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**Publication Date**
1975-03-01
TABLES OF NUCLEAR STRUCTURE AMPLITUDES FOR TWO-NUCLEON TRANSFER INVOLVING LIGHT NUCLIDES (a ≤ 4)

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March 1975

Prepared for the U.S. Energy Research and Development Administration under Contract W-7405-ENG-48

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Abstract

Tables are presented of values of the nuclear structure amplitudes required for construction of form factors for the reactions $\alpha, d$, $^3\text{He}, p$, $^3\text{He}, n$, $(t, n)$ and $(t, p)$, and their inverses.
Introduction

Structure amplitudes for two-nucleon transfer reactions, denoted by

$$ A(a,b)B, b = a-2, B = A+2 $$

are tabulated for light nuclei $$ a \leq 4 $$.

Somewhat abbreviated tables were made available in unpublished reports dated August 1968. The depletion of the supply of these reports and the continued interest in two-nucleon transfer reactions is the occasion for the publication of the original tables, with the addition of some higher shell model configurations.

The structure tables facilitate the construction of the form factor for these reactions, which for a pure two-nucleon configuration

$$ (n_1 l_1 j_1) (n_2 l_2 j_2) $$

is given by

$$ f(j_1 j_2)_{LSJ}(R) = \sum_{N} G_{N} \ u_{NL}(2\nu R^2) $$

(2)

The structure amplitudes, $$ G_{N} $$, are tabulated for the quantum numbers appearing as subscripts on $$ f(R) $$. The function $$ u_{NL} $$ is an oscillator function, defined later, and the size-parameter $$ \nu $$ of the target nucleus is listed in the tables. For more complicated states than pure configurations, the form factor is merely an appropriate linear combination of the above.
In the distorted wave Born approximation (DWBA), the amplitude, when it is evaluated with a zero-range interaction, involves the following integrals over the form factors $f_{LSJ}(R)$:

$$M_{B_{LSJ}} = \frac{i^{-L}}{\sqrt{2L+1}} \left[ \psi_b^{(-)} \left( \frac{A}{A+2} \right) f_{LSJ}(R) Y_L^M(R) \right]^{*} \psi_a^{(+)}(R) dR \quad (3)$$

(which corresponds to Satchler's $\beta_{Sj}^{m}$ of Ref. 2). Most DWBA computer programs allow for the introduction by the user of any chosen form factor.

The cross sections for stripping and pickup reactions are given by

$$\frac{d\sigma}{d\Omega} = \begin{cases} \frac{k_b}{k_a} \left( \frac{2J_B + 1}{2J_A + 1} \right) \left( \frac{d\sigma}{d\Omega} \right)_0 & \text{(stripping)} \\ \frac{k_a}{k_b} \left( \frac{2s_a + 1}{2s_b + 1} \right) \left( \frac{d\sigma}{d\Omega} \right)_0 & \text{(pickup)} \end{cases} \quad (4)$$
with

\[
\left( \frac{d\sigma}{d\Omega} \right)_0 = (2\pi \hbar^2)^{-2} \frac{Aa}{A+a} \frac{(A+2)(a-2)}{A+a} D_0^2 \Omega_{a-2}^2
\]

\[
\times \sum_{LSJT} \sum_{M_L} C_{ST}^2 \left| M_L \right| B_{LSJ}^2
\]

\( (5) \)

where

\[
C_{ST} = b_{ST} C_{A}^{T_A} C_{T}^{T_B} \]

\( (6) \)

and \( Z_A \) and \( Z_B \) obviously denote the isospin \( Z \)-components of nuclei \( A \) and \( B \), and \( b_{ST} \) takes on the following values depending upon the reaction

\[
b_{ST} = \begin{cases} 
2a_0 & \delta_{S0} \delta_{T1} & (t,p) \\
-2a_0 & \delta_{S0} \delta_{T1} & (^3\text{He},n) \\
-\frac{2}{\sqrt{2}} & (a_0 \cdot \delta_{S0} \delta_{T1} + a_1 \delta_{S1} \delta_{T0}) & (t,n) \\
\frac{2}{\sqrt{2}} & (a_0 \cdot \delta_{S0} \delta_{T1} - a_1 \delta_{S1} \delta_{T0}) & (^3\text{He},p) \\
4a_1 & \delta_{S1} \delta_{T0} & (\alpha,d)
\end{cases}
\]

\( (7) \)
The first two change sign for the inverse reactions. Generally the overall signs are unimportant since they are squared in the cross section. The exceptions are:

1) If the optical potential used to describe the distortion in the relative motion wave functions $\psi_b^{(-)}$ and $\psi_a^{(+)}$ have a spin-orbit term, then the sums on $L$ and $S$ are not incoherent as we have written them. The correct generalization is given by Satchler in Eq. (13) and (17) of Ref. 2.

