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Measurement of $B \to K^+\gamma$ Branching Fractions and Charge Asymmetries

The branching fractions of the exclusive decays $B_s^0 \to K^{*0} \gamma$ and $B^+ \to K^+ \gamma$ are measured from a sample of $(22.74 \pm 0.36) \times 10^6 B\bar{B}$ decays collected with the BABAR detector at the PEP-II asymmetric $e^+ e^-$ collider. We find $\mathcal{B}(B_s^0 \to K^{*0} \gamma) = [4.23 \pm 0.40 \text{(stat)} \pm 0.22 \text{(syst)}] \times 10^{-5}$, $\mathcal{B}(B^+ \to K^+ \gamma) = [3.83 \pm 0.62 \text{(stat)} \pm 0.22 \text{(syst)}] \times 10^{-5}$ and constrain the CP-violating charge asymmetry to be $-0.170 < A_{CP}(B_s^0 \to K^{*0} \gamma) < 0.082$ at 90% C.L.

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In the standard model (SM) the exclusive decays $B \rightarrow K^* \gamma$ proceed dominantly by the electromagnetic loop “penguin” transition $b \rightarrow s \gamma$. Many extensions of the SM provide new virtual high-mass fermions and bosons that can appear in the loop, causing deviations in the inclusive rate for $b \rightarrow s \gamma$ [1]. The sensitivity of the exclusive rates to these effects is limited by the uncertainty in the SM calculation. However, there has been considerable theoretical progress recently [2]. The precision measurement of the exclusive branching fractions $\mathcal{B}(B^0 \rightarrow K^{*0} \gamma), \mathcal{B}(B^+ \rightarrow K^{*+} \gamma)$ is needed to test and improve these calculations. The non-SM processes can also interfere with the SM decay to cause CP-violating charge asymmetries at a level as high as 20% [3]. The CP-violating charge asymmetry from SM contributions alone is expected to be $<1\%$.

In this Letter, measurements of the exclusive branching fractions, $\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)$ in the $K^{*0} \rightarrow K^+ \pi^- , K_S^0 \pi^+$ modes, and $\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)$ in the $K^{*+} \rightarrow K^+ \pi^0 , K_S^0 \pi^+$ modes, are presented. Here $K^*$ refers to the $K^*(892)$ resonance and the charge conjugate decays are implied unless otherwise stated. The $K^{*0} \rightarrow K^+ \pi^-$ and $K^{*+} \rightarrow K^+ \pi^0 , K_S^0 \pi^+$ modes are used to search for CP-violating charge asymmetries.

The data were collected with the BABAR detector [4] at the PEP-II asymmetric $e^+e^-$ (3.1 GeV) $- e^- (9$ GeV) storage ring [5]. The results in this paper are based upon an integrated luminosity of $\approx 20.7$ fb$^{-1}$ of data corresponding to $(22.74 \pm 0.36) \times 10^9 B\overline{B}$ meson pairs recorded at the $Y(4S)$ resonance (“on-resonance”) and $\approx 2.6$ fb$^{-1}$ at 40 MeV below this energy (“off-resonance”). The number of $B\overline{B}$ meson pairs is determined from the ratio of the number of hadronic events to muon pairs in on- and off-resonance data [4]. We assume that the $Y(4S)$ decays equally to neutral and charged $B$ meson pairs.

We use Monte Carlo simulations of the BABAR detector based on GEANT 3.21 [6] to optimize our selection criteria and to determine signal efficiencies. Events taken from random triggers are used to measure the beam backgrounds. These simulations take account of varying detector conditions and beam backgrounds.

