression tests to a mean total stress of 160 MPa on unsaturated, clay specimens compacted to reach initial degrees of saturation ranging from 0.6 to 1.0 but the same initial void ratio. The same matric suction measured in the compacted specimens was imposed during drained compression using the axis translation technique, and the evolution in the degree of saturation of the unsaturated specimens were monitored during the tests.

**BACKGROUND**

**Compression of Unsaturated Soils to High Stresses**

Similar to saturated soils, unsaturated soils change in volume due in response to changes in effective stress. Because unsaturated soils are a three-phase system, mechanical deformation and
hydraulic changes of unsaturated soil take place simultaneously under external loads, with the
degree of saturation playing an important role in the soil behavior (Sun et al. 2010). To account
for the role of degree of saturation in volumetric response, the effective stress approach of Bishop
(1959) was adopted in this study. In this case, the mean effective stress $p'$ in unsaturated soil can
be expressed as follows:

$$p' = p_{\text{net}} + \chi \psi$$  \hspace{1cm} (1)

where $p_{\text{net}}$ is the mean net stress equal to the mean total stress in excess of the pore air pressure
(i.e., $p-u_a$), $\psi$ is the matric suction ($u_a-u_w$), and $\chi$ is the effective stress parameter which is assumed
to be equal to the effective saturation for simplicity ($\chi = S_e$) following the approach of Bolzon and
Schrefler (1995). The effective saturation is defined as follows:

$$S_e = \frac{S_r - S_{r,\text{res}}}{1 - S_{r,\text{res}}}$$  \hspace{1cm} (2)

where $S_r$ is the degree of saturation and $S_{r,\text{res}}$ is the residual saturation.

Based on the observations from compression tests on compacted clay to high stresses by Mun
and McCartney (2015), hypothetical representations of the drained, isotropic compression curves
of unsaturated clay under different initial conditions are shown in Figure 1. Initial compression of
compacted, unsaturated soil is essentially elastic until reaching an apparent preconsolidation stress.
The initial compression response follows a recompression line (RCL) with having a slope $\kappa$ that
does not depend on the suction magnitude until reaching the mean effective preconsolidation stress
$p'_c (A'-B')$. The value of $p'_c$ depends on both the soil structure associated with compaction (typically
quantified by a compaction water content wet or dry of optimum) as well as the amount and
distribution of water in the specimen (typically quantified by the matric suction or degree of
saturation). After passing the preconsolidation stress, the slope of the VCL for unsaturated soils
may also depend on the magnitude of matric suction or degree of saturation. Some studies have observed that the slope of the VCL of unsaturated soils differs from that of unsaturated soils, but does not depend on the magnitude of suction (Sivakumar 1993; Uchaipichat and Khalili 2009). Other studies have proposed that the slope of the VCL for unsaturated soils will decrease with suction (Alonso et al. 1990) or will vary nonlinearly with effective saturation (Zhou et al. 2012a). Regardless of the trends in the slope of the VCL with matric suction, the results of Mun and McCartney (2015) indicate that the VCLs for unsaturated soils should converge with the VCL for saturated soil at the point of pressurized saturation where air is either expelled from the soil in drained conditions or locally dissolved into the pore water (Hilf 1948). The hypothetical curves in Figure 1 follow the trends in the limited set of data to high stresses presented by studies such as Jotisankasa et al. (2007) and Mun and McCartney (2015). The point of pressurized saturation for a soil will occur at a given mean effective stress depending on the initial degree of saturation of the soil (air-void closure, Point C'). After pressurized saturation, it is expected that the soils under constant suction conditions will follow a single compression curve (C'-D'). Mun and McCartney (2015) found that at extremely high mean effective stresses (i.e., greater than approximately 10 MPa) the shape of the compression curve follows a different shape (D'-E') than the bilinear compression curves conventionally used in soil mechanics. Specifically, the slope of the VCL appears to become steeper with increasing mean effective stress as shown in Figure 1. However, this is an artifact of plotting the compression curve on a logarithmic scale, and will actually follow an exponential decay trend if the mean effective stress is plotted on a natural scale. At high stresses, the soil will start to transition toward void closure, which may be induced by the deformation of the clay particles to close the voids, or drainage of hydroskopically-bound water (Point E'). If the unsaturated specimens having different initial degrees of saturation all have the same initial soil
structure, then the curve at high stresses is expected to be the same as that of the saturated specimen, as shown in Figure 1.

Mun and McCartney (2015) observed that the transition to void closure may depend on the initial soil structure induced by compaction. Specifically, they evaluated specimens that had different initial degrees of saturation associated with compaction (and different soil structures), and even though all of the unsaturated specimens were pressure-saturated at high stresses the shapes of their compression curves differed. It should be noted that the hypothetical curve in Figure 1 purposely does not extend to an equilibrium void ratio at void closure, but only shows the transition to void closure due to the current lack of data on full void closure. In other words, due to current experimental limitations, it is not clear whether soil specimens will approach a non-zero void ratio when compressed to extremely high mean stresses, or if they will approach a zero void ratio at some finite mean stress. Although Akers (2001) tested dry sands and mentioned that the soil specimen reaches a void closure, it was found that the compression curve for sand reaches an asymptotic value at mean stresses of 600 MPa and the conditions describing full void closure (i.e., a zero void ratio or a finite void ratio) were not clarified.

