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Introduction

In many hadron-hadron collisions at high center of mass energies most of the final state hadrons are collimated within cylinders of fixed transverse momentum along the beam direction with an average transverse momentum of 0.2 GeV/c. The very short nature of the potential is responsible for the high $p_T$ reactions which do not contribute significantly to the total cross section. Consequently, soft hadronic reactions ($p_T \leq 1$ GeV/c) populate most of the final state particles and most of the cross section. The relevance of the constituent picture of hadrons in the framework of QCD to soft hadronic reactions has been demonstrated through many experimental and theoretical investigations.\(^1\) Since it is not yet possible to calculate the QCD predictions for these reactions, many of the theoretical studies are based on phenomenological models motivated by QCD.

Among several proposed models to explain the bulk of data available on soft hadron reactions, the so called "valon parton" model\(^2\) have been successfully applied to many hadron-hadron and hadron-nucleus reactions.\(^3\) Valons as defined in this model are conglomerations of valence quarks, sea quarks and gluons. In a bound state situation valons saturate all of the momentum carried by a hadron, i.e., a proton contains three valons UUD, shown by capital letters to distinguish them from quarks, and a meson is bound state of a valon-anti valon, e.g., $\pi^+$ is a U$\bar{D}$, etc.

In this paper our aim is to study the properties of valon distribution and structure functions in strange and non-strange mesons in the framework of valon parton model. Mesons present many
attractive features for studying soft hadronic reactions because of their simple $qq$ structure as opposed to $qqq$ structure of baryons. In the followings, we first present some brief review of the essential formalism of the valon parton model, then we describe our method for determination of valon distribution functions and structure functions in pions and kaons respectively.

The static (bound state) properties of valons are characterized by their wave functions in hadrons where they are considered as constituent quarks. On the other hand, they respond to a dynamical probe in a way specified by their structure function which represents the internal structure of cluster quarks. The precise knowledge of both properties as mentioned above is very crucial in determination of the subsequent behavior of quarks and produced hadrons in a scattering process. Of course, the ultimate solution should be presented by QCD to connect these perturbative and non-perturbative regions. Nevertheless, one may try to gain some understanding of the underlying mechanism just by resorting to phenomenological view points in the context of valon parton model.

Let us consider the nucleon as our example. We denote by $G_{VN}(y)$ the valon distribution function in the nucleon, where $y$ is the fraction of momentum carries by valon $V$. Then, the average momentum $y_V$ is

\[ \bar{y}_V = \int y G_{VN}(y) \, dy \]

and

\[ \sum_V \bar{y}_V = 1. \]

The nucleon structure function can be written as convolution of the valon distribution and valon structure function,

\[ F_N(x) = \sum_V \int_0^1 dy \, G_{VN}(y) F_V(x/y) \]

where $F_V(x)$ is the structure function of valon $V$. By this function we mean, when a valon undergoes a collision, it is no longer an indivisible point-like object. The virtual cloud of partons are excited, however, the fast and slow partons behave differently. Now for this quark cluster, one can write,

\[ F_V(x) = \sum_q \int_0^1 dy \, G_{qVN}(y) F_q(x/y) \]

where $G_{qVN}(x)$ is the distribution of quarks in the valon $V$, and $F_q(x)$ is the quark structure function. Assuming a point-like structure for quarks (which is appropriate to the kind of reactions we are considering), then $F_q(x) = \delta(x-1)$ and Eqs. (2) and (3) give,

\[ F_V(x) = \int x G_{VN}(x) \]

\[ F_N(x) = \sum_V \int x G_{VN}(x) / y G_{VN}(x/y) \]

Denoting the quark distribution in the nucleon by $G_{qN}(x)$, the sum in Eq. (5) separates into two parts: the valence quark distribution

\[ xG_{vN}(x) = \int x \int y G_{VN}(y) / y G_{VN}(x/y) \]

and sea quark distribution

\[ xG_{sN}(x) = \int x \int y G_{VN}(y) / y G_{SV}(x/y). \]
Therefore, the quark distributions have been expressed in terms of valon distributions in hadrons.

