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THE MICRO FOUNDATIONS OF REAL ESTATE RETURNS AND APPRAISAL: STATICS AND DYNAMICS

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THE MICRO FOUNDATIONS OF REAL ESTATE RETURNS AND APPRAISAL:
STATICS AND DYNAMICS

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Abstract

The evaluation of real estate assets and portfolio composition is complicated by the infrequent trading of real properties and the heterogeneity of those properties. For this reason techniques of appraisal have arisen, to estimate market values and hence return series and asset covariances.

This paper considers the role of property appraisal in a market where costly and incomplete information and the condition of sale can lead to considerable price dispersion.

The paper presents a dynamic model of the real estate market in which the appraiser's function is to convey information, optimally extracted from observed market prices, to agents for use in subsequent transactions. This task is undertaken in a market comprised of agents with information imperfections, varying expectations and differing search costs, all of which we take to be characteristic of the real estate market. It is shown that the appraiser conveys the optimally filtered information to subsequent market participants. As a result of this signal function extraction, it is shown that short run equilibrium price dispersions resulting from information imperfections is reduced or eliminated in the long run.
I. INTRODUCTION

Real estate markets differ from most other markets in that purchases and sales of individual properties occur only infrequently. Thus the current market value of the stock of real capital, or the worth of any investor's holdings of real estate, must be inferred from limited information about recent transactions.

Not surprisingly, methods of property appraisal have been developed to make the process of imputation of market value more consistent and to make inferences of value more replicable, and professional associations have arisen to add credence to appraisal procedures. Nevertheless, to a considerable extent, the process of property appraisal is still considered an art practiced by the initiated.

This paper presents the theoretical foundation of real estate appraisal. This foundation is based upon the microeconomics of buyer and seller behavior in a market for differentiated commodities. Specifically we characterize the information conveyed by the sales of property for the problem of imputing the market value of other properties not offered for sale.

The information conveyed by the sale of a given property at some market price depends crucially upon the
process by which price offers are made by potential buyers and sellers in the market. This, in turn depends upon the information gathering and decision making processes of actors in the market.

The theoretical underpinning of our analysis is the notion of price dispersion in market equilibrium. A variety of theoretical models have been developed which lead to such dispersion, for example, Reinganum [1979], based upon firms' varying production costs; Salop and Stiglitz [1976], based upon differing search costs; Wilde and Schwartz [1979], based upon consumers' inherent propensities to search. As noted by Burdett and Judd [1983], it appears to be crucial for price dispersion in equilibrium that there exist some ex post difference in the information available to buyers or sellers.

We take information imperfections, varying expectations, and differing search costs to be characteristic of the real estate market -- for both housing and investment properties. This leads to price dispersion in short run equilibrium -- in which the transactions prices for identical properties vary.

The optimal use of transactions information in inferring the price at which other similar properties would be sold is the technique of appraisal. The contents of
an appraisal, in turn, provide information to subsequent buyers and sellers. From this perspective, appraisal is the dynamic process which updates and "filters" optimally all available information on market prices. Real estate appraisal thus has the function of reducing or eliminating price dispersion by extracting the price signal from the "noisy" transactions which occur in an imperfect market. Under certain conditions, discussed below, the appraisal-transactions process converges to a unique value of the price of a given property or a specified class of properties.

Section II below places the assessment problem in the broader context of real estate investment analysis. Section III presents the basic model of microeconomic behavior of real estate actors. Section IV characterizes the appraisal problem more specifically. In this section we also indicate the equivalence of the so-called income and "comparables" approaches to appraisal. Section V indicates the dynamic path of optimal property appraisal and presents a concrete simulation of the process. Conclusions are presented in Section VI.

II. EQUITY RETURNS AND ESTIMATES

Methods of imputation of market value are more important for real estate than for other components of in-
vestment portfolios, and reliable value imputations are crucial to profitability. In the absence of sales, rates of return must be imputed rather than observed, and the correlation of returns across investment categories must be inferred from evidence on current operating income, the sales of comparable properties, or from historical trends.