The selection criteria for this analysis are optimized to maximize $S/S_0$ where $S$ is the number of signal candidates expected, assuming the central values of the previous measurement $\mathcal{B}(B^0 \rightarrow K^{*0} \gamma, B^+ \rightarrow K^{*+} \gamma) = [4.55 \pm 0.72 \times 0.68 (\text{stat}) \pm 0.34 (\text{syst}) , 3.76 \pm 0.83 (\text{stat}) \pm 0.28 (\text{syst})] \times 10^{-5}$ [7], and $B$ is the expected number of background candidates determined from Monte Carlo and confirmed with off-resonance data. Quantities are computed in both the laboratory frame and the center-of-mass frame of the $e^+e^-$ system. Those computed in the center-of-mass frame are denoted by an asterisk; e.g., $E_{\text{beam}} = 5.29$ GeV is the on-resonance center-of-mass energy of the $e^+$ and $e^-$ beams.

We require a high-energy radiative photon candidate with energy $1.5 < E_\gamma < 4.5$ GeV in the laboratory frame and $2.30 < E_\gamma^* < 2.85$ GeV in the center-of-mass frame. A photon candidate is defined as a localized energy maximum [4] in the calorimeter acceptance $-0.74 < \cos \theta < 0.93$, where $\theta$ is the polar angle to the detector axis. It must be isolated by 25 cm from any other photon candidate or track and have a lateral energy profile consistent with a photon shower. We veto photons from a $\pi^0(\eta)$ by requiring that the invariant mass of the combination with any other photon of energy greater than 50(250) MeV not be within a $\approx$2.7(2.2) sigma window of the nominal $\pi^0(\eta)$ mass, 115(508) < $M_{\gamma\gamma}$ < 155(588) MeV/c$^2$.

The $K^*$ is reconstructed from $K^*+, K_S^0, \pi^-, \pi^0$, and $\pi^0$ candidates through the four modes $K^{*0} \rightarrow K^+ \pi^- , K_S^0 \pi^+$ and $K^{*+} \rightarrow K^+ \pi^0 , K_S^0 \pi^+$. The $K^+$ and $\pi^-$ track candidates are required to be well reconstructed in the drift chamber and to originate from a vertex consistent with the $e^+e^-$ interaction point (IP). The $K_S^0$ candidates are reconstructed from two oppositely charged tracks coming from a common vertex displaced from the IP by at least 0.2 cm and having an invariant mass within $\approx 3.3$ sigma of the nominal $K_S^0$ mass. $489 < M_{\pi^+\pi^-} < 507$ MeV/c$^2$. A track is identified as a kaon if it is projected to pass through the fiducial volume of the particle identification detector, an internally reflecting ring-imaging Cherenkov detector (DIRC) [4], and the cone of Cherenkov light is consistent in time and angle with a kaon of the measured track momentum. A charged pion is identified as a track that is not a kaon. The $\pi^0$ candidates are reconstructed from pairs of photons, each with energy greater than 30 MeV, and are required to have 115 < $M_{\gamma\gamma}$ < 150 MeV/c$^2$ and $E_{\gamma\gamma} > 200$ MeV. A mass-constraint fit to the nominal $\pi^0$ mass is used to improve the resolution of its momentum. The $K^*$ reconstruction is completed by requiring the invariant mass of the candidate pairs to be within 100 MeV/c$^2$ of the $K^{*0}/K^{*+}$ mass.

The $B$ meson candidates are reconstructed from the $K^*$ and $\gamma$ candidates. The background is predominantly from continuum $q\bar{q}$ production, where $q$ can be a $u, d, s, \text{ or } c$ quark, with the high-energy photon originating from initial-state radiation or from $\pi^0$ and $\eta$ decays. The background from other nonradiative $B$ meson decays is found to be negligible from Monte Carlo simulation. We exploit event topology differences between signal and background to reduce the continuum contribution. We compute the thrust axis of the event excluding the $B$ meson daughter candidates. Figure 1 shows the distribution of $|\cos \theta_H|$ for signal Monte Carlo events and off-resonance data, where $\theta_H$ is the angle between the high-energy photon candidate and the thrust axis. In the center-of-mass frame, $B\overline{B}$ pairs are produced approximately at rest and produce a uniform $|\cos \theta_H|$ distribution. In contrast, $q\bar{q}$ pairs are produced back-to-back in the center-of-mass frame which results in a $|\cos \theta_H|$ distribution peaking at 1. We require $|\cos \theta_H| < 0.8$. We further suppress backgrounds using the angle of the $B$ meson candidate’s direction with respect to the beam axis, $\theta_H$, and the helicity angle of the $K^*$ decay, $\theta_H$. The helicity angle is defined as the angle between either one of the $K^*$ daughters’ momentum
vectors computed in the rest frame of the $K^+$ and the $K^*$ momentum vector in the parent $B$ meson rest frame. It follows a $\sin^2\theta_H$ distribution for the signal and peaks slightly towards $\pm 1$ for $q\bar{q}$ background. The $B$ meson candidate’s direction also follows a $\sin^2\theta_B$ for the signal and is approximately flat for the $q\bar{q}$ background. We require $|\cos\theta_B| < 0.80$ and $|\cos\theta_H| < 0.75$.