2) If various reaction channels are coupled then the relative signs of these amplitudes must be retained.

We must still define the potential strengths $a_0$ and $a_1$ for singlet and triplet transfer. They are given by

$$a_0 = \frac{3}{4} A_{01} + \frac{1}{4} A_{10} \quad (8)$$

$$- a_1 = \frac{3}{4} A_{01} + \frac{1}{4} A_{10} \quad (9)$$

This follows from the assumption that the potential responsible for the transfer is

$$V = \sum_{\alpha\beta} V(r_{\alpha\beta}) \quad (10)$$

where $\alpha$ is a nucleon that is transferred from $a$ and $\beta$ is one that remains in $b = a-2$, and that this nucleon-nucleon potential may depend on the spin and iso-spin state of the interacting nucleons in the way

$$V(r_{\alpha\beta}) = v(r_{\alpha\beta}) \sum_{ST} A_{ST} p_{ST}^{\alpha\beta} \quad (11)$$

where $p_{ST}^{\alpha\beta}$ projects onto the spin-isospin state $S,T$ of nucleons $\alpha$ and $\beta$. 
The zero-range assumption is made with respect to the approximation

\[ \sum_{\alpha \beta} v(r_{\alpha \beta}) \approx g \delta(\rho) \quad (12) \]

where \( \rho \) is the coordinate connecting the center of mass of the two nucleons \( \alpha = 1, 2 \) that are transferred, to the center of mass of the nucleus \( b \) (a nucleon if \( a = 3 \) and deuteron for \( a = ^4\text{He} \)). For a potential that is independent of the state of the interacting particles, \( A_{ST} = 1 \).

Theory

a) Selection Rules

The selection rules on the spin and iso-spin that is carried by the transferred pair of nucleons in such reactions as (1) depend upon the particular symmetries of the nuclei involved.

The symmetry of the wave functions that we assume throughout for the light nuclei is that of their dominant component. For example the deuteron is known to have both an \(^3\text{S}\) and \(^3\text{D}\)-component in the relative motion of neutron and proton. However the D-component is small and we assume that the transfer reaction can be described exclusively in terms of the S-component. Naturally this leads to some error and the effects of this kind of neglect are under investigation. But it is an effect that clearly does not fall into the realm of tables such as these, which are intended for straightforward general use.

To demonstrate how the selection rules follow from the symmetry of the nuclei is very simple. Consider for example the \((p, ^3\text{He})\) reaction. We assume for all light nuclei up to \( a = 4 \) that the relative motion
between any pair of nucleons has zero orbital angular momentum ($\ell = 0$),
which is referred to as S-state. Since this assumption for the space
coordinates tells us that the space part of the wave function is
symmetric, then according to the Pauli-principle the spin part must be
antisymmetric if the pair are like nucleons to make the wave function
antisymmetric. The two spin states available to nucleons are $S = 0$ and 1,
which are respectively antisymmetric and symmetric, so that in light
nuclei the space-spin state of like pairs is singlet-even, which is
denoted by $^1S$. The isospin must be symmetric ($T = 1$) in order to preserve
the overall antisymmetry.

This state of space-spin-isospin can be denoted by $^{13}S$ (standing for
$S = 0$) (singlet), $T = 1$ (triplet) and S-state in the space coordinate).

The spin-state of a neutron-proton pair in He$^3$ can be easily in-
ferred now. Since the two protons have spin $S = 0$, their spin directions
with respect to the neutron is random, and therefore the $S = 0$ and 1 states
are equally probable. (The fact that the triplet-even force is stronger
the the singlet-even introduces a small correction to this statement.)
Consequently, both $S = 0$ and 1 are transferred in the p,He$^3$ reaction.