A large literature comparing real estate holdings with other investments concludes, in general, that: real estate provides a somewhat higher risk-adjusted return when compared to other investment instruments; the inclusion of real estate in a portfolio of investments can substantially reduce portfolio risk, and; real estate is a good hedge against inflation.¹

The conclusions of this entire literature depend upon the construction of one or more real estate return series which can be compared with similar indices for other investments. Many researchers have relied upon professional appraisals to represent market value.² Indeed, this may be the standard practice.³

¹ Recent studies include Fama and Schwert [1972], Webb and Sirmans [1980], Miles and McCue [1982, 1984], Ibbotson and Siegel [1984], and Brueggeman, et al [1984].


Several difficulties may restrict the usefulness of return series computed from unadjusted appraisal data. For example, it is commonly asserted by researchers that appraisal data is subject to "smoothing" by the application of professional rules-of-thumb, thereby reducing the variance of the prices reported for a sample of appraisals relative to a sample of the actual sales of identical properties. If true, such an assertion is disquieting, since all measures of risk, as well as the diversification potentials of assets, are based on variance measures. Thus, reliance upon an artificially smoothed series will necessarily underestimate the riskiness of an asset, and will distort the correlation of its return with the returns to other assets as well. An ideal index will eliminate, or at least minimize, such distortions. The creation of such an index requires a better understanding of the appraisal process as well as the process which generates the prices at which trades are observed.

The infrequent trading of real estate and the absence of spot prices is a characteristic of the real estate market which is not likely to be altered. Thus any thorough attempt to construct a real estate return index must resort to some form of imputation. Because an appraisal is an estimate of a price generated from observ-
ing other market transactions, errors will exist. The deficiencies of using appraisal data containing such errors are well recognized, but their cause is not well understood. For example, Hartzell, Hekman and Miles [1986] assert that appraisal smoothing is due to a flaw in appraisal methodologies. Although outright errors may be a source of smoothing, a difference in information available to housing market actors will, by itself, also produce smoothed estimates of the appraised values of properties. Suppose, for example, that a potential purchaser has access to an appraisal before purchasing a property at auction. The appraisal is based upon the set of information \( I_a \), which, given the timing of the transaction, is a subset of the buyer’s information at the subsequent time of sale, \( I_b \). Then \( P \), a random variable representing the price of the property, will be related to its expected value according to

\[
\begin{align*}
(1) \quad P &= E[P|I_a] + e_a \\
&= P_a + e_a \\
P &= E[P|I_b] + e_b \\
&= P_b + e_b
\end{align*}
\]

where \( P_a \) and \( P_b \) are the appraiser’s and the buyer’s price estimates, the expected value of the market price given the information sets, \( I_a \) and \( I_b \), and the \( e \)’s are the estimation errors. Taking the expectation of the appraiser’s expression conditional on \( I_b \) and using the law of iterative expectations we obtain:
(2) \[ E[P|I_b] = E[P|I_a] + E[e_a|I_b] \]

\[ p_b = p_a + e \]

where \( e \), equal to \( E[e_a|I_b] \), is, in general, nonzero. Thus, the variance of the price estimate made by the buyer must be larger than that made by the appraiser. Appraisal smoothing in this context arises, not from methodological errors, but purely due to differences in information sets or in the timing of transactions.

III. BUYERS, SELLERS, AND PRICE DETERMINATION

As suggested in the previous section, appraisers filter information revealed by buyers and sellers in concluding real estate transactions to estimate the market prices of similar properties. Buyers and sellers, in turn, use the information provided by an appraisal to estimate the selling prices of these properties. In this section, we consider one part of this recursive process — the conclusion of a transaction conditional upon a real estate appraisal.

We consider a group of potential buyers and sellers of identical properties. Each buyer and seller has access to an independent appraisal of market value for each property.\(^4\). It is assumed that each seller has more

\(^4\) For expositional convenience only we concentrate on the case where a third party appraiser provides information to potential buyers and sellers. It should be clear
information about his own property than the third party appraiser. Since potential buyers have access to the appraisal data, they have no less information than that reflected in the appraisal report.

As discussed below, differences in information or costs will result in differences in threshold (minimum selling) prices \( P^s \) for sellers and differences in reservation (maximum offer) prices \( P^r \) for buyers. An observed transaction in the market \( P_T \) depends upon the relative bargaining power of each agent or the condition of the sale:

\[
P_T = \omega P^r + (1-\omega) P^s \quad 0 \leq \omega \leq 1, \quad P^r \geq P^s.
\]

where \( \omega \) and \( (1-\omega) \) represent the relative bargaining power of the buyer and seller respectively.

