Lawrence Berkeley National Laboratory
Recent Work

Title
PRELIMINARY STUDIES ON BEAM PROGRAMMING IN A CYCLOTRON

Permalink
https://escholarship.org/uc/item/1970n542

Author
Willax, Hans A.

Publication Date
1960-09-30
PRELIMINARY STUDIES ON BEAM PROGRAMMING IN A CYCLOTRON

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
PRELIMINARY STUDIES ON BEAM PROGRAMMING IN A CYCLOTRON

Hans A. Willax

September 30, 1960
PRELIMINARY STUDIES ON BEAM PROGRAMMING IN A CYCLOTRON

Hans A. Willax

Lawrence Radiation Laboratory
University of California
Berkeley, California

September 30, 1960

ABSTRACT

In order to investigate the influence of the electric field on the particle motion in the center of a cyclotron, a geometric method for approximate orbit construction has been used in combination with analog measurements of the electric field.

The results show that perturbations of the electric field, as induced by the ion source and the ion-extraction device, can be of important influence on the beam behavior and the beam quality in a cyclotron.

Possibilities for "Beam Programming" are discussed briefly under these aspects.
PRELIMINARY STUDIES ON BEAM PROGRAMMING IN A CYCLOTRON

Hans A. Willax *

Lawrence Radiation Laboratory
University of California
Berkeley, California

September 30, 1960

I. Introduction

Generally in conventional cyclotrons it is not possible to extract more than a few per cent of the relatively intense internal beam. As Howard and Livingstone explain, this is a consequence of the poor spatial and energy definition of the beam at the extraction radius. A very large percentage of the beam is finally lost by hitting structural material within the machine. For high-energy cyclotrons at which intense external beams are desired, this can be a source of many difficulties. The beam quality at the extraction radius should be an optimum in machines of this type. As Gordon and Blosser point out, a very well-defined internal beam would be required, especially for regenerative extraction. One desires good radial separation of the orbits, good axial confinement, and a small energy distribution.

According to theoretical investigations by Smith and Garren only relatively small radial deviations can be controlled by the radial focusing in an isochronous cyclotron with AG focusing. It therefore seems to be useful to provide radial beam definition on the first few turns of the beam in the center of the cyclotron before axial AG focusing takes place. (See also Miller and Lind.)

A. H. Morton and W. I. B. Smith, and Smith alone, as well as Powell, Blosser and Irwin, Allen, et al., show that the beam quality in a cyclotron can be much improved by an arrangement of defining slits in the first few turns. With such an arrangement in an 8-Mev proton cyclotron Morton and Smith obtained an extraction efficiency greater than 95% and an energy spread of the external beam of only 0.5%.

Defining slits or plates in the center region of a cyclotron can have several functions:

1. All ions that leave the ion source with too large an angular deviation are trapped before they gain much energy.

*Guest from the Federal Institute of Technology, Zürich, Switzerland.
2. All ions that leave the ion source with an unfavorable phase and all ions that run on orbits with displaced centers are trapped.

3. Axial defocusing of the E field within the dee gap can be diminished by slits in plates placed across the dee aperture.

Defining slits are useful as long as the beam orbits in the cyclotron stay fixed. In a cyclotron, however, in which the beam orbits are to be changed for accelerating particles with different e/m to variable final energies, beam defining with slits can present problems.

In order to predict the cyclotron-beam behavior, it seems necessary to obtain some knowledge of the main parameters that can cause poor beam quality in the center of the machine. Among the parameters to consider are the influence of space charge, inhomogeneities in the magnetic field, and the precession of orbit centers due to harmonics in the magnetic field, as well as the influence of the electric field on the low-energy ions on the first few revolutions. Since the influence of the electric field in the center of the cyclotron seemed to be the reason for many observed phenomena on cyclotron beams (see Powell, W. I. B. Smith, Blosser, Taylor, Jones, and others, Suwa, and Kikuchi), we tried to obtain a qualitative judgement of some of these effects. Theoretical investigations on this matter were made by several authors.

Experimental results on some effects such as phase bunching and orbit-center displacement sometimes do not agree with the theoretical prediction. A possible reason for this disagreement is that most of these investigations were based on simplified assumptions of the E-field shape around the ion source, in order to get a clear analytic expression for the phase behavior or for the behavior of the orbit-center movement (assumption of "uniform field").

In actual geometries, however, the E-field shape in the center of the cyclotron can not be described by a simple function of spatial coordinates. This is especially true for a single-dee geometry. If the E-field shape is known from measurements taken in an electrolytic tank, orbit calculations can be done on a digital computer by using point-by-point data or a more complicated approximation formula for the E field.

Orbit calculations with computers seem to be too expensive for qualitative preliminary investigations. An attempt was made to investigate the E-field influence on particle orbits by using a geometrical approximation method for the orbit construction in the median plane of the cyclotron. This method does not require the use of a computer even for complicated E-field shapes, and the accuracy is sufficient for a first approximation.