The selection rules for the ($\alpha$,d) reaction contain an additional
element. Following reasoning as above, the n-n pair and the p-p pair are
each in $^{13}S$ states. The spins of each pair being zero, the individual
spin orientation of a neutron with a proton is as likely to be singlet
as triplet. However the outgoing deuteron has spin $S = 1$. Therefore
only the $S = 1$ part in the n-p spin state of the $\alpha$-particle contributes
to the reaction.
The selection rules are summarized by the table

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Spin(S)</th>
<th>Isospin(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(α,d)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(t,p), (3He,n)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(t,n), (3He,p)</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

b) Mathematical Development

The purpose of this section is to present sufficient of the theory as to define the meaning of the quantities that are tabulated, the parameters used in their calculation, and the assumptions that are made to reduce the DWBA expression for the amplitude to a tractable form. The basic assumptions that underlie the DWBA have been stated and discussed explicitly elsewhere.3,4

The starting point for this discussion is

$$T = \int \psi_b^* (R_b) \left\langle \phi_{b^{-2}}(a^{-2})\phi_{A+2} \mid V \mid \phi_a(a) \phi_{A}(A) \right\rangle \psi_a^* (R_a) \quad \text{d}r_1 \text{d}r_2 \text{d}r_b \text{d}A. \quad (13)$$

Here the angle bracket denotes integration over spin and isospin coordinates, and we write explicitly the radial integrations. The notation is necessarily symbolic. The wave function $\phi_a(a)$, denotes the wave
function for the internal state of the nucleus \( a \) including its spin, isospin and the \( Z \)-projections of these.

The functions \( \psi^{(\pm)} \) denote distorted waves,\(^2\) and the coordinates that they depend on, \( R_a \), and \( R_b \), are the vectors separating \( a \) from \( A \) and \( b \) from \( B \) as shown in Fig. 1.

![Fig. 1](image)

The nuclide \( a = b + 1 + 2 \) has C.M. position vector \( R_a \) measured from \( A \). While nuclide \( b \) has C. M. position vector \( R_b \) measured from \( B \).

The integration is over nucleon coordinates. We always denote the transferred nucleons positions by \( r_1 \) and \( r_2 \), while \( d\mathbf{r}_b \) stands for

\[
d\mathbf{r}_b = \begin{cases} 
  dr_3 & \text{if } a = 3 \\
  dr_3 \, dr_4 & \text{if } a = 4
\end{cases}
\]

and

\[
dA = dr_{A+1} \cdots dr_{A+a}
\]

denotes integration over the \( A \) coordinates in nucleus \( A \).

The wave functions need to be written more explicitly now. Since \( b \) is a nucleon or deuteron depending on whether \( a = 3 \) or \( a = ^4\text{He} \),
\[
\phi_b(a-2) = \begin{cases} 
\chi_{1/2}(\sigma_3) & \varepsilon_b, a = 3 \\
\chi_{1/2}(\tau_3) & \varepsilon_b, a = 3 \\
\chi_{1}(\sigma_3, \sigma_4) & \varepsilon_0(\tau_3, \tau_4), \psi_d(r_{34}), a = 4 
\end{cases}
\]

where \( \chi \) is used to denote a spin or isospin spinor and \( \psi_d \) is the radial part of the deuteron, assumed as discussed in section 4 to be a pure S-state in the relative coordinate \( r_{34} \).

For the nucleus \( a \), it is common to assume a Gaussian wave function in the relative coordinates \( r_{12}, r_{13}, \ldots \). Such a wave function however can be very conveniently written in terms of the 1S harmonic oscillator functions

\[
\phi_a(a) = \psi_a(\text{space}) \chi_a(\text{spin-isospin})
\]

where

\[
\psi_a = \begin{cases} 
\phi_{10}(4\eta_{12}^2) & \psi_{10}(4\eta_{12}^2) \\
\phi_{10}(8\eta_{12}^2) & \phi_{10}(8\eta_{12}^2), a = 4 \\
\phi_{10}(3\eta_{12}^2) & \phi_{10}(4\eta_{12}^2) \\
\phi_{10}(4\eta_{12}^2) & \phi_{10}(4\eta_{12}^2), a = 3 
\end{cases}
\]

and \( \phi_{n\ell}(\nu r^2) \) denotes a harmonic oscillator and spherical harmonic

\[
\phi_{n\ell}^{m}(\nu r^2) = u_{n\ell}(\nu r^2) Y_{\ell}^{m}(\hat{r}).
\]

