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NONLINEAR TWO-DIMENSIONAL POTENTIAL PLASMA WAKE WAVES

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Abstract

The conditions for potential description of the wake waves, generated by flat electron driving bunch in cold plasma, are derived. The nonlinear equation for potential, valid for small values of that, is obtained and exact solutions are found for two-dimensional nonlinear plasma wake-waves. In particular, at some boundary conditions, corresponding to blow-out regime, the solution in form of solitary wave is found. In all cases considered, the values of the electric field in wake waves is by order of magnitude equal to \( \frac{m_0 p v_0}{e} \sim \frac{\pi M_v}{\lambda_p \text{ cm}} \).

1 INTRODUCTION

Analysis of the one dimensional longitudinal, transverse and coupled transverse longitudinal plain nonlinear waves in cold relativistic plasma are given in the review [1] (see therein the references on original works). One dimensional nonlinear longitudinal waves, generated by the driving bunches with the infinite transverse dimensions, were considered in [2]-[8].

In the present work the two-dimensional nonlinear wake waves, generated by the flat electron bunch, are discussed.

The corresponding linear problem was considered in [9],[10] and was found, in correspondence with previous result [11], that the magnetic field in wake wave in linear approximation is equal zero. This result connected with the absence of the energy flow in the wake wave and the absence the vortexes in plasma electron motion in linear approximation.
In the one dimensional nonlinear treatment [2]-[8] the magnetic field in the wake wave is also zero (by construction), due to the symmetry of the problem relative to transverse displacements.

In two-dimensional wake wave the magnetic field is zero only when vortexes connected with the plasma electron motion are zero, which in this case is an additional requiremment on the type of the motion of the plasma electrons. The wake waves in this case are potential, i.e. electric field components \( E_z, E_y \) can be expressed as a gradient of one scalar function \( \varphi(y, z) \), and components of the plasma electrons current also can be expressed through one scalar function \( \psi(y, z) \).

The approximate nonlinear (up to terms \( \sim \varphi^3 \)) equation for potential \( \varphi \) can be obtained, using Maxwell equations and approximate equations of the motion. Equation for potential has an exact solutions with the separated variables.

Among the solutions, which are finite, there are nonlinear waves by cnoidal nature and, at some boundary conditions, which associated with the blow-out regime, there exists the solution (on separatirix), in form of the solitary wave.

2 VORTEX-FREE WAKE WAVE

Consider the wake wave generated in the cold neutral plasma, with the immobile ions, by the flat electron bunch, which has horizontal dimensions \( 2a \) much larger than vertical dimension \( 2b \), longitudinal dimension is \( 2d \). The charge density in the bunch is \( n_b \), electron plasma density is \( n_0 \), and we consider both overdense and underdense regimes.

Bunch is moving along \( z \)-axis with the constant velocity \( v_0 < c \). All the physical quantities in the question are considered as a function of vertical coordinate \( y \) and \( \tilde{z} = z - v_0 t \). An electrical field, generated by the bunch \( |E_x| \ll E_y \neq 0, E_z \neq 0 \) and magnetic field \( B_z = 0, |B_y| \ll |B_x| = |B| \neq 0 \).

Introduce the dimensionless variables and arguments by

\[
\begin{align*}
\bar{E} &= \sqrt{4\pi n m v_0^2} E' = \frac{\omega m v_0}{e} \bar{E}' \\
\bar{B} &= \sqrt{4\pi n m v_0^2} B' = \frac{\omega m v_0}{e} \bar{B}'
\end{align*}
\]
\[ z', y' = k\dot{z}, ky, k^2 = \frac{\omega^2}{v_0^2} = \frac{4\pi ne^2}{mv_0^2} \]  \hspace{1cm} (2)

where \( n \) is the arbitrary electron density, which is convenient to choose equal \( n = n_b \) in the underdense \((n_b > n_0)\) case and \( n = n_0 \) in the overdense \((n_b < n_0)\) case.