In order to check the method, investigations were made applying the dee geometry and acceleration conditions (H field and rf voltage) of the Lawrence Radiation Laboratory 88-inch cyclotron. For the approximate construction of the E-field shape in the median plane, the results of analog measurements of the electric potential in a vertical cross section of the
cyclotron were used. For closer investigations the E-field shape should be
evaluated more carefully (for example, in an electrolytic tank). The "beam"
behavior could be investigated by drawing orbits for particles with different
starting phases. In order to obtain reasonable accuracies in orbit drawings,
most of the particle orbits were constructed for nitrogen 3+ ions (the radius
for \( \text{N}^{3+} \) is approximately twice that for \( \text{D}^+ \)).

Information was also obtained on the beam behavior if an extractor
electrode were utilized, according to the proposal of K. Ehlers. 11

An artificial defining of the cyclotron beam characteristics -- such
as the orbit centering, the radial and axial betatron amplitudes, and the phase
relationship of the beam pulse -- is usually called "Beam Programming."
Some possibilities for this beam defining in the center of a cyclotron are dis­
cussed qualitatively.

II. The Electric Field for a "Single-Dee" Geometry
in the Cyclotron

Murray and Ratner show a method for computing E fields within
the dee gap for two-dee cyclotron systems. 23 The E-field shape along the
edge of a single dee can also be computed; however, difficulties arise if a
dummy dee at liner potential is used. For our investigations, the vertical
section of the cyclotron (see sketch, Fig. 1) was drawn on conductivity paper
(GE) with silver paint and the equipotential lines were measured by a probe,
using a bridge circuit. The results are given in Figs. 2 through 9.

The accuracy of this method is limited because of different conduc­
tivities of the paper in orthogonal directions (deviation about 5%). Therefore,
two measurements were made on papers at right angles to each other for
comparison of the results. The curves 1a and 1b in Fig. 10 show the devi­
ations in the median plane. The resulting error can be neglected for these
investigations.

As can be seen from Figs. 2 through 5, the dummy dee causes a
compression of the equipotential lines in the x direction. The displacement
of these lines is especially large on the right-hand side of the center line
(\( x < 0 \)), as the dummy dee is brought closer.

As Fig. 10 shows, the potential gradient in the median plane is at
a minimum when the dummy dee is at infinity (no dummy dee). Each approach
of the dummy increases the gradient. For all possible distances of the dummy
dee from 2 in. to infinity, the potential curves lie between curve 2 and curves
1a and b.

For comparison of the electrical focusing conditions in all figures,
an E-field line was drawn which ends 1/4 in. within the dee edge. One sees
from Figs. 2 and 3 that the curvature of this line does not change greatly
within the possible axial beam range (\( |z| < d/2 \)) when the dummy is 2 in. from
the dee. This is illustrated in Fig. 11. However, the absolute value of the
field strength, and therefore the absolute value of the $z$ component of the E field, is changed. Curve 1 of Fig. 11 shows the maximum $z$ component of the E field at a distance $z = 1/2$ in. For a dee voltage of 70 kv the maximum $z$ component of the E field on the dummy-dee side ($x < 0$) of the center plane ($x = 0$) increases from 1.4 kv/cm to 2.75 kv/cm if the dummy is used (curve 2). This means a relative increase of about 95%. The relative increase of the $z$ component of the E field on the dee side ($x > 0$) is only about 20%.

If a solid plate is arranged opposite the dee, the shape of the equipotential lines is obvious (Figs. 6, 7, 8, 9). Figure 9 shows that a lip-shaped offset can be helpful for axial electric focusing.

From the results of potential measurements in the $x - z$ plane one can estimate the shape of the equipotential lines in the median plane about the ion source. For this purpose one can assume that any perturbation of the field in the median plane vanishes at a radial distance $2d$ (where $d$ = dee aperture) or $2d'$ ($d'$ = liner aperture) from the edge of the field-perturbing object (such as ion source, defining slits, and so on). Comparison to the results of Miller and Lind, who made exact measurements in an electrolytic tank, justifies this assumption.

If an object with a certain dc potential, such as an extractor electrode is present in the rf field, both gradients can be superimposed vectorially. The resulting E field at a certain location is the vector sum of a stable vector and a vector whose absolute value changes with time (see also Fig. 17).

III. Method of Approximate Orbit Construction in the Center of a Cyclotron

We confine our investigations to the median plane. For the particle motion (see Fig. 13) in the median plane of a cyclotron the following differential equations are valid:

$$ \ddot{x} = \frac{e}{mm} \left\{ E_x(x, y) \cos (\omega_{rf} t + \phi_0) + \dot{y} H_z(x, y) \right\}, \quad (1) $$

$$ \ddot{y} = \frac{e}{mm} \left\{ E_y(x, y) \cos (\omega_{rf} t + \phi_0) - \dot{x} H_z(x, y) \right\}.$$

where $x$ points into the dee, $y$ is parallel to the dee edge, and $z$ is perpendicular to the median plane (see Fig. 1):

$$ x = \frac{dx}{dt} $$
$$ \dddot{x} = \frac{d^2 x}{dt^2} $$
$$ y = \frac{dy}{dt} $$
$$ \dddot{y} = \frac{d^2 y}{dt^2} $$
\( \phi_0 \) = starting phase of the particle,

\( q \) = number of charge units,

\( e/m \) = charge-to-mass ratio of the proton,

\( M \) = atomic weight,

\( E_x \) = component of the electric field strength in the \( x \) direction (analog \( E_y \)),

\( H_z \) = magnetic field strength (\( z \) component).