Their precise definition can be found, for example in the appendix of Ref. 5. For the present it is important to note the size parameter \( \nu \),

\[
u_n(\nu r^2) = (\nu r^2)^{3/2} e^{-\frac{1}{2} \nu r^2} P(\nu r^2)
\]

where \( P \) is a polynomial function. For the representation of a single-particle state in a nucleus of mass \( A \), the choice of
\[ \nu = A^{-1/3}(F^{-2}) \]  

(21) 

is appropriate. For the light nuclides \( a = 3 \) and \( 4 \) values of \( \eta \) in Eq. (18) can be determined from their rms radius, as determined from electron scattering and are given in the table below\(^5\) and are used in the evaluation of the structure amplitudes.

<table>
<thead>
<tr>
<th></th>
<th>( ^4\text{He} )</th>
<th>( ^3\text{He} )</th>
<th>( ^3\text{H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle r^2 \rangle^{1/2} )</td>
<td>1.61</td>
<td>1.97</td>
<td>1.68</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.233</td>
<td>0.206</td>
<td>0.242</td>
</tr>
</tbody>
</table>

The spin-isospin part of the wave function (17) of \( a \), under the assumption concerning the symmetries discussed in part a of this section, is

\[ \chi_a \equiv \chi_{S_a=0, T_a=0} \]

\[ = \sum_{S,T} \frac{(-)^S}{\sqrt{2}} \frac{1-(-)^{S+T}}{2} \left[ \chi_S(\sigma_1 \sigma_2) \chi_S(\sigma_3 \sigma_4) \right]_{0}^{0} \]

\[ = \left[ \chi_T(\tau_1 \tau_2) \chi_T(\tau_3 \tau_4) \right]_{0}^{0}, \quad a = 4 \]

(22)
or

\[ \chi_a = \chi_{a,s=1/2, t_a=1/2} \]

\[ = \sum_{S, T} \frac{(-1)^S}{\sqrt{2}} \left[ \chi_S(\sigma_1, \sigma_2) \chi_{1/2}(\sigma_3) \right]_{S, a} \left[ \chi_T(\tau_1, \tau_2) \chi_{1/2}(\tau_3) \right]_{T, a} \]

(23)

The square brackets denote vector coupling. These spin-isospin functions are antisymmetric under exchange of any pair of nucleon coordinates

\[ \chi_a(1,2,3) = -\chi_a(2,1,3) \text{ etc.} \]  

(24)

For the wave function of the heaviest nucleus, \( B = A + 2 \) we introduce a parentage expansion based on the "core" nucleus \( A \). Anticipating that this is to be used in (13) where the \( dA \) integral will project out of \( \phi_B \) the part having the \( A \) nucleons in the state \( \phi_A \), we write it as

\[ \phi_B(A+2) = \phi_{J_B, T_B} \]

\[ = \sum_{j_1, j_2, JT} \beta_{j_1, j_2}^{J_T} \left[ \phi_{J_A, T_A}^{(A)} \psi_{j_1, j_2}^{J_T(1,2)} \right]^{J_B, T_B} \]

(25)

Such a parentage expansion for \( B \) can be obtained explicitly, if for example a shell model wave function for \( B \) is available. In some cases the coefficients would be simply the configuration amplitudes of a shell model expansion for the two particle state \( \psi_{j_1, j_2} \). For pure configurations they are fractional parentage coefficients. Generally speaking each nucleus is a special case for which
the $\beta$'s can be derived, from an assumed or calculated nuclear wave function. Examples of the use of such wave functions in the present context and fractional parentage coefficients can be found in Ref. 5 and 6.

