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ACCOMPANYING ALPHA DECAY

M. S. Rapaport, F. Asaro, and I. Perlman

April 1974

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Radioactivity $^{238}_{\text{Pu}}$, $^{210}_{\text{Po}}$; measured $\alpha$-$\kappa$ x-ray coin.

Deduced electron shake-off abundances, energy distributions.

Hartree - Fock - Slater treatment.
K-SHELL ELECTRON SHAKE-OFF ACCOMPANYING ALPHA DECAY

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ABSTRACT

The α spectra associated with K-shell electron shake-off in $^{210}$Po and $^{238}$Pu decay have been determined by K x-ray − α coincidence measurements. Although the shapes of the spectra generally agree with the theoretical expectations, some discrepancies are observed. The abundances per α particle of the total K electron shake-off effect were also determined in these measurements and found to be $(1.65 \pm 0.16) \times 10^{-6}$ for $^{210}$Po and $(0.75 \pm 0.09) \times 10^{-6}$ for $^{238}$Pu. These results are also compared with theoretical predictions and further experimental and theoretical studies are suggested.
The phenomenon by which an electron in a given orbital is excited into the continuum (shake-off) during nuclear decay was first predicted by Migdal and Feinberg and later by Levinger. Since then much theoretical and experimental work related to \( \beta^- \), \( \beta^+ \) and E.C. decay has been done. The various experimental works involved measurements of x-ray – \( \beta \) coincidences, \( \beta \) and x-ray intensities, relative x-ray and \( \gamma \)-ray intensities and relative intensities of x-rays in parent and daughter nuclei. Shake-off of L electrons accompanying internal conversion in the K shell was also observed. The agreement between theory and experiment has in general been good.

Essentially, all of the measurements of electron shake-off during \( \alpha \) decay have been made on \( ^{210} \text{Po} \) with one unpublished result on \( ^{238} \text{Pu} \). In the \( ^{210} \text{Po} \) experiments the x-ray abundances were measured and any excess over that expected from the internal conversion of the 803 keV \( \gamma \)-ray was assumed to be due to electron shake-off. The measurement of the K-shell effect in \( ^{238} \text{Pu} \) decay was very similar, but more involved because of additional gamma rays. The agreement between experiment and theory has not been good. Generally, the experimental results for the K shell probability were about 60% of the theoretical predictions, and the discrepancy was about twice the stated experimental error.

The present work was undertaken to measure directly that part of the alpha spectrum connected with the electron shake-off effect in the K shell and to determine the differential shape of this spectrum and compare it with theoretical predictions. We hoped to improve the precision of previous measurements of the total K shell probability and delineate more clearly the discrepancy between experiment and theory.
II. EXPERIMENTAL WORK

A. Equipment

The general experimental procedure was to measure the alpha spectra of \(^{210}\)Po and \(^{238}\)Pu which were in coincidence with K x-rays.

1. K X-Ray Side

The K x-rays were detected with a solid state detector of pure Ge which had a full-width-at-half-maximum (F-W-H-M) of 1.0 keV for a 122 keV \(\gamma\)-ray and an overall detection efficiency of 13.5\% at that energy.

The output was amplified and fed into a single channel analyzer. In the \(^{210}\)Po measurement, this analyzer was set on the \(K_\alpha\) x-rays, which comprise 78\% of the total. In the \(^{238}\)Pu measurement, however, the single channel analyzer was set on the \(K_\beta\) x-rays, which are only 23\% of the total K x-rays, in order to eliminate the \(\alpha\) spectrum in coincidence with the 100 keV \(\gamma\)-ray. In this latter experiment the \(\gamma\)-ray output of the preamplifier was gain-stabilized. The output of the single channel analyzer was part of a fast-slow triple coincidence system. The block diagram is shown in Fig. 1.

2. \(\alpha\) Side

The \(\alpha\) particles were detected with Au-Si surface-barrier type detectors with geometries of about 2-3\%. The intense bombardment of the detector by the \(\alpha\) activity of the sources resulted in a deterioration of resolution over the course of the experiments. In the \(^{210}\)Po measurement the F-W-H-M changed from 22.5 keV at the beginning of the experiment to 30.0 keV at the end. For the \(^{238}\)Pu measurement the F-W-H-M changed from 30.0 to 36.5 keV.