As the impact of suction on the mean effective stress in Equation (1) may become negligible when reaching high mean total stress values, especially when the soil is close to saturated conditions, the compression curves in terms of mean total stress and mean effective stresses may have a similar shape during drained compression to stresses above the point of pressurized saturation. Although a total stress analysis could be used, an effective stress analysis is used for the full range of stresses for consistency. This observation should not be generalized to undrained compression, as mean total and effective stresses will be very different due to pore fluid compression (Mun and McCartney 2016).
Limitations of Existing Elasto-Plastic Models for Unsaturated Soils

Before developing a new constitutive model for unsaturated soils, it is important to consider if available constitutive relationships for the isotropic compression response of unsaturated soils can be extended to high stresses. Most of the commonly-used constitutive models are based on the Cam-Clay model using independent stress state variables (Alonso et al. 1990; Josa et al. 1992; Cui et al. 1995; Wheeler and Sivakumar 1995; Sheng et al. 2008), or effective stresses (Wheeler et al. 2003). The Barcelona Basic Model (BBM) presented by Alonso et al. (1990) is one of the most widely used models for unsaturated soils in terms of independent stress state variables. Although the BBM is capable of considering the role of suction hardening (e.g., changes in the preconsolidation stress with suction), problems have been encountered in matching the model to experimental data (Wheeler et al. 2002). However, one of the fundamental issues in the BBM that prevents it from being extended to high stresses is that the relationship between preconsolidation stress and suction is linked to the slope of the VCL for unsaturated soils such that the VCLs all converge at a single point. For the model parameters selected by Alonso et al. (1990), the VCLs will start from a single point at low mean net stresses and will diverge at high stresses. This does not permit the phenomenon of pressurized saturation to be simulated. Wheeler et al. (2002) showed that it may be possible to choose parameters to make the VCLs converge at high stresses, but they will all converge at a single mean net stress. This also does not permit the phenomenon of pressurized saturation to be simulated as higher stresses are required to reach pressurized saturation for soils with a lower initial degree of saturation than those that are initially closer to saturation.

Several other studies have considered the compression response of unsaturated soils using the Cam Clay model but describing the stress state using the generalized-effective stress (Kohgo et al. 1993; Bolzon et al. 1996; Loret and Khalili 2002; Gallipoli et al. 2003; Wheeler et al. 2003;
Georgiadis et al. 2005; Tarantino 2007; Khalili et al. 2008; Sun et al. 2010). These effective-stress models successfully capture the complex volume change behavior of unsaturated soils under mean effective stresses less than 10 MPa, and use a smaller number of material parameters than required when using the independent state variable approach. Furthermore, the effective-stress models permit consideration of the role of hydraulic hysteresis (Wheeler et al. 2003) by incorporation of the soil-water retention curve (SWRC) into the model formulation. Although the development and calibration of these different volume change models were focused on reproducing key features of unsaturated soils such as suction-induced hardening, collapse/swelling behavior, and the influence of hydraulic hysteresis, these models haven’t been used to evaluate pressurized saturation or the behavior at high stresses.

The studies of Zhou et al. (2012a, 2012b) used the concept of bounding-surface plasticity to consider the nonlinear compression curves commonly exhibited by unsaturated, compacted soils. Zhou et al. (2012a) proposed a volume change equation and an approach to describe the change in the effective saturation during drained compression to define the compression curve of unsaturated soils. Although they do not fully support the unsaturated soil behavior at high stresses, the possibility for the pressure-saturation of unsaturated soils under constant suction condition can be considered in their model. Accordingly, this model was used as the basis of the new constitutive relationship for high stresses developed in this study.

**EXPERIMENTAL APPROACH**

**Testing Material**

A low plasticity clay (CL), referred to as Boulder clay, was selected as a test material for this study (liquid limit of 41, plastic limit of 18, and plasticity index of 23). The specific gravity $G_s$ was measured to be 2.70. The maximum dry unit weight corresponding to the standard Proctor
compaction effort 17.4 kN/m$^3$ and the optimal water content is 17.5%. The soil water retention curve (SWRC) for the Boulder clay specimen used in the compression tests was inferred using the Transient Water Release and Imbibition Method (TRIM) of Wayllace & Lu (2012). The SWRC for Boulder clay is shown in Figure 2, along with the parameters $\alpha_G$ and $n_G$ of the van Genuchten (1980) SWRC model.

