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2000-05-01
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This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and National Science Foundation Grant PHY-95-14797.
EFFECTIVE COSMOLOGICAL CONSTANTS FROM SUPERGRAVITY

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A derivation of the Randal–Sundrum's model as an effective description of the dynamics of the low-energy spectrum of string theories is given. The geometry can be described in terms of effective cosmological constants in the bulk and on the brane when the dilaton coupling vanishes and these cosmological constants are quantized.

1 Compact and non-compact extra dimensions

Despite the fact that the Standard Model (SM) of electroweak and strong interactions is a renormalizable quantum field theory and thus can be extrapolated a priori to any energy scale, there are phenomenological as well as theoretical evidences for a physics beyond this model. Notably a physics that would describe gravity at a quantum level. So far string theory is the only consistent theory in this direction and at low energy the lightest string excitations can be described as an effective quantum field theory such as the SM nevertheless with a caveat regarding the space-time dimension because (super)string theories require ten or eleven dimensions. And since the works of Kaluza and Klein, we know that, if there exists some extra-dimensions to our universe, an infinity of massive states will be associated to each usual 4D field. Because these KK modes have not yet been observed, necessarily their masses must be beyond the experimental range of energies resolved in accelerators (∼1 TeV). That is why the size of extra-dimensions cannot exceed such a ridiculously tiny scale (∼1 TeV⁻¹ ∼ 10⁻¹⁹ m). However recent progresses in string theories¹ have corrected this old scenario suggesting that the Standard Model gauge interactions are confined to a four dimensional hypersurface while gravity can still propagate in the whole bulk space-time. Since the gravity has not yet been tested for energy beyond 10⁻⁴ eV, the bounds on the size of extra-dimensions are now much lower (∼1 mm)² and this lack of experimental data allows for a modification of gravitational interactions at submillimetric distances. However this analysis was not yet complete essentially because it assumes a particular factorizable geometry associated to the
higher-dimensional space-time being a direct product of a 4D space-time with
compact space. Recently this last assumption has been overcome 3 unwarping
a very rich potential of physical effects. The most exciting one reveals
the non-incompatibility between non-compact extra-dimensions and experi-
mental gravity. The toy model constructed by Randall and Sundrum (RS)
considers a 3-brane embedded in a 5D bulk at energy low enough to model
the dynamics of all the fields but the metric itself by a negative cosmological
constant in the bulk, \( \Lambda_{bh} \) and a positive cosmological constant or tension on
the brane, \( \Lambda_{br} \). If these two quantities are adjusted such that:

\[
\Lambda_{bh} = -\kappa^2 \Lambda_{bh}^2 / 6 \tag{1}
\]

there exists a solution to Einstein equations with a \( \mathbb{Z}_2 \) symmetry in the trans-
verse coordinate, \( y \), and a 4D Poincaré invariance on the brane:

\[
ds^2 = e^{-\frac{\Lambda_{bh}}{\kappa^2} \ln |y|} \, dx^2 + dy^2 \tag{2}
\]

The fluctuation modes around this background contains a normalizable bound
state which can be interpreted as the usual 4D graviton. Of course, there still
exists an infinite tower of KK modes, even a continuum spectrum without gap,
but the shape of their wave functions is such that they almost do not overlap
with the 4D graviton and thus maintain the deviations to the Newton’s law
in limits which are still very far from experimental bounds.

When some matter characterized by its energy density, \( \rho_{br} \), is introduced
on the brane, the previous configuration is no longer static and the brane will
expand. It has been shown 4 that the bulk contribution exactly cancels the
non-standard extrinsic expansion of the embedded brane 5 and a usual FRW
cosmology is recovered up to small corrections.