Commonly, all components of the electric and magnetic field are functions of \( x \) and \( y \). The equations of motion can be integrated if these functions are known. However --- especially in the machine center --- they become complicated in many cases.

The method of geometric orbit construction is one of approaching the real orbit with its steadily changing curvature by a series of arcs around a moving center of motion. For this approximation method, where the drawing accuracy gives a certain limit in the over-all accuracy, it is sufficient to have a plot of electric potential lines rather than a plot of the \( E \) field (see Fig. 13). Since we are interested in the first turns only, in which range the magnetic field will not change drastically with radius, the following assumptions are justified:

1. The magnetic field in the observed range should be constant:
   
   \[ H_z(x, y) = \text{constant} = H \]

2. The rf frequency \( \omega_{rf} \) of the dee voltage corresponds to the cyclotron frequency in this range:
   
   \[ \omega_{rf} = \omega_0 = \frac{q e}{M m} \cdot H. \]

3. The energy of the particle stays well below the relativistic range.

Assume a particle with momentum \( p \) is in transit through an electric field whose potential is shown in Fig. 12. (This field will be changing corresponding to \( E = E_0 \cos (\omega t + \phi_0) \)). We assume that instead of a steady momentum change between \( P_1 \) and \( P_2 \), there is a sudden momentum change \( \Delta p_1 \) at the point \( A_1 \) (half way between \( P_1 \) and \( P_2 \)) which corresponds to the \( E \)-field influence between points \( P_1 \) and \( P_2 \). Thus the actual \( E \)-field influence between \( P_1 \) and \( P_2 \) is concentrated at point \( A_1 \) by this method. Since the particle runs in a constant magnetic field, its orbit is a circle with the radius \( r_1 \) corresponding to its momentum \( p_1 \) around the center of motion \( C_1 \) until it reaches the point \( A_1 \). After the momentum change at \( A_1 \) the particle proceeds with a momentum \( p_2 \) on a circular orbit around the new orbit center \( C_2 \) with the radius \( r_2 \) until it reaches point \( A_2 \). There another sudden momentum change is given according to the total field influence between \( P_2 \) and \( P_3 \).

**Time Variation of the Electric Potential**

During the time required for the particle to pass from \( A_1 \) to \( A_2 \), the absolute \( E \)-field strength changes from \( E_0 \cos (\omega t_1 + \phi_0) \) to \( E_0 \cos (\omega t_2 + \phi_0) \)
(where \( t_1 \) and \( t_2 \) are the times at which the particle passes \( A_1 \) and \( A_2 \)). In a constant magnetic field the time interval a particle needs in order to travel a certain angular interval \( \Delta \theta \) is constant and independent of the particle energy, as long as the relativistic mass increase can be neglected. Since \( \omega_{rf} = \omega \), the rf phase changes the same amount \( \Delta \theta \) during this time interval. To find the new E-field influence between \( P_2 \) and the approximated point \( P_3 \), one has to take into account this time variations of the electric potential. If the chosen angular intervals \( \Delta \theta \) are small enough, one can use the potential difference between \( P_2 \) and \( P_3 \) observed at the time the particle passes \( A_2 \).

In general, to find the point \( P_{n+1} \) from point \( P_n \), the arc with the radius \( r_n \) around the center \( C_n \) can be extended by \( \Delta \theta/2 \) beyond the point \( A_{n+1} \). In most cases this approximation is of sufficient accuracy to estimate the acceleration potential between \( P_n \) and \( P_{n+1} \).

**Momentum Change and Motion of Orbit Center**

To find the center of motion \( C_{n+1} \) from \( C_n \), one must determine the direction of momentum change at \( A_n \), which is parallel to the direction of the average E field between \( P_n \) and \( P_{n+1} \), and the direction of the orbit-center motion. They are perpendicular to each other (see Appendix).

The following simplified equations can be used to find the direction and magnitude of the orbit-center displacement \( \Delta r_n \) that corresponds to a sudden momentum change at point \( A_n \).