Let the ex post competitive price of a particular class of identical properties be characterized by a \( q \) factor model.

\[
P = P (X_1, X_2, \ldots, X_q) = P(\Omega)
\]

\( P \) is the market value of members of this class of properties and the \( X \)'s are the factors. The factors in-

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from the model, however, that it is equally applicable to confidential or "in house" appraisal services provided to potential traders.
clude the physical and financial characteristics of a particular property; \( \Omega \) represents the full set of information about the property. Equation (4) can be interpreted as the reduced form of the market price generating process or as a so-called hedonic price equation under complete information. \( P \) is a random variable, since some elements of \( \Omega \) include realizations of variables following a random process, (for example random physical characteristics whose realizations are observable at any given time). There are \( m \) buyers, \( n \) sellers and one appraiser, each with information comprising some subset of \( \Omega \). No agent knows \( \Omega \), nor necessarily its dimension; each must therefore form an expectation of \( P \) conditional on his information set. Each agent \( u \) (out of the \( m+n+1 \) agents) can form a price estimate for any property \( v \) (out of the \( n \) identical properties) based upon his own set of information about that property, \( \mathbb{E}(P_v|I_v^u) \).

a. The Seller's Threshold Price

Since each seller has more information regarding his own property than the appraiser, each seller makes his own expectation of \( P \) conditional on his richer information set. Let \( P_i^S = \mathbb{E}(P|I_i^S) \) be the estimate of \( P \) by seller \( s \) for his own property \( i \). This estimate can be interpreted as the threshold price of the seller; he would be unwilling to sell his property at any price
below this level. Thus a distribution of information among sellers will result in a distribution of threshold prices for identical properties.\(^5\) Let the distribution and density function of the threshold price be \(G(P^s)\) and \(g(P^s)\) respectively.

b. The Buyer’s Reservation Price

The buyer is also assumed to have access to the appraiser’s information. Realizing that the appraiser’s estimate is based upon incomplete information, each buyer may allocate resources to obtain additional information. Thus the information set of each buyer is no less than that of the appraiser. Define buyer \(b\)'s threshold price as \(P_{j^b} = E[P|I_{j^b}]\) for any property \(j=1,...,n\). This estimate can be interpreted as the buyer’s reservation price; he would be unwilling to pay any more. Once again, buyers will have a distribution of information, resulting in a distribution of threshold prices. Let this distribution and its a density function be \(F(P^b)\) and \(f(P^b)\), respectively.

\(^5\) Other factors may, by themselves, also lead to a distribution of seller threshold prices. For example, sellers may offer at different prices precisely because there is a positive probability that a buyer will not observe a lower price -- that is, costly information on the part of buyers alone may lead to a distribution of sellers’ threshold prices.
A buyer searches for the lowest price by sampling from the distribution of threshold prices $g(P^S)$. Given that there is cost to search, he will continue searching until the marginal expected gain from obtaining an extra observation is equated with the marginal cost of search. Under well known conditions (including, for example knowledge of the distribution. See Lippman and McCall, 1976), an optimal stopping rule exists. The searching procedure moreover will have the reservation price property; that is, if a price offered by the seller is above the reservation price, the buyer will continue searching. If the offer is below, the buyer will stop searching and conclude a transaction. The reservation price is, of course, lower than the threshold price.

Specifically, consider an elementary sequential search model with an infinite horizon and no discounting. It is assumed that each risk neutral buyer has rational expectations (See Burdett and Judd, 1983) -- that is, each buyer knows the distribution $G(P^S)$. A representative buyer engages in search; he draws a price $P_i^S$ from the cumulative distribution $G(P^S)$. Successive draws are assumed to be mutually independent. Each buyer has a stopping rule which maximizes his expected profit, $\Pi$, defined as:

$$E[\Pi] = E[\min (P_1^S, \ldots, P_N^S)] + NC$$
where recall is permitted, \( N \) is the random stopping time, and \( c \) is the cost per search. Define \( \delta \) to be the reservation price, which can be interpreted as the expected gain from search as dictated by the best stopping rule. By definition, for a draw which results in the realization \( P_1^S \), the decision rule is:

\[
\begin{align*}
\text{if } P_1^S &\leq \delta \quad \text{buy the property} \\
\text{if } P_1^S &> \delta \quad \text{do not buy and continue searching.}
\end{align*}
\]