**High Pressure Isotropic Testing of Unsaturated Clay**

A series of suction-controlled isotropic compression tests under mean total stresses up to 160 MPa were conducted for clay specimen with different initial degrees of saturation ($S_r$ of 1.00, 0.92, 0.84, 0.72, 0.62) under drained conditions. Specimens having a diameter and height of 71.1 mm were compacted at different gravimetric water contents to a dry unit weight of 17.5 kN/m$^3$, which corresponds to the initial void ratio of an estimated value of 0.51. This approach permits consideration of the role of the initial degree of saturation and compaction conditions on the compression response. Details of the specimens are summarized in Table 1. The specimen with $S_r$ of 1.00 was prepared using a similar compaction water content as the specimen with $S_r$ of 0.92. Specifically, the specimen with $S_{r,0}$ of 1.0 was compacted to an initial void ratio of 0.508 with a compaction water content of 17.3% then saturated to a water content of 18.9% using upward imbibition within the isotropic cell under vacuum after which a back pressure of 210 kPa was applied, while the specimen with $S_{r,0}$ of 0.92 was prepared to an initial void ratio of 0.506 with compaction water content of 17.2%. This led to slight differences in the initial effective saturation at compaction (0.890 for the saturated specimen and 0.887 for the specimen with $S_{r,0}$ of 0.92), which has an impact on the compression curves of these specimens at high stresses as will be discussed later.
The experiments were performed in a high pressure isotropic loading apparatus developed by Mun and McCartney (2015), who provide a detailed description of the device, testing procedures, and system calibration. The device is capable of controlling the total stress and tracking specimen volume changes using a high-pressure syringe pump, and permits control of the suction in unsaturated specimens using the axis translation technique with back-pressure saturation. Specifically, positive pore air and water pressures are applied independently to the base of the specimen through a sintered metal porous disk and a high-air entry ceramic disk, respectively. All water was de-aired prior to testing. Although air will gradually diffuse into the pore water during the compression tests due to their duration, this is not expected to have a major effect on the drained compression response as the pore air and pore water are free to move in and out of the specimen during compression and the pore air is never constrained.

In this study, the suctions applied to the specimens correspond to the initial suctions measured in the compacted specimens using a UMS T5 tensiometer following procedures described by Mun and McCartney (2015). Skempton’s B parameter was measured to be greater than 0.95 for the saturated specimen before drained compression was started. The initial suction values for the specimens measured using the tensiometer are shown in Figure 2. These values correspond well with the drying-path SWRC. After suction application, mean total stresses were applied isotropically to the specimens at a constant rate using the syringe pump up to a cell pressure of 160 MPa. Although this stress range may not be sufficient to reach full void closure, it is sufficient to see the transition in the shape of the compression curve toward void closure. A constant volumetric strain rate of 1%/hr was applied up to a stress of 10 MPa, after which the rate was reduced by a factor of 10 to 0.1%/hr for the remainder of the compression test. This approach considers the fact that the drainage rate decreases as the void ratio and hydraulic conductivity decrease. As will be
noted in the discussion of the results, the slower rate was still not sufficient to ensure fully drained conditions at the highest mean stresses.

**Drained Compression Curves**

The drained compression curves for Boulder clay specimens having initial degrees of saturation ranging from 1.0 to 0.6 are shown in Figure 3(a) with the mean effective stress plotted on a logarithmic scale. The curves in this figure confirm the generalized shapes of the compression curves of unsaturated soils shown in Figure 1, with regards to the effect of suction hardening on the mean effective preconsolidation stress (i.e., an increase in $p'_c$ for specimens with lower $S_{r,0}$), the increase in slope of the VCLs for specimens with lower $S_{r,0}$, the occurrence of pressurized saturation at greater mean effective stress values for specimens with lower $S_{r,0}$, and a deviation in behavior from the conventional bi-linear compression curve to an asymptotic relationship reflecting the transition to void closure. The compression curves for the unsaturated specimens converge with the compression curve for the saturated specimen at mean effective stresses ranging from approximately 1000 to 6000 kPa. During an unloading cycle at 30 MPa, each specimen showed a relatively linear unloading response with the same slope as the initial portion of the compression curve. When reloading the specimen up to 160 MPa after this unloading cycle, the specimens exhibited a compression curve that has a different shape than that expected from the conventional bilinear compression curves that is proposed to represent the transition to void closure. An asymptotic transition to void closure at high mean effective stress is more clearly observed when plotting these same compression curves with the mean effective stress on a natural scale, as shown in Figure 3(b). At high stresses, the trends in the compression curves indicate that the specimens with lower initial degrees of saturation (i.e., specimens compacted dryer of optimum) have a slower transition to void closure, likely due to the initial soil structure associated
with compaction. During unloading after compression to 160 MPa, a nonlinear unloading curve is observed that gradually approaches the slope of the curve observed when unloading from 30 MPa. As will be discussed later, this nonlinearity is likely observed because unloading was too fast for the specimen to draw water back into the smallest pores due to the reduction in hydraulic conductivity at extremely high stresses.

During drained compression of a saturated soil specimen in general, it is expected that the change of the volume of voids $\Delta V_v$ will be equal to the change in the volume of water flowing out of the specimen $\Delta V_w$ as all of the voids are filled with water. On the other hand, the change in the volume of voids for an unsaturated soil specimen in general will likely be affected first by the compression of the air voids, after which a more substantial amount of water outflow will occur upon reaching the point of pressurized saturation. A plot of $\Delta V_v$ versus $\Delta V_w$ is shown in Figure 4(a) for the saturated and unsaturated specimens of Boulder clay. For most of the stress range, a 1:1 relationship between the two variables is observed for the saturated specimen. As expected, the results for unsaturated specimens show a significant change in the volume of voids at the beginning of stress application with negligible water outflow. However, the trend in the curves for the unsaturated specimens approach a 1:1 slope after the points of pressurized saturation. At high mean stresses both the saturated and unsaturated specimens deviate upwards from the 1:1 line, which may be due to the lack of full drainage during compression. During subsequent unloading, all of the curves show a steep decrease in the volume of voids combined with additional water outflow instead of water absorption. This unloading behavior further indicates that the excess pore water pressures from application of the highest stresses were not completely dissipated.