Given

\[
r_n = \frac{144}{H} \left( \frac{M}{q} \right)^{1/2} U_n^{1/2},
\]

(2) where

\[
r_n = \text{orbit radius (in cm) at point } P_n \text{ of the particle with an atomic weight } M, \text{ the number of charges } q, \text{ the energy } U_n \text{ (in electron volts) in a constant magnetic field } H \text{ (gauss)},
\]

the magnitude \( \Delta r_n \) of the orbit-center displacement can be described by

\[
\Delta r_n = (\Delta r_{n(tang)}^2 + \Delta r_{n(norm)}^2)^{1/2},
\]

(3)

where

\[
\Delta r_{n(tang)} = \text{increase (or decrease) of the orbit radius at point } A_n \text{ due to the energy gain (or energy loss)} \Delta U_n \text{ of the particle between } P_n \text{ and } P_{n+1}.
\]

\[
\Delta r_{n(tang)} = \frac{144}{H} \left( \frac{M}{q} \right)^{1/2} \frac{\Delta U_n}{2(U_n + \Delta U/2)^{1/2}}, \text{ (in cm)}
\]

(4)

and

\[
\Delta r_{n(norm)} = \text{orbit-center displacement orthogonal to } \Delta r_{n(tang)} \text{ due to the average E-field component normal to the orbit between } P_n \text{ and } P_{n+1}.
\]
\[ \Delta r_{n}(\text{norm}) \approx \frac{r_n \times s \times \overline{E}_{\text{norm}}}{2 U_n}, \quad \text{(in cm)} \]  
(5)

\[ s = \text{path length (in cm) between } P_n \text{ and } P_{n+1}, \]

\[ \overline{E}_{\text{norm}} = \text{average E-field component (in } \text{V/cm}) \text{ normal to the orbit between } P_n \text{ and } P_{n+1}. \]

For \( r_{n}(\text{tang}) \ll r_n \) and \( 0 \leq \overline{E}_{\text{norm}} \leq 2 \overline{|E|}_{\text{tang}} \) \((\overline{E}_{\text{tang}} = \text{orbit - tangential component of the E-field})\), the following equations give the components of \( \Delta r_n \) with sufficient accuracy:

\[ \Delta r_{n}(\text{tang}) \approx r_{n+1} - r_n; \]

(6)

\[ \Delta r_{n}(\text{norm}) \approx \Delta r_{n}(\text{tang}) \times \tan \overline{\alpha}_n, \]

(7)

where \( \overline{\alpha}_n = \text{average angle between orbit tangent and direction of the E field between } P_n \text{ and } P_{n+1}. \)

From \( \Delta r_{n}(\text{tang}) \) and \( \Delta r_{n}(\text{norm}) \) the new orbit center \( C_{n+1} \) is defined. In this manner an ion orbit can be constructed step by step. Because of the assumption of a sudden momentum change, the final orbit is a series of scallops. By decreasing the angular intervals \( \Delta \theta \) a smoother curve can be obtained.

From this procedure one can obtain graphically the location and center of motion at a certain time for particles with different starting conditions. In principle this method can be applied for any complicated shape of the electric potential and also for a slowly changing magnetic field where, in fact, the assumption of a constant \( H \) field within the curvature of an angular interval \( \Delta \theta \) is valid. The choice of the size of this angular interval \( \Delta \theta \) has to be fitted to the wanted accuracy, the drawing accuracy, and the accuracy of the field data. (\( \Delta \theta \) corresponds to the broadness of the steps in the approximation of the time variation of the electric potential and the x-y variation of the magnetic field.)

Since the potential field used was only an approximation and the general idea of this investigation was a proof of the method to obtain an overall picture of the beam behavior, for the orbit construction shown an angular interval of \( \Delta \theta = 30 \text{ deg} \) has been chosen.

IV. Particle Orbits in a Conventional Ion-Source Geometry

For a single-dee geometry in which the extractor is represented by a simple rod opposite the ion source slit, Fig. 13 shows the electric equipotential lines, the first two turns of a \( N^{+++} \) ion starting in phase \( (\phi_0 = 0) \), and the orbit-center motion for this particle.
Since the electric equipotential lines are concentrated between ion source and extractor within \( \frac{1}{2} \) in., the gradient that accelerates the particle from zero velocity is so large that the first transition time corresponds to only \(-30\) deg phase change; this means that the ion gains almost full dee voltage. (This is derived from integration of the equation of motion.) The center of motion remains at \( C_1, 2, 3, 4 \) until the particle arrives in an accelerating field again.

As one can notice, the center of motion subsequently shifts slowly towards the dee. This is due to the unbalanced perturbation of the electric field, which the ion source causes in this case when no dummy dee is present. Behind the ion source, there is a fairly large \( y \) component of the \( E \) field which is not balanced within the dee. This \( y \) component in the \( E \) field causes the center shift in the \( x \) direction. If the magnetic field were constant, the center shift perpendicular to the dee would cease after the ion orbits got beyond the \( E \)-field perturbation.

By mounting a dummy dee in the center region, this effect can be almost eliminated as will be seen, for example, from Fig. 15. If the \( E \)-field perturbation becomes symmetric to the center line, the center of motion also stays close to this line of symmetry.