For the first realization, \( P_1^S \), the expected return from search is:

\[
(6) \quad E[\min(\delta, P_1^S)] + c
\]

where \( c \) includes the opportunity cost of search which may vary across buyers (and may thus represent the reinvestment opportunities of each buyer).\(^6\) Now since \( \delta \) is defined as the expected gain from using the best stopping rule, it follows that:

\[
(7) \quad \delta = E[\min(\delta, P_1^S)] + c
\]

We can express \( E[\min(\delta, P_1^S)] \) as:

\[
(8) \quad E[\min(\delta, P_1^S)] = \delta \int_0^\infty dG(P^S) + \int_0^\delta P^S dG(P^S)
\]

\[= \delta + \int_0^\delta (P^S - \delta) dG(P^S)\]

\(^6\) For notational convenience, the subscript denoting search cost differences across buyers is suppressed.
Using (7), it follows that:

\[ c = \int_0^\delta (\delta - P^s) \, dG(P^s) = D(\delta) \]

Define the right hand side of (9) as \( D(\delta) \), which is an increasing function of \( \delta \). This reflects the fact that those individuals with a high search costs will have high reservation prices. Conversely, if search cost is low, buyers will be willing to wait longer to draw a lower price.

Since \( D(\delta) \) is monotonic, the distribution of the reservation price can be calculated, given the distribution of search costs. Define the resulting density function of the reservation price as \( 1(P^r) \). This density function characterizes all possible realizations of the reservation price. Not all realizations are feasible, however, since no trade will occur below each buyer's threshold price. Define \( k(P^r) \) as the overall density of the reservation price taking into account this truncation. This can be expressed as:

\[
\begin{align*}
  k(P^r) &= f(p^b) & \text{for } p^b > \delta \\
  k(P^r) &= 1(P^r) & \text{for } p^b \leq \delta
\end{align*}
\]

Search and information, defined this way, determine the market price. Since no seller will sell at a price below the threshold price drawn from \( g(P^s) \) and no trade will occur at price below the buyer's reservation price
drawn from \( k(P^r) \), any observed transaction must be between the threshold price and the reservation price.

Therefore, any price we observe in the market is a drawing from the convolution of \( g(P^s) \) and \( k(P^r) \). Define the density of this convolution as \( h(P_T) \). For the case when \( P^r \) and \( P^s \) are independent, the density of \( P_T \) is:

\[
(10) \quad h(P_T) = \int_0^{P_T} g((1-\omega)P^s)k(P_T - (1-\omega)P^s)dP^s
\]

Equation (10) defines the density function for the observed transactions of identical parcels of real estate.

c. Price Determination: A Simple Example

Suppose that the density of sellers' threshold prices can be characterized by a distribution with parameters \( \alpha \) and \( \lambda \)

\[
(11) \quad g(P^s) = \left[ \frac{\lambda^\alpha}{\Gamma(\alpha)} \right] P^s^{\alpha-1}e^{-\lambda P^s} \quad \text{for } P^s \geq 0
\]

\[
= 0 \quad \text{for } P^s < 0
\]

where

\[
\Gamma(\alpha) = (\alpha-1)!
\]

Then from (9):

\[
(12) \quad C = \int_0^\delta \left[ \frac{\lambda^\alpha}{\Gamma(\alpha)} \right] P^s^{\alpha-1}e^{-\lambda P^s} dP^s
\]

\[
= e^{-\lambda\delta} \left[ \frac{\alpha}{\lambda} + \sum_{i=2}^{\alpha} \left( \frac{i-\delta\alpha-i+1}{\alpha} \right) \right] + \delta - (\alpha/\lambda)
\]
Equation (12) defines an explicit relationship between the cost of search and the reservation price; for a given distribution of search costs it defines the density for the reservation price.\(^7\) For example, if the density function for the search cost is rectangular, then