**Valon distributions in mesons**

The valon distributions in mesons follow a sum rule similar to Eq. (1) for nucleons,

\[ \sum_v \int_0^1 y G_{vn}(y) \ dy = 1 \]  
(8)

Furthermore, for pions, one expects a symmetric distribution, i.e.

\[ G_{vn}(y) = G_{Dn}(y) \]  
(9)

while for kaons the SU(3) breaking has to be considered. Let us parametrize the double valon distribution in the pions and kaons as follows:

\[ G_{v_1v_2}(y_1, y_2) = \alpha \delta(y_1 + y_2 - 1) \]  
(10)

\[ G_{v_1v_2K}(y_1, y_2) = \alpha \delta(y_1 + y_2 - 1) \]  
(11)

Where the \( \delta \)-functions have been introduced for momentum conservation. Integrating Eqs. (10) and (11), the single valon distribution in the pions and kaons become

\[ G_{vn}(y) = y^a(1 - y)^b/B(a + 1, b + 1) \]  
(12)

\[ G_{vK}(y) = y^a(1 - y)^b/B(a + 1, b + 1) \]  
(13)

\[ G_{vK}(y) = y^a(1 - y)^b/B(a + 1, b + 1) \]  
(14)

where \( B \) is the Euler Beta function. We first consider the case of pions.

**Pions:**

The method employed in extracting valon distribution functions in nucleons is described in detail in Reference (3). We are going to use those results plus the recently available comprehensive fits\(^4\) to quark distributions in nucleons and pions at \( Q^2 \gtrsim 4 \text{ GeV}^2 \), to calculate the distribution of valons in the pions. Actually, the connection is quite simple if one knows the structure function of non-strange valons, say U-valons. We assume this function, \( G_{uv}(x) \), to be independent of its host particle, i.e., the internal structure of a particular valon in the proton or pion is the same (it probably differs between strange and non-strange valons, and we will discuss this point later in the next section). Consequently, we proceed by its determination using the results of References (3) and (4) for valon and quark distributions respectively.

According to Eq. (6) the connection between, say, d-quark and D-valon is

\[ x d_1(x, Q^2 = 4) = \int_1^x G_{Dn}(y) \ dy G_{uD}(x/y) \ dy. \]  
(15)

The valence d-quark distribution is given by\(^4\)

\[ x d_1(x, Q^2 = 4) = 2.8 x^{0.76}(1 - x)^4, \]  
(16)

and the D-valon distribution is\(^3\)
We use the following parametrization for \(G_{dD}(z)\), for which the motivation is discussed in detail in Reference (3):

\[
zG_{dD}(z) = A \cdot z^{\alpha} e^{b z}
\]  

(18)

\(A\) is the normalization factor and is given by,

\[
A = \left[ \int_0^1 z^{\alpha-1} e^{b z} dz \right]^{-1}.
\]

(19)

Substituting Eqs. (16)-(19) in Eq. (15), we find the values of \(\alpha\) and \(b\) to be 0.8 and -0.4 respectively. The normalization factor \(A\) in (19) for the given values of \(\alpha\) and \(b\) is \(A = 1\), consequently from Eq. (18) we obtain

\[
zG_{dD}(z) = z^{0.8} e^{-0.4z}.
\]

(20)

Equation (20) is precisely what we need to determine the valon distribution function in the pions, i.e.

\[
xu_{vn}(x, Q^2 = 4) = 0.52 x^{0.4}(1 - x)^{-0.7}
\]

consequently, using Eqs. (22) and (23) along with Eq. (12) we obtain the distribution of valons in pion to be

\[
G_{U/n}(y) = G_{d/n}(y) = 0.66y^{-0.22}(1 - y)^{-0.22}.
\]

(24)

For a large range of \(y\), this equation is consistent with the simple result of Reference (2) which is \(G_{U/n}(y) = 1\). It is interesting to note that if one uses the same line of arguments as in Reference (2), the result of Eq. (24) will emerge approximately. The idea is that in order to obtain a \((1-x)^{-0.7}\) behavior for the valence u-quark near \(x = 1\), the power \(\alpha\) in Eq. (12) must be -0.3, because \(G_{uU}(x)\) is finite near \(x = 1\). This will give \(G_{U/n} = y^{-0.3} (1-y)^{-0.3}\), which is similar to in result in Eq. (24). In Fig. (1), the valon distributions in pion as given by Eq. (24) and Reference (2) are plotted. The authors of Reference (5) predicted similar results using the non relativitic quark model.