Figure 14 shows the situation if the ion extractor is left out. The perturbation of the equipotential lines in the vicinity of the ion source is strongly asymmetric. Therefore, the migration of the center of the orbit towards the dee is very evident. This figure also indicates orbits for particles starting \( 30 \) deg early (\( \phi_0 = -30 \) deg) and \( 30 \) deg late (\( \phi_0 = +30 \) deg). The orbit for a particle starting \( 30 \) deg late and the migration of its center of motion deviate strongly from those for particles with negative and zero starting phase. Comparing the times, however, at which the different particles cross the center plane (\( x = 0 \)), one can notice that in this case the ion starting \( 30 \) deg late advances in phase relative to the particle starting at zero phase. The late-starting particle gains about \( 15 \) deg after the first half turn, and it arrives almost exactly in time with the zero-starting-phase-particle after the first full turn. This effect is well known as "phase bunching," \( 10, 18 \) and occurs in the center of a cyclotron when the low-energy particles have to spend a fairly long part of the rf period (\( \geq 1/4 \) period) in a rf field region where the electric field is mostly orthogonal to the direction of motion.

However, the stronger the phase-bunching effect becomes the more the centers of motion for particles with different starting phases depart from one another. To demonstrate a case in which almost no phase bunching is to be expected, the orbits for particles with the same starting time as in Fig. 14 are drawn in Fig. 15. The field geometry, however, is different. The puller, mounted opposite the ion source slit and \( 1/2 \) in. away produces a high gradient such as to accelerate the ions to fairly uniform initial energy. The dummy dee causes more symmetry in the field perturbation. Since there is no region where the particles see a strong \( E \)-field component orthogonal to their direction of motion, no effective phase bunching occurs. Also, the centers of motion after the first two turns lie much closer together. [These results are in general agreement with calculations and experimental observations of several authors, \( 6, 7, 8, 13, 15, 17, 21 \) and others.]
Radial beam extension, displacement of the final center of motion, center spread, and phase bunching depend very much on the shape of the E field around the ion source. Therefore, it is not difficult to control the parameters by choosing optimal geometry.

It should be mentioned, however, that the effect of a certain E-field shape in the center of the machine is much different for particles with different e/m. As can be seen from Fig. 16, deuterons, for example, make about twice as many revolutions in a certain field perturbation as nitrogen ions. They find a larger range with an E field orthogonal to the orbit; therefore, the phase-bunching effect and the relative center spread are larger than for \( \mathrm{N}^{++} \). To obtain the same effective transit time on the first acceleration gap the distance between the ion source and puller has to be changed. For example, for deuterons at 17 kilogauss the puller distance has to be diminished from 1/2 in. to about 1/3 in. in order to obtain the same transit time as for \( \mathrm{N}^{++} \) ions. For the same particles the starting conditions are also different with different H levels. This indicates the desirability of flexibility in the center geometry for a variable-energy cyclotron.

V. Particle Orbits with an Extractor Electrode

In order to obtain a radially and axially defined and space-charge-compensated beam of one distinct ion charge state for acceleration in a cyclotron, Kenneth Ehlers proposed the inclusion of a separate dee-shaped extractor electrode. This electrode, mounted in the region of the dummy dee and biased negatively by a dc potential of approximately 20 kv, serves to extract the ions from the source. The ions make the first half turn within this electrode where they are shielded from the rf field and leave it through a narrow slit. In Fig. 17 the potential conditions for an idealized simplified setup are shown. The effective acceleration field in the surroundings of the extractor electrode consists of a time-constant component due to the dc extractor potential and a time-varying component which corresponds to the dee potential. Ions that leave the extractor electrode through the slit see an acceleration potential which is decreased by the amount of the dc potential of the extractor. On the dummy-dee side the ions have to pass the fringing field of the extractor. This field tends to bend the orbits inward, and the result is a phase progression on the first two turns.

The shift on the final center of motion towards the dee is relatively large. This again is the result of the asymmetric field perturbation caused by the extractor electrode and the ion source. Also, the spread of the orbit centers for particles with different starting phases seems to become unfortunately large. Some phase bunching results; however. It is somewhat less than the bunching without the extractor electrode (see Fig. 14), since the particles have a starting energy of 20 kv. Particles with a starting phase of more than +10 deg are lost after the first half turn on the extractor itself.

More favorable acceleration conditions could result if the rf gradient on the exit side of the extractor could be increased. This can be accomplished by mounting a shield plate across the dee aperture behind the ion source. The field shape for such a geometry is shown in Fig. 18. In this case, the ions gain
more energy on the first transit through the rf field. Also, the particles with a starting phase $\phi_0 = +30$ deg can pass the extractor electrode on its dummy-dee side. The phase precession, caused by the fringing field of the extractor electrode, is now much less than in the previous case; this results because the particle energy on the first turn is higher and the fringing field is not so effective. Also, the center shift and the center spread for different particles are not much worse than in Fig. 15, where a comparable geometry without the extractor electrode was shown.