\[
\ell (P^r) = \ell (\delta) = \frac{1}{C^*} \left[ 1 - e^{-\lambda \delta} \left( \sum_{i=1}^{\alpha} \frac{(\lambda \delta)^{\alpha-i}}{(\alpha-i)!} \right) \right]
\]

where

\[
C^* = \delta^* - \frac{\alpha}{\lambda} \left[ 1 - e^{-\lambda \delta^*} \right] + e^{-\lambda \delta^*} \left[ \sum_{i=1}^{\alpha-1} \sum_{k=1}^{i} \frac{(\lambda \delta^*)^{i-k} (i-k)!}{(i-k+1)!} \right]
\]

for \(0 \leq \delta \leq \delta^*\)

Taking into account the truncation problem, the reservation price density is:

\[
k(P^r) = l(P^r) \quad \text{for } P_i^r < P_i^b
\]

\[
k(P^r) = f(P^b) \quad \text{for } P_i^r \geq P_i^b
\]

For simplicity let \(\alpha = 2\), and let the search cost assume a gamma distribution:

\(^7\) It can be easily verified that the reservation price is a monotonically increasing and concave function of the search cost, i.e.,

\[\frac{d\delta}{dc} > 0 \text{ and } \frac{d^2\delta}{dc^2} < 0 \text{ for } \delta > 0\]
(15) \( g(c) = \Gamma(c; \alpha, \lambda) = \Gamma(c; 2, 1) = ce^{-c} \)

then

(16) \( \lambda(P) = \lambda(\delta) = m(\delta) e^{-m(\delta)} \left[ 1 - e^{-\lambda \delta (\lambda \delta + 1)} \right] \)

where

\[ m(\delta) = (1/\lambda) \left[ e^{-\lambda \delta (\lambda \delta + 2)} + \lambda \delta - 2 \right] \]

Given the above expressions, the convolution of the seller threshold price distribution and the overall reservation price distribution can be derived for \( P_i^r < P_i^b \):

(17) \[ h(P_T) = \frac{2}{\omega (1-\omega)^2} \int_0^{P_T} n(P^r) \exp(-n(P^r)) \left[ 1 - (\lambda P_T/\omega + 1) \right] \exp(-\lambda P_T/\omega) (P_T - P^r) \exp[-\lambda (P_T - P^r)/(1-\omega)] dP^r \]

where

\[ n(P^r) = [1/\omega \lambda] \left[ (P^r \lambda + 2\omega) \exp(-\lambda P^r/\omega) + P^r \lambda + 2\omega \right] \]

Figure 1 presents a schematic of the reservation and threshold price distributions, equations (11) and (16), drawn for \( \lambda = 1 \) and \( \alpha = 2 \). The bottom panel of the figure shows the density of transactions prices equation (17) associated with the reservation and threshold price distributions. The figure is drawn for the case of equal bargaining power between buyers and sellers, \( \omega = 0.5 \).
IV. THE APPRAISERS' PROBLEM

As noted in equation (1) the price of any property is related to the price estimated by any actor \( k \) according to

\[
P = E[P|I_k] + e_k
\]

(18)

where \( e \) represents errors attributable to the lack of knowledge of \( \Omega \) and is orthogonal to the information set. This is a general representation since any stationary random variable can be expressed as the sum of a conditional expected value and an orthogonal error term. Consider three actors: a buyer (\( k=b \)), an appraiser (\( k=a \)) and a seller (\( k=s \)). Since the errors made by these actors are composed of common elements of \( \Omega \), they are correlated with each other, but not with the search cost. Let

\[
E(e^k) = \mu_k
\]

\[
E(e^k e^j) =
\begin{cases} 
\sigma_{kj} & \text{for } k=j \\
\sigma_k^2 & \text{for } k=j
\end{cases}
\]

The reservation price \( p^r \) can be decomposed into:

\[
p^r = p_b - \epsilon \quad \text{where } \epsilon \geq 0
\]

(20)

Since \( p_b = P - e_b \) we define \( e^r \) as:
Now consider the appraiser's problem. The appraiser is required to make an estimate of the competitive price of a property. Using the comparable method, he updates his prediction based upon the knowledge of the last sale price, \( P_T \). This price, however, is some function of the buyer's and seller's expected prices, the cost of search, the condition of sale and the relevant distribution parameters. The appraiser thus must extract the relevant signal from the "noisy" transacted price. After performing signal extraction, the appraiser uses this information to update his prediction in an optimal manner.