The \(\alpha\) detector output was first sent to a gain-stabilizer. Then part of the output was fed into the triple coincidence system, part was fed into a
unit which scaled down the counting rate by a factor of 20 and part was fed to a 400 channel pulse height analyzer through a bias amplifier and a linear gate. The linear gate was triggered via a mixer gate by either pulses from the triple coincidence system or the scaled-down α singles pulses. The pulse height analyzer was also gated by the triple coincidence system via the gain-stabilizer so that only coincidence pulses were stored and not the scaled-down singles used for gain stabilization.

The net effect of the electronic arrangement was that α pulses which were in coincidence with the K x-ray gate could register on the pulse height analyzer with a minimum of accidental coincidences \(72 \times 10^{-9}\) sec resolving time) and without any gain change during the experiment.

B. Source Preparation

The \(^{238}\)Pu was chemically purified by dissolving in 12 M HCl, loading onto an anion column (DOWEX AG 1 × 8%), washing the column with 12 M HCl containing some HNO\(_3\), and eluting the \(^{238}\)Pu off the column with a solution 12 M in HCl and .44 M in HI. The eluent was evaporated to dryness and then vaporized in vacuum from a tungsten filament onto a .002 inch thick mylar foil. The source which had been collimated to an area 5/16 inch in diameter during vaporization was invisible and had an activity of \(~1.2 \times 10^7\) α dis/min.

Two vials of \(^{210}\)Po were purchased from New England Nuclear. The \(^{210}\)Po was catalogued as carrier-free and of natural origin although investigation at the conclusion of the experiment showed it was prepared by the reaction and decay: \(^{209}\)Bi(n,γ)\(^{210}\)Bi \(\beta^-\) 5 day \(^{210}\)Po. The \(^{210}\)Po from one of the vials was further purified by fuming to near dryness with concentrated HNO\(_3\), loading onto a cation column (DOWEX 50) with .2 M HCl, washing with 2 M HNO\(_3\), and eluting the
\[ ^{210} \text{Po} \] with 2 M HCl. The eluent was evaporated to dryness and vaporized like the \[ ^{238} \text{Pu} \] onto a .002 inch thick mylar foil. The source was \( \sim 1.7 \times 10^7 \) \( \alpha \) dis/min and was invisible.

C. Results

1. \[ ^{238} \text{Pu} \]

The \[ ^{238} \text{Pu} \] was measured in the coincidence unit for a total running time of 15 days. The \( \alpha \) singles spectra were measured and recorded every day as were the coincident spectra. The \( \gamma \) singles spectra were monitored continuously. The \( \alpha - K \) x-ray coincidence spectra for the one day runs were summed and this total spectrum is presented in Fig. 2. The abscissa is the analyzer channel in which the coincidences appeared, and it is roughly linear with the \( \alpha \) particle energy. The ordinate is the total number of observed coincidences in the 15 day period. The highest energy peaks, \( \alpha_0 \) and \( \alpha_{44} \), are due to accidental coincidences between the most intense \( \alpha \) groups and radiation in the \( K \) x-ray gate. The most intense peak, \( \alpha_{296} \), is due to true coincidences with \( K \) x-rays from conversion of the 153 keV \( \gamma \)-ray and with the Compton background of this \( \gamma \)-ray in the \( K \) x-ray gate region. The pertinent part of the \[ ^{238} \text{Pu} \] decay scheme is shown in Fig. 3.