2 Effective cosmological constants

How interesting the RS model can be, at the present stage it seems lacking in
generality and suffers from apparently \textit{ad hoc} fine-tuning between the cosmolo-
gical constants \( \Lambda_{bh} \) and \( \Lambda_{br} \). It has been proposed 6 that these cosmological
constants could describe the dynamics of a scalar field. We have addressed
this question at a more fundamental point of view taking into account the
low-energy spectrum of string excitations which are described by 10D super-
gravity. The bosonic spectrum contains the metric, a scalar field (the dilaton)
and numerous differential forms and the effective action takes the general
form:

\[
S_{\text{eff}} = \int d^D x \sqrt{|g|} \left( \frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2} \partial_a \Phi \partial^a \Phi - \sum_n e^{\alpha_n} F_{n+2}^2 \right), \tag{3}
\]
The coefficients $a_n$ are explicitly determined by a string computation. The *bulk* effective action can couple to some *branes* and the total action is:

$$S = S_{\text{eff}}^{\text{brane}} + S_{\text{eff}}^{\text{brane}}$$

which the equations of motion for the metric, the dilaton and the forms are derived from. There exists a solution to these equations of motion with particular symmetry namely a Poincaré invariance in $d_b = p + 1$ dimensions identified as the longitudinal dimensions of a $p$-brane and also a rotational invariance in the $d_+ = D - d_b$ dimensional transverse space. It is expressed as $(\mu, \nu = 0 \ldots d_b - 1$ and $i, j = 1 \ldots d_+)$:

$$ds^2 = H^{2n_x} \eta_{\mu \nu} dx^\mu \otimes dx^\nu + H^{2n_y} \eta_{ij} dy^i \otimes dy^j ;$$

where $H$ is a only function of the radial distance, $r = \sqrt{g^{ij} \eta_{ij}}$, in the transverse space. The consistency of the whole set of equations of motion determines the powers $n_x$ and $n_y$:

$$n_x = \frac{-2(d_b - 2) \kappa^2}{(D - 2) A_{WZ}}$$

$$n_y = \frac{2d_b \kappa^2}{(D - 2) A_{WZ}}$$

and the coefficient $A_{WZ}$ which has to be related to the dilaton coupling by:

$$A_{WZ}^2 = \frac{2\kappa^2}{\ell^2} \frac{d_b(d_b - 2)}{d_b + d_+ - 2} + \frac{\alpha^2}{2} .$$

The function $H$ is harmonic in the transverse space:

$$H = l + \frac{Q}{r^{d_+ - 2}}$$

where $l$ is an arbitrary dimensionless constant and $Q$ is a constant with a dimension $d_+ - 2$ in length that can be expressed in terms of $\kappa_D^2$.

In some cases, the geometry associated to these solutions can be described *à la* RS in terms of cosmological constants in the bulk and on the brane. In these cases, the Einstein tensor should be derived from two constants $\Lambda_{bk}$ and $\Lambda_{br}$:

$$G^{\mu \nu} = -\kappa^2 \left( \Lambda_{bk} + \lambda_{br} \frac{\delta_{d_+}(y)}{\sqrt{g_1}} \right) g^{\mu \nu}$$

$$G^{ij} = -\kappa^2 \Lambda_{bk} \left( 2g^{ij} \frac{y^i y^j}{R^2} - g^{ij} \right)$$

The Einstein tensor takes this form if and only if the dilaton coupling vanishes\(^a\) and thus $\Lambda_{bk}$ and $\Lambda_{br}$ are given by:

$$\Lambda_{bk} \propto -N^2/(2-d_+) \kappa_D^{-2D/(D-2)}$$

$$\Lambda_{br} \propto N \kappa_D^{-2d_b/(D-2)}$$

\(^a\)Note that the vanishing value of the dilaton coupling is compatible with the supersymmetry for special case only such as the D3-brane of IIB supergravity for example.
where \( N \) is the number of coincident branes stacked at the origin.

In the case of co-dimension one brane, the numerical factors in (10) lead to the RS fine-tuning. However, according to (6), this case requires an imaginary Wess–Zumino coupling, \( A_{WZ} \), which would destroy unitarity. This result is also suggested by the difficulty to embed the RS scenario in a supersymmetric theory.

The generalization to higher co-dimension overcomes this difficulty but the Planck scale on the brane diverges meaning that gravity decouples on the brane. However, these solutions can be used to construct new configurations in a method quite similar to RS by cutting the region near infinity and gluing a copy to the region near the origin: gravity is then localized on a jump brane surrounding the SM brane.

**Acknowledgment** This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by the NSF under grant PHY-95-14797.

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