To proceed with the evaluation of (13) we define also the potential $V$. It acts between the nucleons of the separated fragments of $a$

$$V = \begin{cases} V(r_{13}) + V(r_{23}), & a = 3 \\ V(r_{13}) + V(r_{14}) + V(r_{23}) + V(r_{24}), & a = 4 \end{cases}$$

For the moment we ignore its possible dependence on the spin and isospin of the interacting nucleons, which more general form is written in (11). In any case we suppose that $V$ can be approximated by a zero-range interaction acting between the centers of mass of the separated fragments of $a$, which is the coordinate $\rho$ in Fig. 1.

$$V \approx g \delta(\rho).$$

The effect of this zero-range assumption is

$$R_b \rightarrow \frac{A}{A+2} R, \quad R_a \rightarrow \rho.$$

The following changes of integration variables (which each have unit Jacobian) can be made

$$d\rho \cdot d\rho \cdot d\rho = d\rho \cdot d\rho \cdot d\rho = \begin{cases} d\rho \cdot d\rho \cdot d\rho, & a = 3 \\ d\rho \cdot d\rho \cdot d\rho, & a = 4 \end{cases}$$
We mention one other factor at this point. The expression for $T$, Eq. (13) was written without antisymmetrizing between nucleons between $A$ and $a$, or $B$ and $b$. We are prepared to neglect the exchange integrals which would be introduced by taking this into account. However we keep the statistical weight arising from the many different equal direct integrals.\textsuperscript{7} This is

$$
\left[ \frac{A+2}{2} \right]^{1/2} \left[ \frac{a}{2} \right]^{1/2}
$$

(30)

With the above information and assumptions we now can write $T$ as

$$
T = \left[ \frac{A+2}{2} \right]^{1/2} \left[ \frac{a}{2} \right]^{1/2} \sum_{j_1j_2} C_A^j M_{j_1}^B Z_a Z_b T_{Bj_1} T_{Aj_2}
$$

$$
\frac{1}{\sqrt{2}} \frac{1 - (-1)^{S'+T'}}{2} T_{Bj_1} T_{Aj_2} \sum_{j_1j_2} \beta_j Z_a Z_b
$$

$$
\langle \psi_b^* \left( \frac{A}{A+2} \right) | \psi_{(j_1j_2)}^\dagger \rangle M_Z(1,2) \phi_{10}(a\eta r_{12}^2) X_S(1,2) X_T(1,2)
$$

(31)

where

$$
D_0 = g \int \phi_{10} \left[ (4a-8)\eta^2 \rho^2 \right] \frac{\delta(\rho)}{\rho^2} \rho^2 d\phi d\eta
$$

$$
= g \sqrt{4\pi} \left[ \frac{2 (4a-8)\eta^2}{\Gamma(3/2)} \right]^{3/2}
$$

(32)
and

\[ \Omega_{a-2} = 1 \quad \text{if } a = 3 \]

\[ = \int \phi_d^*(r_{34}) \phi_{10} (4\eta^2 r_{34}^2) \, dr_{34} \quad \text{if } a = 4 \quad (33) \]

These factors obviously affect the magnitude of the cross section but not the angular distribution.

We also define for convenience the factor

\[ b_{ST} = \left[ \begin{array}{c} a \\ 2 \end{array} \right] \frac{1}{\sqrt{2}} \frac{1}{2} \frac{1}{2} \left( \frac{S+T}{2} \right) \frac{T}{C_2} \frac{t_b}{Z_b} \frac{t_a}{Z_a} \frac{1}{\sqrt{2S+1}} \quad (34) \]

which could be called the light particle spectroscopic amplitude. For the general central force mixture (11), the analogous quantity was defined in Eqs. 7-9.\(^*\)

The remaining integral in (31) contains the form factor for the reaction

\[ \text{reaction} \]

\[ * \text{In Eq.7, the factors } 2a_0 \text{ and } -2a_1 \text{ for the } a = 3 \text{ cases and } -4a_1 \text{ for } a = 4, \text{ count the number of interactions times their strengths. For a pure Wigner force, as (12), these factors go to unity. This gives the correspondence with (34).} \]
Its physical meaning is quite clear. The left side, including the sum \( j_1 j_2 \), describes the state of motion of nucleons 1 and 2 in the nucleus \( B = A+2 \). The right side describes their state of motion as they exist in the light nucleus \( a \). Therefore it projects out of their motion in the nucleus \( B \) that part whose relative motion corresponds to their motion in nucleus \( a \), and describes how their center of mass moves in \( B \) when they possess this correlation. For this reason it is sometimes referred to as a projected wave function. Its norm is generally much less than unity. For correlated states it is concentrated in the surface region. 8