Following [12],[9] introduce BFTCh-transformation of the variables

\[ V_z = \frac{\beta_{ez}}{\beta - \beta_{ez}}, V_y = \frac{\beta_{ey}}{\beta - \beta_{ez}} \beta = \frac{v_0}{c}, \beta_{ez} = \frac{v_{ez}}{c} \]  \hspace{1cm} (3)

\[ \frac{n_e}{n} = \frac{\beta N}{\beta - \beta_{ez}} = N(1 + V_z); \]  \hspace{1cm} (4)

\( \quad (N \rightarrow \frac{n_0}{n}, \beta_{ez} \rightarrow 0, \text{when} \ z, y \rightarrow +\infty). \)

the Maxwell equations can be rewritten in the following form:

\[ \begin{align*}
& (a) \quad \frac{\partial B}{\partial y} = \beta NV_z + \beta \frac{\partial E_z}{\partial z} + \beta \frac{n_b}{n} \\
& (b) \quad \frac{\partial (B + \beta E_y)}{\partial z} = -\beta NV_y \\
& (c) \quad \frac{\partial (\beta B + E_y)}{\partial z} = \frac{\partial E_z}{\partial y} \\
& (d) \quad \frac{\partial E_z}{\partial z} + \frac{\partial E_y}{\partial y} = \frac{n_0 - n_b}{n} - N(1 + V_z)
\end{align*} \]  \hspace{1cm} (5)

The continuity equation \( \frac{\partial n}{\partial z} = \frac{\partial Nv_e}{\partial y} \) follows from (5.d),(5.a),(5.b). Using (5.a), (5.b),(5.c) we have

\[ \frac{\partial^2 B}{\partial y^2} + (1 - \beta^2) \frac{\partial^2 B}{\partial z^2} = \text{rot}_z(\beta N\vec{V}) + \text{rot}_z(\beta \frac{n_b}{n}) \]  \hspace{1cm} (6)

which means that the magnetic field is zero in plasma (linear or nonlinear) wake wave only when

\[ \text{rot}(\beta N\vec{V}) = 0 \]  \hspace{1cm} (7)

i.e. the plasma electrons motion is vortex-free.
In the following, we consider the region of the space, occupied by wake wave i.e. \( z < -d \) (we drop out subscripts prime on \( z \) and \( y \)). Maxwell equations (5) for wake waves under condition (7) can be obtained putting in (5) \( B = 0 \) and \( n_b = 0 \).

\[
\begin{align*}
(a) & \quad \frac{\partial E_z}{\partial z} = NV_z \\
(b) & \quad \frac{\partial E_y}{\partial z} = -NV_y \\
(c) & \quad \frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y} \\
(d) & \quad \frac{\partial E_y}{\partial y} = \frac{n_0}{n} - N
\end{align*}
\]

Then from (8.c) follows that
\[
\vec{E} = -\text{grad} \varphi
\]

i.e. the wake fields under condition (7), as it must be, are potential.

3 THE BASIC EQUATION FOR THE POTENTIAL. EXACT GENERAL SOLUTION

Consider Maxwell equations (5) for wake waves, when \( z < -d, B = 0, n_b = 0 \). From (5.a), (5.d) and (9) in this case we have

\[
N = \frac{n_0}{n} + \frac{\partial^2 \varphi}{\partial y^2}
\]

\[
\frac{\partial^2 \varphi}{\partial z^2} = \left( \frac{n_0}{n} + \frac{\partial^2 \varphi}{\partial y^2} \right) V_z
\]

From hydrodynamic equation of the plasma wake wave electrons motion, using (1), (2), (3) it is possible to obtain the relativistic equation of motion for the \( V_z \) component of the generalized velocity:

\[
-\frac{\partial V_z}{\partial z} + V_y \frac{\partial V_z}{\partial y} = -W^{1/2} \left[ E_z \left( 1 + 2V_z + \frac{V_z^2}{\gamma^2} \right) + \beta^2 V_z V_y E_y \right],
\]

\[
W \equiv 1 + 2V_z + \frac{V_z^2}{\gamma^2} - \beta V_y^2;
\]
Neglecting terms with the squares of generalizid velocity, compared to the terms with the first power of that, the expression (12) converted to

$$\frac{\partial V_z}{\partial z} \approx E_z(1 + 3V_z) \quad (13)$$

The solution of this equation, using (9), is

$$V_z = \frac{1}{3}(e^{-3\varphi} - 1) \approx -\varphi + \frac{3}{2}\varphi^2 + \ldots \quad (14)$$

with the condition \(\varphi = 0\), when \(V_z = 0\).