VI. Electric Axial Focusing

The electric axial focusing on the first few beam revolutions in a cyclotron has been investigated by several authors. In a conventional geometry electric axial focusing cannot be obtained for particles that cross the symmetry plane of the dee gap before the dee voltage has reached its maximum, whereas good electric focusing results for particles with a reasonable phase lag. Smith has effectively improved axial focusing conditions in the Canberra cyclotron by increasing the particle path length on the first half turn from 180 deg to 200 deg by means of turning back the ion source-extractor geometry 20 deg. Thus the particles cross the gap about 20 deg late with respect to the rf period after the first half turn.

In a single-dee geometry one could also use the asymmetry effect which the lack of a dummy dee would cause on $E_x$ as a function of the $x$ coordinate (see Fig. 11). As can be derived from Fig. 19 A, B, C, there would be a net electric focusing force on the first few beam revolutions on particles in phase if a dummy dee were mounted only on the side where the particles enter the dee ($y < 0$). A resulting orbit-center displacement (caused by the unbalanced $E$-field perturbation thus created) could be controlled by corresponding movement of the ion source and the dee, in order to bring the effective center of motion into the magnetic center of the cyclotron.

VII. Beam Programming

In order to obtain good quality of an intense beam at the extraction radius in a cyclotron, it is necessary to (a) optimize the starting conditions of the beam, (b) define its axial and radial betatron amplitude, (c) bring the average center of motion in coincidence with the magnetic center, and (d) adjust the phase relation of the beam pulse. It is obvious that this beam defining should be done in the center region.

The axial beam extension and the position with respect to the median plane can be defined by the ion source-puller geometry. Since space charge results in an axial defocusing, electric focusing for the initial acceleration of the particles should be provided.

In the very center of a cyclotron only electric axial focusing can be provided. One possibility is to mount slit plates across the dee and dummy dee in the range of the first few turns. However, unbalanced $E$-field
perturbations can be induced in this way, which result in a displacement of the center of motion. Another possibility is to pass the beam pulse through the acceleration gaps with a reasonable phase lag. This method can be especially useful if a magnetic-field bump is provided in the center which causes phase advance besides magnetic focusing after the first few revolutions.

Since the shape of the electric potential changes fairly fast with distance \( z \) from the median plane, and since the beam quality is very much affected by the E-field shape on the first few turns, it seems helpful to keep the \( z \) extension of the beam small from the start \( (|z| \lesssim d/8)\).

For defining the beam radially one has to decide whether a strong phase bunching -- with a resulting larger center spread and, therefore, a larger radial betatron amplitude -- or a small radial betatron amplitude -- with a corresponding larger phase spread of the beam pulse-- would be desirable. If the phase control is good enough over the whole acceleration range, a larger phase spread can be taken into account. In general, some compromise would be necessary.

For definition of the radial betatron amplitude due to radial divergence of the beam leaving the ion source, defining slits can be provided on the first few turns. Beam defining by means of slits is most effective where the largest beam extension can be expected. Betatron-defining slits, therefore, should be placed after \( 1/4+n \) turns (where \( n \) = integer).

Phase-defining slits are most effective after about \( 1/2+n \) turns, as can be deduced from the graphs.

In general one should try to keep the slit plates out of a region where they cause an unbalanced E-field perturbation. Because of difficulties in adjustment, a complicated slit system is not very desirable.

In a variable-energy heavy-ion cyclotron it should be possible to move the dee, dummy dee, ion source, puller, and defining slits separately in order to adjust the beam starting conditions to different \( H_p \) of the particles and to bring the final orbit center into coincidence with the magnetic center of the machine.

For an electronically controlled beam programming, the extractor electrode proposed by Kenneth Ehlers for space-charge compensation could be used. In this small dc extractor electrode, which shields the beam completely from the rf, a small separate slit electrode for radial beam defining could be mounted. It should be possible to pulse or rf-bias this slit electrode to a couple of kv in order to pass the beam through a proper exit slit into the cyclotron at a certain time only. Thus, the time-controlled injection of a space-charge-neutralized beam, well defined axially and radially, may be achieved. Besides providing a good phase control, this beam injection system makes it easy to vary the duty cycle of the cyclotron without pulsing the ion source or the rf generator. For different \( H_p \) of the ions extracted from the source, the extractor potential must be variable.

Such an electronically controlled beam-programming device would probably be especially useful for a variable-energy heavy-ion cyclotron,
where a system of defining slits in the cyclotron-beam path produces many difficulties.

VIII. Summary

The shape of the electric field across the dee gap in a single-dee cyclotron geometry has been measured by an analog method and the influence of a dummy dee at several distances was evaluated.

The dee and liner geometry of the Lawrence Radiation Laboratory 88-inch cyclotron was used. Approximate equipotential lines in the median plane for possible ion source geometries were drawn, and the first few orbits of N ions and deuterons at typical cyclotron conditions were constructed, using a geometrical approximation method. This method allows an evaluation of the movement of the orbit center, the phase shift, and the energy of the particles after a couple of turns in the median plane. The results, which are in general agreement with many observed beam phenomena in cyclotrons, give a qualitative understanding of the influence of the E field on beam quality in the center of a cyclotron.