The \( \alpha_{296} \) coincident peak (Fig. 2) is broader than the \( \alpha_0 \) and \( \alpha_{44} \) accidental peaks and this is probably due to a combination of effects including shifts in the threshold of the bias amplifier and a non-linearity in this region of the pulse height analyzer. The broad \( \alpha \) distribution (Fig. 2) in the region of channels 175-260 is broader than \( \alpha_{296} \), tails substantially more on the low energy side and, if we exclude the shake-off effect, would not correspond to any \( \alpha \) groups of \[ ^{238} \text{Pu} \] expected to be in coincidence with the \( K_\beta \) x-ray gate. A measurement was made of the maximum amount of the 100 keV \( \gamma \)-ray of \[ ^{238} \text{Pu} \] which
could be in the gate region. It indicated that only a negligible proportion of
the coincidences in the region of channels 175-260, could be due to this γ-ray.
To determine if these observed coincidences had the proper maximum energy for a

\(^{238}\text{Pu}\) α particle which ejected a K electron with about zero kinetic energy we
extrapolated their high energy side (Fig. 2) and that of \(^{238}\text{Pu}\) α to \(\sim 1/4\) of
their peak height. There was a difference in energy of \(115\pm10\ \text{keV}\) which agrees
with the K binding energy of uranium, 115.6 keV.

Thus, the distribution in Fig. 2 in the region of channels 175-260 should
be due to the electron shake-off effect of the main alpha groups. The distribution
is spread out over so many channels because there are two major α groups involved,
\(\alpha_0\) and \(\alpha_{44}\), and because the shake-off electrons carry off energy causing a spread
in α particle energy and a tailing on the low energy side. There was a total of
271 coincidences (\(\sim 264\) true and \(\sim 7\) accidentals) measured in the 15 day experiment
in the region of interest (channel 175-260). From the true coincidence counting
rate, the α singles counting rate, the K x-ray side detector efficiency (including
geometry), the fraction of total K x-rays in the gate and the K shell fluorescence
yield, the abundance of ejected K shell electrons is \((0.75\pm0.09) \times 10^{-6}\) per \(^{238}\text{Pu}\)
α particle.

The nomenclature for the normal α groups shown in Fig. 2 is the usual
one with the energy of the excited state being a subscript to the α symbol, e.g.
the \(^{238}\text{Pu}\) α groups populating the 44 keV excited state in \(^{234}\text{U}\) is designated
\(^{238}\text{Pu}\) α\(_{44}\) or simply α\(_{44}\). We suggest for the α groups ejecting orbital electrons
in their passage through the Coulomb field, that the shell designation of the
ejected electrons be added as a subscript before the excited state energy. Thus,
the \(^{238}\text{Pu}\) α group which populates the 44 keV state in \(^{234}\text{U}\) and which also causes
a K electron to be ejected would be designated \(^{238}\text{Pu}\) α\(_{K,44}\) or simply α\(_{K,44}\).
2. \( ^{210}\text{Po} \)

The \( ^{210}\text{Po} \) was also measured in the coincidence unit for a total running time of 15 days. The experiment was very similar to that for \( ^{238}\text{Pu} \) except that a larger fraction of the K x-ray peak could be used in the gate as there are no gamma rays in \( ^{210}\text{Po} \) decay near the gate energy. The various coincidence runs in the 15 day period were summed and the total spectrum is shown in Fig. 4. The highest energy peak, \( \alpha \), is due to accidental coincidences with the main \( \alpha \) group. The only other known \( \alpha \) group of \( ^{210}\text{Po} \) populates the 803 keV excited state of \( ^{206}\text{Pb} \) and has a very low abundance of \( 1.07 \times 10^{-5} \). The peak at \( \sim \) channel 340 (Fig. 4) is broader than \( \alpha \) and tails more on the low energy side.

The linearity of the amplifier-analyzer system for \( \alpha \) particles was carefully checked with \( ^{240}\text{Pu} \) and \( ^{242}\text{Pu} \) and was found to be linear within 1% in the region of interest. Thus, the increased peak width is not due to non-linearity in the energy scale and is very likely caused by the kinetic energy carried off by the K electrons ejected during the \( \alpha \) decay process as in \( ^{238}\text{Pu} \) decay. By extrapolating the two peaks (Fig. 4) in the same fashion as for the \( ^{238}\text{Pu} \) experiment, we found the difference in energy was 88±1 keV in excellent agreement with K shell binding energy of lead, 88.0 keV.