Substituting (14) in (11) and leaving only terms up to third power on \(\varphi\) we have the basic equation for \(\varphi\)

$$\frac{\partial^2 \varphi}{\partial z^2} + \varphi \frac{\partial^2 \varphi}{\partial y^2} + \frac{n_0}{n} \varphi - \frac{3n_0}{2n} \varphi^2 = 0 \quad (15)$$

The eq. (15) permits to search the solution with the separable arguments

$$\varphi(y, z) = \varphi_1(y)\varphi_2(z) \quad (16)$$

$$\frac{\varphi''_2 + \frac{n_0}{n}\varphi_2}{\varphi^2_2} = -(\varphi''_1 - \frac{3n_0}{2n}\varphi_1) \equiv -k \quad (17)$$

where \(k\) is a separation constant. The equations for \(\varphi_1\) and \(\varphi_2\) are:

$$\varphi''_1 - \frac{3n_0}{2n}\varphi_1 = k \quad (18)$$

$$\varphi''_2 + \frac{n_0}{n}\varphi_2 + k\varphi^2_2 = 0 \quad (19)$$

Due to the symmetry of the problem the solution of equation (18) must be symmetric on \(y; \varphi_1'(y = 0) = 0\) (due to \(E_y = 0\) at \(y = 0\)). The solution of the linear problem [9],[10] is concentrated in the region of the "trace", falling outside it exponentially. Adopting the same picture of the potential flow for considering case too, and demanding the continuity of \(\varphi_1\) and \(\varphi_1'\) at \(y = \pm b, \varphi_1 \to 0, \text{when } y \to \pm \infty\), we have the following solution of eq. (18)
\[ \varphi_1(y) = -A \left( \cosh \sqrt{\frac{3n_0}{2n}} b + \sinh \sqrt{\frac{3n_0}{2n}} b \right) + A \cosh \sqrt{\frac{3n_0}{2n}} y, \]  
\[ -b \leq y \leq b, \]  
\[ \varphi_1(y) = -A \sinh \sqrt{\frac{3n_0}{2n}} b - \sqrt{\frac{3n_0}{2n}} |y - b|, \]  
\[ |y| \geq b \]  
and \( A \) is an arbitrary constant. The equation (19) is the equation for nonlinear oscillator, with nonlinear part of the force proportional to \( \varphi^2 \) (for mathematical pendulum the first nonlinear term is proportional to \( \varphi^3 \) see e.g. [13]). The general solution of this equation is given in the implicit form by

\[ -(z + d) = \int_{\varphi_2}^{\varphi_0} \frac{d\varphi}{\sqrt{2[h - F(\varphi_2)]^{1/2}}}, \]  
where \( h \) is an energy constant, defined by

\[ h = \frac{1}{2} \varphi_2^2 + \frac{n_0}{2n} \varphi_2^2 + \frac{k}{3} \varphi_2^3 \]  
and determined from boundary condition at \( z = -d, \varphi_2(-d) \equiv \varphi_0, \varphi_2'(-d) \equiv \varphi_0' \). The function \( F(\varphi_2) \) is

\[ F(\varphi_2) = \frac{n_0}{2n} \varphi_2^2 + \frac{k}{3} \varphi_2^3 \]  
For \( k < 0 \) \( h_s \) is the separatrix, \( h_s = \frac{1}{6k^2} \left( \frac{n_0}{n} \right)^3 \), which is tangent to \( F(\varphi_2) \) at its maximum point; \( F(\varphi_2) \) has three real roots: double root equal zero and one root at \( B = \frac{3n_0}{2n|k|} \). The roots of the equation

\[ h - F(\varphi_2) = 0, \]  
are \( \varphi_i (i = 1, 2, 3) \)