These results can be summarized as follows:

A. Influence of the Dummy Dee

In a single-dee geometry the electric potential across the dee gap is an asymmetric function of the distance from the dee edge. With a dummy dee at a distance of about the dee aperture an almost complete symmetry can be obtained.

B. E-Field Influence on the Particle Motion in the Median Plane

1. The center of motion moves according to the energy gain of the particle. The direction of the center motion is orthogonal to the direction of the E field, which causes the momentum change. Its absolute value per transit through the same geometric distance in the rf field is proportional to \( \Delta U / r \), or approximately \( \Delta U \times \Delta \tau \) (where \( \Delta U \) = voltage difference at the time the particle crosses the range under consideration, \( r \) = orbit radius, and \( \Delta \tau \) = transit time through the range under consideration).

From this can be deduced:

a. An orbit center motion orthogonal to the dee edge is caused by an E-field component parallel to the dee edge.

b. The absolute value of the center motion orthogonal to the dee edge corresponds to the time the particle spends in the region where E has a component parallel to the dee edge. It is especially large on the first few revolutions.

c. If the E-field perturbation is unbalanced with respect to the symmetry plane in the cyclotron, an actual displacement of
the final center of motion orthogonal to the dee edge can be expected. Especially, unbalanced perturbations around the ion source can cause such an effective orbit-center displacement.

2. An electric field, which is orthogonal to the direction of particle motion causes a deviation of the cyclotron frequency of the particle, in other words a phase shift of these particles relative to the rf.

This phase shift is also related to the time the particle spends in such an E field orthogonal to the orbit.

a. If the E field orthogonal to the direction of motion varies with time (cyclotron rf field), "phase bunching" can occur.

b. The more time the particle spends in a rf field orthogonal to the orbit the larger is the phase-bunching effect.

c. Particles with different starting phases, but bunched in time by the rf field have at the same time of observation different centers of motion. Phase bunching is connected with a simultaneous spread of the orbit centers.

By "shaping" the E field in the center region of a cyclotron, it should be possible to control center displacement as well as phase bunching and correlated center spread (or in other words, radial betatron amplitude at larger radii.)

C. The E-Field Influence on the motion of Particles with a Displacement from the Median Plane

1. The geometry of the E field changes rather fast with increasing deviation from the median plane. Particles that deviate from the median plane not only experience an axial force, but also see different acceleration conditions. Therefore, in order to assure good quality of the beam, the axial beam extension should be kept \( \leq d/4 \) on the first turns.

2. Electric axial focusing in a single-dee geometry can be obtained by passing the beam pulse through the acceleration gap with a reasonable phase lag. Another method would be to mount focusing slit-plates across the dee aperture (grid focusing) on the side where the particles enter the dee, or perhaps only a dummy dee also on this side.

D. Beam Programming

In order to obtain a good quality internal beam in a variable-energy cyclotron the following items should be considered:

1. Adjustment of the starting geometry for particles with different e/m at different H levels.
2. Proper shape of the E field around the ion source and adjustment of the whole center geometry for making the orbit center coincide with the magnetic center of the machine.

3. Selection of particles with proper radial betatron amplitude by means of a defining slit (most effective position after \(1/4 + n\) turns).

4. Selection of particles with proper phases by means of a defining slit (most effective position after \(1/2 + n\) turns).

5. Axial beam confinement (ion source-puller geometry).

For betatron defining and phase defining, probably only one slit has to be provided.

A pulsed slit electrode within a shielded dc extractor electrode could be used for electronically controlled beam injection.
References


2. M. M. Gordon, Beam Deflection with the Aid of a Nonlinear Resonance, in Sector-Focused Cyclotrons, p. 234 (see Ref. 15).


7. W. I. B. Smith, Beam Defining Slits and Focusing Grids Near the Ion Source, in Sector-Focused Cyclotrons, p. 183 (see Ref. 15).

8. W. B. Powell, The University of Birmingham Radial-Ridge Cyclotron, in Sector-Focused Cyclotrons, p. 12 (see Ref. 15).


13. C. J. Taylor, Livermore Cyclotron Beam Features, in Sector-Focused Cyclotrons, p. 208 (see Ref. 15).

14. R. J. Jones, Ion Source and Beam Quality Studies, in Sector-Focused Cyclotrons, p. 211 (see Ref. 15).


22. H. G. Blosser, Beam Quality Measurements and Focusing Grid Studies, in Sector-Focused Cyclotrons, p. 203 (see Ref. 15).
Fig. 1. Coordinates in a cyclotron center.
Fig. 2. Equipotential lines in a vertical cross section of a single-dee geometry. Liner aperture 5-3/4 in.; dee aperture 2 in.; dee-liner distance 1-3/8 in.
Fig. 3. Equipotential lines with a dummy dee at liner potential. Distance between dee and dummy dee: 2 in.
Fig. 4. Equipotential lines with a dummy dee at liner potential. Distance between dee and dummy dee: 3 in.
Fig. 5. Equipotential lines with a dummy dee at liner potential. Distance between dee and dummy dee: 4 in.
Fig. 6. Equipotential lines with a solid plate on liner potential opposite the dee for 2 in. distance.
Fig. 7. Equipotential lines with a solid plate on liner potential opposite the dee and 1.5 in. distance.
Fig. 8. Equipotential lines with a solid plate on liner potential opposite the dee for 1 in. distance.
Fig. 9. Equipotential lines with a solid plate and a lip.
Fig. 10. Potential in the median plane as a function of X coordinate, derived from Figs. 2 and 3.