Since the $B$ mesons are produced via $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$, the energy of the $B$ meson in the center-of-mass frame is the beam energy, $E^\text{beam}_B$. This is compared to the measured energy of the $B$ meson daughters by defining $\Delta E^* = E^*_K + E^*_\gamma - E^*_\text{beam}$. The distribution of $\Delta E^*$ is peaked at zero for the signal with a width dominated by the resolution of the photon candidates. It is asymmetric due to energy leakage from the calorimeter. We require $-200 < \Delta E^* < 100$ MeV for the $K^+\pi^-$, $K^0_S\pi^0$ modes and $-225 < \Delta E^* < 125$ MeV for the modes containing a $\pi^0$, namely $K^+\pi^0$ and $K^0_S\pi^0$. The beam-energy substituted mass is defined as $m_{ES} = \sqrt{E^2_{\text{beam}} - p^2_B}$, where $p_B^2$ is the momentum vector of the $B$ meson candidate calculated from the measured momenta of the daughters. The $m_{ES}$ distribution for the signal is well described by an asymmetric resolution function [8], with an approximately Gaussian core dominated by the resolution of the beam energy measurement, and an asymmetric tail caused by the energy leakage from the calorimeter for the photon candidates. For the modes containing a single photon candidate, namely $K^+\pi^-$ and $K^0_S\pi^0$, we can remove the tail in $m_{ES}$ by rescaling the measured photon energy $E^*_\gamma$ by a factor $\kappa$, determined for each event, so that $E^*_K + \kappa E^*_\gamma - E^*_\text{beam} = 0$. The signal for these modes is then described by a Gaussian. The background is empirically described by a threshold function [9] for each mode. We select candidates with $m_{ES} > 5.2$ GeV/$c^2$.

Figure 2 shows the $m_{ES}$ distribution for each of the four modes. An unbinned maximum-likelihood technique is used to fit the $m_{ES}$ distributions for signal [8] and background [9] contributions. The signal mean and width are allowed to vary in the fit for the high statistics $K^+\pi^-$ and $K^+\pi^0$ modes. The fitted width is slightly larger than the predicted Monte Carlo value. In the lower statistics modes we fix the width to the Monte Carlo value adjusted for the small difference observed in the high statistics modes. We fit the on-resonance data with a signal plus background shape, and simultaneously the on-resonance sideband and off-resonance samples with the same background function, using a common fit parameter. The off-resonance data sample is required to pass the same selection criteria as the on-resonance data sample except that we remove the kaon particle identification requirement to gain statistics in the $K^+\pi^-$ and $K^+\pi^0$ modes. The on-resonance sideband sample is selected with the same criteria as the on-resonance data sample, except that we require $150 < \Delta E^* < 400$ MeV in the $K^0_S\pi^0$ and $K^+\pi^0$ modes, and $100 < \Delta E^* < 500$ MeV in the $K^+\pi^-$ and $K^0_S\pi^0$ modes. The signal yields with statistical errors from the fit are given in Table I.

