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A MODEL FOR PION ABSORPTION IN NUCLEI

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ABSTRACT

To describe pion absorption in nuclei, we introduce a model which assumes that the incident pion is absorbed by the row of target nucleons along the beam direction. The momentum distribution of these nucleons after absorbing the pion is assumed to be determined by the phase space. The experimental data on the inclusive proton cross section are well reproduced.
Recently, systematic investigations of pion absorption in nuclei have been carried out.\textsuperscript{1-3} These experiments were performed with both $\pi^+$ and $\pi^-$ incident on nuclear targets of masses $12 \leq A \leq 181$. The energy of the pion is in the region of the $\Lambda(3,3)$ resonance. It is observed that the pion absorption cross section increasingly dominates the reaction cross section in heavy nuclei. Characteristic features of the proton inclusive spectra are as follows.\textsuperscript{1,3}

1. The proton yields for energy $E_p > 40$ MeV are in the ratio of $\sim 3:1$ for $\pi^+$ vs. $\pi^-$ bombardment, independent of pion energy.

2. The increase in proton yield with target mass is roughly proportional to $A^{2/3}$.

3. The proton spectra with $\pi^+$ and $\pi^-$ at the same bombarding energy and detecting angle are similar in shape, i.e., monotonically decreasing with increasing proton energy and containing little structure.

4. The invariant proton cross section in the rapidity plane implies that protons are emitted almost isotropically from a slowly moving frame. The number of nucleons participating in the absorption process as deduced from the rapidity shifts increases with target mass roughly according to $A^{1/3}$. For a given target, the effective number of participating nucleons is independent of the pion energy and is only slightly larger for $\pi^-$ than for $\pi^+$.

The above features are unexpected from the conventional understanding that a pion is absorbed in nuclei by a two-nucleon mechanism, as
widely studied in the literature.\textsuperscript{4-7} There were cascade calculations for pion absorption in nuclei,\textsuperscript{8} the predictions are, however, qualitatively different from the experimental data. The authors in Ref. 1 have therefore suggested the need of a new understanding of pion absorption in nuclei.

If pion absorption in nuclei indeed involves many nucleons, then a microscopic study of the absorption process is certainly prohibitively complicated. We have therefore attempted to study this problem in a simple model. We hope to obtain from this study a gross picture of the pion absorption mechanism in nuclei.

Very generally, the proton inclusive cross section can be written as\textsuperscript{9}

\[ \frac{d^3 \sigma}{dp^3} = \sum_N \sigma(N) F_N(p) \]  

(1)

where \( p \) is the momentum of the proton, and \( N \) is the number of participating nucleons. The quantity \( \sigma(N) \) is the cross section for \( N \) nucleons absorbing the incident pion; and \( F_N(p) \) is the final momentum distribution of the \( N \) participating nucleons, and is normalized to unity.

Since the average number of participating nucleons ranges from three to six as deduced from the experimental data, we expect that the phase space available to these nucleons will be the dominant factor in determining the outgoing proton spectra. This is further supported by the experimental observation that the proton spectra are void of any significant structures. We thus assume that the momentum distribution function \( F_N(p) \) is purely statistical, and write it as\textsuperscript{10}
\[ F_N(p) = I_{N-1} \left( \frac{p_o - p}{2m}, E - \frac{p^2}{2m} \right) / I_N(p_o, E) \]  

with

\[ I_N(p_o, E) = \prod_{i=1}^{N} \int d^3 \vec{p}_i \, \delta(3) \left( p_o - \sum_{i=1}^{N} \vec{p}_i \right) \delta \left( E - \sum_{i=1}^{N} \frac{p^2_i}{2m} \right) \]

\[ = \frac{(2\pi m)^{3/2} \Gamma(3/2(N-1))}{N^{3/2}} \left( \frac{E - \frac{p_o^2}{2N m}}{3/2N - 5/2} \right) \]  

and a similar expression for \( I_{N-1} \). In the above, \( p_o \) and \( E \) are the momentum and total energy (including the rest mass) of the incoming pion. The nucleon mass is denoted by \( m \). In Eq. (3), it is assumed that nucleons move non-relativistically. This is certainly valid for the energy range we are working with. For \( N = 2 \), \( F_N(p) \) is a delta function. For reasons to be explained later the contribution to the proton inclusive cross section from \( N = 2 \) amounts to 15% or less in heavy nuclei. We neglect it in our preliminary calculations. We have also implicitly neglected the final state interactions of the emitted protons with the target nucleus. The authors in Ref. 1 have pointed out that for nucleons with energy \( \sim 100 \) MeV, the mean free path in nuclei should be \( \sim 4-9 \) fm. It is therefore reasonable to neglect the final state interactions.

As to the geometrical aspect, we have the following picture. Since we are mainly interested in the energy range of the pion around the \( \Delta(3,3) \) resonance, a virtual \( \Delta \) is most probably formed when the incident pion hits the first nucleon. This virtual \( \Delta \) then propagates through the medium and collides with other nucleons. During its propagation, the virtual \( \Delta \) may convert into a nucleon. We assume for simplicity that this leading particle propagates in a straight trajectory parallel to the beam.
direction. Such a one-dimensional cascade model for pion-nucleus interaction has recently been studied by Hüfner and Thies.\textsuperscript{11} The number of collisions the leading particle suffers for a given impact parameter $b$, is then given by $N(b) = 1$, where

$$N(b) = 2\sqrt{R^2 - b^2} \rho \sigma$$

with $\rho$ the nuclear density, $\sigma$ the total cross section of the leading particle with a target nucleon, and $R$ the nuclear radius. Let $Q$ be the probability that no pion results from the collision of the leading particle with a nucleon, then the probability that all the $N(b)$ nucleons participate in the absorption process is given by $Q^{N(b)-1}$. The proton inclusive cross section can then be written as

$$\frac{d^3\sigma}{dp^3} = 2\pi \int_0^R db \ b N_\pm(b) Q^{N(b)-1} F_{N(b)}(p)$$

where $N_\pm(b) = N(b) Z/A \pm 1$, corresponding to the number of protons for $\pi^+$ and $\pi^-$ induced reactions, respectively.