There are several implications of this loading rate on the shape of the compression curves. First, it is likely that the final void ratio at the highest mean effective stress will be overestimated
as complete drainage did not occur. Preliminary tests found that the final void ratio for the saturated specimen following the two-rate testing approach followed in this study was 30% lower than that from a test performed at a volumetric strain rate of 1%/hour for the entire range of stresses. Although this percent difference is relatively high, the difference in the magnitude of the final void ratio is relatively small, on the order of 0.03. Second, the nonlinear shapes of the curves when unloading from 160 MPa observed in Figure 3(a) are likely due to the combination of the facts that the specimens were not able to fully reabsorb water and because excess pore water pressure from the preceding compression phase had not fully dissipated. Although the rate of 0.1%/hour for the higher stresses was selected for practical considerations, the optimal rate for a constant rate of strain testing could be defined using the enhanced CS2 model proposed by Fox and Pu (2012), which considers changes in hydraulic conductivity with void ratio.

The curves in Figure 4(a) can help better understand the process of pressurized saturation. Specifically, the initial volume of air in each clay specimen calculated from phase relationships could be compared with the volume of voids obtained from the bending points in the curves for the unsaturated specimens in Figure 4(a) where appreciable water outflow started. The comparison between these volumes is shown in Figure 4(b). The 1:1 relationship in this figure indicates that the initial compression response is dominated by compression of the air-filled voids, and that the bend in the curve corresponds approximately to the point of pressurized saturation. Because the deviations of the curves from the 1:1 line in Figure 4(a) do not occur until reaching relatively high stresses, the change in the volume of water outflow shown in Figure 4(a) can be used with confidence when interpreting the changes in effective saturation and mean effective stress during compression below the point of pressurized saturation.
Another interesting comparison that can be drawn from the curves in Figure 4(a) is a comparison between the initial volume of water in each specimen and the water outflow after reaching the maximum mean stress of 160 MPa, as shown in Figure 4(c). As expected, the results indicate that the volume of water outflow is lower for the specimens with lower initial degrees of saturation. However, at the highest mean stress of 160 MPa, the initially unsaturated specimens with lower values of $S_{r,0}$ have a greater fraction of the initial water in their pores than the saturated specimen. This may be due to the differences in soil structure of the specimens compacted drier of optimum with respect to their given compaction efforts.

**DRAINED COMPRESSION MODEL FOR UNSATURATED CLAY**

A goal of this study is to extend the effective stress-based constitutive model of Zhou et al. (2012a, 2012b) to capture the key transition points of unsaturated soil behavior over a wide range of stress levels. First, the components of the constitutive model that are based on the calibrated Zhou et al. (2012a) model are summarized. Next, the experimental compression curves presented in Figure 3 are used to calibrate the parameters for the volume change equation of Zhou et al. (2012a, 2012b). Further, the component of their model that predicts the changes in effective saturation during compression was modified to better predict the points of pressurized saturation observed in the compression tests. To consider the observed behavior for the compacted clay specimens at high mean effective stresses, a new empirical relationship for the transition to void closure was then defined and combined with the model of Zhou et al. (2012a, 2012b) to predict the drained compression curves over the full range of mean effective stresses.

**Zhou et al. (2012) Model Formulation and Calibration**

The drained compression curves shown in Figure 3(a) indicate that the unsaturated soils exhibit elastic behavior until reaching a mean effective preconsolidation stress ($p'_c$). Similar to
conventional elasto-plastic frameworks, Zhou et al. (2012a) assumed that changes in void ratio in
the elastic region can be expressed as follows:

$$\Delta e = \kappa \cdot \ln \frac{p'_{f}}{p'_{i}} - 1 \quad p'_{0} \leq p'_{i} < p'_{f} \leq p'_{c}$$

where $\kappa$ is the elastic compressibility index, $p'_{0}$ is the initial mean effective stress at the beginning
of compression, and $p'_{i}$ and $p'_{f}$ are initial and final values representing an increment in mean
effective stress in the elastic region. A value of $\kappa$ equal to 0.005 was found to be representative of
the compression curves for Boulder clay in Figure 3(a), and it is assumed that $\kappa$ does not depend
on the initial effective saturation. Zhou et al. (2012b) proposed a relationship for the
preconsolidation stress of unsaturated soils having a given value of $S_{e}$, defined as follows:

$$p_c' = \left( p_{c,sat}' \right) ^{\beta}$$

where $p_{c,sat}'$ is the mean preconsolidation stress obtained from the compression curve for a
saturated specimen, equal to 110 kPa for the saturated Boulder clay specimen, and $\beta$ is a parameter
that depends on the effective saturation, as follows:

$$\beta = \frac{\lambda_0 - \kappa}{\lambda(S_e) - \kappa}$$

where $\lambda(S_e)$ is a compressibility parameter that can be defined as follows:

$$\lambda(S_e) = \lambda_0 - (1 - S_e)^{\alpha_1} \cdot (\lambda_0 - \lambda_d)$$

where $\lambda_0$ is the compression index describing the VCL for saturated soil ($S_e = 1.0$), $\alpha_1$ is a fitting
parameter that defines the variation of compression index with the effective saturation, and $\lambda_d$ is
the compression index for the soil under residual saturation conditions.
During drained compression of unsaturated soils, the value of $S_e$ will increase because the change in the volume of voids is typically greater than the water expelled from the voids (i.e., the air-filled voids are compressed before the water-filled voids). Zhou et al. (2012a) proposed that the change in the effective saturation $\Delta S_e$ caused by changes in matric suction or volume can be quantified using the following expression:

$$
\Delta S_e = \frac{\Delta S_e}{\Delta y} + \frac{D_e \Delta e}{1 + e_0}
$$

(7)

where the first term on the right-hand side describes the changes in effective saturation with suction, which can be ignored during drained compression under constant matric suction, and the second term describes the volumetric strain during isotropic compression. $D_e$ is a hydro-mechanical interaction function that depends on the current values of effective saturation and void ratio, as follows:

$$
D_e = \frac{S_e (1 + e)}{e} (1 - S_e)^{a_2}
$$

(8)

where $a_2$ is a fitting parameter that defines the variation of $S_e$ under constant suction.

Zhou et al. (2012a) noted that the increase in soil compressibility with increasing effective saturation during compression corresponds to a debonding effect, and that the compression index of an unsaturated soil compressed under a constant suction will approach the value for saturated conditions. This will cause the specimen to transition from VCLs having slopes described by Equation (6). Specifically, change in void ratio for mean effective stresses greater than the preconsolidation stress can be quantified as follows:

$$
\Delta e = \frac{\lambda(S_e)}{p'} \cdot \frac{\Delta p'}{p'} + \frac{\Delta \lambda(S_e)}{\Delta S_e} \cdot \ln p' \Delta S_e - 1
$$

\[ \text{for } p'_c \leq p' \leq p'_{ps} \]

$$
= \frac{\lambda(S_e)}{p'} \cdot \frac{\Delta p'}{p'} + a_1 \cdot (\lambda_0 - \kappa) \cdot (1 - S_e)^{\psi-1} \cdot \ln p' \Delta S_e - 1
$$

(9)
where $p'_{ps}$ is the mean effective stress at the point of pressurized saturation that will be discussed later, $\Delta p'$ is an increment in mean preconsolidation stress between $p'_c$ and $p'_{ps}$, and $\Delta S_e$ is defined using Equation (7).

The $a_1$ parameter of the Zhou et al. (2012a) model was first calibrated using the compression curves for the saturated specimen of Boulder clay and an unsaturated specimen of Boulder clay for which the changes in effective saturation were calculated during drained compression. In the model of Zhou et al. (2012a), unsaturated specimens will experience an increase in $S_e$ and transition between different virgin compression lines whose slopes are defined by Equation (6) and are defined as follows:

$$ e = N - \lambda(S_e) \ln p' - 1 $$

where $N$ is an intercept value at $p'=1$ that does not depend on the effective saturation that can be defined from a compression test on a saturated specimen. Using the compression curve for the saturated specimen of Boulder clay, a value of $N$ equal to 1.65 and $\lambda_0$ equal to 0.032 were found to provide a good fit. Next, data points obtained from the compression curve for the unsaturated specimen having an initial $S_{e,0}$ of 0.89 (initial $S_{r,0}$ of 0.92) were first selected, having values of $S_e$ equal to 0.89, 0.92, 0.95, 0.99, and 1.00, as shown in Figure 5(a). A value of $a_1$ equal to 0.28 was found to lead to a good match between the intersection of the VCLs defined with Equation (10) having slopes $\lambda(S_e)$ described by Equation (6) with the points on the experimental compression curve for the unsaturated specimen having those values of $S_e$. The evolution of the calibrated compression index relationship $\lambda(S_e)$ is shown in Figure 5(b), indicating a nonlinear increasing trend with $S_e$. A challenge in defining the value of $a_1$ was the definition of $\lambda_d$. Although Zhou et al. (2012a) proposed that $\lambda_d$ can be assumed to equal the elastic compression index $\kappa$ for fine-grained soils, this did not lead to a good match with the test results in Figure 3(a). As a compression
test for Boulder clay under residual saturation was not available, a best fit value of $\lambda_d$ equal to 0.023 was found to provide a good match for all of the other compression curves. Using these calibrated parameters, the experimental compression curves for specimens with different values of $S_{e,0}$ are plotted in terms of $e$-log$p'$ along with lines having slopes predicted from Equation (6) for different effective saturation values in Figure 5(c). The VCL lines for each $S_e$ value intersect the experimental compression curves for the specimens with different values of $S_{e,0}$ with the corresponding measured values of $S_e$ on each curve.