**Kaons:**

For kaons the situation is somewhat different. The main differences are: (i) \(SU(3)\) breaking among the constituents of kaons, (ii) lack of knowledge with regard to quark distributions in kaons at different values of \(Q^2\). However, the following technique can be employed to get the distribution of non-strange valence quarks from which we shall attempt to calculate the respective valon distribution.
The details of this method are given in Reference (5) and here we present a brief account of it.

Let us define the moments of non-singlet (in this case valence quarks) distribution in hadron \( h \) as

\[
M^n_{\text{NS}h}(Q^2) = \int_0^1 x^{n-1} q(x, Q^2) \, dx
\]  

(25)

where \( q(x, Q^2) \) is the quark (valence quark) distribution in hadron \( h \). Then to all orders of perturbative QCD but to leading twist order, one can write

\[
\frac{M^n_{\text{NS}h}(Q^2_1)}{M^n_{\text{NS}h}(Q^2_2)} = \frac{M^n_{\text{NS}h}(Q^2_1)}{M^n_{\text{NS}h}(Q^2_2)} \frac{M^n_{\text{NS}h}(Q^2_2)}{M^n_{\text{NS}h}(Q^2_1)}
\]  

(26)

which implies that the ratio of moments at different values of \( Q^2 \) is particle independent. As long as the quark distributions do not contain any corrections of order \((1/Q^2)^n\), one can choose the values of \( Q^2 \) in (26) arbitrarily. Solving Eq. (26) for \( M^n_{\text{NS}h}(Q^2) \), we get

\[
M^n_{\text{NS}h}(Q^2_1) = \frac{M^n_{\text{NS}h}(Q^2_2)}{M^n_{\text{NS}h}(Q^2_1)} \frac{M^n_{\text{NS}h}(Q^2_1)}{M^n_{\text{NS}h}(Q^2_2)}
\]  

(27)

Let us choose \( Q^2_1 = 4 \text{ GeV}^2 \) and \( Q^2_2 = 20 \text{ GeV}^2 \). Then Eq. (27) gives the moments of non-singlet quark distributions (in this case \( u_{\nu K}(x, Q^2) \)) at \( Q^2 = 4 \text{ GeV}^2 \) if we know its moments at \( Q^2 = 20 \text{ GeV}^2 \). The rest of the quantities in Eq. (27) are given using the \( Q^2 \)-dependent quark distributions of Reference (4). Therefore, we proceed to determine \( M^n_{\nu K}(Q^2_2 = 20) \).

The valence \( u \)-quark distribution at \( Q^2 = 20 \text{ GeV}^2 \) can be calculated using the respective quark distribution in the pions which is given by

\[
x_{u_{\nu K}}(x, Q^2 = 20) = 0.514 x^{0.381}(1 - x)^{0.895}
\]

(28)

and the result of CERN-NA36 experiment shown in Fig. (2). A fit to the data of Fig. (2) results in the following distribution for \( u \)-quark in kaons,

\[
x_{u_{\nu K}}(x, Q^2 = 20) = 0.656 x^{0.438}(1 - x)^{1.169}
\]

(29)

We now return to Eq. (27) where all of the quantities on right hand side are given through Eqs. (23), (25), (28) and (29). Finally, the inverse transformation of the moments of Eq.(27) gives the valence \( u \)-quark distribution at \( Q^2 = 4 \text{ GeV}^2 \) as:

\[
x_{u_{\nu K}}(x, Q^2 = 4) = 0.680 x^{0.466}(1 - x)^{0.968}
\]

(30)

The difference between Eqs. (29) and (30) near \( x = 1 \) is mainly due to gluon radiation which causes the quark distributions to become steeper as \( Q^2 \) goes higher.
The procedure for the determination of valon distributions is similar to what we used for pions, i.e., first we make the connection through Eq. (6),

\[ x_{\nu K}(x, Q^2 = 4) = \int_x^1 G_{\nu K}(y) G_{\nu U}(x/y) \, dy \]  

with

\[ zG_{\nu U}(z) = z^{0.8}e^{-0.4z}. \]

Then using Eqs. (13) and (30) we obtain,

\[ G_{\nu K}(y) = 0.5y^{-0.4}(1 - y)^{-0.2} \]  

This equation also implies (see Eq. (14))

\[ G_{\sigma K}(y) = 0.5y^{-0.2}(1 - y)^{-0.4} \]

Figure (3) shows plots of valon distributions in the kaons given by Eqs. (32) and (33).