The different solutions (22) of the equation (19) defined by the value of the \( h \), which in turn, depends on the boundary values \( \varphi_0 \) and \( \varphi_0' \). Physical solutions \( c_1 \leq \varphi_2 \leq c_2 \).
exist, when \( h \leq h_s \), i.e. \( c_s \leq c_1 \leq 0 \), where 
\[
c_s = -\frac{n_0}{2|k|n}, \quad \text{and} \quad 0 \leq c_2 \leq c_m = \frac{n_0}{|k|n},
\]
(\( c_m \) corresponds to the local maximum of the function \( F(\varphi_2) \) which is equal to \( h_s \)). The third root of the eq (25) is \( c_3: c_m \leq c_3 \leq B = \frac{3n_0}{2|k|}\)

4 BOUNDARY CONDITIONS

Obtained solution depends on three constants \( \varphi'_0, \varphi'_0 \) and \( A \) (or \( k \), see (21)).

For the definition of these constants it is necessary to have three physical quantities given at \( z = -d \) and \( y = 0 \). They can be \( N_0 = \frac{n_d}{n_0}, V_{z0} \) and \( E_{z0} \) \((E_{y0} = 0, V_{y0} = 0 \) automatically due to the symmetry of the \( \varphi_1(y) \)).

From (8.d) and (9) at \( z = -d, y = 0 \) follows 
\[
\frac{n_d}{n} = \frac{n_0}{n} + \frac{3n_0}{2n} A \varphi_0
\]  
(26)

From (9) follows:
\[
E_{z0} = -A \varphi'_0 \left[ 1 - \cosh \left( \sqrt{\frac{3n_0}{2n}} b \right) - \sinh \left( \sqrt{\frac{3n_0}{2n}} b \right) \right]
\]  
(27)

And from (14),(20) follows
\[
A \varphi_0 \left[ 1 - \cosh \left( \sqrt{\frac{3n_0}{2n}} b \right) - \sinh \left( \sqrt{\frac{3n_0}{2n}} b \right) \right] = -V_{z0}
\]  
(28)

or, using (21) and (26)
\[
k \varphi_0 = \frac{n_d - n_0}{n} + \frac{3n_0}{2n} V_{z0}
\]  
(29)

Notice, that it is possible to express physical quantities only through combinations
\( A \varphi_0 \) or \( k \varphi_0, A \varphi'_0 \)

The constant \( h \) (23), also can be expressed through the roots of the equation (25):
\[
h = \frac{k}{3} c_1 c_2 c_3 = -\frac{\alpha_1 \alpha_2 \alpha_3}{3k^2} \left( \frac{n_0}{n} \right)^3
\]  
(30)

\(-1/2 \leq \alpha_1 \leq 0, 0 \leq \alpha_2 \leq 1, 1 \leq \alpha_3 \leq 3/2 \)
For the separatrix $c_1 = c_s, \alpha_1 = -1/2, c_2 = c_3 = c_m, \alpha_2 = \alpha_3 = 1$ and

$$h = h_s = \frac{1}{6k^2}\left(\frac{n_0}{n}\right)^3$$

(31)

Consider the case when $n = n_b, n_0/n_b < 1, n_d = 0$ and $V_{zo}$ and $E_{z0}$ are small.

This conditions corresponds to the blow-out regime in underdense plasma, considered in [14] by computational analysis. From (27) and (28) in that case follows

$$\varphi' = \frac{V_{zo}}{E_{z0}} \varphi_0$$

(32)

From (23), (29), (30) when $k < 0$ for the definiteness, follows

$$h \approx h_s - 1/2 \frac{E_{z0}^2 n_0}{V_{z0}^2 k^2 n_b} (1 - 3V_{z0})$$

(33)

and when $E_{z0} \to 0, h \to h_s,$

$$\frac{n_0}{2|k|n_b} < \varphi_0 = \frac{n_0}{|k|n_b} - \frac{3n_0}{2n|k|V_{z0}} < \frac{n_0}{|k|n_b}$$

(34)

i.e. the $\varphi_0$ lies on separatrix for $0 < V_{z0} < 1$

For the values $h > h_s$ and $h < 0$ as it is evident, the solutions for $\varphi_2$ have an infinite values. When $\varphi_0 > c_2$ the solution became unphysical, even for $0 \leq h < h_s$