The optimal updating procedure for an appraiser, given an initial information set \( I_a \) and an additional piece of information, \( P_T \), is the so-called least squares or recursive projection (See Sargent, [1979, chapter 10] or Samuelson, [1965, 1973]):

\[
(22) \quad E[P|I_a, P_T] = E[P|I_a] + E[(P - E[P|I_a])|(P_T - E[P_T|I_a])].
\]

That is, the new prediction is made by augmenting the previous estimate by the expected value of the last period error, conditional on the errors of predicting the new information given the previous information set. The last term of (22) is the updating component:
(23) \[ E[(P-E[P|I_a])|(P_T-E[P_T|I_a])] \]

From (1) we know that the first term of (23) is the error of the appraiser's estimate.

(24) \[ P - E[P|I_a] = e_a \]

Upon substitution of (20) and (24) into (21), the observed price can be expressed as:

(25) \[ P_T = P - \omega e^r - (1-\omega)e^s \]

\[ = E[P|I_a] + e_a - \omega e^r - (1-\omega)e^s \]

Taking conditional expectations of (25) with respect to the information set \( I_a \) we get:

(26) \[ E[P_T|I_a] = E[E[P|I_a]|I_a] = E[P|I_a] \]

since \( e^k \) is orthogonal to \( I_a \) for \( k=s,r \). Hence from (25) and (26),

(27) \[ P_T - E[P_T|I_a] = e_a - \omega e^r - (1-\omega)e^s \]

which is the second term of (23). Thus the updating component can be written as:

(28) \[ E[(P-E[P|I_a])|(P_T-E[P_T|I_a])] = E[e_a|e_a - \omega e^r - (1-\omega)e^s] \]
This conditional expectation projection can be expressed as a linear projection, analogous to the linear regression model (Samuelson [1965]). Thus

\[(29) \quad E[P|I_a, P_T] = E[P|I_a] + \beta[P_T - E[P|I_a]]\]

where from (28) the coefficient \(\beta\) is:

\[(30) \quad \beta = \frac{\sigma_a^2 - \omega \sigma_{ar} - (1-\omega) \sigma_{as}}{\sigma_a^2 + \omega \sigma_r^2 + (1-\omega) \sigma_s^2 - 2 \omega \sigma_{ar} - 2(1-\omega) \sigma_{sa} + 2\omega(1-\omega) \sigma_{sr}}\]

This result can be made more intuitive; rewrite (29) as

\[(31) \quad E[P|I_a, P_T] = \beta P_T + (1-\beta) E[P|I_a]\]

The updated appraisal for properties in this class is a weighted average of the price recorded for the last transaction and the appraiser's previous estimate, with the weights depending on the second moments of the error distributions. Since the variance of the errors collapse to zero as the system becomes more and more informative, the informational content of the system is summarized in its variance.

Consider the simple case in which each has identical information \(e_a=e_b=e^s\). For this case, \(\beta=0\) and the updated

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\(^8\) That is, \(E[Y|X] = aX + v\), where \(a = \text{cov}(X,Y)/\text{var}(X)\) and \(v\) is orthogonal to \(X\).
price is identical to that of the initial appraisal estimate. Since no additional information is contained in the transaction price, no weight is assigned to $P_T$. Conversely, if both the buyer and seller have perfect information but the appraiser does not, then $\beta=1$, and all the weight is assigned to the transacted price and none to the initial appraisal. Under these circumstances, the best prediction of a property is the last sale price of a comparable property.

V. APPRAISAL AND MARKET DYNAMICS

The dynamics of price determination in the market depend upon both aspects of the model developed in this paper -- buyers and sellers making transactions based on their own information and the information provided by an appraisal, and appraisers using transactions prices to update their current estimates of market value. In this section, we present a simulation of both aspects of the process and the dynamics of price determination. Specifically, we show that over time, as the seller obtains more information on the market from the appraiser, then the appraiser’s optimal use of transactions data leads to a decrease in the variance in transactions prices.