Using the calibrated parameters, the predicted preconsolidation stress values are compared with the experimental values for specimens with different initial effective saturations in Figure 6(a), and a good match between the predicted and experimental values is observed. In addition, the predicted compression curves for the range of mean effective stresses in Equation (9) are shown in Figure 6(b). One issue with the model of Zhou et al. (2012a) is that they mentioned that the compression line for an unsaturated soil will gradually approach that of the saturated soil, but will never completely intersect. However, the experimental data in Figure 3(a) confirms that the curves for saturated and unsaturated specimens will converge at the point of pressurized saturation. To investigate the model of Zhou et al. (2012a) at high stresses, the predicted compression curves in Figure 6(b) were used to calculate the changes in effective saturation using Equation (7). These predictions were then compared with the measured changes in effective saturation to define the value of $a_2$. Although the point of pressurized saturation (i.e., $S_e = 1$) may be estimated using Equations (7) and (9) by finding the change in void ratio required to reach a change in $S_e$ that would lead to saturation, a limitation of the model of Zhou et al. (2012a) is that it does not provide a continuous expression for the compression process up to and beyond the point of pressurized saturation. Specifically, for lower values of $a_2$, the predicted effective saturation will exceed 1.0.
On the other hand, the predicted value of $S_e$ will never converge to 1.0 for the case of higher values of $a_2$. Furthermore, the predicted values suddenly drop after reaching a maximum value due to the form of Equation (7). The effect of the fitting parameter $a_2$ on the changes in effective saturation during compression is shown in Figure 7(a). Accordingly, the predicted $S_e$ should be fixed to 1.0 after reaching the point of pressurized saturation which can be considered by adding the following condition to Equation (7):

$$
\Delta S_e = \begin{cases} 
\frac{\Delta S_e}{\Delta \psi} + \frac{D_s \Delta e}{1 + e_0} & \text{for } S_e < 1.0 \\
0 & \text{for } S_e = 1.0
\end{cases}
$$

Based on this assumption, the changes in effective saturation for unsaturated clay under different initial conditions were predicted in Figure 7(b). A fitting parameter $a_2$ equal to 0.035 was found to provide a good match to the trends in effective saturation during drained compression of unsaturated specimens of Boulder clay. A good match is observed for all of the unsaturated Boulder clay specimens except for the driest specimen (i.e., $S_{e,0}$ of 0.47), although the difference for this specimen is not significant. Equation (7) can also be combined with Equation (9) to define the relationship between the mean effective stress and effective saturation, as shown in Figure 7(c). This figure can be used to define the points of pressurized saturation (i.e., the value of $p'_s$ at which $S_e$ equals 1.0) for specimens with different initial effective saturations. A good match is observed between the measured $p'_s$ values and the predicted values from model of Zhou et al. (2012a) subjected to the additional condition in Equation (11).

**High Stresses Model Formulation and Calibration**

As explained above, the compression curves for unsaturated soils are expected converge with the saturated compression curve after reaching the point of pressurized saturation. Inspection of the compression curves in Figure 3(a) indicates that after reaching the respective value of $p'_s$ for
each specimen, the compression curves for all of the specimens (regardless of the initial effective
saturation) will have a slope $\lambda_0$ with increasing mean effective stress between $p'_{ps}$ and a given mean
effective stress that can be referred to as the mean effective stress at void closure initiation, $p'_{vci}$. Accordingly, the compression curve for all soils in this stress range can be expressed as follows:

$$\Delta e = \lambda_0 \cdot \ln \frac{p'_{f}}{p'_{i}} - 1 \quad (p'_{ps} \leq p'_{i} < p'_{f} \leq p'_{vci})$$

where $\lambda_0$ is the slope of the VCL for saturated soil, and the value of $p'_{ps}$ can be defined using curves such as those in Figure 7(c) for a specimen with a given $S_{e,0}$. The value of $p'_{vci}$ is assumed to depend on the initial soil structure. Although the all of the unsaturated specimens of Boulder clay are pressure saturated in the region described by Equation (12), their behavior deviates from the conventional bilinear shape of the compression curve described by Equation (12) at different values of $p'_{vci}$. Based on the compression curves in Figure 3(a), it was observed that the value of $p'_{vci}$ is related with the effective saturation at compaction $S_{e,compaction}$, as follows:

$$p'_{vci} (S_{e,compaction}) = p'_{vci} (1.0) S_{e,compaction}^{-\xi}$$

where $\xi$ is fitting parameter considering the initial compaction conditions and $p'_{vci}(1.0)$ is an intercept value for the power law relationship. The value of $S_{e,compaction}$ is used in Equation (13) instead of $S_{e,0}$ because the saturated soil specimen was compacted at a lower effective saturation, then wetted to reach $S_{e,0}=1.0$ before commencing the drained compression test. The relationship between $p'_{vci}$ and $S_{e,compaction}$ for Boulder clay is shown in Figure 8(a). A value of $\xi$ for Boulder clay was found to be equal to 1.24 and $p'_{vci}$ for saturated soil ($p'_{vci} (1.0)$) was found to be 7000kPa.