To complete the reduction of (35) we transform \( \Psi(1,2) \) to center of mass and relative coordinates. Since

\[
\Psi(j_1 j_2) J_T(1,2) = \frac{\chi_T^Z(1,2)}{\sqrt{2(1+\Delta j_1 j_2)}} \left\{ \Psi(j_1 j_2) J_T(1,2) + (-)^T \Psi(j_1 j_2) J_T(2,1) \right\}
\]

is an antisymmetrized two-particle wave function with

\[
\Psi(j_1 j_2) J_T(1,2) = \begin{bmatrix} \phi_{n_1 j_1} (1) & \phi_{n_1 j_1} (2) \end{bmatrix}^M
\]

and

\[
\Psi(j_1 j_2) J_T(2,1) = \begin{bmatrix} \phi_{n_2 j_2} (1) & \phi_{n_2 j_2} (2) \end{bmatrix}^M
\]
\( \phi_{n\ell j}(r) = \text{single-particle wave function} \)

\[
\chi^m_{ij} = \left[ Y_{\ell_1}^{\pm}(\hat{r}) Y_{\ell_2}^{\pm}(\hat{r}) \right] \chi_{1/2}^{\pm}(r) \quad \text{(for example)}
\]

\[
\Delta_{j_1, j_1'} = \delta_{\ell_1 \ell_2} \delta_{\ell_1' \ell_2'} \delta_{j_1 j_1'}
\]

The transformation can be accomplished by transforming \( \phi_{n\ell j} \) to \( \chi \) and then using the Moshinsky transformation from \( \chi \) to \( \chi_{1/2} \) and then onto \( \chi_{1/2} \) as shown in Eqs. (39).

The result for \( F \) is:

\[
F_{JST}^{M} = \sum_{L} C_{LM} S J_{1} M_{S} M_{j} \left[ L S J (R) \right] Y_{L}^{*}(\hat{r}) \chi_{1/2}^{+}(r)
\]

This shows that the angular integration \( \int_{12} \) in Eq. (35) project onto the \( S \)-state of relative angular momentum, \( \lambda = 0 \).

The result for \( F \) is:

\[
F_{JST}^{M} = \frac{1 - (-)^{S+T}}{2} \sum_{L} C_{LM} S J_{1} M_{S} M_{j} \left[ L S J (R) \right] Y_{L}^{*}(\hat{r}) \chi_{1/2}^{+}(r)
\]
where the form factor \( f_{LSJ}(R) \) is

\[
f_{LSJ}(R) = \sum_N G_{LSJ} u_{NL} (2\nu R^2)
\]

(42)

\[
G_{LSJ} = \sum_{j_1j_2} r_{j_1j_2}^{l_1l_2} G_{NLSJ}
\]

(43)

with

\[
G_{NLSJ} = \Omega_n \left( \frac{2}{1+\Delta_{j_1j_2}} \right)^{1/2} \begin{bmatrix} 1/2 & j_1 \\ j_2 & L_S J \end{bmatrix} (n0NL; L|n_{1}l_{1}n_{2}l_{2}; L) \]

(44)

\[
\Omega_n = \int \phi_{n0}^* \left( \frac{1}{2} \nu r_{12}^2 \right) \phi_{10} (a_{1}r_{12}^2) \, dr_{12}.
\]

(45)

This last factor is evaluated explicitly in Ref. 5. The structure amplitudes which are tabulated in this work are the \( G_{NLSJ} \). We have omitted the label \( T \), because according to the first factor on the right side of (41), \( S+T = \text{odd} \). Also the label \( n \) is omitted because a condition for the non-vanishing of the Moshinsky bracket in (44) is

\[
2(n+N)+L = 2(n_1+n_2) + l_1+l_2.
\]