To simplify the simulation, let the buyer’s estimate of the price at each period be identical to that of the appraiser’s estimate in the previous period. Information
is introduced into the model by letting the variance of the seller's price estimate decrease at a constant rate over time. At any time $t$, the variance of the price estimates made by the appraiser, $V(K_t)$ is:

\[(32) \quad V(K_t) = \beta_t^2 V(P_t^T) + (1-\beta_t)^2 V(K_{t-1}) \]

where $\beta_t$, assuming independence between the buyer and the seller's errors, is

\[(33) \quad \beta_t = \frac{(1-\omega)V(K_{t-1})}{(\omega-1)^2 V(K_{t-1}) + (1-\omega)^2 V(P_t^S)} \]

The variance of the observed distribution of transaction price, $V(P_t^T)$, is therefore

\[(34) \quad V(P_t^T) = \omega^2 V(K_{t-1}) + (1-\omega)^2 V(P_t^S) \]

Thus, for any initial distribution of appraised prices, with variance $V(K_0)$, a decreasing variance of the seller's error leads to reduction in the variance of transactions prices over time.

Figure 2 presents a simulation of the variance of transaction prices based on equation (34). For various values of $\omega$, it presents the variance of observed transactions prices in the market over time for 50 periods. The figure is drawn assuming $V(K_0)=1$ and $\lambda_0=0.5$. In each period $\lambda$ increases by 0.01. Clearly the filtering
process leads to a reduction in the variance of prices in the market. The variance of transactions prices is clearly larger when \( \omega \) is larger, that is when the condition of the sale favors the seller. This is to be expected, given our assumption about the dynamics of information.

Figure 3 presents similar information. It presents a graph of the density of transactions prices as the market becomes better informed, i.e. as \( \lambda \) increases. Clearly the dispersion of prices is reduced and the mean approaches the price \( P \).

VI. CONCLUSION

This paper has presented a theory of the market for real estate in which property appraisal performs an important efficiency enhancing role. Profit oriented, but imperfectly informed, actors in the market make varying offers to buy and sell properties, leading to a short run equilibrium in which there is some distribution of market prices for identical properties. The role of the appraiser is to provide information so that the variance of this price distribution is reduced. The appraiser does this by updating his own estimate of the value of comparable properties every time a transaction is observed. Under quite general conditions, we have shown that the
recursive process linking appraisers to potential buyers and sellers of property reduces the market imperfections which arise, for example, from costly search and uncertain income projections by market actors.

The key to the model is the updating rule which the appraiser employs to extract the price signal from the "noisy" transactions made by imperfectly informed actors in the market. The optimal updating rule specifies the appropriate weighting of the information in a given transaction with the stock of prior information available to the appraiser. This stock of information is the experience and human capital of the appraiser, which forms the basis for signal extraction.

The stylized model emphasizes the difference in information available to individual buyers and sellers, who make transactions only infrequently, and the appraiser, whose expertise comes from observing many transactions. The model indicates that, although no actor is fully informed, in a stationary world the dynamics of the market lead to a convergence of transaction prices.

The model can clearly be generalized to more realistic circumstances. For example, idiosyncratic aspects of buyers or sellers (e.g. "distress sales") can be introduced (by imposing some distribution on \( \omega \)); excess supply or demand in local markets can be modeled by modifying
the convolution equations appropriately. Finally, the optimal updating rule can be made more realistic by employing a full fledged Kalman filter algorithm. Indeed, it appears that these theoretical notions can be used to improve practice in the computerization of appraisal information.

Clearly, this analysis is only a first step in relating the actions of real estate appraisers to the economics of information.
REFERENCES


FIGURE 1

Distributions of Threshold Prices, Reservation Prices and Transactions Prices Arising from Gamma Distribution of Search Costs: \( \Gamma(c; \alpha, \lambda) = \Gamma(c;2,1), \omega=0.5 \)
FIGURE 2

Dispersion of Transactions Prices
Over Time: \( V(K_0) = 1, \lambda_0 = 0.5, \)
\( \lambda_t = \lambda_{t-1} + 0.01 \)

Variance of Transaction Prices
FIGURE 3

Distribution of Transactions Prices Arising from Gamma Distribution of Search Costs: $\Gamma(c;2,\lambda)$ as $\lambda$ Increases, $\omega = 0.5$