There was a total of 1,347 coincidences (\( \sim 1,285 \) true and \( \sim 62 \) accidental) in the region of the \( \alpha \) shake-off peak, \( ^{210}\text{Po} \). Calculated in the same way as for the \( ^{238}\text{Pu} \) experiment the abundance of ejected K shell electrons is \( (1.65\pm.16) \times 10^{-6} \) per \( ^{210}\text{Po} \) \( \alpha \) particle. As a check on the correctness of our geometry calibration, we calculated the K conversion coefficient of \( \alpha \) from its abundance in the coincidence run (Fig. 4) and the tabulated singles abundance, \( 1.07 \times 10^{-5} \). The resulting value \( (8.1\pm1.4) \times 10^{-3} \) is in good agreement with the theoretical \( 8.08 \times 10^{-3} \).
III. DISCUSSION

Migdal treated the $\alpha$ decay shake-off phenomenon as an example of an adiabatic perturbation since the velocities of the $\alpha$ particles are much smaller than the velocities of the inner electrons. Levinger modified Migdal's treatment by including the recoiling nucleus in the perturbation. This reduced very much the probability of the shake-off taking place. It was shown,\(^1\)\(^9\),\(^2\)\(^0\) however, that Levinger's method of including the recoiling nucleus in the perturbation was not correct.

According to Migdal the probability of ionizing one of the electrons is given by:

$$
\frac{8v^2}{3(E_k - B)} |<k, l = 1| \frac{1}{r^2} |1s>|^2 dE_k
$$

+ much smaller terms

where $v$ is the velocity of the $\alpha$ particle, $B$ is the binding energy of a $1s$ electron and $E_k$ is the kinetic energy carried off by the ionized $1s$ electron. The matrix element can be readily calculated with hydrogenic wave functions and the probability equation becomes:

$$
\frac{11.2}{3Z^6} \frac{4Z}{k} \frac{\text{Arctan} \frac{k}{Z}}{(1 + \frac{k^2}{Z^2})^5 \left(1 - e^{-2\pi Z/k}\right)} dE_k
$$

where $Z$ is the charge of the daughter nucleus and $k = \sqrt{2E_k}$. One gets the total ionization probability by numerical integration.

A more realistic set of wave functions would be of the self-consistent type. We used the tabulated\(^2\)\(^1\) Hartree-Fock-Slater central potentials to solve the Schrödinger equation for the continuum electrons. We applied the Numerov\(^2\)\(^2\) integration method until the solution became asymptotic. The two points at
small \( r \) required to generate the solutions were taken to be hydrogenic. The asymptotic solution is known to be\(^{23}\)

\[
P_{k\ell}(r) \approx \left( \frac{2}{\pi k} \right)^{1/2} \cos[kr + k^{-1}\ln(2kr) - \frac{1}{2} \pi(\ell + 1) - \delta_{\ell}] . \tag{3}
\]

where \( \ell \) is the orbital angular momentum quantum number and \( \delta_{\ell} = \arg \Gamma(\ell + 1 + i \frac{2}{k}) \) is the complex phase of the \( \Gamma \)-function. Using the derivative of (3) we can write

\[
P_{k\ell}^2(r) = \left( \frac{2}{\pi k} \right)^{1/2} \cos[2kr + k^{-1}\ln(2kr) - \frac{1}{2} \pi(\ell + 1) - \delta_{\ell}] . \tag{4}
\]

Equation (4) is independent of \( r \) for large \( r \). Thus, the numerical solution for the continuum wave functions can be normalized by requiring Eq. (4) to hold for large \( r \). With these solutions and the tabulated\(^{21}\) \( ls \) wave functions we calculated the matrix elements in Eq. (1) and obtained the probability vs. energy relation. The total probability was obtained by numerical integration.

Table I summarizes the experimental results and our theoretical results on the shake-off effect in \( \alpha \) decay.

Using our calculations of shake-off probability as a function of electron energy and the experimental average peak shape in the \( ^{210}\)Po \( \alpha \) singles spectrum, we determined the shape of the \( \alpha \) spectrum associated with the shake-off of K electrons which would be expected from Migdal's theory. The theoretical shape was normalized to the same peak height as the experimental curve and is shown as a dashed line in Fig. 4. The \( \alpha \) singles spectrum had a small perturbation about 300 keV below the peak due to instrumental effects and this is reflected in both the calculations and the coincidence spectrum. As seen in Fig. 4, there
is a definite discrepancy between the experimental and theoretical curves. The probability of electron shake-off decreases more rapidly than the theoretical prediction as the electron energy increases (i.e., as the $\alpha$ particle energy decreases). Ovechkin and Tsenter\textsuperscript{20} observed the same effect in comparing electron energy measurements with Migdal's calculations, but the authors felt their experimental work was not sufficiently precise to indicate a definite discrepancy.