The form factor for a pure two-nucleon configuration is given by (2) with \( G \) defined by (44) and for the general state by (42,43). This completes the derivation for the structure amplitudes from which the form factors for two-nucleon transfer can be constructed.
We conclude by mentioning how the tail region of the form factor can be improved. The harmonic oscillator function \( u_{NL} \) in (42) decays rapidly in this region. One can correct this defect by matching \( f_{LSJ}(R) \) to a Hankel function

\[
-\frac{L}{\pi} H_L^{(1)}(\kappa R)
\]

where

\[
\kappa^2 = \frac{4m^*}{\hbar^2} \varepsilon_B
\]

with \( 2m^* \) the reduced mass of the pair of nucleons in \( B = A+2 \), and \( \varepsilon_B \) their effective binding energy to the core \( A \). Frequently this is taken to be their separation energy. Actually this could be reduced somewhat owing to the self binding or correlation energy of the pair. This would be the lesser of the correlation energy of the pair in \( B \) or in \( a \). In the latter case the average correlation energy is \( \varepsilon_a/(\gamma_a^2) \) with \( \varepsilon_a \) the binding energy of \( a \). There are other prescriptions for handling the tail region. For example one could use Saxon-Woods single-particle wave functions bound at one half the two-particle separation energy.\(^5\) This procedure is more complicated numerically and since the prescription above achieves the same physical result, it is preferred.
References

Conventions

Any nuclear structure calculation is performed with the conscious (or unconscious) adoption of a number of conventions. In the subsequent use of wave functions from such calculations, the same conventions must be used, or compensating phase factors introduced. Below are listed the conventions adopted here, which are commonly but not universally employed.

1) All radial functions \( u_{n\ell j}(r) \) have positive slope at the origin.
2) Spherical harmonics have Condon-Shortley phases.
3) The order of spin-orbit coupling is \( j = \ell + 1/2 \) and not \( j = 1/2 + \ell \). The tabulated values of \( G \) for the configuration \((j_1, j_2)\) can be converted to the latter convention by multiplying the \( G \)'s by \((-)^{\sigma} \) where \( \sigma = \ell_1 + j_1 + \ell_2 + j_2 + 1 \).
4) If the order \((j_2, j_1)\) is needed, while the table lists the order \((j_1, j_2)\) the \( G \)'s for the one order are given in terms of those of the other order by multiplying the tabulated values by \((-)^{\rho} \) where \( \rho = j_1 + j_2 + J + S + 1 \).

5) The tables are constructed under the assumption that the wave function of the two transferred nucleons in nucleus \((A+2)\) in the reaction (1) have a prescribed isospin \( T \). That means that they are totaly antisymmetric in space-spin-isospin coordinates. Of course the wave function of a pair of like nucleons must be antisymmetric in space-spin coordinates and it automatically has prescribed isospin \( T = 1 \). However for some nuclei the assumption of overall antisymmetry need not hold for a neutron-proton pair. If the wave functions describing \( A+2 \), where the transferred nucleons
are unlike, do not have a prescribed isospin, the entries in the table should be divided by $\sqrt{2}$ whenever the two nucleons occupy different single particle states $(n_1l_1j_1) \neq (n_2l_2j_2)$.

Explanation of Tables

Three tables are needed, dictated by the size of the light nucleus a. They are given in the order

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha,d$</td>
<td>$^3\text{He},p$</td>
<td>$t,n$</td>
</tr>
<tr>
<td>$^3\text{He},n$</td>
<td></td>
<td>$t,p$</td>
</tr>
</tbody>
</table>

Within each table, the pure two-nucleon configurations are arranged in order as they would appear in light to heavy nuclei. The structure amplitudes, $G_N$, vary slowly with nuclear size so that for each two-nucleon configuration, the amplitudes are given for only three nuclear sizes, listed according to mass number $A$, for masses in the region appropriate to the configuration. A short study of the tables will quickly reveal the overlapping way in which they have been arranged.

There are restrictions on the spin ($S$) that the transferred particles can carry, the reasons for which are discussed in Theory. Thus for example, the amplitudes for both of the reactions ($^3\text{He},p$) and ($^3\text{He},n$) are listed in the same table because of the common nucleus $^3\text{He}$. However only the $S = 0$ part is relevant to the ($^3\text{He},n$) reaction. This fact is mentioned in the heading of each table.
S, J, L

Intrinsic, total and orbital angular momentum carried by the transferred pair of nucleons.

G(N)

Structure amplitudes (N = 1, 2, ... ) for the specified two-nucleon configuration \( (n_1^l_1 j_1) \) \( (n_2^l_2 j_2) \) LSJ.