For mean effective stresses greater than $p'_{vci}$, the compression curves of the specimens follow a shape similar to an exponential decay function that reflects the transition to void closure. Similar to the value of $p'_{vci}$, the shape of this curve is also assumed to depend on the initial soil structure
of the specimen induced by compaction. Although one would expect that soils that have the same initial soil structure would all follow the compression curve for saturated specimens, this behavior was not noted in the data for compacted Boulder clay in Figure 3(a). Instead, the compression curves clearly indicate that the specimens compacted drier of optimum (lower initial $S_{e,compaction}$) show a lower rate of volume change at high mean effective stresses. Based on this observation, it is hypothesized that the compression behavior of soil to high stresses depends more on the initial soil structure than on the impact of matric suction on the effective stress state. Accordingly, an exponential decay function is used to represent the shape of the compression curve at high stresses:

$$e = e_{vci} \times e^{-\Lambda \left( p' - p_{vci}^{'}(S_{e,compaction}) \right)} \quad \left( p_{vci}^{'} < p' \right)$$

(14)

where $e_{vci}$ is the void ratio at the point of void closure initiation (VCI) for a specimen with a given $S_{e,compaction}$ that can be calculated by subtracting the changes in void ratio from Equations (3), (9) and (12) from the initial void ratio $e_0$, and $\Lambda$ is a soil structure function that depends on the compaction conditions. The value of $\Lambda$ is assumed to be a function of the effective saturation at compaction, as follows:

$$\Lambda = A \cdot \left[ \left( S_{e,compaction} - S_{e,ref} \right) + B \right] + B$$

(15)

where $A$ and $B$ are fitting parameters. In order to fit the compression curves for Boulder clay, these fitting parameters were found to be $10^{-5}$ and -0.12, respectively. Trends in the soil structure function calculated using Equation (15) for specimens of Boulder clay with different values of $S_{e,compaction}$ are shown in Figure 8(b).

It is hypothesized that if a soil specimen is compacted at a given value of $S_{e,compaction}$ then the effective saturation is changed to a different value, the structure parameter described by Equation (15) would still correspond to the value at compaction and thus empirically reflect the initial soil
structure. This should ideally be the case when comparing the compression curves for the specimens with \( S_{e,0} \) of 1.00 and 0.89, as they have similar values of \( S_{e,\text{compaction}} \) (0.890 and 0.887, respectively). However, these small differences in \( S_{e,\text{compaction}} \) between the two specimens were observed to have an effect on the shape of the compression curves at high stresses. Although the value of \( p'_{vci} \) was found to be similar for both soil specimens, the slight differences in initial conditions led to slightly different compression curves at high stresses. Another potential uncertainty is that the compression behavior of the drier soil specimens under high stresses may not be fully representative of drained conditions due to the different decreases in hydraulic conductivity of soil specimens during compression (i.e., potentially different rate effects for the different unsaturated specimens).

One important feature of Equation (14) is that the void ratio at extremely high stresses will tend toward a value of zero at the point of void closure. However, the results in this study only extend to 160 MPa, so it cannot be confirmed whether soils will converge to a non-zero void ratio at extremely high stresses or if full void closure (\( e=0 \)) is possible.

**EVALUATION OF MODELED COMPRESSION CURVES**

A summary of the parameters of the model defined to fit the drained compression curves of the compacted specimens of Boulder clay over a wide range of mean effective stress shown in Figure 3(a) is given in Table 2. The actual initial conditions (e.g., \( S_{e,0}, e_0, p'_0 \)) from the experiments shown in Table 1 were used as model inputs. A total of 11 parameters are required for the model to capture the nonlinear compression behavior of unsaturated soil over a wide range of mean effective stress, including the new phenomena that haven’t been considered in previous models such as the point of pressurized saturation, the transition to void closure, and the effect of soil structure induced by different initial compaction conditions on the shape of the compression curve.
at high stresses. Comparisons between the model predictions (dashed lines) and the measured compression curves (solid lines) are shown in Figures 9(a) and 9(b) for specimens having the different initial effective saturations, with the mean effective stress on logarithmic and natural scales, respectively, for comparison in the shapes of the curves. The model matched the experimental data well for specimens at high effective saturations. The unloading line from high mean effective stresses was assumed to be elastic even though the experimental results indicate a nonlinear rebound due to the impact of incomplete drainage. The same model predictions are shown in Figures 9(c) and 9(d) with the mean effective stress on logarithmic and natural scales, respectively, without the experimental data to better observe the trends in the model predictions for different initial effective saturations.

**CONCLUSIONS**

A series of drained, isotropic compression tests were performed on unsaturated, compacted clay specimens having different initial degrees of saturation up to a mean total stress of 160 MPa. A constitutive model was established to characterize transition points observed in the compression curves. The existing effective stress-based model of Zhou et al. (2012a, 2012b) was used to capture the suction hardening effect on the preconsolidation stress and the shape of the compression curve up to the newly defined point of pressurized saturation. To characterize the behavior of the soil specimens at high stresses, this model was combined with an empirical relationship based on the experimental data to account for the impact of different initial soil structures induced by compaction conditions. Overall, the developed model was observed to provide a good match to the drained compression curves for unsaturated soils with different initial effective saturations over a wide range of mean effective stresses. The following specific conclusions can be drawn from this study:
• A suction hardening effect on the mean effective preconsolidation stress is observed in the drained compression curves of unsaturated, compacted soils interpreted in terms of mean effective stress.