Curve 1 a, b: single dee (crossed coordinates of conductivity paper).
Curve 2: dummy dee at 2 in. distance (Fig. 3).
Fig. 11. $z$ component of $\vec{E}$ at $z = \pm 1/2$ as a function of $X$, derived from Figs. 2 and 3.

Curve 1: single dee.
Curve 2: dummy dee at 2 in. distance.
Fig. 12. Method of orbit construction in a cyclotron rf field.

Example: Particle: $^14_7$N

$V = 100$ kv
$H = 16$ k gauss

time at $A_1$: 45 deg

initial energy of the particle: 200 kev
Fig. 13. First two revolutions of a $N^{14}(3^+)$ particle in a simplified cyclotron field (full scale).

Geometry: single dee, ion source, puller at 1/2 in. distance.
Starting phase: $\phi_0 = 0$ deg
Time intervals: 30 deg (1/12 rf period).
Fig. 14. First two revolutions of $^14N^{+++}$ particles with starting phases $\phi_0 = -30^\circ$, $0^\circ$, $+30^\circ$ (full scale).

Geometry: single dee, ion source without puller.
Fig. 15. First two revolutions of $N^{14++}$ particles with starting phases $\phi_0 = -30$ deg, 0 deg, +30 deg (full scale).

Geometry: dee, dummy dee, ion source, puller at 1/2 in. distance.
Fig. 16. First two revolutions of a deuteron in a simplified cyclotron field (full scale).

Geometry: dee, dummy dee, ion source, puller at 1/2 in. distance.
Starting phase: 0 deg.
Fig. 17. First two revolutions of $^{14+}$ particles in a simplified cyclotron field, when a dc extractor electrode is used (full scale).

Geometry: dee, dummy dee, ion source, extractor electrode.
Injection time: $\phi_0 = -30\,\text{deg}, 0\,\text{deg}, +30\,\text{deg}$. 

$N^{14+}\ H=16\ \text{kgauss}$

$U_{rf} = 70\ \text{kV} \ U_{ex} = -20\ \text{kV}$
Fig. 18. First two revolutions of $^{14}\text{N}^{3+}$ particles in a simplified cyclotron field, when a dc extractor electrode is used (full scale).

Geometry: dee with shielding plate, dummy dee, ion source, extractor electrode.

Injection time: $\phi_0 = -30$ deg, 0 deg, +30 deg.
Fig. 19. Electric axial focusing conditions in the center region of a cyclotron (simplified).

A. single-dee geometry
B. dee, dummy-dee, or two-dee geometry
C. dee, half-dummy-dee geometry

The axial electric force on an imaginary particle with a constant displacement $z \approx \pm d/4$ is shown qualitatively for these cases. A constant orbit radius was assumed. Solid lines: particle enters dee; interrupted lines: particle leaves dee.
Appendix: Geometric Orbit Construction

Displacement of the center of curvature of nonrelativistic particle orbits by an electric field orthogonal to a magnetic field.

In App. I the magnetic field should be orthogonal to the plane of drawing.

Basic assumption: The momentum of the particle is large compared to the momentum change caused by the E field under consideration:

\[ r >> \Delta r \]

and \[ U >> \Delta V \]

from App. 1 can be derived:

\[ E = \frac{V_2 - V_1}{d} = \frac{\Delta V}{d} \]

\[ d \approx s \cdot \cos \alpha \]

\[ E_{||} \approx \frac{\Delta V}{s} \]

\[ E_\perp \approx \frac{\Delta V}{s} \cdot \tan \alpha \]

\[ \Delta r_{\text{norm}} = (r_1 + \Delta r_{\text{tang}}) \cdot \tan \gamma \]

where \[ \Delta r_{\text{tang}} = \frac{r_1 \cdot \Delta V}{2 \, U_1} \]

(derivative of the equation for the orbit radius (2))

with \[ \tan \gamma = \frac{s \cdot E_\perp}{2 \, U} = \frac{s \cdot E_\perp}{U_1 + U_2} \approx \frac{\Delta V \cdot \tan \alpha}{2 \, U_1 + \Delta V} \]

becomes \[ \Delta r_{\text{norm}} \approx r_1 (1 + \frac{\Delta V}{2 \, U_1}) \cdot \frac{\Delta V \cdot \tan \alpha}{2 \, U_1 + \Delta V} \], or

with sufficient accuracy \( \approx \frac{r_1 \cdot \Delta V \cdot \tan \alpha}{2 \, U_1} \)

\[ \approx \Delta r_{\text{tang}} \cdot \tan \alpha \]
Appendix I
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.