NN1 NN2

Principle quantum numbers of a two-particle configuration

\[ [NN1 = 2(n_1 - 1) + \ell_1] \]

MASS

Mass number of target nucleus.

NU

Size parameter, \( \nu = A^{-1/3} F^{-2} \) which appears in the single-particle radial functions from which the form factor is constructed (Eq. 20, 39).

\( 1 \, d_{5/2}^{1/2} \, 2 \, s_{1/2} \)

A two-nucleon configuration in spectroscopy notation \( n\ell j \)

\( (n \geq 1) \)
Examples

Example 1. Suppose it is postulated that a \((p, {}^3\text{He})\) transfer reaction near mass 40 to a \(4^+\) state involves the pure configuration \(\left[f_{7/2}^2\right]_{J=4}\), with a passive zero-spin core. Since \(1f_{7/2}\) belongs to the 3'rd oscillator shell we look for the \(\text{NN}_1 = \text{NN}_2 = 3\) section of the tables. From known selection rules or the tables, we find that \(S = 0\) alone is allowed, whence according to Eq.(41), such a configuration has \(T = 1\). The projected wave function \(f_{LSJ}\) is

\[
f_{404}(R) = 0.0189 \, u_{14}(2\nu R^2) + 0.1169 \, u_{24}(2\nu R^2)
\]

with \(\nu = 0.292 \, \text{F}^{-2}\).

Example 2. As an extension of example 1 suppose the wave function is

\[
\psi_{JT=1} = \alpha \left[f_{7/2}^2\right]_{J=4} \left[f_{7/2}^2 2p_{1/2}\right]_{J=4} + \beta \left[f_{7/2}^2 2p_{1/2}\right]_{J=4}.
\]

Since \(T=1\), only the \(S=0\) part of the table is relevant and we have for \(G_{NLSJ}\)

\[
G_{404} = 0.0189 \, \alpha + 0.0044 \, \beta
\]

\[
G_{2404} = 0.1169 \, \alpha + 0.3016 \, \beta
\]
in place of the amplitudes quoted in example 1. That is

\[ f_{404}(R) = G_{1404} \ u_{14}(2\nu R^2) + G_{2404} \ u_{24}(2\nu R^2). \]

**Example 3.** Suppose the state of example 2 is assumed to have no particular symmetry. This does not affect the first configuration since it automatically has \( T=1 \), but the second configuration is affected. According to the conventions we have for \( G_{NLSJ} \)

\[ G_{1404} = 0.0189 \ \alpha + 0.0044 \ \beta/\sqrt{2}, \]
\[ G_{2404} = 0.1169 \ \alpha + 0.3016 \ \beta/\sqrt{2}. \]

But the second configuration can now be reached with spin \( S=1 \) also:

\[ G_{1414} = 0.0050 \ \beta/\sqrt{2}, \]
\[ G_{2414} = 0.3372 \ \beta/\sqrt{2}. \]

The projected wave functions corresponding to the transfer of the pair in these two center-of-mass states are therefore

\[ f_{404} = G_{1404} \ u_{14}(\rho) + G_{2404} \ u_{24}(\rho), \]
\[ f_{414} = G_{1414} \ u_{14}(\rho) + G_{2414} \ u_{24}(\rho), \]

where \( \rho = 2\nu R^2. \)

**Example 4.** Suppose that the postulated wave function for a \( 3^+ \) state near mass 40 is \( \left[ f_{7/2}^2 \right]_{J=3} \). From known selection rules, or the tables we find \( S=1 \) for the \((p,^4\text{He})\) reaction. Two values of \( L \) are allowed, 2 and 4. The corresponding projected wave functions \( f_{LSJ} \) are
where \( p = 2\nu R^2 \), and \( \nu = 0.292 \, \text{F}^{-2} \).

**Example 5.** The same function as in example 2 would appear, if written in the convention in which the spin-orbit coupling is

\[ s + \lambda = j \] as

\[ \psi_{JT} = \alpha \left( \bar{F}_{7/2}^2 \right)_{J=4} - \beta \left( \bar{F}_{7/2}^2 \bar{J}_{1/2} \right)_{J=4}, \quad T=1 \]

where bars denote use of this latter convention. Application of the rules on conventions leads to the same \( G \)'s as before.
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