As a consistency check we plot in Fig. 3a the $\Delta E^*$ projection for the $K^+\pi^-$ mode after requiring $5.27 < m_{ES} < 5.29$ GeV/$c^2$. A comparison of the observed $\Delta E^*$ distribution with Monte Carlo shows good agreement. We also plot $M_{K^+\pi^-}$ in Fig. 3b after requiring $5.27 < m_{ES} < 5.29$ GeV/$c^2$, $-200 < \Delta E^* < 100$ MeV, and $0.7 < M_{K^+\pi^-} < 1.1$ GeV/$c^2$. We fit with a relativistic Breit-Wigner plus linear background shape and determine that the signal is consistent with coming from the $K^* (892)$.

The efficiency for the selection of $B \rightarrow K^*\gamma$ candidates is given in Table I. The branching fraction is determined from the yield, the efficiency and the total number of $B\bar{B}$ events in the sample. The cross-feed from the other $B \rightarrow K^*\gamma$ modes and the down-feed from $B \rightarrow X_{s}\gamma$ is...
are estimated with Monte Carlo assuming the measured branching fractions from the CLEO Collaboration [7,10] for each mode and subtracted from the signal yield.

The total systematic error is the sum in quadrature of the components shown in Table II. The systematic uncertainty in the signal yield derives from uncertainties in the signal line shape, and cross-feed and down-feed contributions. The uncertainty in the signal line shape results from the $m_{ES}$ width difference described above. To gain statistics in the off-resonance data sample used to fit the background function for the $K^+\pi^-$ and $K^+\pi^0$ modes we relax the kaon identification requirement and consequently assign a systematic uncertainty to the assumption that the background shape is unaffected. The error in the assumed branching fractions and final-state modeling for $B \to X_s \gamma$ [10] gives a systematic error in the estimated down-feed from these modes. The tracking efficiency is computed by identifying tracks in the silicon vertex detector and observing the fraction that is well reconstructed in the drift chamber. We estimate the $K_S^0$ efficiency uncertainty by comparing the momenta and flight-distance distributions in data and Monte Carlo. The kaon identification efficiency in the DIRC is derived from a sample of $D^{++} \to D^0 \pi^+, D^0 \to K^- \pi^+$ decays. The photon and $\pi^0$ efficiencies are measured by comparing the ratio of events $N(\tau^0 \to h^- \pi^0)/N(\tau^0 \to h^- \pi^0\pi^0)$ to the previously measured branching ratios [11]. The photon isolation and $\pi^0/\eta$ veto efficiency are dependent on the event multiplicity and are tested by “embedding” Monte Carlo–generated photons into both an exclusively reconstructed $B$ meson data sample and a generic $B$ meson Monte Carlo sample. The $\Delta E^*$ resolution is dominated by the photon-energy resolution so that uncertainties in the calorimeter energy resolution and overall energy-scale cause an uncertainty in the efficiency of the $\Delta E^*$ requirement. The photon-energy resolution is measured in data using $\pi^0$ and $\eta$ meson decays and $e^+e^- \to e^+e^-\gamma$ events. The energy-scale uncertainty is estimated by using a sample of $\eta$ meson decays with symmetric energy photons; the deviation in the reconstructed $\eta$ mass from the nominal $\eta$ mass provides an estimate of the uncertainty in the measured single photon energy.

The $B \to K^+\gamma$ samples, except for the $K_S^0\pi^0$ sample, are used to search for $CP$-violating charge asymmetries $A_{CP}$, defined by

$$A_{CP} = \frac{\Gamma(B \to K^+\gamma) - \Gamma(B \to K^+\gamma)}{\Gamma(B \to K^+\gamma) + \Gamma(B \to K^+\gamma)}.$$  

The flavor of the underlying $b$ quark is tagged by the charge of the $K^+$ or $K^{*+}$ in the decay. The probability of a double misidentification of the kaon and pion, which would result in a dilution of the asymmetry, is estimated to be $0.0026 \pm 0.0008$ and has been neglected. The on-resonance sample for each mode is divided into two $CP$-conjugate samples and the signal yield for each is extracted with the same fitting technique as for the branching