## 5 FINITE NONLINEAR SOLUTIONS

Consider first the case when $c_1 = c_s, c_2 < c_2$. From general solution (22), using known expressions for the elliptic integrals and elliptic functions [15, 16], we have

$$\varphi_2(z) = c_1 + (c_2 - c_1)sn^2 Z_1,$$

(35)

where

$$Z_1 \equiv F(\gamma_0, q) + \frac{1}{2} \sqrt{\frac{2|k|(c_3 - c_1)}{3}} (z + d)$$

(36)

$$\gamma_0 = \arcsin \sqrt{\frac{\varphi_0 - c_1}{c_2 - c_3}},$$

(37)

$$q = \sqrt{\frac{c_2 - c_1}{c_3 - c_1}} = \sqrt{\frac{c_2 - c_1}{c_3 - c_1}}$$
and \( F(\gamma_0, q) \) is the elliptic integral of the first kind, and \( snz \)-elliptic function. Using (9),(20)-(21),(26) and (35)-(36) it is possible to obtain

\[
E_y = \left( \frac{2n_0}{3n} \right)^{1/2} sh\left( \frac{3n_0}{2n} y \right) \left[ \alpha_1 + (\alpha_2 - \alpha_1) sn^2 z_1 \right] \times \left[ csh\left( \frac{3n_0}{2n} b \right) + sh\left( \frac{3n_0}{2n} b \right) \right]^{-1}
\]

\[
E_z = \left( \frac{2}{3} \right)^{3/2} \left( \frac{n_0}{n} \right)^{1/2} \left( 1 - csh\left( \frac{3n_0}{2n} y \right) \right) \left[ csh\left( \frac{3n_0}{2n} b \right) + sh\left( \frac{3n_0}{2n} b \right) \right]^{-1} \times (\alpha_2 - \alpha_1)(\alpha_3 - \alpha_2)^{1/2} snz_1 cnz_1 dnx_1
\]

for \(-b \leq y \leq b, z < -d\). In (39) it is possible to use the formula

\[
snz_1 cnz_1 dnx_1 = \frac{1}{2} sn2z_1 (1 - q^2 sn^4 z_1),
\]

where

\[
q^2 = \frac{c_2 - c_1}{c_3 - c_1} = \frac{\alpha_2 + |\alpha_1|}{\alpha_3 + |\alpha_1|}.
\]

The elliptic integral \( F(\gamma_0, q) \) entering in (36) depends on \( q \) (41) and \( \gamma_0 \). Using (29) we have

\[
\gamma_0 = \arcsin \left( \frac{k n_0 \varphi_0 - \alpha_1}{\alpha_2 - \alpha_1} \right) = \arcsin \left( \frac{1 + |\alpha_1| - \frac{nd}{n_0} - \frac{3}{2} V_0}{\alpha_2 + |\alpha_1|} \right)
\]

From (38)-(39),(41)-(42) it follows that physical quantities \( E_y, E_z \) does not depend on separation constant "k" and hence the "k" can be choosen arbitrary, for example, \(|k| = 1\). Then all constants \( A, \varphi_0, \varphi'_0, h \) expressed through boundary values of the physical quantities using (21),(23),(27),(28). The roots of the eq (25) and hence \( \alpha_1, \alpha_2, \alpha_3 \) entering in (38),(39),(41),(42) also can be expressed through physical boundary values.

The period \( \lambda_n \) of the nonlinear waves is given by

\[
\omega \lambda_n = 2 \int_{c_1}^{c_2} \frac{d\varphi_2}{\sqrt{2[h - F(\varphi_2)]^{1/2}}};
\]

\[
(43)
\]
\[ \lambda_n = \frac{4v_0}{\omega} \left( \frac{3n}{2n_0} \right)^{1/2} \frac{1}{(\alpha_3 + |\alpha_1|)^{1/2}} \times \]
\[ \times F \left( \frac{\pi}{2}, \frac{(\alpha_2 + |\alpha_1|)^{1/2}}{(\alpha_3 + |\alpha_1|)} \right) \]

In the linear case
\((n = n_0, n_b \ll n_0), h \to 0, \alpha_1 \to 0, \alpha_2 \to 0, \alpha_3 \to 3/2\) and \(\lambda_n \to \lambda_p = \frac{2\pi v_0}{\omega_p} \).