• The slopes of the compression curves for unsaturated soils are greater than that of a saturated soil so that the compression curves of unsaturated soils will converge with that of saturated soil at the points of pressurized saturation that depend on the initial effective saturation. This is represented indirectly through the model of Zhou et al. (2015a) using a compression index that considers changes in effective saturation during compression.

• The point of pressurized saturation was observed to occur at higher stresses for specimens with an initially lower effective saturation. This point was predicted well using the model of Zhou et al. (2015b) after an additional constraint is added to the model.

• The transition toward void closure was predicted well using an exponential function with parameters that are sensitive to the initial soil structure induced by compaction.

ACKNOWLEDGMENTS

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APPENDIX I. REFERENCES


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Table 1: Summary of results from the drained isotropic compression tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0$</td>
<td>0.508 0.506 0.507 0.514 0.514</td>
</tr>
<tr>
<td>$S_{r,0}$</td>
<td>1.00 0.92 0.84 0.72 0.62</td>
</tr>
<tr>
<td>$S_{e,0}$</td>
<td>1.00 0.89 0.76 0.59 0.47</td>
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<tr>
<td>$S_{e,compaction}$</td>
<td>0.890 0.887 0.76 0.59 0.47</td>
</tr>
<tr>
<td>$w_0$</td>
<td>18.9* 17.2 15.7 13.5 11.6</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.005 0.005 0.005 0.005 0.005</td>
</tr>
<tr>
<td>$p'_c$ (kPa)</td>
<td>110 350 500 520 680</td>
</tr>
<tr>
<td>$p'_{ps}$ (kPa)**</td>
<td>- 1,100 1,800 3,100 6,100</td>
</tr>
<tr>
<td>$p'_{vci}$ (kPa)</td>
<td>7,000 7,500 9,000 13,000 18,000</td>
</tr>
</tbody>
</table>

*Compacted at $w_0 = 17.3\%$ then saturated to 18.9% using upward flow under vacuum

**$p'_ps$ is the mean effective stress at the point of pressurized saturation observed in each test

Table 2: Drained compression model parameters for Boulder clay having $e_0 = 0.51$

<table>
<thead>
<tr>
<th>Component model</th>
<th>Parameter</th>
<th>Value</th>
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<td>Zhou et al. (2012) elasto-plastic model</td>
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<tr>
<td></td>
<td>$\lambda_0$</td>
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</tr>
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<td></td>
<td>$\lambda_d$</td>
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<td>-</td>
</tr>
<tr>
<td></td>
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<td>kPa</td>
</tr>
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<td></td>
<td>$a_2$</td>
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<td>Transition to void closure model</td>
<td>$p'_{vci}$</td>
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<td>MPa</td>
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<tr>
<td></td>
<td>$p'_{vci}$</td>
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<td></td>
<td>$\xi$</td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td>$A$</td>
<td>$10^{-5}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
<td>-0.12</td>
<td>-</td>
</tr>
</tbody>
</table>

*Estimated from the compression curve for saturated Boulder clay
Figure 1

Void ratio

Drained recompression

Increase in preconsolidation (yield) stress $p'_c$ with suction

Saturated

Unsaturated

Pressurized saturation ($p'_{ps}$)

Transition to void closure (Dependent on initial soil structure)

Mean effective stress, $\ln(p')$
Wetting path: 
\[ \alpha = 0.26 \, \text{kPa}^{-1}, \, n = 1.97 \]
\[ \theta_s = 0.35; \, \theta_r = 0.10 \]

Drying path: 
\[ \alpha = 0.01 \, \text{kPa}^{-1}, \, n = 1.81 \]

\[ \psi_{ae} = 40 \, \text{kPa} \]
Figure 5

(a) Compressibility parameter, $\lambda(S_e)$:

$\lambda(S_e) = \lambda_0 - (1 - S_e)^n \cdot (\lambda_0 - \lambda_d)$

- $\lambda_0 = 0.032$
- $\lambda_d = 0.023$
- $a_1 = 0.28$

(b) $S_e$

(c) $e$ vs. $p'$ (kPa)
Figure 6

\[ p'_{c,S_e} = \left( p'_{c,\text{sat}} \right)^\beta \]

\[ \beta = \frac{\lambda_0 - \kappa}{\lambda(S_e) - \kappa} \]

\( \lambda_0 = 0.032 \)

\( \kappa = 0.005 \)

(a)

(b)
Figure 7

(a) $S_{e,0} = 0.76$
- $a_2 = 0.011$
- $0.035$
- $0.070$

(b) $D_e = \frac{S_e (1+e)}{e} (1-S_e)^{a_2}$
- $a_2 = 0.035$

(c) $D_e = \frac{S_e (1+e)}{e} (1-S_e)^{a_2}$
- $a_2 = 0.035$
Figure 8

(a) $p'_v(S_{e,0,compaction}) = p'_v(1.0) \cdot S_{e,0,compaction}^{-\xi}$

(b) $\Lambda = A \cdot \{S_{e,0,compaction} (1-S_{res}) + S_{res}\} + B$