From Eq. (4), it is easily seen that $N(b) \approx 2$ for only a small range of the impact parameter in heavy targets. This justifies our neglect of two-nucleon absorption mechanisms mentioned previously. However, it certainly introduces appreciable errors for light targets.

Since the proton spectra are measured up to the kinematical limit, it is crucial to take into account the intrinsic Fermi motion of the target nucleons. This is approximately included by adding to the total energy of the $N$ participating nucleons the energy $P^2/2Nm$, where \textsuperscript{12}

$$P^2 = \frac{N(A-N)}{A-1} \frac{3}{5} k_F^2$$
with \( k_F \) as the Fermi momentum. Equation (6) is the mean square momentum of the group of nucleons \( N \) in a nucleus with \( A \) nucleons.

For our calculations, we used the normal values \( p = 0.17 \text{ fm}^{-3} \), \( k_F = 1.36 \text{ fm}^{-1} \), and \( R = 1.2 A^{1/3} \text{ fm} \). The cross section \( \sigma \) is taken to be 40 mb, slightly larger than the p-p total cross section \( \approx 25 \text{ mb} \) at the corresponding energy range. We choose this value for \( \sigma \) because it gives the average number of participating nucleons \( \bar{N} \approx 4/3 R p \), that is consistent with the experimentally deduced value. The parameter \( Q \) introduced previously for describing the probability that no pion is produced when the leading particle collides with a nucleon is taken as a free parameter. We have found from our calculations that with \( Q = 0.9 \) a good description of the experimental data is obtained.

The calculated results are shown in Figs. 1 and 2. In Fig. 1a, we show, as a function of the mass number, the integrated proton inclusive cross section induced by 220 MeV \( \pi^+ \) absorption. The ranges of angular and energy integrations are \( 30^\circ - 150^\circ \) and \( 40 \text{ MeV} - 250 \text{ MeV} \), respectively. Except for the \( ^{12}C \) nucleus, the calculated results agree very well with the data. For heavy targets, the calculated integrated cross section is sensitive to the parameter \( Q \). With \( Q = 1 \), the increase of \( \sigma_{\pi^+} \) with \( A \) is much steeper than the experimental data show, and the theoretical value for the target \( ^{181}Ta \) is a factor of two larger than the observed value. The underestimate of the integrated proton inclusive cross section for \( ^{12}C \) nucleus is probably due to the neglect of the two-nucleon absorption contribution in the calculation.

In Fig. 1b, the ratio \( \bar{R} = \sigma_{\pi^+}/\sigma_{\pi^-} \) for \( \pi^+ \) and \( \pi^- \) induced integrated proton inclusive spectra is shown. It is seen that \( \bar{R} \approx 3 \) and
agrees reasonably with the data. This quantity depends only weakly on the parameter $Q$. In Fig. 1c, the average number of participating nucleons is plotted against the target mass number. It is calculated according to

$$\bar{N} = \frac{\int_{Q}^{R} db \ b \ N(b) \ Q^{N(b)-1}}{\int_{0}^{R} db \ b \ Q^{N(b)-1}}$$

(7)

The theoretical values, which again depend weakly on $Q$, agree well with the experimentally determined values.

In Fig. 2, we show the inclusive proton energy spectra at two laboratory angles $30^\circ$ and $150^\circ$, resulting from the absorption of 160 MeV $\pi^+$ in $^{58}$Ni. The solid curve is from the theoretical calculation. Except for large proton kinetic energies, the data are well explained by the model calculation. The high energy part of the proton spectra can be better fitted with a larger Fermi momentum such as $k_F = 1.6 \ fm^{-1}$ than used here.

The quantities shown in Fig. 1 are global and are determined mainly by the geometry in the model. The favorable results from our calculations compared with the experimental data suggest strongly that the eikonal approximation to the propagation of the leading particle seems reasonable. The fact that $Q$ is only slightly smaller than unity suggests further that the row of nucleons along the beam direction seems to absorb the incident pion collectively.

The reasonable reproduction of the experimental inclusive proton angular and energy spectra as shown in Fig. 2 suggests the adequacy of a statistical approximation to the momentum distribution of the participating
nucleons after absorbing the incident pion. The fine structure of the experimental spectra may then contain the non-statistical aspect of the process.

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REFERENCES


FIGURE CAPTIONS

Fig. 1. (a) As a function of the target atomic mass, the integrated proton cross section for 220 MeV $\pi^+$; (b) the ratio of integrated proton yields seen with $\pi^+$ to those seen with $\pi^-$; (c) average number of nucleons participating in the process; circles are from experimental data while solid curves are from calculations.

Fig. 2. Proton energy spectra from 160 MeV $\pi^+$ on Ni at laboratory angles 30° and 150°. Circles are from experimental data and solid curves from calculations.
Fig. 1

\[ \sigma_{\pi^+} \text{(b)} \]

\[ \bar{R} \]

\[ \bar{N} \]

\[ A \]

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$\pi^+(160 \text{ MeV}) + ^{58}\text{Ni} \rightarrow p + X$

Fig. 2

$\frac{d^2\sigma}{dE \cdot d\Omega}$ ($\mu$b/sr-MeV²) vs $E_p$ (MeV)

$30^\circ$

$150^\circ$