![FIG. 3.](a) The $\Delta E^*$ projection for $B^0 \to K^0\gamma$, $K^0 \to K^+\pi^-$ candidates. The curve is the Monte Carlo expectation with a linear background. (b) The $M_{K^+\pi^-}$ projection for $B^0 \to K^0\gamma$, $K^0 \to K^+\pi^-$ candidates with the $M_{K^+\pi^-}$ mass cut relaxed. The curve is a fit to a relativistic Breit-Wigner function with linear background.)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Efficiency</th>
<th>No. signal events</th>
<th>No. cross-feed events</th>
<th>No. down-feed events</th>
<th>$\mathcal{B}(B \to K^+\gamma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+\pi^-$</td>
<td>14.0</td>
<td>135.7 ± 13.3</td>
<td>0.4 ± 0.1</td>
<td>0.6 ± 0.1</td>
<td>4.24 ± 0.41 ± 0.22</td>
</tr>
<tr>
<td>$K_S^0\pi^0$</td>
<td>1.4</td>
<td>14.8 ± 5.6</td>
<td>0.4 ± 0.1</td>
<td>1.0 ± 0.2</td>
<td>4.10 ± 1.71 ± 0.42</td>
</tr>
<tr>
<td>$K_S^0\pi^+$</td>
<td>3.9</td>
<td>28.1 ± 6.6</td>
<td>0.7 ± 0.2</td>
<td>1.2 ± 0.2</td>
<td>3.01 ± 0.76 ± 0.21</td>
</tr>
<tr>
<td>$K^+\pi^0$</td>
<td>4.3</td>
<td>57.6 ± 10.4</td>
<td>1.2 ± 0.2</td>
<td>2.6 ± 0.4</td>
<td>5.52 ± 1.07 ± 0.38</td>
</tr>
</tbody>
</table>

Table II. The systematic uncertainties in the measurement of $\mathcal{B}(B \to K^+\gamma)$. 

<table>
<thead>
<tr>
<th>% Uncertainty in $\mathcal{B}(B \to K^+\gamma)$</th>
<th>$K^+\pi^-$</th>
<th>$K_S^0\pi^0$</th>
<th>$K_S^0\pi^+$</th>
<th>$K^+\pi^0$</th>
</tr>
</thead>
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<tr>
<td>$m_{ES}$ line shape</td>
<td>...</td>
<td>7.4</td>
<td>1.7</td>
<td>1.9</td>
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<tr>
<td>Background shape</td>
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<td>...</td>
<td>...</td>
<td>3.8</td>
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<tr>
<td>Down-feed modeling</td>
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<td>1.5</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Photon efficiency</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Photon distance cut</td>
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<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\pi^0$ efficiency</td>
<td>...</td>
<td>2.5</td>
<td>...</td>
<td>2.5</td>
</tr>
<tr>
<td>$\pi^0/\eta$ veto efficiency</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Energy resolution</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
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<tr>
<td>Energy scale</td>
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<td>1.0</td>
</tr>
<tr>
<td>$K^+/\pi^-$ tracking efficiency</td>
<td>2.4</td>
<td>...</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>$K_S^0$ efficiency</td>
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<td>4.5</td>
<td>4.5</td>
<td>...</td>
</tr>
<tr>
<td>Kaon identification</td>
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<td>...</td>
<td>...</td>
<td>1.0</td>
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<tr>
<td>MC statistics</td>
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<td>2.4</td>
<td>1.5</td>
<td>2.1</td>
</tr>
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<td>$B$ counting</td>
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<tr>
<td>Total</td>
<td>5.3</td>
<td>10.3</td>
<td>6.7</td>
<td>7.0</td>
</tr>
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</table>
fraction measurements. In the fit the background shape
and normalization, as well as the signal peak and width,
are constrained to be the same for both CP-conjugate
samples. The measured asymmetries and the asymmetry
of the background in the sideband regions defined by
$-200 < \Delta E^* < 100$ MeV, $5.2 < m_{ES} < 5.27$ GeV/c$^2$
are given in Table III.