Taking into account the numerical values of the elliptic function, entering in (38), (39), it is easy to notice that magnitudes of the electrical field components are of the order of \(w_p'; v_0, \omega_p = 4\pi v_0 n_0\), and in the considered case the nonlinearity essentially changes only the waves length (43) and shape of the fields (38), (39).

In the case \(h = h_s\) the changes due to nonlinearity are more drastic.

As we see, this case corresponds to the conditions
\(n_b \gg n_0, n_e(z = -d, y = 0) \equiv n_d = 0\) and \(V_{z0} = 0\) and \(E_{z0}\) are small \((E_{z0} = 0, \text{finally})\) which resembles the blow-out regime in underdense plasma [14].

In this case
\(c_2 \to c_3 \to c_m = \frac{n_0}{|k|n}, c_1 \to c_s = -\frac{n_0}{|k|n}\) and
\(h - F(\varphi_2) \to \frac{|k|}{3}(\varphi_2 - \frac{n_0}{|k|n})^2(\varphi_2 + \frac{n_0}{|k|n}).\) From (22) using [15] it follows:

\[ \varphi_2 = -\frac{n_0}{2|k|n} + \frac{3n_0}{2|k|n} \tan^2 \left[ \frac{1}{2} \theta_0 - \frac{1}{2} \sqrt{\frac{n_0}{n}} (z + d) \right], \quad (44) \]

where

\[ \theta_0 = \ln \left( \frac{(\varphi_0 + \frac{n_0}{2|k|n})^{1/2} + (\frac{3n_0}{2|k|n})^{1/2}}{(\varphi_0 + \frac{n_0}{2|k|n})^{1/2} - (\frac{3n_0}{2|k|n})^{1/2}} \right) \]

\[ \ln \left( \frac{(1 - V_{z0})^{1/2} + 1}{(1 - V_{z0})^{1/2} - 1} \right) = 2 \arctan (1 - V_{z0})^{1/2}, V_{z0} > 0 \]

From (9), using (44)-(45), (21), (34), it follows

\[ E_y = -\frac{\sqrt{\frac{3n_0}{2n} s} h \sqrt{\frac{3n_0}{2n} y}}{(c s h \sqrt{\frac{3n_0}{2n} b} + s h \sqrt{\frac{3n_0}{2n} b})} \times \quad (46) \]
\[
E_z = -\sqrt{\frac{n_0}{n}} \left[ 1 - \frac{\cosh \sqrt{\frac{3n_0 y}{2n}}}{(\cosh \sqrt{\frac{3n_0 b}{2n}} + \sinh \sqrt{\frac{3n_0 b}{2n}})} \right] \times \frac{\left[ (1 - V_{z0})^{1/2} - \tanh \frac{1}{2} \sqrt{\frac{n_0}{n}} (z + d) \right] \left[ 1 - (1 - V_{z0})^{1/2} \right]}{\cosh^2 \left( \frac{1}{2} \sqrt{\frac{n_0}{n}} (z + d) \right) \left[ 1 - (1 - V_{z0})^{1/2} \tanh \frac{1}{2} \sqrt{\frac{n_0}{n}} (z + d) \right]^3}
\]

for \(|y| \leq b, -\infty < z + d \leq 0\). From (46) and (47) it is evident that \(E_y, E_z\) are also does not depend on separation constant \(k\), and defined by boundary value \(V_{z0}\) and parameter \(2b\) (width of the bunch). Expression (46) and (47) describes the solitary wave (part of it, due to a range of the values of the argument \(z + d\)). For large values of the \(z(k_p|z| \gg 1, k_p = \frac{w_k}{v_0})\) the expressions (46) and (47) for \(E_y, E_z\) are exponentially small; when \(z + d \simeq 0\) (near to the rear end of the driving bunch) \(E_y, E_z\) are by the order of the magnitude \(\sim \frac{nw_p v_0}{e}\).

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