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Abstract

Possible Light Scattering Experiment to Detect the Three Dimensional Wigner Lattice

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ABSTRACT

It is proposed that a possible experiment to detect the three dimensional Wigner electron lattice is the study of Brillouin scattering of photons by the LA and TA modes of vibration of the lattice.
The Wigner lattice is a body-centered cubic lattice of electrons whose formation in a solid is predicted to occur at a sufficiently low value of the electron density. Considering semiconductors with variable extrinsic electron densities as potential hosts for the electron lattice, the possibility of experimental detection of the three-dimensional Wigner lattice arises. The aim of this Letter is the discussion of a possible experiment for this purpose. To the author's knowledge, this is the first paper discussing such an experiment.

Considering electrons in the presence of a uniform background of fixed positive charge, Wigner argued that, as their potential energy \( V \) of interaction became large compared to their kinetic energy \( T \), the electrons would become localized in space and form a lattice. If \( n \) is the electron density in a solid of dielectric constant \( \varepsilon \), then the ratio

\[
\frac{V}{T} = \frac{C(4\pi/3)^{1/3} n^{-1/3} (m^*e^2/e^2)}{C(r_o/a^*)} = \frac{Cr_s}{},
\]

where \( m^* \) is the electron effective mass, \( (4\pi r_o^3/3) = (1/n) \), \( a^* = (h^2/c/m^*e^2) \) is a generalized Bohr radius, and \( C \) is a constant of order unity. Eq. (1) shows that \( (V/T) \) increases as \( r_s \) increases, leading to crystallization of the electron gas at sufficiently large values of \( r_s \) larger than a critical value \( r_c \).

The ground state energy of the electron gas and lattice has been the subject of a number of calculations. The values of \( r_s \) obtained are between 14 and 20, suggesting that the Wigner lattice will be stable at electron densities corresponding to values of \( r_s \) greater than about 15. The requisite low values of the electron density can (in principle) be achieved in extrinsic germanium.

If the Wigner lattice does form, how might it be detected experimentally? Using \( n = 10^{14} \text{ cm}^{-3} \), corresponding to \( r_s = 19 \) in germanium, the lattice constant \( a \) of the bcc electron lattice is given by \( n = (2/a^3) \), or \( a = 2.7 \times 10^{-5} \text{ cm} \). The electron gas in a semiconductor of simple band structure has a plasma oscillation mode of frequency \( \omega_p \), where \( \omega_p^2 = (4\pi n^2/m^*\varepsilon) \). Plasmons in the electron
gas in semiconductors have been detected by Brillouin scattering experiments, in which an incident photon of frequency $\omega_o$ is inelastically scattered with the creation of a plasmon of frequency $\omega_p$ and a scattered photon of frequency $\omega_s$. If the electron gas in a semiconductor does crystallize, it is expected that transverse acoustic vibrations would be possible in addition to the longitudinal acoustic branch. This expectation is borne out in the calculation of Clark of the vibrational spectrum of an electron lattice in the presence of a uniform background of positive charge. The electron lattice is thus expected to have vibrational modes whose phonon aspects may be thought of approximately as longitudinal (LA) and transverse (TA) "plasmons". The former are the analog of longitudinal plasma oscillations in the electron gas, but the latter should be something new.

Clark's result is that the frequencies $\omega_L$ and $\omega_T$ of the LA and TA modes are given by

$$\omega_L = (1/\sqrt{2\pi})\Lambda_L \omega_p; \quad \omega_T = (1/\sqrt{2\pi})\Lambda_T \omega_p,$$

where $\omega_p$ is the plasma frequency of the electron gas of density $n$, etc. The quantities $\Lambda_L$ and $\Lambda_T$ are reduced frequencies whose values were calculated at various special points ($\Gamma$, $H$, $P$) of the Brillouin Zone. It was found that the frequency of the zone center LA mode of the lattice is the same as the plasma frequency $\omega_p$ of the electron gas of the same density but the zone edge (points $H$ and $P$) LA and TA modes of the electron lattice have frequencies of about $0.6\omega_p$.

One may contrast the expected features of the Brillouin scattering spectrum of the electron lattice with those observed for scattering by plasmons in the electron gas. In the latter case, the central feature of the spectrum is a peak at $\Delta\omega = (\omega_o - \omega_p) = \omega_p$ due to scattering by plasmons. For the electron crystal, one would also expect to observe Brillouin scattering at $\Delta\omega = \omega_p$ due to the
zone-center LA vibrational mode of $\omega_p$. However, it is expected that the additional vibrational modes due to the formation of the electron lattice would result in new and additional features in the Brillouin scattering spectrum, and that these features would not be present in the spectrum for the electron gas. Of special interest are the transverse vibrations of the electron lattice; these would be expected to couple strongly to incident photons. It is of particular interest and importance that, for the electron lattice, one would expect Brillouin scattering by LA and TA modes whose wave vectors are significantly different from zero. The reason is that the lattice constant of the electron crystal is very large compared to those found in usual crystals, resulting in a much smaller Brillouin Zone than those generally encountered. Thus, for the electron lattice, the zone-edge values of the phonon wave vector are roughly equal to the wave vectors of visible and near infrared photons, so LA and TA vibrational modes with wave vectors equal to the zone-edge values would take part in the Brillouin scattering process.

It is concluded that the Brillouin scattering spectrum (i.e., intensity as a function of photon frequency shift $\Delta \omega$ for a given scattering direction) of the electron lattice should show new features in addition to scattering by the LA ($k = 0$) mode at $\Delta \omega = \omega_p$ found for the electron gas. Since both the LA and TA vibrations with wave vectors covering the whole Brillouin Zone would be expected to take part, spectral structure at values of $\Delta \omega$ smaller than $\omega_p$ is to be anticipated.

In summary, it is reasonable to expect significant qualitative differences between the Brillouin scattering spectrum due to plasmons in the electron gas and the features anticipated for the spectrum of the electron lattice. It is therefore proposed that a possible experiment to detect the three dimensional Wigner electron lattice is the study of Brillouin scattering of photons by the LA and TA modes of vibration of the electron lattice.
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REFERENCES

1. R. Dalven, (to be published).
3. See Reference 1 for full references to these calculations.
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