The systematic uncertainty in the asymmetry is due to
possible detector effects that cause a different reconstruc-
tion efficiency for the two CP conjugate decays. This
uncertainty has been estimated with data from a num-
ber of known charge-symmetric processes and is given in
Table III.

Finally, we combine the measured branching fractions
for the individual modes using a weighted average,
$\mathcal{B}(B^+ \rightarrow K^{*+} \gamma) = [3.83 \pm 0.62(\text{stat}) \pm 0.22(\text{syst})] \times 10^{-5}$,
$\mathcal{B}(B^0 \rightarrow K^{*0} \gamma) = [4.23 \pm 0.40(\text{stat}) \pm 0.22(\text{syst})] \times 10^{-5}$.
The weighting uses the quadratic sum of the statisti-
cal and uncorrelated systematic errors, and the combined
error takes into account the correlated systematic errors.
The weighted average of the measured CP-violating
charge asymmetries is $A_{CP}(B \rightarrow K^+ \gamma) = -0.044 \pm 0.076(\text{stat}) \pm 0.012(\text{syst})$. We constrain $-0.170 < A_{CP}(B \rightarrow K^+ \gamma) < 0.082$ at 90\% C.L.

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NFR (Norway), MIST (Russia), and PPARC (United King-
dom). Individuals have received support from the Swiss
NSF, A.P. Sloan Foundation, Research Corporation, and
Alexander von Humboldt Foundation.

\begin{table}[h]
\centering
\caption{The measured $A_{CP}$ in signal and background
samples.}
\begin{tabular}{|c|c|c|}
\hline
Mode & $A_{CP}$ (signal) & $A_{CP}$ (background) \\
& ($\pm \text{stat} \pm \text{syst}$) & ($\pm \text{stat}$) \\
\hline
$K^+ \pi^-$ & $-0.049 \pm 0.094 \pm 0.012$ & $-0.011 \pm 0.104$ \\
$K^0_\ell \pi^+$ & $-0.190 \pm 0.210 \pm 0.012$ & $-0.080 \pm 0.080$ \\
$K^+ \pi^0$ & $0.044 \pm 0.155 \pm 0.021$ & $-0.022 \pm 0.105$ \\
\hline
\end{tabular}
\end{table}

\footnote{Also with Università di Perugia, Perugia, Italy.}
\footnote{Also with Università della Basilicata, Potenza, Italy.}

[1] See, for example, S. Bertolini, F. Borzumati, and
A. Masiero, Phys. Lett. B 192, 437 (1987); H. Baer and
M. Brhlik, Phys. Rev. D 55, 3201 (1997); J. Hewett and
J. Wells, Phys. Rev. D 55, 5549 (1997); M. Carena \textit{et al.},

and M. Neubert, hep-ph/0110078; Z. Ligeti and M.B.


[4] \textit{BABAR} Collaboration, B. Aubert \textit{et al.}, hep-ex/0105044
[Nucl. Instrum. Methods (to be published)].


[6] “\textit{GEANT} Detector Description and Simulation Tool,” CERN

5283 (2000).

[8] The signal for the $K^+ \pi^0$ and $K^0_\ell \pi^0$ modes is fit with the
function $dN/dm_{ES} = A_8 \exp \left(-0.5[\ln^2(1 + \Lambda (m_{ES} -
m_0)]/\tau^2 + \tau^2)\right)$ where $\Lambda = \sinh(\tau \sqrt{m_4})/(\sigma \tau \sqrt{m_4})$, the
peak position is $m_0$, the width is $\sigma$, and $\tau$ is the tail
parameter. The $K^+ \pi^-$ and $K^0_\ell \pi^+$ modes are fit with a
Gaussian distribution.

[9] The background for each mode is fit with
$dN/dm_{ES} = A_{BBM} m_{ES} \exp \left(-\zeta (1 - m_{ES}^2/E_{beam}^2)\right)$,
a function introduced by the ARGUS Collaboration,


70, 1207 (1993).