A distinctly different type of theoretical treatment was published recently by J. S. Hansen\textsuperscript{19} after the present work was concluded. Hansen obtained a value of $2.02 \times 10^{-6}$ for the total K shell ionization probability for $^{210}$Po which is somewhat closer in agreement with the experimental result than any of the theoretical values. Although not discussed in the present work, Hansen's calculations for L and M shells were in far better agreement with the experimental results than any other theory.

IV. SUMMARY AND SUGGESTIONS

It is clear that Migdal's theory still predicts ionization probabilities approximately twice as high as observed experimentally. The use of H-F-S type wave functions has little effect on the total ionization probabilities. Also, the energy distributions calculated with H-F-S type wave functions are almost identical with those calculated with the hydrogenic type. The theoretical energy distribution of the ejected electrons does not fall off as rapidly with increasing energy as the experimental one.

It is possible the discrepancies mentioned above could be reduced by using relativistic Hartree-Fock wave functions. These wave functions should be made very accurate at small distances since this is the region where most of the strength of the matrix elements lies.
An experimental study of $^{148}$Gd similar to the ones presented in this paper should indicate if Migdal's theory gives the proper dependence on both charge and alpha particle energy. Very large discrepancies exist between Migdal's theory and experimental shake-off measurements for the L and M shells. More precise measurements in which the abundance (L, M, and N subshells) and electron energy dependence (L subshells) are determined for each subshell could help indicate the sources of these discrepancies. Measurements of the abundance of N shell effect, which should be substantially larger than that in the M shell, could indicate what would be the "best" alpha peak shape or resolution which present-day high-resolution $\alpha$ spectrometers could obtain.

ACKNOWLEDGMENTS

We are very grateful to Duane Mosier of the Lawrence Berkeley Laboratory for many consultations and detailed suggestions on the types of equipment used in these experiments.
FOOTNOTES AND REFERENCES

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†I. Perlman is now at the Hebrew University in Jerusalem.

15. C. M. Lederer, private communication.


Table I. Probability of electron shake-off from the K shell.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Ref.</th>
<th>Experiment</th>
<th>Stated Error</th>
<th>Theory (Hydrogenic)</th>
<th>Theory (H-F-S)</th>
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<td>210Po</td>
<td>24</td>
<td>1.5 × 10^{-6}</td>
<td>±33%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>2.0</td>
<td>±16%</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>26</td>
<td>1.6</td>
<td>±31%</td>
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<td></td>
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<td></td>
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<tr>
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<td>2.87 × 10^{-6}</td>
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</tr>
<tr>
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FIGURE CAPTIONS

Fig. 1. Block diagram of electronics used for α - x-ray coincidence measurements.

Fig. 2. $^{238}$Pu α spectrum in coincidence with uranium $K_{\beta}$ x-rays.

Fig. 3. Partial decay scheme of $^{238}$Pu.

Fig. 4. $^{210}$Po α spectrum in coincidence with lead $K_{\alpha}$ x-rays. ---- Theoretical shape normalized to peak height.
Fig. 1

- Gain stabilizer
- L. amp
- 400 channel analyzer
- \( \gamma \) monitor
- Source
- Preamp.
- X-ray
- Preamp.
- Gain stabilizer
- L. amp.
- L. amp.
- Cross over
- S.C.A.
- Linear gate
- Biased amp.
- Gate mixer
- Scale down
- Coinc. events
- Coinc. circuit
- Scaler
- 400 channel analyzer
- Fig. 1
Fig. 2

Coincidence per channel vs. Channel number

238Pu

\[ a_{296} \]

\[ a_{K,44} \]

\[ a_{K,0} \]

acc.

\[ a_{44} \]

\[ a_{0} \]
Fig. 3
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