The average momentum fraction carried by U-valon and S-valon are

\[ \bar{x}_U = 0.43 \]  

\[ \bar{x}_S = 0.57 \]

which indicates the ratio of momentum fractions carried by respective valons to be

\[ \frac{\bar{x}_S}{\bar{x}_U} = \frac{4}{3} = 1.33 \]  

If this ratio is approximately equal to the ratio of masses of respective valons, then we must have

\[ \frac{M_S}{M_U} = 1.33 \]

In the literature different values for the ratio (36) have been assumed which have a range between 1 and 2.
Inclusive Reactions

An interesting area of the application of the results presented so far is the inclusive meson production in meson proton collisions. The invariant cross section for such reactions (after integrating over $p_T$) can be written as

$$\frac{x}{\sigma_{\text{in}}} \frac{d\sigma}{dx} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F_{q\bar{q}}(x_1, x_2) R(x_1, x_2; x)$$

(37)

where $F_{q\bar{q}}$ is the joint quark distribution and $R$ is the recombination function which gives the probability of quark $q(x_1)$ and anti quark $\bar{q}(x_1)$ to recombine and produce a meson at $x = x_1 + x_2$. $\sigma_{\text{in}}$ is the total inelastic cross section for the reaction under consideration. The valon parton model gives prescriptions to calculate $F_{q\bar{q}}$ and $R$, however, in this paper, we like to use a simpler version of the model for the purpose of illustration. According to simple recombination model, the spectrum of the final state particle in a single particle inclusive reaction is similar to the distribution of the common valence quark between the beam and final state particle. That means, at small $p_T$, in a reaction like $K^- p \rightarrow \pi^- + X$, we have

$$\frac{x}{\sigma_{\text{in}}} \frac{d\sigma}{dx} \sim xu_{\nu K}(x) \sim (1 - x)^{-1}$$

(38)

The data\(^9\) indicates a slightly steeper distribution than (38), i.e. $(1 - x)^{-1.4}$. This is actually related to the fact that the simple recombination model does not introduce any correlation between $q$ and $\bar{q}$ in $F_{q\bar{q}}$, which in general results in a steeper distribution.

Another interesting example is the inclusive production of fast $\phi$'s in $K^+ p$ reactions. The data\(^10\) is taken at 70 GeV/c and the invariant cross section for $x \approx 0.5$ is

$$\frac{d\sigma}{dx}_{K^+ \rightarrow p} \sim (1 - x)$$

(39)

In this case the common quark between the beam ($K^+$) and final state particle ($\phi$) is the $s$-quark. Thus, according to simple recombination model, the distribution of $s$-quarks in the kaons must be flatter than $u_v$-quarks near $x = 1$. This has been reflected in the valon distributions given by Eqs. (32) and (33).

As we mentioned above, the valon parton model presents a well prescribed method to calculate joint quark distributions. However, for reactions like $K^- p \rightarrow \bar{K}^0 + X$, one also needs to structure function of strange valons, i.e. $G_{s\bar{s}}(x)$, to determine the evolution of quarks from strange valons. The authors of Reference (7) used low-$p_T$ data to determine that function. It is interesting to investigate for some other methods independent of data at low-$p_T$. Precise measurements of the kaon structure function at high $Q^2$ will be very interesting. The method we already described for the determination of $u_v$-quark distribution in the kaon can be easily applied to calculate $s_v$-quark distribution which is related to $S$-valons through Eq. (6). Therefore, one can extract the structure of strange valons using that relationship.
Summary

In this paper we studied some of the methods for the determination of valon distribution functions and valon structure functions in pions and kaons. Our final results are given by Eqs. (22), (24), and (32). We indicated the complications with the structure function of the strange valons, and the need for future experiments to determine this function from hard scattering data. The implications of our results to single particle inclusive reactions were studied in the context of simple recombination model, and it was shown, semi-qualitatively, that they agree with existing data.

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References

2. A thorough and complete introduction to valon parton model can be found in R. C. Hwa’s talk in Reference 1, p. 137.

Figure Captions

Fig. (1). Valon distribution function in pion given by Eq. (24), solid line, and Reference (2), dashed line.
Fig. (2). Data for the ratio $R = u_{v/n}(x, Q^2 = 20)/u_{v/K}(x, Q^2 = 20)$ from CERN-NA3 experiment. The solid line is our fit to this data.
Fig. (3). The strange (solid line) and non-strange (dashed line) valon distribution in kaons.
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